Exercise 2

Deadline: 28. November, 2021

Please send your results as a zip file either to **tewodros_amberbir.habtegebrial@dfki.de** or **torben.fetzer@dfki.de**.
Use the **Latex template** provided in Exercise_0 for the report.
Hand-written results will not be accepted!

Theory

1. Homography Definition

In the lecture a planar homography was introduced as $h : \mathbb{P}^2 \mapsto \mathbb{P}^2$.

(a) Give the definition for a general homography $h : \mathbb{P}^n \mapsto \mathbb{P}^n$.

(b) How many degrees of freedom does it have?

2. Line Preservation

Let $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3 \in \mathbb{P}^2$ be three points on a line. Show that a non-singular matrix $\mathbf{H} \in \mathbb{R}^{3\times3}$ that maps the points in projective space preserves this property.
(Hint: Use the implicit definition of a line $ax + by + c = 0$, thus $\mathbf{l}^\top \mathbf{x}_i = 0$ with $\mathbf{l} = (a, b, c)^\top$.)

3. Camera Center in Wold Coordinates

We are following the convention that a point $\mathbf{X}_W$ in homogeneous world coordinates is transformed to camera coordinates as follows: $\mathbf{X}_C = [\mathbf{R} \,|\, \mathbf{t}]\, \mathbf{X}_W$.
The camera center in world coordinates $\mathbf{C}_W$ can be expressed in terms of the extrinsic camera parameters by

$$\mathbf{C}_W = -\mathbf{R}^\top \mathbf{t}.$$

(a) Prove this property. (Hint: Where is the camera center in camera coordinates?)

(b) In this context, illustrate the meaning of $\mathbf{t} = -\mathbf{R}\mathbf{C}_W$, i.e. from where to where does this vector point?

Practical Part

For the following tasks, you find the required intrinsic parameters of the camera $(\alpha_x, \alpha_y, x_0, y_0, s)$ and the homographies $\mathbf{H}_i$ in the file `data/ex2.mat`.

1. Relative Rotation Estimation from Homography

A homography between two images taken with the same camera can be used to compute the relative rotation $\mathbf{R}_{rel}$ when the camera has undergone a pure rotation, i.e. no translation between the two views. Homography $\mathbf{H}_1$ (from `ex2.mat`) was obtained from a perfect pure rotation, whereas $\mathbf{H}_2$ was computed after imperfect (manual) rotation of the camera. Write a function `compute_relative_rotation` that:

(a) loads a homography from `ex2.mat`

(b) computes $\mathbf{R}_{rel}$ and prints it to the console

(c) checks whether $\mathbf{R}_{rel}$ fulfills the properties of a rotation matrix

(d) if necessary, corrects $\mathbf{R}_{rel}$ and prints the new rotation matrix to the console

Apply `compute_relative_rotation` to $\mathbf{H}_1$ and $\mathbf{H}_2$. Why does the rotation matrix computed from $\mathbf{H}_2$ need correction?

2. Camera Pose Estimation from Homography

(a) A homography between a plane in the world coordinate system and a camera image can be used to compute rotation $\mathbf{R}$ and translation $\mathbf{t}$ of the camera. Homography $\mathbf{H}_3$ was computed from the corners of a (fully visible) checkerboard. The checkerboard lies in the xy-plane of the world coordinate system centered around the origin. Write a function `compute_pose` to determine $\mathbf{R}$ and $\mathbf{t}$ from $\mathbf{H}_3$ and print them to the console.

(b) The third element of $\mathbf{t}$ in part (a) above might be negative. What does this mean in this particular case (consider the location of the checkerboard corners)? Why can this happen?

Remark:
Make sure your code executes the tasks above sequentially by simply calling `python main.py` (include `ex2.mat` alongside `main.py` in your `.zip` file).

Good Luck!