EXPERIMENT 5

STUDY OF A PARALLEL *LCR* CIRCUIT

Structure

5.1 INTRODUCTION

In Experiment 4 you have studied the frequency response of a series *LCR* circuit. While performing the experiment, you have learnt that the circuit delivers power only in a limited band of frequencies around its resonant frequency. Hence, it can be used for tuning to a particular channel on TV or radio receiver.

What happens if we connect the capacitor, inductor and resistor in a parallel arrangement as shown in Fig. 5.1, and apply an ac voltage? We observe that we get the same resonant frequencies as in a series *LCR* circuit, but the behaviour of current in this case is opposite.

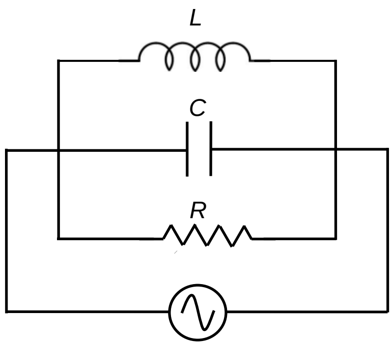


Fig. 5.1: **Parallel** *LCR* **circuit.**

The parallel combination of L and C is commonly termed as the *tank circuit*. At resonant frequency, f_r, the reactances of L and C are equal but opposite. Hence, the energy stored in

Since minimum current flows through the circuit at resonant frequency, the resonant frequency of parallel *LCR* circuit is sometimes called **anti-resonant frequency.**

the circuit keeps on circulating within these two components in the form of alternating current of frequency f_r. Effectively, at this frequency, minimum current flows from the source and the power delivered from the *LCR* circuit is minimum. That is, we are able to reject a frequency band about f_r from the signal and hence such a circuit can be used as a **band stop filter**. If we tap the signal across the capacitor or inductor, we are able to get a signal of a selected band of frequency. Hence, parallel *LC* (tank) circuits are commonly used in **oscillators** (which are used as sinusoidal frequency generators in different electronics applications).

In the first part of this experiment, you will study the frequency response of a parallel *LCR* circuit. You will determine the resonant frequency, band width and calculate the quality factor of the circuit. You know that the inductor is made up of winding of metal wire. Hence, it possesses finite resistance. So, in practice, inductor is a combination of an inductor in series with some resistor. This fact affects the performance of a parallel *LCR* circuit. The resonant frequency of practical parallel *LCR* circuit is not exactly the same as that of a series *LCR* circuit. In the second part of the experiment, you will study the effect of resistance value in the circuit on the resonant frequency.

Expected Skills

After performing this experiment, you should be able to

- ❖ build a parallel *LCR* circuit;
- ❖ estimate its ideal resonant frequency;
- ❖ determine the resonant frequency of a practical circuit;
- ❖ determine the band width of the circuit from frequency-impedance plot and calculate the quality factor; and
- ❖ investigate the effect of the value of *R* on the resonant frequency of the parallel *LCR* circuit.

You will require the following components and apparatus for this experiment.

Apparatus required

An inductor (any value in 50 mH – 100 mH range), a capacitor (any value in 0.01 μF to 1 μF range), resistance box (10 Ω – 1000 Ω), audio frequency oscillator, ac milliammeter, ac voltmeter and connecting wires.

Let us first present a brief overview of the theory of parallel *LCR* circuit and obtain the expression for resonant frequency of the ideal and practical parallel *LCR* circuits.

5.2 THEORY GOVERNING PARALLEL *LCR* CIRCUITS

We begin this discussion by a parallel *LCR* circuit comprising ideal components. We analyse it using the approach of admittances of the components rather than impedances that we considered in the series *LCR* analysis. You may know that admittance is defined as $Y = \frac{1}{X}$, where *X* is the impedance.

Let us first obtain the expression for the resonant frequency of an ideal parallel *LCR* circuit. Then we will consider the effects of practical components on circuit performance.

5.2.1 Resonant Frequency of an Ideal Parallel *LCR* Circuit

Let the circuit shown in Fig. 5.1, comprise a parallel combination of an *ideal* inductor, an *ideal* capacitor and a resistor.

