BIL 105E – Introduction to Scientific and Engineering Computing (C)

Spring 2016-2017

Homework 4
Treasure Hunt
Example On Paper

In this document, you can find an example for the homework. Purpose of this document is to give you illustrations of what happens for each instructions and show you some basics about linear algebra.

Let the following line is the input of the program:

m 1 3 c 1 -1 1 1 m 2 2 r 30 m -1 4 l 60 m 1 5 d

1. First instruction is m which is move

$$m\,1\,3 \rightarrow move\begin{bmatrix}1\\3\end{bmatrix}$$

Treasure hunter starts at point (0,0). He should move using this vector above. New coordinates would be (1*basis_vector1, 3*basis_vector2). Basis vectors are the vectors that defines our coordinate system. We can show these vectors in a matrix A standard x-y coordinate system has the following basis vectors.

$$basis\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$\begin{bmatrix}1\\0\end{bmatrix}$ is the first basis vector (x axis)

$\begin{bmatrix}0\\1\end{bmatrix}$ is the second basis vector (y axis)

When you move, you move respect to the basis. A matrix,vector multiplication is required for this operation.

$$newcoordinates=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}*\begin{bmatrix}1\\3\end{bmatrix}+previouscoordinates$$

Previous coordinates were (0,0)
New coordinates become (1,3).
Check Figure 1.
In figures:
Red arrow : first basis vector
Green arrow : second basis vector
Blue arrow : move operation

