EXPERIMENT 7

VERIFICATION OF THEVENIN'S AND NORTON'S THEOREMS

Structure

7.1 INTRODUCTION

From your school physics courses, you are familiar with electrical circuits. You may have verified Ohm's law in your school physics laboratory using resistances boxes. Electrical circuits are very common in our world – ranging from the household circuits carrying electricity to the circuits in sophisticated electronic equipment, computers, communication networks, etc. These circuits contain passive elements like resistors, capacitors, inductors or active electronic devices such as *p-n* junction diodes, bipolar junction transistors and many other types of electronic devices. While working with an electrical circuit, it is important to know the currents, voltages, power in it and its frequency response. While performing Experiments 3 to 6, you have measured currents and voltages as well as the frequency responses of *RC* and *LCR* electrical circuits and worked with filters.

If the circuit is simple and contains only resistive elements, we can analyse the currents and voltages in them using Ohm's law and Kirchoff's current and voltage laws. However, when the circuits are complicated, we need additional methods to analyse them. There are network theorems that make analysis of the currents and voltages in complicated electrical circuits easier by simplifying such circuits. You will now experimentally verify two such theorems, namely, Thevenin's and Norton's theorems for dc circuits. In the next experiment, you will be verifying superposition and maximum power transfer theorems. You may not have learnt these theorems in your physics theory courses so far. Therefore, in this experiment we will first explain the basic physics underlying the theorems. Then you will learn how to build the circuit connections for the experiment and verify both theorems for DC currents in circuits containing resistive elements.

Expected Skills

After performing this experiment, you should be able to:

- ❖ verify Thevenin's theorem; and
- ❖ verify Norton's theorem.

The apparatus required for this experiment is listed below.

You may be more familiar with the term circuit. A network and circuit mean the same in electrical literature and are used interchangeably.

Apparatus required

Two regulated power supplies (0-30 V), resistors (100 - 1000 Ω), variable resistors, ammeters (0 -10 mA and 0-30 mA), multimeter, connecting wires.

7.2 THEVENIN'S AND NORTON'S THEOREMS

Let us begin this section by formally introducing the basic terminology related to an electrical network.

7.2.1 Basic Network Terminology

Let us begin by asking: **What is an electrical network**?

Any interconnection of electrical elements such as resistors, capacitors, inductors, generators/power supplies is called an **electrical network**. A specific path between two points in an electrical network is called a **branch**. An electrical network may have many branches where each branch may have *R*, *L*, *C* elements or other types of linear elements. A **linear element is one in which current is proportional to the voltage applied**.

An electrical network may have two, three or four terminals as shown in Figs. 7.1 to 7.3. A 2-terminal network has two terminals as shown in Figs. 7.1a to c.

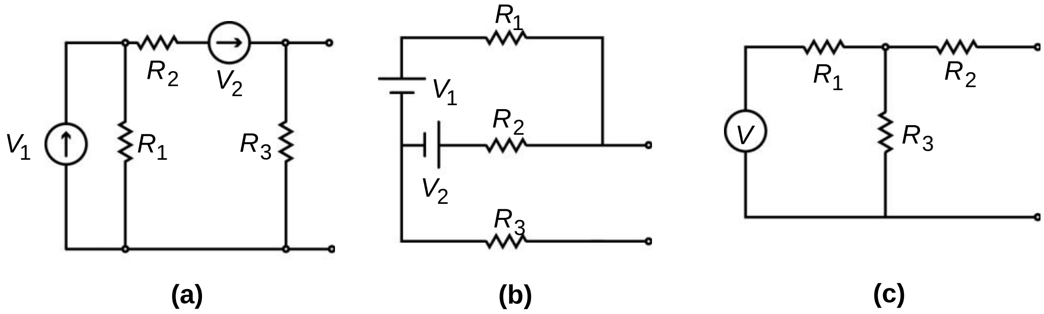


Fig. 7.1: **Some examples of 2-terminal electrical networks.**

A linear network having two distinct pairs of terminals (1, 1; 2, 2) is called a four terminal network (Fig. 7.2).