The total admittance of the circuit in parallel arrangement of the components is given by

$$Y = Y_1 + Y_2 + Y_3$$

Component	Admittance
Inductor	$\frac{1}{j\omega L}$
Capacitor	$j\omega C$
Resistor	$\frac{1}{R}$

where Y_1, Y_2 and Y_3 are the admittances corresponding to *L*, *C* and *R*, respectively. Therefore, we can write

$$Y = \frac{1}{j\omega L} + j\omega C + \frac{1}{R}$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

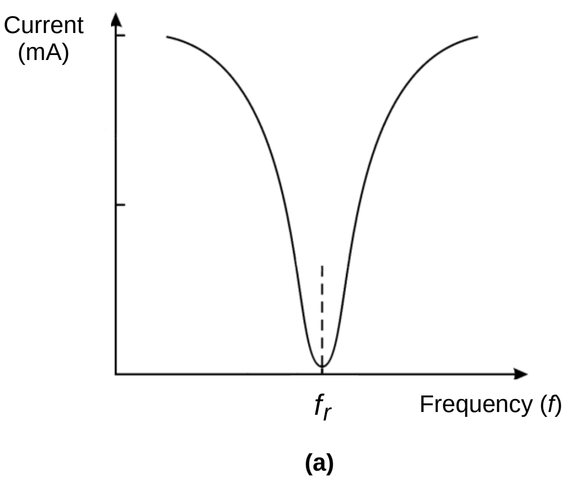
At resonant frequency $\omega_r\ (= 2\pi f_r)$, the imaginary part of admittance is zero.

$$\therefore \quad \omega C - \frac{1}{\omega L} = 0 \qquad (Y_L = Y_C)$$

$$\therefore \quad f_r = \frac{1}{2\pi\sqrt{LC}} \tag{5.1}$$

Note that this expression of resonant frequency is the same as in the case of series resonance circuit.

Typical plots of current versus frequency and impedance versus frequency for a parallel *LCR* circuit are shown in Fig. 5.2a and b respectively. If the source voltage is constant, the impedance curve of the circuit will have a maximum at f_r, as shown in Fig. 5.2b. Thus, minimum current flows through the circuit at f_r. So, we get a dip in the current-frequency plot.



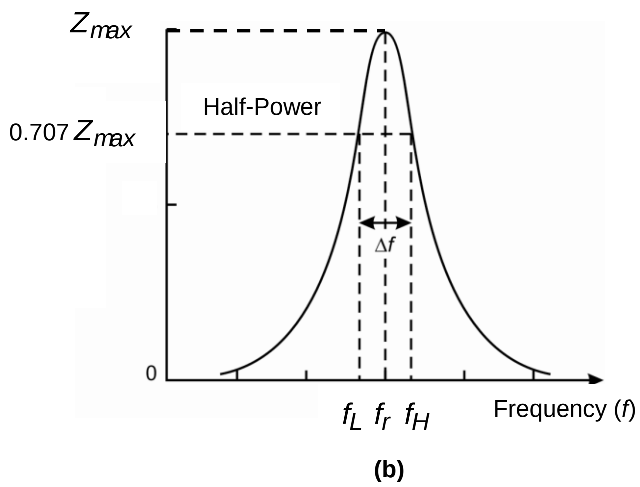


Fig. 5.2: **a) Current; b) impedance versus frequency plots of parallel *LCR* circuit.**

The bandwidth $(\Delta f = f_H - f_L)$ can be obtained from the impedance versus frequency plot (in Fig. 5.2b) by finding the frequencies corresponding to the impedance value of $0.707 \times Z_{max}$.

The quality factor of the circuit can be calculated by the following relation:

$$Q = \frac{f_r}{\Delta f} \tag{5.2}$$

In practice, the inductor (which is formed by winding a wire in the form of a coil) is always accompanied by a finite value of resistance of the wire. We now discuss, in brief, the effect of the internal resistance of the inductor on circuit performance.

5.2.2 Significance of Resistance of the Inductor

Eq. (5.1) gives the resonant frequency of the circuit comprising ideal components. Now, if we consider a practical inductor, the parallel *LC* circuit will get modified as shown in Fig. 5.3. Here we are considering only the *LC* part of the circuit shown in Fig. 5.1, which contributes in determination of resonant frequency.

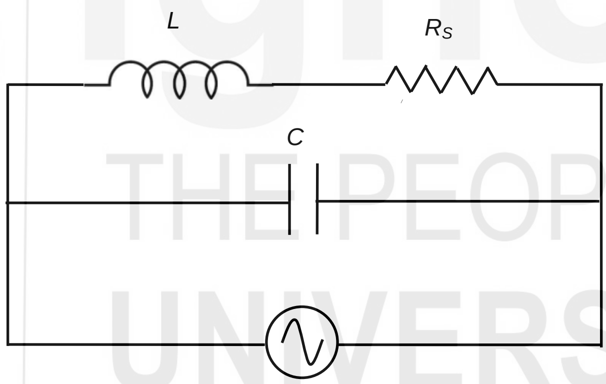


Fig. 5.3: **Parallel** *LCR* **circuit with internal resistance of inductor,** R_S.

