EXPERIMENT 8

VERIFICATION OF SUPERPOSITION AND MAXIMUM POWER TRANSFER THEOREMS

Structure

8.1 INTRODUCTION

In Experiment 7, you have verified Thevenin's and Norton's theorems for dc circuits containing resistors. You have learnt that a complicated electrical circuit can be replaced by a Thevenin equivalent circuit that has a voltage source in series with a resistance. Or, it can be replaced by a Norton equivalent circuit that has a current source in parallel with a resistance. You have also learnt how to calculate and measure Thevenin voltage and Thevenin resistance, and Norton current and Norton resistance.

In this experiment, you will be verifying two more important network theorems, namely, the superposition theorem and the maximum power transfer theorem. Both these theorems are important in network analysis. Superposition theorem helps in converting any circuit into equivalent Thevenin or Norton circuit.

The maximum power transfer theorem is used to design electrical networks in which we would like to maximize power transfer to the load. You will learn more about their applications in Secs. 8.2 and 8.3. You may not have learnt these theorems in your physics theory courses so far. Therefore, in this experiment we will first explain the basic physics underlying these theorems. Then you will learn how to set up the circuit connections for the experiment and verify both theorems.

Expected Skills

After performing this experiment, you should be able to:

- verify superposition theorem; and
- verify maximum power transfer theorem.

The apparatus required for this experiment is listed below.

Apparatus required

Two regulated power supplies (0-30 V), resistance boxes (100-1000 Ω), voltmeter (0-10 V), ammeters (0-10 mA and 0-30 mA), connecting wires.

8.2 SUPERPOSITION THEOREM

The superposition theorem is used to simplify electrical networks in which two or more current sources are present. It provides us with a method to determine currents in a circuit having many sources by taking one source at a time and replacing the other sources by their internal resistances. Let us first state the theorem.

SUPERPOSITION THEOREM

Superposition theorem states that in any linear, active, bilateral network having **more than one source, the response across any element is the algebraic sum of the responses obtained from each source considered separately, with all other sources replaced by their internal resistance.**

It is applicable only to linear networks, i.e., networks having elements for which Ohm's law holds.

An alternative and more practical statement of the theorem is as follows:

The current in any given branch of a linear circuit having many sources is obtained by determining the currents in that branch produced by each source acting alone, with all other sources replaced by their internal resistance. The total current in that branch is the algebraic sum of the currents in that branch due to each individual source.

We now prove the superposition theorem with the help of an example. Let us consider the circuit shown in Fig. 8.1 with two voltage sources and only resistive elements. We would like to use superposition theorem to determine the currents I_1, I_2 and I flowing in the three branches of the circuit.

Let us determine the currents I_1, I_2 and I flowing in the two loops (1 and 2) of the circuit. Applying Kirchhoff's current law to node A, we have

$$I = I_1 + I_2 \qquad (8.1a)$$

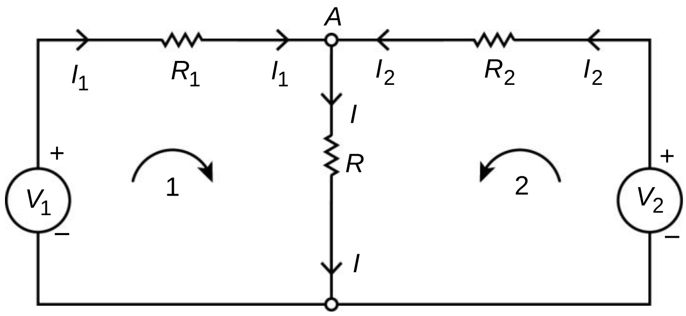

Fig. 8.1: **An electrical circuit having two voltage sources.**

You must have learnt Kirchhoff's laws in your school physics. You may like to revise them. Here we state them briefly for reference:

Kirchhoff's current law (first law) states that the sum of the currents flowing into a node (or a junction) must be equal to the sum of the currents flowing out of it.

