Maximum Matchings

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problem definition

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- We define

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$$\text{iff } \forall e_1, e_2 \in M : e_1 \cap e_2 = \emptyset$$

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- In an unweighted graph, a maximum matching is a matching of maximum cardinality
- In a weighted graph, a maximum matching is a matching such that the sum over the included edges is maximum
- BGL does not provide weighted matching algorithms

Maximum Matchings

general unweighted case

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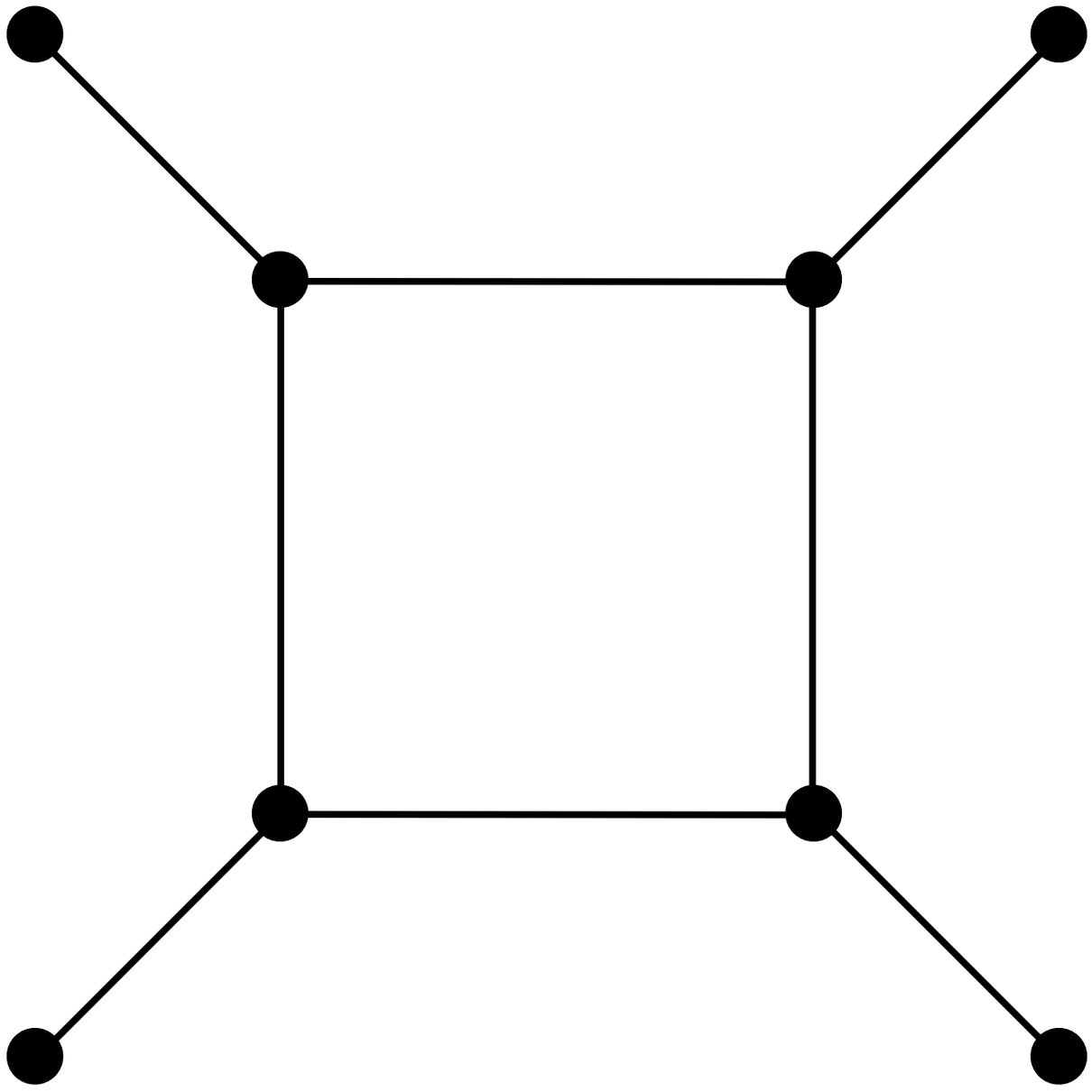
- An *augmenting path* is an alternating path that starts and ends at a free vertex
- We can improve any matching by "flipping" the edges along an augmenting path
- A matching is a maximum (not only maximal) matching, iff there is no augmenting path!

Maximum Matchings

general unweighted case example



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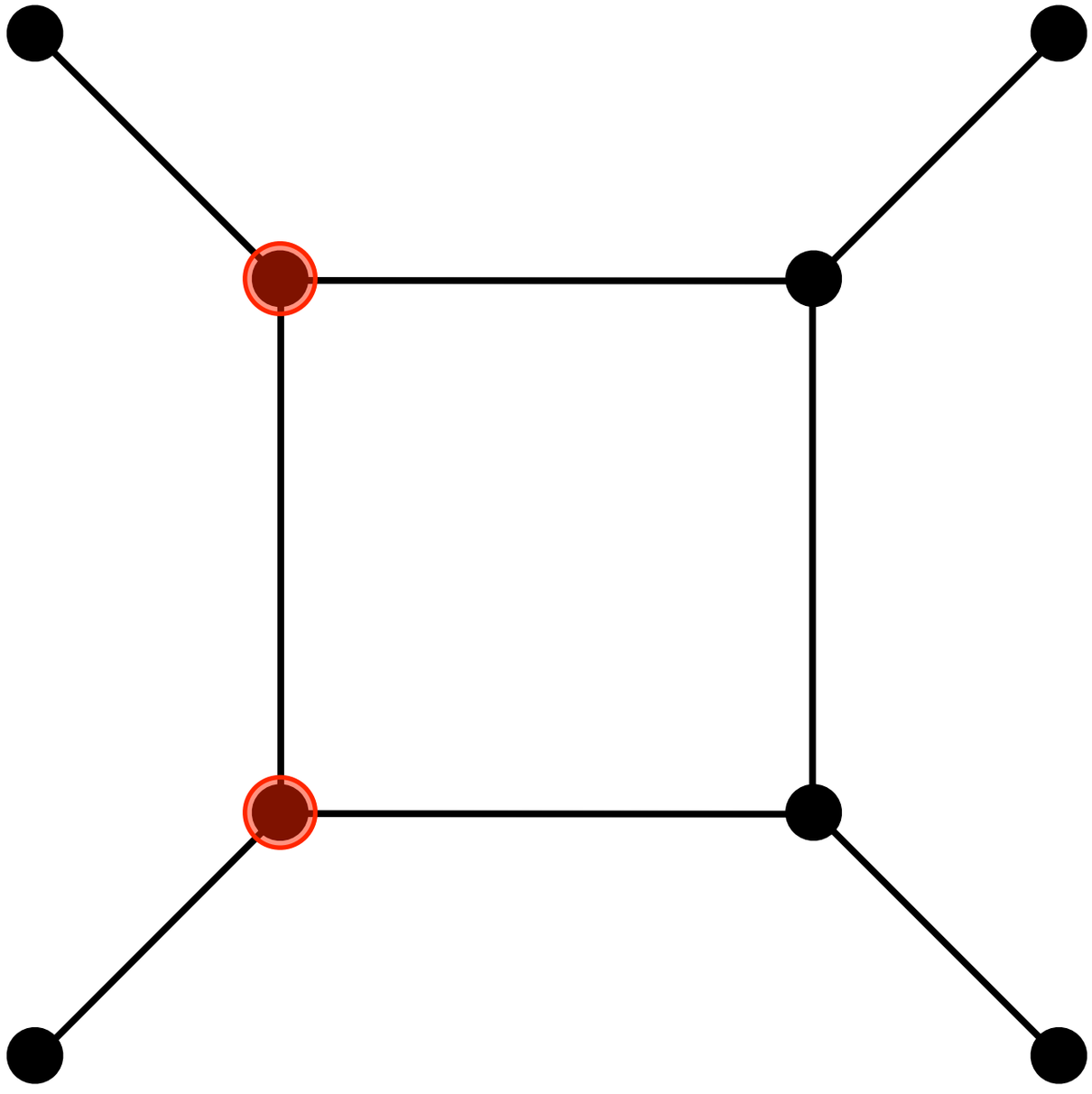