The impedance of the inductor branch (*RL*) of the circuit is

$$Z_{RL} = jX_L + R_S \tag{5.3}$$

$$Y_{RL} = \frac{1}{Z_{RL}} = \frac{1}{R_S + jX_L} = \frac{R_S}{R_S^2 + X_L^2} - j\frac{X_L}{R_S^2 + X_L^2} \tag{5.4}$$

We can rewrite Eq. (5.4) as

$$Y_{RL} = \frac{1}{\dfrac{R_S^2 + X_L^2}{R_S}} + \frac{1}{j\left(\dfrac{R_S^2 + X_L^2}{X_L}\right)} \tag{5.5}$$

$$= \frac{1}{R_E} + \frac{1}{jX_{LE}}.$$

where $R_E = \dfrac{R_S^2 + X_L^2}{R_S}$ and $X_{LE} = \dfrac{R_S^2 + X_L^2}{X_L}$ are effective resistance and inductive reactance, respectively; which appear in the circuit as parallel combination. So we can redraw the circuit in Fig. 5.3 as shown in Fig. 5.4.

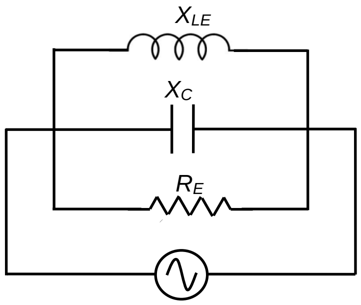


Fig. 5.4: **Parallel *LCR* circuit with effective resistance R_E of *L* in parallel arrangement.**

Now, the admittance of this circuit is

$$Y_T = \frac{1}{R_E} + j\left(\frac{1}{X_C} - \frac{1}{X_{LE}}\right) \tag{5.6}$$

At resonant frequency $\dfrac{1}{X_C} = \dfrac{1}{X_{LE}}$

or $$X_C = X_{LE} \tag{5.7}$$

Substituting for X_{LE}, we get

$$X_C = \frac{R_S^2 + X_L^2}{X_L} \tag{5.8}$$

Simplifying Eq. (5.8), we get

$$R_S^2 + X_L^2 = X_C X_L = \frac{1}{\omega_r C} \times \omega_r L = \frac{L}{C}$$

$$\therefore \quad X_L^2 = \frac{L}{C} - R_S^2$$

or $$X_L = \sqrt{\frac{L}{C} - R_S^2}$$

or $$2\pi f_r L = \sqrt{\frac{L}{C} - R_S^2}$$

$$\therefore \quad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_S^2}{L^2}} \tag{5.9}$$

So now you must have understood the significance of the inductor resistance in determination of resonant frequency of a parallel *LC* circuit. Since the frequency decreases with increase in R_S, you can observe a resonant frequency shift by connecting another variable resistor in series with L (and R_S).

With this theoretical understanding, you can perform your experiment. In the first part of the experiment, you will determine the resonant frequency and bandwidth of a parallel *LCR* circuit assuming L to be ideal. In the second part of the experiment, you will investigate the effect of resistance in series with L on the resonant frequency.

5.3 DETERMINATION OF RESONANT FREQUENCY AND BANDWIDTH OF A PARALLEL *LCR* CIRCUIT

Procedure

1. Connect the circuit with L, C and R in parallel combination as shown in Fig. 5.5. Similar to the series *LCR* experiment choose the values of L and C such that the resonant frequency [calculated using Eq. (5.1)] is in the range of 1 kHz to 2 kHz. Choose value of R to be about 200 Ω. Note these values of L, C and R in the Observation Table 5.1.

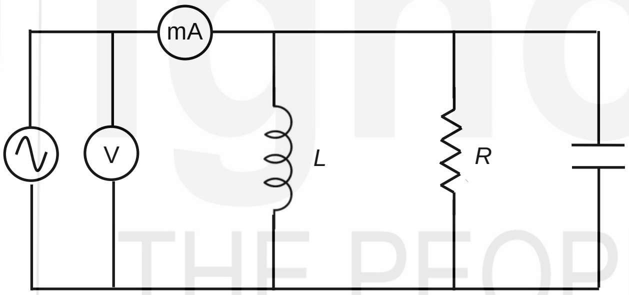


Fig. 5.5: **Parallel** *LCR* **circuit for determination of** f_r **and bandwidth.**