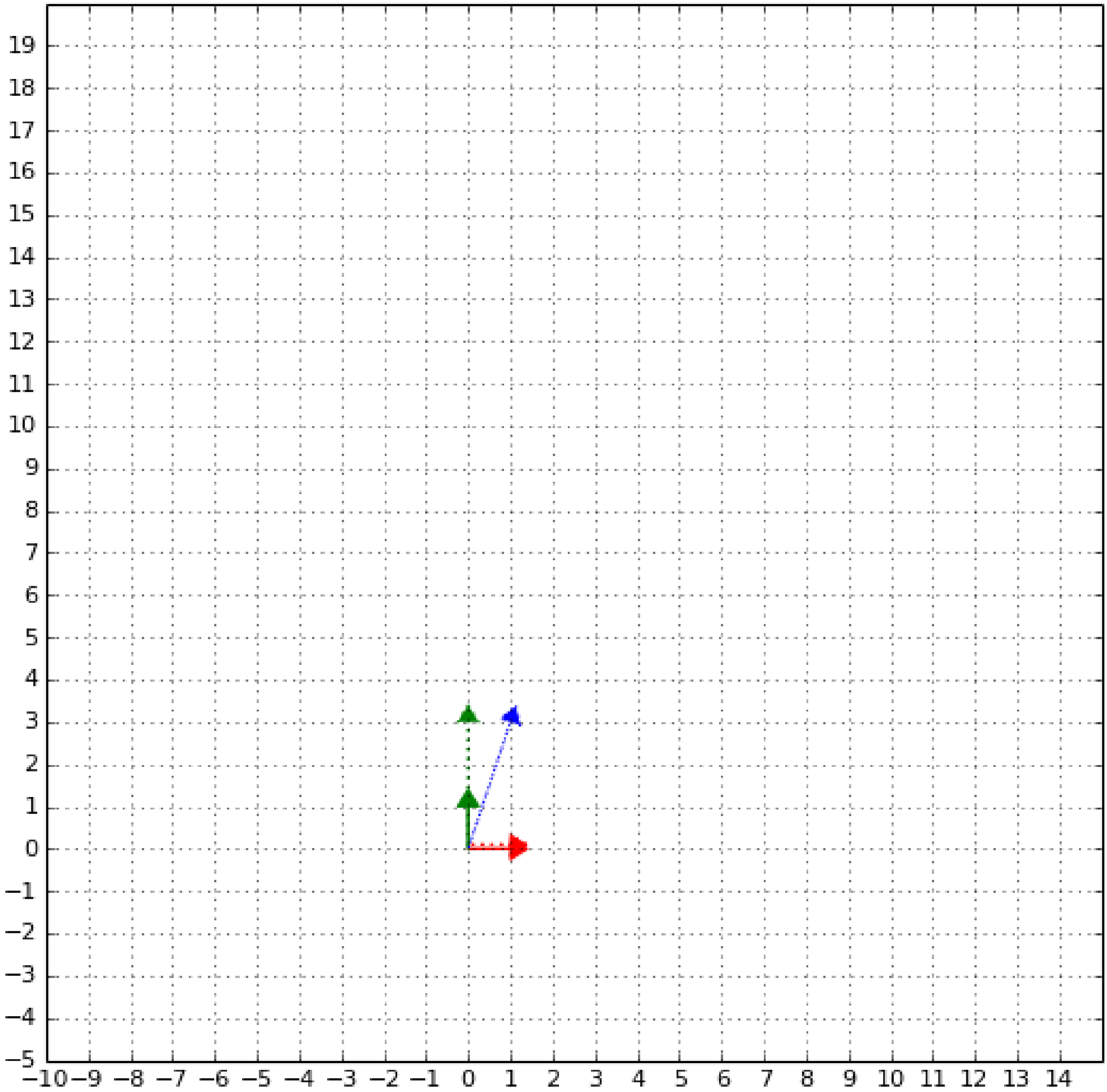

Figure 1. (m 1 3)

2. Next instruction is *c* which is change basis.

$$\text{c 1 -1 1 1} \rightarrow changebasis\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

This instruction changes the basis we use for move operations. Our new basis becomes:

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Check Figure 2.

