Section 5: Environmental Issues, Programmes and Policies: Environmental Issues, United Nations and the Environment, Environmental Policies with Special Reference to India.

Tutorials (1 Credit): Students are required to do tutorials equivalent to 1 Credit provided in the Self Learning Material.

9.1.5 Mathematics

Mathematics is deeply rooted in the activities of our daily life. At the same time, it is also known as an abstract discipline. Indeed, mathematics has two aspects – functional and philosophical. It has wide applications in various areas of the sciences, social sciences, humanities, engineering, technology, commerce and agriculture. It is also a philosophy with intrinsic beauty and logical validity. In view of this mathematics is being introduced as a discipline in the Bachelor's Degree programme. We are offering the following mathematics courses as part of this programme. The pre-requisite for any of these courses is knowledge of mathematics that is imparted at the senior secondary (+2) level or an equivalent level.

Course Code: BMTC-131	Course Title: CALCULUS	Credits: 6

This is a first level course, consisting of five blocks, and is intended as a short introduction to calculus. Calculus is increasingly being recognised, and accepted, as a powerful tool in the exact sciences and social sciences. Its power stems from two sources – the derivative and the integral. In this course, we shall acquaint you with the basic techniques of differential and integral calculus. We shall also briefly trace the historical development of calculus.

We shall begin the course with essential preliminary concepts, in the first block. You will be introduced the concepts of 'limit', 'continuity' and 'derivative' in the second and third block. We shall discuss the geometrical significance and application of the derivative in the fourth block. The fifth block focuses on the other important concept of calculus, namely, integral.

Definition and examples of sets and subsets, Venn diagrams, Complementation, Intersection, Union, Distributive laws, De Morgan's laws, Cartesian Product, Relations and Functions, Composition of Functions and Binary Operation, Operations (inverse, composite). Cartesian system, Graphs of functions, Equation of a line, Symmetry, change of axis, polar coordinates. Definition and examples of complex numbers, Geometric representation and polar representation, algebraic operations, De Moivre's theorem, trigonometric identities, n^{th} roots of a complex number. Basic Theory of Equations: Relations between roots and coefficients; Descartes rule of signs, Solution of equations up to bi-quadratic equations.

Real number line, Supremum and Infimum, Absolute value, Interval and some special types of functions (even, odd, monotonic, periodic). Definition of Limits, Algebra of limits, One-sided limits, The concept of infinite limits (infinite limits as the independent variable $x \to a \in R$, one-sided infinite limits, limits as the independent variable tends to ∞ or $-\infty$). Continuity, algebra of continuous functions, Types of discontinuity.

Derivatives of some simple functions, algebra of derivatives, the chain rule, continuity versus derivability. Derivatives of the various trigonometric functions, derivative of inverse of a function. The inverse function theorem, derivatives of inverse trigonometric functions, use of transformations. Derivative of exponential function, logarithmic functions, hyperbolic functions, inverse hyperbolic functions, methods of differentiation (derivative of x^r, logarithmic differentiation, derivatives of functions defined in terms of a parameter, derivatives of implicit functions).

Higher order derivatives: Second and third order derivatives, nth order derivatives, Leibnitz Theorem, Taylor Polynomials. Indeterminate forms: L'Hopital's rule for $\frac{0}{0}$ form, L'Hopital's rule for $\frac{\infty}{\infty}$ form, other types of indeterminate forms (indeterminate forms of the type $\infty - \infty$, indeterminate forms of the type $0.\infty$, indeterminate forms of the type 0^0, ∞^0, 1^∞) Ups and Downs: Rolle's Theorem, Lagrange's mean value theorem, Maxima-minima of functions (Definitions and examples, a necessary condition for the existence of extreme points, Sufficient conditions for the existence of extreme points, first derivative test, second derivative test), Monotonicity, Curvature, Tangents and Normals, Angles of intersection of two curves, Concavity / Convexity, points of inflection. Classifying singular points, Asymptotes (Parallel to the axes, Oblique asymptotes), Tracing of curve.