Kirchhoff's voltage law (second law) states that the algebraic sum of all voltages around any **closed** loop in a circuit must be equal to zero.

You may take the help of your Counsellor if you have any difficulty in applying these laws.

Applying Kirchhoff's voltage law to loop 1, we get

$$I_1 R_1 + IR = V_1 \tag{8.1b}$$

Substituting for I from Eq. (8.1a) in Eq. (8.1b), we get

$$I_1 R_1 + I_1 R + I_2 R = V_1 \tag{8.1c}$$

or
$$I_1 = \frac{V_1 - I_2 R}{R_1 + R} \tag{8.2}$$

Next, we apply Kirchhoff's voltage law to loop 2 and get

$$I_2 R_2 + IR = V_2 \tag{8.3a}$$

Substituting for I from Eq. (8.1a) in Eq. (8.3a), we get

$$I_2 R_2 + I_1 R + I_2 R = V_2 \tag{8.3b}$$

or
$$I_2 = \frac{V_2 - I_1 R}{R_2 + R} \tag{8.4}$$

Therefore,

$$I = I_1 + I_2 = \frac{V_1 - I_2 R}{R_1 + R} + \frac{V_2 - I_1 R}{R_2 + R} \tag{8.5}$$

We now take only the source V_1 and short circuit the source V_2 as shown in Fig. 8.2. Then we determine the currents I_1', I_2' and I' in all three branches of this circuit using Kirchhoff's laws.

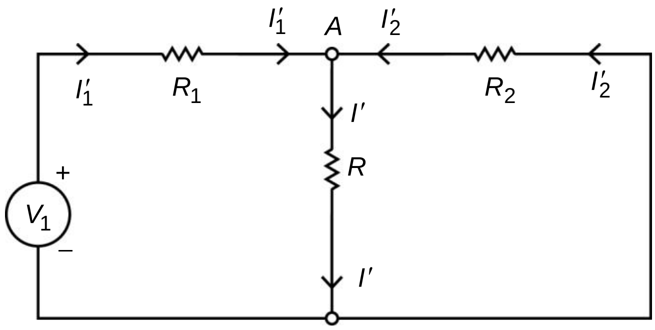

Fig. 8.2: **Currents in the circuit when only one source V_1 is taken.**

We follow the same procedure as we did for writing Eqs. (8.1) to (8.5).

Applying Kirchhoff's current law to node A, we have

$$I' = I'_1 + I'_2 \tag{8.6a}$$

Applying Kirchhoff's voltage law to loop 1, we get

$$I'_1 R_1 + I' R = V_1 \tag{8.6b}$$

Substituting for I' from Eq. (8.6a) in Eq. (8.6b), we get

$$I'_1 R_1 + I'_1 R + I'_2 R = V_1 \tag{8.6c}$$

or $$I'_1 = \frac{V_1 - I'_2 R}{R_1 + R} \tag{8.7}$$

Next, we apply Kirchhoff's voltage law to loop 2 and get

$$I'_2 R_2 + I' R = 0 \tag{8.8a}$$

Substituting for I' from Eq. (8.6a) in Eq. (8.8a), we get

$$I'_2 R_2 + I'_1 R + I'_2 R = 0 \tag{8.8b}$$

or $$I'_2 = -\frac{I'_1 R}{R_2 + R} \tag{8.9}$$

$$\therefore \quad I' = I'_1 + I'_2 = \frac{V_1 - I'_2 R}{R_1 + R} - \frac{I'_1 R}{R_2 + R} \tag{8.10}$$

We now short circuit the source V_1 and take only the source V_2 as shown in Fig. 8.3 and determine the currents I''_1, I''_2 and I'' in all three branches using Kirchhoff's laws.

