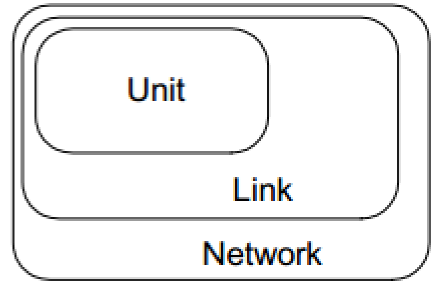
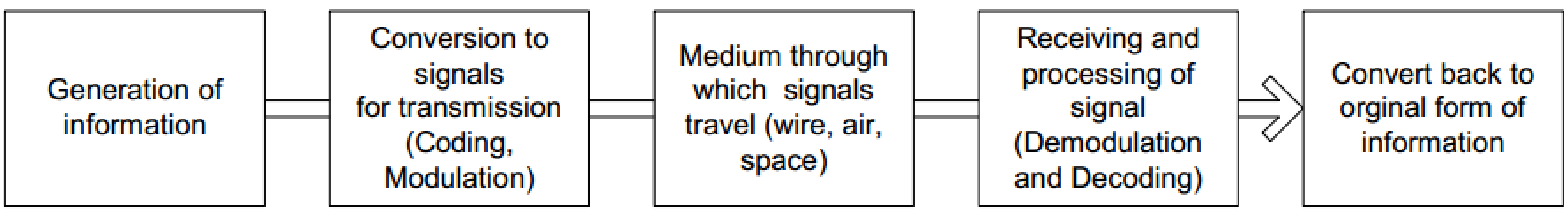
Tutorial 1
Basic concepts in signal analysis, power, energy and spectrum

2013/12

Communications



- Unit
  - Performance issues of a component such as a filter, an amplifier, an antenna.
- Link
  - Waveform or signal issues.



- Network
  - Issues of network design, congestion, and access control.

Signal Types

- Analog Signals
  - Nature
  - Continuous
- Discrete Signals
  - any signal that has values only at specific time interval
  - Morse code messages : naturally occurring
- Digital Signals
  - The signal that only takes on a specific set of values.
  - Special case: binary signals

Signal Types

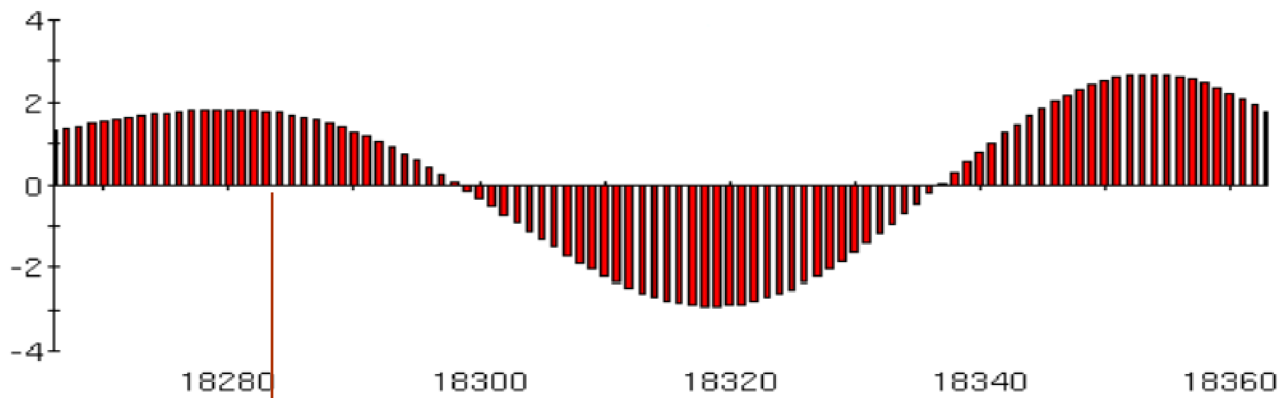


Fig. 1.3 – A discrete signal with varying amplitude at each time instant (sample number).

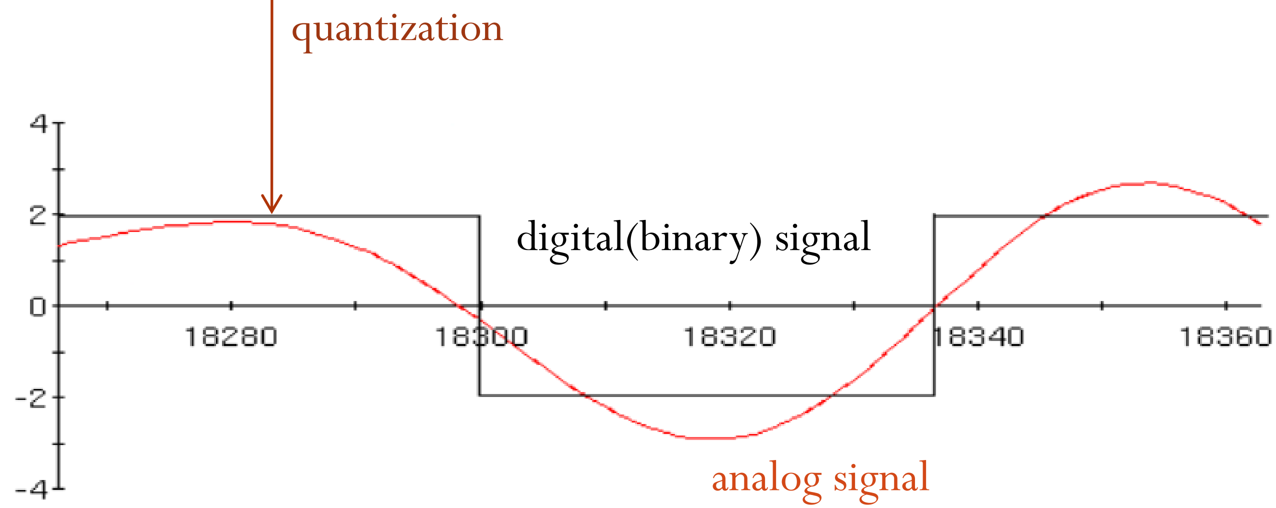


Fig. 1.4 – An analog signal with an equivalent digital (binary) signal (sample number).

