# CHAPTER 5

## Appendix

↳ Characteristic Portfolio of Alpha

→ Portfolio A:

$$h_A = \frac{V^{-1} \cdot \alpha}{\alpha^T \cdot V^{-1} \cdot \alpha} \quad (5A.1)$$

$$\sigma_A^2 = h_A^T \cdot V \cdot h_A \quad (5A.2)$$

$$\alpha = \frac{V \cdot h}{\sigma_A^2} \quad (5A.3)$$

↳ Information Ratios

→ Portfolio Q : Fully invested portfolio with maximum sharpe

→ Proposition 1

Portfolio A has properties:

① zero beta, typically has long and short positions.

② maximum information ratio.

③ total and residual risk:

$$\omega_A = \sigma_A = \frac{1}{IR} \quad (5A.7)$$

④ Any portfolio P that can be written as:

$$h_P = \beta_P \cdot h_B + \alpha_P \cdot h_A \quad \text{with } \alpha_P > 0 \quad (5A.8)$$

has $IR_P = IR$

⑤ Portfolio Q is a mixture of the benchmark and portfolio A

$$h_Q = \beta_Q \cdot h_B + \alpha_Q \cdot h_A \quad (5A.9)$$

where

$$\beta_Q = \frac{f_B \cdot \sigma_Q^2}{f_Q \cdot \sigma_B^2} \quad (5A.10) \qquad \alpha_Q = \frac{\sigma_Q^2}{f_Q \cdot \omega_A^2} \quad (5A.11)$$

⑥ Total holdings in portfolio A:

$$e_A = \frac{\alpha_C \cdot \omega_A^2}{\sigma_C^2} \quad (5A.12)$$

⑦ $\theta_P$ residual return on any portfolio P.

$$IR_P = IR_Q \cdot Corr\{\theta_P, \theta_Q\} \quad (5A.13)$$

⑧ $$IR = \frac{\alpha_Q}{\omega_Q} = SR \cdot \left[\frac{\omega_Q}{\sigma_Q}\right] \quad (5A.14)$$

⑨ $$\alpha = IR \cdot \left[\frac{V \cdot h_A}{\omega_A}\right] = IR \cdot \mathbf{MCRR_Q} \quad (5A.15)$$

↳ active managers must always check the marginal contributions. For eg., if they have an IR of 0.50, then half the marginal contributions should equal their alphas.

⑩ $$SR_B^2 = SR^2 - IR^2 \quad (5A.16)$$

↳ Optimal Policy and Optimal Value added

→ Portfolio A is key to finding optimal

$$VA = Max\{h_P^T \cdot \alpha - \lambda_R \cdot h_P^T \cdot VR \cdot h_P\} \quad (5A.25)$$

where, $VR = V - \beta \cdot \sigma_B^2 \cdot \beta^T$ (residual covariance matrix)

→ Proposition 2

optimal solution to (5A.25) is given by:

$$h_P = \beta_P \cdot h_B + \left[\frac{IR}{2 \cdot \lambda_R \cdot \omega_A}\right] \cdot h_A \quad (5A.26)$$

value added and residual volatility of optimal solution:

$$VA = \frac{IR^2}{4\,\lambda_R} \quad (5A.27) \qquad \omega_P = \frac{IR}{2 \cdot \lambda_R} \quad (5A.28)$$

Proof

$$\frac{\partial VA}{\partial h_P} = \alpha - 2 \cdot \lambda_R \cdot (V - \beta \cdot \sigma_B^2 \cdot \beta^T) \cdot h_P$$

↳ setting to 0 yields: $\alpha = 2 \cdot \lambda_R \cdot (V - \beta \cdot \sigma_B^2 \cdot \beta^T) \cdot h_P$ (5A.29)

$\alpha = 2 \cdot \lambda_R \cdot V \cdot h_P - 2 \cdot \lambda_R \cdot \beta \cdot \sigma_B^2 \cdot \beta^T \cdot h_P$ ——— $\beta_P = \beta^T \cdot h_P$

$\alpha + 2 \cdot \lambda_R \cdot \beta \cdot \sigma_B^2 \cdot \beta_P = 2 \cdot \lambda_R \cdot V \cdot h_P$ (5A.30) ——— $\alpha = \frac{V \cdot h_P}{\sigma_A^2} = IR\left[\frac{V \cdot h_P}{\omega_A}\right]$ and $\beta = \frac{V \cdot h_B}{\sigma_B^2} \Rightarrow \beta \cdot \sigma_B^2 = V \cdot h_B$

$\Rightarrow IR\left[\frac{V\,h_A}{\omega_A}\right] + 2 \cdot \lambda_R \cdot \beta_P \cdot V\,h_B = 2 \cdot \lambda_R\,V\,h_P$ (5A.31) ——— multiply by $V^{-1}$ and divide by $(2 \cdot \lambda_R)$

$\Rightarrow \frac{IR \cdot h_A}{2 \cdot \lambda_R \cdot \omega_A} + \beta_P \cdot h_B = h_P$ ——— yields (5A.26)

↳ The Active Position Y: No cash and no beta

→ Portfolio A might have large ± cash exposure

→ Portfolio C (From appendix of Ch.2): minimum risk fully invested portfolio

Residual holdings of C:

$$\mathbf{h}_{CR} = \mathbf{h}_C - \beta_C \cdot \mathbf{h}_B \qquad (5A.36)$$

→ Portfolio Y:

$$\mathbf{h}_Y = \frac{\mathbf{h}_A}{\omega_A} - \left[\frac{IR_C}{IR}\right] \cdot \left[\frac{\mathbf{h}_{CR}}{\omega_C}\right] \qquad (5A.37)$$

→ <u>Proposition 3</u>

Portfolio Y has properties:

① zero beta

② total and residual variance:

$$\omega_Y^2 = 1 - \left[\frac{IR_C}{IR}\right]^2 \qquad (5A.38)$$

③ alpha:

$$\alpha_Y = IR \cdot \left[1 - \left[\frac{IR_C}{IR}\right]^2\right] \qquad (5A.39)$$

④ zero cash position $e_Y = 0$

⑤ Information ratio:

$$IR_Y = IR \cdot \sqrt{1 - \left[\frac{IR_C}{IR}\right]^2} = IR \cdot \sqrt{1 - Corr\{\theta_Q, \theta_C\}} \qquad (5A.40)$$

↳ Optimal No Active Beta and No Active Cash

→ Problem:

$$Max\{\mathbf{h}_P^T \cdot \boldsymbol{\alpha} - \lambda_R \cdot \mathbf{h}_P^T\, \mathbf{VR} \cdot \mathbf{h}_P\} \quad s.t \quad \boldsymbol{\beta}^T \cdot \mathbf{h}_P = 1 \text{ and } \mathbf{e}^T \cdot \mathbf{h}_P = e_B \qquad (5A.41)$$

→ <u>Proposition 4</u>

Optimal solution to (5A.41):

$$\mathbf{h}_P = \mathbf{h}_B + \left[\frac{IR}{2 \cdot \lambda_R}\right] \cdot \mathbf{h}_Y \qquad (5A.42)$$

→ <u>Proposition 6</u>

→ Information ratio is linear in the input alphas.

→ One difficulty in practice: alphas are so optimistic that they overwhelm any reasonable constraint on risk.

Given a set of alphas, $IR_0 = \sqrt{\boldsymbol{\alpha}^T \cdot \mathbf{V}^{-1} \cdot \boldsymbol{\alpha}}$.

If we find $IR_0 = 2.46$, we multiply alphas by 0.75/2.46. These alphas can now be put into an optimization program.