> *restart*; *libname*;

"D:\Maple 2017\lib" **(1)**

In order for Maple to load the GRTensor library, we must first add the directory it was installed in to the Maple libpath variable.

> *libname* := "D:/grtensor/released/grt3/lib", *libname*;

libname := "D:/grtensor/released/grt3/lib", "D:\Maple 2017\lib" **(2)**

Now the GRTensor module can be loaded using the with() command.

> *with*(*grtensor*);

"GRTensor III v2.1.10 Oct 3, 2017"

"Copyright 2017, Peter Musgrave, Denis Pollney, Kayll Lake"

"Latest version is at http://github.com/grtensor/grtensor"

"For help ?grtensor"

"Support/contact grtensor3@gmail.com"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *cmcompare*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *gralterd*, *grapply*, *grarray*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grt2DG*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*] **(3)**

Help for the GRTensor commands is now available. e.g. you can try ?spacetime

The spacetime command is used to specify the input for GRTensor. The most common use is to enter the line element of the spacetime you want to calulate objects in.

In this example we will follow the development in Schutz Ch 10 for a general spherically symmetric spacetime with co-ordinates r, theta, phi and t. The intent is to solve Einstein's equation and "discover" the Schwarzschild solution so the metric begins with general functions. With reference to (10.7) the spacetime command is used to enter this general spherical metric. Note that the dependence of the functions alpha, beta and gamma on r is explicitly indicated.

>

> *spacetime*(*spherical*, *coord* = [*r*, theta, phi, *t*], *ds* = -exp(2·Phi(*r*))· *d*[*t*]^2 + exp(2 · Lambda(*r*))·*d*[*r*]^2 + *r*^2·(*d*[theta]^2 + sin(theta)^2·*d*[phi]^2));

```
Calculated ds for spherical (0.000000 sec.)
```

CPU Time = 0.

For the spherical spacetime:

Line element

$$ds^2 = \left(e^{\Lambda(r)}\right)^2\, dr^2 + r^2\, d\theta^2 + r^2 \sin(\theta)^2\, d\phi^2 - \left(e^{\Phi(r)}\right)^2\, dt^2 \quad \textbf{(4)}$$

The metric spherical is now available for use.

The most common operations in GRTensor are the definition, calculation and simplification of the components of tensors in the spacetime. Definitions are provided for all of the commonly used tensors. A full list can be found on the ?grt_objects help screen.

To indicate a tensor object in GRTensor a Maple function expression is used. The name of the function is the tensor name and the arguments specify the number and type of indices. For example the covariant (indices down) metric tensor is g(dn,dn); the cotravariant version of the same tensor is g(up,up).

To calculate one of the pre-defined tensor objects the command grcalc() is used. The result of the calculation is not automatically displayed (because in some case the expression may be very large and require simplification). It can be displayed with grdisplay(). In cases where the output is expected to be small a tensor can be calulated and displayed using grcalcd().

We first demonstrate grcalc() by calculating the Christoffel symbols.

```
> grcalcd(Chr(dn, dn, up));
Calculated g(dn,dn,pdn) for spherical (0.000000 sec.)
Calculated Chr(dn,dn,dn) for spherical (0.000000 sec.)
Calculated detg for spherical (0.000000 sec.)
Calculated g(up,up) for spherical (0.000000 sec.)
Calculated Chr(dn,dn,up) for spherical (0.000000 sec.)
```

$$\mathit{CPU\ Time}\ = 0.$$

For the spherical spacetime:

Christoffel symbol of the second kind (symmetric in first two indices)

$$\Gamma_{r\,r}{}^{r} = \frac{d}{dr}\Lambda(r)$$

$$\Gamma_{r\,\theta}{}^{\theta} = \frac{1}{r}$$

$$\Gamma_{r\,\phi}{}^{\phi} = \frac{1}{r}$$

$$\Gamma_{r\,t}{}^{t} = \frac{d}{dr}\Phi(r)$$

$$\Gamma_{\theta\,\theta}{}^{r} = -\frac{r}{\left(e^{\Lambda(r)}\right)^2}$$

$$\Gamma_{\theta\,\phi}{}^{\phi} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\Gamma_{\phi\,\phi}{}^{r} = -\frac{r\sin(\theta)^2}{\left(e^{\Lambda(r)}\right)^2}$$

$$\Gamma_{\phi\,\phi}{}^{\theta} = -\sin(\theta)\cos(\theta)$$

$$\Gamma_{t\,t}{}^{r} = \frac{\left(e^{\Phi(r)}\right)^2\left(\frac{d}{dr}\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} \qquad \textbf{(5)}$$

