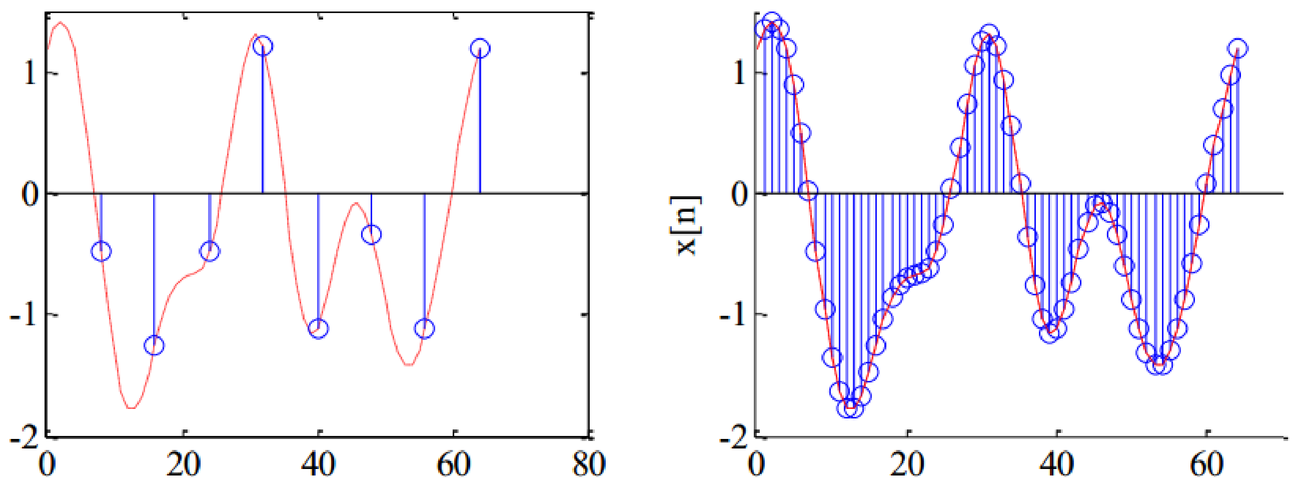
Tutorial 6
Discrete Time Signals and Fourier Series

2014/1

Sampling of Signals

- Suppose we have an analog signal and we wish to create a discrete version of it.

When the rate is not quick enough when the rate is too fast



(See Matlab Program 1)

Figure 1 – Continuous and a discrete signal

- There is an optimum sampling rate which captures enough information without overdoing it.

Sampling of Signals

- **Sampling Theorem**
  - For any analog signal containing among its frequency contents a maximum frequency of f_{max}, the underlying signal can be represented faithfully by N equally spaced samples, provided the sampling rate is at least two times f_{max} samples per second.
- A Maximum **Sampling Period** $T_s = \frac{1}{2f_{\max}}$ seconds
- **Sampling Frequency** $F_s = \frac{1}{T_s} = 2f_{\max}$ samples/second

$$f_{\max} = \frac{1}{2T_s} Hz$$

$$\omega_{\max} = 2\pi f_{\max} = \frac{\pi}{T_s} \, radians/\sec$$

Sampling of Signals

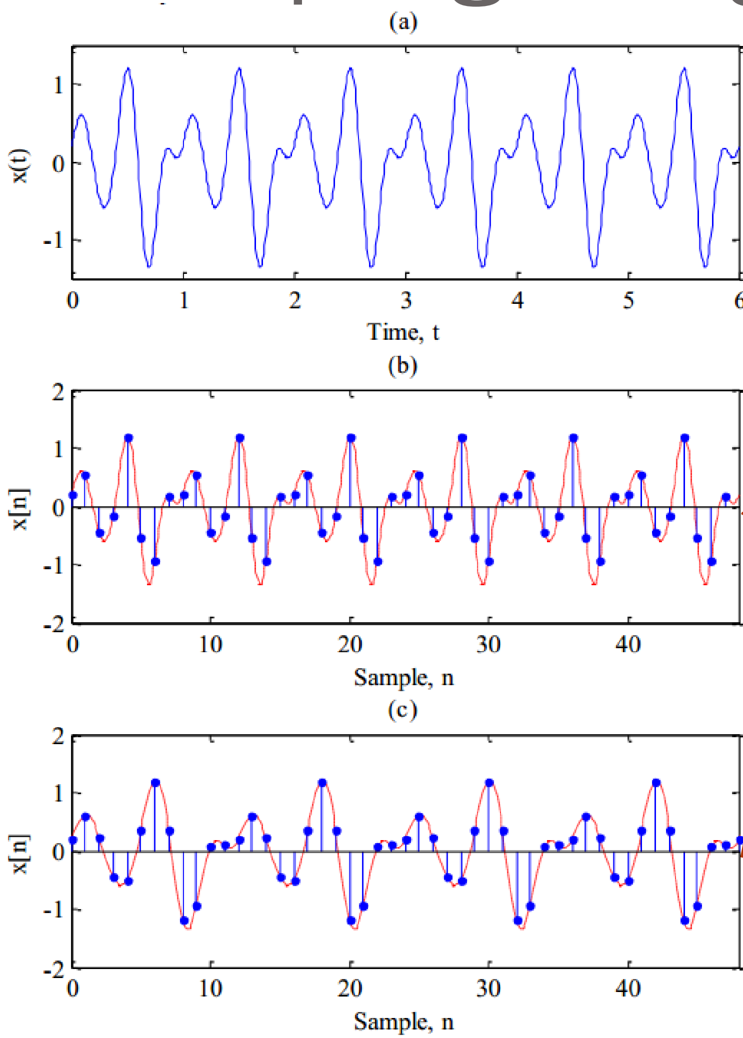


Figure 2 – (a) A continuous signal, (b) sampled at 8 samples per second and (c) sampled at 12 samples per second.

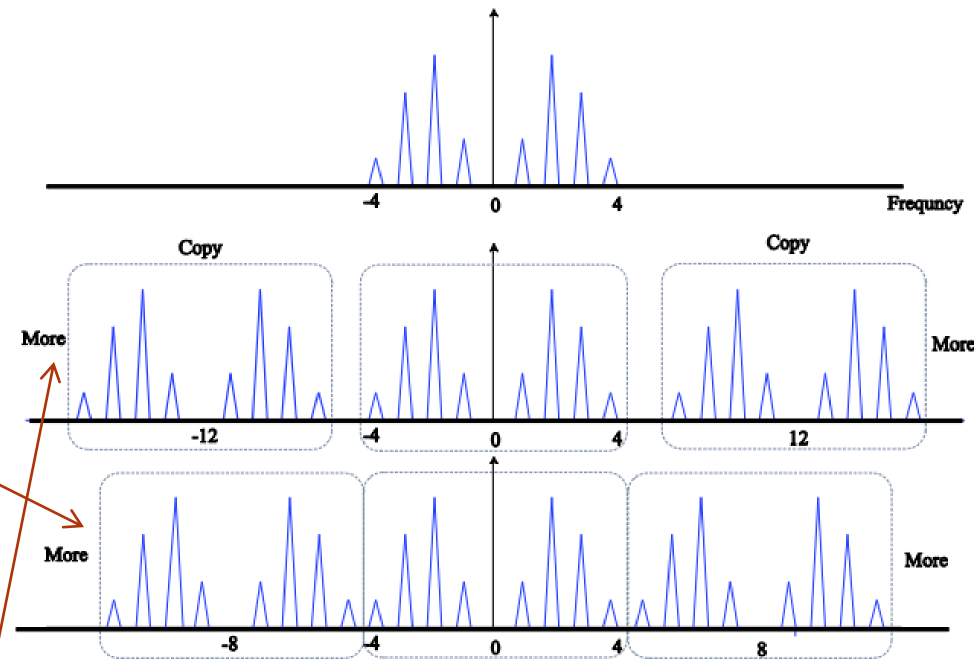


Figure 3 – Replicating spectrums as a consequence of sampling

- If we sample the continuous signal at a rate less than twice the maximum frequency, **Nyquist rate**, the spectrum would begin to overlap. The problem is called aliasing.

