Tutorial 6-2
Fourier Transform of aperiodic and periodic signals – Chapter 4

2014/2

Review

- Assume that a signal is repeating by creating what is called a periodic extension, $x_n[k]$ with a squiggle over x.

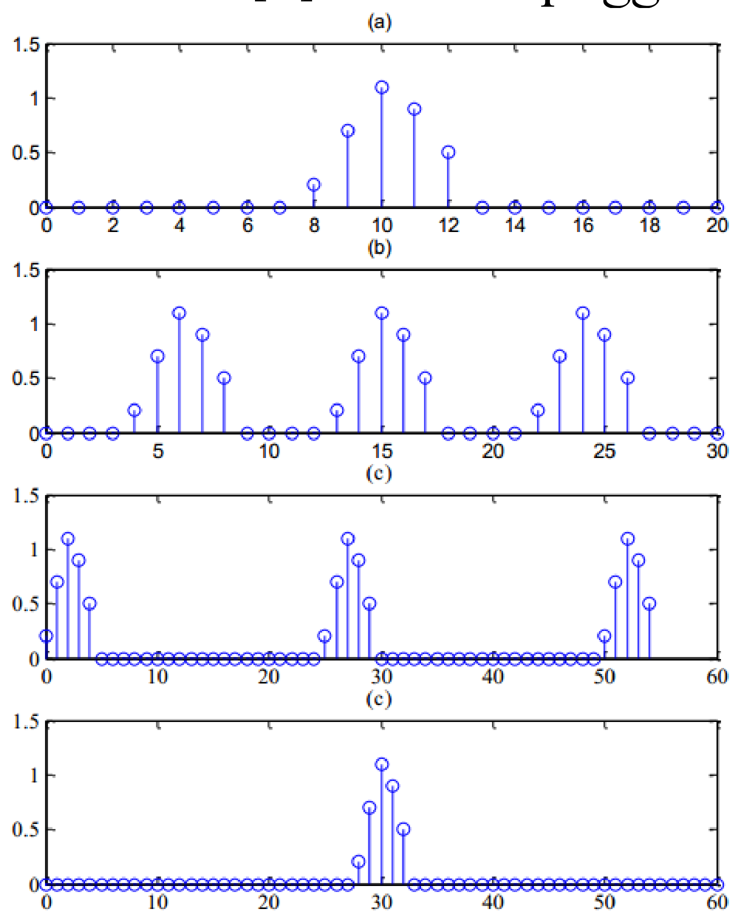


Figure 4.1 – Going from a periodic to a non-periodic signal

Review

- FS for a continuous signal $x(t) = \sum^{\infty} C_n \, e^{-jn\omega_0 t}$
  - The Fourier coefficients $C_n = \frac{1}{T}\int_0^T x(t)\, e^{j\omega_n t}\, dt$
    - $w_0 = 2\pi/T_0$: the fundamental frequency
    - n: the index of the harmonic
    - nw_0: the nth harmonic
    - T: the period for the continuous use
    - (K_0: the period for the discrete use, k: the sample/bin number, $2\pi/K_0$: the frequency space between the bin)

Review

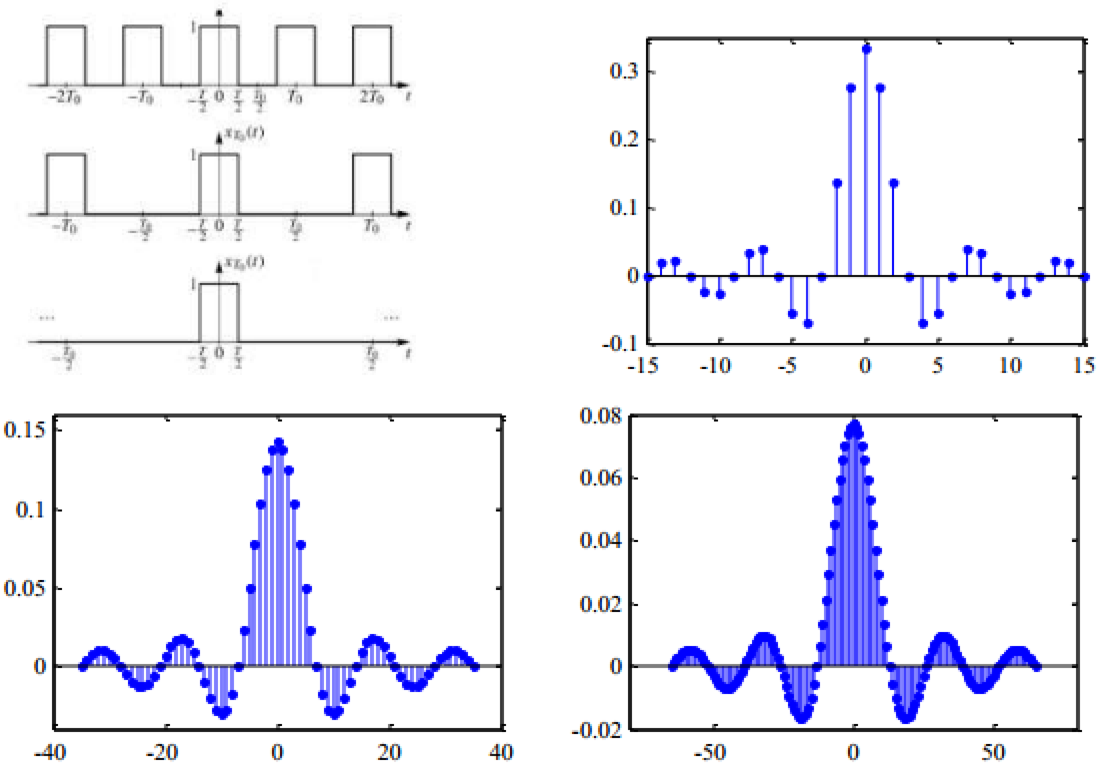


Figure 4.2 - Stretching the period, makes the fundamental frequency smaller, which makes the spectral lines move closer together.

Key idea: Increasing the period of a signal allows us to create an aperiodic version of the signal. The increasing period brings harmonics closer together, so that the spectrum of an aperiodic signal becomes continuous.

Continuous-time Fourier Transform (CTFT)

- The Fourier series coefficients (FSC) of a continuous time signal

$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{j\omega_n t}\, dt$$

  - $w_n = nw_0$: the nth harmonic or n times the fundamental frequency
  - $1/T = w_0/2\pi$

- $C_n = \lim\limits_{T\to\infty} \dfrac{\Delta\omega}{2\pi} \displaystyle\int_{-T/2}^{T/2} x(t)\, e^{-j\omega_n t}\, dt$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n\, e^{j\omega_n t} \quad => \quad x(t) = \lim_{T\to\infty} \sum_{n=-\infty}^{\infty} \overset{\text{Fourier transform}}{\boxed{\left\{ \frac{\Delta\omega}{2\pi} \int_{-T/2}^{T/2} x(t) e^{-j\omega_n t} dt \right\}}} e^{j\omega_n t}$$

Continuous-time Fourier Transform (CTFT)

- the *formula for the coefficients of a non-periodic signal.*

$$\boxed{X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

Fourier transform of a continuous-time non-periodic

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega}$$

Inverse Fourier Transform

- In frequency from,
  - Forward Fourier Transform $X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}$
  - Inverse Fourier Transform $x(t) = \int_{-\infty} X(f)\, e^{j2\pi ft} df$

Comparing Fourier Series and Fourier Transform

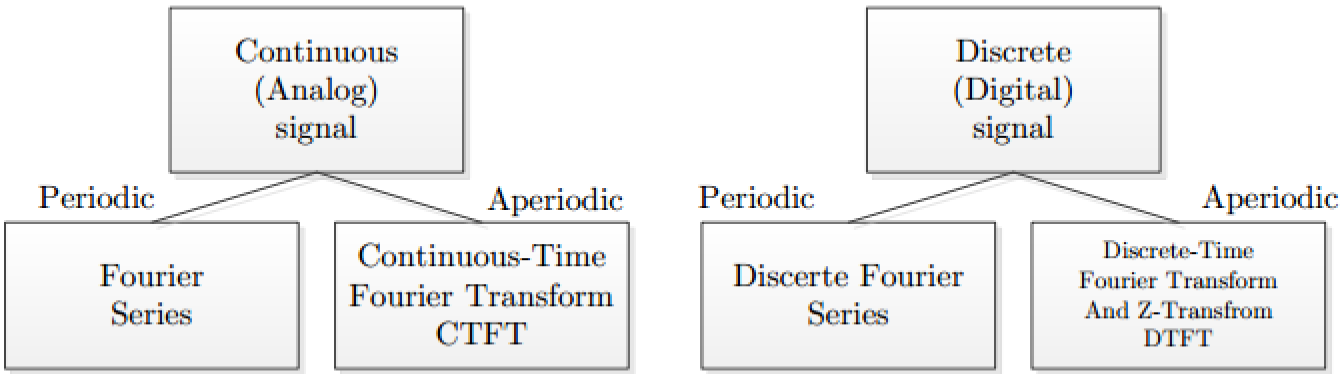
- 

Figure 4.3 – Fourier series and Fourier Transform

- The Fourier Series is supposedly valid only for periodic *continuous* signals and periodic *discrete* signals.
- When the signal is non-periodic, we use the Fourier Transform.

Comparing Fourier Series and Fourier Transform

- $$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-j\omega_n t} dt \quad FSC$$

  $$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt \qquad FT$$

  - They are nearly the same except that the term 1/T is missing from the latter.
    - We developed FT by assuming T goes to infinity, and then 1/T=△f and mapped it to a continuous variable w by turning it into dw.
  - The frequency w is continuous for the FT.
- $$x(t) = \sum_{n=-\infty}^{\infty} C_n\, e^{j\omega_n t} \qquad FS$$

  $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \quad FT$$

  - dw was associated with the time-domain formula or the inverse FT. So the forward FT moved to the inverse FT in form of 2π.

CTFT of Aperiodic Signals

- Example 4-1

  What is the FT of a single impulse function located at origin?

