Foundations of Machine Learning

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Week 4 Lab: Concept Check Exercises

Subgradients

1. ($\star$) If $f : \mathbb{R}^n \to \mathbb{R}$ is convex and differentiable at x, the $\partial f(x) = \{\nabla f(x)\}$.

2. Fix $f : \mathbb{R}^n \to \mathbb{R}$ and $x \in \mathbb{R}^n$. Then the subdifferential $\partial f(x)$ is a convex set.

3. (a) True or False: A subgradient of $f : \mathbb{R}^n \to \mathbb{R}$ at x is normal to a hyperplane that globally understimates the graph of f.

   (b) True or False: If $g \in \partial f(x)$ then $-g$ is a descent direction of f.

   (c) True or False: For $f : \mathbb{R} \to \mathbb{R}$, if $1, -1 \in \partial f(x)$ then x is a global minimizer of f.

   (d) True or False: Let $f : \mathbb{R}^n \to \mathbb{R}$ and let $g \in \partial f(x)$. Then $\alpha g \in \partial f(x)$ for all $\alpha \in [0, 1]$.

   (e) True or False: If the sublevel sets of a function are convex, then the function is convex.

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x_1, x_2) = |x_1| + 2|x_2|$. Compute $\partial f(x_1, x_2)$ for each $x_1, x_2 \in \mathbb{R}^2$.