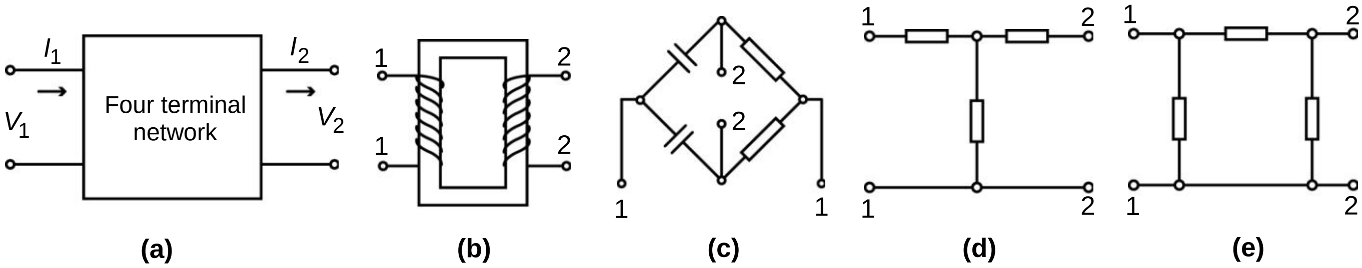


Fig. 7.2: **Some examples of 4-terminal electrical networks.**

However, if one of the 1, 1 terminals is common to the 2, 2 pair as in Figs. 7.2d and e, the circuit is known as a 3-terminal network (Fig. 7.3). In Figs. 7.3a and b, there are three terminals.

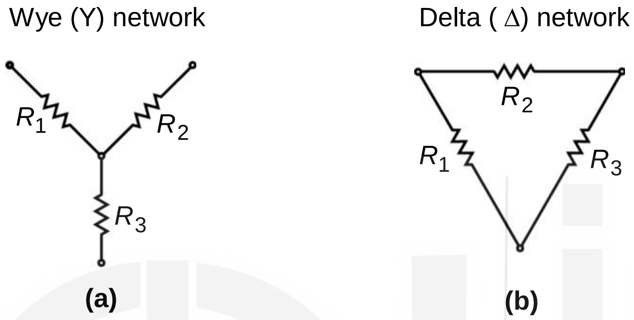


Fig. 7.3: **Some examples of a 3-terminal electrical network.**

You must become familiar with the terms related to electrical networks. We give them in Table 7.1.

Table 7.1: Basic terminology related to electrical networks

Term	Description
Passive network	A network containing circuit elements without any source of energy such as a battery/power supply.
Active network	A network containing sources of energy or generators along with other elements.
Network element	Any individual circuit element with two terminals which can be connected to another circuit element.
Branch	A specific path between two points in an electrical network.
Node	A point at which two or more elements are joined together.
Junction point	A point where three or more branches meet. The junction points are also the nodes of the network.
Loop	A closed path in the network which originates from a particular node and terminates at the same node. In a loop, the nodes are not traversed twice except the initial point, which is also the final one. Other paths can be included inside a loop.
Mesh	Mesh is a set of branches forming a closed path in a network in such a way that if one branch is removed then the remaining branches do not form a closed path. A **mesh** has no other paths inside it. In other words, it is a loop with no other loops inside it.

Let us explain these terms with the help of an example. Refer to Fig. 7.4. The electrical network shown in it is an *active network* as it contains sources of energy, V_1 and V_2. The resistors (R_1 and R_2), capacitor (C) and inductor (L) are the *network elements*.

In the figure, *A-B*, *B-C*, *C-F*, etc. are the *branches* of the electrical network. The points *B* and *E* are the *junction points* and *A*, *B*, *C*, *D*, *E* and *F* are the *nodes* in the network. The paths *ABEDA*, *ABCFEDA*, *BCFEB*, etc., are the *loops* of the network. The path *ABEDA* is a mesh and *BCFEB* is another mesh in the circuit.

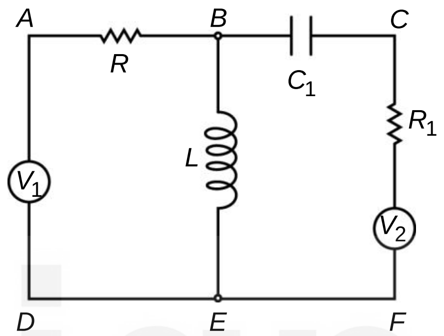


Fig. 7.4: **Examples of various terms used in electrical networks.**

Now that you are familiar with the network terminology, you may like to solve an SAQ to fix these ideas in your mind.

SAQ 1 – **Network terminology**

Are the electrical networks shown in Figs. 7.1a, b, c, 7.2 and 7.3 active or passive? Identify the network elements, branches, nodes, junction points, loops and meshes in the networks shown in these figures.

So far, you have learnt the basic terms used in network/circuit analysis. Actually, network/circuit analysis is the process of finding the voltages across, and the currents through, every component in the network/circuit. There are many different theorems and methods for calculating these values. Mostly, these theorems and methods apply to linear networks, i.e., networks consisting of elements that are linear. Recall that you have learnt in this section that a *linear element is one in which current is proportional to the voltage applied*.