Signal Types

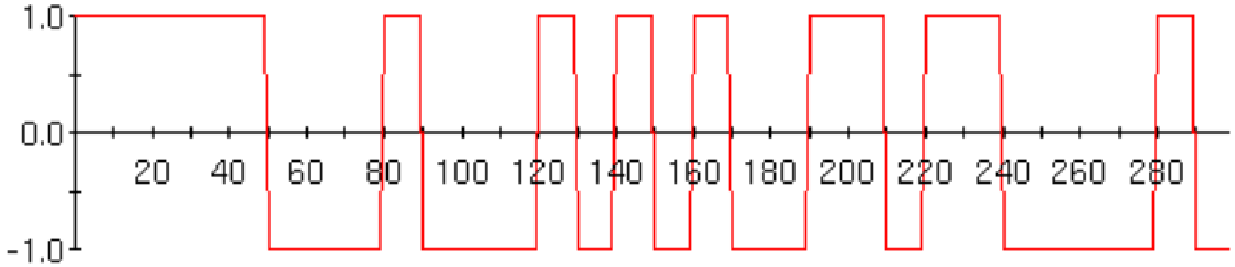


Fig. 1.6 – A binary signal

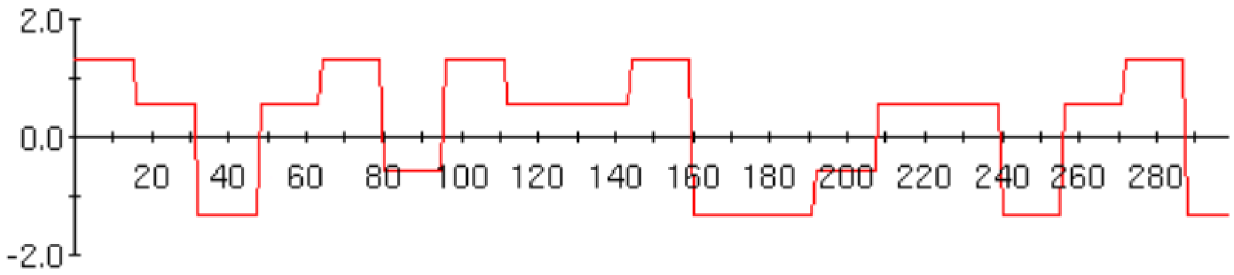


Fig. 1.7 – A 3-level signal

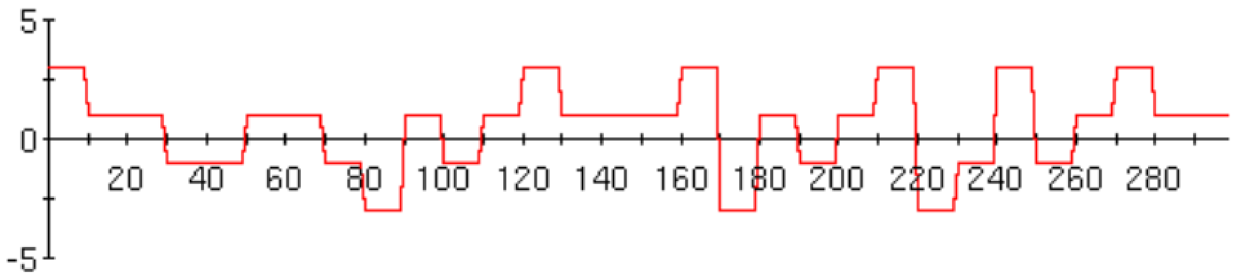
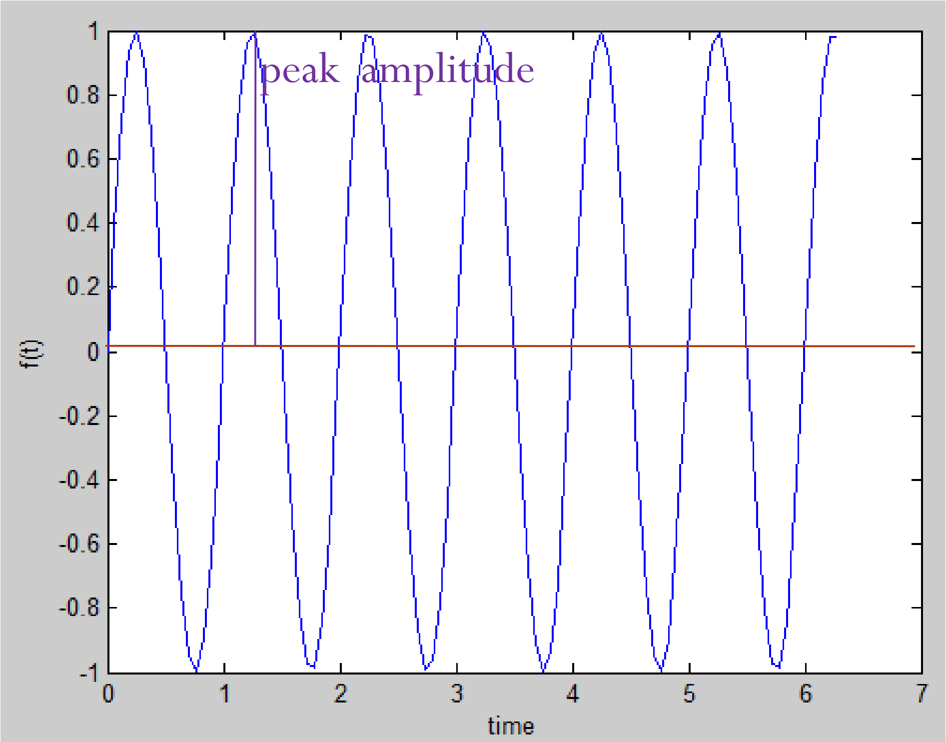


Fig. 1.8 – A 4-level signal

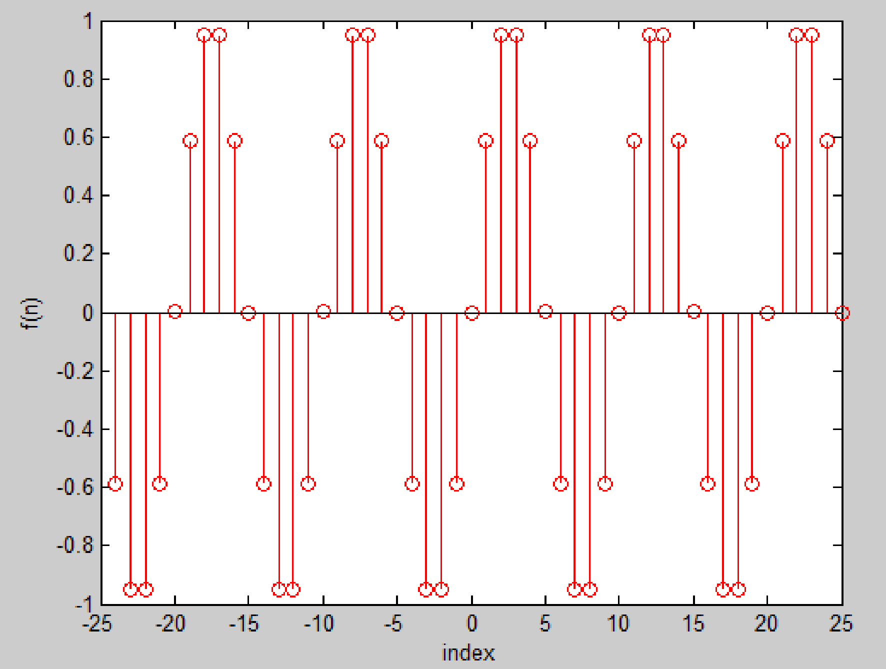
Analog signal

- Sine wave (or sinusoid)
  - $f(t)=A*\sin(2\pi ft+\theta)$
    - A: peak amplitude, t: time, θ: Phase, f:frequency



