COMP 430/530: Data Privacy and Security - Fall 2023
Homework Assignment # 2

Introduction

This assignment consists of 4 parts (100 + 10 pts bonus). Parts 1 and 2 are theoretical; you can write your answer on paper and scan it, or you can type your answer using LaTeX. Parts 3 and 4 contain implementation questions. Here, use the Python skeleton files we provide as your starting point. Ensure that you adhere to the function names and parameters we ask for. For questions that require plots or discussion, you should submit your answers in a separate report (pdf file).

Part 1: Mini Questions [15 pts (5+10)]

(a) Consider that we have a database of Turkish residents and their COVID vaccination status. We open this database to queries, but implement a differential privacy (DP) interface such that all queries are answered with DP. Now suppose that one of the residents in the database, Ali, publishes an Instagram post which shows his personal vaccination status (e.g., photo of his vaccination card or screenshot from the vaccination app on his phone). Has differential privacy been violated? Why or why not?

(b) Let $A_1, A_2, ..., A_n$ be n independent algorithms which satisfy $\varepsilon_1, \varepsilon_2, ..., \varepsilon_n$-DP respectively. Does their sequential composition (i.e., applying one after the other in successive fashion) satisfy $(\sum_{i=1}^{n} \varepsilon_i)$-DP? *Hint: This is a simplified version of sequential composition in which $A_1, A_2, ..., A_n$ are independent. Use the fact that the algorithms are independent.*

Part 2: Privacy Proofs [30 pts]

Consider two values v_i, v_j. There are many ways to measure the distance $d(v_i, v_j)$ between them, e.g., absolute value distance, Euclidean distance, etc. A **metric** is a measure of distance which satisfies the following properties:

(1) Non-negativity: $d(v_i, v_j) \geq 0$, for all v_i, v_j
(2) Identity of indiscernibles: $d(v_i, v_j) = 0$ if and only if $v_i = v_j$
(3) Symmetry: $d(v_i, v_j) = d(v_j, v_i)$ for all v_i, v_j
(4) Triangle inequality: $d(v_i, v_k) \leq d(v_i, v_j) + d(v_j, v_k)$

Now consider that we are in a Local Differential Privacy (LDP) scenario. Each user has a true value v coming from a finite universe $\mathcal{U}$. A distance metric d is known for $\mathcal{U}$. The notion of metric-based LDP (MLDP) is defined as follows.

Definition 1 (α-MLDP) *A randomized algorithm $\mathcal{A}$ satisfies α-MLDP, where $\alpha > 0$, if and only if for any inputs $v_1, v_2 \in \mathcal{U}$:*

$$\forall y \in Range(\mathcal{A}): \quad \frac{Pr[\mathcal{A}(v_1) = y]}{Pr[\mathcal{A}(v_2) = y]} \leq e^{\alpha \cdot d(v_1, v_2)}$$

where Range($\mathcal{A}$) denotes the set of all possible outputs of algorithm $\mathcal{A}$.

Notice that MLDP is a modified version of the original LDP definition. In MLDP, indistinguishability of v_1, v_2 is dependent on not only the privacy parameter α but also $d(v_1, v_2)$. Since MLDP is different from LDP, we need to design new algorithms to achieve MLDP.

Let Ψ be a perturbation algorithm $\Psi : \mathcal{U} \to \mathcal{U}$, i.e., it takes as input some value $v \in \mathcal{U}$ and perturbs it to some value $y \in \mathcal{U}$. Given $v \in \mathcal{U}$, the probability that Ψ produces y as its output is:

$$\Pr[\Psi(v) = y] = \frac{e^{\frac{-\alpha \cdot d(v,y)}{2}}}{\sum_{z \in \mathcal{U}} e^{\frac{-\alpha \cdot d(v,z)}{2}}}$$

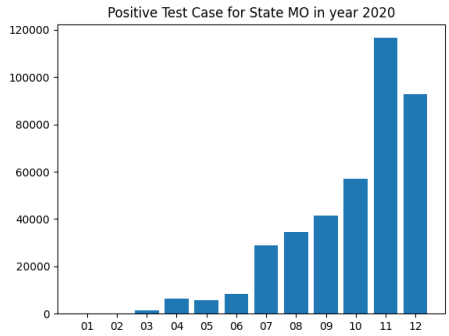
Prove that this Ψ satisfies α-MLDP.