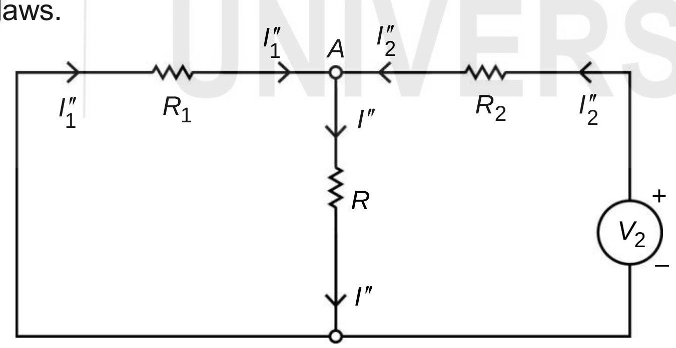

Fig. 8.3: **Currents in the circuit when only one source V_2 is taken.**

Applying Kirchhoff's current law to node A, we have

$$I'' = I''_1 + I''_2 \tag{8.11a}$$

Applying Kirchhoff's voltage law to loop 1, we get

$$I''_1 R_1 + I'' R = 0 \tag{8.11b}$$

Substituting for I'' from Eq. (8.11a) in Eq. (8.11b), we get

$$I''_1 R_1 + I''_1 R + I''_2 R = 0 \tag{8.11c}$$

or $$I_1'' = -\frac{I_2''R}{R_1 + R} \qquad (8.12)$$

Next, we apply Kirchhoff's voltage law to loop 2 and write

$$I_2'' R_2 + I'' R = V_2 \qquad (8.13a)$$

Substituting for I'' from Eq. (8.11a) in Eq. (8.13a), we get

$$I_2'' R_2 + I_1''R + I_2''R = V_2 \qquad (8.13b)$$

or $$I_2'' = \frac{V_2 - I_1''R}{R_2 + R} \qquad (8.14)$$

$$\therefore \quad I'' = I_1'' + I_2'' = \frac{V_2 - I_1''R}{R_2 + R} - \frac{I_2''R}{R_1 + R} \qquad (8.15)$$

So we have determined the currents in both loops when the respective voltage sources are shorted.

Now, according to the superposition theorem, we must have

$$I_1 = I_1' + I_1'' \qquad (8.16a)$$

$$I_2 = I_2' + I_2'' \qquad (8.16b)$$

and $$I = I' + I'' \qquad (8.16c)$$

Let us verify these results. To do so, we add Eqs. (8.10) and (8.15):

$$I' + I'' = \frac{V_1 - I_2' R}{R_1 + R} - \frac{I_1'R}{R_2 + R} + \frac{V_2 - I_1'' R}{R_2 + R} - \frac{I_2''R}{R_1 + R}$$

or $$I' + I'' = \frac{1}{R_1 + R}\left(V_1 - I_2' R - I_2''R\right) + \frac{1}{R_2 + R}\left(V_2 - I_1''R - I_1'R\right) \qquad (8.17)$$

Now compare Eqs. (8.5) and (8.17) and you can see that these will be satisfied if and only if Eqs. (8.16a to c) hold. Therefore, we have

$$I_1 = I_1' + I_1'', \quad I_2 = I_2' + I_2'' \quad \text{and} \quad I = I' + I''$$

Hence, we have proved the superposition theorem.

8.3 MAXIMUM POWER TRANSFER THEOREM

As you have learnt in the introduction, the maximum power transfer theorem is used to design electrical networks in which we would like to maximize power transfer to the load. As such, it does not give us a method to analyse electrical circuits. Rather we can use this theorem to determine the load resistance for which power delivered can be maximised in an electrical circuit.

For example, in transmission of signals or radio frequencies, it is important to design final stage amplifiers that deliver maximum power to the antenna or transmission line. In electrical vehicle design, we need to maximise the power delivered to the motor and for that we use this theorem. Let us state the theorem.

