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Exercise Sheet CGAL Week III

Algorithms Lab

Exercise 3 – *Portfolios*

In a little country of Bankland, people are so rich, that they do not know what to do with their money. The definitely do not want to spend it for some cheap fun like going to parties, but rather want to invest it wisely to become even richer.

There are several assets into which one may invest. Starting with Bankland's own government bonds or Neighborland's government which involve a bit more risk and ending with a true gambling such as betting on a zero in roulette.

Let us be more precise now. We have n assets called (for the lack of imagination) $1, \ldots, n$. Each asset i has a cost c_i and an expected return r_i per unit. To model risk, we will use variance and covariances (in the same meaning as for random variables) – each pair of assets i and j has a covariance v_{ij} (variance of an asset i is the covariance v_{ii}; the covariances are symmetric, i.e. $v_{ij} = v_{ji}$ and positive semidefinite, i.e. the matrix $(v_{ij})_{i,j}$ is positive semidefinite). If we buy a portfolio of assets $1 \ldots n$ in the amounts $\alpha_1, \ldots, \alpha_n$ then the total variance of the portfolio is calculated as

$$V = \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j v_{ij}$$

and the total expected return $R = \sum_i \alpha_i r_i$.

Every one person in Bankland has different requirements on their portfolio. They have different starting capital, require different minimum expected return and as well accept some maximum risk (total variance of the portfolio). Your goal is to decide whether there exists a portfolio fulfilling their requirements. You may assume, that each asset can be bought in fractional amounts (i.e., smaller than one unit) but you can only buy nonnegative amounts of each asset.

Input The input consists of several test cases. Each of them starts with a line consisting of two integers n and m ($1 \leq n \leq 100$, $1 \leq m \leq 10$). The value n is the number of total assets available and m is the number of people asking you for the portfolio. The following n lines consist of 2 integers each. The i-th line consists two numbers c_i r_i, the cost and expected return of the i-th asset ($1 \leq r_i, r_j \leq 10^6$). The following n lines describe the covariances. The i-th line consists of n integers $v_{i1} \ldots v_{in}$ ($-10^6 \leq v_{ij} \leq 10^6$).

Each of the following m lines describes the individual investors and consists of three numbers C R V, where C is the maximum cost of the portfolio, R is the minimum expected return and V is the maximum variance of the portfolio. The matrix $(v_{ij})_{i,j}$ is symmetric and positive semidefinite (as such covariance matrices tend to be). The input is terminated by a line `0 0`.

Output For each of the test cases, the output should consist of m lines, each containing either the word “`Yes.`” or “`No.`” depending whether the portfolio for the investor exists (=“`Yes.`”) or not (=“`No.`”). Do not include any other whitespaces or symbols.

Sample Input

```
1 2
2000 160
10000
1000 80 2500
1000 80 2499
3 7
2000 160
2000 160
2000 60
10000 0 0
0 10000 0
0 0 400
1000 80 2500
1000 80 1250
1000 80 1249
1000 60 466
1000 60 465
1000 30 64
1000 30 63
0 0
```

Sample Output

```
Yes.
No.
Yes.
Yes.
No.
Yes.
No.
Yes.
No.
```