Exercise 2
3D Computer Vision

Jingyuan Sha

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Part I
Theory

1 Homography Definition

1. A homography is a non-singular, line preserving projective mapping $h : P^n \mapsto P^n$. (from slide page 9.)
2. the degree of freedom is $(n+1)^2 - 1$

2 Line Preservation

Let mapping $h(\mathbf{x}_i) = \mathbf{H}\mathbf{x}_i$, where $\mathbf{x}_i$ lies on line $\mathbf{l}$ such as $\mathbf{l}\mathbf{x}_i = 0$. Then we have

$$\mathbf{l}\mathbf{H}^{-1}\mathbf{H}\mathbf{x}_i = 0$$

Then the mapping point $\mathbf{H}\mathbf{x}_i$ lies on line $\mathbf{l}\mathbf{H}^{-1}$. Thus the mapping points in projective space preserves lines.

3 Camera Center in World Coordinates

1. The camera matrix

$$\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

can map a homogeneous world coordinate to camera coordinate. Let R_C be the rotation matrix with respect to the world coordinate axes. Then, camera matrix is equal to the inverse of world coordinate matrix, that is

$$\begin{aligned}
\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} R_C & C_W \\ 0 & 1 \end{bmatrix}^{-1} \\
&= \left(\begin{bmatrix} I & C_W \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_C & 0 \\ 0 & 1 \end{bmatrix} \right)^{-1} \\
&= \left(\begin{bmatrix} R_C & 0 \\ 0 & 1 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} I & C_W \\ 0 & 1 \end{bmatrix} \right)^{-1} \\
&= \begin{bmatrix} R_C^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & -C_W \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} R_C^T & -R_C^T C_W \\ 0 & 1 \end{bmatrix}
\end{aligned}$$

Then we have

$$\begin{aligned}
R &= R_C^T \\
t &= -R_C^T C_W
\end{aligned}$$

That is $C_W = -R_C \cdot t = -R^T \cdot t$

2. the vector **t** points the position of world origin in camera coordinates.

Part II
Implementation