Introduction to Integration: UPF, LPF, Definite integral, properties, Fundamental theorem of calculus (without proof.), Standard integrals, Algebra of integrals. Methods of Integration, Reduction Formula. Applications of Integration: Area under a curve, area bounded by a closed curve, length of a plane curve, Volume and Surface area of a solid generated by revolution.

Course Code: BMTC-132	Course Title: Differential Equations	Credits: 6

This course of Differential Equation assumes the knowledge of the course BMTC-131 on Calculus. The studies in this course are divided into four blocks.

Block-1 deals with functions of two and three real variables. The purpose of this block is to provide the basis for studying the remaining blocks of the course. We have given a brief discussion on 3D-cordinate system and discussed the algebraic and geometrical structure of R^2 and R^3. The notions of limit, continuity and differentiability are extended for functions of 2 and 3 variables. This block also covers chain rule and homogeneous functions.

We have started Block-2 with the essentials and the basic definitions related to the study of differential equations. After discussing various methods of solving first order ordinary differential equations (ODEs) we have formulated some of the problems of physical and engineering interest in terms of first order linear differential equations. In Block-3 we have laid specific stress on the applications of second order ODEs.

In Block-4 we have discussed simultaneous, total and partial differential equations (PDEs). Here we have classified the first order PDEs into linear, semi-linear, quasi-linear and non-linear PDEs and discussed the various types of solutions/integrals of these PDEs.

All the concepts discussed are followed by a lot of examples as well as exercises. These will help you get a better grasp of the techniques discussed in this course.

3D-Cartesian Coordinate System, Spherical Coordinate System and Cylindrical Coordinate System, Cartesian products, Properties of $\mathbf{R}^n$ (Linear Space Properties), Distance in $\mathbf{R}^2$ and $\mathbf{R}^3$, Functions from $\mathbf{R}^n$ to $\mathbf{R}$ (n = 2, 3), Limit of functions from $\mathbf{R}^2 \to \mathbf{R}$ and from $\mathbf{R}^3 \to \mathbf{R}$, Repeated Limits, Properties of Limits, Continuity of functions from $\mathbf{R}^2 \to \mathbf{R}$ and from $\mathbf{R}^3 \to \mathbf{R}$, Algebra of continuous functions. First Order Partial Derivatives, Geometrical Meaning, Continuity and Partial Derivatives, Differentiability of functions from $\mathbf{R}^2 \to \mathbf{R}$, Differentiability of functions from $\mathbf{R}^3 \to \mathbf{R}$, Higher Order Partial Derivatives, Equality of Mixed Partial Derivatives (Euler's , Schwarz's and Young's Theorem without proof), Chain rule for finding partial derivatives of composit functions, Total Derivative, Homogeneous functions and Euler's theorem.

Basic concepts in the theory of differential equations, Family of curves and differential equations, Differential Equations arising from physical situations. Separation of Variables, Homogeneous equations, Exact equations, Integrating factors. Classification of first order differential equations (DE), General solutions of linear non-homogeneous equation, Method of Undetermind coefficient, Method of Variation of Parameters, Equations reducible to linear form, Applications of linear DEs. Equations which can be factorized, Equations which cannot be factorized (solvable for x, y, independent or dependent variable is absent, homogeneous in x and y, Clairaut's and Riccati's equations).

General form of linear ordinary differential equation, Condition for the existence of unique solution, linear dependence and independence of the solution of DEs, Method of solving homogeneous equation with constant coefficients; Method of undetermined coefficients – Types of non-homogeneous terms for which the method is applicable (polynomial, exponential, sinusoidal etc.), Observations and Constraints of the method. Variation of parameters, Reduction of order, Euler's equations. Differential operators, General method of finding Particular Integral (PI), Short method of finding PI, Applications – Mechanical Vibrations, Electric Circuits.

