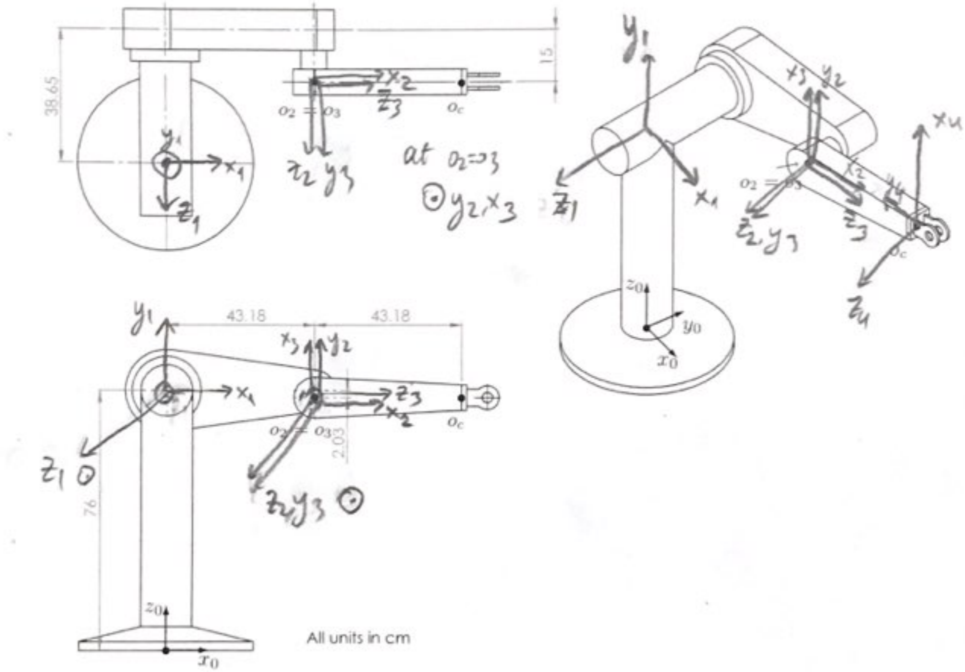


Figure 1: Schematics of the PUMA560 robot

By extracting the 3×3 matrix R_6^0 and the 3×1 vector o_6^0 from H_6^0, the forward kinematics computation is complete. Recall the geometric meaning of R_6^0 and o_6^0: the columns of R_6^0 are the unit axes of the end effector frame 6 represented in the coordinates of frame 0: $R_6^0 = [x_6^0 \mid y_6^0 \mid z_6^0]$; the vector o_n^0 is the origin of frame 6 expressed in the coordinates of frame 0.

2.2 Inverse Kinematics

Given a desired position $o_d^0 \in \mathbb{R}^3$ and desired orientation $R_d \in \mathrm{SO}(3)$ of the end effector, the inverse kinematics problem is to find $(\theta_1, \ldots, \theta_6)$ such that $R_6^0(\theta_1, \ldots, \theta_6) = R_d$ and $o_6^0(\theta_1, \ldots, \theta_6) = o_d^0$.

Since the PUMA 560 has six links and a spherical wrist, one can solve the inverse kinematics problem by the technique of kinematic decoupling. In kinematic decoupling, the problem is divided in two parts: inverse position and inverse orientation.

Inverse position. The position of the wrist centre o_c only depends on the angles of the first three joints, $(\theta_1, \theta_2, \theta_3)$. The idea is to compute the desired location of the wrist centre and then find $(\theta_1, \theta_2, \theta_3)$ accordingly.