(See Matlab Program 2)

$f_1=1$ $f_2=2$ $f_3=3$ $f_4=f_{max}=4$

$$x(t) = .25\sin(2\pi t) + .7\cos(4\pi t) - .5\cos(6\pi t) + .15\sin(8\pi t)$$

Specifying a Discrete Signal

- If a continuous signal is referred to as x(t) then a discrete sampled signal is written as $x(kT_s)$
  - K=0, ±1, ±2,… : the sample number
  - T_s : the sampling period

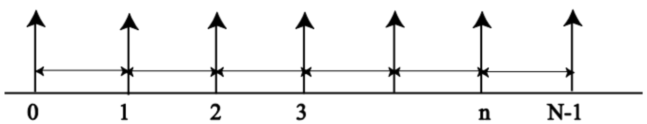


Figure 4 – Discrete signal samples

- To create a discrete signal from a continuous signal
  - 1. the continuous signal is multiplied by an impulse train of the sampling period.
  - 2. multiply the sampled signal once again by an impulse train, point by point.

"combing" the signal to create a discrete signal.

$$x(kT_s) = x(t) \times \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$ (continuous)

$$x[k] = \sum_{k=-\infty}^{\infty} x(kT_s)\delta(t - kT_s)$$ (discrete)

Specifying a Discrete Signal

- Two ways to specify a sampled signal
  - By sample numbers. (Figure 5(b))
  - By its phase. (Figure 5(c))
- Example. $x(t) = \sin(2\pi f_0 t), \quad f_0 = 1.$

$$x[-10] = \sin[2\pi(-10 \div 5)] = 0$$

$$x[-9] = \sin[2\pi(-9 \div 5)] = \sin[-3.6\pi] = .951$$

$$x[-8] = \sin[2\pi(-8 \div 5)] = \sin[-3.2\pi] = .588$$

$$x[-7] = \sin[2\pi(-6 \div 5)] = \sin[-2.4\pi] = -.951$$

$$x[-6] = \sin[2\pi(-5 \div 5)] = \sin[-2\pi] = 0$$

$$x[-5] = \sin[2\pi(-4 \div 5)] = \sin[-1.6\pi] = .951$$

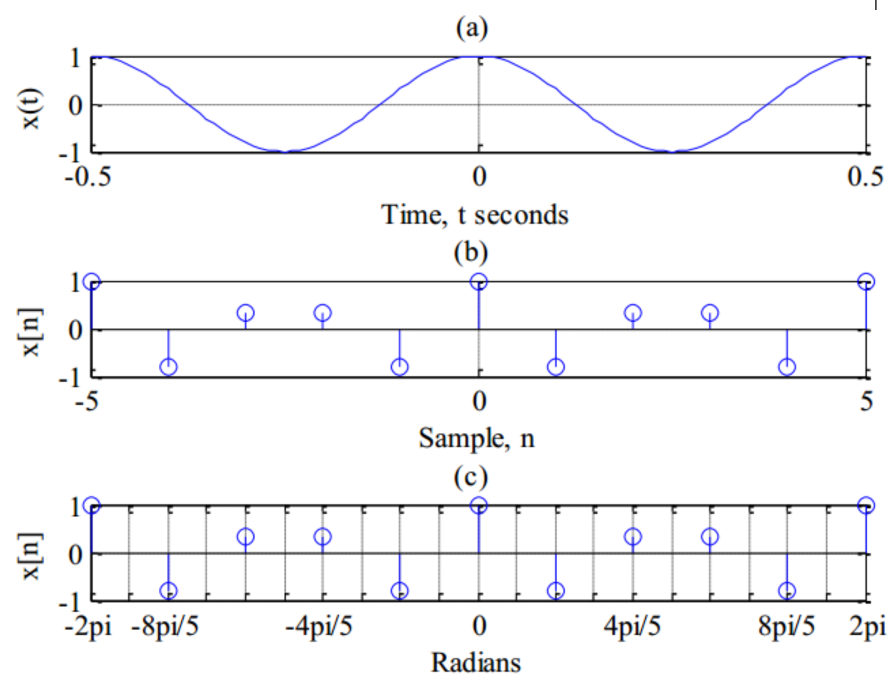


Figure 5 – Sampling of a continuous signal

(Matlab Program 3)

Parameters of a Discrete Signal

- To create a discrete signal, we sample this signal with sample time of T_s. The Sampling Period T_s is independent of the fundamental period T_o of the continuous signal and no larger than the Nyquist threshold $\frac{1}{2} f_{max}$.
- If we know the signal frequency f_0 and the sampling frequency F_s, we can also write the signal this way replacing T_s with $1/F_s$.

$$x[k] = \sin\left[\left(\frac{2\pi f_0}{F_s}\right)k\right]$$

- **Digital Frequency** (phase advance)

$$\Omega_0 = \frac{2\pi f_0}{F_s} \quad \frac{radians \times cycles / second}{sample / second} \to \frac{radians}{sample}$$

  - each sample moves the signal by the value of digital frequency.
  - Ex. a signal of frequency 10 Hz sampling with a sampling frequency Fs = 30 Hz, then its digital frequency $\Omega_0 = \frac{2\pi \times 10}{30} = 2\pi / 3$

Parameters of a Discrete Signal

- K_0: a ratio of the sampling frequency to the fundamental frequency or the maximum frequency.
  - ex. $K_0 = \frac{F_s}{f_0} = \frac{30}{10} = 3$
- If there are K_0 samples in one period, then K_0 times the digital frequency Ω must equal 2π. We define the period of the signal in samples as: $K_0\Omega_0 = 2\pi \;\; or \;\; K_0 = \frac{2\pi}{\Omega_0}$
  - The fundamental frequency is given by $\Omega_0 = \frac{2\pi}{K_0}$
- Suppose the underlying signal is periodic, the sampled discrete signal is also periodic if and only if N is an integer.

$$N = mK_0 = m\frac{T_0}{T_s} = m\frac{F_s}{f_0}$$

fo

  - ex. A signal with fundamental frequency of 5 which is sampled at a rate of 20, will have a fundamental period of 4 and as such this sampling rate would result in a periodic discrete signal.

Discrete Signal Periodicity

- The definition of a periodic signal for a discrete signal is

$$x[k] = x[k + K_0]$$

  - Ex. Show that the periodicity of a discrete sinusoid sin(Ωn).

    $\underline{\sin(k\Omega)} = \sin(\Omega(k+K_o)) = \underline{\sin(k\Omega+K_o\Omega)}$

    only if $K_o\Omega=2\pi$, the equation holds.

