EXPERIMENT 4

STUDY OF A SERIES *LCR* CIRCUIT

Structure

4.1 INTRODUCTION

In your school physics course, you have learnt about resistors, inductors, capacitors and know how these are used as passive elements in an electrical circuit. For instance, a resistor is used as heating element in an electric iron; a capacitor filters ac component in dc and an inductor and a capacitor are combined to tune to a particular frequency of a radio station. These elements are called *passive* since they cannot provide any power amplification to a signal. Basically all these components offer opposition to flow of current through them.

The measure of opposition to the flow of current in a dc circuit is specified in terms of *resistance* and for an ac circuit, we use the term *impedance*. The impedance of a resistor is independent of frequency. The impedances offered by capacitors and inductors are frequency dependent and are expressed in terms of their *reactances* (capacitive and inductive) as shown in Table 4.1.

Table 4.1: Impedances and reactances of various components

Component	Impedance	Reactance
Resistor	$Z_R = R$	$X_R = R$
Capacitor	$Z_C = -\frac{j}{\omega C}$	$X_C = \frac{1}{\omega C}$
Inductor	$Z_L = j\omega L$	$X_L = \omega L$

Due to the frequency dependence of reactances, *L* and *C* play an important role in ac circuits when placed individually, together or in combination with *R*. In this experiment, you will get an opportunity to study the behaviour of these components with variable frequency signals. You will also study the frequency responses of these components individually as well as when all of them are connected in series.

You may recall from your school physics course that in case of *R*, the current and voltage are in phase with each other. But in case of *L*, the voltage leads the current in phase by $\pi/2$ and in case of *C*, it lags by $\pi/2$, as shown in Fig. 4.1.

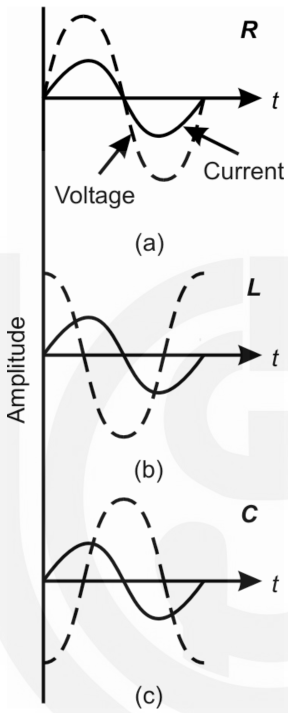

Fig. 4.1: **Phase responses of a)** *R*; **b)** *L*; **c)** *C*.

In case of a series *LCR* circuit, the frequency response curve exhibits a resonant frequency with a spread around it. This spread, determined by the total circuit impedance, is a measure of the **quality factor**, *Q* of the circuit. It is defined as the ratio of the resonant frequency and the bandwidth of the resonance curve (at half-power points). In this experiment, you will also study the dependence of *Q* on *R* in a series *LCR* circuit.

Expected Skills

After performing this experiment, you should be able to

- ❖ understand the frequency response of a resistor, an inductor and a capacitor;
- ❖ build a series *LCR* circuit;
- ❖ draw the frequency response of a series *LCR* circuit;
- ❖ determine the resonant frequency and bandwidth of series *LCR* circuit;
- ❖ calculate the quality factor of a series *LCR* circuit; and
- ❖ determine the dependence of *Q* on *R* in a series *LCR* circuit.

You will require the following apparatus and components to perform this experiment.

Apparatus required

Audio frequency (AF) oscillator, resistors (in 100 – 500 Ω range), inductors (in mH range), capacitors (in μF range), ac milliammeter (0-25 mA), ac voltmeter (0-10V), connecting wires and sand paper.

Before starting the experiment, we take an overview of frequency responses of individual passive elements and their series combination.

4.2 CHARACTERISTICS OF A SERIES-*LCR* CIRCUIT

From your school physics course, you may recall that the combinations of *RC*, *RL* and *LC* are used to filter out unwanted frequencies from a desired signal. A frequency in a very narrow band can be selected by a series or parallel *LCR* combination. To understand this, refer to Fig. 4.2, where we have depicted frequency dependence of reactances of *R*, *L* and *C*.

