EXPERIMENT 3

STUDY OF A SERIES *RC* CIRCUIT

Structure

3.1 INTRODUCTION

You have learnt about circuit elements like resistor, capacitor and inductor in your school physics course. Capacitor and inductor are capable of storing energy within them. A combination of resistor-capacitor is very useful in day-to-day applications. Due to frequency dependent impedance offered by the capacitor, *RC* circuits are also used as filters for allowing only preferred frequencies to pass through.

If we charge a capacitor by an external voltage source, we can use this stored energy to power the other circuit elements when the voltage source is absent. So it can work as a dc voltage provider in the power supply circuits. We need to accumulate the charge on the capacitor in this application and it requires some time to build this charge on the capacitor plates. The charging time of the capacitor depends on the value of resistance and capacitance in the circuit. The product $R \times C$ is called the **time constant** of the circuit.

In this experiment you will be studying the charging and discharging of a capacitor in a series *RC* circuit and find the time constant of this circuit using two different methods.

For a large time constant circuit you will use a dc voltage source for charging and measure the voltage across the capacitor while it is charging and discharging. In case of small time constant circuit, you will use a square wave generator to observe the charging-discharging cycle with the help of a cathode ray oscilloscope (CRO).

Expected Skills

After performing this experiment, you should be able to

- ❖ identify polarities of polar capacitor;
- ❖ build a series circuit using a resistor, capacitor and dc voltage source / function generator;
- ❖ estimate the time constant of the built circuit;
- ❖ set a CRO for observing charging/discharging of a capacitor with function generator; and
- ❖ take readings of charging/discharging of a capacitor with dc voltage source using voltmeter/multimeter.

For performing this experiment, you will need following apparatus and components:

Apparatus required

Capacitors of two values (typically 10 nF and 1000 μF), resistors of two values (typically 10 kΩ and 100 kΩ), dual beam/trace CRO, square wave (function) generator, dc voltage source (0 – 10V), multimeter / voltmeter, stop-watch, bread board for circuit assembly, connecting wires.

3.2 CHARGING/DISCHARGING OF A CAPACITOR IN A SERIES *RC* CIRCUIT

Before you do the experiment, you should learn the theory underlying charging and discharging of a capacitor. You should also learn about the relevant current-voltage equations.

3.2.1 Capacitor Charging Equations

It is customary to indicate dc values of current, voltage, charge etc. with capital letters (I, V, Q, etc.) while time dependent values by small letters (i, v, q, etc.)

From school physics and Unit 11 of BPHCT-133, you know that the charge (Q) stored on a capacitor plate and its voltage (V_C) are related by the equation

$$Q = CV_C \qquad (3.1)$$

where C is the capacitance of the capacitor.

Consider the *RC* circuit shown in Fig. 3.1. It consists of a capacitor C and a resistor R connected in series with dc voltage source V_0 through a switch S.

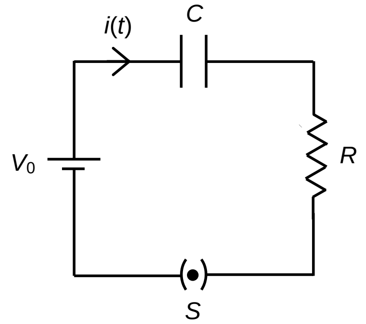


Fig. 3.1: **Series *RC* circuit for charging of a capacitor.**

When *S* is closed at time $t = 0$, current $i(t)$ begins to flow through the circuit and the capacitor gets charged. Here, $i(t)$ is time dependent current, which is maximum at $t = 0$ and reduces to zero when the capacitor is completely charged. The time dependent capacitor voltage $v_C(t)$ can be written as

$$v_C(t) = \frac{q(t)}{C} \tag{3.2}$$

where $q(t)$ is charge on the capacitor at time *t*.