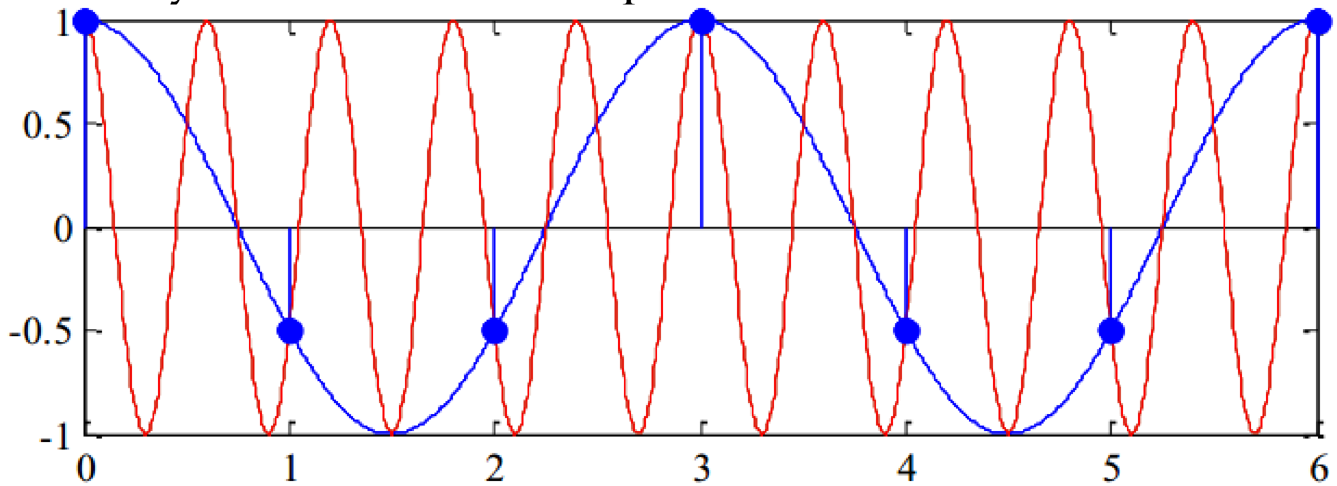


Figure 6 – A discrete signal can represent any number of continuous signals.

(Matlab Program 4)

Discrete Signal Periodicity

- Example 3-1

  What is the digital frequency of this signal? What is its period?

$$x[k] = \cos\left[\frac{2\pi}{3}k + \frac{\pi}{3}\right]$$

  - The digital frequency $\Omega_0 = 2\pi/3$
  - Its period $K_0 = 2\pi/(2\pi/3) = 3$

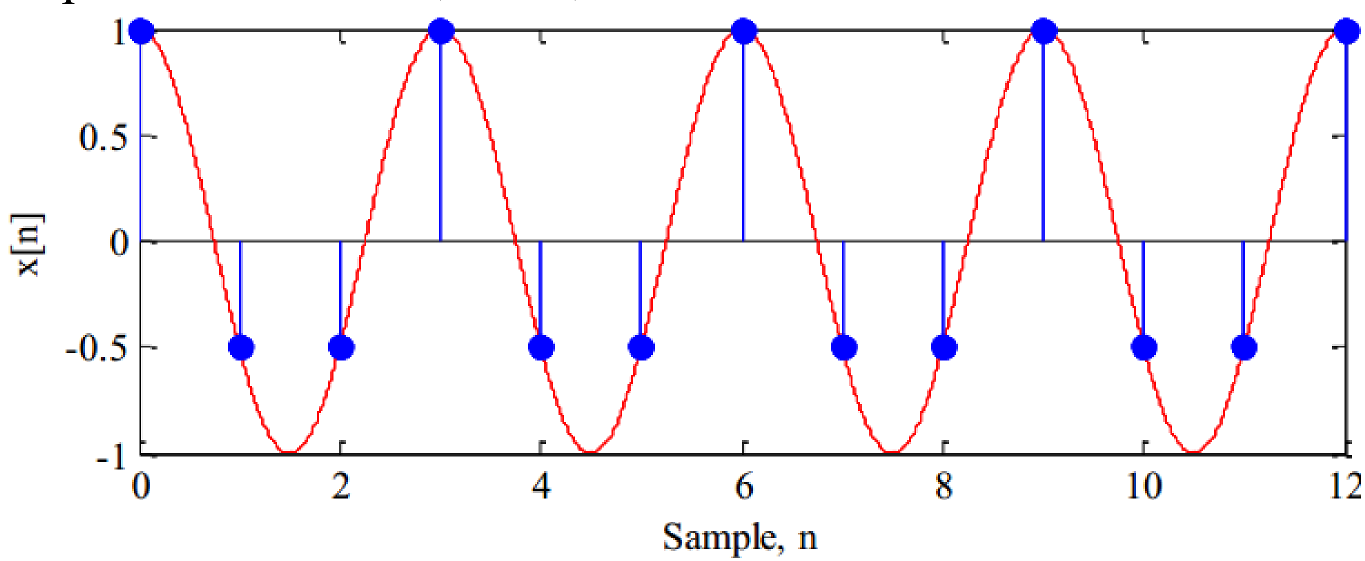


Figure 7 – Discrete signal of example 3-1

(Matlab Program 5)

Discrete Signal Periodicity

- Example 3-2

  What is the period of this discrete signal? Is it periodic?

$$f[k] = \sin\left(\frac{3\pi k}{4} + \frac{\pi}{4}\right)$$

  - The digital frequency $\Omega_0 = 3\pi/4$
  - The period of the signal is: $3\pi m = m4 \rightarrow K_0 = 8,\ m = 2$

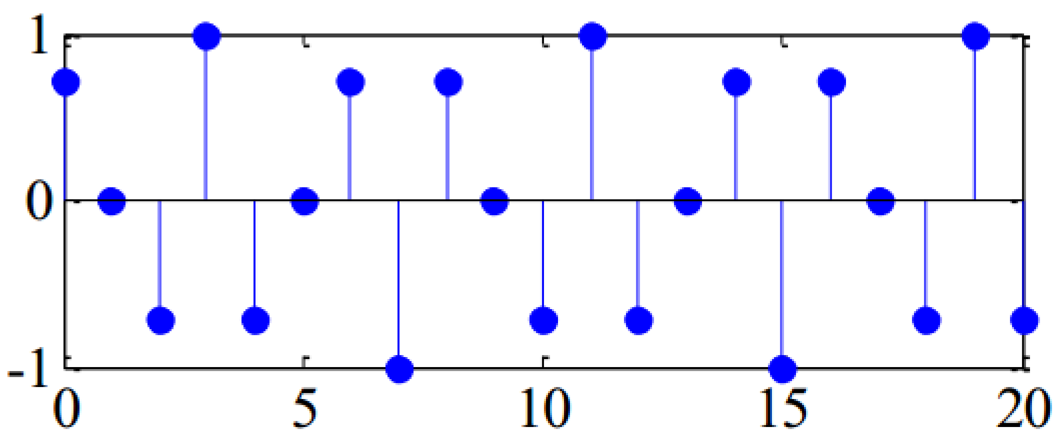


Figure 8 – Discrete signal of Example 3-2

Discrete Signal Periodicity

- Example 3-3

  Is this discrete signal $f[n] = \sin[.5n + \pi]$ periodic?