Note from Fig. 4.2 that at lower frequencies, capacitive reactance X_C is large and inductive reactance X_L is small. Most of the voltage drop in a circuit containing *L*, *C* and *R* in series combination is then across the capacitor. At high frequencies, the inductive reactance is large but the capacitive reactance is low and most of the voltage drop is then across the inductor. In between these two extremes, there is a frequency at which the capacitive and inductive impedances are equal, which means their reactances are exactly equal but act in opposition and cancel each other. This frequency is called *resonant frequency*. We have denoted it by f_r.

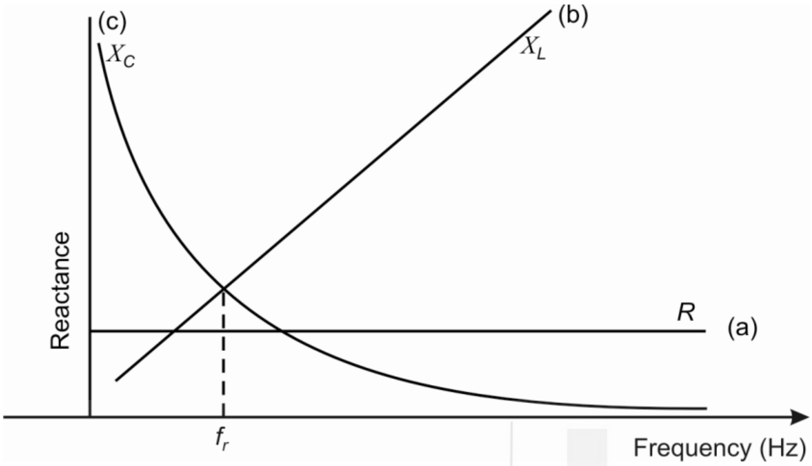

Fig. 4.2: **Frequency dependence of reactance for a)** *R*; **b)** *L*; **c)** *C*.

The resonant frequency is defined by (read the margin remark)

$$f_r = \frac{1}{2\pi\sqrt{LC}}. \qquad (4.1)$$

At resonant frequency f_r, reactances of *L* and *C* are equal. It means

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

i.e.

$$4\pi^2 f_r^2 = \frac{1}{LC}$$

Hence

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

In a series *LCR* circuit, at resonance condition, the impedance is minimum as only the resistance *R* in the circuit opposes the flow of current. The current at resonant frequency is equal to the applied voltage divided by the circuit resistance, and thus can be very large if the resistance is low.

For a fixed applied voltage, the expected qualitative variation in circuit current with frequency is shown in Fig. 4.3. Note that current is maximum at the resonant frequency and decreases on both sides around it, giving a bell-shaped curve.

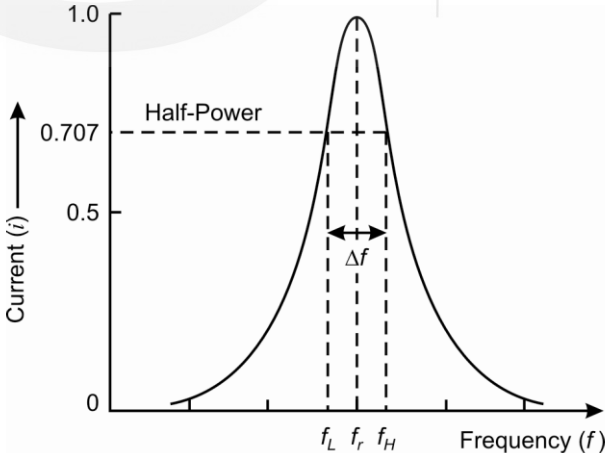

Fig. 4.3: **Resonance curve for a series** *LCR* **circuit:** f_r **and** $\Delta f (= f_H - f_L)$ **respectively denote resonant frequency and bandwidth.** f_H **and** f_L **denote the higher and lower frequencies respectively of the bandwidth.**