Curves and surfaces in space, Formation of simultaneous DEs, Methods of solution – Method of Multipliers, One Variable absent, Applications – Particle motion in phase-space, Electric Circuits. Total Differential Equations – Definition and examples, Integrability condition (only statement and illustration), Methods of Integration (By Inspection, Variable separable, One variable separable, Homogeneous equation). Origin, Classification (order, degree, linear, semi-linear, non-linear) of linear first order PDEs, Formation of Linear Equations of the First Order and types of their solutions, Lagranges Method, Solutions of non-linear PDEs – The Complete integral, Compatible system of first order equations, Charpits method, Standard forms.

Course Code: BMTC-133	**Course Title: REAL ANALYSIS**	**Credits: 6**

This course is mainly designed assuming that you have a working knowledge of the concepts covered in our first level calculus course. As the name suggests, real analysis is one of the core branches of mathematics which gives a more rigorous and abstract treatment to concepts such as limit, continuity, differentiability and integrability. These concepts are widely applied in several fields such as the sciences, engineering and economics. The aim of this course is also to acquaint you with the language of mathematics, which is required for a clear presentation of various arguments in mathematics. The whole material is divided into six booklets called blocks.

The first block initiates you into the world of analysis. Sequences and series are introduced and discussed in Blocks 2 and 3. In Block 4 a formal study of limits, continuity and differentiability is laid out. Block 5 introduces you to the concepts of Riemann integrability, and discusses some properties of a Riemann integrable functions. In the last block, Block 6, the sequences and series of functions and their point wise convergence and uniform convergence are discussed.

Mathematical Statements and Symbols, Terms (necessary, sufficient, converse, negation, if, only if, if and only if, there exists, for all), Mathematical reasoning, Methods of Proof (Contrapositive, Counter example, Contradiction), Field Structure, Ordered Field, Boundedness, Supremum and Infimum, Order Completeness, Archimedian property, Countable and uncountable subsets of R, Neighbourhood, Limit Point, Open and Closed sets, Bolzano Weierstrass Theorem.

Real Sequences, Bounded Sequences, Monotonic Sequences, Subsequences , Convergent Sequences, Cauchy Sequences, Criteria for the convergence of Sequences, Algebra of Convergent Sequences, Cauchy's First Theorem on limits, Cauchy's Second Theorem on Limits, Squeeze Theorem, Monotone Convergence Theorem.

Infinite Series, Convergence criteria, Cauchy Convergence criteria, Comparison test, Cauchy Root Test, D'Alembert's ratio test, Integral Test, Raabe's test, Gauss's test, Alternating Series, Leibniz test, Absolute and Conditional Convergence, tests for convergence (Abel test and Dirichlet Test), rearrangement of terms.

Concept of limit, Continuous functions, Algebra of Continuous functions, Types of discontinuity, Uniform Continuity, Existence of Derivative, Differentiability versus Continuity, Inverse function theorem, Algebra of derivatives, Darboux's Theorem, Intermediate Value Theorem for Derivatives, Rolle's theorem, Mean Value Theorems (Lagrange, Cauchy and Generalized Mean Value Theorems) Increasing and Decreasing Functions and Sign of the Derivative, Taylor's Theorem, Maxima and Minima (necessary and sufficient conditions for maxima and minima).

Integral as a Limit of Riemann Sums, Necessary and sufficient conditions for Riemann integrability, Algebra of Integrable functions, Special class of integrable functions (monotone functions and continuous functions), The Fundamental Theorem of Calculus, Mean Value theorems.

Sequences of functions, Point-wise Convergence, Uniform Convergence, Series of functions, Point-wise Convergence, Uniform Convergence, Term by term Integration, Term by term differentiation, Power series, radius of convergence.

Course Code: BMTC-134	**Course Title: Algebra**	**Credits: 6**

This course comprises 4 blocks. Through this course you will be familiarised with several algebraic systems, namely, groups, rings and fields. The first two blocks focus on group theory, and in the next two blocks you will study rings and fields.

As you go through the course, you will get acquainted with various methods of algebra. You will also see how several different algebraic objects can actually be studied in one go by using these methods. This study will, of course, help you develop your abilities to think mathematically and to appreciate the beauty of mathematics.

