AKUL VAISHNANI
3C018
Roll NO :- 102103487

Page No.
Date

Predictive Analysis using Statistics

A.1) Given random sample $(x_1, \ldots, x_n)$

$$\rightarrow L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(\frac{-(x_i-\mu)^2}{2\sigma^2}\right)}$$

→ Taking natural log of likelihood function,

$$\Rightarrow \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2} \ln(2\pi\sigma^2) \right)$$

→ To find MLE, differentiate log likelihood w.r.t θ_1 & θ_2

$$\Rightarrow \text{i)} \frac{d}{d\theta_1} \left(\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i-\mu}{\sigma^2} \right) \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0$$

$$\therefore \boxed{\frac{\theta_1}{\mu} = \frac{1}{n} \sum_{i=1}^{n} X_i}$$

for θ_2; $\frac{d}{d\theta_2}\left(\ln L(\theta_1,\theta_2) = \sum_{i=1}^{n}\left(\frac{-(x_i-\theta_1)^2}{2\theta_2^2} + \frac{1}{2\theta_2}\right)\right.$

$= 0$

$$\Rightarrow \sum_{i=1}^{n} \frac{(x_i-\theta_1)^2}{\theta_2^2} - \frac{n}{\theta_2} = 0$$

$$\Rightarrow \frac{\theta_2^2}{\theta_2} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$$\Rightarrow \theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$ & Sample Variance

Page No.			
Date			

A; 2, To find the MLE of Θ for binomial [redacted] distribution $B(n, \Theta)$ where m is a known +ve integer

$$\rightarrow L(\Theta) = \prod_{i=1}^{n} \binom{m}{x_i} \Theta^{x_i} (1-\Theta)^{m-x_i}$$

Taking ln,

$$\Rightarrow \ln(L(\Theta)) = \sum_{i=1}^{n} \left(\ln \binom{m}{x_i} + x_i \ln(\Theta) + (m-x_i) \ln(1-\Theta) \right)$$

$$\rightarrow \frac{\partial}{\partial \Theta} \left(\ln(L(\Theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\Theta} - \frac{m-x_i}{1-\Theta} \right) \right) = 0$$

$\Rightarrow$ Solving for Θ

$$\sum_{i=1}^{n} \frac{x_i}{\Theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\Theta}$$

$$\Rightarrow \sum_{i=1}^{n} x_i(1-\Theta) = \sum_{i=1}^{n} (m-x_i)(\Theta)$$

$$\Rightarrow \Theta = \frac{1}{m} \sum_{i=1}^{n} x_i \quad (1-\Theta \approx \Theta)$$

$\therefore$ MLE of Θ is sample mean of observations