Analog signal

- discrete version of sine wave
  - $f(n)=\sin(2\pi nT)$, $n=0,\pm 1, \pm 2,\ldots$
    - **n:** the nth time tick, **period T:** the intervel between ticks
  - Sampling Frequency = fs = 1/Ts (Hz)
  - ex. Ts=0.1 sec, an= $f(n)=\sin(2\pi nT)=\sin(\pi n/5)$



Information Signal

- Analog – voice and music
- Digital – stock market and financial data
- A/D converter
  - sample into discrete signals
  - quantization
- Low frequencies – referred to Baseband signals <50 kHz

Carrier Signals

- Carriers
  - the higher frequency signals that facilitate transfer of information over a variety of media.
  - A pure sinusoid of a particular frequency
    - analog
    - continuous
- Why use a carrier to transmit the information signal?
  - a signal as it travels degrades in power
    - a high frequency carrier allows us to use smaller antennas
  - Some media are not friendly to all frequencies
- Modulation
  - the process of "carrying the information"
- Requirement
  - a carrier's frequency must be at least 2 times the highest frequency in the information signal

Carrier Signals

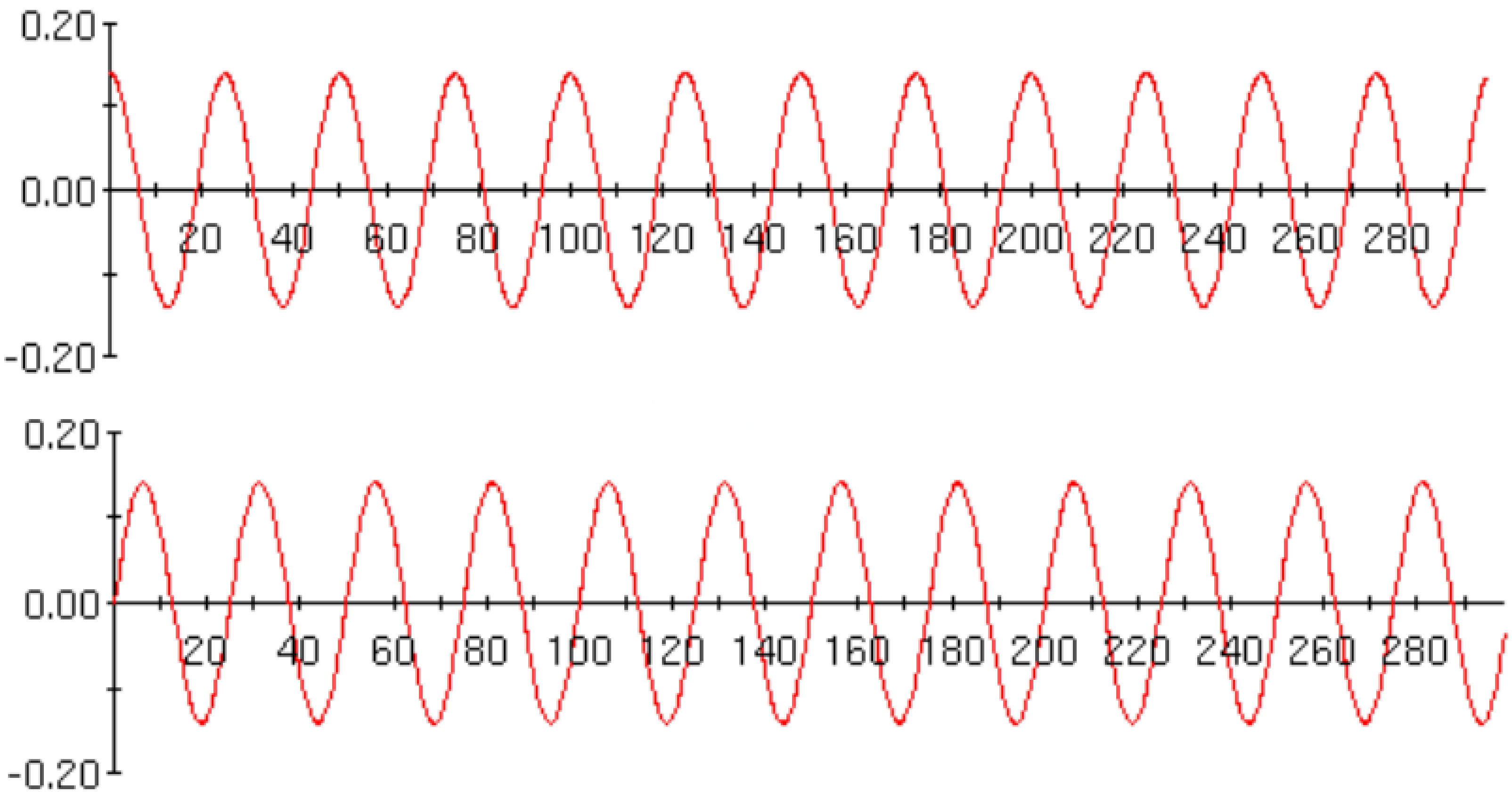


Figure 1.9 – A carrier is a pure sinusoid of a particular frequency.

Modulated Signals

- A carrier that has been loaded with an information signal
- D/A conversion
  - Done at baseband
  - No change in the frequency of the signal

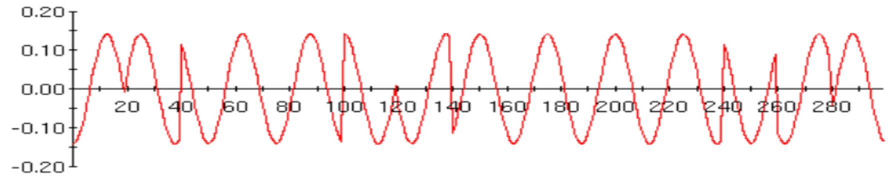


Fig. 1.10a – A modulated carrier signal (digital input)

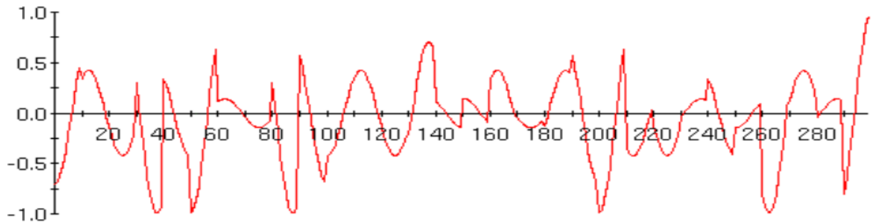


Fig. 1.10b – Another modulated carrier signal (digital input)