  - The fundamental frequency is 0.5.
  - $K_0 = \dfrac{2\pi}{\Omega_0} k = 12\pi k$
  - The continuous signal is periodic but the discrete samples of it is not because K_0 is not a ratio of integers.

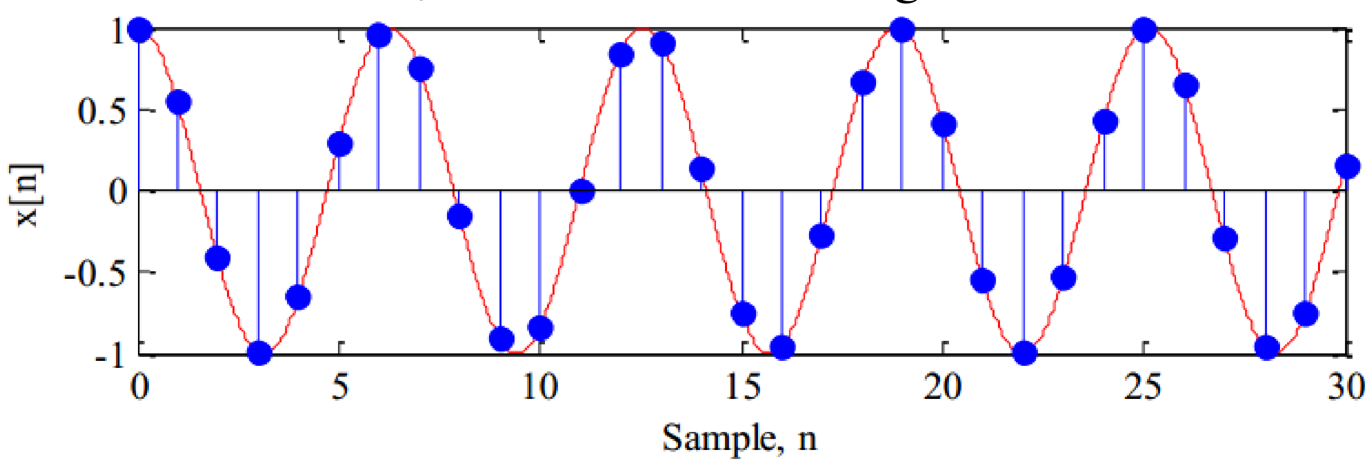


Figure 9 – Non-periodic discrete signal of example 3-3.

(Matlab Program 6)

Basic Function for Discrete-Time Fourier Series

- A Fourier series representation for discrete time signals
  - The discrete complex exponential is written by replacing t with k . We can write this in terms of the digital frequency as:

$$e^{j\omega t} \quad \text{continuous signal}$$

$$e^{j\Omega k} \quad \text{discrete signal}$$

  - In the discrete case, the harmonic relationship is based on phase:

$$\Omega + 2\pi k \rightarrow \Omega$$

  - Every time increases by 2π, we get a new complex exponential given by:

$$e^{j(\Omega+2\pi)k} = e^{j\Omega n} \underline{e^{j2\pi k}} = e^{j\Omega k}$$

$$e^{j2\pi k} = \cos(2\pi k) - \underbrace{j\sin(2\pi k)}_{=0} = \cos(2\pi k) = 1$$

Basic Function for Discrete-Time Fourier Series

- Example 3-4

Show harmonics of the exponential $e^{j\frac{2\pi}{3}t}$ if it is being sampled with sampling period of 0.25 seconds.

- We can write the exponential in discrete form by replacing t with $kT_s=k/4$

$$y[k]=e^{j\frac{2\pi}{12}k}$$

- Let's plot the real part of this signals along with its next two harmonics, which are $e^{j\left(\frac{2\pi}{12}+2\pi\right)k}$ and $e^{j\left(\frac{2\pi}{12}+4\pi\right)k}$

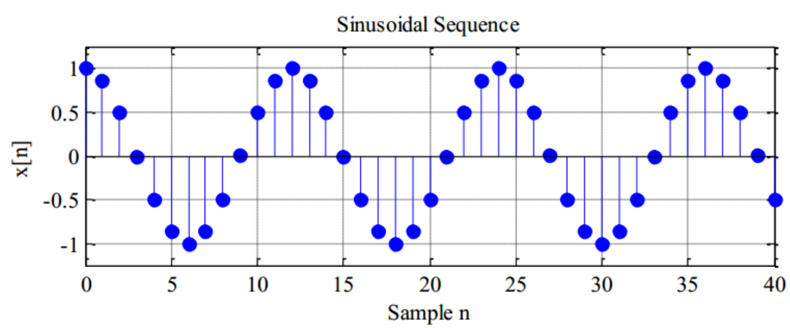


Figure 10 – Three discrete harmonic signals

(Matlab Program 7)

Basic Function for Discrete-Time Fourier Series

- Let's see what happens as digital frequency Ω is varied just within the 0 to 2π range.
  - Take the signal $x[k]=e^{j\frac{2\pi}{6}k}$. Its digital frequency is equal 2π/6 and its period K_0 is equal to 6.
  - We will increase the frequency of this signal in 6 steps.

$$
\begin{aligned}
\phi_0 &= 2\pi(n=0)/6 = 0 \\
\phi_1 &= 2\pi(n=1)/6 = 2\pi/6 \\
\phi_2 &= 2\pi(n=2)/6 = 4\pi/6 \\
&\vdots \\
\phi_5 &= 2\pi(n=5)/6 = 10\pi/6
\end{aligned}
$$

- The variable n steps from 0 to K_0-1. There are N harmonics, and we index them with letter n. Index k remains the index of the sample.

Basic Function for Discrete-Time Fourier Series

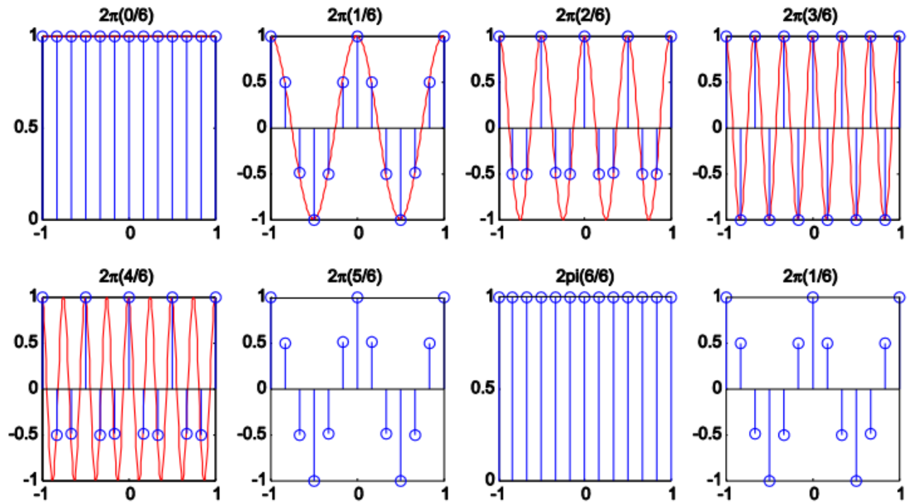


Figure 11 – Discrete signals in between harmonic frequencies

(Matlab Program 8)

- The analog signals are harmonic along the frequency axis whenever $\omega_k = k\omega_0$.
- Discrete signals are harmonic in between these values when specified in terms of phase.
- if we can do the orthogonality test, and we find that they are indeed harmonic to each other.

$$\sum_{n=0}^{k_0-1} \phi_1 \phi_2^* = 0$$

Discrete Time Fourier Series - DTFS

- Key ideas about discrete signals
  - 1. We do not know the underlying analog signal nor its frequency. All we know is that if the sampling frequency is Fs, then we can from a discrete signal unambiguously extract signals of only of frequency half as much.
  - 2. A discrete signal is defined by its digital frequency. The units of digital frequency are in radians per sample. We can think of it as being defined over a circle from 0 to 2π (or $-\pi$ to $+\pi$)
  - 3. A discrete signal of frequency Ω_0 is exactly the same as all its harmonics when $\Omega_0 \pm 2\pi k$ for all k.
  - 4. There are only N distinct discrete–time complex exponential signals that are harmonically related for any given period N. K_0 is the smallest such number and called the fundamental period.