At resonant frequency, the maximum power in the circuit is given by

$$P_{\max} = i_{\max}^2 \times Z = i_{\max}^2 \times R \qquad (4.2)$$

At resonant frequency, the maximum power in a series-*LCR* circuit is given by

$$P_{max} = i_{max}^2 R \qquad \text{(i)}$$

and output power is

$$P_o = i_o^2 R . \qquad \text{(ii)}$$

At half power points, we can write

$$P_o = \frac{1}{2} P_{max} .$$

On combining (i) and (ii), we get

$$i_o^2 = \frac{i_{max}^2}{2}$$

or

$$i_o = \frac{i_{max}}{\sqrt{2}}$$

$$= 0.707 \; i_{max}$$

That is, the current in a series *LCR* circuit at half power points is 0.707 times the maximum

From Fig. 4.3, you may conclude that a series *LCR* circuit will sustain only those frequencies which fall within the width of the bell shape. This phenomenon facilitates frequency selectivity, which is quantified in terms of the *bandwidth* of the circuit. It is defined as the range of frequencies corresponding to half-power points. Physically, it means that the *LCR* circuit operates and delivers more than half the maximum power in this frequency range. In Fig. 4.3, the half-power points correspond to 0.707 times the value of maximum current (read the margin remark).

The frequency difference $\Delta f = f_H - f_L$ is a measure of the bandwidth of the resonance curve. In terms of resonant frequency f_r and bandwidth Δf, we characterise the quality of a circuit by defining the **quality factor** Q of the circuit as

$$Q = f_r \,/\, \Delta f \qquad (4.3)$$

As such, Q determines the sharpness of resonance. Q is usually used in designing electronic circuits in communication engineering and the typical values of Q are of the order of 10^2 to 10^5. Quality factor can also be expressed in terms of component values. Without going into a detailed derivation, we can write their relation in the following form:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad (4.4)$$

A series *LCR* circuit is also called a *series resonant circuit*. It enables us to select the signals of only one frequency and reject all the others. The sharper is the *i-f* curve of an *LCR* circuit, the more selective it is for a particular frequency. The selectivity of an *LCR* series resonant circuit depends on the resistance in the circuit, as is clear from Eq. 4.4. In Fig. 4.4, we have compared the *i-f* curves for two value of *R*. You will note that the curve corresponding to smaller *R* is sharper. That is, the bandwidth is smaller and frequency selectivity would be better in a low resistance ac circuit.

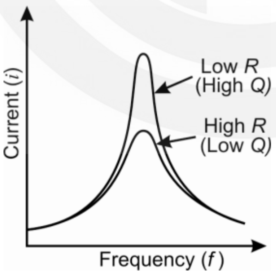

Fig. 4.4: **Dependence of Q on value of resistance.**

We use a series *LCR* circuit in the antenna circuit of radio and TV receivers. By suitably adjusting the values of *L* and *C*, we tune to the desired frequency of a radio or TV station.

SAQ 1 – Tuning using *LCR* Circuit

You want to tune your radio to listen to IGNOU programmes. The frequency of Gyan Vani radio station is located in a densely populated frequency range, that is being used by many broadcasters. What type of series *LCR* circuit should it have: one with a low or a high value of *R*? Justify your answer, giving reason.

You will perform this experiment in two parts. Initially you will study the frequency response of individual components like resistor, capacitor and inductor separately. Then you will connect these components in a series circuit and study its frequency response.

4.3 FREQUENCY RESPONSES OF A RESISTOR, AN INDUCTOR AND A CAPACITOR

You may have realised by now that in an ac circuit three quantities may change: voltage, current and frequency. To study frequency responses of different circuit components, we must have a suitable variable frequency source. For this, in a physics laboratory, we normally use an audio frequency (AF) oscillator, which generates signals of frequencies in the range of 20 Hz to 20,000 Hz. For studying the frequency responses of *R*, *L* and *C*, we keep the current constant and measure voltages V_R, V_L and V_C across *R, L* and *C* respectively, for different values of *f*. For studying the frequency response of a series *LCR* circuit, we keep the voltage fixed and measure current for different values of *f*.

An AF oscillator usually has a number of knobs. The three knobs with which you will work in this experiment are the voltage selector, the frequency range selector and the frequency selector. The voltage selector determines the voltage of the oscillator, while the other two knobs deal with frequency. When the frequency of the output current is varied, the voltage also changes. For keeping the output voltage fixed, it is safer to vary the frequency only in the range 100 Hz to 10,000 Hz. Before starting the experiment, it is important that you familiarise yourself with the various knobs and input and output leads of the oscillator. To fully convince yourself, you could also consult your counsellor.