Using Kirchoff's voltage law, we can write the voltage equation for the circuit as:

$$V_0 = v_R + v_C \tag{3.3}$$

where V_0 is supply voltage, $v_R [= i(t) \times R]$ is the voltage across the resistor and v_C is the voltage across the capacitor. At any time *t*, we have

$$V_0 = i(t)R + v_C(t)$$

or

$$V_0 - v_C(t) = i(t)R \tag{3.4}$$

From Eq. (3.2), the current $i(t)$ can be expressed as

$$i(t) = \frac{dQ(t)}{dt} = C\frac{dv_C(t)}{dt} \tag{3.5}$$

Substituting this value of $i(t)$ in Eq. (3.4), we can write

$$V_0 - v_C(t) = RC\frac{dv_C(t)}{dt} \tag{3.6}$$

Rearranging the terms, we get

$$\frac{dt}{RC} = \frac{dv_C(t)}{V_0 - v_C(t)} \tag{3.7}$$

Integrating Eq. (3.7), we get

$$\frac{t}{RC} = -\ln[(V_0 - v_C(t)] + K \tag{3.8}$$

i.e.

$$-\frac{t}{RC} + K = \ln[V_0 - v_C(t)] \tag{3.8a}$$

where *K* is the constant of integration. At $t = 0$, voltage on the capacitor is zero. Applying this condition in Eq. (3.8a), we get $K = \ln V_0$.

So, we can write

$$-\frac{t}{RC} + \ln V_0 = \ln[V_0 - v_C(t)] \tag{3.9}$$

$$\therefore \quad -\frac{t}{RC} = \ln\frac{V_0 - v_C(t)}{V_0} \tag{3.10}$$

Taking exponentials of both sides, we have

$$e^{-t/RC} = \frac{V_0 - v_C(t)}{V_0}$$

or $$V_0 e^{-t/RC} = V_0 - v_C(t) \qquad (3.11)$$

Hence, the voltage across the capacitor at time t is given by

$$v_C(t) = V_0(1 - e^{-t/RC}) \qquad (3.12)$$

The product RC is called the **time constant** of the circuit and sometimes denoted by greek symbol τ (tao). So, when the capacitor is subjected to a dc voltage, it gets charged and the voltage across it increases from 0 and approaches the maximum value (source voltage) in an exponential manner. After a sufficiently long time ($t = 5\tau$), the capacitor acquires more than 99% of the source voltage across it.

If you consider the variation of voltage with time, you will get an exponential curve as shown in Fig. 3.2.

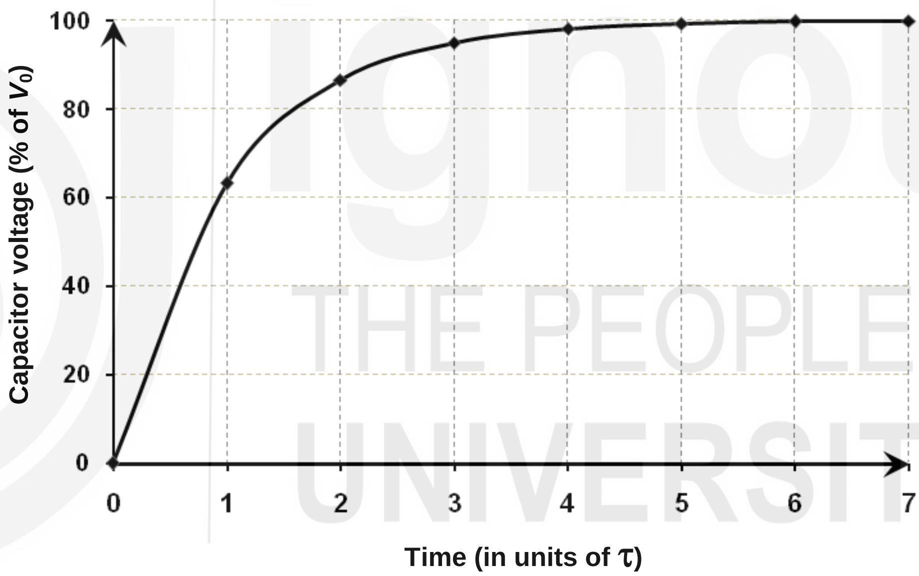


Fig. 3.2: **Capacitor charging curve.**

The voltage across capacitor in terms of percentage of source voltage at different times is given in Table 3.1.

