

Box number: \_\_\_\_\_ (name on back)

Math 60: Section \_\_\_\_\_

Assignment 2

Due Date: 9/19/18

Section 2.3: R17, R23, 24, R31, 33, R38, 40, 42

Section 2.4: R3, 5, 17\*\*, R22, 23, 29a

Section 2.5: R2, R4, 7, R7, R13, 14, R21, 24, 34, R43

Section 2.6: R5, 6, R11, 12, R15, 18, R21

Section 2.3:

*Find the gradient  $\nabla f(\mathbf{a})$ , where  $f$  and  $\mathbf{a}$  are given in Exercises 18-25.*

24.  $f(x, y, z) = \cos z \ln(x + y^2)$ ,  $\mathbf{a} = (e, 0, \pi/4)$

In Exercises 26-33, find the matrix  $D\mathbf{f}(\mathbf{a})$  if partial derivatives, where  $\mathbf{f}$  and  $\mathbf{a}$  are as indicated.

33.  $\mathbf{f}(s, t) = (s^2, st, t^2)$ ,  $\mathbf{a} = (-1, 1)$

40. Find the equations for the planes tangent to  $z = x^2 - 6x + y^3$  that are parallel to the plane  $4x - 12y + z = 7$ .

42. Suppose that you have the following information concerning a differentiable function  $f$ :

$$f(2, 3) = 12, f(1.98, 3) = 12.1, f(2, 3.01) = 12.2.$$

- (a) Give an approximation for the plane tangent to the graph of  $f$  at  $(2, 3, 12)$ .
- (b) Use the result of part (a) to estimate  $f(1.98, 2.98)$ .

Section 2.4:

*Verify the product and quotient rules (Proposition 4.2) for the pairs of functions given in Exercises 5-8.*

5.  $f(x, y) = x^2y + y^3$ ,  $g(x, y) = \frac{x}{y}$

*For the functions given in Exercises 9-17 determine all second-order partial derivatives (including mixed partials).*

17.  $f(x, y, z) = x^2e^y + e^{2z}$

23. Let  $f(x, y) = ye^{3x}$ . Give general formulas for  $\partial^n f / \partial x^n$  and  $\partial^n f / \partial y^n$ , where  $n \geq 2$ .

29a. The three-dimensional **heat equation** is the partial differential equation

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \frac{\partial T}{\partial t} \quad (1)$$

where  $k$  is a positive constant. It models the temperature  $T(x, y, z, t)$  at the point  $(x, y, z)$  and time  $t$  of a body in space.

- (a) We examine a simplified version of the heat equation. Consider a straight wire "coordinatized" by  $x$ . Then the temperature  $T(x, t)$  at time  $t$  and position  $x$  along the wire is modeled by the one-dimensional heat equation

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad (2)$$

Show that the function  $T(x, t) = e^{-kt} \cos(x)$  satisfies this equation. Note that if  $t$  is held constant at value  $t_0$ , then  $T(x, t_0)$  shows how the temperature varies along the wire at time  $t_0$ . Graph the curves  $z = T(x, T_0)$  for  $t_0 = 0, 1, 10$ , and use them to understand the graph of the surface  $z = T(x, t)$  for  $t \geq 0$ . Explain what happens to the temperature of the wire after a long period of time.



Section 2.5:

7. Suppose that the following function is used to model the monthly demand for bicycles:

$$P(x, y) = 200 + 20\sqrt{0.1x + 10} - 12\sqrt[3]{y}. \quad (3)$$

In this formula,  $x$  represents the price (in dollars per gallon) of automobile gasoline and  $y$  represents the selling price (in dollars) of each bicycle. Furthermore, suppose that the price of gasoline  $t$  months from now will be

$$x = 1 + 0.1t - \cos\left(\frac{\pi t}{6}\right) \quad (4)$$

and the price of each bicycle will be

$$y = 200 + 2t \sin\left(\frac{\pi t}{6}\right) \quad (5)$$

At what rate will the monthly demand for bicycles be changing six months from now?

14. Suppose that  $z = f(x + y, x - y)$  has continuous partial derivatives with respect to  $u = x + y$  and  $v = x - y$ . Show that

$$\frac{\partial z}{\partial x} \frac{\partial z}{\partial y} = \left( \frac{\partial z}{\partial u} \right)^2 - \left( \frac{\partial z}{\partial v} \right)^2. \quad (6)$$

*In Exercises 19-27, calculate  $D(\mathbf{f} \circ \mathbf{g})$  in two ways: (a) by first evaluating  $\mathbf{f} \circ \mathbf{g}$  and (b) by using the chain rule and the derivative matrices  $D\mathbf{f}$  and  $D\mathbf{g}$ .*

24.  $\mathbf{f}(x, y, z) = (x^2y + y^2z, xyz, e^z), \mathbf{g}(t) = (t - 2, 3t + 7, t^3)$

34. Suppose that  $y$  is defined implicitly as a function  $y(x)$  by an equation of the form

$$F(x, y) = 0. \quad (7)$$

(For example, the equation  $x^3 - y^2 = 0$  defines  $y$  as two functions of  $x$ , namely,  $y = x^{3/2}$  and  $y = -x^{3/2}$ . The equation  $\sin(xy) - x^2y^7 + e^y = 0$ , on the other hand, cannot readily be solved for  $y$  in terms of  $x$ . See the end of section 2.6 for more about implicit functions.)

(a) Show that if  $F$  and  $y(x)$  are both assumed to be differentiable functions, then

$$\frac{dy}{dx} = -\frac{F_x(x, y)}{F_y(x, y)} \quad (8)$$

provided  $F_y(x, y) \neq 0$ .

(b) Use the result of part (a) to find  $dy/dx$  when  $y$  is defined implicitly in terms of  $x$  by the equation  $x^3 - y^2 = 0$ . Check your result by explicitly solving for  $y$  and differentiating.

Section 2.6:

*In Exercises 2-8, calculate the directional derivative of the given function  $f$  at the point  $\mathbf{a}$  in the direction parallel to the vector  $\mathbf{u}$ .*

6.  $f(x, y, z) = xyz$ ,  $\mathbf{a} = (-1, 0, 2)$ ,  $\mathbf{u} = \frac{2\mathbf{k} - \mathbf{i}}{\sqrt{5}}$

12. A ladybug (who is very sensitive to temperature) is crawling on graph paper. She is at the point  $(3,7)$  and notices that if she moves in the  $\mathbf{i}$ -direction, the temperature increases at a rate of  $3 \text{ deg/cm}$ . If she moves in the  $\mathbf{j}$ -direction, she finds that her temperature decreases at a rate of  $2 \text{ deg/cm}$ . In what direction should the ladybug move if

- (a) she wants to warm up most rapidly?
- (a) she wants to cool off most rapidly?
- (a) she desires her temperature *not* to change?

*In Exercises 16-19, find an equation for the tangent plane to the surface given by the equation at the indicated point  $(x_0, y_0, z_0)$ .*

18.  $2x^z + yz - x^2y + 10 = 0, (x_0, y_0, z_0) = (1, -5, 5)$