  - The Dirac delta function $\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}, \int_{-\infty}^{\infty} \delta(x)\, dx = 1.$
  -

$$X(\omega)\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

$$X(0) = \int_{-\infty}^{\infty} \underbrace{e^{-j(\omega=0)t}}_{=1}\, dt$$

$$= 1$$

$$X(\omega) = \int_{-\infty}^{\infty} \delta(w)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(w) \cdot e^{-j0t} dt$$

$$= 1$$

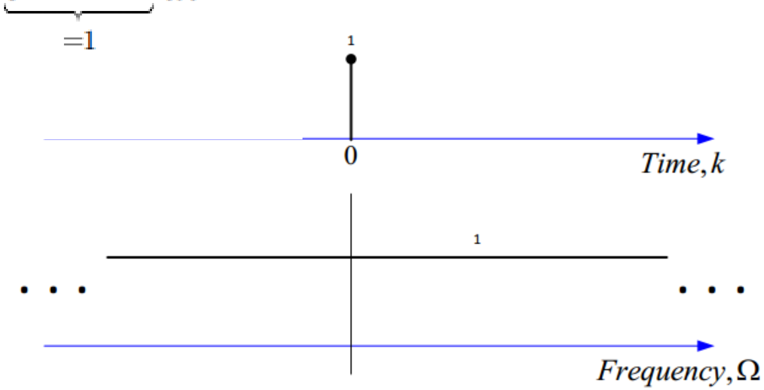


Figure 4.4 – Spectrum of a delta function located at time 0

CTFT of Aperiodic Signals

- What happens if there are two impulse functions?

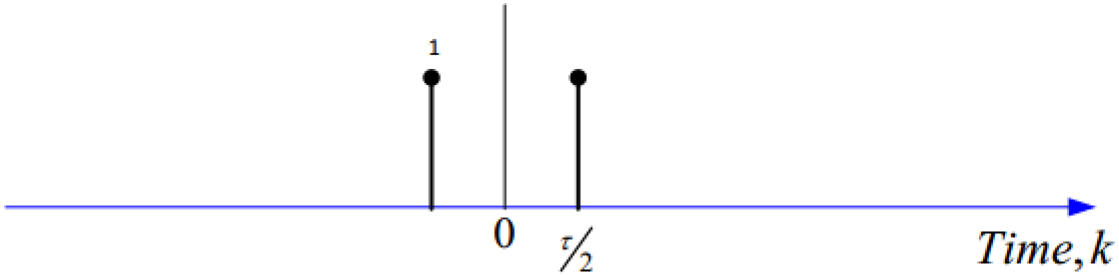


Figure 4.5 – Spectrum of two delta functions

$$X(\omega) = \int_{-\infty}^{\infty} \delta(t - \tfrac{T}{2}) e^{-j\omega t}\, dt + \int_{-\infty}^{\infty} \delta(t + \tfrac{T}{2}) e^{-j\omega t}\, dt$$

$$= e^{-j\omega\frac{T}{2}} + e^{j\omega\frac{T}{2}}$$

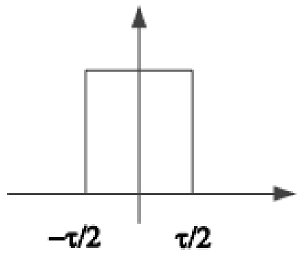
$$= \frac{1}{2}\ \cos\ \omega\tfrac{T}{2}\ - j\sin\ \omega\tfrac{T}{2}\ + \frac{1}{2}\ \cos\ \omega\tfrac{T}{2}\ + j\sin\ \omega\tfrac{T}{2}$$

$$= \cos\ \omega\tfrac{T}{2}$$

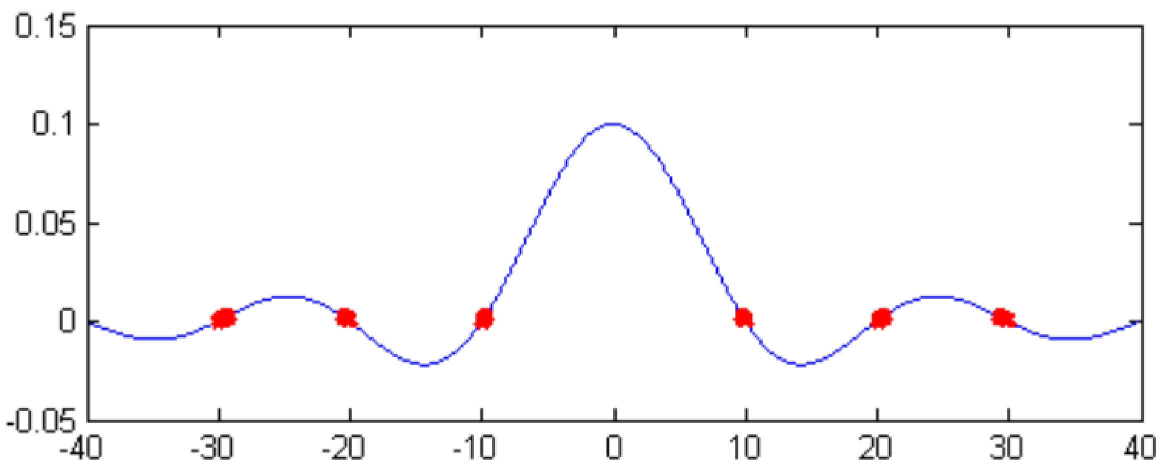
CTFT of Aperiodic Signals

- Example 4-2

  Find the CTFT of a square (rectangular) pulse of amplitude 1v, with a period of τ, located at zero.

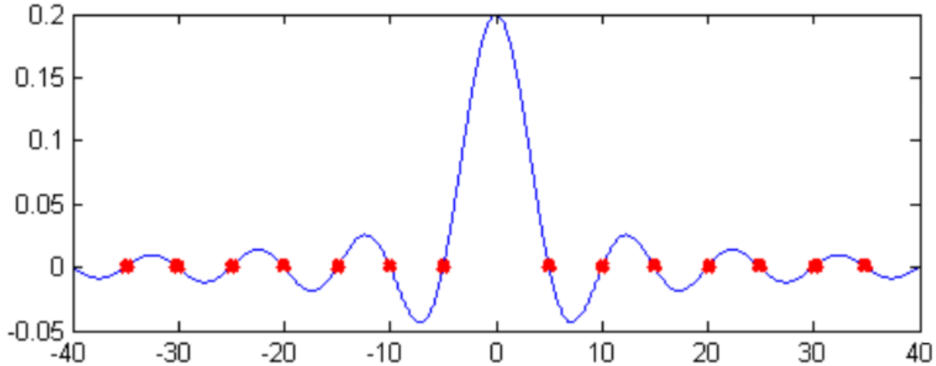


(a)



(b)

CTFT of Aperiodic Signals



(c)

```
%Example 4-2
clf
tau = .2;
w = -250: .01: 250;
xom = tau*sinc(w*tau/(2*pi));
plot(w/(2*pi), xom)
```

Figure 4.6 – Spectrum along a Frequency line
A square pulse has a sinc shaped spectrum. (a) time-domain shape, (b) Spectrum for $\tau = .1$ sec. (c) Spectrum for $\tau = .2$ sec.

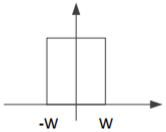
- $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt = \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-j\omega t} dt = \left. \frac{e^{-j\omega t}}{j\omega} \right|_{-\tau/2}^{\tau/2} \Rightarrow X(\omega) = \tau \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)$
- If the pulse were to become infinitely wide, the FT would become an impulse function.
- If it were infinitely narrow, the freq. spectrum would be flat.
- The bi-directional relationship is often writte $1 \xrightarrow[CTFT]{} \delta(\omega)$
  $\delta(\omega) \xrightarrow[CTFT]{} 1$

CTFT of Aperiodic Signals

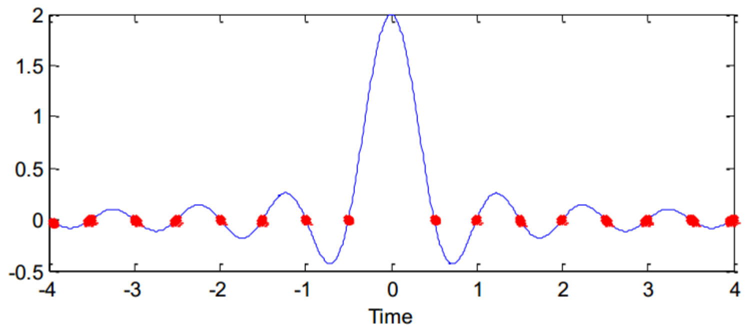
- Example 4-3

  Given a freq. spectrum that looks like a square and it's flat for a certain band from -W to +W Hz. we define the half bandwidth by W. What time-domain signal produces this freq. response?

  - $x(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega = \dfrac{1}{2\pi}\int_{-W}^{W} 1\cdot e^{j\omega t}d\omega = \dfrac{1}{2\pi}\left.\dfrac{e^{j\omega t}}{j\omega}\right|_{-W}^{W} \Rightarrow x(t) = \dfrac{W}{\pi}\operatorname{sinc}\left(\dfrac{W}{\pi}t\right)$

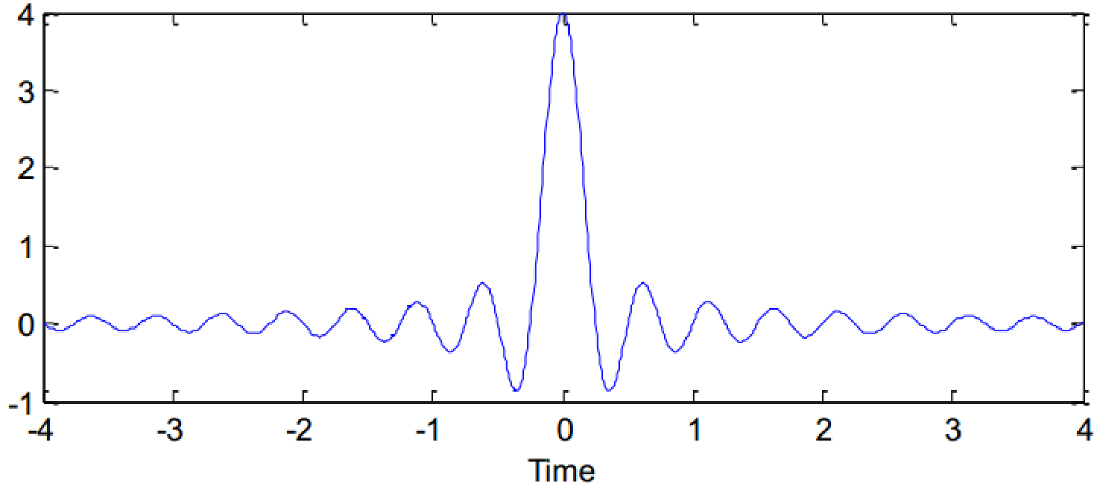


(a)



(b)

CTFT of Aperiodic Signals



(c)

Figure 4.7 – Time domain signal corresponding to the rectangular frequency. To obtain a rectangular frequency spectrum, a sinc pulse shape is required in time-domain. A narrow band signal is slower than a wideband signal in its zero crossings. (a) W = 2π **Hz, (b) W =** 4π **Hz**

CTFT of Aperiodic Signals

- Example 4-4

  Given a single impulse located at w_1 in frequency domain. What signal gives this FT?

