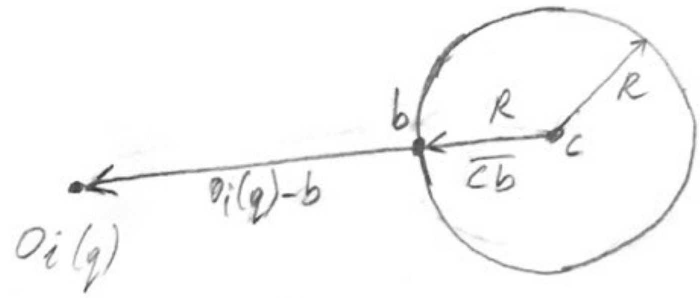
Case 1: sphere with R, centered at $c = (c_x, c_y, c_z)$



$\star$ $\overline{cb}$ is in the direction of $o_i(q)-b$ and $o_i(q)-c$

Now, $$b = c + R \cdot \underbrace{\frac{o_i(q)-c}{\|o_i(q)-c\|}}$$

unit vector in the direction of $o_i(q)-b$ and $o_i(q)-c$ (just to give direction to R)

Thus,

$$\Rightarrow o_i(q) - b = o_i(q) - c - R \cdot \frac{o_i(q)-c}{\|o_i(q)-c\|} = \left(o_i(q)-c\right)\left(1 - \frac{R}{\|o_i(q)-c\|}\right)$$

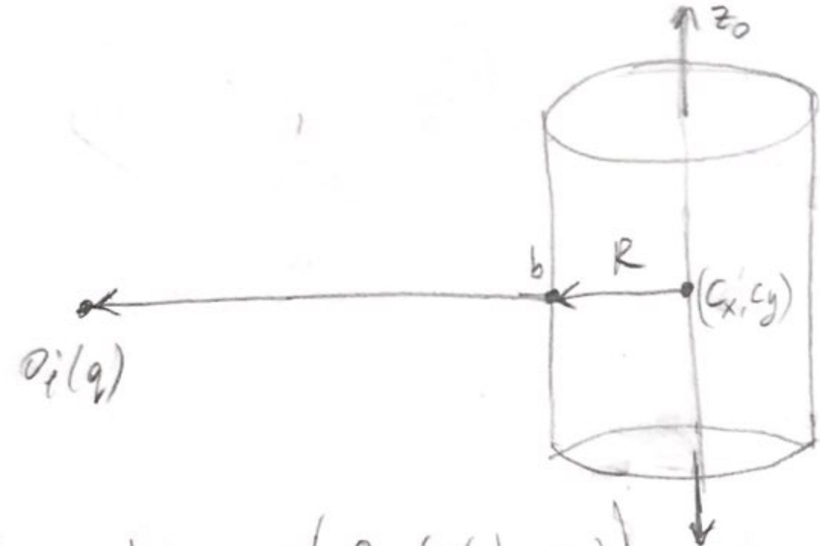
From the figure we see that

$$o_i(q) - c = \overline{cb} + o_i(q) - b$$

And magnitude wise: $\|o_i(q)-c\| = \underbrace{\|\overline{cb}\|}_{R} + \|o_i(q)-b\|$

$$\Rightarrow \|o_i(q)-b\| = \|o_i(q)-c\| - R$$

Case 2: cylinder of inf. height, centered at $c = (c_x, c_y)$, axis parallel to z_0, radius R



Now, $$b_x = c_x + \left(R \cdot \frac{(o_i(q)_x - c_x)}{\|o_i(q)_x - c_x\|} \right)$$

$$b_y = c_y + \left(R \cdot \frac{o_i(q)_y - c_y}{\|o_i(q)_y - c_y\|} \right)$$

$$b_z = o_i(q)_z$$

$$\Rightarrow o_i(q) - b = \begin{bmatrix} (o_i(q)_x - c_x) \\ (o_i(q)_y - c_y) \\ 0 \end{bmatrix} \cdot \left(1 - \frac{R}{\|o_i(q) - c_{(x,y)}\|} \right)$$

$$\Rightarrow \|o_i(q) - b\| = \|o_i(q) - c\| - R \quad \text{(similar to part 1)}$$