2. Remember to connect the ac milliammeter in series and the ac voltmeter in parallel to the audio frequency generator.
3. Set the output voltage (V_{ac}) of the audio generator to 10 V and make sure that this value remains constant throughout this experiment.
4. Begin the experiment at 100 Hz frequency. Note the value of current and frequency in the Observation Table 5.1.
5. Increase the frequency in steps of 100 Hz upto 4 kHz and keep on recording the current value at each frequency. You should initially see a decrease in the current and beyond f_r again an increase.
6. Calculate the value of impedance Z by dividing the voltage (V_{ac}) by the current and note it in the Observation Table 5.1.
7. Plot the graph of impedance versus frequency. Find the resonant frequency corresponding to the maximum value of Z (Z_{max}). This is the observed resonant frequency $f_{r(obs)}$.
8. Calculate the value of $0.707 \times Z_{max}$ and find the corresponding frequencies (f_L and f_H) from the Z versus f plot. Obtain the bandwidth of your circuit:

$$\Delta f = f_H - f_L$$

9. Calculate the quality factor of your circuit:

$$Q = \frac{f_r}{\Delta f}$$

10. Report f_r, Δf and Q in Results and state whether there is any deviation in the values of f_r calculated by you in Step 1 and obtained from graph in Step 7. Comment on your result.

Observation Table 5.1: Determination of resonant frequency and bandwidth

Inductance = L = ………………….mH

Capacitance = C =...........................μF

Resistance = R =...........................Ω

ac Voltage = V_{ac} = ………………….V

S.No.	Frequency f (Hz)	Current I (mA)	Z (Ω) $= V_{ac}/I$
1.	100		
2.	200		
3.	.		
.	.		
.	4,000		

Result: i) Calculated resonant frequency = $f_{r\,(calc)}$ = ………………… Hz

ii) Resonant frequent obtained experimentally = $f_{r\,(obs)}$ = ………………… Hz

iii) Lower cut-off frequency = f_L = ………………… Hz

iv) Upper cut-off frequency = f_H = ………………… Hz

v) Bandwidth Δf = $f_H - f_L$ = ………………… Hz

vi) Quality factor Q = $\dfrac{f_{r(obs)}}{f_H - f_L}$ = …………………

You can now perform the second part of the experiment where you find the dependence of resonant frequency on the value of R.

5.4 STUDY OF DEPENDENCE OF RESONANT FREQUENCY ON THE VALUE OF RESISTANCE

Procedure

1. Connect the circuit as shown in Fig. 5.6. Here you will be connecting three different values of R and studying the current-frequency variation in each case.

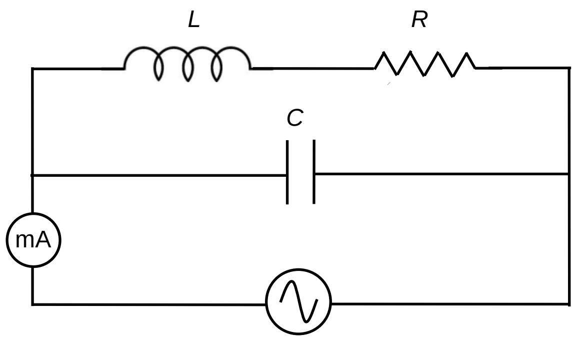


Fig. 5.6: **Circuit to study dependence of f_r on the value of *R*.**

2. Initially connect R = 100 Ω in the circuit. As you have done in the previous part of the experiment, take readings of frequency and current and note them in Observation Table 5.2 for R = 100 Ω.

3. Now change the values of R to 300 Ω and 500 Ω and repeat Step 2 in each case.

4. Plot the frequency versus current curves for all three sets of observations on the same graph paper.

5. Do the curves resemble Fig. 5.2a? Find the value of f_r in each case. Do you observe any change in the resonant frequency with resistance? Is there any systematic trend in the shift of f_r? Comment on your result.

Observation Table 5.2: Effect of resistance on resonant frequency

S.No.	Frequency (Hz)	Current (mA)		
		$R_1 = 100\ \Omega$	$R_2 = 300\ \Omega$	$R_3 = 500\ \Omega$
1.	100			
2.	200			
3.	.			
.	.			
.	.			
.	4,000			

Result: Resonant frequency for R_1 (= 100 Ω) = ………………… Hz
Resonant frequency for R_2 (= 300 Ω) = ………………… Hz
Resonant frequency for R_3 (= 500 Ω) = ………………… Hz

Conclusions

Based on the plot of current versus frequency for different values of R, write your conclusions about the relation between resistance and resonant frequency.