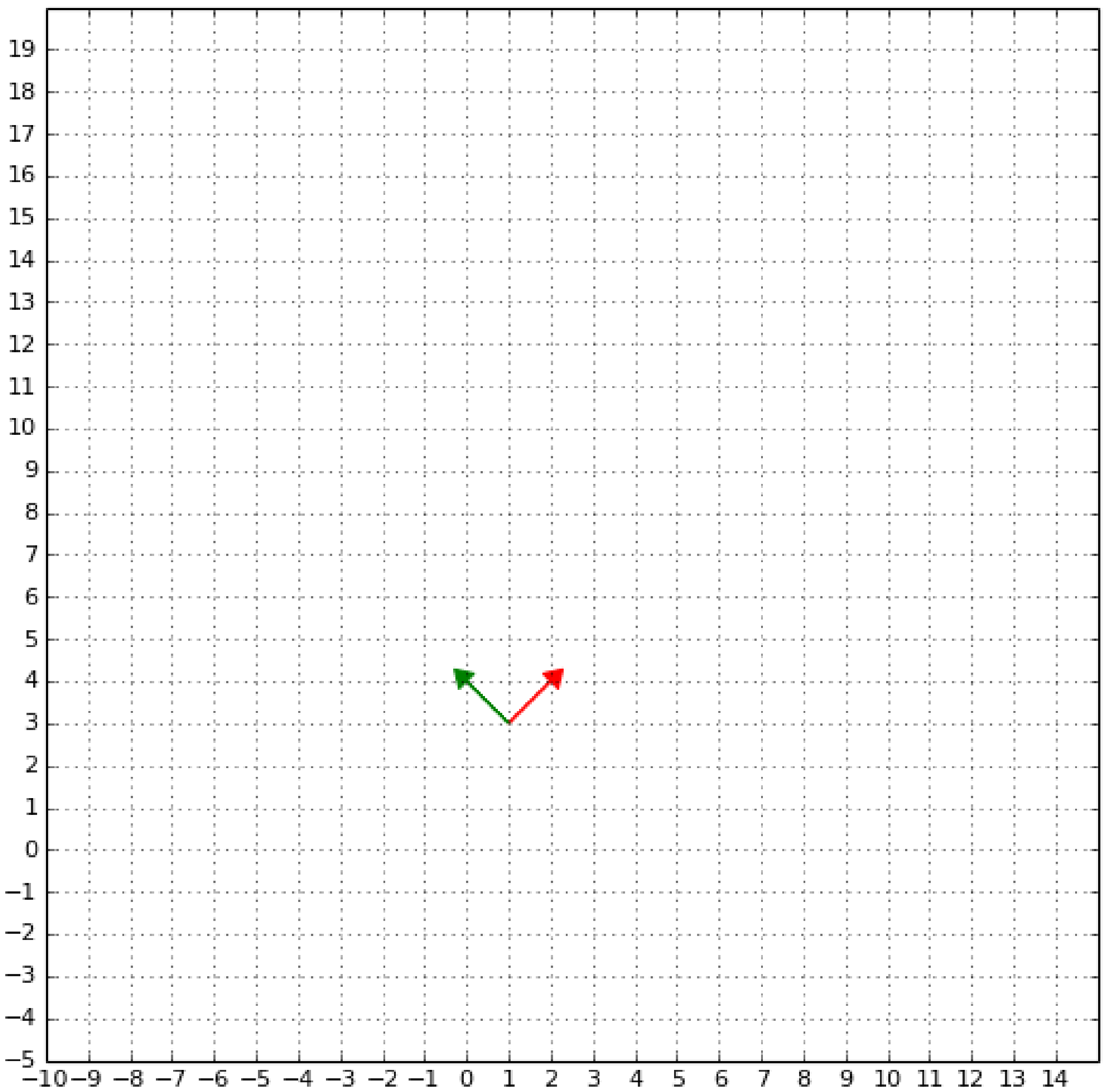

Figure 2. (c 1 -1 1 1)

3. Next instruction is m which is move.

$$m\,2\,2 \rightarrow move\begin{bmatrix}2\\2\end{bmatrix}$$

We can think as we need to move 2 times with basisvector1 and 2 times with basisvector2. This can be shown as a matrix-vector multiplication.

$$newcoordinates=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}*\begin{bmatrix}2\\2\end{bmatrix}+previouscoordinates$$

Previous coordinates were (1,3)
New coordinates become (1,7).
Check Figure 3.