Table 3.1: Voltage across capacitor during charging

Sl. No.	Time in terms of time constant	Voltage across capacitor (% of source voltage)
1.	0	0
2.	$1\tau = 1 \times RC$	63.2%
3.	$2\tau = 2 \times RC$	86.5%
4.	$3\tau = 3 \times RC$	95.0%
5.	$4\tau = 4 \times RC$	98.2%
6.	$5\tau = 5 \times RC$	99.3%

So to charge a capacitor completely upto the source voltage, you must apply this voltage across it at least for the time equal to 5τ.

3.2.2 Capacitor Discharging Equations

Just like the charging process, the discharging of the capacitor is also governed by an exponential relationship. Suppose that in the circuit shown in Fig. 3.1, the capacitor is charged upto voltage V_0. Then the switch *S* is opened and the voltage source is removed. Now, the capacitor is allowed to discharge through the *RC* circuit shown in Fig. 3.3 by closing the switch *S*. Current $i(t)$ starts flowing through the circuit and voltage across the capacitor starts reducing. From Kirchoff's voltage law, we can write the equation for time dependent voltage across the capacitor as

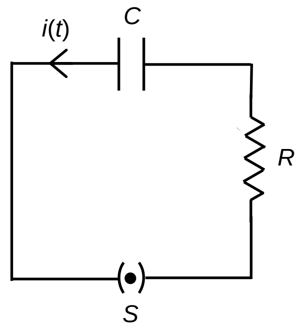


Fig. 3.3: **Discharging of capacitor.**

$$i(t)R + v_C(t) = 0 \tag{3.13}$$

Substituting

$$i(t) = \frac{dq(t)}{dt} = C\frac{dv_C(t)}{dt}$$

in Eq. (3.13), we get

$$RC\frac{dv_C(t)}{dt} + v_C(t) = 0 \tag{3.14}$$

Rearranging the terms, we get

$$\frac{dv_C(t)}{v_C(t)} = -\frac{dt}{RC} \tag{3.15}$$

Integrating the Eq. (3.15), we get

$$\ln v_C(t) = -\frac{t}{RC} + K \tag{3.16}$$

where *K* is the constant of integration. At $t = 0$ the capacitor is fully charged, hence we have $v_C(t) = V_0$. Applying this condition to Eq. (3.16), we get $K = \ln V_0$.

Substituting the value of *K* in Eq. (3.16), we can write

$$\ln v_C(t) = -\frac{t}{RC} + \ln V_0$$

$$\therefore \quad \ln\frac{v_C(t)}{V_0} = -\frac{t}{RC} \tag{3.17}$$

Taking the exponential of both sides, we have

$$\frac{v_C(t)}{V_0} = e^{-t/RC}$$

Hence, the voltage reduction across the capacitor in the discharging process is governed by the equation

$$v_C(t) = V_0\, e^{-t/RC} \tag{3.18}$$

So for the discharging process, you should be able to assert that

i) in 1τ (= 1 × RC) s the capacitor voltage reduces to 1/e = 0.368 of its initial voltage value; and

ii) the time required for the capacitor to discharge almost completely (< 1% of V_0) is more than 5τ (= 5 × R × C).

3.2.3 Significance of Time Constant

You must have realized by now that the time constant of an *RC* circuit is very important in its functioning. The value of the time constant is determined by the product of the values of capacitance and resistance used in the circuit.

For different applications of the *RC* circuits, different values of time constant are preferred.

You know that the time required for complete charging or discharging of a capacitor is equal to atleast 5 τ. Now, if you are working with a circuit using high frequency voltage, the time period for which the charging voltage is available per cycle is short. So, in order to charge the capacitor sufficiently during that available time, you need to have a very small value of time constant ($R \times C$).

On the other hand, if you are charging the capacitor with a low frequency or dc signal, then you have a longer time for it to charge. In such cases, you can use large value of the time constant ($R \times C$).

In this experiment, you will work with *RC* circuits having both small and large values of the time constant.

Now that you have learnt about the time dependence of voltage in a series *RC* circuit, you can perform the experiment.