We now introduce two network theorems that help us simplify complicated electrical networks/circuits so that we can easily determine the currents and voltages in their elements and branches. These are **Thevenin's theorem** and **Norton's theorem**.

7.2.2 Thevenin's Theorem

In your school physics laboratory, you have verified Ohm's law for a simple circuit like the one shown in Fig. 7.5a. From Ohm's law, the current through the load resistance R is just $I = \dfrac{E}{R}$. However, it is not easy to apply Ohm's law to the circuit of Fig. 7.5b to determine the current through the load resistance R_L. We can use Thevenin's theorem to simplify such complicated circuits/networks. Let us state the theorem.

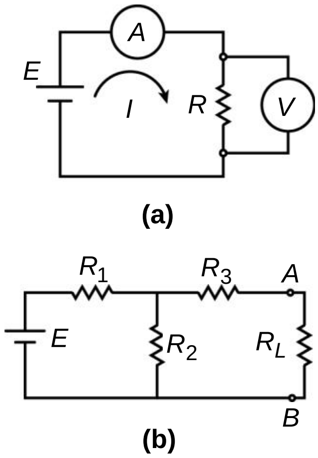


Fig. 7.5: **a) Simple electrical circuit for which it is easy to apply Ohm's law; b) example of a circuit for which it is not easy to apply Ohm's law.**

THEVENIN'S THEOREM

Thevenin's theorem states that **any two-terminal linear network containing voltage/current sources and impedances** can be replaced with **an equivalent circuit consisting of a voltage source E_{Th} <u>in series</u> with a resistance R_{Th}.**

Here E_{Th} (called the **<u>Thevenin voltage</u>**) is the **open circuit voltage between the terminals of the network.**

And R_{Th} (called the **<u>Thevenin resistance</u>**) is the **impedance measured between the terminals with all voltage sources replaced by their internal resistances and all current sources replaced by infinite resistance.**

So, Thevenin's theorem helps us to simplify any complicated circuit to a circuit containing a single voltage source in series with a single resistor.

Any complicated electric circuit such as the one shown in Fig. 7.6a can be replaced by a Thevenin equivalent circuit consisting of a voltage source E_{Th} in series with an equivalent resistance R_{Th} (Fig. 7.6b).

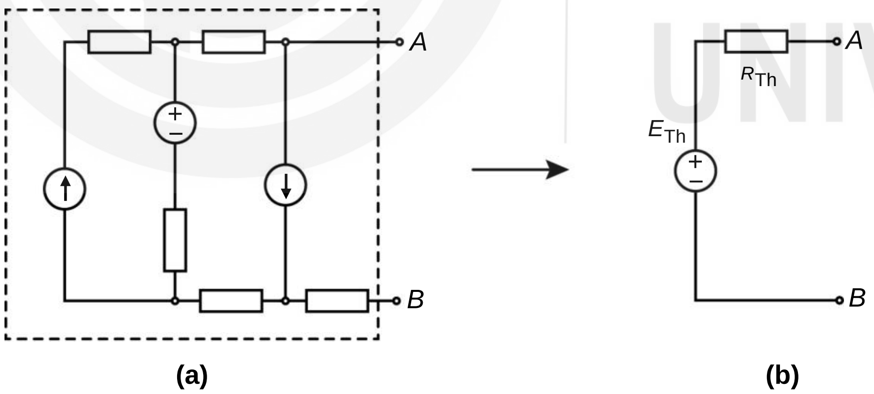


Fig. 7.6: **a) A complicated electrical circuit containing only resistances, and voltage and current sources; b) a Thévenin equivalent circuit for the circuit in part (a) consisting of an equivalent voltage source in series with an equivalent resistance known as Thevenin's resistance.**

The question obviously is: What are the values of E_{Th} and R_{Th} ?

Let us consider a simpler circuit as shown in Fig. 7.5b and apply Thevenin's theorem to it. It is the kind of circuit for which you will be verifying the Thevenin's theorem experimentally.

It can be simplified into a Thevenin's equivalent circuit shown in Fig. 7.6b.

Let us first determine the Thevenin voltage E_{Th} for this circuit.