Given below is the outline of the matter covered in the course:

Integers, divisibility, well ordering principle, division algorithm, Fundamental Theorem of Arithmetic, Principle of induction, binary operation, equivalence relations, partitions, examples (including $\mathbb{Z}_n$); Introduction to operations on Matrices and symmetries; Definition and examples of groups; properties of groups; Subgroups, centre, centralizer as examples, different characterisations of subgroups, intersection, union, sum, product of subgroups; condition for products of subgroups to be a subgroup; Order of an element, order of a^k vis-a-vis o(a), `a and $gag^{-1}$ have the same order'; Cyclic groups, 'Subgroup of a cyclic group is cyclic.', `Order of a subgroup divides the order of a finite cyclic group.',` For each divisor *d* of the order *n* of a finite cyclic group, there is a unique subgroup with that order.', number of elements of a particular order in a finite cyclic group; number of generators for finite and infinite cyclic groups.

Left and right cosets, Lagrange's theorem, converse is true for cyclic groups, not true in general, applications of Lagrange's theorem (Fermat's little theorem, etc.); normal subgroups, properties and characterisation of normal subgroups, Simple groups, `Simple subgroups of abelian groups are cyclic of prime order.'; quotient groups; group homomorphism and isomorphism, automorphism, Fundamental Theorem of Homomorphism, correspondence between subgroups of G/H and subgroups of G that contain H, three isomorphism theorems.

Permutation groups, cycle decomposition, alternating group, Cayley's theorem.

(Optional: Direct product, Sylow theorems (without proof), classifying groups of orders 1 to 10.)

Rings, elementary properties, Z_n, polynomial rings, matrix rings over $\mathbb{R}$, C, Zn, rings of the form $\{a+b\sqrt{n}$, a, b, n are integers and n is not a square$\}$, ring of continuous functions, ring of differentiable functions, Cartesian product of rings is again a ring. Commutative and non-commutative rings; subrings, examples of subrings in matrix rings, polynomial rings, characterisation, properties, algebra of subrings(Intersection, direct product are rings, etc.); ideals, properties of ideals (sum, product, intersection), prime ideal, maximal ideal; quotient rings; Ring homomorphism and isomorphism, properties, isomorphism theorems.

Zero divisors, integral domain, definition of a field, `Every finite integral domain is a field.', characteristic of an integral domain, field of quotients of an integral domain; Ring of Polynomials, properties of R[x] (the polynomial ring over a ring R), the division algorithm in F[X], F a field; roots of polynomials, statement of the Fundamental Theorem of Algebra, field of rational functions.

(Optional: Euclidean Domains, Gaussian Integers, primes, factorisation into primes; principal ideal domains, units, irreducible and prime elements, associates, unique factorisation domains, Eisenstein's criterion for irreducibility of polynomials over $\mathbb{Q}$.)

9.1.6 Physics

Course Code: BPHCT-131	Course Title: Mechanics	4 Credits

Mathematical Preliminaries: Geometrical and algebraic representation of vectors, vector algebra, scalar and vector products, derivatives of a vector with respect to a scalar. First order homogeneous ordinary differential equations, separable and linear first order ordinary differential equations. Second order homogeneous ordinary differential equations with constant coefficients.

Basic Concepts of Mechanics: Newton's laws of motion, frames of reference, straight line motion, motion in a plane, uniform circular motion, 3-d motion. Applications of Newton's law of motion, friction, tension, gravitation, spring-mass system – Hooke's law. Satellite in circular orbit and applications, geosynchronous orbits, basic idea of global positioning system (GPS). Weight and weightlessness. Linear momentum, conservation of linear momentum, impulse, impulse-momentum theorem, motion of rockets. Work and energy, conservation of energy. Kinematics of angular motion, angular displacement, angular velocity and angular acceleration, general angular motion. Dynamics of rotational motion, torque, rotational inertia, kinetic energy of rotation, angular momentum, conservation of angular momentum and its applications. Motion of a particle in a central force field, motion in a plane,