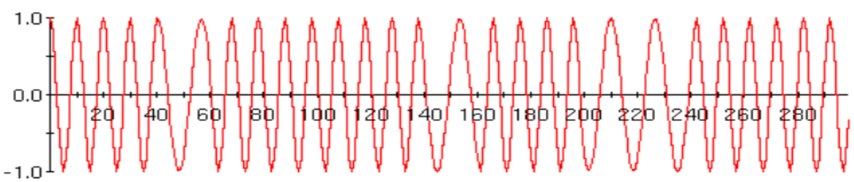
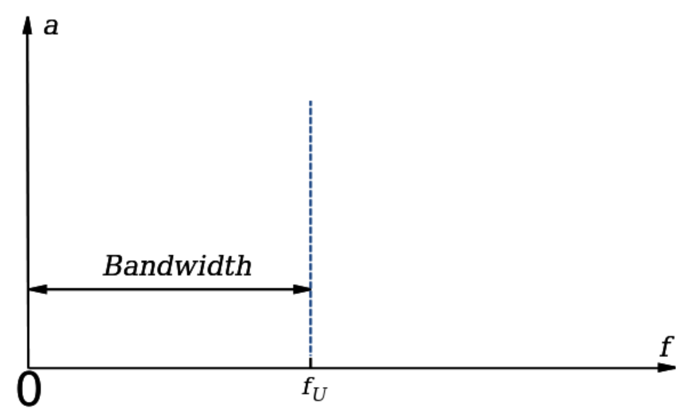


Fig. 1.10c – Yet another modulated carrier signal (analog input)

Bandwidth

- Frequency width, the fatness of the signal
- the difference between the upper and lower frequencies in a continuous set of frequencies
- The bandwidth of a carrier signal is zero
- The bandwidth of the informational signal ∝ the information in a signal
- The bandwidth of the modulated signal = the bandwidth of the information signal



Properties of Signals

- Periodicity

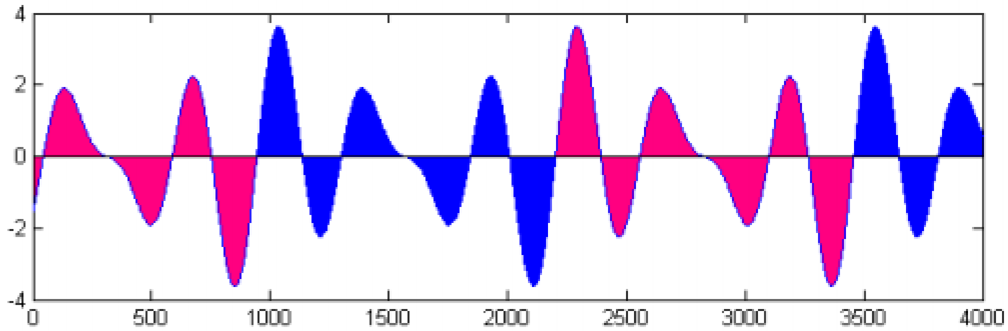


Fig. 1.12 – Carriers are periodic, information signals are not.

- Discrete periodic signal
  - f(t) = f(t±T), T: discrete period time
  - The result of the adding periodic signals is still periodic.
  - Mean : the average value of the signal's samples Xn

$$\mu_x = \frac{1}{N}\sum_{n=0}^{N-1} x_n$$

Power and Energy of Signals

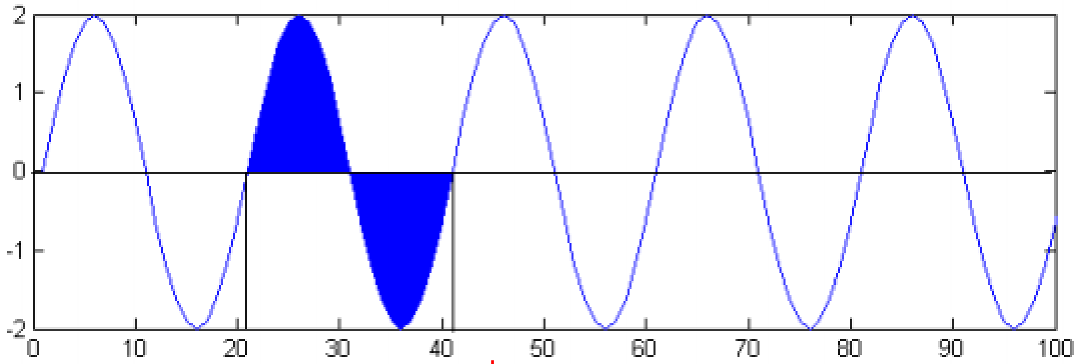
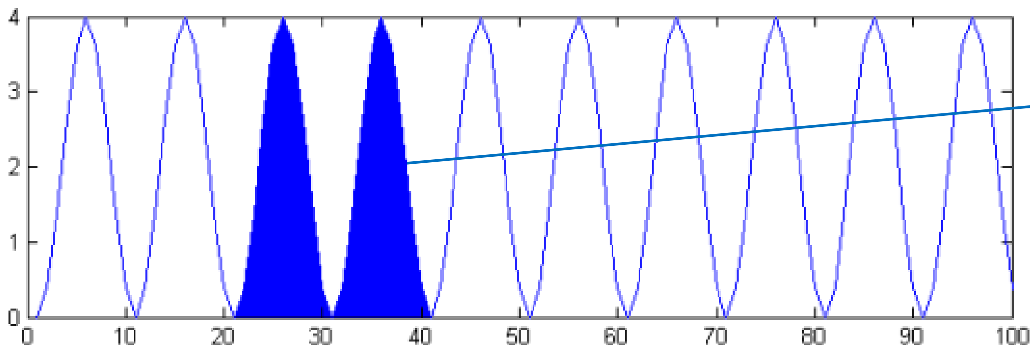


Fig. 1.13 – The area under one period of this signal is zero.

square



The area for one period

= 2*(4*10/2)

= 40 units

Fig. 1.14 – The area under the squared period is non-zero and indicates the power of the signal.

Power and Energy of Signals

- Signal energy
  - the area under the squared signal $E_x = \sum_{n=0}^{N-1} |x_n|^2$
- (average) Signal power
  - the area under the squared signal divided by the number of periods

$$P_x = \frac{E_x}{N} = \frac{1}{N} \sum_{n=0}^{N-1} |x_n|^2$$

- Root mean square
  - The square root of its average power

$$x_{RMS} = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 \dots}{n}} = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} |x_n|^2}$$

  - A measure of the amplitude (or voltage)

Power and Energy of Signals

- Variance
  - The power of the signal with its mean removed

$$\sigma_x^2 = \frac{1}{N}\sum_{n=0}^{N-1}\left|x_n - \mu_x\right|^2 = P_x - \mu_x^2$$

  - For zero-mean signals, $\sigma_x^2 = P_x$
- Instantaneous power $x^2(t)$
  - the instantaneous amplitude squared
- Bit energy (energy per bit)
  - the squared amplitude divided by the bit rate Rb

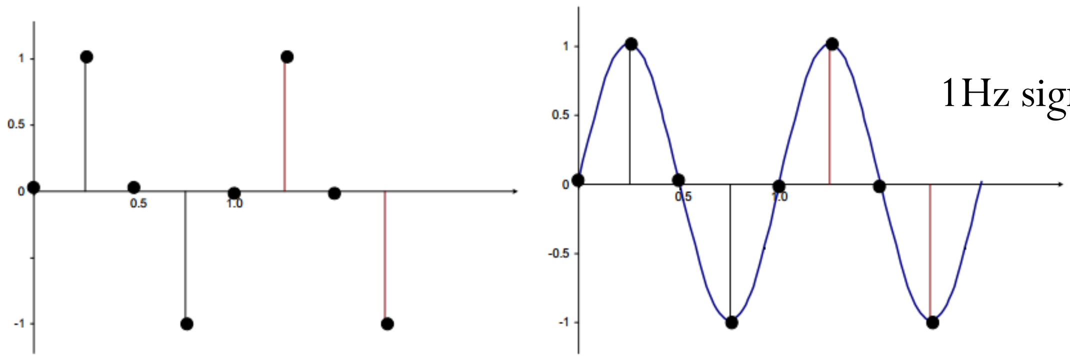
$$E_b = \frac{Avg\left[x^2(t)\right]}{R_b} = C/Rb,\ C = \text{signal power}$$