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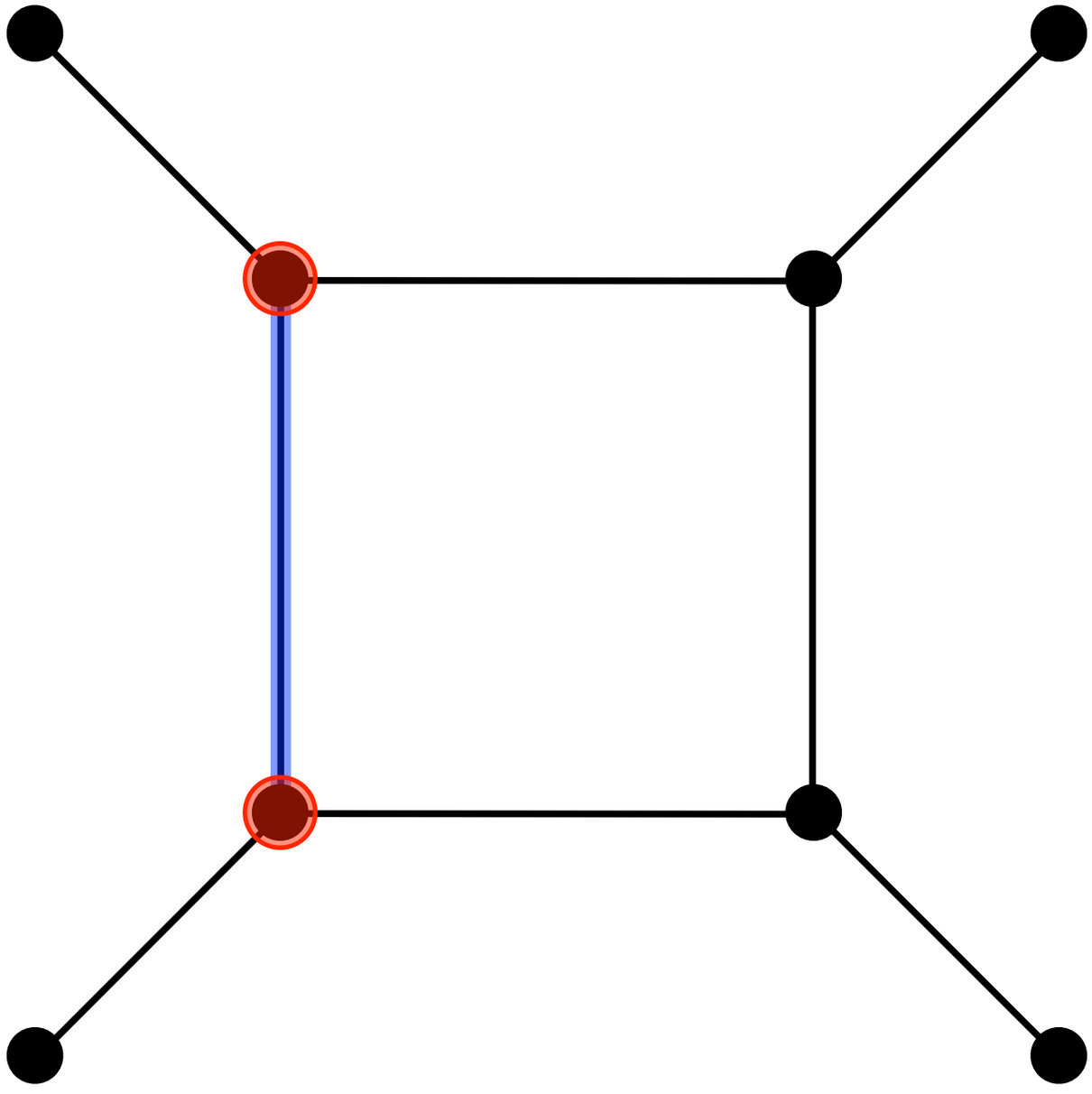

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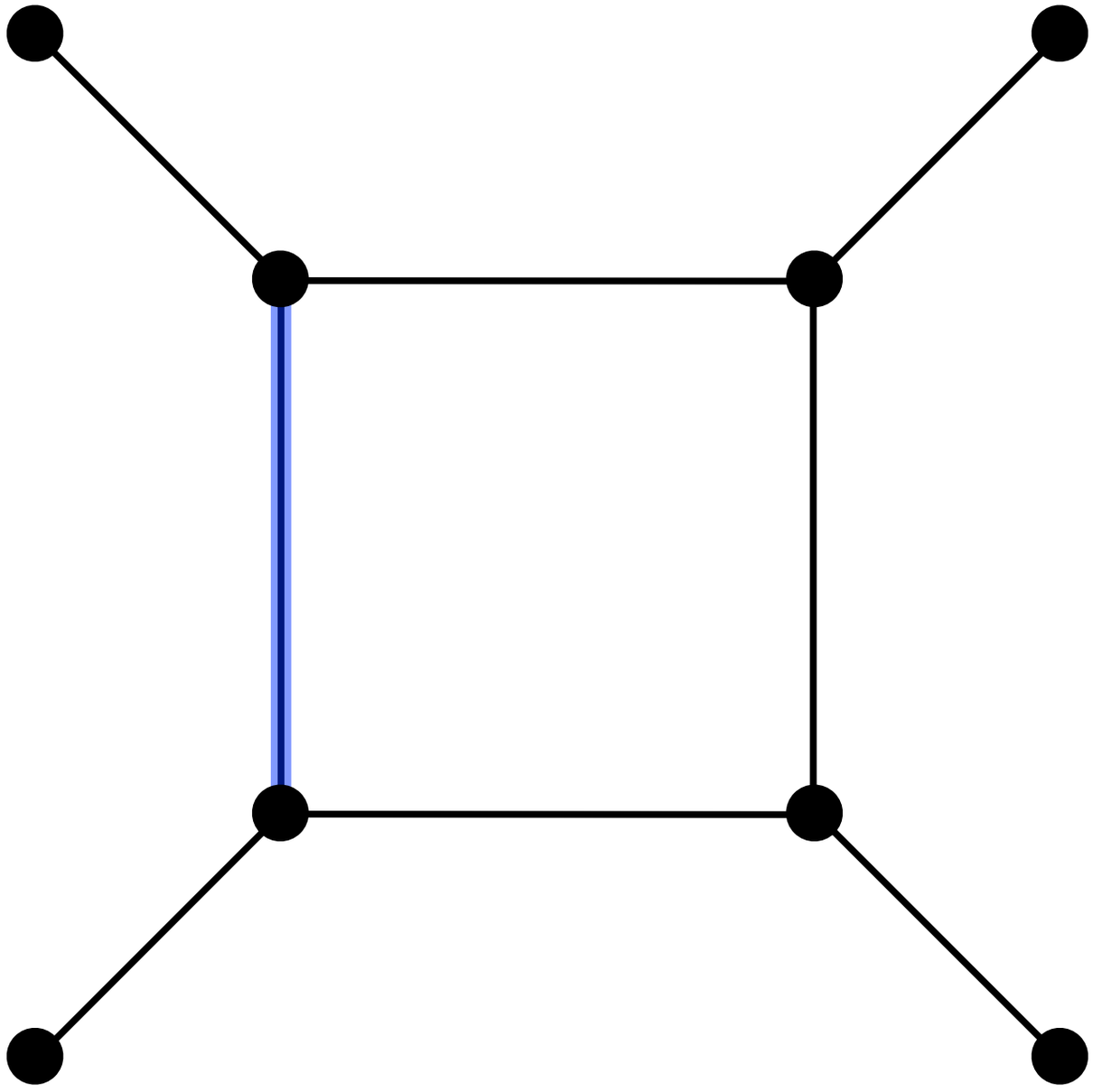

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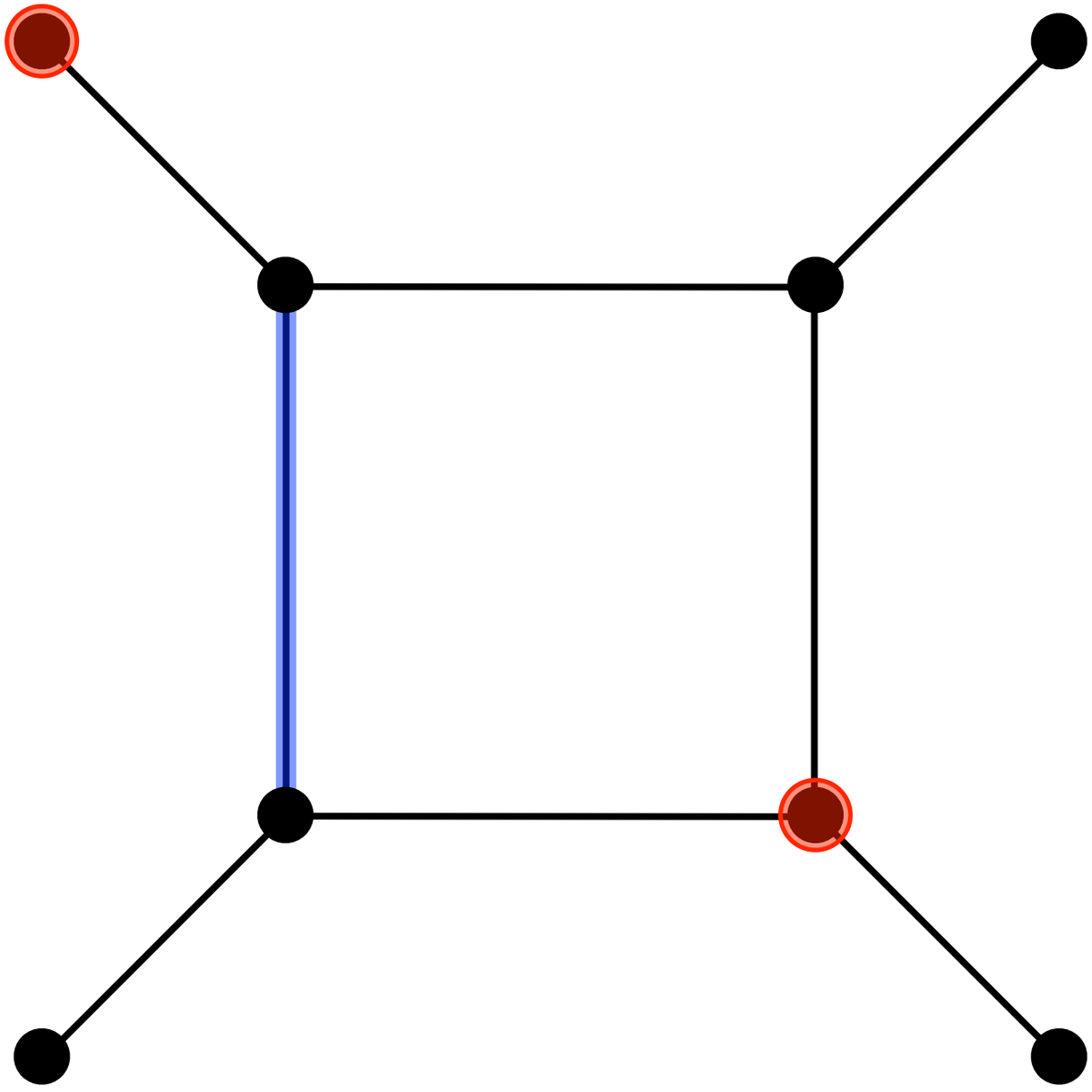