Discrete Time Fourier Series - DTFS

- If $\Omega_0 = \dfrac{2\pi}{5}$, we can write the discrete harmonic complex exponentials as:

$$e^{-j\,\Omega_0 n\,k},$$

$$e^{-j\left(\frac{2\pi}{n}\times 0\right)k}, e^{-j\left(\frac{2\pi}{n}\times 1\right)k}, e^{-j\left(\frac{2\pi}{n}\times 2\right)k}, e^{-j\left(\frac{2\pi}{n}\times 3\right)k}, \cdots, e^{-j\left(\frac{2\pi}{n}\times (K_0-1)\right)k}$$

  - The index n is used to indicate the harmonics. The index k is the time sample.

- The discrete-time representation of the signal is written as the weighted sum of these.

$$x[k] = \sum_{k=0}^{K_0-1} C_n\, e^{j\,(n\,\Omega_0)k}$$

- The complex coefficients, $C_0, C_1, \cdots, C_{(K_0-1)}$ are given as

$$C_n = \frac{1}{K_0}\sum_{n=0}^{K_0-1} x[k]\; e^{-j\,(n\,\Omega_0)k}$$

Discrete Time Fourier Series - DTFS

- we show that the DTFS coefficients of the nth harmonic are exactly the same as the coefficient for a harmonic that is an integer multiple of mK_0 samples away so that: $C_n = C_{(n+mK_0)}$
  - Here m is an integer. The nth coefficient is equal to

$$C_n = \sum_{k=0}^{N-1} x[n] e^{-j\, n\Omega_0\, k}$$

  - The ($n+ mK_0$) coefficient is given by

$$C_{(n+mK_0)} = \sum_{k=0}^{N-1} x[k] e^{-j(n+mK_0)\Omega_0 k} = \sum_{k=0}^{N-1} x[k] e^{-jn\Omega_0 k} e^{-jmK_0\Omega_0 k}$$

$$e^{-jmK_0\Omega_0 k} = e^{-jm2\pi k} = 1$$

  - So we have: $C_{(n+mK_0)} = \sum_{k=0}^{N-1} x[k] e^{-jn\Omega_0 k} = C_n$

Discrete Time Fourier Series - DTFS

- Example 3-5

Find the discrete time Fourier series coefficients of this signal.

$$x[k] = 1 + \sin\left(\frac{2\pi}{10}k\right)$$

- The fundamental period $K_0 = 10$

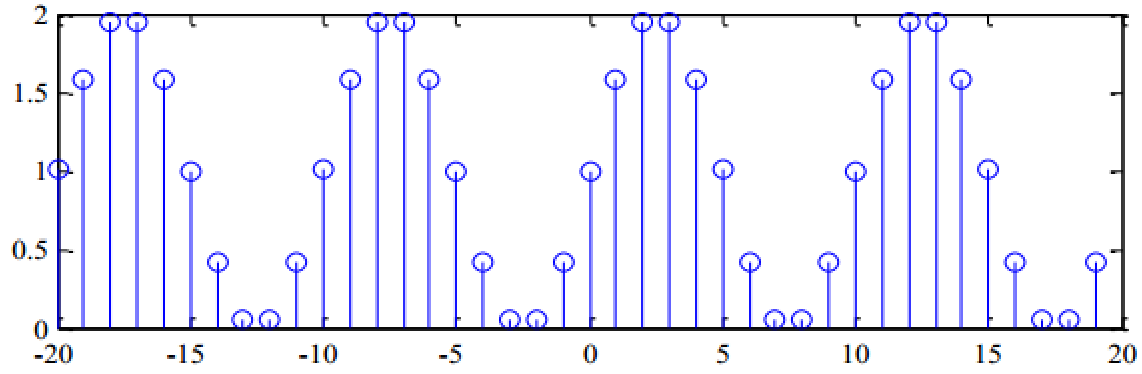


Figure 12 – Signal of example 3-5

- the Euler equivalent expression for this signal

$$x[k] = 1 + \frac{1}{2j}e^{j\left(\frac{2\pi}{10}\right)k} - \frac{1}{2j}e^{-j\left(\frac{2\pi}{10}\right)k}$$

Discrete Time Fourier Series - DTFS

- Example 3-5
  - $C_n = \frac{1}{K_0}\sum_{n=0}^{K_0-1} x[k]\ e^{-j(n\Omega_0)k} \quad = \frac{1}{10}\sum_{n=0}^{9} x[k]\ e^{-j\left(\frac{2\pi}{10}n\right)k}$
  - x[k]= [1.0000, 1.5878, 1.9511, 1.9511, 1.5878, 1.0000, 0.4122, 0.0489, 0.0489, 0.4122]
  - $C_n = C_0 = \frac{1}{10}\sum_{k=0}^{9} x[k]\ \underbrace{e^{-j\ o\Omega_0\ k}}_{1} = \frac{1}{10}\sum_{k=0}^{9} x[k] = 1$
  - $C_1 = \frac{1}{10}\sum_{k=0}^{9} x[k] e^{-j\ 1\cdot\Omega_0\ k} = \frac{1}{2j}$

    $C_{-1} = \frac{1}{10}\sum_{k=0}^{9} x[k] e^{j\ 1\cdot\Omega_0\ k} = -\frac{1}{2j}$
  - The rest of the coefficients from C_2 to C_9 are zero.
  - the coefficients repeat after C_9 so that $C_{(1+9k)} = C_1$.