Random Signals

- The power of the random signal $P_x = E\left[x^2(t)\right]$
  - For zero-mean signals, $P_x = E\left[x^2(t)\right] = Variance = R_x(0)$
    - $R_x(0)$: zero shift
      - a signal multiply by itself = square each instantaneous amplitude
- Stationary signal
  - For a signal x(t), E{x(t)} does not change over time
  - Non-stationary signal: Doppler signal
- Wide-sense stationary(WSS) signal
  - only requires that 1st moment and co-variance do not vary with respect to time.
- Ensemble: One piece of a particular signal
- Ergodic signal
  - the average value of an ensemble = the average of the whole signal
- Fourier transform-based signal processing is only vaild for signals that are both stationary and ergodic

Sampling

- Challenge
  - Figure out what signal was actually sent



1Hz signal

Fig. 1.15 Guessing what was sent based on sampled values.
(a) Received samples, (b) Our assumption of the signal that was sent.

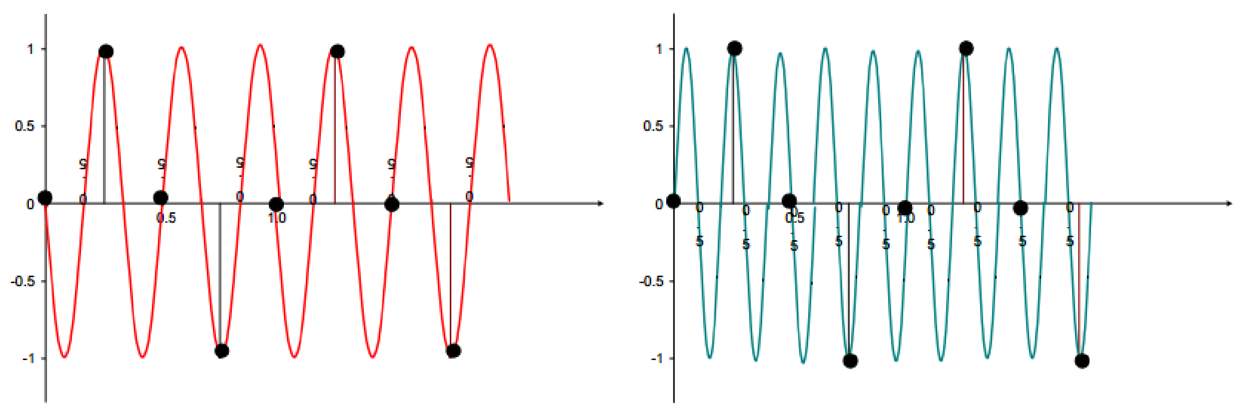


Fig. 1.16 Guessing what was sent based on sampled values.
(a) 2 Hz signal also fits data, (b) 4 Hz signal does too.

Sampling

- Nyquist frequency $f_{l\arg est} = \dfrac{1}{2T_s} = \dfrac{f_s}{2}$
  - The sampling frequency> 2*Nyquist frequency
    - If no, we can't find the signal
  - Ex. f(t) = sin(2π(4)t)+cos (2π(6)t)

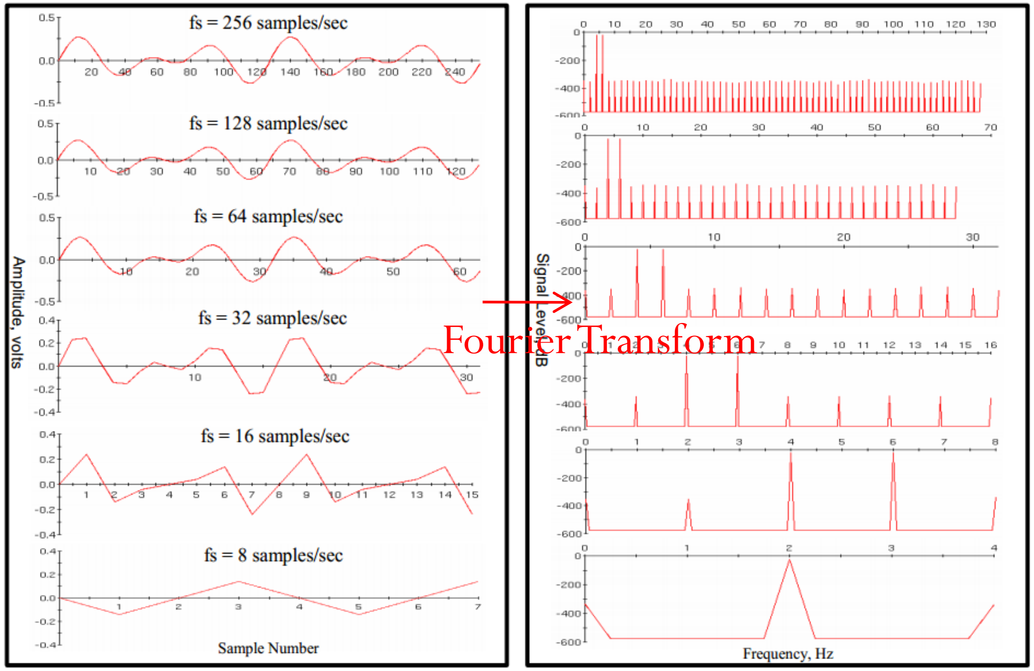


Fig. 1.17 – Sampling speed has to be fast enough (> 2 times the highest frequency embedded) for the processing to detect it and reconstruct the information signal.

Random Signal Distributions

- Probability Density Function (PDF)
  - The function for the probability that the random variable takes on a value in that interval
  - Given a random signal x(t)
    - Mean $\overline{x} = E[x] = \int_{-\infty}^{\infty} x \ p(x) \, dx$
    - Variance $\sigma^2 = E[(x-\overline{x})^2] = \overline{(x^2)} - \overline{x}^2$
      - The measure of how much the voltage varies over time

Random Signal Distributions

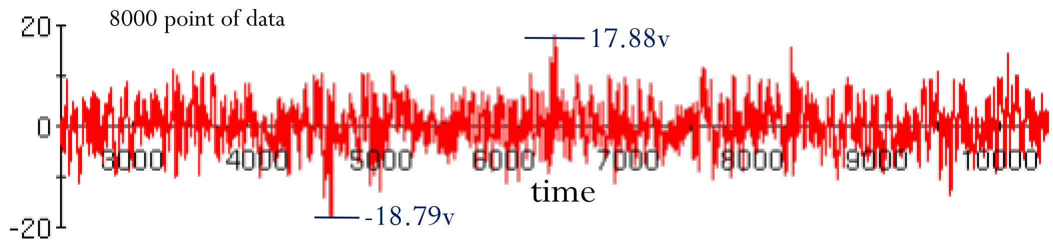


Fig. 1.18 – A data signal that has picked up noise.