Determination of E_{Th}

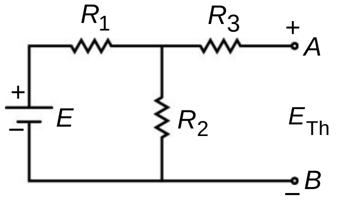


Fig. 7.7: **To determine the Thevenin voltage for the circuit shown in Fig. 7.5b, the load resistor is removed from the circuit. The Thevenin voltage is the open circuit voltage between the terminals *A* and *B*.**

In the first instance, we assume that the emf source in Fig. 7.5b has zero internal resistance. Since E_{Th} is the open circuit voltage between the terminals of the network, we remove the load resistor R_L from the circuit of Fig. 7.5b as shown in Fig. 7.7. The current in the circuit is

$$I = \frac{E}{R_1 + R_2} \qquad (7.1a)$$

Thevenin's voltage is the voltage across the terminals *A* and *B*. Since the current flows only through R_2, the voltage across the terminals *A* and *B* is just the voltage drop across R_2. Therefore,

$$E_{Th} = I R_2 \qquad \Rightarrow \qquad E_{Th} = \frac{E R_2}{R_1 + R_2} \qquad (7.1b)$$

Determination of R_{Th}

To determine R_{Th}, we replace the emf source by its internal resistance. Since we have assumed it to be zero, we short the circuit and also remove the load resistance R_L from the circuit of Fig. 7.5b as shown in Fig. 7.8.

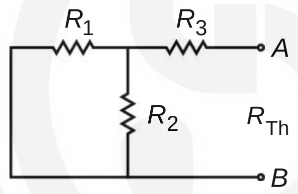


Fig. 7.8: **To determine the Thevenin resistance for the circuit shown in Fig. 7.5b, the voltage source is replaced by a short circuit and the load resistance is removed. The Thevenin resistance is the equivalent resistance in the circuit seen from the terminals *A* and *B*.**

Then the Thevenin resistance is the effective resistance across the terminals *A* and *B*. Note that in Fig. 7.8, R_1 is parallel to R_2 and their combination is in series with R_3. The resultant of the parallel combination of R_1 and R_2 is

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

Therefore, Thevenin resistance is

$$R_{Th} = R_3 + \frac{R_1 R_2}{R_1 + R_2} \qquad (7.2)$$

If the emf source has a finite internal resistance *r* (see Fig. 7.9a), then

$$I = \frac{E}{R_1 + R_2 + r}$$

$$\therefore \qquad E_{Th} = \frac{E R_2}{R_1 + R_2 + r} \qquad (7.3a)$$

Since *r* is in series with R_1 and their combination is parallel to R_2, their resultant is (see Fig. 7.9b), $\left(\frac{1}{R_1 + r} + \frac{1}{R_2}\right)^{-1} = \frac{(R_1 + r) R_2}{R_1 + R_2 + r}$. Further, this resultant resistance is in series with R_3, and therefore, the Thevenin resistance is given by

$$R_{Th} = R_3 + \frac{(R_1 + r) R_2}{R_1 + r + R_2} \qquad (7.3b)$$

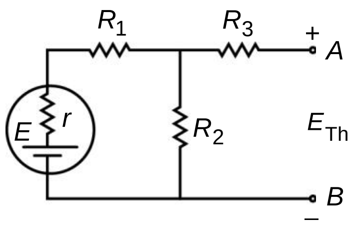


Fig. 7.9: **Taking into account the internal resistance of the emf source for determining a) Thevenin voltage; b) Thevenin resistance in the circuit of Fig. 7.5b.**

So far, you have learnt that Thevenin's theorem is used to simplify a complicated electrical circuit into a simple equivalent circuit consisting of a **single resistance <u>in series</u> with a <u>voltage</u> source**. You may like to solve a numerical problem to obtain a Thevenin equivalent circuit.

SAQ 2 – **Thevenin equivalent**

Suppose in Fig. 7.5b, the values of the resistors are $R_1 = 24\,\Omega$, $R_2 = 8.0\,\Omega$ and $R_3 = 44\,\Omega$, and $E = 40\,\text{V}$. Calculate the Thevenin equivalent voltage and resistance.

Next we discuss Norton's theorem which is used to simplify a complicated electrical circuit into a simple equivalent circuit consisting of *a* **single resistance <u>in parallel</u> with a <u>current</u> source**.

7.2.3 Norton's Theorem

Let us first give a formal statement of the theorem.

NORTON'S THEOREM

Norton's theorem states that **any linear circuit containing several energy sources and resistances can be replaced by a single constant current generator <u>in parallel</u> with a single resistor.**

So, any two-terminal linear network containing voltage/current sources and impedances can be replaced with **an equivalent circuit consisting of a current source** I_N **in parallel with a single resistance** R_N.

