

Constraint Programming Assignment2

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SUB1: Simple linear inequality

We want to define propagators for $a \cdot x + b \cdot y \leq c$.

We firstly rewrite the equation as:

$$a \cdot x \leq c - b \cdot y$$

$$b \cdot y \leq c - a \cdot x$$

now let's examine the case of $a > 0, b > 0$

the propagator for this case would be:

$$p(s) = \{ X \rightarrow \{ n \in s(x) \mid n \leq (c - b \cdot \min(s(y)))/a \}, \\ Y \rightarrow \{ n \in s(y) \mid n \leq (c - a \cdot \min(s(x)))/b \} \} = s'$$

as we are cutting from the higher end of both domains ($n \leq \dots$) we will have either

$\min(s(x)) = \min(s'(x))$ or $\min(s(y)) = \min(s'(y))$, or we have cut out every possible value leaving $s'(x) = \emptyset$ or $s'(y) = \emptyset$. The point is that in all those cases reapplying the propagator will yield the same result.

If we keep doing this for all combinations of signs (or zero equalities) of a and b we will find that we are always cutting the opposite end of the domain so that we have a fixpoint after one propagation. We discover subsumption after only one propagation.

Therefore the propagator is idempotent.

SUB2: Changing Propagation Order

Not true.

Counter Example:

$p1 : x \neq y$

$p2 : x < 2$

$s = \{ \{x \rightarrow \{1,2,3\}\}, y \rightarrow \{1,2,3\} \}$

$p1(p2(s)) = \{x \rightarrow \{1\}, y \rightarrow \{2,3\}\}$

$p2(p1(s)) = \{x \rightarrow \{1\}, y \rightarrow \{1,2,3\}\}$

SUB3: Idempotent Propagator

Propagators always strengthen the store, or leave the store as it is.

This we write as: for any store s and propagator p where $p(s) = s'$, $s \geq s'$.

When we apply a propagator we therefore have two cases:

1. we reach a fix point because the store doesn't change
2. we strengthen the store by removing possible value(s).

In the first case the propagator is clearly idempotent. In the second case, we will remove some possible values and after that be in a situation: either we have more possible values, or we don't have any more possible values. If we have more possible values we reapply the propagator. If we don't have more possible values we have reached a fix point. By induction we find that there will always exist an $n > 0$ so that $p^n(s)$ is idempotent.

Is this true for arbitrary functions on arbitrary sets?

This is not true for example in the case that the set X is \mathbb{R} (the real number set) and the function $f: X \rightarrow X$ is defined as $f(x) = x+1$

Therefore it is not true for arbitrary functions on arbitrary sets.