MATH 1

WEEKS 1-4

① Every set has 2^n subsets.

② Identity Relation → every element just to itself

Reflexive Relation → every element to itself and some more

Symmetric Relation → $(a,b) \in R$ and $(b,a) \in R$

Transitive Relation → $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$

Equivalence Relation → all of the above.

③ Injective function → one-to-one
↳ $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$

Surjective function → onto
↳ range = co-domain

④ Distance between 2 points (x_1, y_1) and (x_2, y_2):

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

⑤ Distance between a point (x_1, y_1) and a line $Ax + By + C$:

$$= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

⑥ Distance between 2 lines $l_1 \Rightarrow y = mx + C_1$; $l_2 \Rightarrow y = mx + C_2$

$$= \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} \quad OR \quad = \frac{|C_1 - C_2|}{\sqrt{1 + m^2}}$$

⑦ Point $p(x, y)$ cutting a line segment into two segments m and n:

$$\frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$$

⑧ Area of a $\triangle ABC$; $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$:

$$= \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

⑨ Slope of a line $= m = \tan\theta = \frac{x_2 - x_1}{y_1 - y_2}$

⑩ $\perp$ lines m_1 and $m_2 \Rightarrow m_1 = \frac{-1}{m_2}$

⑪ Angle of intersection θ b/w two lines with slopes m_1 and m_2

$$\theta = \frac{|m_2 - m_1|}{1 + m_2 \cdot m_1}$$

⑫ Sum of Squared Errors: $\sum_{i=1}^{n} (y_i - mx_i - C)^2$

⑬ Vertex (min/max) of a quadratic func. $x = \frac{-b}{2a}$

(14) Slope of quadratic function at any point x : $2ax + b$

(15) Roots of a quadratic function : $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(16) Intercepts of quadratic function $y = a(x-p)(x-q)$; x-intercepts → p, q

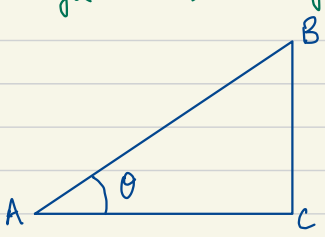
WEEKS 5-8

(1) $\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^{nt} = e^{xt}$

(2) $(a)^{\log_b(c)} = (c)^{\log_b(a)}$

(3) $\log_a(M \cdot N) = \log_a(M) + \log_a(N)$; $\log_a(M/N) = \log_a(M) - \log_a(N)$

(4)



$\sin\theta = \frac{BC}{AB}$ $\cos\theta = \frac{AC}{AB}$ $\tan\theta = \frac{BC}{AC}$

(5) $\sin^2\theta + \cos^2\theta = 1$

(6) $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

(7) If limit of a function $f(x)$ at point a exists, then:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

(8) If $f(x)$ is differentiable at point a, then:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \text{ exists}$$

(9) $(f+g)'(x) = f'(x) + g'(x)$

(10) $(f \cdot g)'(x) = f'(x)\,g(x) + f(x)\,g'(x)$

(11) $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$

(12) $f(g(x))' = f'(g(x)) \cdot g'(x)$

(13) equation of a tangent of a curve at any point a :

$$y = f'(a)(x-a) + f(a)$$

WEEKS 9-11

(1) local minima ⇒ $f''(a) > 0$
local maxima ⇒ $f''(a) < 0$
inconclusive / saddle ⇒ $f''(a) = 0$

(2) Reimann Sums

$$S(P) = \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i \quad \text{where, } x_i^* \text{ is in } [x_{i-1}, x_i]$$

$$\Delta x_i = \frac{b-a}{n}$$

(3) Indefinite Integral $F'(x) = f(x)$

$$\int_a^b f(x)\,dx = F(b) - F(a) = \left[F(x)\right]_a^b$$

④ $\int (f g') \, dx = (fg) \, dx - \int (f'g) \, dx$

⑤ $\int_a^b f(x) \, dx = -\int_b^a f(x) \, dx$

⑥ If $f(x) \geq g(x)$, then $\int_a^b f(x) \, dx \geq \int_a^b g(x) \, dx$

⑦ $\int_a^b f(g(x)) \, g'(x) \, dx = \int_{g(a)}^{g(b)} f(x) \, dx$

⑧ Planar graph → no edge crosses another

⑨ Maximum vertex cover → selected vertices cover all the edges in the graph.

⑩ Independent set → selected vertices do not share any edge ; no edge b/w any two

⑪ Matching → selected edges such that no two edges have a common vertex.

⑫ Degree of vertex → no. of edges ; indegree → incoming edges ; outdegree → outgoing edges

⑬ BFS (Breadth-First Search) → level-by-level
- ↳ uses a queue
- ↳ FIFO

⑭ DFS (Depth-First Search) → exploring a path then backtrack
- ↳ uses a stack
- ↳ LIFO

⑮ Non-tree edge classification
- ① Forward edge (u, v) ⇒ Interval $[pre(u), post(u)]$ contains $[pre(v), post(v)]$
- ② Backward edge (u, v) ⇒ Interval $[pre(v), post(v)]$ contains $[pre(u), post(u)]$
- ③ Cross edge (u, v) ⇒ $[pre(v), post(v)] \cap [pre(u), post(u)] = \phi$

⑯ Transitive Closure A^+

$$A^+[i,j] = \max \{ A^l[i,j] \mid 1 \leq l \leq n \}$$

⑰ $C = A \times B \rightarrow C[i,j] = \sum_{k=0}^{n-1} (A[i,k] \times B[k,j])$

⑱ Single-source shortest path:
- ① non-negative weights → Dikjstra's Algorithm
- ② Negative weights → Bellman-Ford Algorithm

⑲ All-pairs shortest path: Floyd-Warshall Algorithm

⑳ Minimum cost spanning tree:
- ① Prim's Algorithm → start with smallest and "grow"
- ② Kruskall's Algorithm → scan edges in ascending order and "connect"