Here I_N (called **<u>Norton's current</u>**) is the **short circuit current at the output terminals** and R_N (called **<u>Norton resistance</u>**) is the **value of the resistance looking back into the network with all the current sources open circuited.**

The value of R_N is the same as that of the Thevenin resistance and the current I_N is determined by dividing the Thevenin voltage by R_N. Let us apply Norton's theorem to the circuit shown in Fig. 7.5b repeated here as Fig. 7.10. It is the kind of circuit for which you will be verifying Norton's theorem

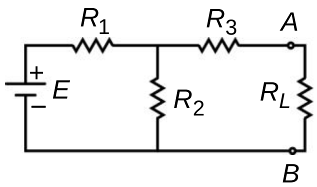


Fig. 7.10

experimentally. It can be simplified into a Norton's equivalent circuit shown in Fig. 7.11b.

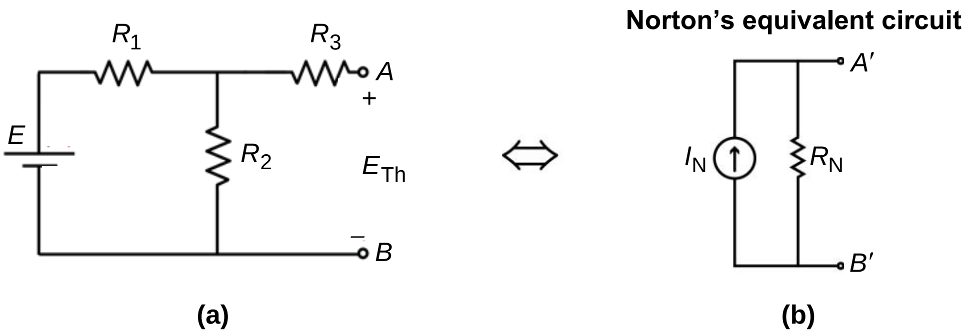


Fig. 7.11: **Norton's equivalent circuit for the circuit shown in Fig. 7.10. a) Thevenin voltage; b) Norton resistance R_N is equal to Thevenin resistance R_{Th} and Norton current I_N is equal to E_{Th}/R_N.**

Let us first assume that the internal resistance of the emf source is zero. Then from Eq. (7.2), the Norton resistance R_N for this circuit is equal to R_{Th}, i.e.,

$$R_N = R_3 + \frac{R_1 R_2}{R_1 + R_2} \tag{7.4}$$

From Eq. (7.1), the Thevenin voltage E_{Th} for this circuit is $E_{Th} = \dfrac{E R_2}{R_1 + R_2}$.

Therefore, Norton current is

$$I_N = \frac{E_{Th}}{R_N} = \frac{1}{R_N}\left(\frac{E R_2}{R_1 + R_2}\right) \tag{7.5}$$

If the emf source has a finite internal resistance r, then using Eqs. (7.3b and 7.3a), we have

$$R_N = R_3 + \frac{(R_1 + r) R_2}{R_1 + r + R_2} \quad \text{and} \tag{7.6a}$$

$$I_N = \frac{E_{Th}}{R_N} = \frac{1}{R_N}\left(\frac{E R_2}{R_1 + R_2 + r}\right) \tag{7.6b}$$

Before verifying Thevenin's and Norton's theorems experimentally, you may like to obtain an equivalent Norton circuit for the circuit of Fig. 7.10 for given values of resistors and voltage.

SAQ 3 – **Norton equivalent**

Suppose, in Fig. 7.10, the values of the resistors are $R_1 = 24\,\Omega$, $R_2 = 8.0\,\Omega$ and $R_3 = 44\,\Omega$, and $E = 40\,\text{V}$. Calculate the Norton equivalent resistance and current.

With this theoretical background, you are ready to do the experiment. So, you can now verify Thevenin's theorem in the first part of this experiment.

7.3 VERIFYING THEVENIN'S THEOREM

In this part of the experiment, you will connect an electrical circuit and simplify it to an equivalent Thevenin circuit. Then you will verify Thevenin's theorem. Before you actually do the experiment, you should take some preliminary steps before connecting the circuits for both these theorems. Note that you will be following the same preliminary steps when you do the next part of the experiment.

PRELIMINARY STEPS
1. Before using the power supply in the experiment, use a multimeter to measure its output to make sure that it is in the range you require.
2. Measure the values of resistances you use with a multimeter for their correct values.
3. Check the zero setting of ammeter and voltmeter.
4. Always switch off the power supply when you make circuit connections or modify the circuit in any way.

Now you should follow the steps described ahead.