Hints: The proof strategy is similar to one of the proofs we saw in class. In the proof, you may need to use the properties of a metric.

Part 3: Analyzing COVID Statistics with DP [35 pts (Task 1: 20 pts, Task 2: 15 pts)]

Consider the COVID-19 dataset provided to you: *covid19-states-history.csv*. This dataset is originally from the Covid Tracking Project, but we modified it for this homework assignment. The CSV file contains daily data on the COVID-19 pandemic for individual US states. Each row corresponds to a state's reported monthly death count, negative test count, and positive test count.

Task 1: Your task is to construct a histogram to examine the monthly positive case counts of COVID-19 for Texas (identified by the state code TX). This histogram should help us understand how positive cases fluctuate over time. Your histogram should look something like:



(a) Implement the function *get_histogram(dataset, state, year)* to construct a non-private histogram of monthly COVID-19 positive test counts. By default this function is configured for state = TX and year = 2020, but it must be adaptable to accommodate different states and years as inputs. The return value of this function should be a Python list containing 12 elements:

[January_positives, February_positives, ..., December_positives]

Draw a histogram using this list and add it to your report (your histogram should have a similar setup compared to the one given above, but the counts can be different).

(b) Implement the function *get_dp_histogram(dataset, state, year, epsilon, N)* to construct a DP histogram of monthly COVID-19 positive test counts. Among the parameters, *dataset, state, year* have the same meaning as the previous part. *epsilon* is the DP privacy parameter. N denotes the

max number of times an individual can test positively for COVID-19 in one month. *get_dp_histogram* should add appropriate Laplace noise to the histogram and return the resulting histogram. Its return value has the same format as *get_histogram*.

Note that, given *epsilon* and N, a critical part of your job is to decide what the appropriate amount of Laplace noise should be. Assume that neighboring datasets are obtained by addition or removal of one individual.

(c) Let H denote the actual histogram and $\hat{H}$ denote the private histogram. The Average Error in $\hat{H}$ can be measured bin-by-bin (bar-by-bar) as follows:

$$AvgErr(\hat{H}, H) = \frac{\sum_b |\hat{H}[b] - H[b]|}{\text{number of bins}}$$

Implement function *calculate_average_error* to compute $AvgErr$ according to this equation. This function should return a float (the error amount).

(d) You design the following experiment to measure the impact of ε on $AvgErr$. For ε values: $\{0.0001, 0.001, 0.005, 0.01, 0.05, 0.1, 1.0\}$, build ε-DP histograms and measure their average errors. Implement this experiment in the *epsilon_experiment* function. Within the function, you should measure error with each ε value 10 times and average the results for statistical significance. The function should return one list of errors (one error value for each ε value).

For this experiment, assume $N = 2$. In your report, provide a table containing your results. Briefly discuss the relationships between ε and error.

(e) You design the following experiment to measure the impact of N on $AvgErr$. For fixed $\varepsilon = 0.5$ and varying $N = \{1, 2, 4, 8\}$, build ε-DP histograms and measure their average errors. Implement this experiment in the *N_experiment* function. Within this function, you should measure error with each N value 10 times and average the results for statistical significance. The function should return one list of errors (one error value for each N value).

For this experiment, assume $\varepsilon = 0.5$. In your report, provide a table containing your results. Briefly discuss the relationships between N and error.

Task 2: We would like to answer the question: "Which month has the highest death count for the given state and year?" using the Exponential Mechanism.

Since some states have high death counts, there can be memory overflows when implementing the Exponential Mechanism in Python. To avoid this issue, it is acceptable to assume this task will be executed only with states that have low death counts, e.g., UT, WY, NV.

(f) Implement function *max_deaths_exponential* to achieve this. This function should internally implement the Exponential Mechanism so that a randomized result will be returned to achieve ε-DP. The return value should be the month with the most deaths in the given state and year.