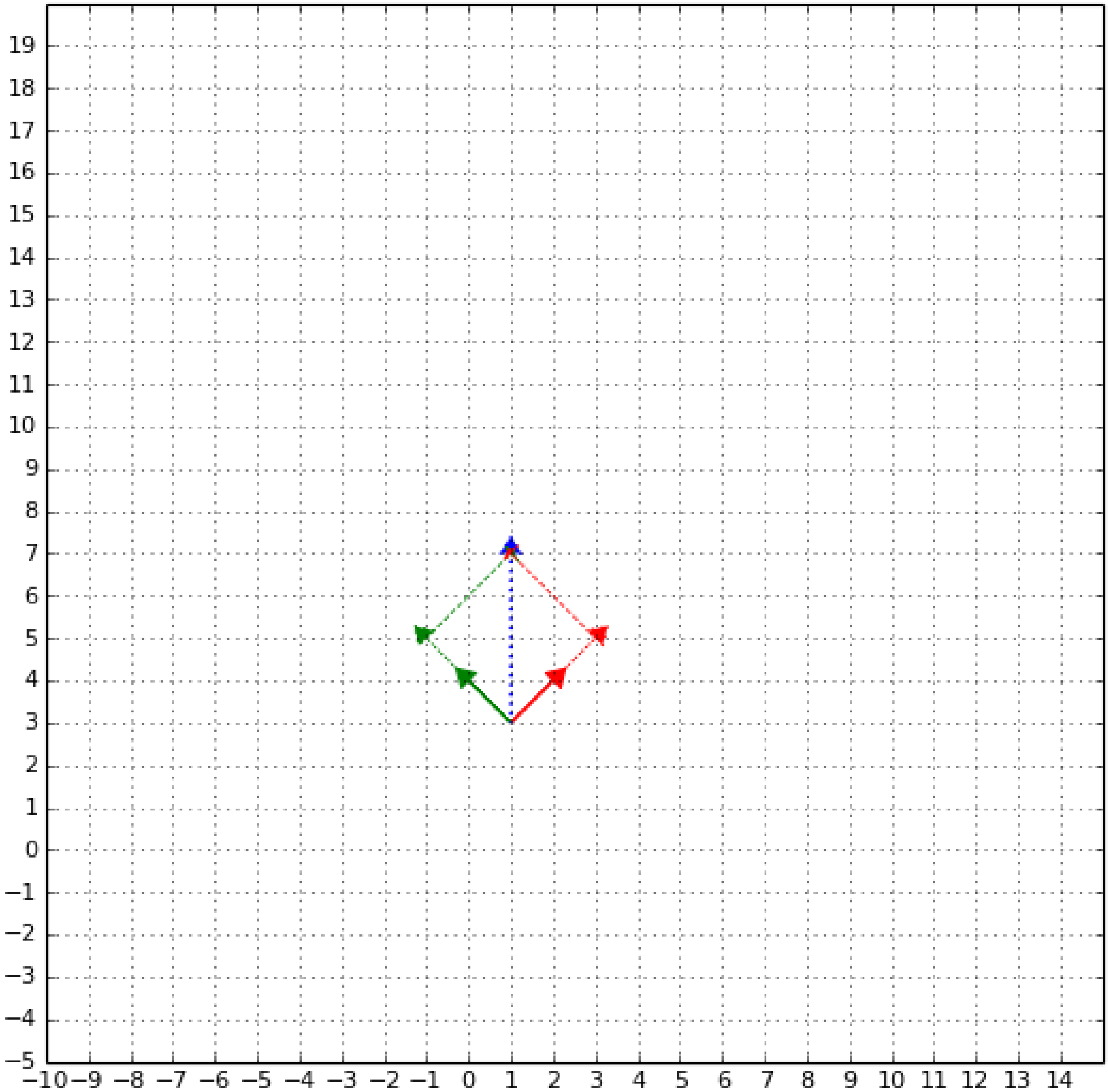

Figure 3. (m 2 2)

4. Next instruction is r which is rotate to right.

$$r\,30 \rightarrow rotateright\,30\,degrees$$

In rotation we need to change our basis with a rotation matrix. Rotation Matrix for right rotation is:

$$\begin{bmatrix} \cos(degree) & \sin(degree) \\ -\sin(degree) & \cos(degree) \end{bmatrix}$$

With a matrix-matrix multiplication we can get the new basis.

$$newbasis = previousbasis * \begin{bmatrix} \cos(30) & \sin(30) \\ -\sin(30) & \cos(30) \end{bmatrix}$$

Previous basis was $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

New basis becomes $\begin{bmatrix} 1.37 & -0.37 \\ 0.37 & 1.37 \end{bmatrix}$

Check Figure 4.