MAXIMUM POWER TRANSFER THEOREM

The **maximum power transfer theorem** states that **the power transferred by a source to the load resistance is maximum when the load resistance is equal to the internal resistance of the source.**

In other words, **to obtain maximum external power from a source with a finite internal resistance, the resistance of the load must equal the resistance of the source as viewed from its output terminals**.

Let us now prove the maximum power transfer theorem. Consider the circuit shown in Fig. 8.4. It has one voltage source *V* having internal resistance *r* connected to a load resistance R_L.

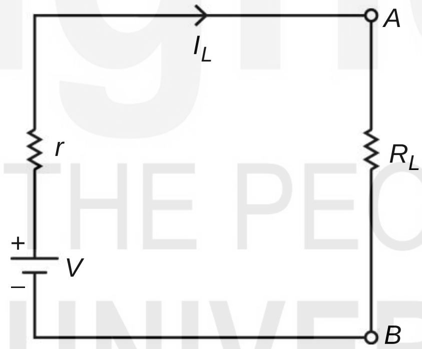

Fig. 8.4: **Circuit for proof of maximum power transfer theorem.**

The current in the circuit is

$$I = \frac{V}{R_L + r} \tag{8.18}$$

The power delivered to the load is

$$P_L = I^2 R_L = \frac{V^2}{(R_L + r)^2} R_L \tag{8.19}$$

We now have to determine the value of R_L for which P_L would be maximum. From your calculus course, you know that for this we need to differentiate P_L with respect to R_L and equate the result to zero:

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left(\frac{V^2 R_L}{(R_L + r)^2}\right) = V^2\left(\frac{1}{(R_L + r)^2} - \frac{2R_L}{(R_L + r)^3}\right) = 0 \tag{8.20a}$$

or $$\frac{1}{(R_L + r)^2} - \frac{2R_L}{(R_L + r)^3} = 0 \tag{8.20b}$$

On solving Eq. (8.20b), we get

$$\frac{2R_L}{(R_L + r)} = 1 \tag{8.20c}$$

or $$R_L = r \tag{8.21}$$

Eq. (8.21) tells us that the power delivered to the load resistance is maximum when the load resistance is equal to the internal resistance of the source. We can also show that the maximum power transfer efficiency of a circuit is 50%. For this, we substitute $r = R_L$ in the expression of P_L to calculate the maximum power delivered to the load:

$$P_{Lmax} = \frac{V^2}{(R_L + R_L)^2} R_L = \frac{V^2}{4R_L}$$

Now, the power generated by the source is

$$P = I^2(R_L + r) = \frac{V^2}{(R_L + r)^2}(R_L + r) \quad \Rightarrow \quad P = \frac{V^2}{(R_L + r)}$$

When $r = R_L$, we get $P = \dfrac{V^2}{2R_L}$

The ratio of maximum power to power generated by the source is then

$$\frac{P_{Lmax}}{P} = \frac{V^2}{4R_L} \times \frac{2R_L}{V^2} = \frac{1}{2} \quad \Rightarrow \quad P_{Lmax} = \frac{P}{2}$$

Thus, the maximum power delivered to the load resistance is only half of the power generated by the source, i.e., the maximum power transfer efficiency is 50%. The remaining half of the power is lost across the internal resistance of the source. With this theoretical understanding of the theorems, you can verify them experimentally. But before you actually do the experiments, we list below some preliminary steps that you should take before doing them.

PRELIMINARY STEPS	
1.	Before using the power supply in the experiment, use a multimeter to measure its output to make sure that it is in the range you require.
2.	Measure the values of resistances you use with a multimeter for their correct values.
3.	Check the zero settings of ammeter and voltmeter.
4.	Always switch off the power supply when you make circuit connections or modify the circuit in any way.

8.4 VERIFYING SUPERPOSITION THEOREM

In this part of the experiment, you will connect an electrical circuit to verify the superposition theorem. Follow the steps described below.