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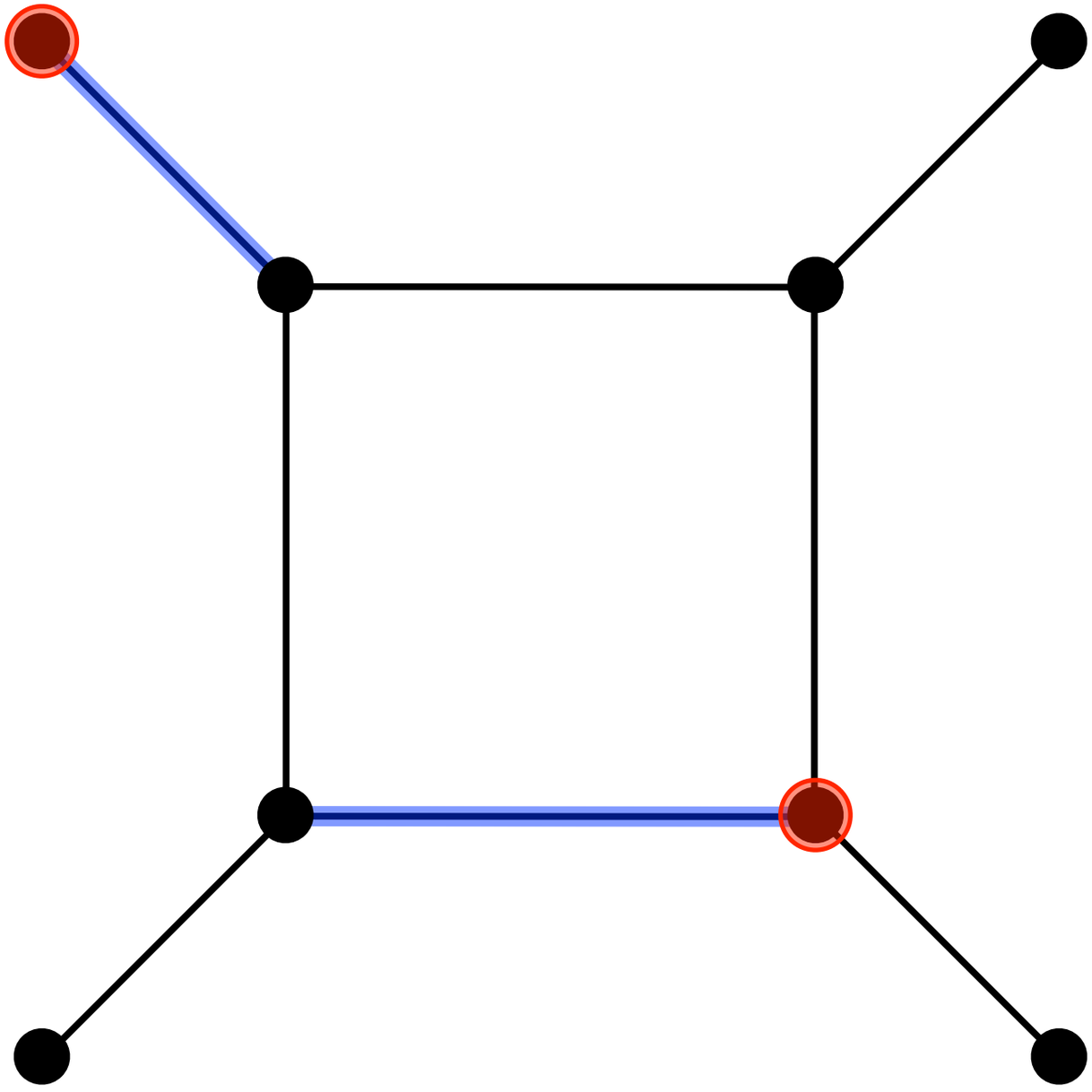

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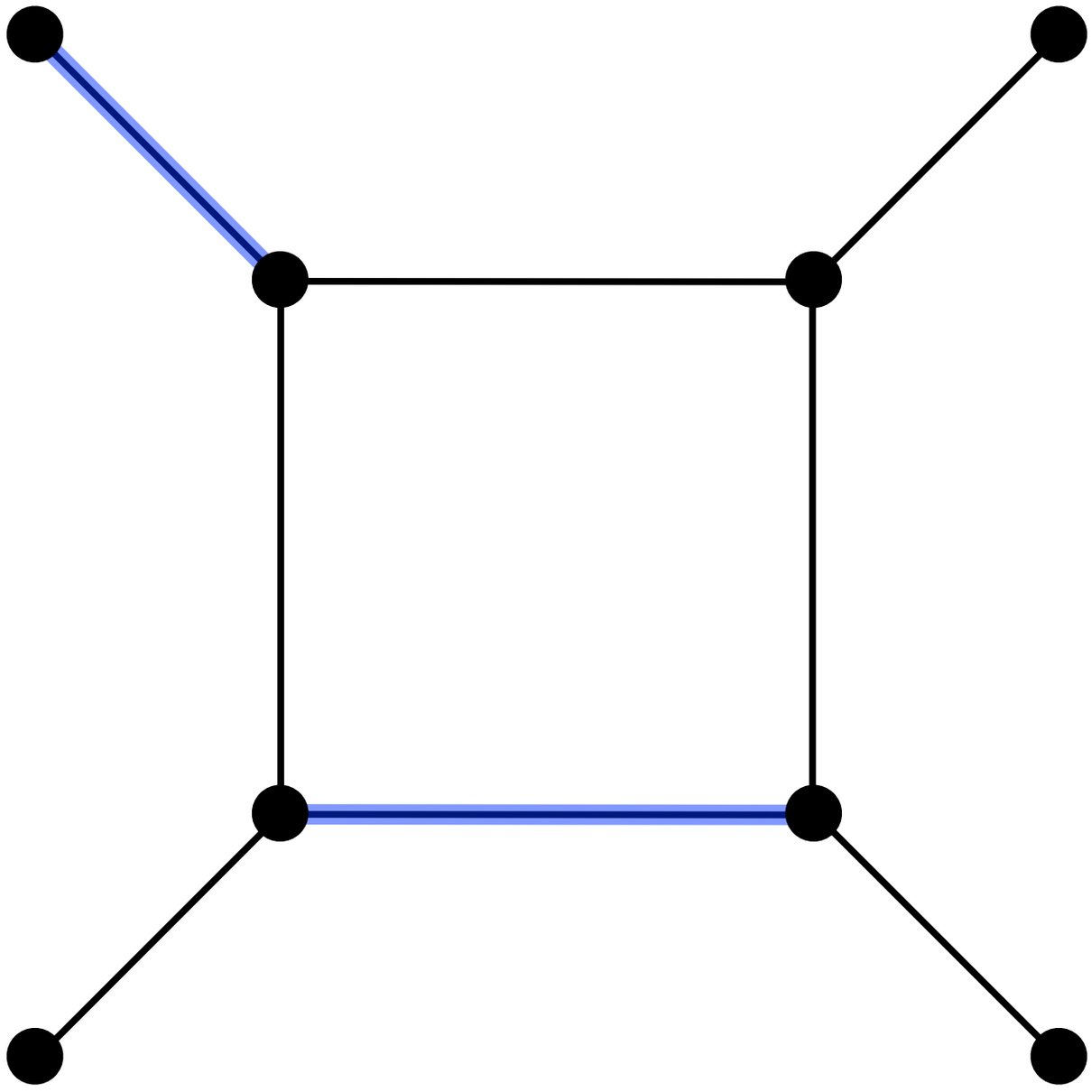