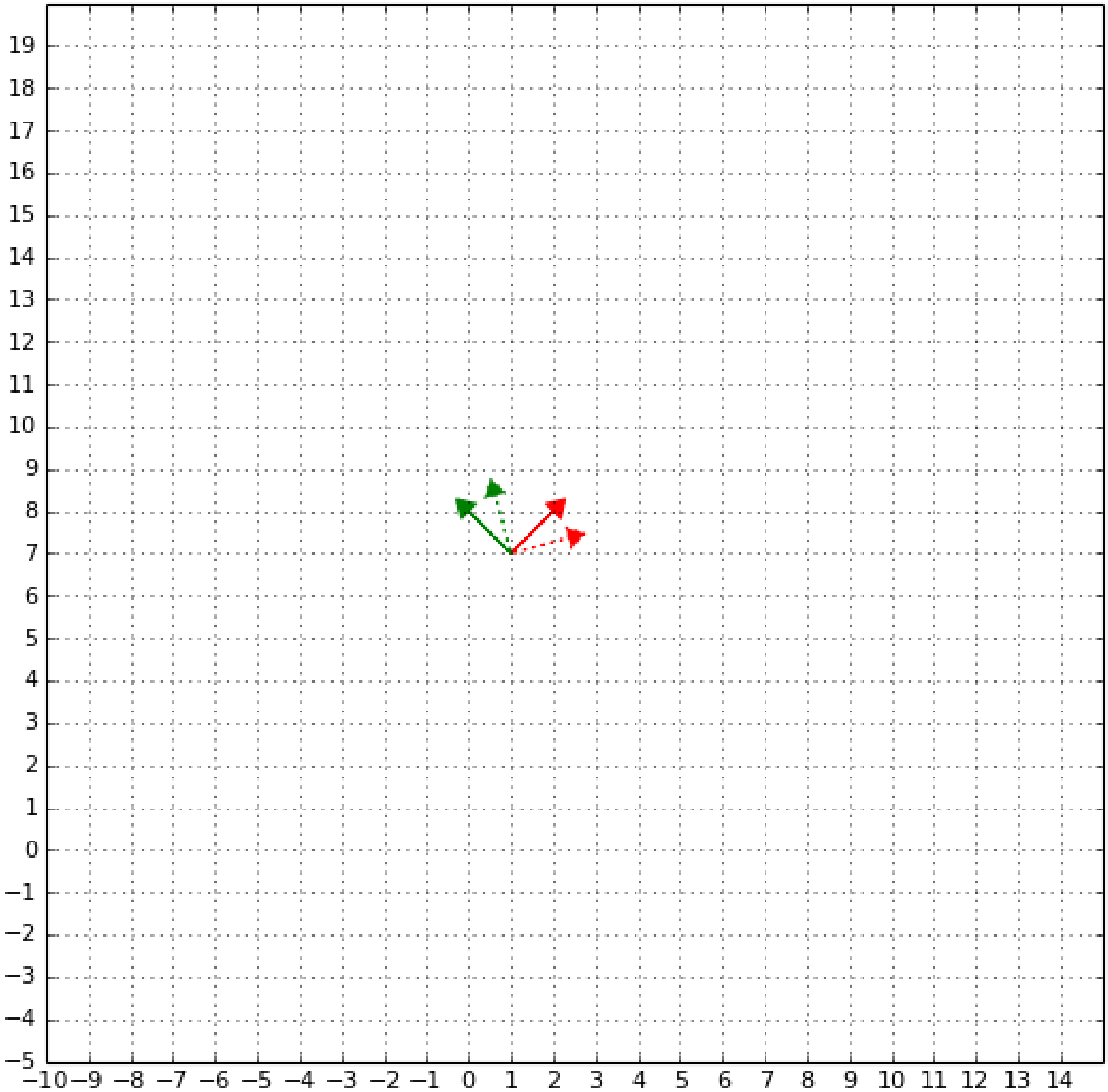

Figure 4. (r 30)

5. Next instruction is m which is move.

$$\text{m -1 4} \rightarrow move\begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

We can think as we need to move -1 times with basisvector1 and 4 times with basisvector2. This can be shown as a matrix-vector multiplication.

$$newcoordinates = \begin{bmatrix} 1.37 & -0.37 \\ 0.37 & 1.37 \end{bmatrix} * \begin{bmatrix} -1 \\ 4 \end{bmatrix} + previouscoordinates$$

Previous coordinates were (1,7)
New coordinates become (-1.83,12.10).
Check Figure 5.