  - Take the inverse FT, denoted by $\mathfrak{I}^{-1}$ and characterize the single impulse as a delta function, $\delta(w-w_1)$.

$$x(t) = \mathfrak{I}^{-1}\{\delta(\omega-\omega_1)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega-\omega_1)e^{j\omega t}d\omega = \frac{1}{2\pi}e^{j\omega_1 t}\Big|_{-\infty}^{\infty} = \frac{1}{2\pi}e^{j\omega_1 t}$$

  - Taking 2π to the other side

$$2\pi\,\delta(w-\omega_1) \xrightarrow[CTFT]{} e^{-j\omega_1 t} \quad \textit{Frequency to Time}$$

$$2\pi\,\delta(t-T_1) \xrightarrow[CTFT]{} e^{-j\frac{2\pi}{T_1}} \quad \textit{Time to Frequency}$$

CTFT of Aperiodic Signals

- Example 4-5

  What is the FT of a cosine wave?

  - $x(t) = \Im^{-1}\{\cos \omega_0 t\}$

$$= \frac{1}{2\pi}\int_0^{-2\pi} \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi}\int_0^{-2\pi} \frac{e^{j(\omega+\omega_0)t}}{2} d\omega + \frac{1}{2\pi}\int_0^{-2\pi} \frac{e^{j(\omega-\omega_0)t}}{2} d\omega$$

$$= \frac{1}{2\pi}\int_0^{-2\pi} \frac{e^{j(\omega+\omega_0)t}}{2} d\omega + \frac{1}{2\pi}\int_0^{-2\pi} \frac{e^{j(\omega-\omega_0)t}}{2} d\omega$$

$$= \pi\delta(\omega+\omega_0) + \pi\delta(\omega-\omega_0)$$

From 4-4,

$$\boxed{2\pi\delta(\omega+\omega_0) \Leftrightarrow \frac{1}{2\pi}\int_0^{-2\pi} e^{j(\omega+\omega_0)t} d\omega}$$

Fourier Transform of A Periodic Signal

- FT of a periodic signal is a discrete form of the FSC.
  - Let's take a periodic signal x(t) with fundamental frequency of $w_0 = 2\pi / T_0$ and write its FS.
    $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$, where C_n are the CTFS coefficients and are given by $C_n = \frac{1}{T_0}\int_{T_0} x(t) e^{-jn\omega_0 t} dt$
  - Let's take the CTFT of both sides of $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$$X(\omega) = \Im\{x(t)\} = \Im\left\{\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}\right\}$$

FT

$$\Rightarrow X(\omega) = 2\pi\left(\sum_{k=-\infty}^{\infty} C_k\right)\delta(\omega - n\omega_0)$$

- **CTFT of an aperiodic signal → aperiodic and continuous**
  CTFT of a periodic signal → discrete and periodic.

CTFT of Periodic Signals

- Example 4-6

  What is the FT of a **periodic** impulse train with period T_0?

$$X(\omega) = 2\pi\left(\sum_{k=-\infty}^{\infty} C_k\right)\delta(\omega - k\omega_0)$$

$$= 2\pi\sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$

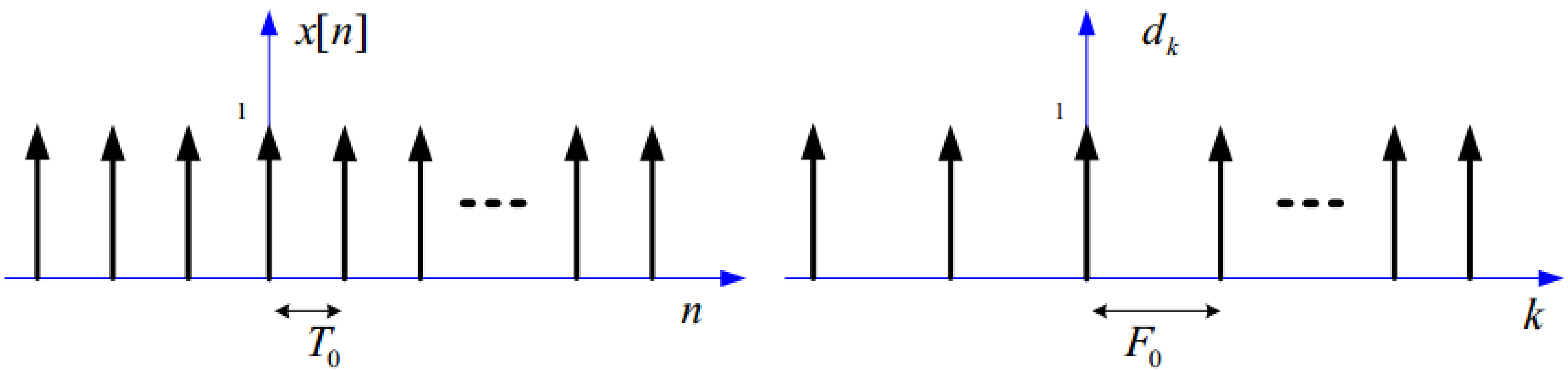
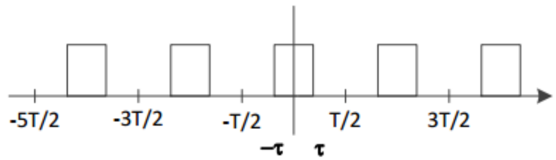


Figure 4.8 – An impulse train and its discrete-time Fourier coefficients

CTFT of Periodic Signals

- Example 4-7

  Find the FT of a **periodic** square pulse train.



$$X(\omega) = 2\pi\left(\sum_{k=-\infty}^{\infty} C_k\right)\delta(\omega - k\omega_0)$$

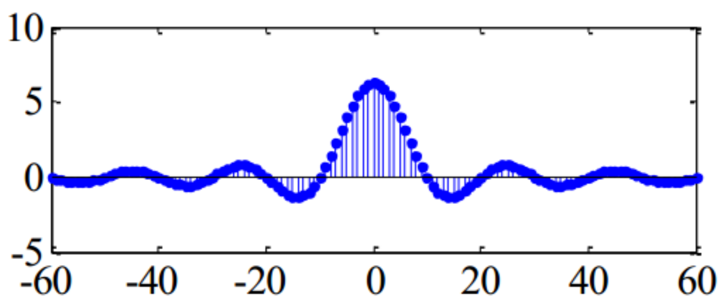


Figure 4.9 – A square pulse train and its discrete-time Fourier coefficients

- The FSC of a square pulse train $C_k = \frac{\tau}{T}\text{sinc}\left(\frac{k\tau}{T}\right)$
- The FT of this periodic signal $X(\omega) = 2\pi\left(\sum_{k=-\infty}^{\infty} C_k\right)\delta(\omega - k\omega_0)$

Discrete-time Fourier transform (DTFT)

- The DTFT, all called the DTFT synthesis equation is given by
  $$X(\Omega) = \sum_{n=-\infty}^{\infty} x[k] e^{-j\Omega nk}$$
  - n: the index of the harmonics
  - k: the index of time
  - $\Omega_0 = 2\pi / K_0$: the fundamental frequency for the discrete case ($w_0 = 2\pi / T_0$ for the continuous case)
- The inverse DTFT, also called the analysis equation is given by $x[k] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega$
- These two equations are called the DTFT pair $x[k] \Leftrightarrow X(\Omega)$
- **CTFT of an aperiodic signal → aperiodic and continuous**
  CTFT of a periodic signal → discrete and periodic.
  DTFT of an aperiodic signal → periodic and continuous
  DTFT of a periodic signal → discrete and periodic.

DTFT of Aperiodic Signals

- Example 4-8

Find the DTFT of the following signal.

$$x[k] = \delta[k-1] + \delta[k] + \delta[k+1]$$

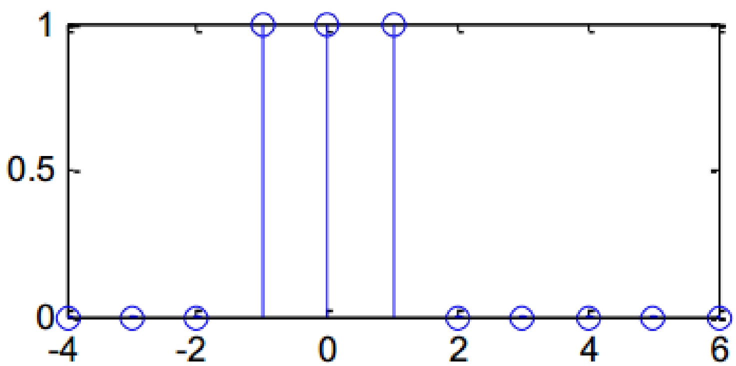


Figure 4.10 – Signal of example 4-8

$$X(\Omega) = \sum_{k=-\infty}^{\infty} \delta(k-1)e^{-j\Omega k} + \sum_{k=-\infty}^{\infty} \delta(k)e^{-j\Omega k} + \sum_{k=-\infty}^{\infty} \delta(k+1)e^{-j\Omega k} = 1 +^{-j\omega} + e^{j\omega} = 1 + 2\cos\omega$$

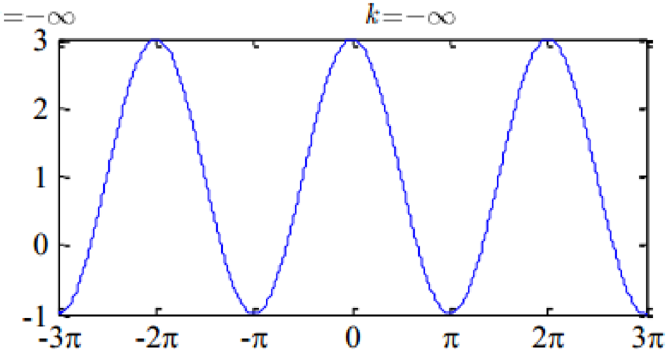


Figure 4.11 – The DTFT of signal 4-8 (a) DTFT

DTFT of Aperiodic Signals

- Example 4-9

  What is the DTFT of this discrete signal?

