```
> restart; libname;
```
"D:\Maple 2017\lib" **(1)**

In order for Maple to load the GRTensor library, we must first add the directory it was installed in to the Maple libpath variable.

```
> libname := "D:/grtensor/released/grt3/lib", libname;
```
libname := "D:/grtensor/released/grt3/lib", "D:\Maple 2017\lib" **(2)**

Now the GRTensor module can be loaded using the with() command.

```
> with(grtensor);
```
"GRTensor III v2.1.10 Oct 3, 2017"

"Copyright 2017, Peter Musgrave, Denis Pollney, Kayll Lake"

"Latest version is at http://github.com/grtensor/grtensor"

"For help ?grtensor"

"Support/contact grtensor3@gmail.com"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *cmcompare*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *gralterd*, *grapply*, *grarray*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grt2DG*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*] **(3)**

Help for the GRTensor commands is now available. e.g. you can try ?spacetime

The spacetime command is used to specify the input for GRTensor. The most common use is to enter the line element of the spacetime you want to calculate objects in.

In this example we use a general metric and define f(r) such that the spacetime metric becomes the charged spherical vacuum; the Reisner-Nordstrom metric. This spacetime has a non-vanishing Einstein tensor since it is not pure vacuum and has some mass-energy due to the charge.

> $f(r) := 1 - \frac{2 \cdot m}{r} + \frac{q^2}{r^2};$

$$f := r \mapsto 1 - \frac{2\,m}{r} + \frac{q^2}{r^2} \quad \textbf{(4)}$$

> $spacetime\left(spherical, coord = [r, \mathrm{theta}, \mathrm{phi}, t], ds = -f(r) \cdot d[t]^2 + + \frac{d[r]^2}{f(r)} + r^2 \cdot \left(d\left[\mathrm{theta}\right]^2 + \sin(\mathrm{theta})^2 \cdot d[\mathrm{phi}]^2 \right) \right);$

```
Calculated ds for spherical (0.000000 sec.)
```
CPU Time = 0.

For the spherical spacetime:

Line element

(5)

$$ds^2 = \frac{d\,r^2}{1 - \frac{2\,m}{r} + \frac{q^2}{r^2}} + r^2\ d\,\theta^2 + r^2 \sin(\theta)^2\ d\,\phi^2 + \left(-1 + \frac{2\,m}{r} - \frac{q^2}{r^2}\right) d\,t^2 \qquad \textbf{(5)}$$

> *grcalcd*(*G*(*dn*, *up*));

```
Created definition for G(dn,up)
Calculated g(dn,dn,pdn) for spherical (0.015000 sec.)
Calculated Chr(dn,dn,dn) for spherical (0.000000 sec.)
Calculated detg for spherical (0.000000 sec.)
Calculated g(up,up) for spherical (0.000000 sec.)
Calculated Chr(dn,dn,up) for spherical (0.000000 sec.)
Calculated R(dn,dn) for spherical (0.000000 sec.)
Calculated Ricciscalar for spherical (0.000000 sec.)
Calculated G(dn,dn) for spherical (0.000000 sec.)
Calculated G(dn,up) for spherical (0.000000 sec.)
```

CPU Time = 0.015

For the spherical spacetime:

G(dn,up)

G(*dn*, *up*)

$$G_a^{\ b} = \begin{bmatrix} -\frac{q^2}{r^4} & 0 & 0 & 0 \\ 0 & \frac{q^2}{r^4} & 0 & 0 \\ 0 & 0 & \frac{q^2}{r^4} & 0 \\ 0 & 0 & 0 & -\frac{q^2}{r^4} \end{bmatrix} \qquad \textbf{(6)}$$

We now introduce the useful GRTensorIII command grdef. grdef allows new tensor objects to be defined. It makes use of a string expression in which tensor indices are places inside curly-braces {} and "upstairs" indices are prefixed with a ^ (caret) symbol. Symbols after a semi-colon are taken to be indices of covariant derivative in alignment with the usual mathematical notation. A new object definition consists of a left-hand side which defines the tensor name and indices (e.g. "Z{a ^b}"). The right-hand side must refer to known tensor objects and must have the same tensor indices as the LHS. The RHS may also have dummy indices to perform summations. For example to define the contracted divergence of the Einstein tensor "G{a ^b; b}".

As an example, we define the contracted divergence of the Einstein tensor.

> *grdef*("divG{a} := G{a ^b; b}");

```
Created a definition for G(dn,up,cdn)
Created definition for divG(dn)
```

Notice that GRTensorIII determined that is needed to define a new object G(dn, up, cdn). This object definition can be created algorithmically, GRIII can create covariant derivatives of known objects as needed.

Note also that indices indicating covariant derivatives are designated using cdn and cup. Indices for partial derivatives are designated pdn. pup.

We now calculate the divergence (even though we know this must be zero, since it is a mathematical identity).

```
> grcalcd(divG(dn));
Calculated G(dn,up,cdn) for spherical (0.000000 sec.)
Calculated divG(dn) for spherical (0.000000 sec.)
```

CPU Time $= 0.$

For the spherical spacetime:

divG(dn)

$divG(dn)$

$$divG_{a} = \textit{All components are zero} \tag{7}$$

\>

As must be true, all components of this new entity are zero. To confirm the intermediate values were computed we can examine the intermediate values:

```
> grdisplay(G(dn, up, cdn));
```

For the spherical spacetime:

G(dn,up,cdn)

$$G_{r}{}^{r}{}_{;r} = \frac{4 q^2}{r^5}$$

$$G_{\theta}{}^{\theta}{}_{;r} = -\frac{4 q^2}{r^5}$$

$$G_{\phi}{}^{\phi}{}_{;r} = -\frac{4 q^2}{r^5}$$

$$G_{t}{}^{t}{}_{;r} = \frac{4 q^2}{r^5}$$

$$G_{\theta}{}^{r}{}_{;\theta} = \frac{2 q^2 \left(2 m r - q^2 - r^2\right)}{r^5}$$

$$G_{r}{}^{\theta}{}_{;\theta} = -\frac{2 q^2}{r^5}$$

$$G_{\phi}{}^{r}{}_{;\phi} = \frac{2 q^2 \left(2 m r - q^2 - r^2\right) \sin(\theta)^2}{r^5}$$

$$G_{r}{}^{\phi}{}_{;\phi} = -\frac{2 q^2}{r^5} \tag{8}$$

\>