Any components not show are zero. We go on to calculate the Einstein tensor

```
> grcalcd(G(dn, dn));
Calculated R(dn,dn) for spherical (0.000000 sec.)
```

```
Calculated Ricciscalar for spherical (0.000000 sec.)
Calculated G(dn,dn) for spherical (0.000000 sec.)
```

CPU Time $= 0.$

For the spherical spacetime:

Covariant Einstein

$G(dn, dn)$

$$G_{ab} = \left[\left[-\frac{\left(e^{\Lambda(r)}\right)^2 - 2r\left(\frac{d}{dr}\Phi(r)\right) - 1}{r^2}, 0, 0, 0 \right], \right.$$
$$\left[0, -\frac{1}{\left(e^{\Lambda(r)}\right)^2}\left(r\left(\left(\frac{d}{dr}\Lambda(r)\right)\left(\frac{d}{dr}\Phi(r)\right)r - \left(\frac{d}{dr}\Phi(r)\right)^2 r - \left(\frac{d^2}{dr^2}\Phi(r)\right)r + \frac{d}{dr}\Lambda(r) - \frac{d}{dr}\Phi(r)\right)\right), 0, 0 \right],$$
$$\left[0, 0, -\frac{1}{\left(e^{\Lambda(r)}\right)^2}\left(\sin(\theta)^2 r\left(\left(\frac{d}{dr}\Lambda(r)\right)\left(\frac{d}{dr}\Phi(r)\right)r - \left(\frac{d}{dr}\Phi(r)\right)^2 r - \left(\frac{d^2}{dr^2}\Phi(r)\right)r + \frac{d}{dr}\Lambda(r) - \frac{d}{dr}\Phi(r)\right)\right), 0 \right],$$
$$\left.\left[0, 0, 0, \frac{\left(e^{\Phi(r)}\right)^2\left(\left(e^{\Lambda(r)}\right)^2 + 2r\left(\frac{d}{dr}\Lambda(r)\right) - 1\right)}{r^2\left(e^{\Lambda(r)}\right)^2} \right] \right] \quad \textbf{(6)}$$

The command gralter allows access to many of the Maple simplification routines. When in doubt, the simplify routine is a reasonable first choice.

> *gralter(G(dn, dn), expand); grdisplay(G(dn, dn));*

```
Component simplification of a GRTensorIII object:

Applying routine expand to object G(dn,dn)
```

CPU Time $= 0.$

For the spherical spacetime:

Covariant Einstein

$G(dn, dn)$

$$G_{ab} = \left[\left[-\frac{\left(e^{\Lambda(r)}\right)^2}{r^2} + \frac{2\left(\frac{d}{dr}\Phi(r)\right)}{r} + \frac{1}{r^2}, 0, 0, 0 \right], \right. \quad \textbf{(7)}$$
$$\left[0, -\frac{r^2\left(\frac{d}{dr}\Lambda(r)\right)\left(\frac{d}{dr}\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} + \frac{r^2\left(\frac{d}{dr}\Phi(r)\right)^2}{\left(e^{\Lambda(r)}\right)^2} + \frac{r^2\left(\frac{d^2}{dr^2}\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} \right.$$

$$- \frac{r \left(\frac{d}{dr} \Lambda(r)\right)}{\left(e^{\Lambda(r)}\right)^2} + \frac{r \left(\frac{d}{dr} \Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2}, 0, 0 \Bigg],$$

$$\Bigg[0, 0, - \frac{r^2 \sin(\theta)^2 \left(\frac{d}{dr} \Lambda(r)\right) \left(\frac{d}{dr} \Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} + \frac{r^2 \sin(\theta)^2 \left(\frac{d}{dr} \Phi(r)\right)^2}{\left(e^{\Lambda(r)}\right)^2}$$