$$x[k] = \delta[k] + 2\delta[k-1] + 4\delta[k-2]$$

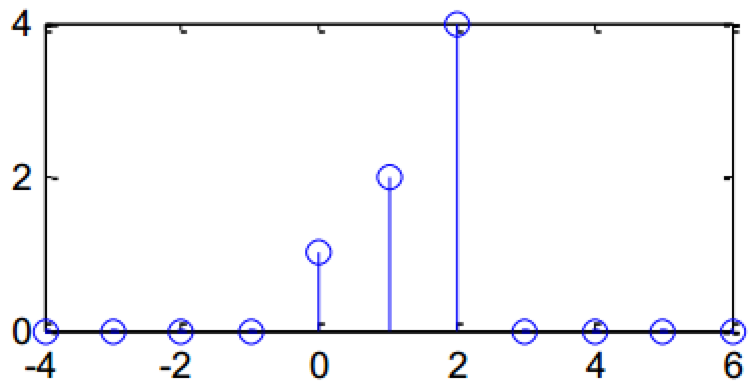
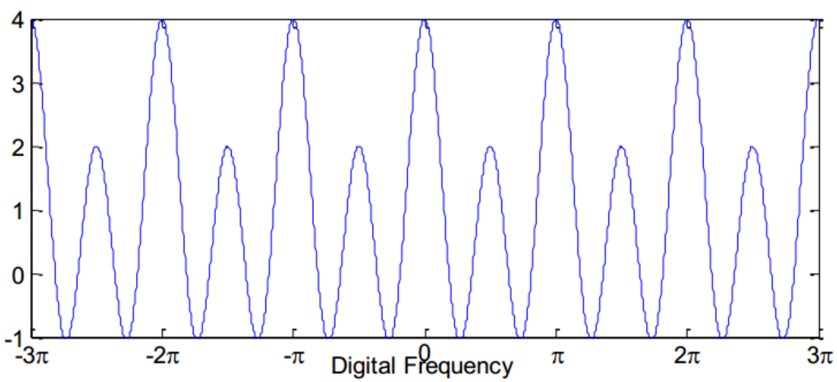
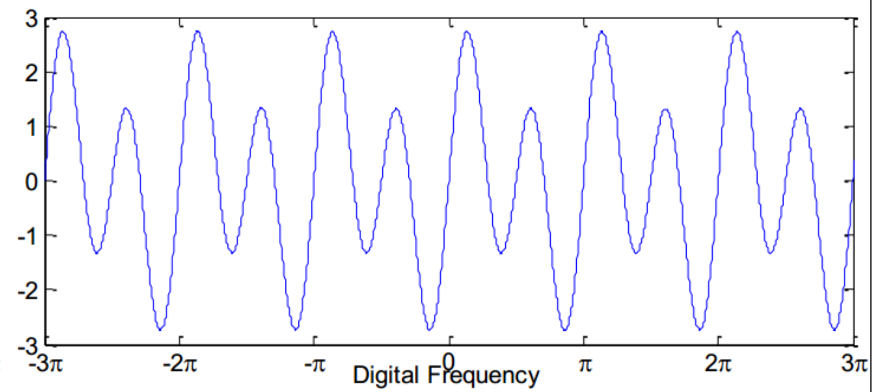


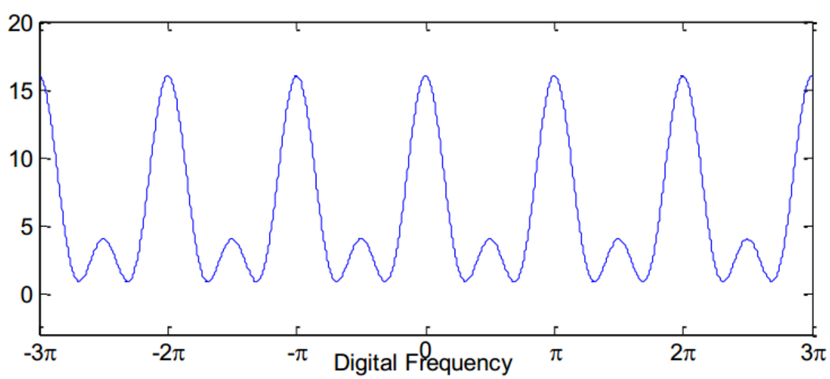
Figure 4.12 – Signal of example 4-9

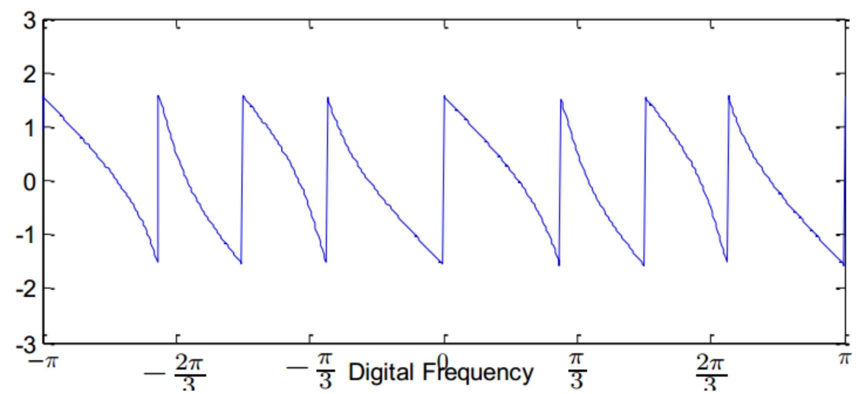
$$
\begin{aligned}
X(\Omega) &= \sum_{k=-\infty}^{\infty} \delta(k) e^{-j\Omega k} + \sum_{k=-\infty}^{\infty} 2\delta(k-1) e^{-j\Omega k} + \sum_{k=-\infty}^{\infty} 4\delta(k-2) e^{-j\Omega k} \\
&= 1 + 2e^{-j2\omega} + 4e^{j4\omega} \\
&= 1 + \cos(2\omega) - j\sin(2\omega) + 2\cos(4\omega) - 2j\sin(4\omega) \\
&= \underbrace{1 + \cos(2\omega) + 2\cos(4\omega)}_{Real} - j\underbrace{(\sin(2\omega) + 2\sin(4\omega))}_{Imag}
\end{aligned}
$$

DTFT of Aperiodic Signals









DTFT of Aperiodic Signals

- Example 4-10

  Find the DTFT of this signal.

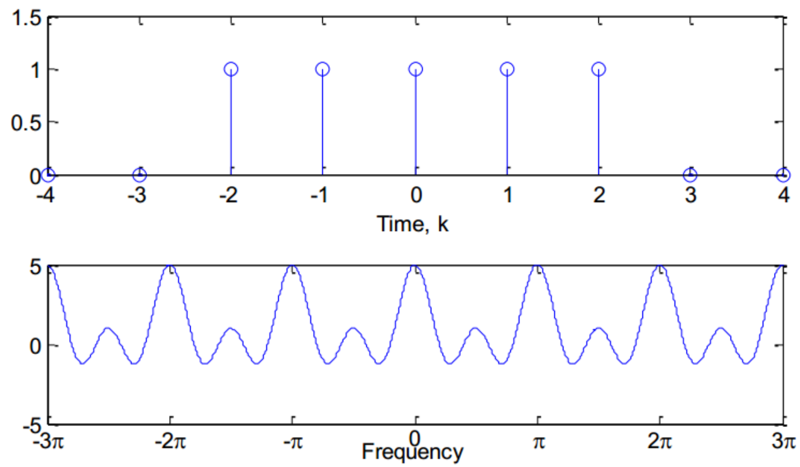


Figure 4.14 – A pulse of length N = 5 and its spectrum

```
% Example 4-10
clf
Kp = 5;
a = [1];
b = [ 0 0 a a a a a 0 0 ];
subplot(2,1,1)
n = -4: 4;
stem(n, b)
xlabel('Time, k')
axis([-4 4 0 1.5])
subplot(2,1,2)
t = -10:.01: 10;
xom = (Kp)*diric(2*t, Kp);
grid on
plot(t, xom)
pilabels
xlabel('Frequency')
hold on
```

- $$X(\Omega) = \sum_{k=-\infty}^{\infty} x[n]e^{-j\Omega n} = \sum_{k=-N}^{N} 1e^{-j\Omega n} = \frac{\sin\left(\frac{2N+1}{2}\Omega\right)}{\sin\left(\frac{1}{2}\Omega\right)}$$
- The spectrum equation looks like a sinc function but it is instead a variation, called the Drichlet function, which is essentially a repeating or periodic sinc function.

DTFT of periodic signals

- Let's take a periodic signal with period K_0 and write its DFS equation.
- $x[k] = \sum_{K_0} C_n e^{jn\Omega_0 k}$, where the coefficients are $C_n = \frac{1}{K_0} \sum_{K_0} x[k] e^{-jn\Omega_0 k}$
- FT of this periodic signal $X(\Omega) = \sum_{n=-\infty}^{\infty} x[k] e^{-j\Omega nk}$
- DTFT of a periodic signal $X(\Omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n \, \delta\left(\Omega - \frac{2\pi n}{K_0}\right)$
  - Continuous
  - repeating

DTFT of periodic signals

- Example 4-12

  Find the FT of the periodic impulse train

$$x[k] = \sum_{n=-\infty}^{\infty} \delta(n - Nk)$$

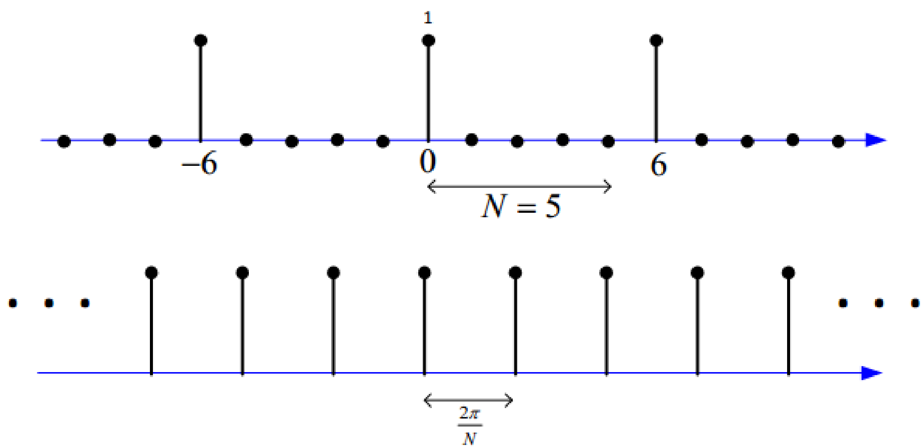


Figure 4.15 – A pulse train and its spectrum

- The Fourier coefficients $d_k = \sum_{k=K_0} x[k]e^{-j\ n\Omega_0\ k} = \frac{1}{K_0}$
- The FT $X(\Omega) = \frac{2\pi}{K_0} \sum_{n=-\infty}^{\infty} \delta\left(\Omega - n\Omega_0\right)$
- The FT is plotted for N=5 and repeats at the fundamental frequency 2π/5

DTFT of periodic signals

- Example 4-12

  Find the DTFT of $x[k] = \cos(\Omega_0 k)$

  - In the Euler form, $\cos(\Omega_0 k) = \frac{1}{2}e^{j\Omega_0 k} + \frac{1}{2}e^{-j\Omega_0 k}$
  - Assume that $\Omega_0 = 2\pi/5$,
    - The coefficients are ½ at k= $\pm$1
    - Applying $X(\Omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n\, \delta\left(\Omega - \frac{2\pi n}{K_0}\right)$ to the coefficients, we get $-\pi \le \Omega < \pi$

$$X(\Omega) = \pi\delta\left(\Omega - \frac{2\pi}{5}\right) + \pi\delta\left(\Omega + \frac{2\pi}{5}\right)$$

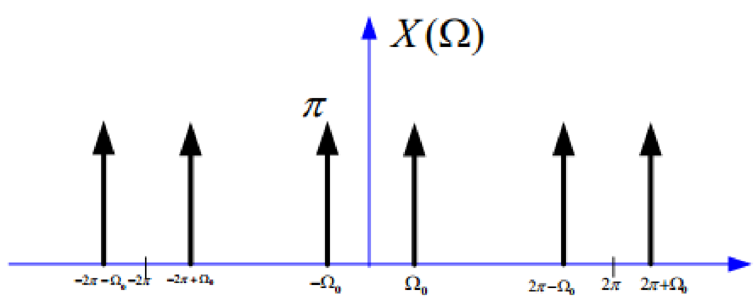


Figure 4.16 DTFT of a discrete cosine wave

DTFT of periodic signals

- Example 4-13

  Find the DTFT of this discrete periodic signal $x[k] = e^{j\Omega_0 k}$

  - The coefficients repeat with frequency Ω_0 , $D_n = \begin{cases} 1 & n = p\Omega_0 \\ 0 & elsewhere \end{cases}$
  - The DTFT of this signal: $X(\Omega) = 2\pi \sum_{n=-\infty}^{\infty} \delta(\Omega - \Omega_0 + 2\pi m)$
  - The DTFT is same as the DTFS coefficients.

