STATS 2

WEEKS 1-4

① Joint PMF of discrete random variables

$$f_{XY}(t_1, t_2) = P(X=t_1 \text{ and } Y=t_2), \quad t_1 \in T_X, \; t_2 \in T_Y$$

② Marginal PMF

$$f_X(t) = \sum_{t' \in T_Y} f_{XY}(t, t')$$

③ Conditional PMF

$$f_{X|Y=t}(t') = \frac{f_{XY}(t', t)}{f_Y(t)}$$

④ X and Y are independent iff:

$$f_{XY}(t_1, t_2) = f_X(t_1) \cdot f_Y(t_2)$$

⑤ Function of 2 random variables X and Y : $Z = g(X, Y)$

$$f_Z(z) = \sum_{(x,y): g(x,y)=z} f_{XY}(x, y)$$

Sum of 2 uniform random variables: $X, Y \sim$ iid Unif $\{1, 2, \ldots, n\}$, $Z = X + Y$

$$f_Z(z) = \begin{cases} \frac{z-1}{n^2}, & 2 \le z \le n+1 \\ \frac{2n-z+1}{n^2}, & n+2 \le z \le 2n \end{cases}$$

⑥ Convolution

Let X and Y → random variables. $Z = X + Y$

$$f_Z(z) = \sum_{x=-\infty}^{\infty} f_{XY}(x, z-x) = \sum_{y=-\infty}^{\infty} f_{XY}(z-y, y)$$

⑦ Sum of 2 independent Poisson random variables X and Y, $X \sim \text{Poisson}(\lambda_1)$, $Y \sim \text{Poisson}(\lambda_2)$. $Z = X + Y$

$$Z \sim \text{Poisson}(\lambda_1 + \lambda_2)$$

$$X \mid Z=n \sim \text{Binomial}\left(n, \frac{\lambda_1}{\lambda_1 + \lambda_2}\right)$$

$$Y \mid Z=n \sim \text{Binomial}\left(n, \frac{\lambda_2}{\lambda_1 + \lambda_2}\right)$$

⑧ Functions of non-overlapping independent random variables are also independent.

⑨ Expectation of a random variable:

$$E[X] = \sum_{x \in T_X} f_X(x) \cdot x$$

⑩ Linearity of Expected value:

① $E[cX] = cE[X]$

② $E[X+Y] = E[X] + E[Y]$

③ $E[aX+Y] = aE[X] + E[Y]$

⑪ Variance of random variable:

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

⑫ Expectation and variance of two independent random variables:

$$E[XY] = E[X] \cdot E[Y]$$

$$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$$

⑬ To standardise a variable X :

$$Y = \frac{X - E[X]}{SD(X)}$$ is standardised variable of X.

⑭ Covariance of two random variables:

$$Cov(X,Y) = E[XY] - E[X] \cdot E[Y]$$

↳ if X and Y are independent, they are uncorrelated.
if X and Y are uncorrelated, they might still be dependent.

⑮ Probability Density Function of a random variable.

$$F_X(x_0) = \int_{-\infty}^{x_0} f_X(x)\, dx$$, where $F_X(x_0) \to$ CDF and $f_X(x_0) \to$ PDF

⑯ If $X \sim Normal(\mu, \sigma^2)$, then $Y \sim Normal(0,1)$, where $Y = \frac{X-\mu}{\sigma}$

WEEKS 5-8

① Functions of continuous random variable → Find CDF and then the derivative for PDF.

② Monotonic differentiable function

$Y = g(X)$ $$f_Y(y) = \frac{1}{|g'(g^{-1}(y))|} f_X(g^{-1}(y))$$

③ Expectation of a function of cont. rand. variable

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$$

④ Expectation and Variance of a cont. rand. variable

$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx \qquad Var(X) = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)\, dx$$

⑤ Conditional probability of discrete given continuous

$$P(X=x \mid Y=y_0) = \frac{f_X(x) \cdot f_{Y|X=x}(y_0)}{f_Y(y_0)}$$

⑥ For uniform joint distribution, $f_{XY}(x,y) = \frac{1}{Area}$, for $x,y \in supp(x,y)$

⑦ Marginal density

$$f_X(x) = \int_{y=-\infty}^{\infty} f_{XY}(x,y)\, dy$$

⑧ Conditional density

$$f_{X|Y=b}(x) = \frac{f_{XY}(x,b)}{f_Y(b)}$$

⑨ Sample mean

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \ldots + X_n}{n}$$

$E[\bar{X}] = \mu$ $\qquad Var(\bar{X}) = \frac{\sigma^2}{n}$

⑩ Sample variance

$$S^2 = \frac{(X_1-\bar{X})^2 + (X_2-\bar{X})^2 + \ldots + (X_n-\bar{X})^2}{(n-1)} \qquad E[S^2] = \sigma^2$$

⑪ Sample proportion

$$S(A) = \frac{\#(X_i \text{ for which A is true})}{n} \qquad E[S(A)] = P(A) \qquad Var(S(A)) = \frac{P(A)(1-P(A))}{n}$$

⑫ Sum of i.i.d.

$E[S] = n \cdot \mu$ $\qquad Var(S) = n \cdot \sigma^2$

(13) MGF of sum of n i.i.d. variables

$Y = X_1 + X_2 + X_3 + \dots + X_n$ $\quad\quad M_Y(\lambda) = (M_X(\lambda))^n$

(14) Sum of n i.i.d. $Exp(\beta)$ = $Gamma(\alpha, \beta)$

(15) Sum of $Normal(0, \sigma^2)$ = $Gamma(1/2, 1/2\sigma^2)$

(16) $\frac{X}{Y} \sim Cauchy(0,1)$, where $X, Y \sim$ i.i.d. $Normal(0, \sigma^2)$

WEEKS 9-11

(1) $Bias(\hat{\theta}, \theta) = E[\hat{\theta} - \theta] = E[\hat{\theta}] - \theta$

(2) $Var(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$

(3) $Risk(\hat{\theta}) = Var(\hat{\theta}) + (Bias(\hat{\theta}, \theta))^2$

(4) Method of moments

→ Epress the parameter in terms of the moments.

→ Use the given sample data to estimate the parameter.

(5) Maximum Likelihood

→ which set of parameter maximises the probability of the respective sample

$$\theta_1, \theta_2, \dots = \arg\max_{\theta_1, \theta_2, \dots} \prod_{i=1}^{n} f_X(x_i : \theta_1, \theta_2, \dots)$$

→ Find the log of likelihood function

→ calculate the derivative of log likelihood function and eqrate to 0.

(6) Hypothesis Testing:

Significance Level = P(Reject H_0 | H_0 is true) → Type I error

Power = P(Reject H_0 | H_A is true) $\quad$ 1- Power → Type II error