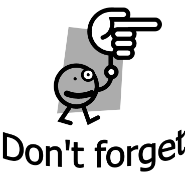
3.3 DETERMINATION OF TIME CONSTANT OF AN *RC* CIRCUIT

Since the nature of voltage (dc, ac, high frequency, etc.) governs the choice of time constant *RC*, you will need to build different circuits and follow different procedures for studying *RC* circuits having large and small time constants.

Let us begin with a series *RC* circuit having a large time constant.

3.3.1 Study of Large Time Constant *RC* Circuit

For this part of the experiment you will require a dc voltage source (power supply) with about 10V output voltage, a capacitor of high value (any value in the range of 470 μF – 1000 μF), resistor of high value (any value in the range of 100 kΩ – 470 kΩ), a dc voltmeter of 0 – 10 V range or a multimeter, a plug key and a stop watch.



You will notice that the high value capacitor is usually in a metal can packing with two leads. One of these leads is marked as '+' and the other as ' – '. This indicates that it is an electrolyte capacitor which is polar in nature and you must pay attention to the polarities of its leads while connecting to the dc voltage source. The voltage rating of the capacitor should always be more than the source voltage you are using.

Procedure:

1. Connect the components in a series *RC* circuit as shown in Fig. 3.4. Note that the capacitor and resistor are connected in series with the voltage source V_0. The voltmeter (or multimeter in dc voltage measurement mode) is connected in parallel with the capacitor. Keep the key *S* open at the beginning.

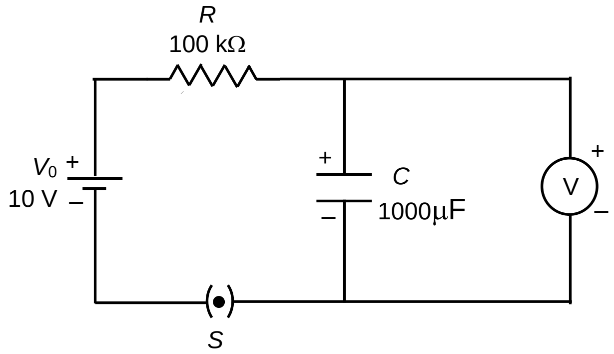


Fig. 3.4: **Circuit for studying charging in a large time constant series *RC* circuit.**

2. If you are using a variable dc voltage supply as source, then set it for the output voltage = 10V.

3. From the nominal values of the resistors and capacitor estimate the value of $R \times C$ (time constant) and note it down in the Observation Table 3.2.

   For example, if you are using $R = 100\ \text{k}\Omega$ and $C = 1000\ \mu\text{F}$, then the time constant

$$\tau = R \times C = 100\ \text{k}\Omega \times 1000\ \mu\text{F} = 100 \times 10^3\ \Omega \times 1000 \times 10^{-6}\ \text{F}$$

$$= 100\ \text{s}$$

4. Before starting the current in the circuit, check that the voltage value read by the voltmeter is zero (i.e., capacitor is completely discharged).

5. Now put the plug in key *S* and start the stop watch.

6. Read the voltage across the capacitor after every 10 s and note it along with time in the Observation Table 3.2.

7. Continue this process till at least 5 times the time constant of your circuit. (In the example given in Step 3, you will continue to take the reading upto 500 seconds).

   You will observe that initially the voltage increases faster, but later on the change in voltage is slower. As the time approaches the value 4*RC*, the voltage value becomes almost constant. At time 5*RC*, you will read the voltage which is very near to the source voltage value.

8. Now remove the plug from the key '*S*', so that the charged capacitor holds the voltage across it.

9. Reset the stop watch.

10. Remove the voltage source and connect its two leads together to get the circuit as shown in Fig. 3.5. Take care while handling the circuit, as the charged capacitor may give shock if touched by hand.