DTFT of periodic signals

- Example 4-14

Find the DTFT of this discrete periodic signal. The signal is periodic with period K_0, and its length of the impulses is K_P samples.

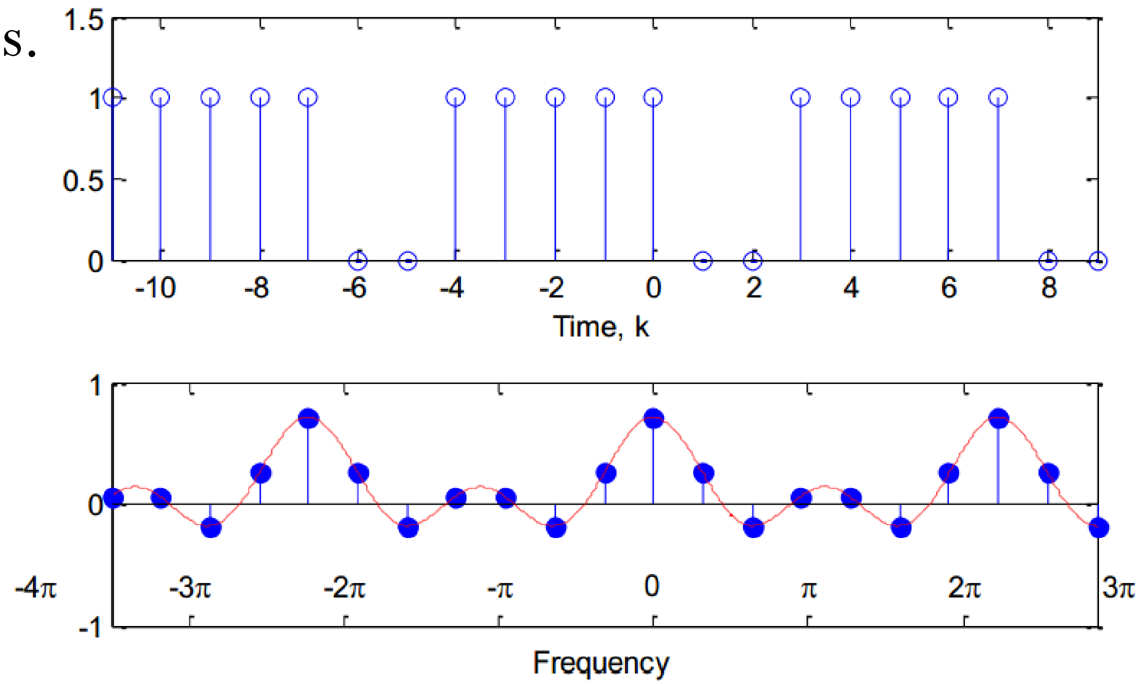


Figure 4-17 DTFT of a periodic signal (a) time domain signal, (b) DTFSC of the signal.

DTFT of periodic signals

- The DTFSC of this signal are discrete

$$\text{Real } C_n = \frac{1}{K_0}\left[\frac{\sin \; K_p n\pi / K_0}{\sin \; n\pi / K_0}\right]$$

- Kp=5 and K_0=7
- The DTFT of this signal
  - multiplying the DTFSC with a pulse train of frequency $\Omega_0 = 2\pi/7$
  - But we these coefficients are already located at a frequency resolution of $2\pi/7$
  - Count the number of samples from $-\pi$ to $+\pi$, we get 7.
- The DTFT of this periodic signal is same as DTFSC except it is scaled by a factor 2π.

Continuous-time Fourier Series

- The CTFS is defined for periodic signals where time is continuous

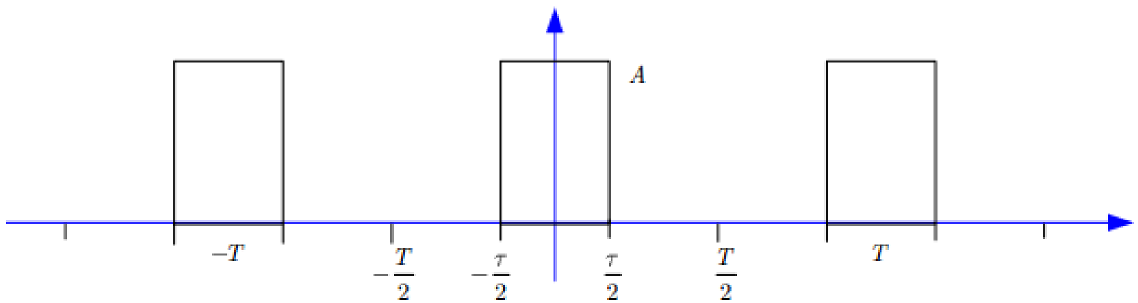


Figure 4.18 A periodic signal with continuous time

$$C_n = \frac{\tau}{T}\operatorname{sinc}\left(\frac{n\tau}{T}\right)$$

- Assume T = 1/2π
- Case 1: τ=T/2=1/4π
  - The fundamental frequency w_0 = 1/T = 2π
  - When τ/T=.5, at f=0, the value of the specturm is .5.
  - At n= ± 2, ±4, ±6…, the spectrum shows zero values which corresponds to frequencies of ± 2π, ±4π, ±6π ….

Continuous-time Fourier Series

- Case 2: $\tau/T=.2$
  - the spectrum is zero at $\pm 5, \pm 10, \pm 15\ldots$, which corresponds to frequencies of $\pm 2\pi, \pm 4\pi, \pm 6\pi \ldots$.
- As the $\tau/T \rightarrow 1$, the spectrum begins to look like an impulse function.
- τ/T: the duty cycle of the signal

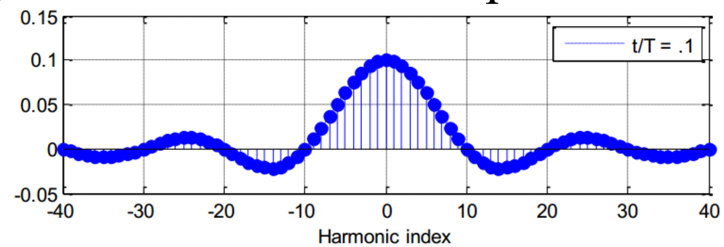
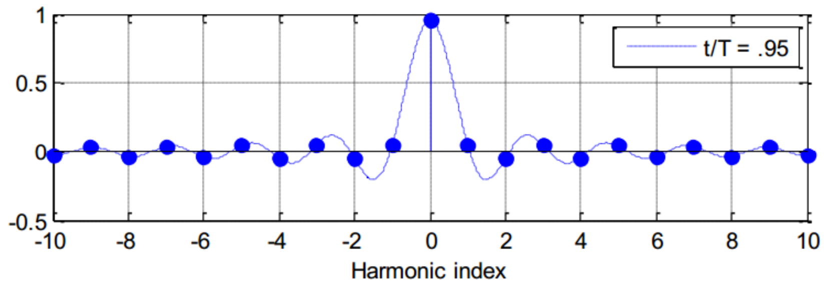
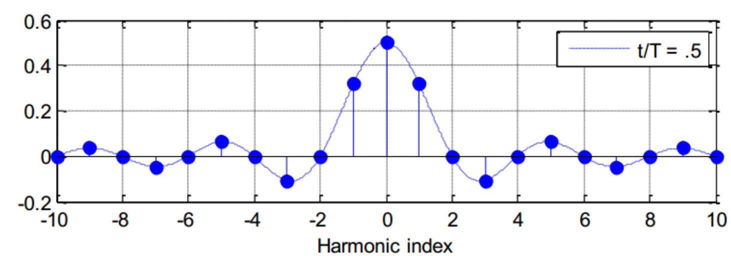
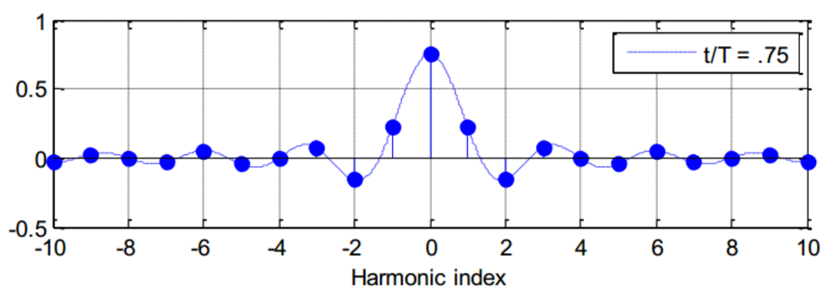
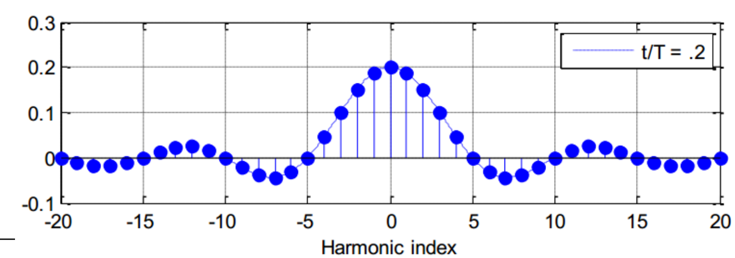


Figure 4.19 The discrete coefficients of the continuous signal as a function of the duty cycle of the signal. As the pulse gets narrow, its CTFSC get more dense.



