Métodos Computacionais em Física

Aula 11

Equações Diferenciais Ordinárias

O problema

$$\frac{dy(x)}{dx} = f(x, y) \quad \rightarrow \quad y = ?$$

Método 1: Euler

$$\frac{d\mathbf{y}(t)}{dt} \simeq \frac{\mathbf{y}(t_{n+1}) - \mathbf{y}(t_n)}{h} = \mathbf{f}(t, \mathbf{y})$$

$$\mathbf{y}_{n+1} \simeq \mathbf{y}_n + h\mathbf{f}(t_n, \mathbf{y}_n)$$

$O(h^2)$

Exemplo

$$\frac{dy}{dx} = 1 - x + 4y$$

$$y(0) = 1$$

$$y(x) = \frac{1}{4}x - \frac{3}{16} + \frac{19}{16}e^{4x}$$

Resolva de x=0 até x=1 com
- h=0.1
- h=0.01
- h=0.001
- h=0.0001

Método 2: Runge-Kutta 2a ordem

$$\mathbf{y}_{n+1} \simeq \mathbf{y}_n + \mathbf{k}_2$$

$$\mathbf{k}_1 = h\,\mathbf{f}(t_n,\ \mathbf{y}_n)$$

$$\mathbf{k}_2 = h\mathbf{f}(t_n + \frac{h}{2}, \mathbf{y}_n + \frac{\mathbf{k}_1}{2})$$

$O(h^3)$

Método 3: Runge-Kutta 4a ordem

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \tfrac{1}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)$$

$$\mathbf{k}_1 = h\mathbf{f}(t_n,\ \mathbf{y}_n)$$

$$\mathbf{k}_2 = h\mathbf{f}(t_n + \frac{h}{2},\ \mathbf{y}_n + \frac{\mathbf{k}_1}{2})$$

$$\mathbf{k}_3 = h\mathbf{f}(t_n + \frac{h}{2},\ \mathbf{y}_n + \frac{\mathbf{k}_2}{2})$$

$$\mathbf{k}_4 = h\mathbf{f}(t_n + h,\ \mathbf{y}_n + \mathbf{k}_3)$$

$O(h^5)$

Equações de 2a ordem

$$ma = F(v, x, t)$$

$$\frac{dv}{dt} = \frac{1}{m} F(v, x, t)$$

$$\frac{dx}{dt} = G(v, x, t) = v$$

Equações de ordem N

$$\frac{dx}{dt} = F_0(x^{(N)}, ..., x'', x, t)$$

$$\frac{dx'}{dt} = F_1(x^{(N)}, ..., x'', x, t)$$

$$\frac{dx''}{dt} = F_2(x^{(N)}, ..., x'', x, t)$$

...

$$\frac{dx^{(N-1)}}{dt} = F_{N-1}(x^{(N)}, ..., x'', x, t)$$

Lançamento de projéteis

$$m\mathbf{a} = m\mathbf{g} - b\mathbf{v}$$

$$\frac{dv_x}{dt} = -\frac{bv_x}{m}$$

$$\frac{dv_y}{dt} = -g - \frac{bv_y}{m}$$

$$\frac{dx}{dt} = v_x$$

$$\frac{dy}{dt} = v_y$$