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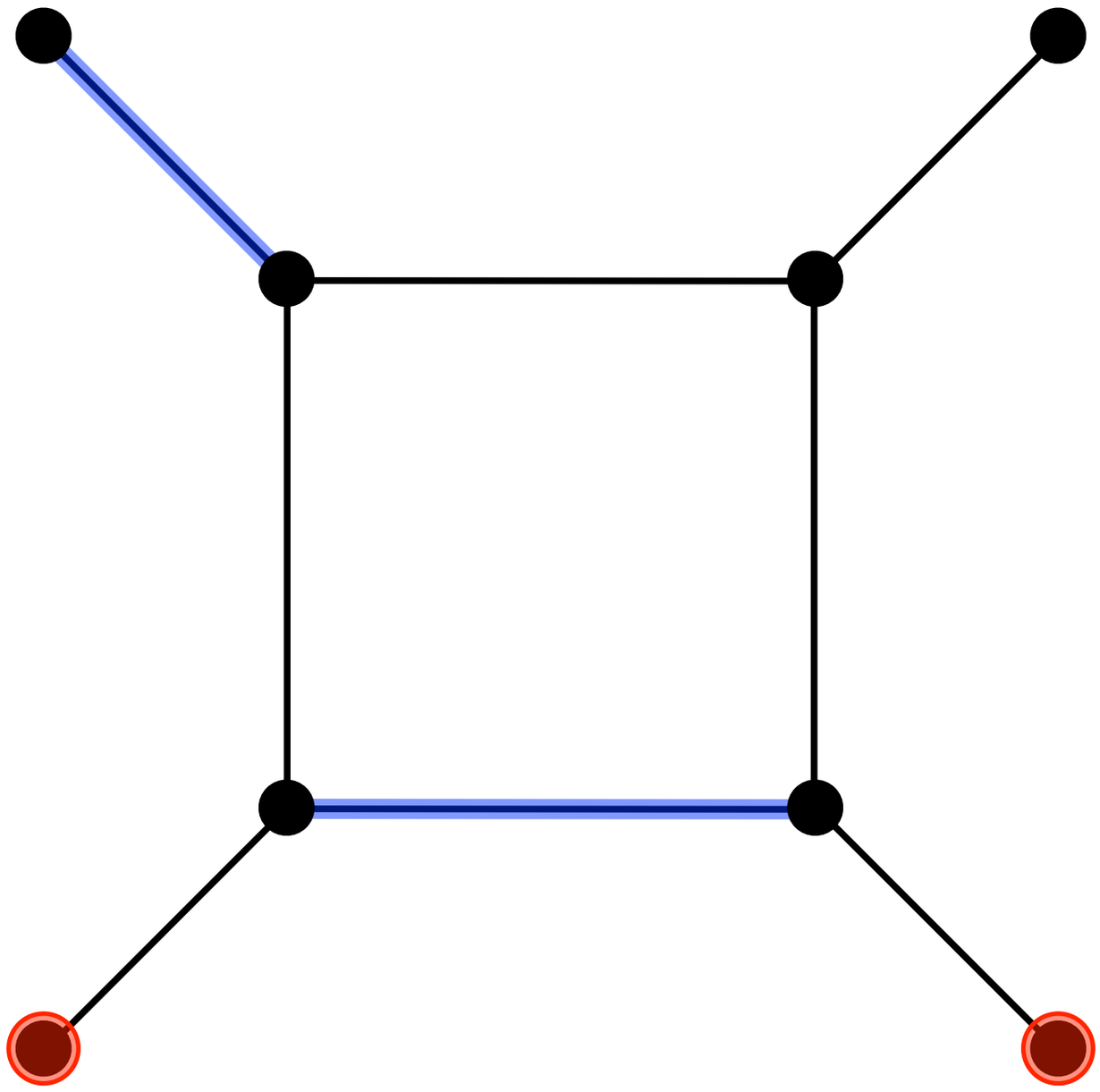

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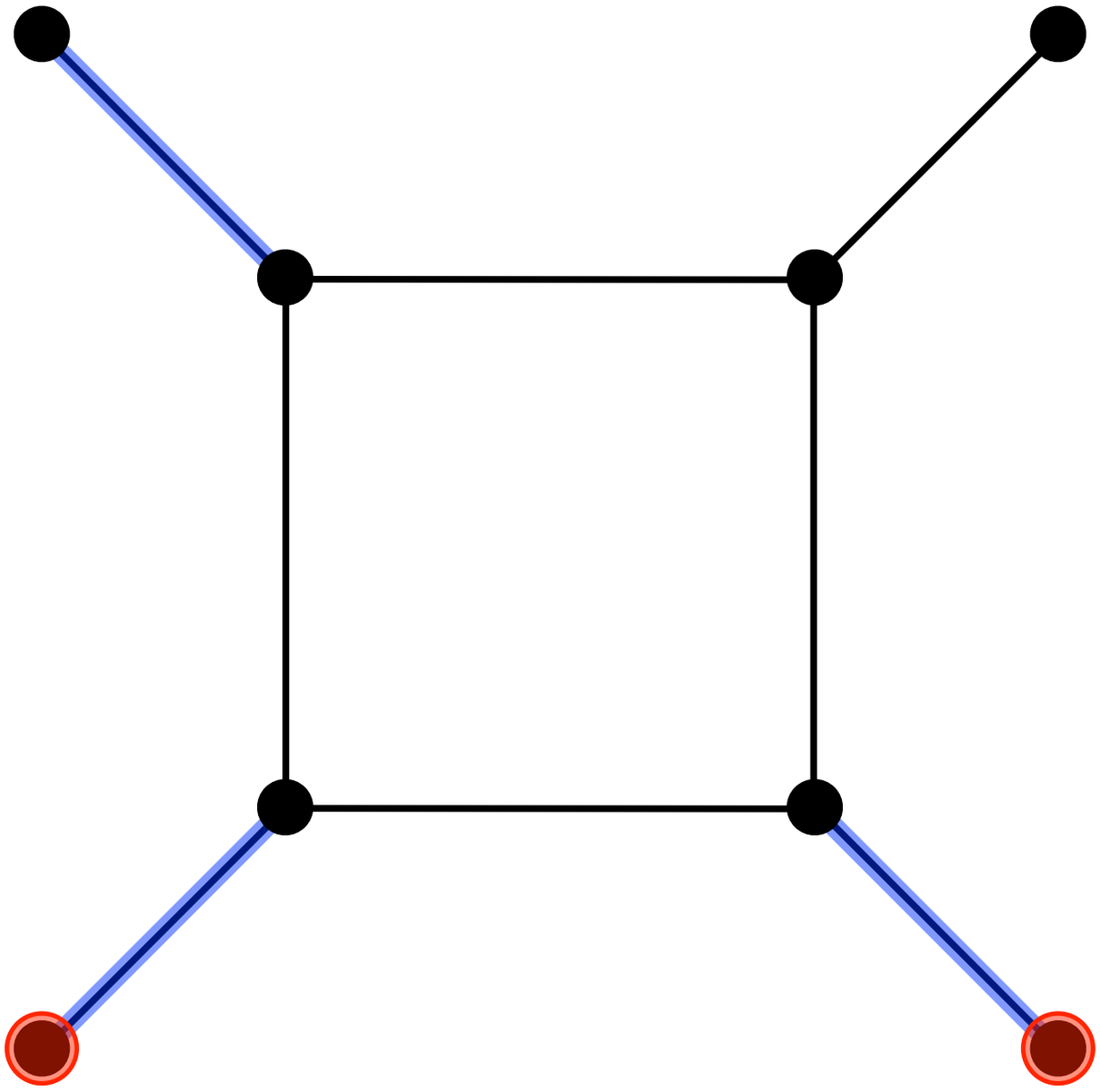