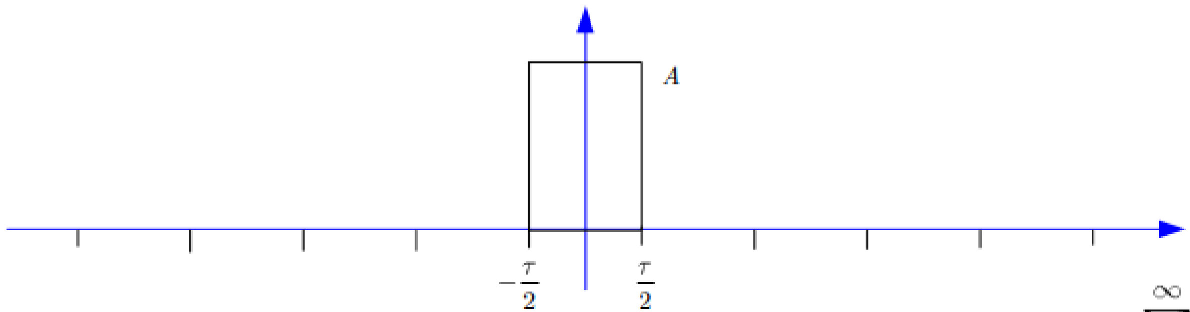






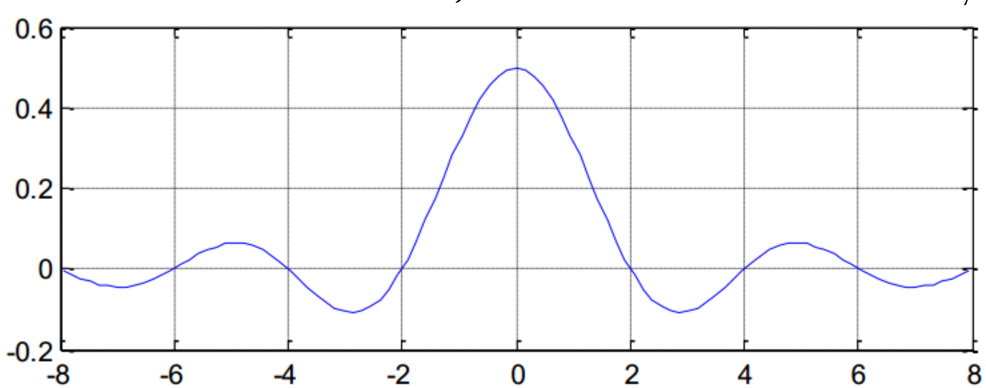
Continuous-time Fourier Transform

- Let's look at just one period of the same signal.
  - continuous but also aperiodic.



$$X(\Omega) = 2\pi \sum_{n=-\infty}^{\infty} D_n \delta \ \ \Omega - n\Omega_0$$

  - The CTFT of this signal $X(\omega) = \tau \,\mathrm{sinc}\left(\frac{\omega\tau}{2\pi}\right)$
  - Assume that $\tau = \pi$, The zeros occur every 2 Hz. $X(\omega) = \pi \,\mathrm{sinc}\left(\frac{\omega}{2}\right)$



Continuous-time Fourier Transform

- For $\tau = 2\pi/5$, we get crossings every 5 Hz.
- For $\tau = \pi/5$, we get crossings every 10 Hz.

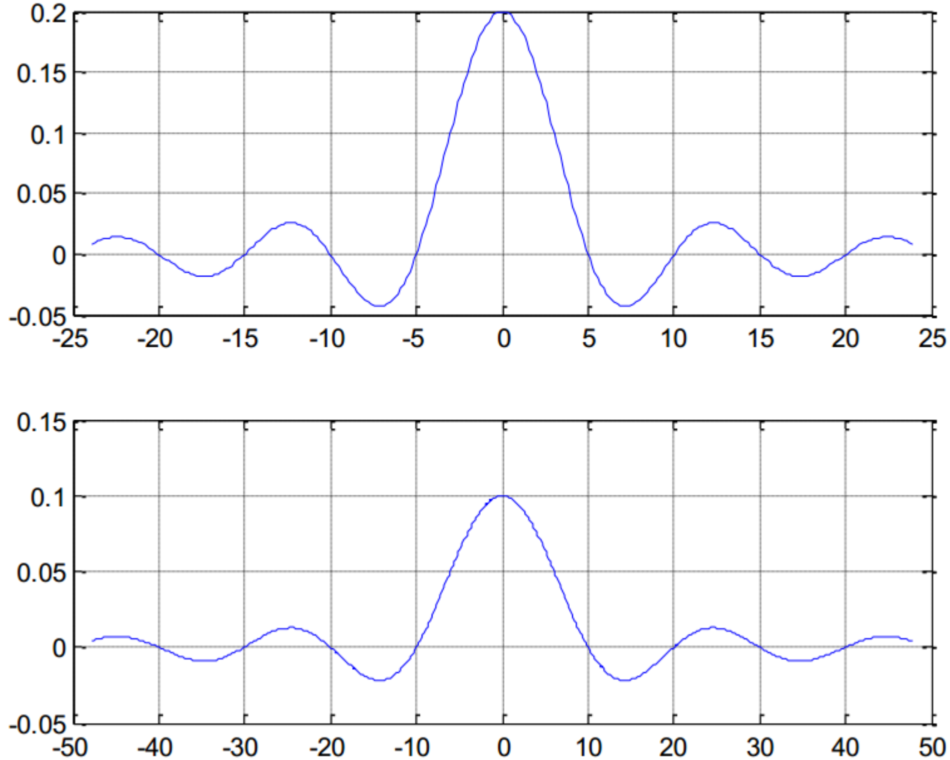


Figure 4.20 The CTFT of the continuous but aperiodic square pulse as a function of the width the square pulse. As the pulse gets narrow, its lobes in the spectrum get wider.

Discrete-time Fourier series, DTFS

- Let's look at the same signal, in a discrete form.

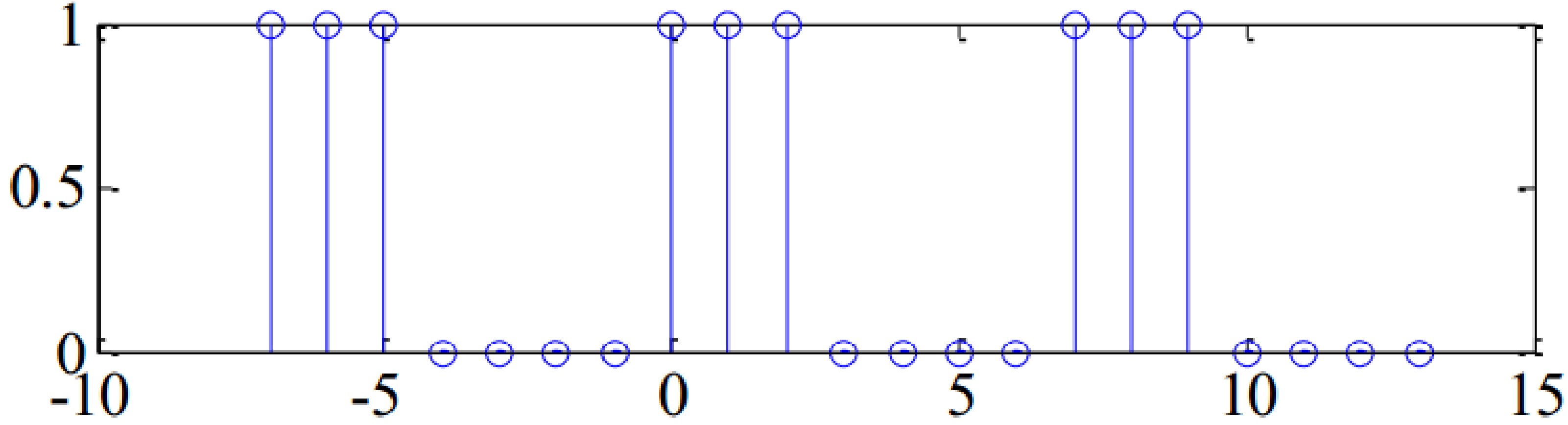


Figure 4.21 The discrete-time periodic signal.

Discrete-time Fourier series, DTFS

- The spectrum contains a Drichlet function.
  - Case1
    - Impulses located 7 bins apart, each of which are 2π/7 Hz apart
    - The period is 1/7 seconds and as such in the frequency domain.
    - In the frequency domain, the frequency pulses have a period of 2π which corresponds to the period of 7 samples.

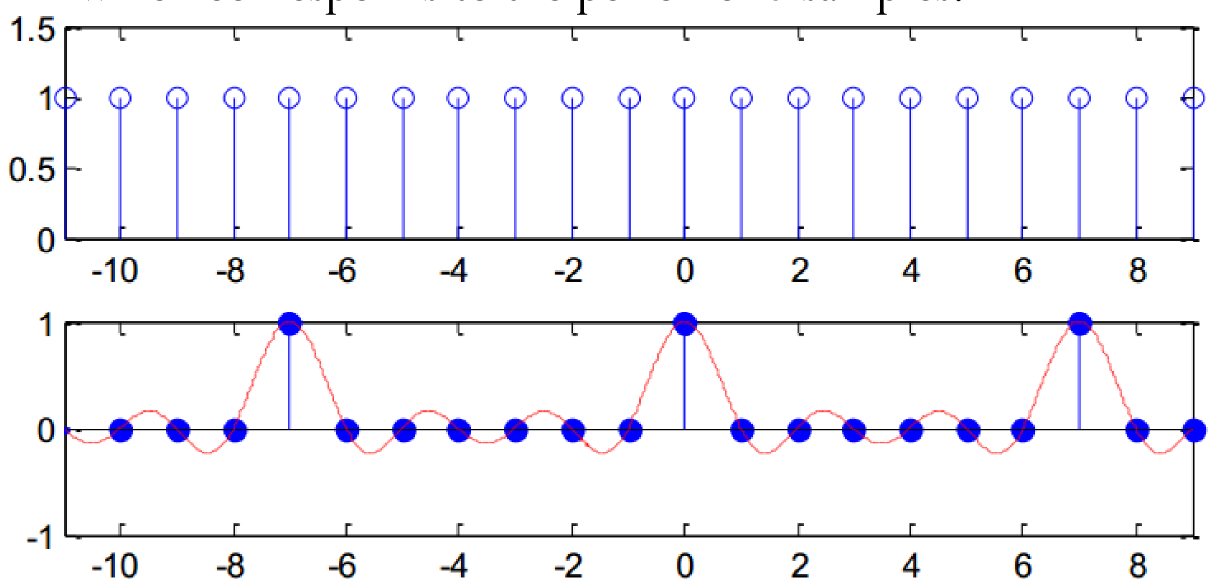


Figure 4.22 The discrete-time periodic signal and its DTFSC - pulse size $K_p = 7, K_0 = 7$

Discrete-time Fourier series, DTFS

- Case 2
  - the pulse size is 6 samples in time domain lasting 6/7 seconds
  - In frequency domain each bin is 2π/7=.897Hz. bandwidth is 7/6 Hz.
  - As long as the pulse width is >π, we will see the sinc function tails.