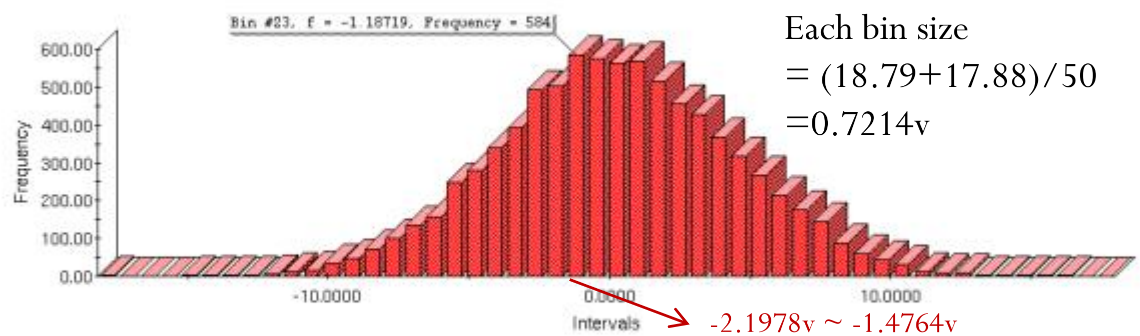


Fig. 1.19 – Histogram developed from the signal amplitude values.

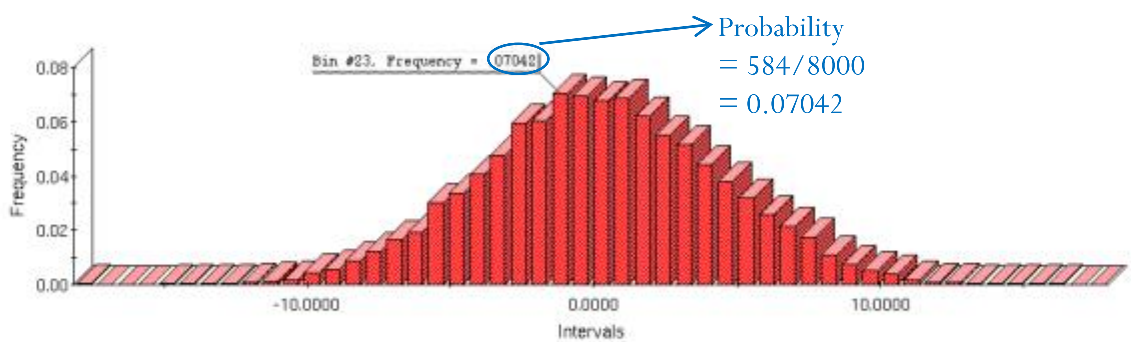


Fig. 1.20 – Normalized histogram approaches a Probability Density Function

Random Signal Distributions

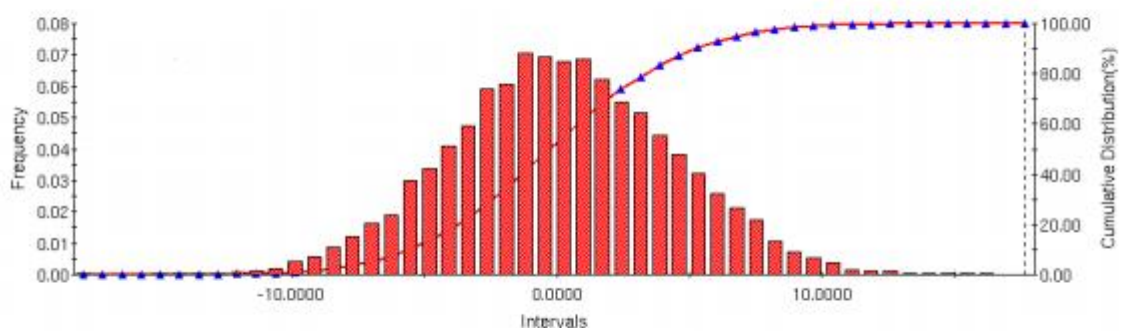


Fig. 1.21 –Probability Densit $F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$ **ability Function**

- $P(x_1 < x < x_2) = \int_{x_1}^{x_2} p(x)\,dx \xrightarrow{\text{normalize}} P(-\infty < x < \infty) = \int_{-\infty}^{\infty} p(x)\,dx = 1$
- Cumulative Probability Function (CDF)
  - $F_X(b) = \int_{-\infty}^{b} f_X(t)\,dt$

Random Signal Distributions

- Power Spectral Density (PSD)
  - PSD describes how the power of a signal or time series is distributed with frequency

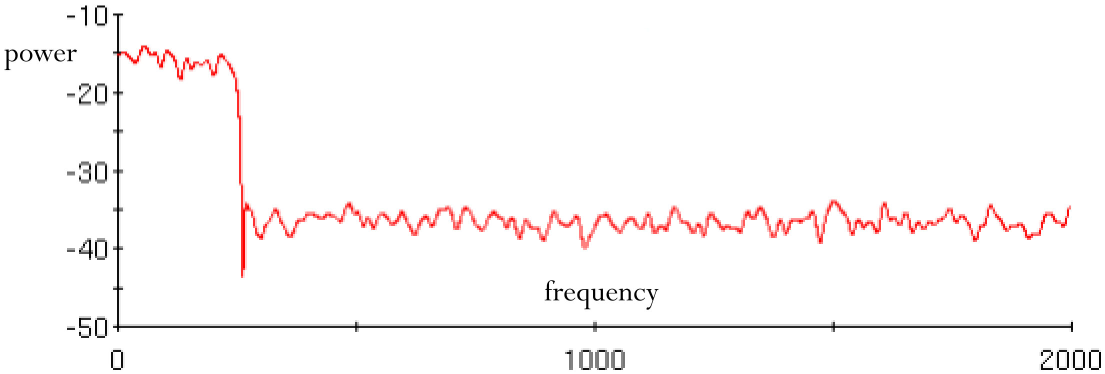


Fig. 1.22 –Power Spectral Density (PSD) of signal in (1.20)

  - PSD of a signal is same as its power spectrum

Random Signal Distributions

- Uniform Distribution

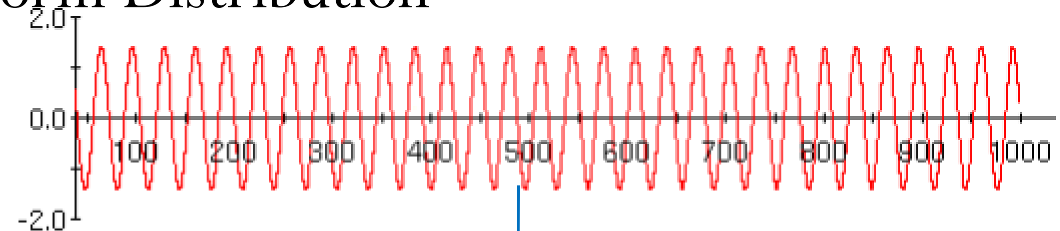


Fig. 1.24 – A sinusoid.