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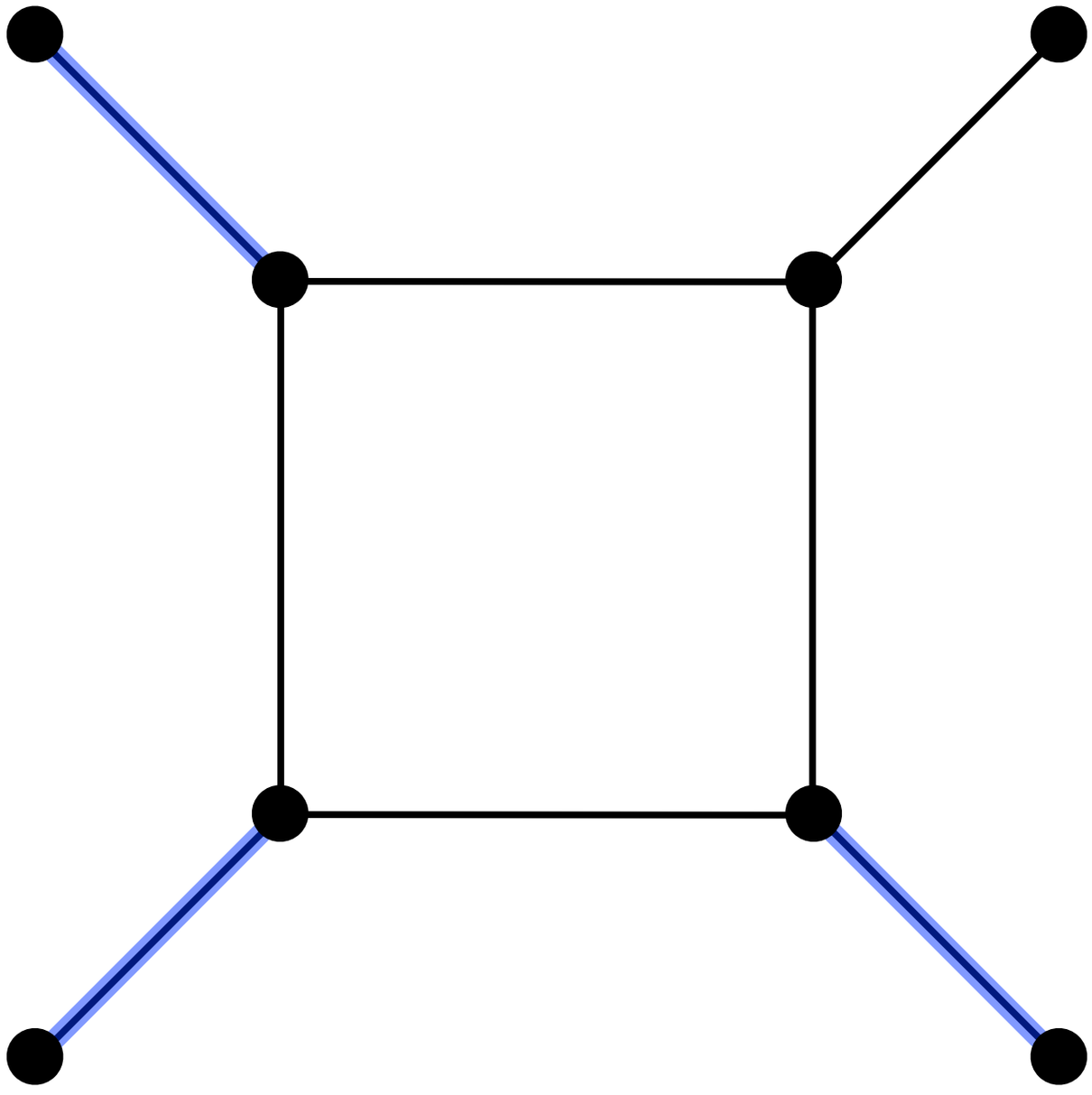

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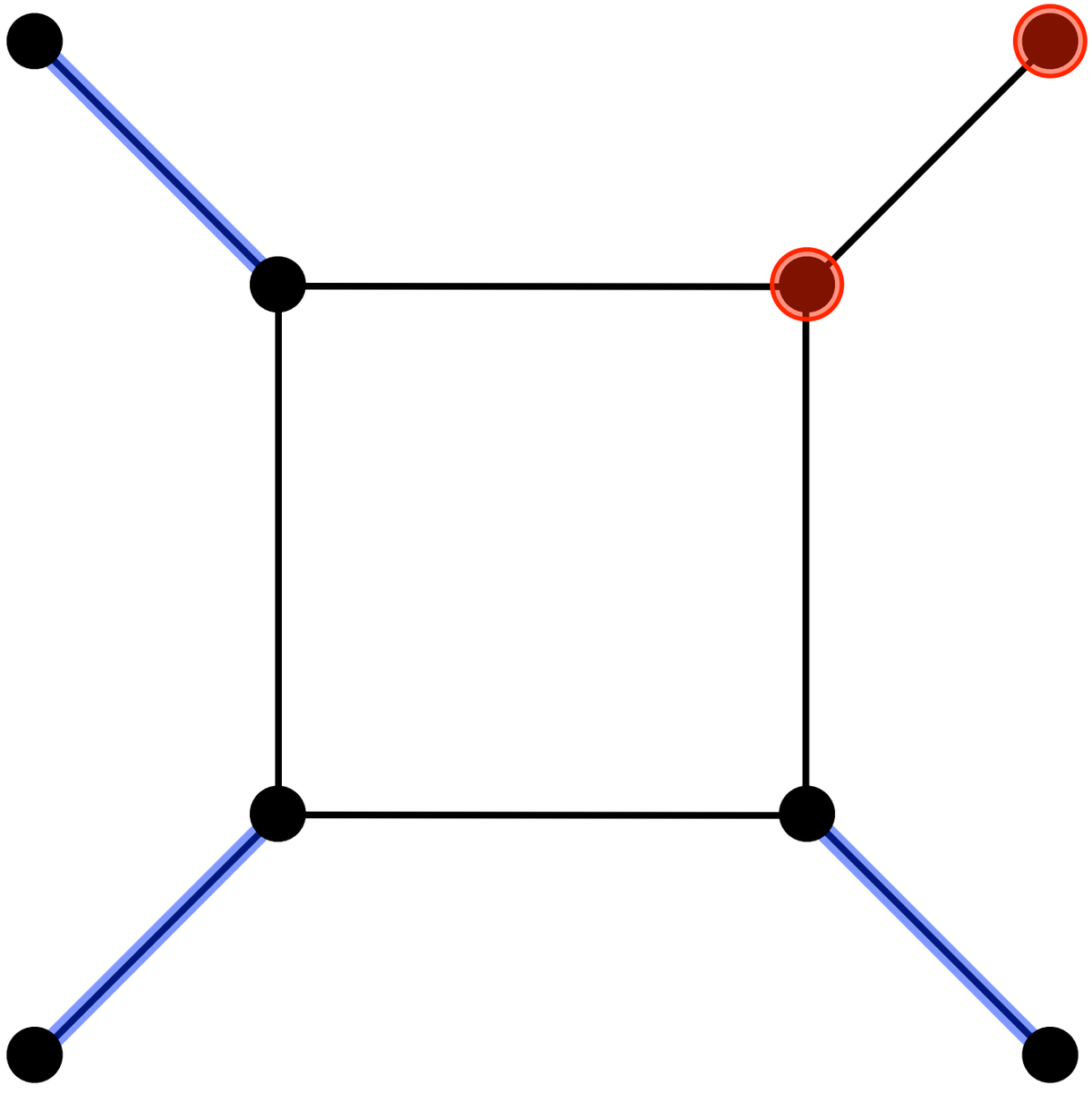

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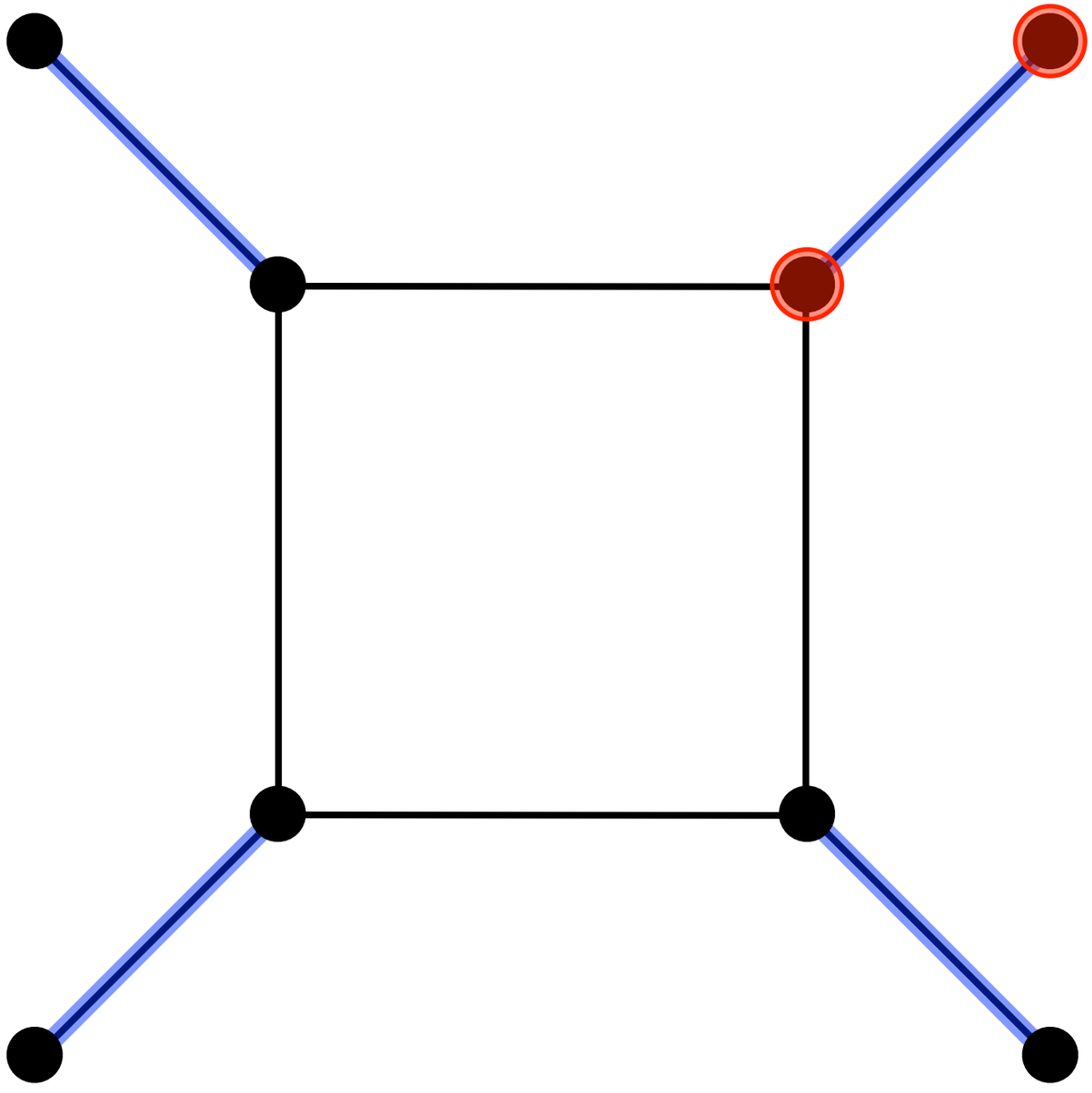