Now we describe the procedure you should follow to perform this part of the experiment.

Procedure

1. Place the AF oscillator on the table and connect it with the resistor and ac milliammeter, as shown in Fig. 4.5a. V is an ac voltmeter, which is connected across the resistor to measure output voltage.

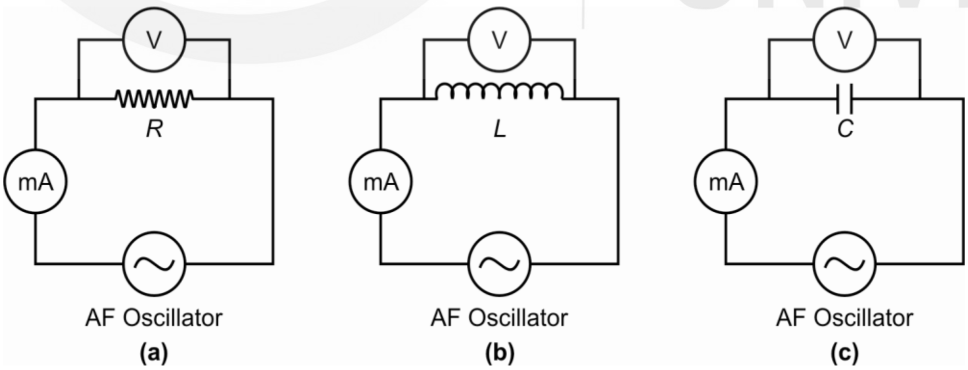

Fig. 4.5: **Circuit diagram for studying frequency responses of a)** *R*; **b)** *L*; **c)** *C*.

2. Connect the power supply cord of AF oscillator to the ac mains supply.
3. Switch on the mains supply. Fix the output voltage at a (low) value so that current *i* in the circuit is within the mA range. This may be checked by using Ohm's law.
4. Note the value of current *i* in the circuit from the milliammeter and record it in Observation Table 4.1. You should keep this value of current unchanged in this part of the experiment.

5. Set the frequency (f) of AF oscillator at to 300 Hz and measure the voltage V_R across the resistor R. Record both f and V_R in Table 4.1.
6. Increase the frequency in the steps of 200 Hz and record the frequency and voltage readings in the Observation Table 4.1. Repeat this process for at least 10 different frequencies.

Observation Table 4.1: Frequency response of a resistor

Value of resistor R =.....................Ω

Current through the resistor, i =.....................mA

S.No.	Frequency f (Hz)	Voltage across the resistor V_R (V)	Reactance (Ω) V_R / i
1.	300		
2.	500		
3.	.		
.	.		
.	.		
10.	.		

7. Replace R by an inductor L in the circuit (Fig 4.5b) and repeat steps 3-6. Note that the output voltage (V_L) has to be low so as to limit the flow of current through the inductor circuit. Record your readings in Observation Table 4.2.

Observation Table 4.2: Variation of voltage across an inductor with frequency

Value of self inductance, L =......................mH

Current through the inductor, i =......................mA

S.No.	Frequency f (Hz)	Voltage across the resistor V_L (V)	Reactance (Ω) V_L / i
1.	300		
2.	500		
3.	.		
.	.		
.	.		
10.	.		

8. Replace the inductor L by a capacitor C in the circuit (Fig. 4.5c) and repeat steps 3-6 taking the same precautions. Record your readings in Observation Table 4.3.

Observation Table 4.3: Variation of voltage across a capacitor with frequency

Value of capacitance, C =…………………..μF

Current through capacitor, i =……..…………..mA

S.No.	Frequency f (Hz)	Voltage across the resistor V_C (V)	Reactance (Ω) V_C / i
1.	300		
2.	500		
3.	.		
.	.		
.	.		
10.	.		

9. Plot a graph of reactance (*y*-axis) vs. frequency (*x*-axis) for resistor, capacitor and inductor (preferably) on the same graph paper.

How does the nature of frequency response curve change for different passive elements in an ac circuit? Do your graphs match with the plots in Fig. 4.2? If not, discuss your results with your counsellor.