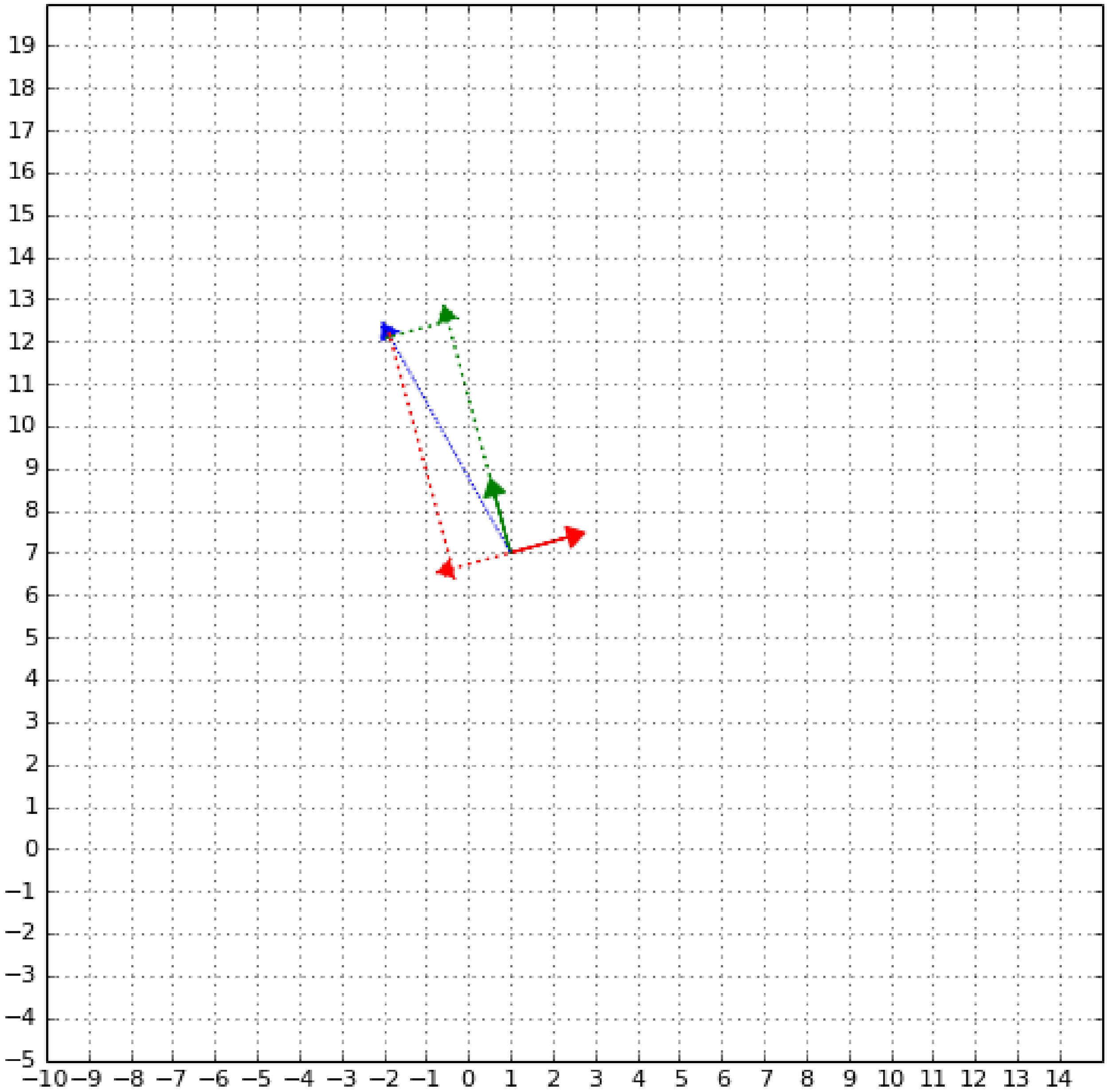

Figure 5. (m -1 4)

6. Next instruction is l which is rotate to left.

$$l \;\; 60 \rightarrow rotateleft\, 60 \; degrees$$

In rotation we need to change our basis with a rotation matrix. Rotation Matrix for left rotation is:

$$\begin{bmatrix} \cos(degree) & -\sin(degree) \\ \sin(degree) & \cos(degree) \end{bmatrix}$$

With a matrix-matrix multiplication we can get the new basis.

$$newbasis = previousbasis * \begin{bmatrix} \cos(30) & -\sin(30) \\ \sin(30) & \cos(30) \end{bmatrix}$$

Previous basis was $\begin{bmatrix} 1.37 & -0.37 \\ 0.37 & 1.37 \end{bmatrix}$

New basis becomes $\begin{bmatrix} 0.37 & -1.37 \\ 1.37 & 0.37 \end{bmatrix}$

Check Figure 6.