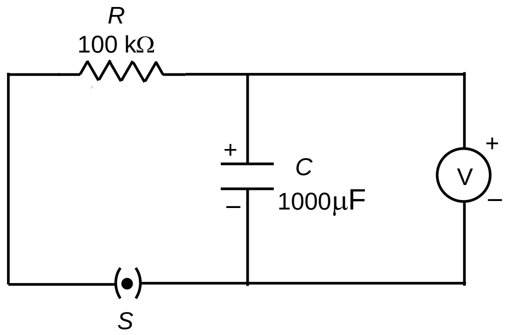


Fig. 3.5: **A circuit for studying the discharging of a capacitor.**

11. The voltage across the capacitor should be nearly at the same value that you noted last in Step 7.
12. Now insert the plug in the key *S* and start the stop watch.
13. Similar to the charging cycle, keep taking the voltage reading after every 10 s and note it in the Observation Table 3.3.
14. Continue to take the readings for more than 5*RC* seconds.
15. Take two graph papers and plot the charging and discharging voltages by taking time (s) along *x*-axis and voltage (V) along *y*-axis. The two plots would look like the ones shown in Figs. 3.6a and b.

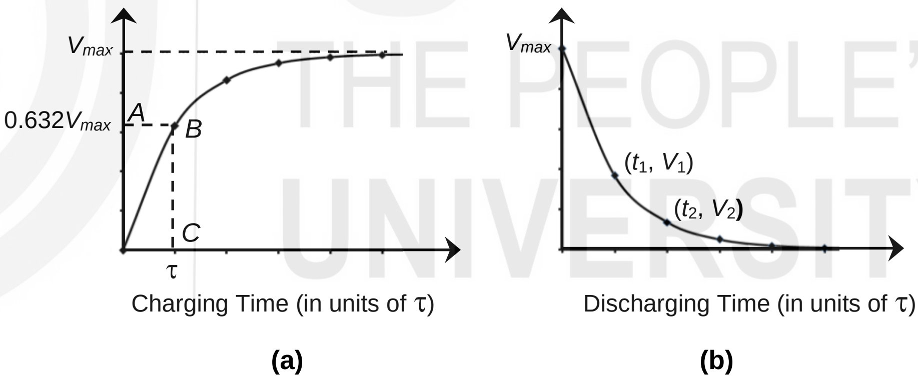


Fig. 3.6: **Typical voltage plots of a) charging; b) discharging cycles of a capacitor**.

16. From the voltage plot of the charging capacitor, you can find out the actual value of time constant τ of the circuit using the values given in Table 3.1.

    You know that in the time period τ during charging cycle, the voltage on the capacitor is 63.2% of maximum voltage. Mark the voltage corresponding to $0.632 \times V_{\text{max}}$ on *y*-axis (point *A* in Fig. 3.6a). Draw a horizontal line (parallel to *x*-axis) from this point so that it intersects the charging curve you have plotted (at point *B*). Now from point *B* draw a vertical line (parallel to *y*-axis), so that it touches the *x*-axis (point *C*). The time corresponding to point *C* is the time constant τ of your circuit. Report it in the Results at (a).

17. Compare this value of τ with the value of the product *RC*, you have calculated from the nominal values of the components (*R* and *C*).

18. You can use the discharging curve also to determine τ for the series *RC* circuit. For this choose two points on the discharging curve, say, (t_1, V_1) and (t_2, V_2) as shown in Fig. 3.6b. From Eq. (3.18) you can write

$$V_1 = V_{max}\, e^{-t_1/\tau} \quad \text{and} \quad V_2 = V_{max}\, e^{-t_2/\tau}$$

$$\therefore \quad \frac{V_1}{V_2} = e^{(t_2 - t_1)/\tau}$$

$$\text{or} \quad \tau = \frac{t_2 - t_1}{\ln(V_1/V_2)} \tag{3.19}$$

If you choose the two points on the curve such that

$$\frac{V_1}{V_2} = e = 2.72$$

$$\text{or} \quad V_2 = V_1/e$$

then $\ln\left(\frac{V_1}{V_2}\right) = 1$ and you can directly get

$$\tau = (t_2 - t_1)\,\text{s} \tag{3.20}$$

In this method you will notice that Eq. (3.19) is independent of the maximum voltage V_{max} and hence it does not depend on when you started the measurement. Report this value of τ in the Results at (b).

19. Is there any difference between the calculated $R \times C$ and graphically obtained value of τ?

If yes, try to find out the reason for the same and write it under the Conclusions.