Discrete Time Fourier Series - DTFS

- Example 3-6

  Compute the DTFSC of this discrete signal.

$$x[k]=0.5+0.25\cos\left(\frac{2\pi}{5}k\right)-0.6\sin\left(\frac{2\pi}{4}k\right)$$

  - The period, K_0 of the cosine is 5 and the period, K_0 of sine is 4. Period of the whole signal is 20 (least common multiple).
  - The fundamental frequency $\Omega_0=\dfrac{2\pi}{20}=\dfrac{\pi}{10}$
  - $C_n=\dfrac{1}{K_0}\displaystyle\sum_{n=0}^{K_0-1}x[k]\,e^{-j(n\Omega_0)k}$

$$=\frac{1}{20}\sum_{n=0}^{19}x[k]\,e^{-j\left(\frac{\pi}{10}n\right)k}$$

$$C_0=C_n=0.5$$

$$C_4=.125 \qquad C_{-4}=.125$$

$$C_5=.3j \qquad C_{-5}=-.3j$$

  - $x[k]=\dfrac{1}{20}\left[x[0]e^{-j(2\pi/5)}\right]$
  - $x[k]=0.5+0.125\left(e^{\left(\frac{2\pi}{5}k\right)}+e^{-\left(\frac{2\pi}{5}k\right)}\right)+0.3j\left(e^{\left(\frac{2\pi}{4}k\right)}-e^{-\left(\frac{2\pi}{4}k\right)}\right)$

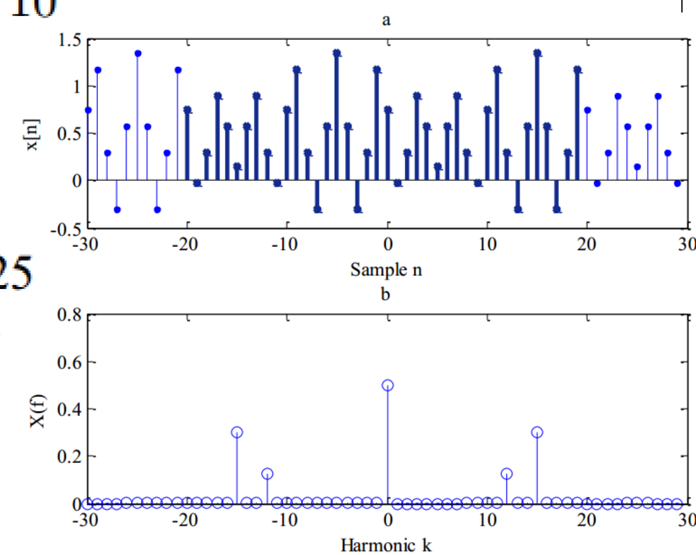


Figure 13- Signal of example 3-6

Discrete Time Fourier Series - DTFS

- Example 3-7

  Compute the DTFS of this periodic discrete signal. The signal repeats with period 4 and has two impulses of amplitude 2 and 1.

  - The fundamental frequency $\Omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

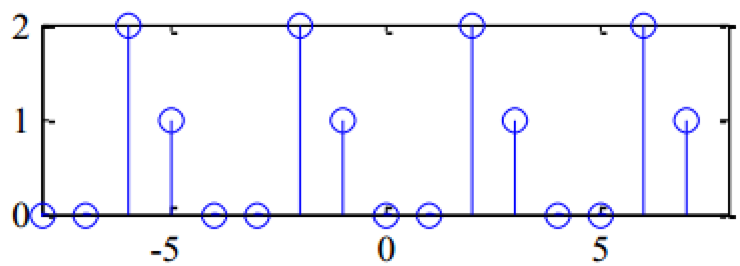


Figure 14 – Signal of example 3-7

  - We write the expression for the DTFSC $C_n = \sum_{k=0}^{3} x[k] e^{-j\left(\frac{\pi}{2}n\right)k}$

$$e^{-j\left(\frac{\pi}{2}\right)nk} = \left(\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} - j \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} \right)^{kn} = (-j)^{nk}$$

Discrete Time Fourier Series - DTFS

- Example 3-7
  - Now substitute this into the DTFSC equation and calculate the coefficients, knowing there are only n = 4 harmonics in the signal because the number of harmonics are equal to the fundamental period of the signal.

$$C_n = \frac{1}{4}\sum_{k=0}^{3} x[k](-j)^{kn}$$

  - $C_0 = \frac{1}{4}\ 2+1\ = \frac{3}{4} \qquad for\ n = 0,\ k = 0,1,2,3$

    $C_1 = \frac{1}{4}(2-j1) = \frac{1}{2} - \frac{j}{4} \qquad for\ n = 1,\ k = 0,1,2,3$

    $C_2 = \frac{1}{4}\ 2-1\ = \frac{1}{4} \qquad for\ n = 2,\ k = 0,1,2,3$

    $C_3 = \frac{1}{4}\ 2+j1\ = \frac{1}{2} + \frac{j}{4} \qquad for\ n = 2,\ k = 0,1,2,3$

Discrete Time Fourier Series - DTFS

- Example 3-7
  - We can setup the DTFSC equation in matrix form by setting the basic exponential to a constant and then writing it in terms of two variables, the index n and k.

$$e^{j\Omega_0} = W$$

$$e^{-jn\Omega_0 k} = W^{-nk}$$

  - $$C_n = \frac{1}{K_0} x[k] \begin{bmatrix} W^{-0\times0} & W^{-1\times0} & W^{-2\times0} & W^{-3\times0} \\ W^{-0\times1} & W^{-1\times1} & W^{-2\times1} & W^{-3\times1} \\ W^{-0\times2} & W^{-1\times2} & W^{-2\times2} & W^{-3\times2} \\ W^{-0\times3} & W^{-1\times3} & W^{-2\times3} & W^{-3\times3} \end{bmatrix}_{n\times k}$$
  - We used Matlab to compute the coefficients.

$$\begin{bmatrix} 2 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} = \begin{bmatrix} .75 & .5-.25j & .25 & .5+.25j \end{bmatrix}$$

Matlab Program 11

Discrete Time Fourier Series - DTFS

- Example 3-8

Find the discrete-time Fourier series coefficients of this signal.

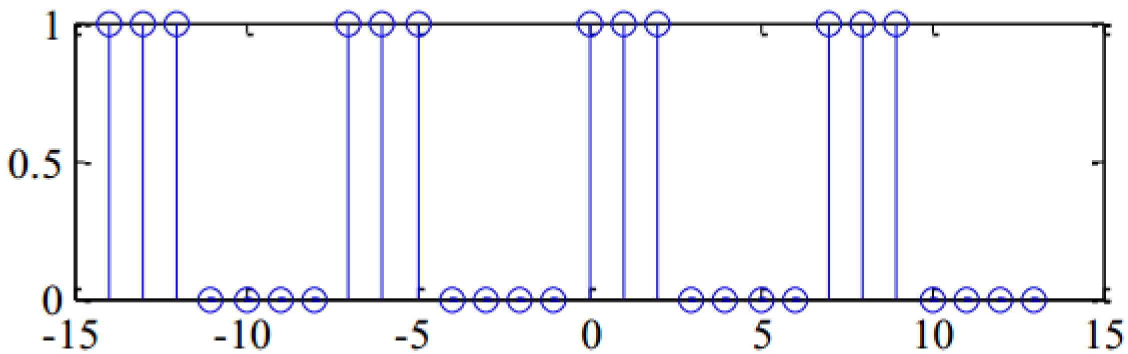


Figure 15 – Signal of example 3-8 with N = 3. K0 = 7

- $$C_n = \frac{1}{K_0}\sum_{k=-N}^{N} 1 \cdot e^{-jn\Omega_0 k} + \frac{1}{K_0}\sum_{k=N+1}^{K_0-N-1} 0 \cdot e^{-jn\Omega_0 k} = \frac{1}{K_0}\left[e^{-jn\Omega_0 N}\frac{1-e^{-jn\Omega_0(2N+1)}}{1-e^{-jn\Omega_0}}\right]$$
- $$C_n = \frac{1}{K_0}\left[\frac{\sin\ Nn\pi / K_0}{\sin\ n\pi / K_0}\right]$$