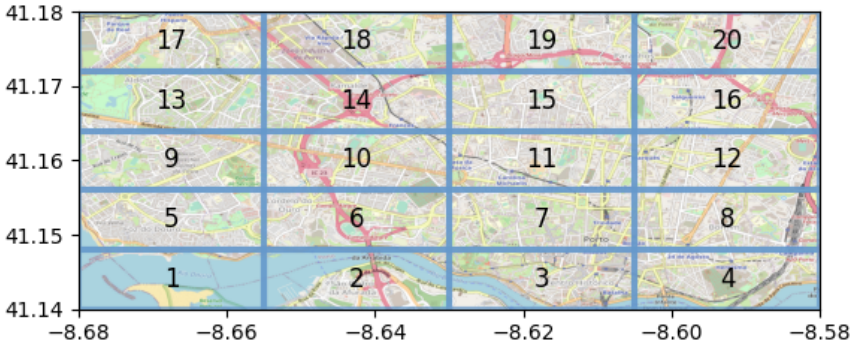
If you are wondering what is the right sensitivity value to use here, we urge you to consider how many times a person can die. ☺

(g) You design the following experiment to measure the impact of ε on the accuracy of the mechanism you implemented in the previous part. For ε values in $\{0.0001, 0.001, 0.01, 0.05, 0.1, 1.0\}$, run *max_deaths_exponential* 10000 times with each ε, and measure its accuracy as the percentage of times it returns the correct answer. Implement this experiment in the *exponential_experiment* function.

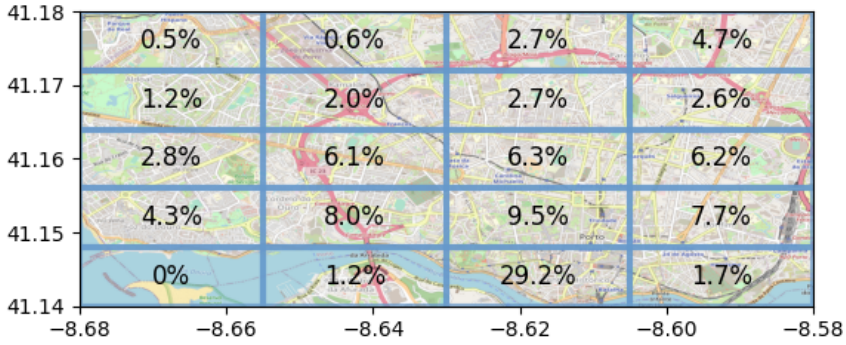
Provide a graph or table of your results in your report: accuracy vs ε. Briefly discuss your observations regarding the relationship between accuracy and ε.

Part 4: LDP Implementation [30 pts (8*3 protocols, 6 pts for experimental analysis)]

You are given a file *taxi-locations.dat* which contains the locations of taxis in the city of Porto. Although the locations were originally in (latitude, longitude) format, we modified them by placing a grid on the city of Porto, and discretized each location by replacing it with the corresponding cell it falls into. For example, the first row in *taxi-locations.dat* contains the integer "3" which means the first taxi is located in cell number 3, the second row contains the integer "10" which means the second taxi is located in cell number 10, etc. The following plot shows how the city was divided into 20 cells using a 5x4 grid.



Imagine that the current location (current grid cell) of each taxi is stored locally on the taxi's on-board navigation system. A central server, acting as the data collector, aims to learn the current distribution of taxis in Porto. In other words, the server would like to recover a distribution such as the one shown in the figure below, which denotes that 0% of taxis are in cell 1, 1.2% of taxis are in cell 2, 29.2% of taxis are in cell 3, and so forth.



To protect the taxis' location privacy, Local Differential Privacy (LDP) will be used. We will simulate data collection under LDP using 3 LDP protocols: GRR, RAPPOR, OUE.

Protocol 1: Generalized Randomized Response (GRR)

(a) Consider that we are using the GRR protocol. Implement *perturb_grr(val, epsilon)* for user-side perturbation. Given a single user's true value *val* (current cell as integer), *perturb_grr* returns the output that the user reports to the server.

(b) Implement *estimate_grr(perturbed_values, epsilon)* for server-side estimation. *estimate_grr* takes as input all users' perturbed values in a list and outputs the estimation result: For each cell $c \in [1,20]$, what is the percentage of taxis in cell c? *Note: If you end up with negative percentages for some cells due to LDP noise, that's OK, keep them that way.*

(c) Implement *grr_experiment(dataset, epsilon)*. This function simulates data collection for the whole user population, with GRR as the protocol and ε as the privacy parameter. Measure the error in the true percentages vs LDP percentages using the *AvgErr* metric from Part 3. The return value of *grr_experiment* should be *AvgErr* (a single float).