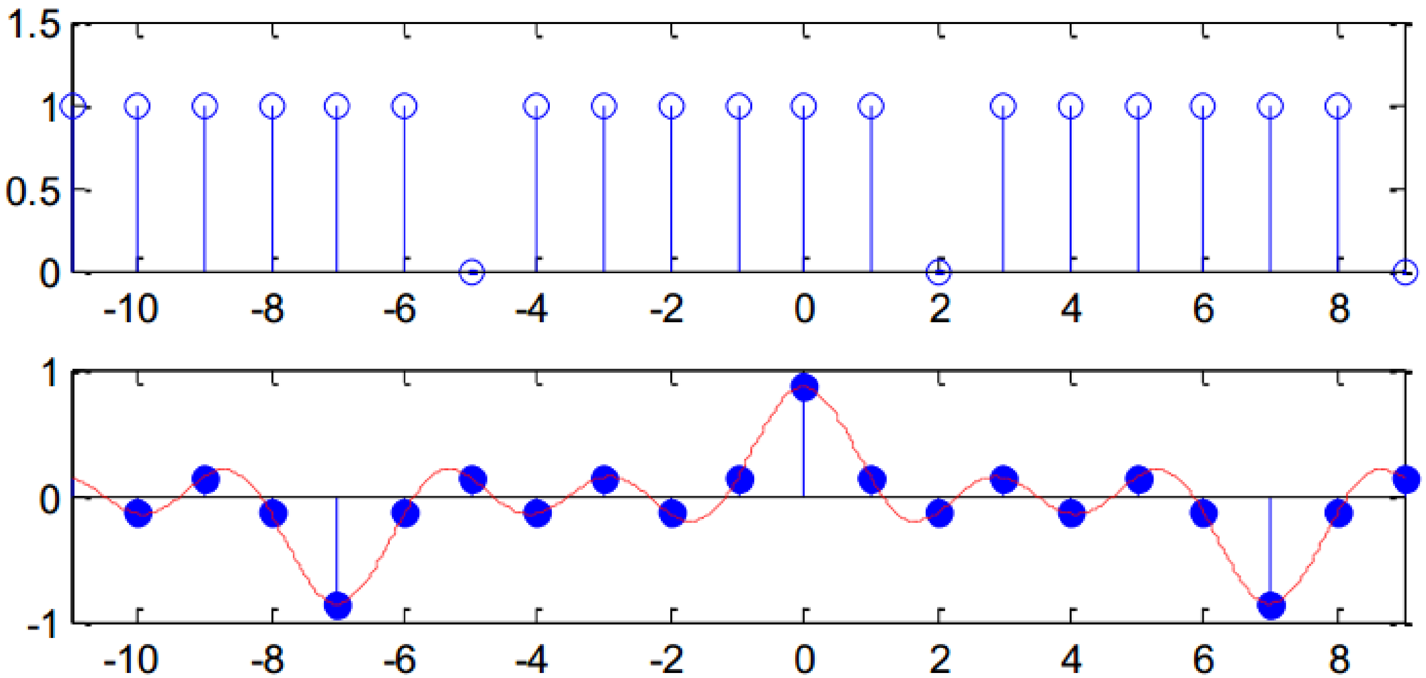


Figure 4.23 The discrete-time periodic signal and its DTFSC - pulse size $K_p = 6, K_0 = 7$

Discrete-time Fourier series, DTFS

- Case 3

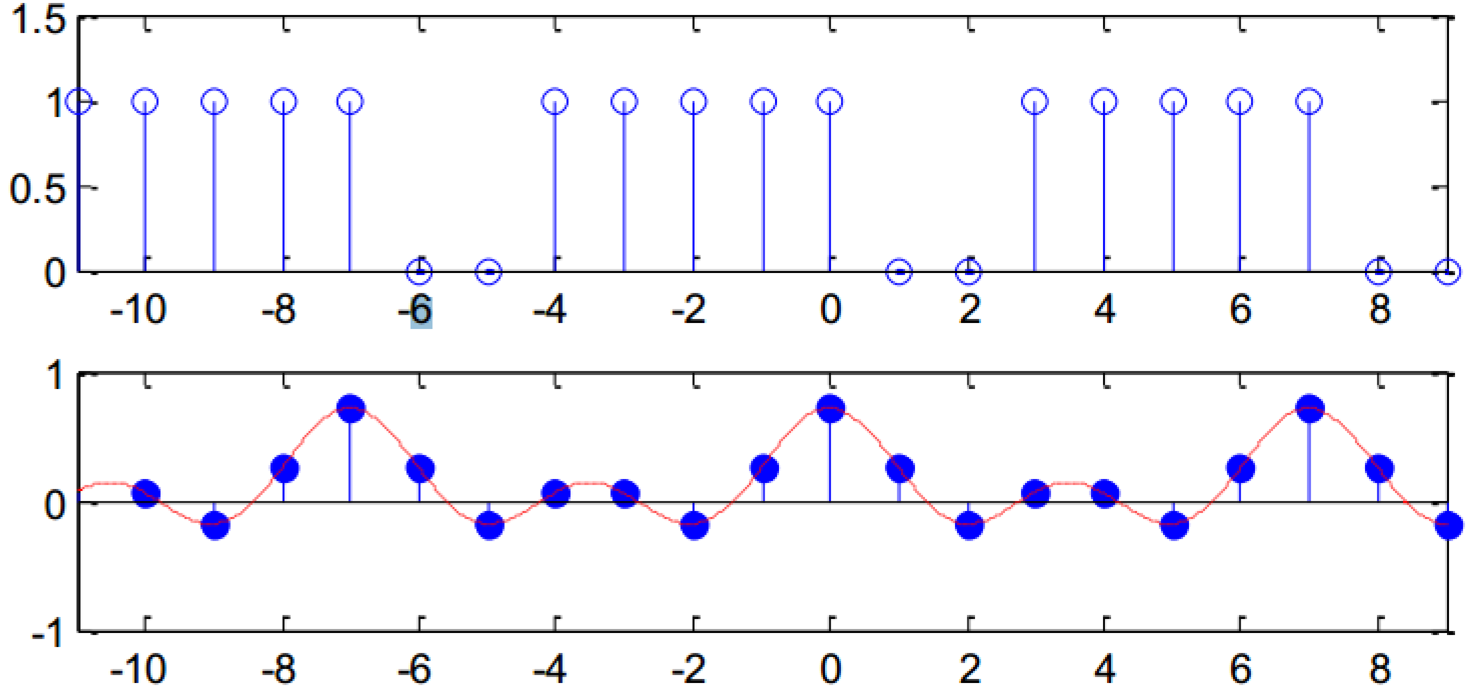


Figure 4.24 The discrete-time periodic signal and its DTFSC - pulse size $K_p = 5, K_0 = 7$

Discrete-time Fourier series, DTFS

- Case 4

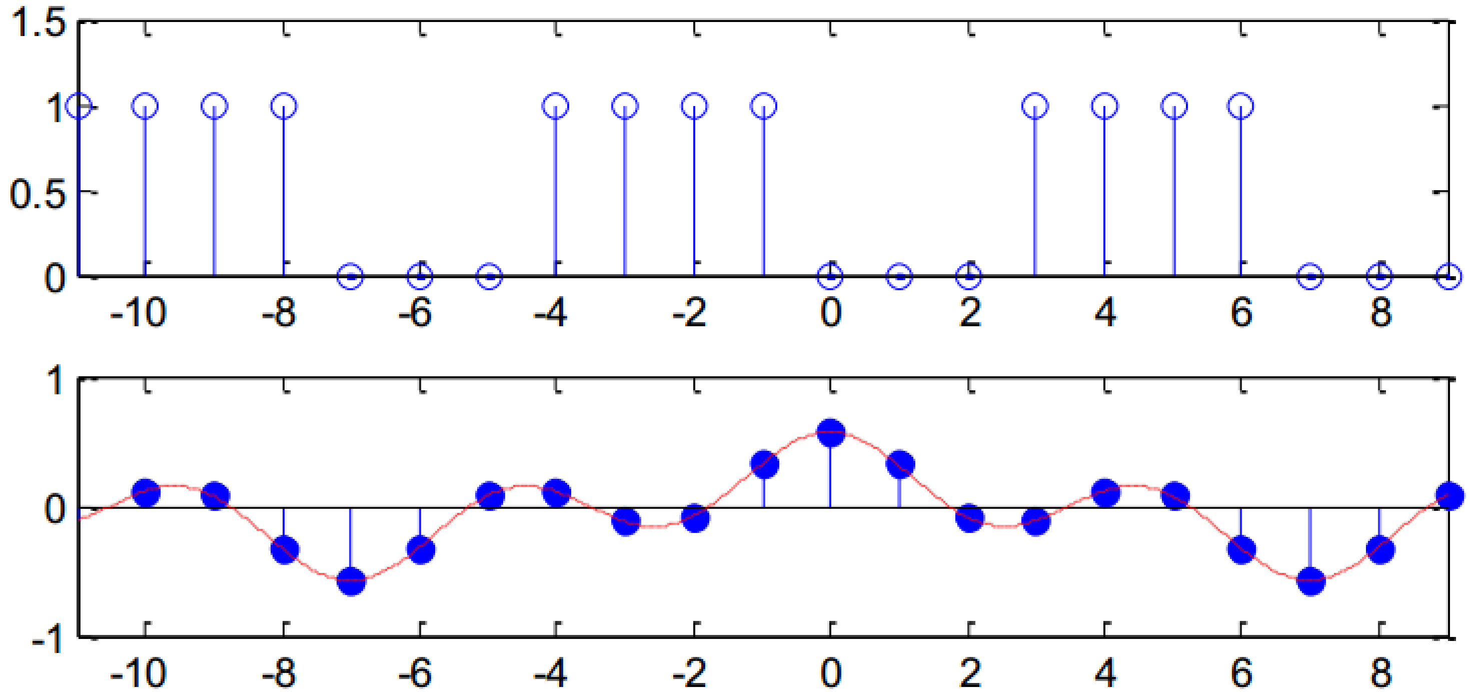


Figure 4.25 The discrete-time periodic signal and its DTFSC - pulse size $K_p = 4, K_0 = 7$