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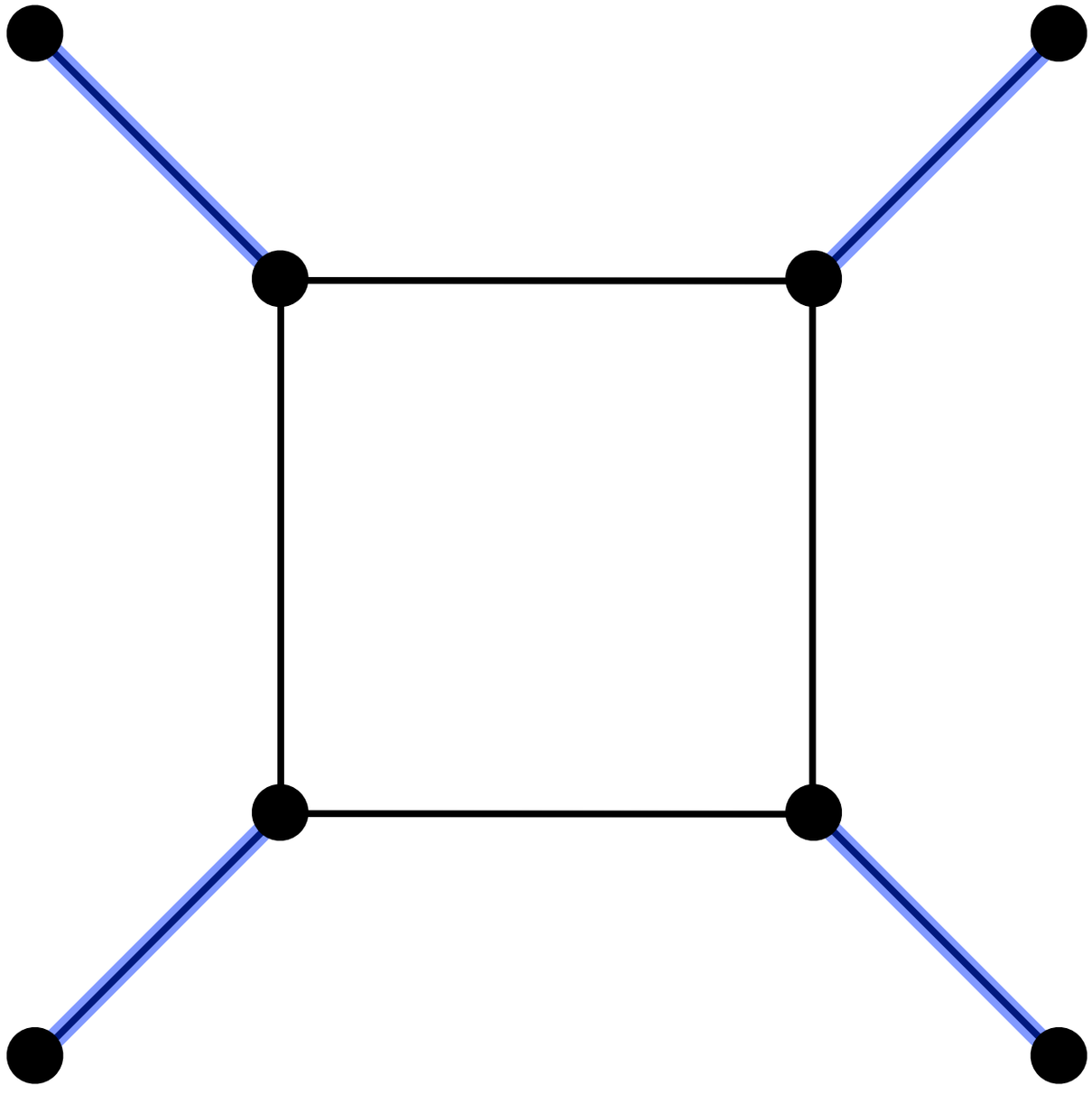
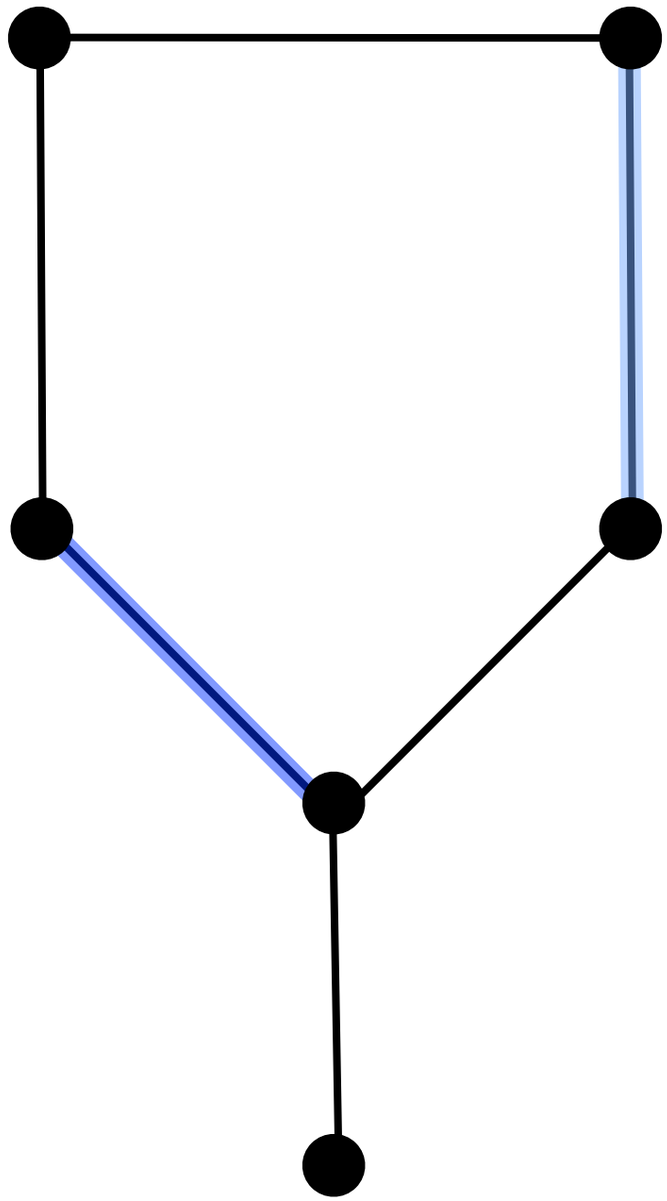


Maximum Matchings

but...