Observation Table 3.2: Charging of capacitor

(i) Value of Resistance = ... Ω

(ii) Value of Capacitance = ...μF

(iii) Estimated value of time constant = $\tau = R \times C$ = ... s

S.No.	Time (*s*)	Capacitor voltage V_C (V)
1.	0	0.0
2.	10	⋮
3.	20	⋮
4.	30	⋮
⋮	⋮	⋮
⋮	$5 \times \tau = 5 \times R \times C$	⋮

Observation Table 3.3: Discharging of capacitor

S.No.	Time (*s*)	Capacitor voltage V_C (V)
1.	0	$\vdots$
2.	10	$\vdots$
3.	20	$\vdots$
$\vdots$	$\vdots$	$\vdots$
$\vdots$	$5 \times \tau = 5 \times R \times C$	$\vdots$

Results: a) Time constant τ determined from charging curve = ... s

b) Time constant τ determined from discharging curve = ... s.

Conclusions:

i) Does the value of time constant estimated from the values of *R* and *C* match with the ones obtained from (a) charging and (b) discharging curve?

ii) If not, give the reason for the difference.

iii) Calculate the error propagated in the calculation of $R \times C$ by considering the uncertainty in the values of *R* and *C* based on their tolerances. (You have learnt about finding the tolerance in the value of *R* based on the colour code of the tolerance band in Experiment 1).

iv) Now comment about the match between the observed and the estimated values of time constant.

3.3.2 Study of Small Time Constant *RC* Circuit

In the first part of this experiment you applied a dc voltage to charge the capacitor, when the time constant $\tau = (R \times C)$ was large. But in many applications, you need to work with high frequency signals. In such cases, the time period available for charging the capacitor is small (due to fast varying signal voltage), and hence you should use small time constant *RC* circuits. It means that the values of resistors and capacitors are small in such cases.

You will appreciate that in case of such fast time-varying signal you cannot take the readings of capacitor voltage manually as done in the earlier part of the experiment. In such a situation, you can use a cathode ray oscilloscope (CRO) to observe the charging-discharging of the capacitor.

You can perform this part of the experiment if time permits.

To perform the experiment, you will require a signal (function) generator, resistor (~ 10 kΩ), capacitor (10-47 nF), double beam/trace CRO, connecting wires and tracing papers.

Procedure:

You have learnt to use the CRO in the laboratory course on Mechanics for observing Lissajous figures.

1. Connect the circuit as shown in Fig. 3.7.

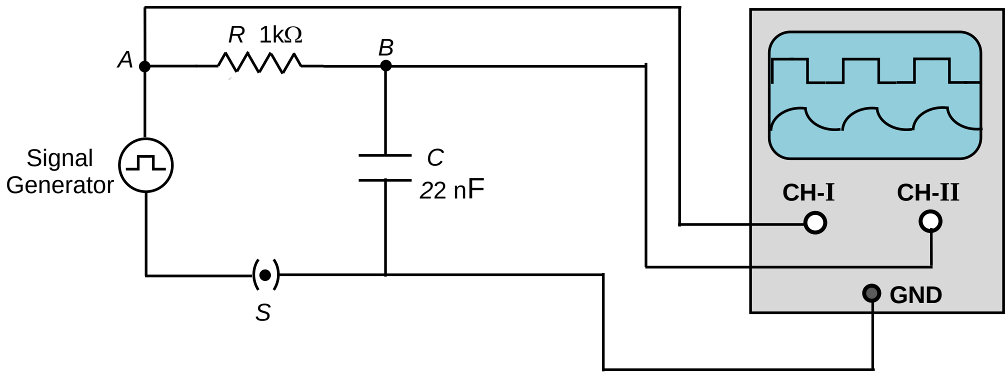


Fig. 3.7: **Set up for the study of small time constant** *RC* **circuit.**

In CH-I input of the CRO, connect the signal from the function generator (point *A*) and give the capacitor voltage (point *B*) to CH-II input. Select the dual beam mode of the CRO.