Discrete-time Fourier series, DTFS

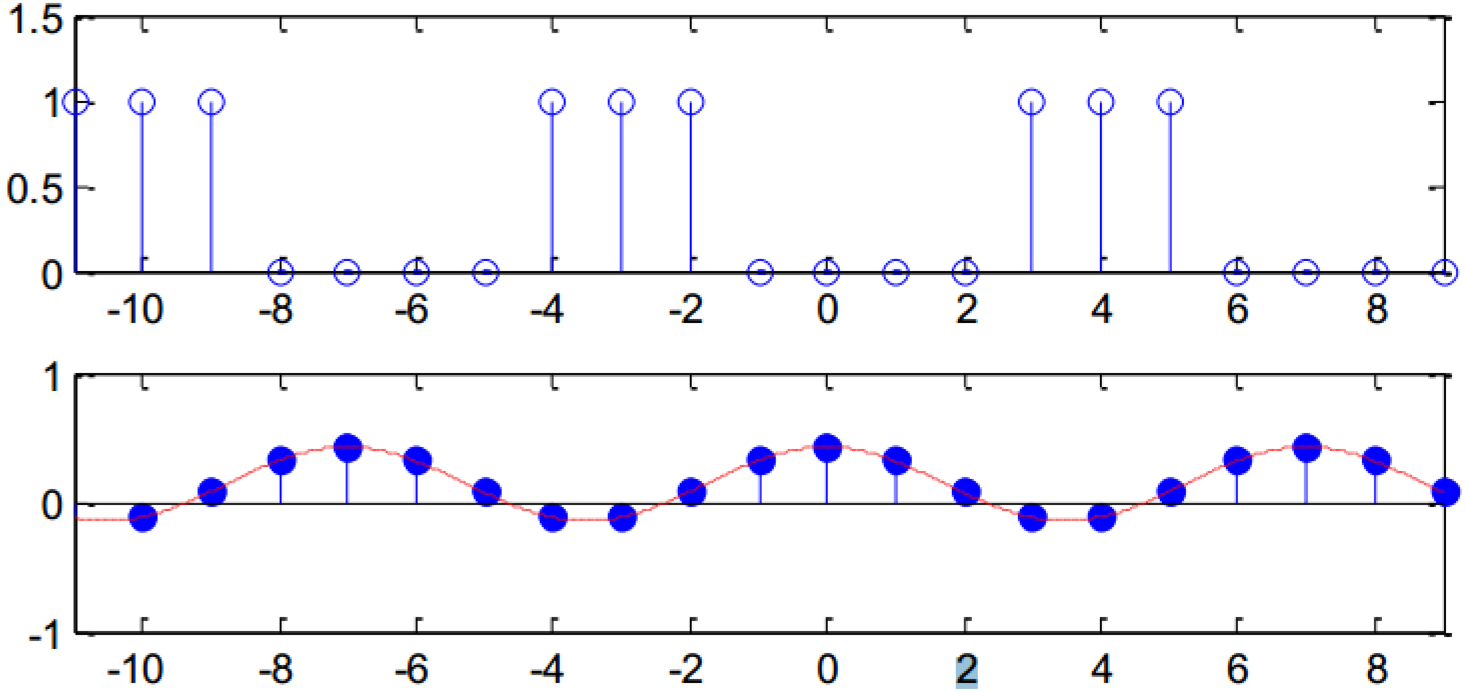


Figure 4-25 The discrete-time periodic signal and its DTFSC - pulse size

$K_p = 3, K_0 = 7$

Discrete-time Fourier series, DTFS

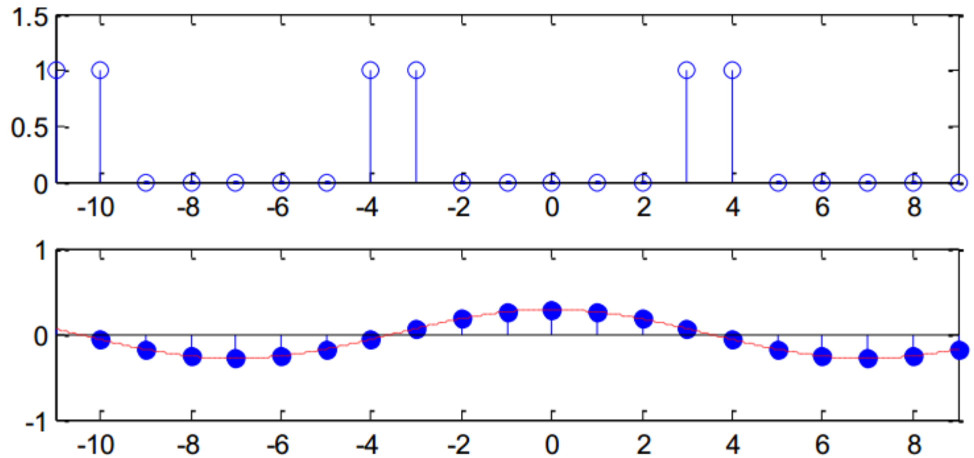


Figure 4.27 The discrete-time periodic signal and its DTFSC - pulse size
$K_p = 2, K_0 = 7$

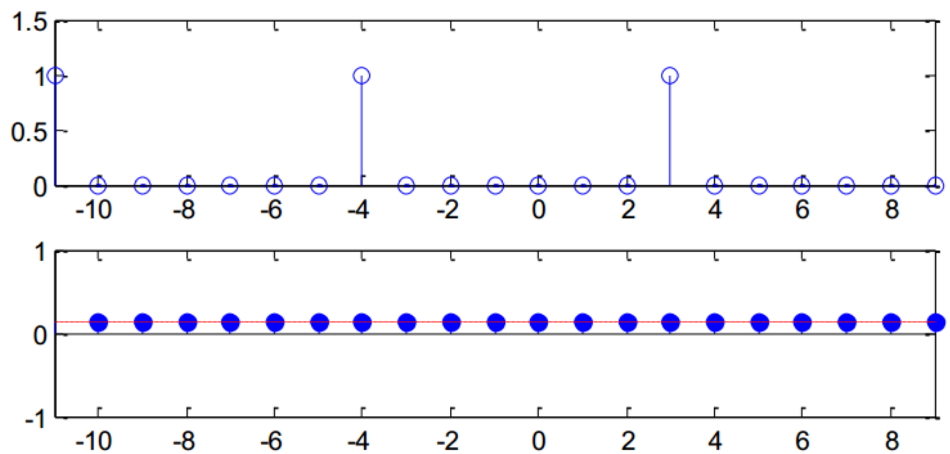


Figure 4.28 The discrete-time periodic signal and its DTFSC - pulse size
$K_p = 1, K_0 = 7$

DTFT of a periodic signal

- a longer pulse has smaller frequency so the spectrum is not aliased.

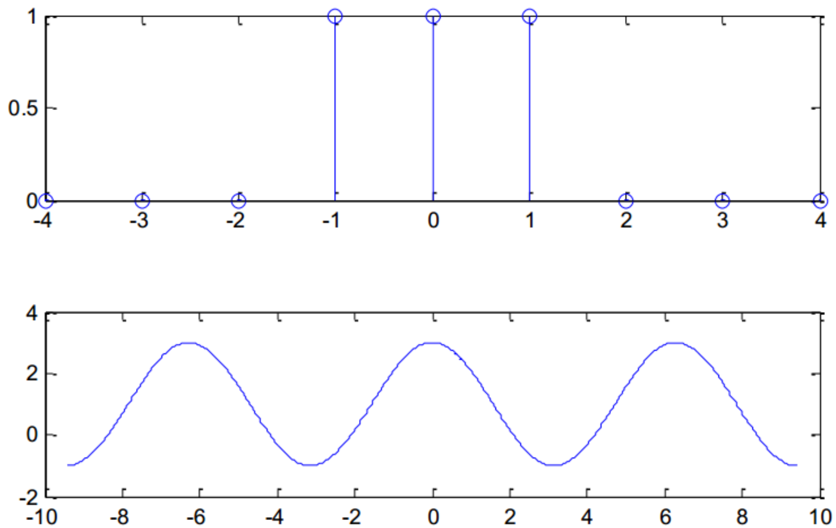


Figure 4.29 The discrete-time aperiodic signal and its DTFSC - pulse size = 3

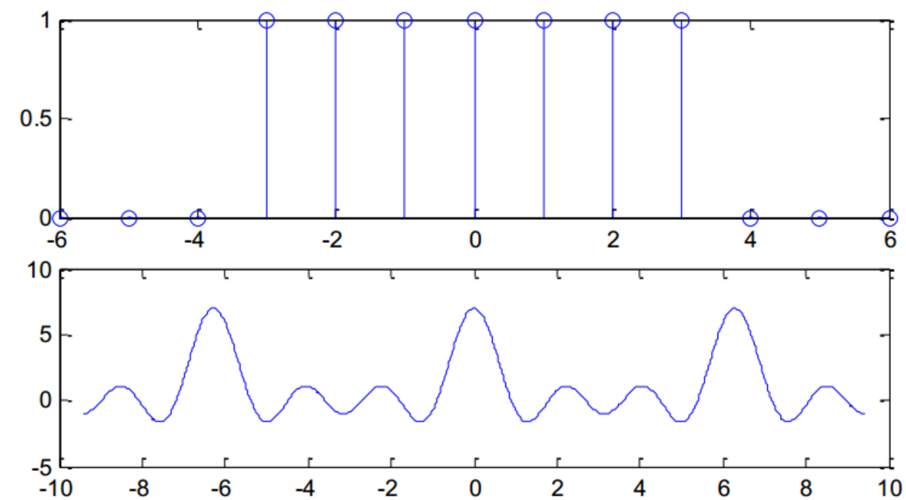


Figure 4.30 - The discrete-time aperiodic signal and its DTFSC - pulse size = 7

- If DTFT assumes that the signal has an infinite period, then why does it matter how long the pulse is relative to the number of points shown?

$$X(\Omega) = \sum_{k=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{k=-N}^{N} 1 e^{-j\Omega n} = \frac{\sin\left(\frac{2N+1}{2}\Omega\right)}{\sin\left(\frac{1}{2}\Omega\right)}$$

DTFT of a periodic signal

- Why is it not being interpreted as a train of impulses? Why aren't we getting a spectrum same as 4.26?

Summary

- 1. **Fourier series** is **not** intended for **aperiodic** signals.
- 2. **Fourier transform** is an extension of the Fourier series and applies to **aperiodic** signals by assuming that the period of the signal is infinite.
- 3. This assumption results in a spectrum that is **continuous** since the fundamental **frequency** is now **zero**.
- 4. The continuous-time Fourier Transform (**CTFT) of aperiodic signals** is **continuous.**
- 5. The discrete-time Fourier Transform (DTFT) is developed in exactly the same way as the CTFT assuming that fundamental period approaches infinity.
- 6. This also results in a continuous spectrum but on that repeats same as DTFSC.
- 7. We can use the DTFSC to compute a DTFT of a periodic signal.
- 8. The DTFT of a periodic signal is a sampled version of the DTFSC.