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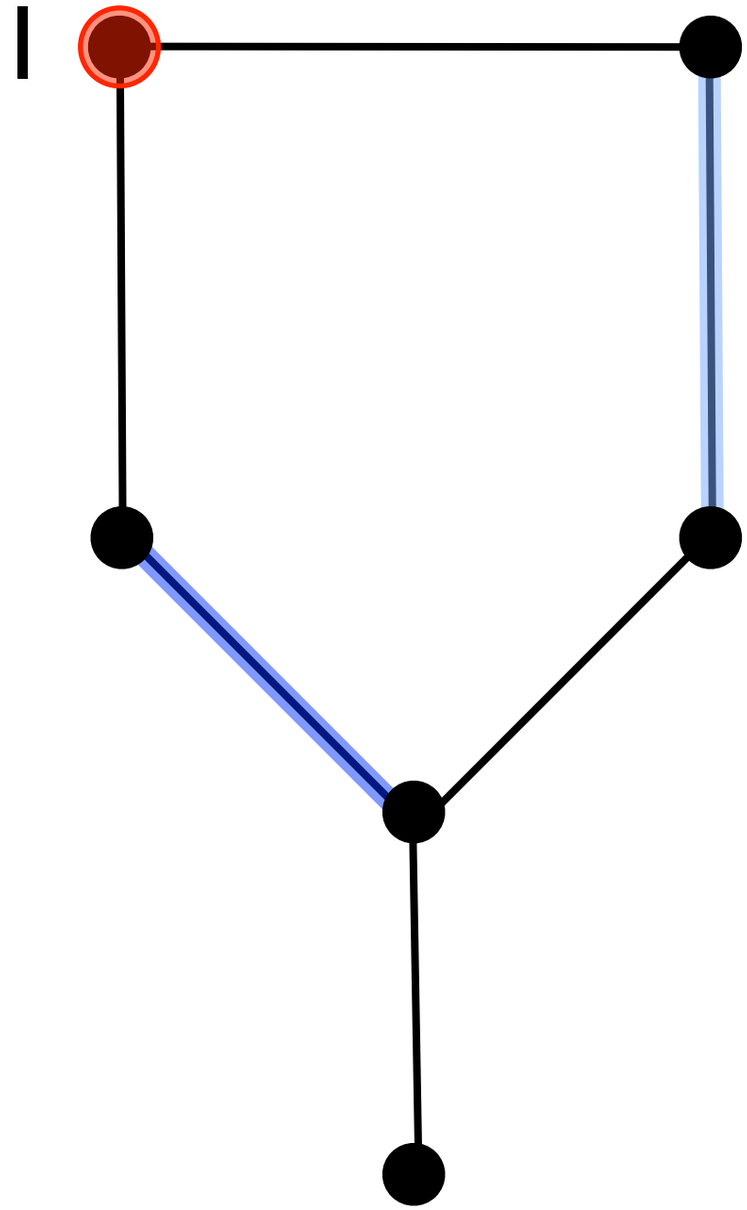

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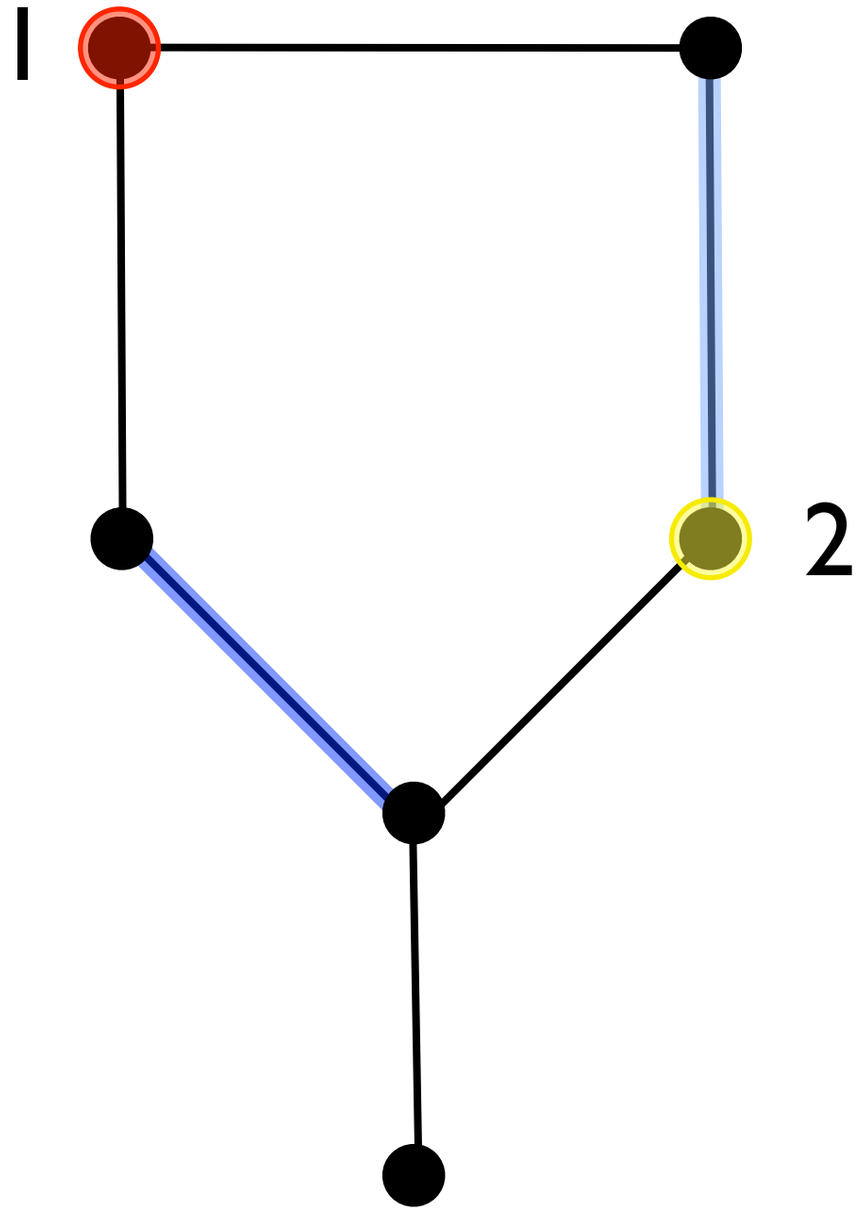

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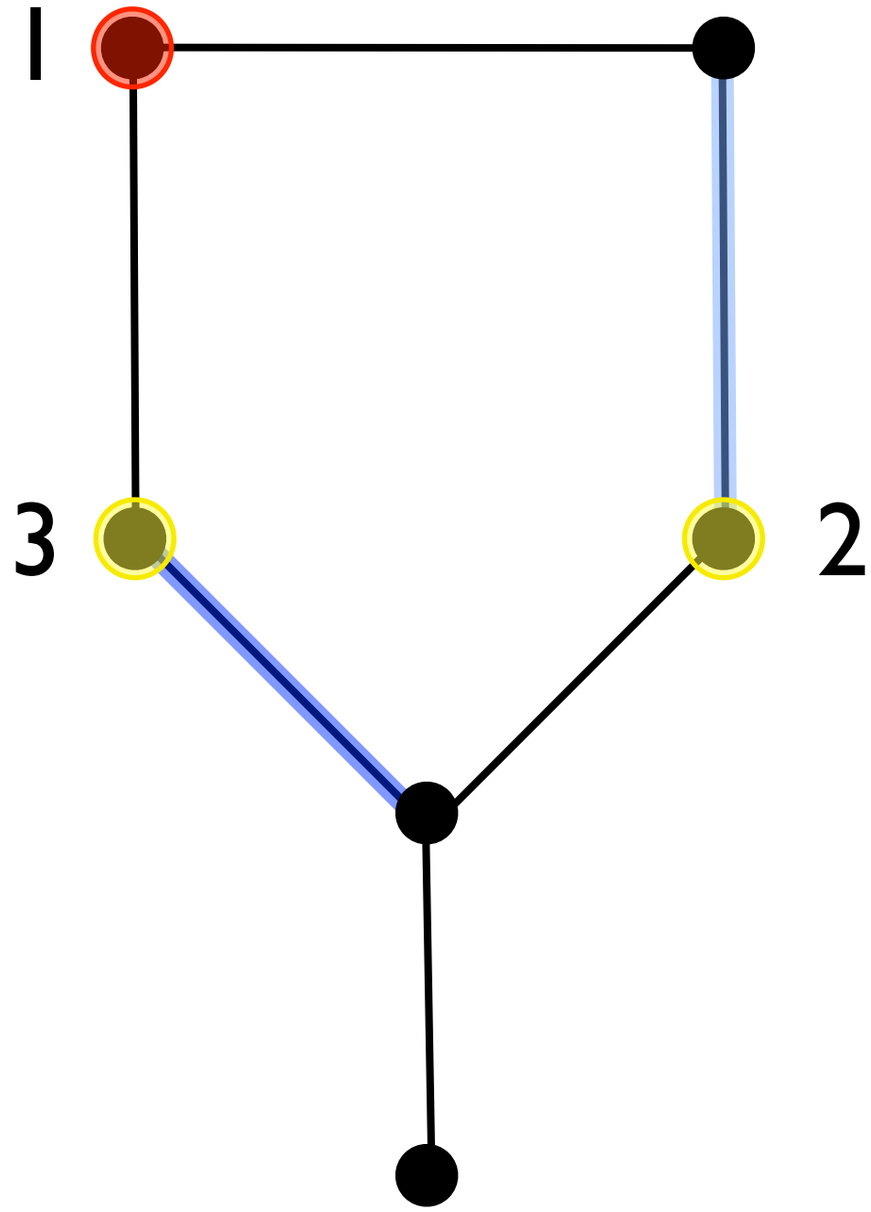