Discrete Time Fourier Series - DTFS

- Example 3-8

Matlab program 12

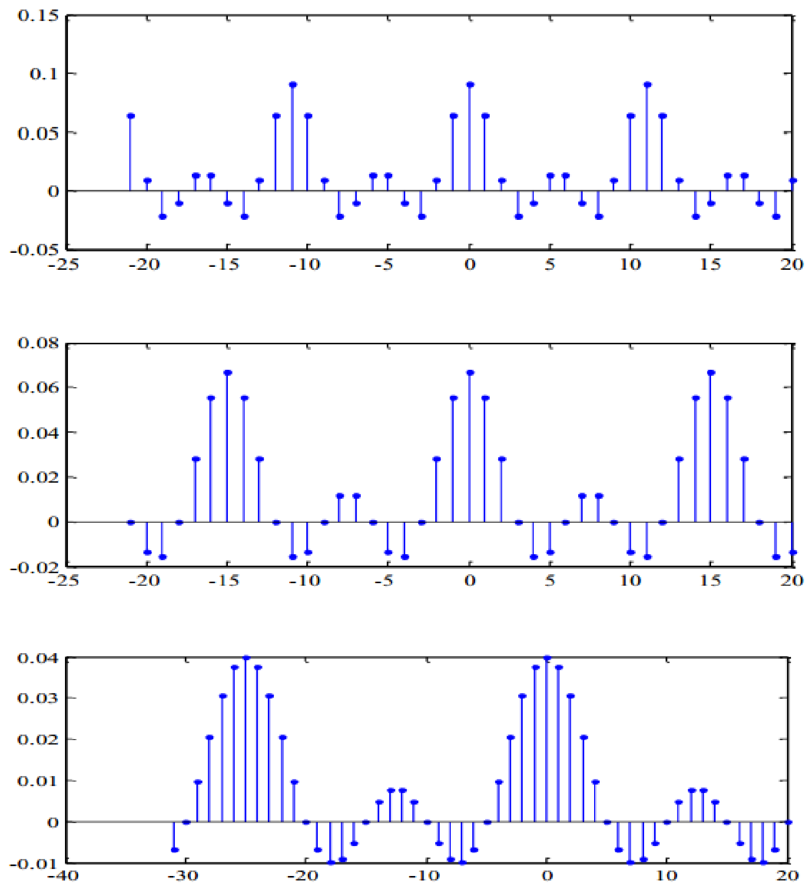


Figure 16 – Coefficients of the periodic pulses, (a) with N= 3, K0 = 7, (b) N =3, K0 = 15, (c) N = 3, K0 = 2

Summary

- A discrete signal can be created by sampling a continuous signal with an impulse train of desired sampling frequency.
- The sampling frequency should be greater the two times the highest frequency in the signal of interest.
- The fundamental period of a discrete signal, given by K_0 must be an integer for the signal to be periodic.
- The fundamental discrete frequency of the signal, given by Ω_0 is equal to $2\pi/K_0$
- The period of a digital frequency is an integer multiple of. Harmonic discrete frequencies vary by integer multiple of 2π , such that Ω and $\Omega+2\pi k$ are harmonic and identical.
- Because discrete harmonic frequencies are identical, we cannot use them to represent a discrete signal.

Summary

- In stead we divide the range from 0 to 2π by K_0 and use these digital frequencies as the basis set.
- Hence there are only $N = K_0$ harmonics available to represent a discrete signal. The Fourier analysis is limited to these N harmonics.
- Beyond the 2π range of harmonic frequencies, the discrete-time Fourier series coefficients, (DTFSC) repeat.
- In contract, the continuous-time signal coefficients are periodic and do not repeat.
- Sometimes we can solve the coefficients using closed from solutions but in a majority of the cases, matrix methods are used to find the coefficients of a signal.
- Matrix method is easy to setup but is computationally intensive.

Matlab Programs 1

```
%Program Chapter 3 - Program 1

f0=1;
Fs = 32;
Ts1 = 1/Fs;
t = 0: Ts1: 2;
clf;
figure(1) % heavy sample
xt=cos(2*pi*t)- .3 + .6*sin(3*pi*t+.5)+.5*cos(4*pi*t)-
.3*cos(5*pi*t+.25);
ylabel('x[n]');
xlabel('Sample');
hold on
plot(t/Ts1,xt,'-. r' );
n = 0: 2*Fs;
xn1=cos(2*pi*n*Ts1)- .3 + .6*sin(3*pi*n*Ts1+.5)+.5*cos(4*pi*n*Ts1)-
.3*cos(5*pi*n*Ts1+.25);
stem(n, xn1);
hold off

figure(2) % light sample
plot(t/Ts1,xt,'-. r' );
hold on
n = 0: 2*4;
Ts1 = 1/4;
xn1=cos(2*pi*n*Ts1)- .3 + .6*sin(3*pi*n*Ts1+.5)+.5*cos(4*pi*n*Ts1)-
.3*cos(5*pi*n*Ts1+.25);
stem(n*8, xn1);
ylabel('x(t)');
xlabel('Sample');
hold off
```

Matlab Programs 2

```
%Chapter 3 - Program 2

t = 0: .01: 6;
x = .25*sin(2*pi*1*t)+.7*cos(2*pi*2*t)-
.5*cos(2*pi*3*t)+.15*sin(2*pi*4*t);
clf;
figure(1);
plot(t, x)
title('(a)')
ylabel('x(t)')
xlabel('Time, t')

figure(2);
n = 0: 47;
fs1 = 8;
xn8 = .25*sin(2*pi*1*n/fs1)+.7*cos(2*pi*2*n/fs1)-
.5*cos(2*pi*3*n/fs1)+.15*sin(2*pi*4*n/fs1);
plot(t*8,x, '--r')
ylabel('x[n]')
xlabel('Sample, n')
title('(b)')
hold on
stem(n, xn8, '.')
axis([ 0 48 -2 2]);
hold off

figure(3)
n2 = 0: fs2*6-1;
fs2 = 12;
xn12 = .25*sin(2*pi*1*n2/fs2)+.7*cos(2*pi*2*n2/fs2)-
.5*cos(2*pi*3*n2/fs2)+.15*sin(2*pi*4*n2/fs2);
plot(t*12, x , '--r')
ylabel('x[n]')
xlabel('Sample, n')
title('(c)')
hold on
stem(n2, xn12, '.')
axis([ 0 48 -2 2]);
hold off

figure(4);
n = 0: 47;
clf;
xnd = (1/48)*fft(xn8);
xnd2 = abs(fftshift(xnd));
plot(n, xnd2)
```

Matlab Programs 3

```
%Chapter 3 - Program 3

t = -.5: .01: .5;
y1 = cos(4*pi*t);
clf;

subplot(3,1,1)
plot(t, y1)
grid;
title('(a) ');
xlabel('Time, t seconds');
ylabel('x(t)');
axis;

subplot(3,1,2)
n = -5: 5;
y2 = cos(4*pi*n*.2);
stem(n, y2)

grid;
title('(b) ');
xlabel('Sample, n');
ylabel('x[n]');

subplot(3,1,3)
%n = -5: 5;
n2 = -2*pi: 2*pi/5: 2*pi;
y2 = cos(2*n2);
stem(n2, y2)
axis([-2*pi 2*pi -1 1]);
grid;
title('(c) ');

xlabel('Radians');
ylabel('x[n]');
axis([-2*pi 2*pi -1. 1.])
% Define x-ticks and their labels..
set(gca,'xTick',-2*pi: pi/5: 2*pi)
set(gca,'xTickLabel',{'-2pi', '', '-8pi/5', '', '', '', '-4pi/5', '',
'', '', '0', '', '', '', '4pi/5', '', '', '', '8pi/5' , '', '2pi'})
```

