models.) Such rules suggest a set of values of K_p, T_i, and T_d that will give a stable operation of the system. However, the resulting system may exhibit a large maximum overshoot in the step response, which is unacceptable. In such a case we need series of fine tunings until an acceptable result is obtained. In fact, the Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning, rather than giving the final settings for K_p, T_i, and T_d in a single shot.

Ziegler–Nichols Rules for Tuning PID Controllers. Ziegler and Nichols proposed rules for determining values of the proportional gain K_p, integral time T_i, and derivative time T_d based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on the plant. (Numerous tuning rules for PID controllers have been proposed since the Ziegler–Nichols proposal. They are available in the literature and from the manufacturers of such controllers.)

There are two methods called Ziegler–Nichols tuning rules: the first method and the second method. We shall give a brief presentation of these two methods.

First Method. In the first method, we obtain experimentally the response of the plant to a unit-step input, as shown in Figure 8–2. If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped, as shown in Figure 8–3. This method applies if the response to a step input exhibits an S-shaped curve. Such step-response curves may be generated experimentally or from a dynamic simulation of the plant.

The S-shaped curve may be characterized by two constants, delay time L and time constant T. The delay time and time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line $c(t) = K$, as shown in Figure 8–3. The transfer

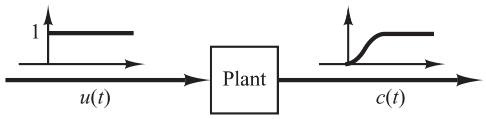

Figure 8–2 Unit-step response of a plant.

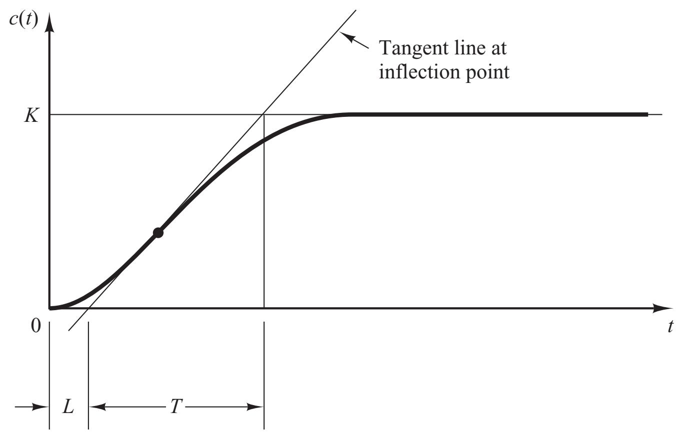

Figure 8–3 S-shaped response curve.

Table 8–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d according to the formula shown in Table 8–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

Second Method. In the second method, we first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure 8–4), increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.) Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally

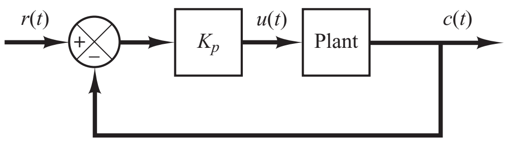

Figure 8–4 Closed-loop system with a proportional controller.

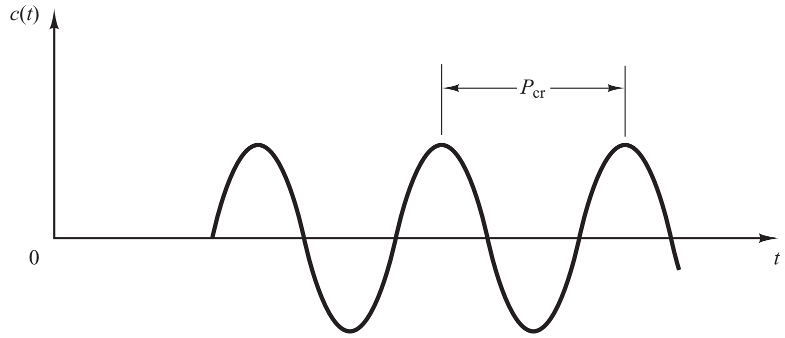

Figure 8–5 Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

determined (see Figure 8–5). Ziegler and Nichols suggested that we set the values of the parameters K_p, T_i, and T_d according to the formula shown in Table 8–2.

Table 8–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$
\begin{aligned}
G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\
&= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\
&= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}
\end{aligned}
$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

Note that if the system has a known mathematical model (such as the transfer function), then we can use the root-locus method to find the critical gain K_{cr} and the frequency of the sustained oscillations ω_{cr}, where $2\pi/\omega_{cr} = P_{cr}$. These values can be found from the crossing points of the root-locus branches with the $j\omega$ axis. (Obviously, if the root-locus branches do not cross the $j\omega$ axis, this method does not apply.)

Comments. Ziegler–Nichols tuning rules (and other tuning rules presented in the literature) have been widely used to tune PID controllers in process control systems where the plant dynamics are not precisely known. Over many years, such tuning rules proved to be very useful. Ziegler–Nichols tuning rules can, of course, be applied to plants whose dynamics are known. (If the plant dynamics are known, many analytical and graphical approaches to the design of PID controllers are available, in addition to Ziegler–Nichols tuning rules.)

EXAMPLE 8–1 Consider the control system shown in Figure 8–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

Since the plant has an integrator, we use the second method of Ziegler–Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5) + K_p}$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

the Routh array becomes as follows:

$$\begin{array}{lcc} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & \\ s^0 & K_p & \end{array}$$

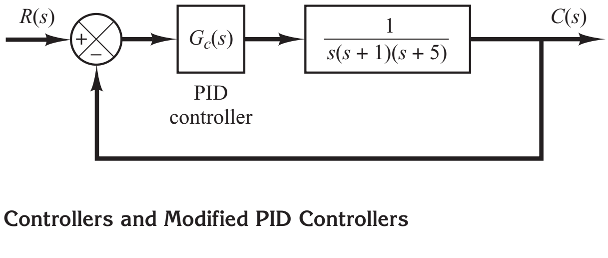

Figure 8–6 PID-controlled system.