1. Connect an electrical circuit as shown in Fig. 7.12 keeping the power supply off. Keep the voltage control knob of the power supply at minimum and its current control knob at maximum and then switch on the power supply. Choose the values of R_1, R_2, R_3 and R_L in the range of 1 kΩ to 10 kΩ. To begin with, you can choose $R_1 = R_2 = 500\,\Omega$, $R_3 = 250\,\Omega$ and $R_L = 500\,\Omega$.

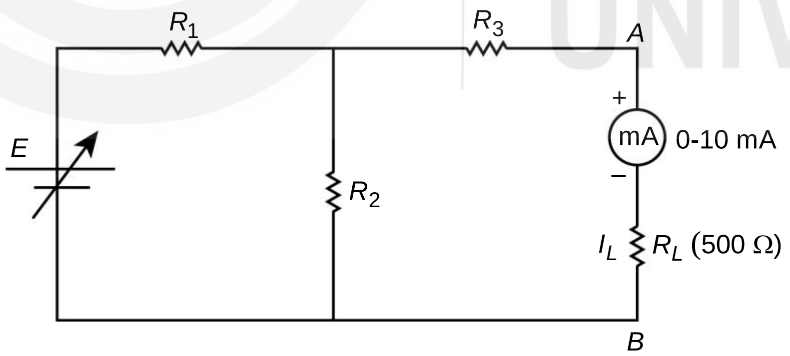


Fig. 7.12: **Circuit connections for verifying Thevenin's theorem.**

2. Set a particular value of voltage E, say 10 V or 12 V, using the power supply. Then using the ammeter, measure the load current I_L in the circuit through the load R_L. Note the values of R_1, R_2, R_3, E and I_L in the Observation Table 7.1.

3. To obtain the Thevenin equivalent of this circuit, switch off the power supply. Remove the load resistor and open the circuit as shown in Fig. 7.13. Then again switch on the power supply keeping the voltage the same as in step 2.

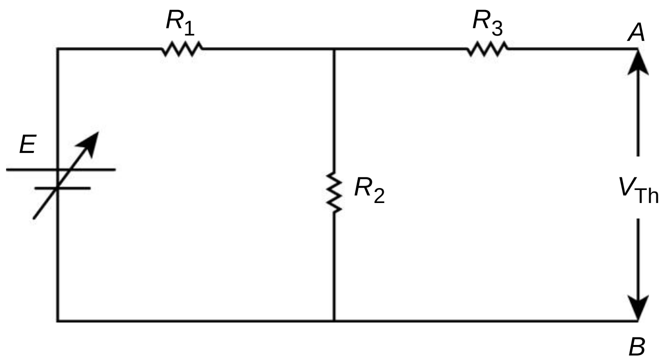


Fig. 7.13: **Circuit connections for measuring Thevenin voltage.**

4. Measure the open circuit voltage, i.e., the voltage across the terminals *A* and *B* with the help of a multimeter/voltmeter. This is the measured Thevenin voltage V_{Th}. Record this measured value of V_{Th} in Observation Table 7.1.

5. To measure Thevenin resistance, switch off the power supply, remove it and connect the circuit as shown in Fig. 7.14. Measure Thevenin resistance R_{Th} using a multimeter. It is the resistance in the circuit as seen from *A* and *B*.

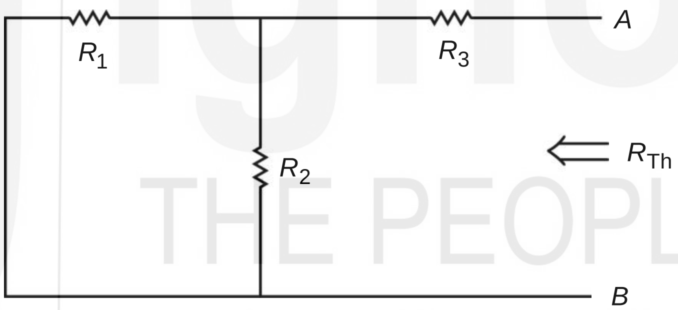


Fig. 7.14: **Circuit connections for measuring Thevenin resistance.**

6. Make the connections for the equivalent Thevenin circuit as shown in Fig. 7.15 using the measured values of Thevenin voltage V_{Th} and Thevenin resistance R_{Th}. Connect the same value of R_L that you have used in step 2.