2. Calculate the time constant τ (= $R\times C$) from the values of the components you are using.

3. Switch on the CRO and you should get two horizontal line traces on the screen. Adjust their positions using *y*-position knobs so that they are separated from each other.

4. Switch on the function generator. Select the square wave output and adjust the voltage level to 10V. You should be able to see this signal on the CH-I of the CRO as shown in Fig. 3.8a.

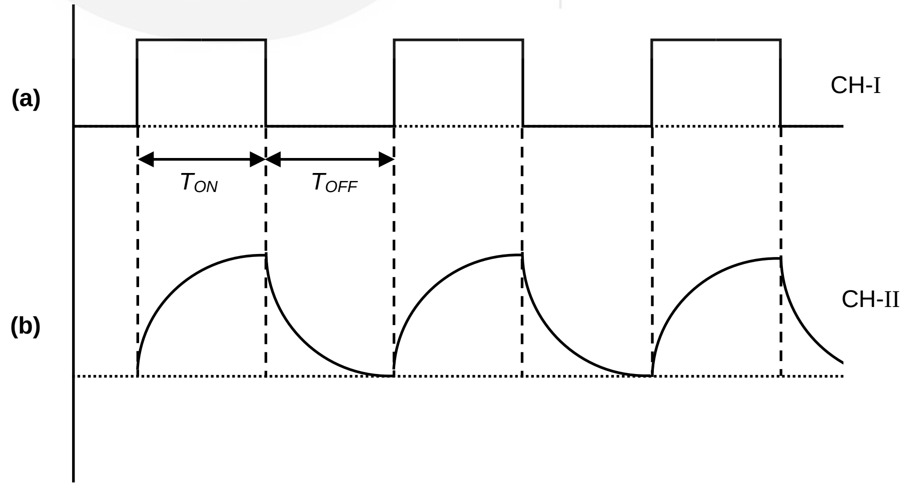


Fig. 3.8: **CRO display of a) input square wave** (CH-I)**; b) voltage across capacitor** (CH-II).

5. In step 2 you have found out the time constant (τ) of the circuit. You know that you need a dc signal for at least $5\times R\times C$ seconds to charge a capacitor completely.

Consider the square wave generated by the signal generator as a repeating (pulsating) dc voltage. Hence you should select the time period of your signal such that it gives high voltage for a period of at least $5 \times R \times C$ seconds.

For example, if you use the same values of components as mentioned in Fig. 3.7 then the time constant of the circuit is

$$\tau = R \times C = 1\ \text{k}\Omega \times 22\ \text{nF} = 1 \times 10^3 \times 22 \times 10^{-9}\ \text{s}$$
$$= 22 \times 10^{-6}\ \text{s} = 22\ \mu\text{s}.$$

So five times the time constant = 110 μs or $\approx$ 0.1 ms.

Effectively, for the square wave shown in Fig. 3.8a, T_{ON} should be 0.1 ms. Naturally, for a square wave $T_{ON} = T_{0FF}$ = 0.1 ms.

Hence,

$$T = T_{ON} + T_{0FF} = 0.1 + 0.1 = 0.2\ \text{ms}.$$

i.e., the frequency of the signal generator should be set at

$$f = \frac{1}{T} = \frac{1}{0.2\ \text{ms}} = 5\ \text{kHz}$$

If you are using any other values of *R* and *C*, you should calculate the required frequency following the steps above, and set the signal generator to give that frequency.

6. You will get the input voltage and voltage across capacitor plots on CH-I and CH-II as shown in Figs. 3.8a and b.

7. Take a tracing paper, cut it to the size of CRO screen and carefully trace the input and output traces on it. Note down the values of *R*, *C* and *f* on this paper.

   You should also note down the chosen time/div and voltage/div on the CRO.

   You should get the height (maximum voltage) of both the plots to be almost equal.

8. Now increase the frequency by 5 times (i.e. the period of on time of square wave cycle is almost equal to the time constant τ of your circuit.) Again trace the curves on another tracing paper and note the respective value of frequency along with maximum voltage and time period on the axis.

9. Paste both the tracing papers in your experiment report sheet / lab book.

10. Write down the conclusions you can draw from the nature of charging-discharging curves in the two cases.