Protocol 2: RAPPOR

(d) Now consider that we are using RAPPOR instead of GRR. There exists an encoding step in RAPPOR before perturbation, which should be implemented in *encode_rappor(val)* function. Given a single user's true value *val* (integer), it should return the encoded bitvector as a list, e.g.: [0, 1, 0, 0, ..., 0]. Then implement *perturb_rappor(encoded_val, epsilon)* for user-side perturbation. Given an encoded bitvector, it should return the perturbed bitvector as a list, e.g.: [1, 1, ..., 0].

(e) Implement *estimate_rappor(perturbed_values, epsilon)*. *estimate_rappor* takes as input all users' perturbed bitvectors as a list [bitvector 1, bitvector 2, ..., bitvector n], and outputs the estimation result: For each cell $c \in$ [1,20], what is the percentage of taxis in cell c? *Note: If you end up with negative percentages for some cells due to LDP noise, that's OK, keep them that way.*

(f) Implement *rappor_experiment(dataset, epsilon)*. This function simulates data collection for the whole user population, with RAPPOR as the protocol and ε as the privacy parameter. Measure the error in the true percentages vs LDP percentages using the *AvgErr* metric from Part 3. The return value of *rappor_experiment* should be *AvgErr* (a single float).

Protocol 3: Optimized Unary Encoding (OUE)

(g) Now consider that we are using OUE. Implement OUE's encoding step in function: *encode_oue(val)*. Then implement *perturb_oue(encoded_val, epsilon)* for user-side perturbation. Given an encoded bitvector, it should return the perturbed bitvector as a list: [1, 1, ..., 0].

(h) Implement *estimate_oue(perturbed_values, epsilon)*. *estimate_oue* takes as input all the users' perturbed bitvectors as a list [bitvector 1, bitvector 2, ..., bitvector n], and outputs the estimation result: For each cell $c \in$ [1,20], what is the percentage of taxis in cell c? *Note: If you end up with negative percentages for some cells due to LDP noise, that's OK, keep them that way.*

(i) Implement *oue_experiment(dataset, epsilon)*. This function simulates data collection for the whole user population, with OUE as the protocol and ε as the privacy parameter. Measure the error in the true percentages vs LDP percentages using the *AvgErr* metric from Part 3. The return value of *oue_experiment* should be *AvgErr* (a single float).

Experimental Analysis

Error analysis: Conduct the following experiment: Simulate data collection with GRR, RAPPOR, and OUE for different ε values: ε = 0.01, 0.1, 0.5, 1, 2. Provide a table containing your results in your report. Briefly discuss the results: How are the protocols' errors impacted by ε? Is there a protocol that is always better?

Visual analysis: We provided you a function *plot_grid* which takes as input a list of percentages (e.g., [0, 1.2, 29.2, 1.7, ...]) and plots them on the map of Porto as shown on the previous page. Pick one of the protocols (GRR or RAPPOR or OUE), and for each different ε value, plot the percentages on the map. Add the plots to your report and discuss your observations from the plots.

Submission

When you are finished, submit your homework via Blackboard:

- Move all of your relevant files (including Python files, pdf report, etc.) into a folder named **your_KU_ID**.
- Compress this folder into a single zip file. Do not use compression methods other than zip.
- Upload your zip file to Blackboard.

Notes and reminders:

- After submitting, download your submission and double-check that: (i) your files are not corrupted, (ii) your submission contains all the files you intended to submit. If we cannot run your code due to missing files or functions, we cannot give you credit!
- You must upload your code in py files, we do not accept Python notebooks (ipynb extension).
- This homework is an individual assignment. All work needs to be your own. Submissions will be checked for plagiarism (including comparing to previous years' assignments).
- Your report should be a pdf file. Do not submit Word files or other file formats which may only be opened on Windows or Mac.
- Only Blackboard submissions are allowed. Do not e-mail your assignment to the instructor or TAs.
- Do not change the names or parameters of the functions we will grade.
- If your code does not run (e.g., syntax errors) or takes so long that grading it becomes impossible, you may receive 0 for the corresponding part.

Good Luck!