$$+ \frac{r^2 \sin(\theta)^2 \left(\frac{d^2}{dr^2} \Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} - \frac{r \sin(\theta)^2 \left(\frac{d}{dr} \Lambda(r)\right)}{\left(e^{\Lambda(r)}\right)^2} + \frac{r \sin(\theta)^2 \left(\frac{d}{dr} \Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2}, 0$$

$$\Bigg],$$

$$\Bigg[0, 0, 0, \frac{\left(e^{\Phi(r)}\right)^2}{r^2} + \frac{2 \left(e^{\Phi(r)}\right)^2 \left(\frac{d}{dr} \Lambda(r)\right)}{r \left(e^{\Lambda(r)}\right)^2} - \frac{\left(e^{\Phi(r)}\right)^2}{r^2 \left(e^{\Lambda(r)}\right)^2} \Bigg]\Bigg]$$

[Digression: the matrix output form here is a bit awkward. This can be controlled by one of the GRTensor options. Anne: I may just build a new version with a better default...]

> *groptions*( );

```
grOptionAlterSize       = false
grOptionCoordNames      = true
grOptionDefaultSimp     = 8
grOptionDisplayLimit    = 5000
grOptionLLSC            = true
grOptionMapletInput     = true
grOptionMetricPath      = (not assigned)
grOptionqloadPath       = (not assigned)
grOptionTermSize        = 100
grOptionTrace           = true
grOptionTimeStamp       = true
grOptionVerbose         = false
grOptionWindows         = true

grOptionDefaultSimp values: 0=None, 1=simplify, 2=simplify[trig],
  3=simplify[power] 4=simplify[hypergeom], 5=simplify[radical],
  6=expand, 7=factor, 8=normal, 9=sort, 10=simplify[sqrt]
  11=simplify[trigsin]
```

> *grOptionTermSize* := 50;

$$grOptionTermSize := 50 \quad \textbf{(8)}$$

> *grdisplay*(*G*(*dn*, *dn*));

For the spherical spacetime:

Covariant Einstein

G(*dn*, *dn*)

$$G_{r\,r} = -\frac{\left(e^{\Lambda(r)}\right)^2}{r^2} + \frac{2\left(\frac{d}{dr}\,\Phi(r)\right)}{r} + \frac{1}{r^2}$$

$$G_{\theta\,\theta} = -\frac{r^2\left(\frac{d}{dr}\,\Lambda(r)\right)\left(\frac{d}{dr}\,\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} + \frac{r^2\left(\frac{d}{dr}\,\Phi(r)\right)^2}{\left(e^{\Lambda(r)}\right)^2} + \frac{r^2\left(\frac{d^2}{dr^2}\,\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} - \frac{r\left(\frac{d}{dr}\,\Lambda(r)\right)}{\left(e^{\Lambda(r)}\right)^2} + \frac{r\left(\frac{d}{dr}\,\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2}$$

$$G_{\phi\,\phi} = -\frac{r^2\sin(\theta)^2\left(\frac{d}{dr}\,\Lambda(r)\right)\left(\frac{d}{dr}\,\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} + \frac{r^2\sin(\theta)^2\left(\frac{d}{dr}\,\Phi(r)\right)^2}{\left(e^{\Lambda(r)}\right)^2} + \frac{r^2\sin(\theta)^2\left(\frac{d^2}{dr^2}\,\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2} - \frac{r\sin(\theta)^2\left(\frac{d}{dr}\,\Lambda(r)\right)}{\left(e^{\Lambda(r)}\right)^2} + \frac{r\sin(\theta)^2\left(\frac{d}{dr}\,\Phi(r)\right)}{\left(e^{\Lambda(r)}\right)^2}$$

$$G_{t\,t} = \frac{\left(e^{\Phi(r)}\right)^2}{r^2} + \frac{2\left(e^{\Phi(r)}\right)^2\left(\frac{d}{dr}\,\Lambda(r)\right)}{r\left(e^{\Lambda(r)}\right)^2} - \frac{\left(e^{\Phi(r)}\right)^2}{r^2\left(e^{\Lambda(r)}\right)^2} \tag{9}$$

We can compare the results to (10.14) to (10.17)

>