4.4 FREQUENCY RESPONSE OF A SERIES *LCR* CIRCUIT

You have studied the frequency responses of individual circuit components in the previous part of the experiment. Let us now investigate the behaviour of a circuit obtained by series combination of *L*, *C* and *R* as shown in Fig. 4.6.

Procedure

1. Choose the values of *L* and *C* such that the resonant frequency f_r lies around 1 kHz or 2 kHz. You can calculate f_r using Eq. (4.1).

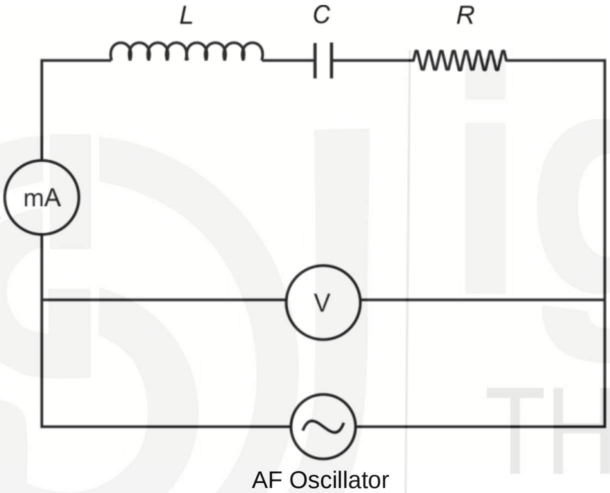

Fig. 4.6: **Circuit diagram for studying the frequency response of a series** *LCR* **circuit.**

2. Connect the circuit as shown in Fig. 4.6 and note down the values of *L*, *C* and *R* in Observation Table 4.4.
3. Set the voltage at low value. You have to keep this value fixed throughout this part of the experiment.
4. Start your experiment with the frequency of 100 Hz and record the corresponding circuit current in Observation Table 4.4.
5. Increase the frequency in steps of 100 Hz and note the corresponding current upto 3 kHz frequency. You will note that current in the circuit increases initially, attains a maximum value and begins to decrease thereafter.
6. Plot *f* along the *x*-axis and *i* along *y*-axis. Is the curve bell shaped? Note the resonant frequency f_r corresponding to the peak of the graph.
7. Find the i_{max} value corresponding to f_r.
8. Calculate $i_{max}/\sqrt{2}$ (= 0.707 i_{max}) and draw a horizontal line on the plot corresponding to this value.
9. Note the values of f_H and f_L corresponding to intersection of this line with the plotted curve.

10. Calculate bandwidth $\Delta f = f_H - f_L$ and quality factor $Q = f_r / \Delta f$.

Observation Table 4.4: Variation of current with frequency in a series *LCR* circuit

Value of resistance R in the circuit =......................Ω

Value of self-inductance L of the inductor =......................mH

Value of capacitance C of the capacitor =.......................μF

S.No.	Frequency, *f* (Hz)	Current, *i* (mA)
1.	100	
2.	200	
3.	.	
.	.	
.	.	
.	3,000	

Results:

Resonant frequency f_r =.....................Hz

Resonance current i_{max} =......................mA

Frequencies corresponding to half power points $(i = 0.707\, i_{max})$:

f_H =......................Hz and f_L =......................Hz

Bandwidth, $\Delta f = f_H - f_L$ =......................Hz

Quality factor $Q = \dfrac{f_r}{\Delta f}$ =......................

4.5 DEPENDENCE OF QUALITY FACTOR ON RESISTANCE

Now let us study the effect of R on the bandwidth and effectively the quality factor of the series *LCR* circuit. For this purpose, you should use at least three different values of resistance and take the observations of circuit current with frequency variation as in Sec. 4.4. Record your readings in Observation Table 4.5.

Observation Table 4.5: Frequency response of a series *LCR* circuit for different resistances

Value of self-inductance L =.....................mH

Value of capacitance C =.....................μF

S.No.	Frequency, *f* (Hz)	Current (*i*) in the circuit (mA)		
		R_1 = …. Ω	R_2 = …. Ω	R_3 = …. Ω
1.	100			
2.	200			
3.	.			
.	.			
.	.			
.	3,000			

Plot frequency versus current graph for all three resistances. Note down the values of f_r, f_H and f_L for each curve and calculate quality factor in each case. What is your conclusion about the dependence of Q on R?