- Compute the vector

$$o_d^0 - R_d \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix}.$$

LAB 1

DH Table

*All units in cm & rads

LINK	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	76	θ_1
2	43.23	0	-23.65	θ_2
3	0	$\pi/2$	0	θ_3
4	0	$-\pi/2$	0.4318	θ_4
5	0	$\pi/2$	0	θ_5
6	0	0	20	θ_6

$a_1 = \sqrt{43.18^2 + 2.03^2}$

$= 43.237$

$d_1 = -38.65 + 15 = -23.65$

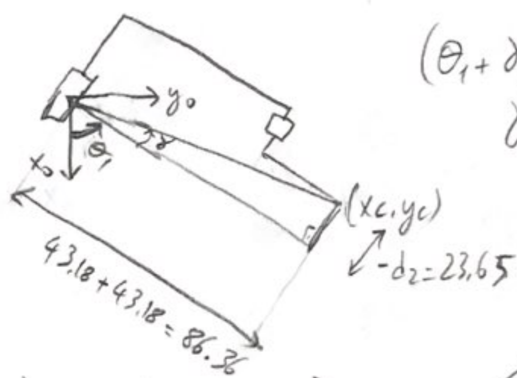
using spherical joint DH table and $d_6 = 20$

Inverse Kinematics

Now that we have the DH table, we can look into IK

Find θ_1:

top view →



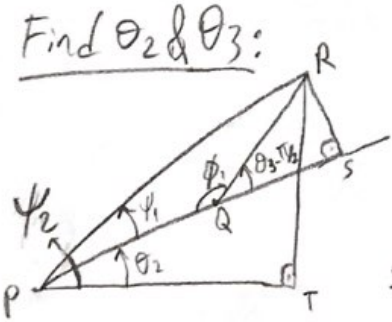
$(\theta_1 + \gamma) = \operatorname{atan2}(y_c, x_c)$

$\gamma = \operatorname{atan}(23.65/86.36)$

$= \arcsin\left(-d_2/\sqrt{x^2+y^2}\right)$

$\Rightarrow \theta_1 = \operatorname{atan2}(x_c, y_c) - \operatorname{atan}(23.65/86.36)$

Find θ_2 & θ_3:



Law of cosines on PQR:

$(\overline{PT}^2 + \overline{RT}^2) = \overline{PQ}^2 + \overline{RQ}^2 - 2\overline{PQ}\,\overline{RQ}\cos(\pi - \theta_3 + \pi/2)$

$\Rightarrow \sin\theta_3 = \dfrac{\overline{PT}^2 + \overline{RT}^2 - \overline{PQ}^2 - \overline{RQ}^2}{2\overline{PQ}\,\overline{RQ}}$

* $\overline{PT}^2 = (x_c^2 + y_c^2)\cos^2\gamma$
* $\overline{RT} = z_c - d_1$ * $\overline{PQ} = a_2$ * $\overline{RQ} = d_4$

$$\Rightarrow \sin\theta_3 = \frac{(x_c^2+y_c^2)\cdot\cos\gamma + (z_c-d_1)^2 - a_2^2 - d_4^2}{2a_2d_4} = D$$

$$\Rightarrow \theta_3 = \operatorname{atan2}\left(D, \pm\sqrt{1-D^2}\right)$$

Now $\theta_2 = \Psi_2 - \Psi_1$

$$\Psi_1 = \operatorname{atan2}\left(d_4 \sin(\theta_3 - \pi/2), a_2 + d_4\cos(\theta_3-\pi/2)\right)$$

$$\Psi_2 = \operatorname{atan2}\left(z_c - d_1, (x_c^2+y_c^2)\cos\gamma\right)$$

And $\theta_2 = \Psi_2 - \Psi_1$.

We now know $\theta_1, \theta_2, \theta_3$, still need $\theta_4, \theta_5, \theta_6$:

we have the formula for H_i^{i-1} given.

$\rightarrow$ Compute $H_3^0 = H_1^0 H_2^1 H_3^2$, extract R_3^0

$\rightarrow$ Compute $(R_3^0)^T R_d = R_6^3$

From here, the $\theta_4, \theta_5, \theta_6$ could be written in terms of R_d & o_d. This was done in the lab as well following the outlined steps.