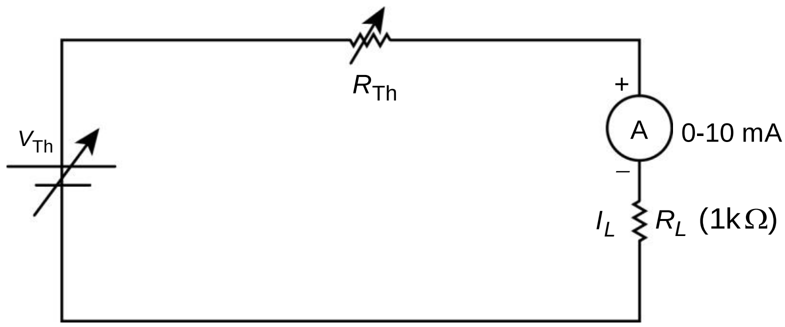


Fig. 7.15: **Connections for equivalent Thevenin circuit.**

7. Measure the load current I_L in the equivalent Thevenin circuit using the ammeter.

   Enter your measurements of the load current I_L in Observation Table 7.1.

Observation Table 7.1: Measured and calculated values for Thevenin equivalent circuit.

S. No.	Source voltage (V)	R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	Measured		Calculated		Load current I_L (mA)	
					V_{Th} (V)	R_{Th} (Ω)	E_{Th} (V)	R_{Th} (Ω)	Circuit of Fig. 7.12	Thevenin equivalent circuit

8. Calculate theoretical values of Thevenin voltage (E_{Th}) and Thevenin resistance (R_{Th}) using Eqs. (7.1 and 7.2), or Eqs. (7.3a and b) if you know the internal resistance of the power supply. Enter them in the Observation Table 7.1.

9. Repeat steps 4 to 9 at least 5 times for different sets of R_1, R_2, R_3 and source voltage and enter all measured and calculated values for each set in Observation Table 7.1. Create more rows in your practical notebook for your readings.

Compare the measured and calculated values of Thevenin voltage and Thevenin resistance for each set of resistors R_1, R_2 and R_3. Compare the load currents in the original circuit and its Thevenin equivalent.

a) What conclusions do you draw?

b) Have you been able to verify Thevenin's theorem?

c) What are the sources of error in the experiment?

Record your findings in your practical notebook. Discuss them with your Counsellor and write your analysis of the results.

Results and Analysis of Results: Record in your practical notebook.

7.4 VERIFYING NORTON'S THEOREM

In this part of the experiment, you will verify Norton's theorem. Remember to take the preliminary steps described in Sec. 7.3 before you start this part of the experiment. Then follow the steps described below for making circuit connections and simplifying the circuit to an equivalent Norton circuit.

1. Keep the power supply off and connect the electrical circuit shown in Fig. 7.16. Keep the voltage control knob of the power supply at minimum and its current control knob at maximum and then switch on the power

supply. Keep the same resistance values as for the first part of the experiment.

2. Set a particular value, E, of voltage using the power supply and using the ammeter, measure the load current I_L through the load R_L in this circuit. Record the measurements in Observation Table 7.2.

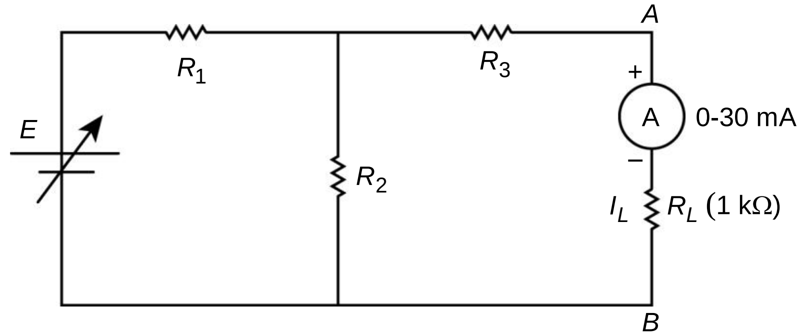


Fig. 7.16: **Circuit connections for verifying Norton's theorem.**

3. To obtain the Norton equivalent of this circuit, switch off the power supply, remove the load resistance and connect the circuit as shown in Fig. 7.17.

4. Switch on the power supply and keeping the voltage value the same as in step 2, measure the short circuit current through the ammeter. This is the Norton current I_N. Record the measured value of I_N in Observation Table 7.2.