+ -10dBc uniformly distributed noise

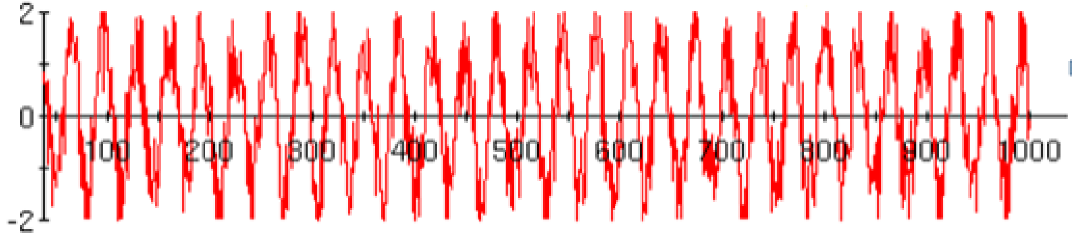
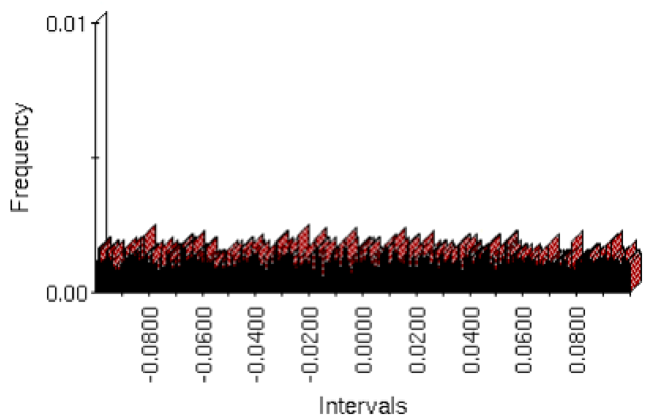


Fig. 1.25 – Transmitted signal which has picked up uniformly distributed noise distributed between -0.1 and +0.1 v.

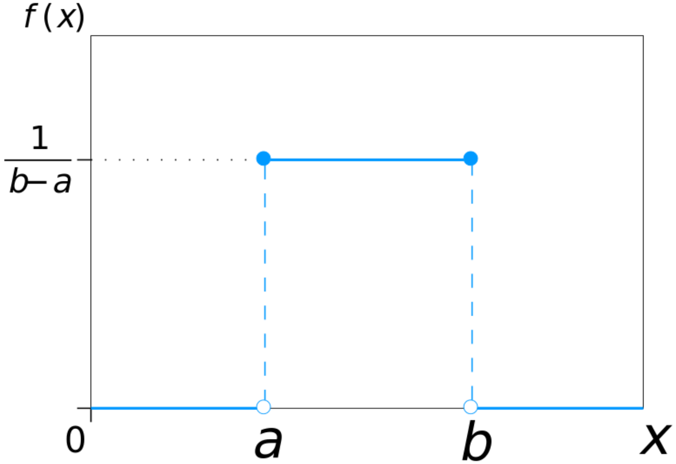


1.26 – Uniformly distributed noise distributed between -0.1 and +0.1

Random Signal Distributions

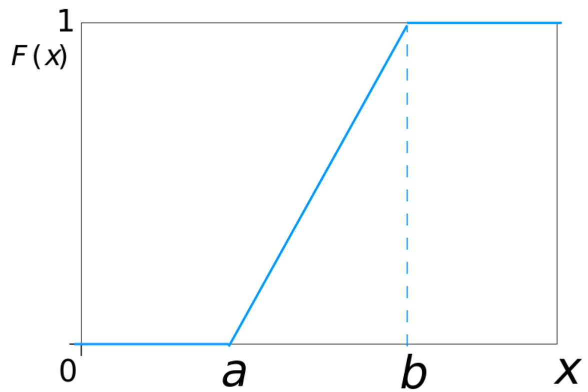
- Properties of uniformly distributed noise
  - $Mean = \dfrac{a+b}{2}$
  - $Variance = \dfrac{(b-a)^2}{12}$
  - PDF

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & x \ge b \end{cases}$$



  - CDF

$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x \ge b \end{cases}$$



Random Signal Distributions

- Normal Distribution (Gaussian Distribution)

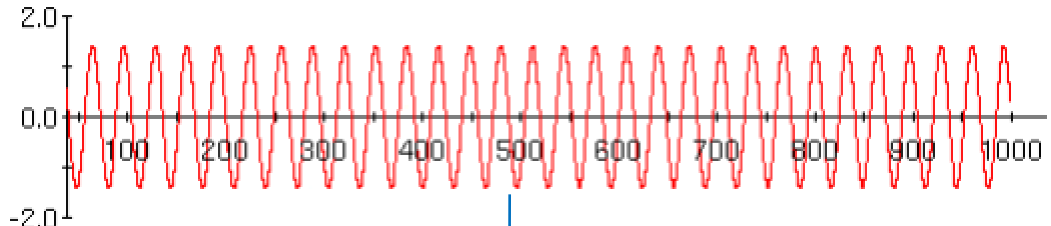
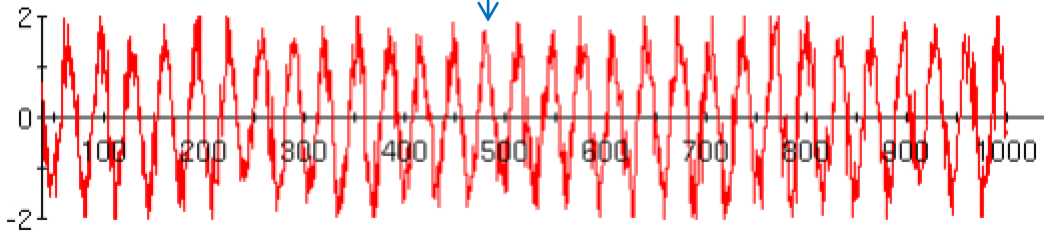
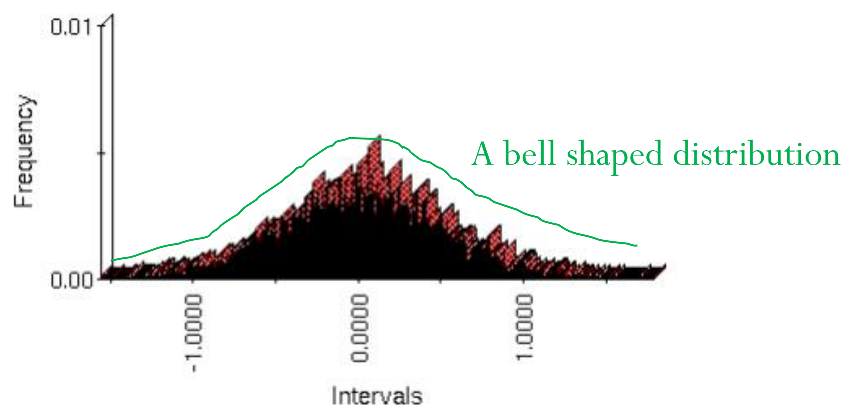


Fig. 1.24 – A sinusoid.

