PHYS 305: Computational Physics II

Winter 2018

Homework #5

(Due: March 16, 2018)

Each problem is worth 10 points. E-mail your solutions to `steve@physics.drexel.edu` *with a subject including PHYS 305 and the Homework number. The e-mail should have as an attachment a zip (or tar) file containing a PDF document containing all discussion, results, and graphs requested, and files containing Python scripts for all programs written.*

1. Do Exercise 7.3 on the course web site.

2. Extend the finite-well calculation carried out in class as follows. In scaled units, Schrödinger's equation is

$$-\frac{d^2\psi}{dx^2} = (z - U)\,\psi\,,$$

where

$$U = \begin{cases} 0 & (|x| < 1)\,, \\ U_0 & (|x| > 1)\,. \end{cases}$$

We will search for even and odd solutions separately. We shoot from the center ($x = 0$) to the edge ($x = 1$) of the well.

For even solutions, the central boundary conditions are $\psi(0) = 1, \psi'(0) = 0$ (we can always scale ψ to satisfy the normalization condition). For odd solutions, we take $\psi(0) = 0, \psi'(0) = 1$. The boundary condition at $x = 1$ is that the solution match smoothly onto the exterior solution $\psi \sim e^{-\eta x}$ (with η as defined in class: $\eta^2 = U_0 - z$), so $\psi' + \eta\psi = 0$ at $s = 1$. The free variable z is the scaled energy; the error is $g(z) = \psi'(1) + \eta\psi(1)$.

For any choice of U_0, find all bound solutions. Then, by looping over U_0, for $U_0 = 0, \ldots, 100$ in steps of 0.1, plot your solutions for the scaled energy z as functions of scaled potential U_0.

3. Find the first 10 energy eigenvalues and eigenfunctions of the harmonic oscillator with

$$U(x) = x^2$$

by shooting from $x = 0$ to $x = x_0 = 6$. Be sure to include both even and odd solutions. Plot all eigenfunction solutions on a single graph, clearly indicating the energies associated with each.

What happens to the eigenvalues and eigenfunctions you calculate if x_0 is reduced to 4?

4. Find all bound solutions of the Schrödinger equation with potential

$$U(x) = -e^{-|x|^{1/2}}.$$

Plot the eigenfunctions and state the eigenvalues.