Matlab Programs 4

```
%Chapter 3 - Program 4

f0=2;
Fs = 6;
t = 0: .001: 1;
n = 0: Fs*t;
n2 = 0: Fs

xt1=cos(2*f0*pi*t);
y = cos(2*f0*pi*n2/Fs)
xt2= cos(2*5*pi*f0*t);
figure(1)

plot(t*Fs, xt1, t*Fs, xt2, 'r')
hold on
stem(n2, y, 'filled')
```

Matlab Programs 5

```
%Chapter 3 - Program 5

f0=1;
Fs = 3;
t = 0: .001: 4;
n = 0: Fs*t;
n2 = 0: Fs*4

xt1=cos(2*f0*pi*t);
y = cos(2*f0*pi*n2/Fs)
figure(1)
grid;
plot(t*Fs, xt1, 'r')
xlabel('Sample, n')
ylabel('x[n]')
hold on

stem(n2, y, 'filled')
```

Matlab Programs 6

```
%Chapter 3 - Program 6


f0=.5/pi;
Fs = 1;
t = 0: .001: 30;
n2 = 0: Fs*30
xt1=cos(2*f0*pi*t);
y = cos(2*f0*pi*n2/Fs)
 figure(1)
 grid
plot(t*Fs, xt1, 'r')
xlabel('Sample, n')
ylabel('x[n]')
hold on

stem(n2, y, 'filled')
```

Matlab Programs 7

```
%Chapter 3 - Program 7
n = 0:40;
w = 2*pi/12;
phase = 0;
A = 1.0;
HShift = 2; %change this (even numbers only) to see effect of shift
x = A*cos((w+(HShift*pi))*n - phase);
clf;
stem(n,x, 'filled');    % Plot the generated sequence
axis([0 40 -1.25    1.25]);
grid;
title('Sinusoidal Sequence');
xlabel('Sample n');
ylabel('x[n]');
axis;
```

Matlab Programs 8

```
%Chapter 3 - Program 8

n=-12:12;
 N=9;
 w0=2*pi/N;

axis([-12.5 12.5 -1.1 1.1]);

k=0;
Phi0n=exp(j*w0*k*n);
subplot(3,4,1);
stem(n,real(Phi0n),'Marker','.');xlabel('1')
t= -10: 1/18: 10;
plot(t, cos(w0*k*t))
axis([-12.5 12.5 -1.1 1.1]);

k=1;
hold on
Phi1n=exp(j*w0*k*n);
subplot(3,4,2);stem(n,real(Phi1n),'Marker','.');xlabel('2')
t= -10: 1/18: 10;
plot(t, cos(w0*k*t))
axis([-12.5 12.5 -1.1 1.1]);
hold off
k=2;
Phi2n=exp(j*w0*k*n);
subplot(3,4,3);stem(n,real(Phi2n),'Marker','.');xlabel('3')
axis([-12.5 12.5 -1.1 1.1]);
k=3;
Phi3n=exp(j*w0*k*n);
subplot(3,4,4);stem(n,real(Phi3n),'Marker','.');xlabel('4')
axis([-12.5 12.5 -1.1 1.1]);
k=4;
Phi4n=exp(j*w0*k*n);
subplot(3,4,5);stem(n,real(Phi4n),'Marker','.');xlabel('5')
axis([-12.5 12.5 -1.1 1.1]);
k=5;
Phi5n=exp(j*w0*k*n);
subplot(3,4,6);stem(n,real(Phi5n),'Marker','.');xlabel('6')
axis([-12.5 12.5 -1.1 1.1]);
k=6;
Phi6n=exp(j*w0*k*n);
subplot(3,4,7);stem(n,real(Phi6n),'Marker','.');xlabel('7')
axis([-12.5 12.5 -1.1 1.1]);
k=7;
Phi7n=exp(j*w0*k*n);
subplot(3,4,8);stem(n,real(Phi7n),'Marker','.');xlabel('8')
axis([-12.5 12.5 -1.1 1.1]);
k=8;
Phi8n=exp(j*w0*k*n);
subplot(3,4,9);stem(n,real(Phi8n),'Marker','.');xlabel('9')
axis([-12.5 12.5 -1.1 1.1]);
k=9;
Phi9n=exp(j*w0*k*n);
subplot(3,4,10);stem(n,real(Phi9n),'Marker','.');xlabel('10')
axis([-12.5 12.5 -1.1 1.1]);
k=10;
Phi10n=exp(j*w0*k*n);
subplot(3,4,11);stem(n,real(Phi10n),'Marker','.');xlabel('11')
axis([-12.5 12.5 -1.1 1.1]);
k=11;
Phi11n=exp(j*w0*k*n);
subplot(3,4,12);stem(n,real(Phi11n),'Marker','.');xlabel('12')
axis([-12.5 12.5 -1.1 1.1]);
```

Matlab Programs 9

```
% Chapter 3 - Program 9
nmin = -10;
nmax = 9;
ND = abs(nmin)+nmax+1;

n = nmin: nmax;
x1 = 0.5 +  0.25*cos(2*pi*n/5) -  0.6*sin(2*pi*n/4);
clf
subplot(2,1,1)
stem(n,x1, '.');
pt = sum(x1.^2)*1/20
x1
title('a')
ylabel('x[n]')
xlabel('Sample n')

xnd = (1/ND)*dft(x1, ND);
subplot(2,1,2)
pt = sum(x1.^2)*1/20
x1
title('a')
ylabel('x[n]')
xlabel('Sample n')

xnd = (1/ND)*dft(x1, ND);
subplot(2,1,2)
```

Matlab Programs 10

```
%Chapter 3 - Program 10
%Figure 12
a0 = [ 0 0 0 0]
d = [ .2 .7 1.1 .9 .5]
xom = [ a0 a0 a0 a0 a0 a0 a0 d a0 a0  a0  a0 a0 a0  a0  ]

n = 0: length(xom)-1;
N = 256;
figure(1)
stem(n, xom)
title('(d)')
figure(2)
X = fft(xom, N);
plot(abs(fftshift(X)))
w = 6*pi * (0:(N-1)) / N;
w2 = fftshift(w);
plot(w2)
w3 = unwrap(w2 - 2*pi);
plot(w3)
plot(w3, abs(fftshift(X)))
xlabel('radians')
plot(w3/pi, abs(fftshift(X)))

xlabel('radians / \pi')
```

Matlab Programs 11

```
%Chapter 3, Problem 11
om = 2*pi/4;
W = exp(-j*om)
for n = 1:4
    for k = 1: 4
       m(n,k) = W^((n-1)*(k-1))
    end

end
m
x = [ 2 1 0 0];
(1/4)*x*m
```

Matlab Programs 12

```
% Chapter 3 - Program 12
N = 5;
K0 = 11;
n = -21:20;
coff = (1/K0)*diric(n*2*pi/K0, N);
stem(n, coff, '.')
```