# CHAPTER 11

→ Appendix

↳ One Forecast N assets

$K = N$ forecasts. Assume IC is the same for each forecast:

$$\text{Cov}\{r_n, g_n\} = IC \cdot \omega_n \cdot \text{Std}_{TS}\{g_n\} \qquad (11A.1)$$

$$\text{Cov}\{r_n, g_m\} = IC \cdot \omega_n \cdot \rho_{nm} \cdot \text{Std}_{TS}\{g_m\} \qquad (11A.2)$$

where $\rho_{nm}$ is correlation between $g_n$ and $g_m$

$$\text{Cov}\{\mathbf{r}, \mathbf{g}\} = IC \cdot \boldsymbol{\omega} \cdot \boldsymbol{\rho} \cdot \mathbf{Std} \qquad (11A.3)$$

$$\text{Var}\{\mathbf{g}\} = \mathbf{Std} \cdot \boldsymbol{\rho} \cdot \mathbf{Std} \qquad (11A.4)$$

$\boldsymbol{\omega}$ and $\mathbf{Std}$ are diagonal matrices

substituting into basic forecasting formula:

$$\boldsymbol{\phi} = IC \cdot \boldsymbol{\omega} \cdot \mathbf{Std}^{-1} (\mathbf{g} - E\{\mathbf{g}\}) \qquad (11A.5)$$

↳ Two Forecasts N assets

$K = 2N$. $\mathbf{g} = \{\mathbf{g}_1, \mathbf{g}_2\}$

Simplifying assumptions:

$$\text{Var}\{\mathbf{g}\} = \mathbf{Std} \cdot \begin{bmatrix} \boldsymbol{\rho} & \rho_{12} \cdot \boldsymbol{\rho} \\ \rho_{12} \cdot \boldsymbol{\rho} & \boldsymbol{\rho} \end{bmatrix} \cdot \mathbf{Std} \qquad (11A.7)$$

$$\text{Cov}\{\mathbf{r}, \mathbf{g}\} = \boldsymbol{\omega} \cdot [IC_1 \cdot \mathbf{I} \quad IC_2 \cdot \mathbf{I}] \begin{bmatrix} \boldsymbol{\rho} & 0 \\ 0 & \boldsymbol{\rho} \end{bmatrix} \cdot \mathbf{Std} \qquad (11A.8)$$

Correlation matrix for $\mathbf{g}_1$ is identical to that of $\mathbf{g}_2$. Correlation b/w every $g_{1n}$ and $g_{2n}$ is a scalar constant $\rho_{12}$. Corr. b/w every $g_{1n}$ and $r_n$ is $IC_1$ and b/w $g_{2n}$ and $r_n$ is $IC_2$.

Substituting (11A.7) and (11A.8) into the basic forecasting formula:

$$\phi = \text{Cov}\{r, g\} \cdot \text{Var}^{-1}\{g\} \cdot (g - E\{g\})$$

$$= \boldsymbol{\omega} \cdot \left[ \left( \frac{IC_1 - \rho_{12} \cdot IC_2}{1 - \rho_{12}^2} \right) \cdot \mathbf{I} \quad \left( \frac{IC_2 - \rho_{12} \cdot IC_1}{1 - \rho_{12}^2} \right) \cdot \mathbf{I} \right] \cdot \mathbf{Std}^{-1} \left[ \mathbf{g} - E\{\mathbf{g}\} \right] \qquad (11A.9)$$

↳ Multiple Forecasts N assets

Easier to understand if we transform $\mathbf{g}$ into a set of orthogonal forecasts $\mathbf{y}$

$$\text{Var}\{\mathbf{g}\} = \mathbf{H}^T \mathbf{H} \qquad (11A.10)$$

Cholesky Decomposition

$$\mathbf{y} = (\mathbf{H}^T)^{-1} \cdot [\mathbf{g} - E\{\mathbf{g}\}] \qquad (11A.11)$$

where $\mathbf{y}$ are standardized and uncorrelated raw forecasts

$$\text{Cov}\{\mathbf{r}, \mathbf{g}\} = \text{Cov}\{\mathbf{r}, \mathbf{y}\} \cdot \mathbf{H} \qquad (11A.12)$$

so, refined forecast:

$$\boldsymbol{\phi} = \boldsymbol{\omega} \cdot \text{Corr}\{\mathbf{r}, \mathbf{y}\} \cdot \mathbf{y} \qquad (11A.13)$$

↳ Testing Alpha Scaling

↳ Uncertain ICs