1. Keep the power supply off and connect the circuit – 1 as shown in Fig. 8.5. Now keep the voltage control knob of the power supply at minimum and its current control knob at maximum. Then turn the power supply on.

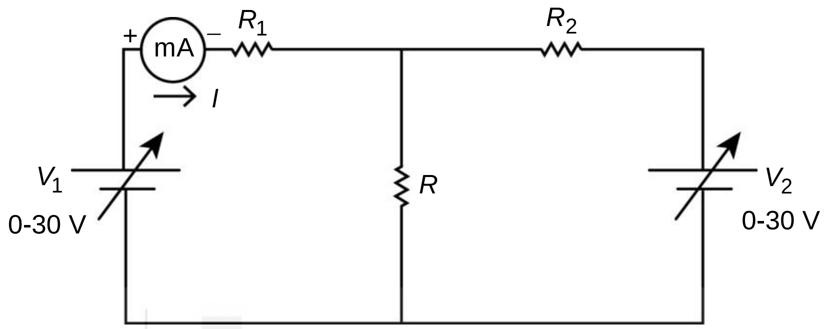

Fig. 8.5: **Circuit – 1 connections for verifying superposition theorem.**

2. Using the power supply, set particular values of voltages for V_1 and V_2. You may take them to be 7 V (say) and 5 V, respectively. You may use three resistors having equal resistance of 500 Ω in the circuit. Using the ammeter (of the range 0 – 10 mA), measure the current I in the circuit. Note down the readings for V_1, V_2 and I in Observation Table 8.1.

3. Now modify the circuit connections of Fig. 8.5 and make circuit – 2 connections as shown in Fig. 8.6. Note that in this circuit, V_1 is undisturbed, the power supply for V_2 is removed and a wire is connected in its place. Keep the value of V_1 and all resistors the same as in step 2. Note down the readings for V_1 and I' in Observation Table 8.1.

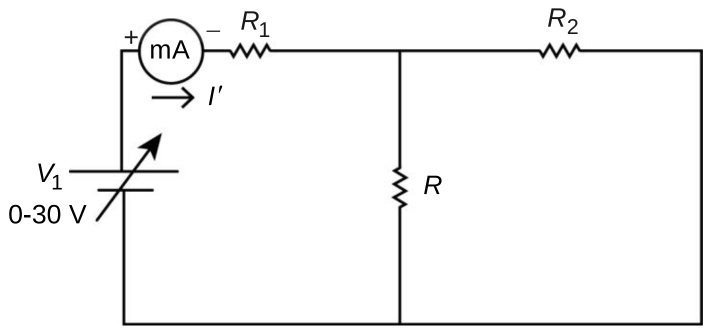

Fig. 8.6: **Circuit – 2 connections for measuring current I'.**

4. Next make the circuit – 3 connections as shown in Fig. 8.7. Note that now the power supply for V_1 has been removed and it has been replaced by a wire to short the circuit. Restore the original value of V_2 and keep the values of all resistors the same as in step 2.

5. Note down the readings for V_2 and the ammeter reading for I'' in Observation Table 8.1.

6. Complete Observation Table 8.1 by filling in the value of $(I' + I'')$. If $I = I' + I''$, then superposition theorem is verified.