+ A normally distributed noise



1.27 – Transmitted signal which has picked up normally distributed noise of variance 0.1.

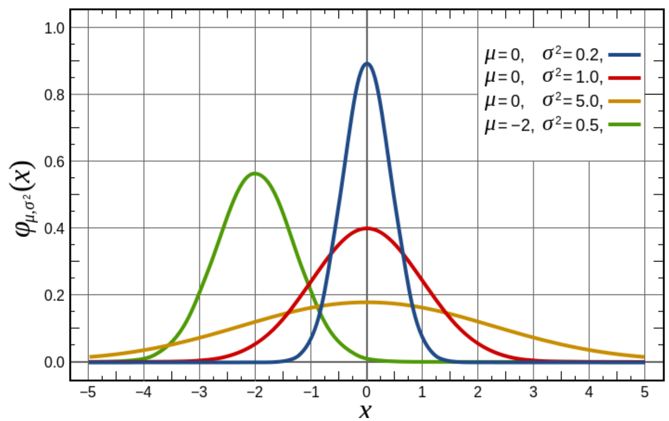


1.28 – The bell shaped probability distribution of a normally distributed random variable.

Random Signal Distributions

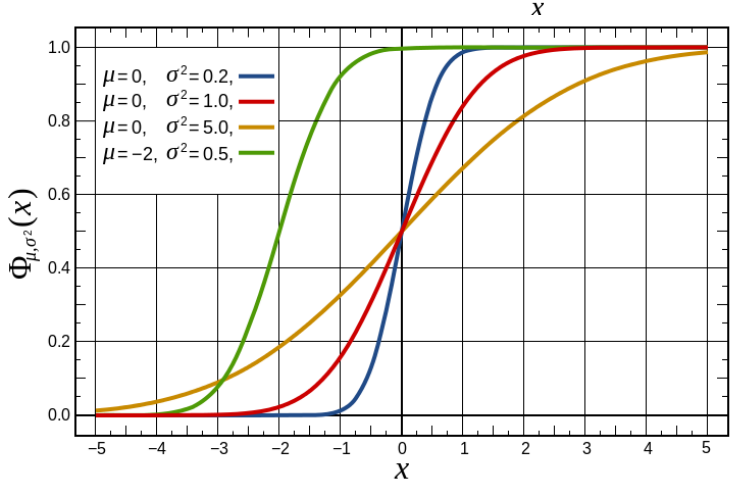
- Properties of normally distributed noise
  - Mean μ
  - Variance σ^2
  - PDF

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



  - CDF

$$\frac{1}{2}\left(1 + erf\,\frac{x-\mu}{\sigma\sqrt{2}}\right)$$



Transforms in Signal Processing

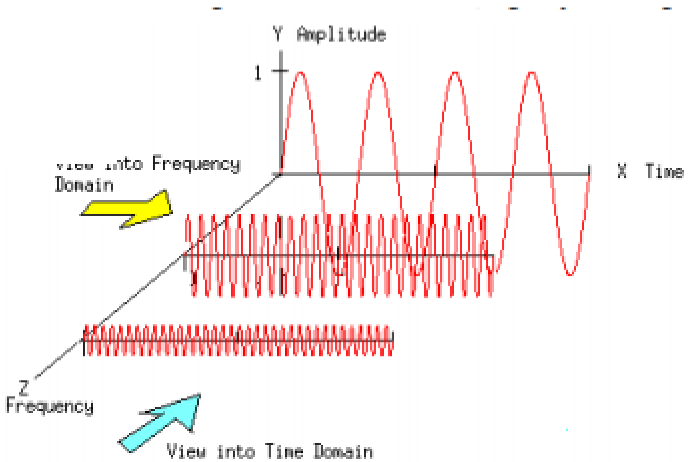


Fig. 1.29 – Frequency and time domain of a signal

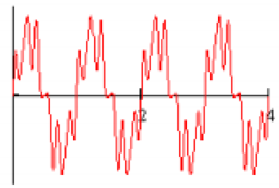


Fig. 1.30a – A periodic signal composed of three sinusoids.

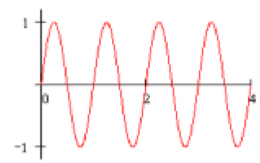


Figure 1.30b - Sine wave 1

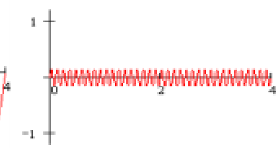


Figure 1.30c - Sine wave 2

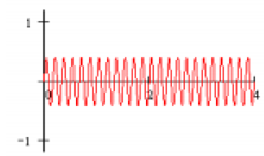


Figure 1.30d - Sine wave 3

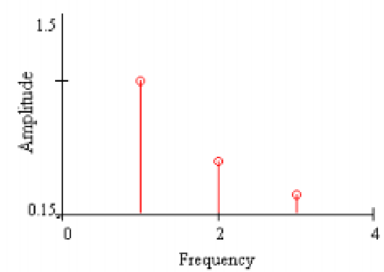


Fig. 1.31 – View into frequency domain a signal consisting of three sine waves of frequencies 1, 2 and 3 Hz.

- Fourier Transform (FT)
  - The sum of various sinusoid
  - De-constructor
- Discrete Fourier Transform (DFT)
  - operates on discrete samples

Transforms in Signal Processing

- Leplace Transform
  - A special case of FT

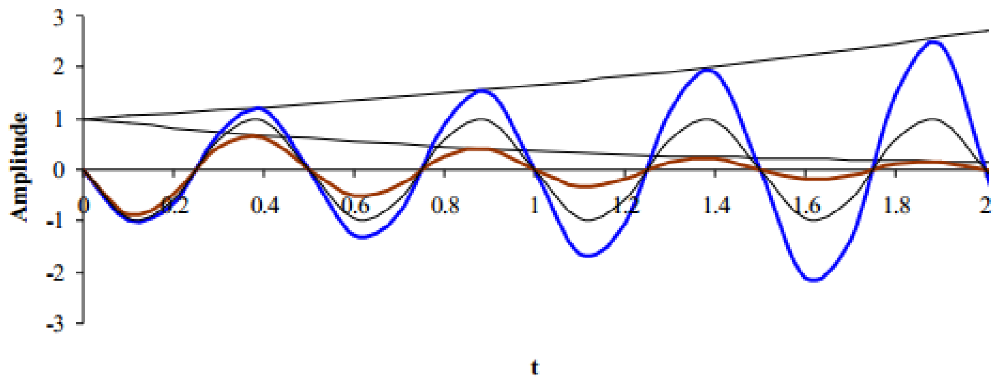


Fig. 1.32 – A signal that is not stationery can be represented by a sum of exponential the amplitude of which is not constant.

  - often used in the analysis of transient signals
    - transient signals : Signals which last for a very short period of time
- Z-transform
  - A special case of DFT
  - often used for discrete transient and non-stationery signals