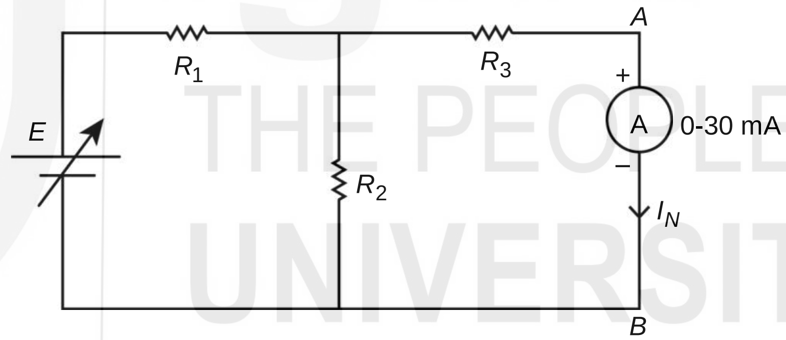


Fig. 7.17: **Circuit connections for measuring Norton current.**

5. To determine Norton resistance R_N, switch off the power supply and remove it from the circuit. Remove the load resistor and connect the circuit as shown in Fig. 7.18. Measure Norton resistance R_N across the terminals A and B using a multimeter.

   Record the measured value of R_N in Observation Table 7.2.

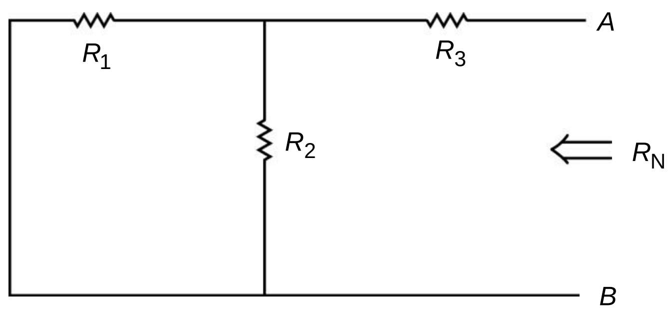
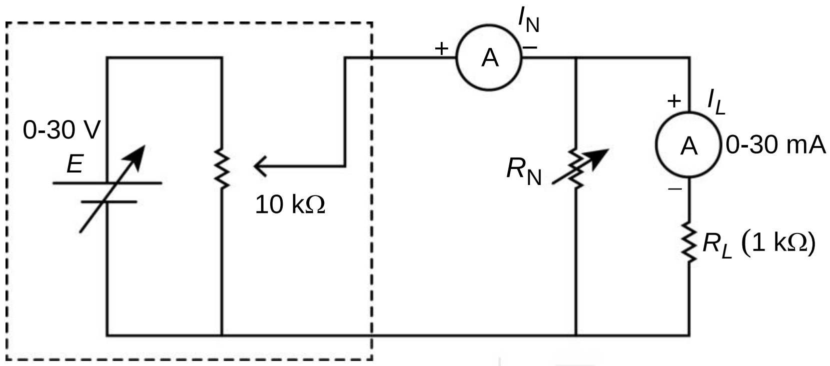


Fig. 7.18: **Circuit connections for measuring Norton resistance.**

6. Make the connections for the equivalent Norton circuit as shown in Fig. 7.19 using the measured values of Norton current I_N and Norton resistance R_N. Keep the value of the load resistor the same as in step 2. Connect the Norton resistance R_N in the circuit. Set Norton current I_N in the circuit by setting an appropriate voltage with the help of the power supply. Note the ammeter reading for the load current I_L. Record your measurement in Observation Table 7.2.



Constant current source

Fig. 7.19: **Circuit connections for Norton equivalent circuit.**

7. Using Eqs. (7.4, 7.5 or 7.6a and b if you know the internal resistance of the power supply), calculate theoretical values of Norton current I_N and Norton resistance R_N. Record these values in Observation Table 7.2.

8. Repeat the measurements at least 5 times for different sets of source voltage, R_1, R_2 and R_3. Enter all measured and calculated values for each set in Observation Table 7.2 in your practical notebook. Create more rows in your practical notebook for your readings.

Observation Table 7.2: Measured and calculated values for Norton equivalent circuit

S. No.	Source voltage (V)	R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	Measured		Calculated		Load current I_L (mA)	
					I_N (mA)	R_N (Ω)	I_N (mA)	R_N (Ω)	Circuit of Fig. 7.16	Norton Equivalent circuit

Compare the values of measured and calculated values of Norton current and Norton resistance for each set of resistors R_1, R_2 and R_3. Compare the load currents in the original circuit and the equivalent Norton circuit.

a) What conclusions do you draw?

b) Have you been able to verify Norton's theorem?

c) What are the sources of error in the experiment?

Record your findings in your practical notebook. Discuss them with your Counsellor and write your analysis of the results.

Results and analysis of results: Record in your practical notebook.