Maximum Matchings
general unweighted case

- We need to take care of odd cycles (*blossoms*) by compressing them

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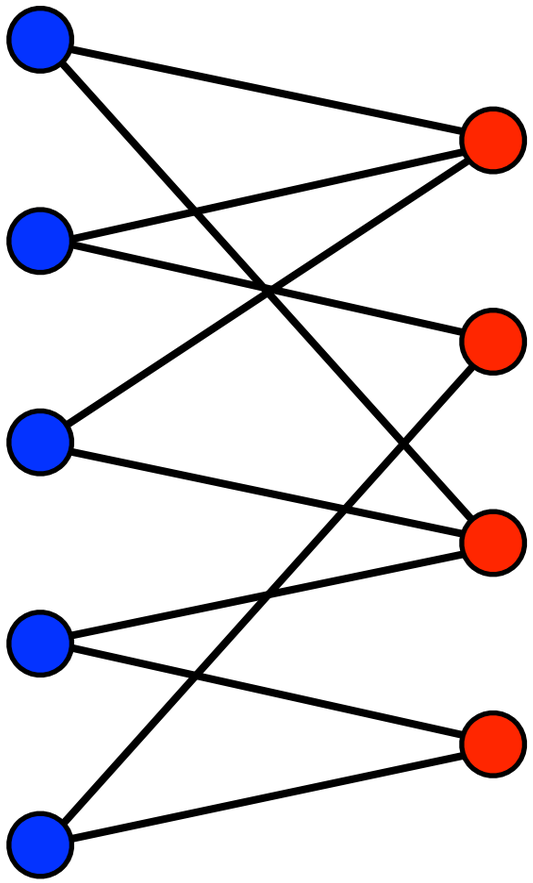
- Edmonds' algorithm does that efficiently

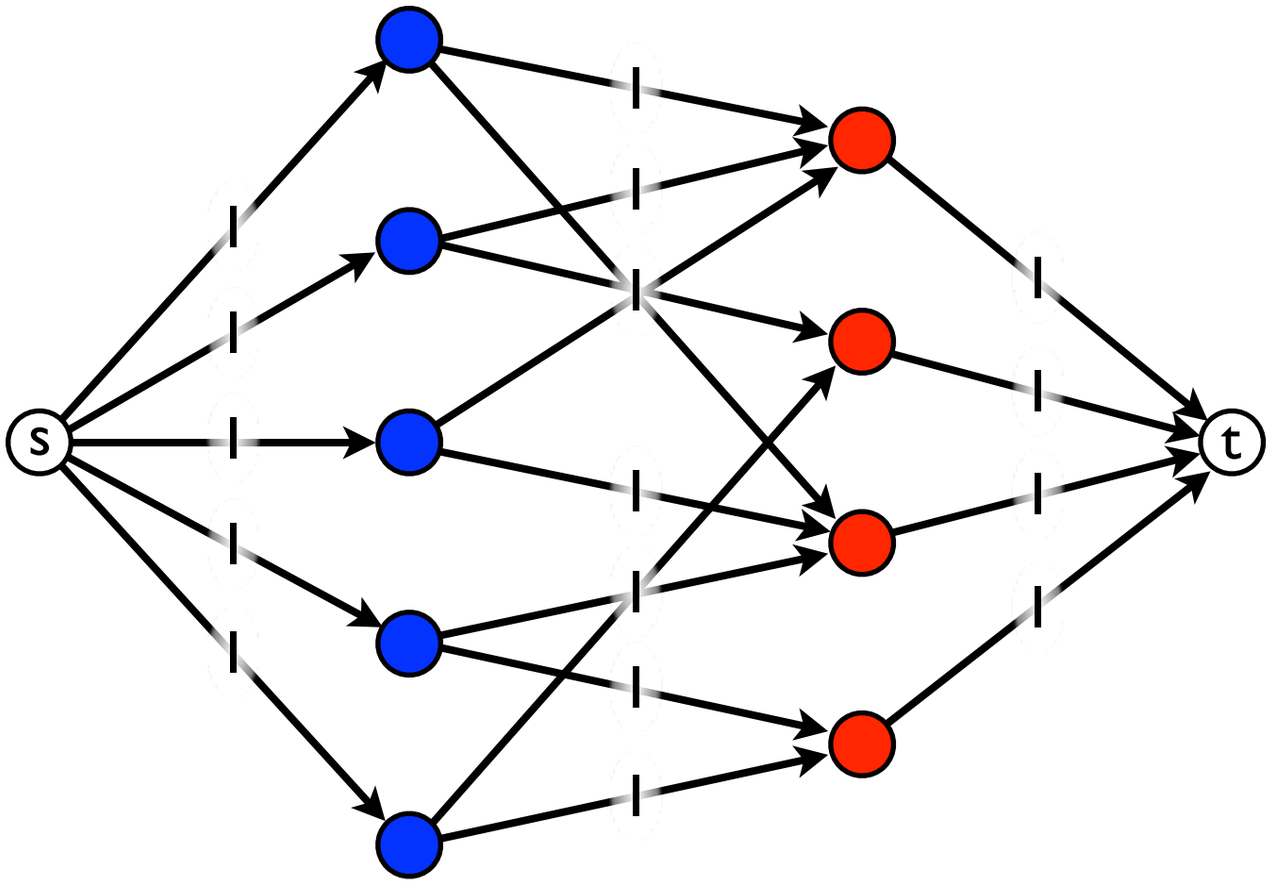
Maximum Matchings

unweighted bipartite case



Maximum Matchings

unweighted bipartite case



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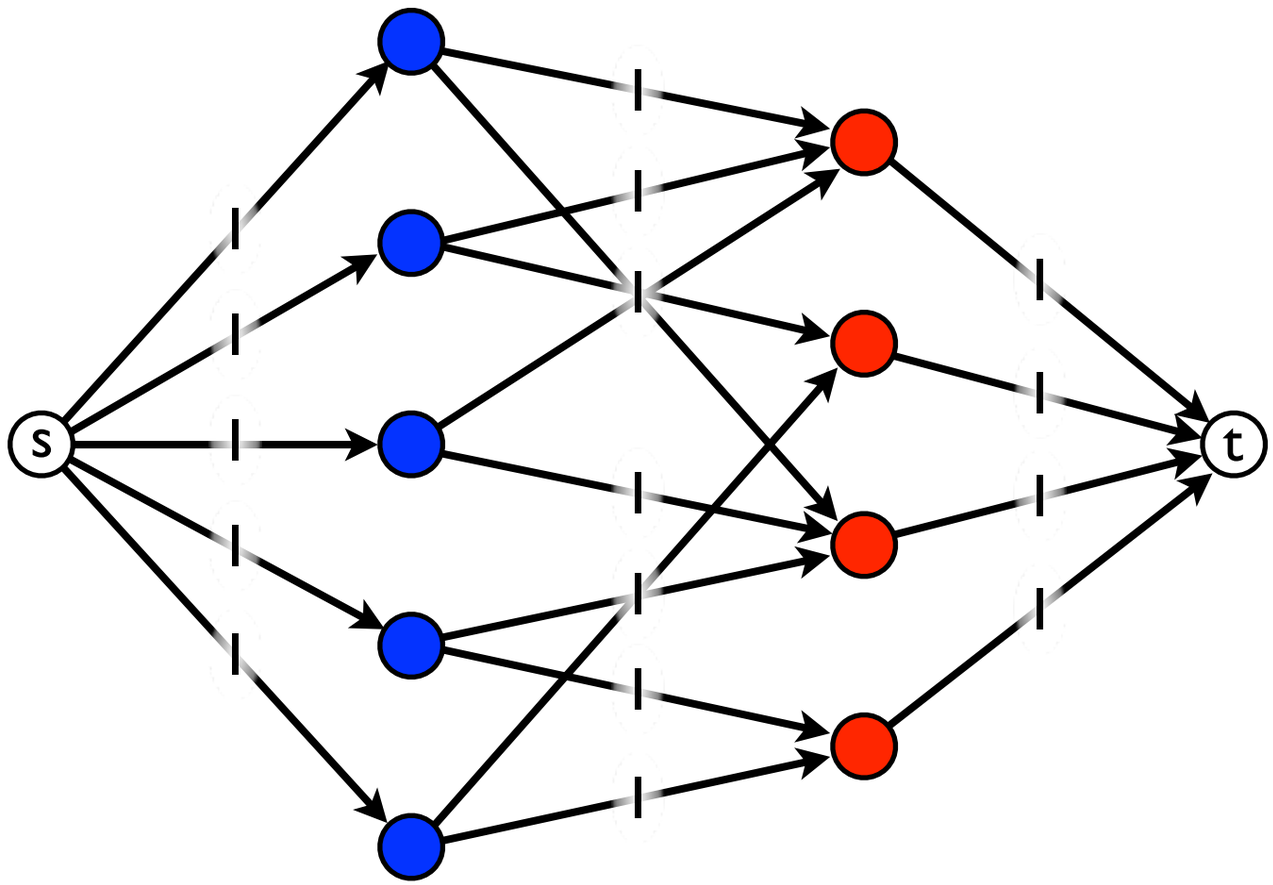

⇒ maximum flow = cardinality of maximum matching

Maximum Matchings

unweighted bipartite case

method	graph type	runtime
flow	bipartite unweighted	$O(VE)$
BGL Edmonds	any unweighted	$O(VE \cdot \alpha(V, E))$
Edmonds	any	$O(\sqrt{V}E)$
Mucha, Sankowski	any	$O(V^{2.376})$

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invoking algorithm

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