Exercise 3
3D Computer Vision

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1 Theory

1. let r_1, r_2 be the first column of $\mathbf{R}_0$ and second column of $\mathbf{R}_0$, $\mathbf{x}_0 = (X, Y)$, then $\mathbf{x}_1$ can be written as follows
$$\mathbf{x}_1 = \mathbf{K}[r_1 r_2 \mathbf{t}_0] \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

2. (a) Since $\mathbf{K}, \mathbf{R}, \mathbf{t}$ are known, thus we can calculate fundamental matrix F.
   (b) Based on $\mathbf{F}, \mathbf{x}_i$, we can get epipolar lines $\mathbf{l}_i$ for both images.
   (c) Determine the corresponding ray of $\mathbf{x}_i$ in each image by epipolar lines and extrinsic matrix.
   (d) The 3D point X is determined by the intersection of rays.

3. Epipole can be computed as follows:
$$\mathbf{e}_0 = \mathbf{P}_0\mathbf{C}_1, \ \mathbf{e}_1 = \mathbf{P}_1\mathbf{C}_0$$
   The relation between epipolar line and epipole is
$$\mathbf{l}_1 = \mathbf{F}_0\mathbf{x}_0, \ \mathbf{l}_0 = \mathbf{F}_1\mathbf{x}_1,$$

4. We can use 8-Point algorithm to compute $\mathbf{F}$ given $n \geq 8$ corresponding points $\mathbf{x}_i$ and $\mathbf{x}_i' i = 1, ..., n$, then the fundamental matrix F is the solution of equation system
$$\mathbf{x}_i^{'T}\mathbf{F}\mathbf{x}_i = \mathbf{0}, \ 1 \leq i \leq n \tag{1}$$

5. The fundamental matrix can be computed with intrinsic parameter and poses under equation
$$F = K^{'-T}[t]_\times RK^{-1}$$

6. A fundamental matching problem of matching technique is aperture problem. It is make sense to match features in this way when the object moves align with the direction of indicated direction.