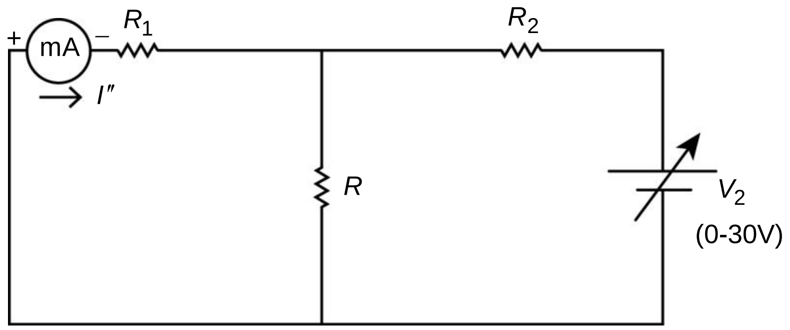

Fig. 8.7: **Circuit – 3 connections for measuring I''.**

Observation Table 8.1: Currents in circuits – 1, 2 and 3

S. No.	R_1, R_2, R	V_1 (V)	V_2 (V)	I (mA)	I' (mA)	I'' (mA)	$I' + I''$ (mA)

7. Repeat steps 1 to 6 for two more sets of voltages and resistances V_1, V_2, R_1, R_2 and R.

Have you been able to verify the superposition theorem?

What is the error in your result in the value of I? What are the sources of error? Discuss them with your Counsellor and write your analysis of the results.

Results and analysis: Record your findings in your practical notebook.

8.5 VERIFYING MAXIMUM POWER TRANSFER THEOREM

In this part of the experiment, you will verify maximum power transfer theorem. Follow the steps described below for making circuit connections and taking measurements.

1. Keep the power supply off and connect an electrical circuit as shown in Fig. 8.8. Now keep the voltage control knob of the power supply at minimum and its current control knob at maximum. Then turn the power supply on. To begin with, take $V = 10\,\text{V}$, $R_S = 500\,\Omega$ and the first value of $R_L = 100\,\Omega$.

2. Note the values of V, V_L in the Observation Table 8.2.

3. Complete Observation Table 8.2 by calculating $\frac{V_L^2}{R_L}$.

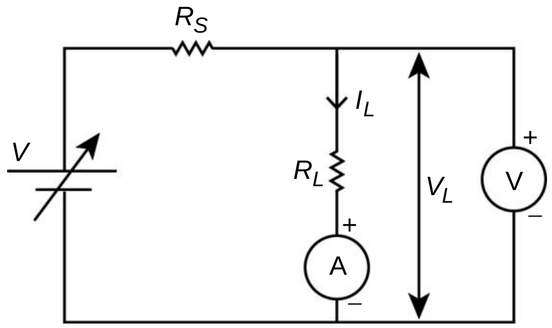

Fig. 8.8: **Circuit connections for verifying maximum power transfer theorem.**

4. Now increase the value of the load resistance R_L in steps of 100 Ω (200 Ω, 300 Ω, …, 1000 Ω) and repeat the steps 2 and 3 at least 5 times. Record the readings in Observation Table 8.2. Create more rows in your practical notebook for your readings.

Observation Table 8.2: Measured load currents, voltages and calculated values of power transferred

Source resistance R_S = ... Ω Source voltage V = ... V

S. No.	R_L (Ω)	Output voltage V_L (V)	Power in load $P_L = \frac{V_L^2}{R_L}$ (W)

5. Now plot a graph between the power transferred P_L (on the y-axis) and load resistance R_L (on the x-axis). Note that the power transferred increases with the increase in load resistance, reaches a maximum value and then decreases with further increase in R_L as shown in Fig. 8.9.

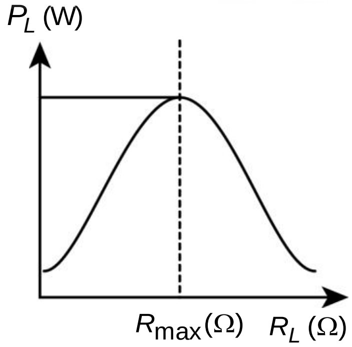

Fig. 8.9: **Graph of power transferred to load resistance versus load resistance.**

For which value of R_L is the power transferred maximum? Read the value R_{max} from the graph. Compare the values of R_{max} and R_S. If $R_{max} = R_S$, the maximum power transfer theorem is verified.

What are the sources of error in the experiment? Discuss them with your Counsellor and write your analysis of the results.

Results and analysis: Record your findings in your practical notebook.