PHYS 305: Computational Physics II

Winter 2018

Homework #3

(Due: February 19, 2018)

Each problem is worth 10 points. E-mail your soultions to `steve@physics.drexel.edu` *with a subject including PHYS 305 and the Homework number. The e-mail should have as an attachment a zip (or tar) file containing a PDF document containing all discussion, results, and graphs requested, and files containing Python scripts for all programs written.*

1. (a) Do Exercise 6.2 on the course web page. Specifically, carry out three integrations of the Sun–Earth–Jupiter system, varying Jupiter's mass M_J and orbital semi-major axis R_J as follows: (i) $M_J = 0.01, R_J = 3.0$, (ii) $M_J = 0.02, R_J = 2.1$, (iii) $M_J = 0.03, R_J = 2.0$. Take $\epsilon = 0$ and run to $t = 1000$. In each case, plot Earth's orbital eccentricity e as a function of time and determine the maximum eccentricity e_{max} reached over the course of the calculation.

   (b) Repeat the orbital calculations (without the plots!) in part (a) for a grid of initial conditions in M_J and R_J: $M_J = 0.01, \ldots, 0.2$ in steps of 0.01, and $R_J = 1.2, \ldots, 2.5$ in steps of 0.1. Take $\epsilon = 0$ and run to $t = 2000$. For definiteness, let's call an orbit for which $e_{max} > 0.5$ an unstable outcome. Plot the outcome of each run as a small circle on the $M_J - R_J$ plane, indicating stability or instability by color, and using the size of the circle to indicate the eccentricity of a stable orbit.

   Note: this may take half an hour or more to run. Do the calculation first with a coarser grid in M_J and R_J to get the graphics right!

   If your computer can stand it, do each (M_J, R_J) run four times, choosing the initial phase of Jupiter's orbit to be $0, \pi/2, \pi$, and $3\pi/2$, and average the resulting e_{max} values.

   (c) What can you conclude about the region of stability in this diagram?

2. (a) Set up a three-body system in the "Pythagorean" configuration described in Exercise 6.3. Use $\epsilon = 0.1$ and $\delta t = 0.001$, and run to time $t = 50$. Plot the x-coordinates of all three bodies on a single graph.

   (b) Now simultaneously run two versions of the system, one with the same initial conditions as in part (a), the other with the initial y-coordinate of particle 2 equal to 0.05. Plot the rms "distance" D between the two simulations, defined as

   $$D = \sqrt{\frac{1}{M}\sum_{i=0}^{2} m_i\,|\mathbf{x}_i - \mathbf{x}'_i|^2},$$

   as a function of time, where $\mathbf{x}$ refers to the original system, $\mathbf{x}'$ to the modified one, and M is the total mass. Can you interpret this plot?

3. (a) Write a script to create a simple N-body system consisting of $N = 150$ identical particles with total mass 1, initially distributed in a homogeneous sphere of radius 1, with initial speeds $v = 0.7$ distributed isotropically in direction (as discussed in class). Choose an initial `numpy` random seed of 12345, and be sure to place the system in the center of mass frame.

   (b) Now create another similar system and separate the two so that their respective centers of mass are at $(2, 0.5, 0)$ and $(-2, -0.5, 0)$, and their center of mass velocities are $(-0.5, 0, 0)$ and $(0.5, 0, 0)$, putting them on a collision course.

   (c) Run the combined system for 30 time units, taking $\epsilon = 0.1$ and $\delta t = 0.001$. Plot as functions of time (i) the total energy, (ii) the total kinetic energy, and (iii) the rms "size" of the system

$$R = \sqrt{\frac{1}{M} \sum_{i=0}^{2} m_i \left| \mathbf{x}_i \right|^2}.$$

   Also plot snapshots of the system at times $t = 0, 1, 2, 5, 10$, and 30.