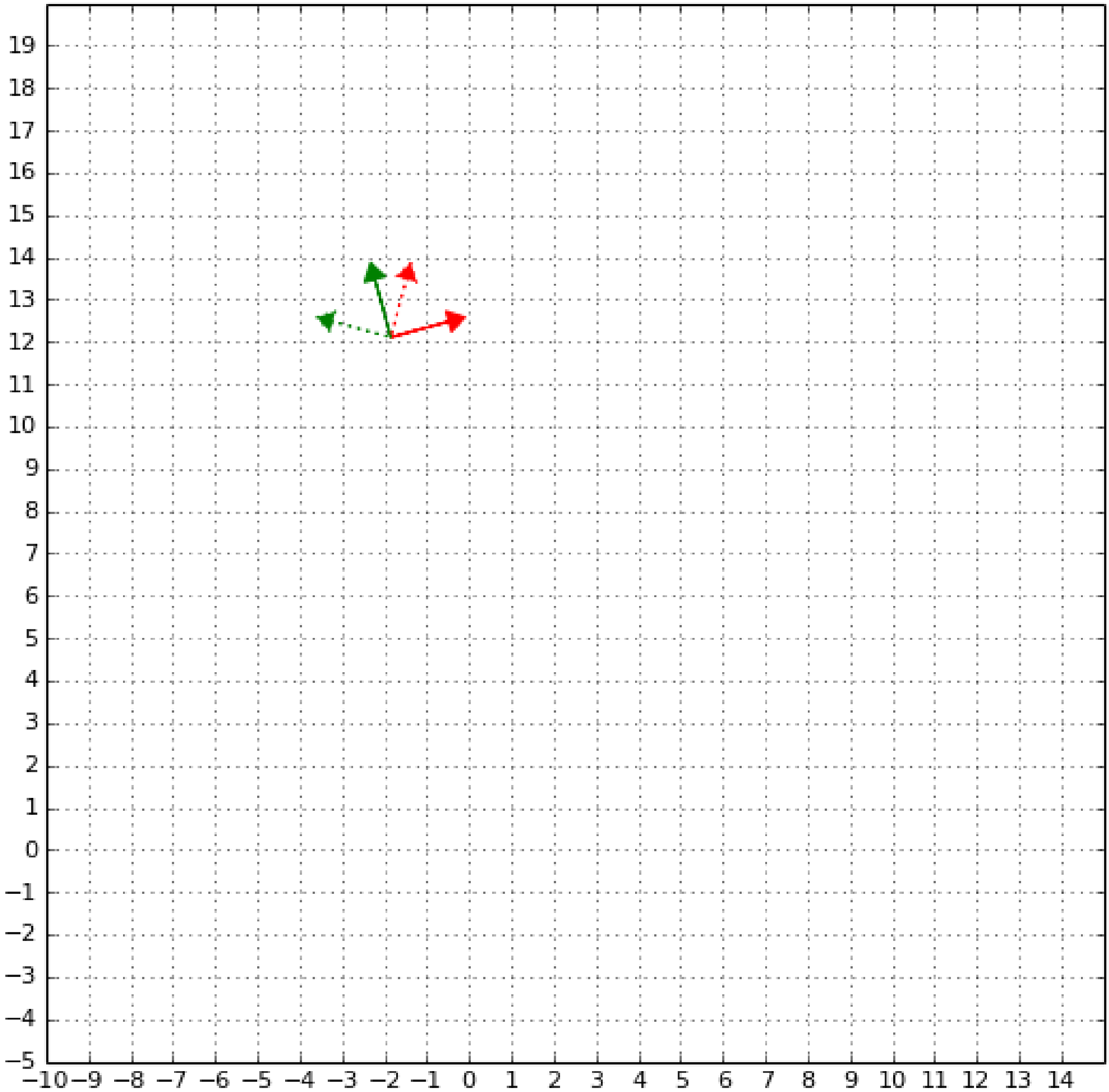

Figure 6. (l 60)

7. Next instruction is m which is move.

$$\text{m 1 5} \rightarrow move\begin{bmatrix}1\\5\end{bmatrix}$$

We can think as we need to move 1 times with basisvector1 and 5 times with basisvector2. This can be shown as a matrix-vector multiplication.

$$newcoordinates = \begin{bmatrix}0.37 & -1.37\\1.37 & 0.37\end{bmatrix} * \begin{bmatrix}1\\5\end{bmatrix} + previouscoordinates$$

Previous coordinates were (-1.83,12.10)
New coordinates become (-8.29,15.29).
Check Figure 7.

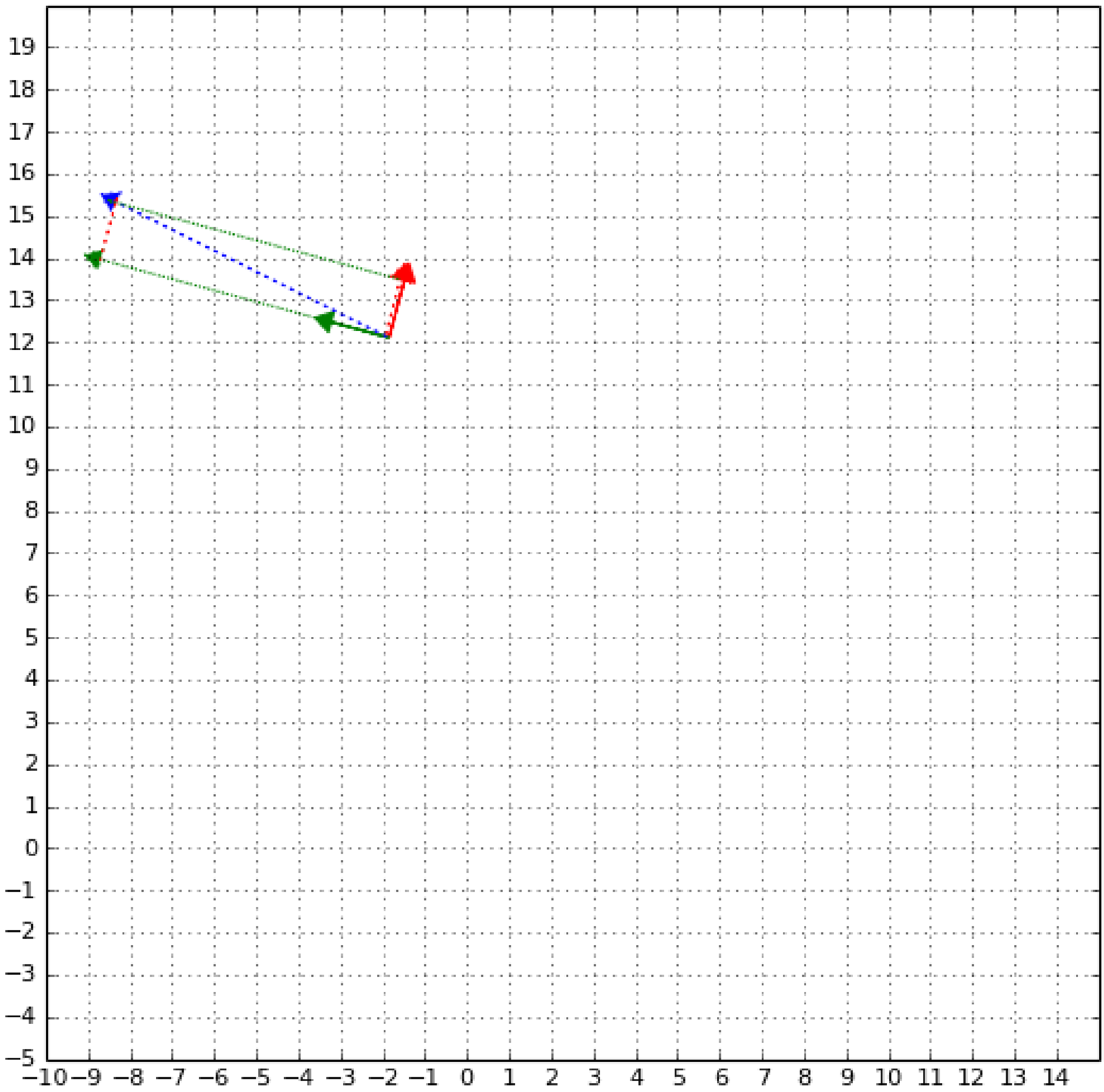

Figure 7. (m 1 5)

8. Next instruction is d which is dig. When treasure hunter digs, program terminates with printing the coordinates of the treasure.

Coordinates: -8.29,15.29