Box number: _____ (name on back)
Math 60: Section _____
Assignment 6
Due Date: 10/19/18

Section 6.1: 2, 9, 21, 31, 34, 40
Section 6.2: 13, 15
Section 7.2: 6, 14, 22, 28
Section 7.3: 4, 6

Section 6.1 #2. In Exercises 2-7, calculate $\int_{\mathbf{x}} f\, ds$ where f and $\mathbf{x}$ are as indicated.

$$f(x, y, z) = xyz, \mathbf{x}(t) = (t, 2t, 3t), 0 \leq t \leq 2$$

Section 6.1 #9. In Exercises 8-16, calculate $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$ where the vector field $\mathbf{F}$ and the path $\mathbf{x}$ are given.

$$\mathbf{F} = (y+2)\mathbf{i} + x\mathbf{j}, \mathbf{x}(t) = (\sin t, -\cos t), 0 \leq t \leq \frac{\pi}{2}$$

Section 6.1 #21. Let $\mathbf{F} = (x^2 + y)\mathbf{i} + (y - x)\mathbf{j}$ and consider the two paths

$$\mathbf{x}(t) = (t, t^2), 0 \leq t \leq 1 \text{ and}$$

$$\mathbf{y}(t) = (1 - 2t, 4t^2 - 4t + 1), 0 \leq t \leq \frac{1}{2}.$$

(a) Calculate $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$ and $\int_{\mathbf{y}} \mathbf{F} \cdot d\mathbf{s}$.

(b) By considering the image curves of the paths $\mathbf{x}$ and $\mathbf{y}$, discuss your answers in part (a).

Section 6.1 #31. Evaluate $\int_C yz\,dx - xz\,dy + xy\,dz$, where C is the line segment from $(1,1,2)$ to $(5,3,1)$.

Section 6.1 #34. Tom Sawyer is whitewashing a picket fence. The bases of the fenceposts are arranged in the xy-plane as the quarter circle $x^2 + y^2 = 25, x, y \geq 0$, and the height of the fencepost at point (x, y) is given by $h(x, y) = 10 - x - y$ (units are feet). Use a scalar line integral to find the area of one side of the fence. (See Figure 6.16.)

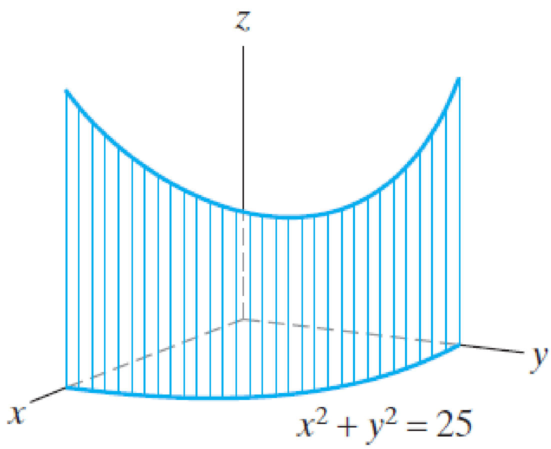

Figure 6.16 The picket fence of Exercise 34. The base of the fence is the quarter circle $x^2 + y^2 = 25$, x, $y \geq 0$.

Section 6.1 #40. You are traveling through Cleveland, famous for its lake-effect snow in winter that makes driving quite treacherous. Suppose that you are currently located 20 miles due east of Cleveland and are attempting to drive to a point 20 miles due west of Cleveland. Further suppose that if you are s miles from the center of Cleveland, where the weather is the worst, you can drive at a rate of at most $v(s) = 2s + 20$ miles per hour.

(a) How long will the trip take if you drive on a straight-line path directly through Cleveland? (Assume that you always drive at the maximum speed possible.)

(b) How long will the trip take if you avoid the middle of the city by driving along a semicircular path with Cleveland at the center? (Again, assume that you drive at the maximum speed possible.)

(a) Repeat parts (a) and (b), this time using $v(s) = (s^2/16) + 25$ miles per hour as the maximum speed that you can drive.

Section 6.2 #13. Evaluate $\oint_C (x^4y^5 - 2y)\,dx + (3x + x^5y^4)\,dy$, where C is the oriented curve pictured in Figure 6.29.

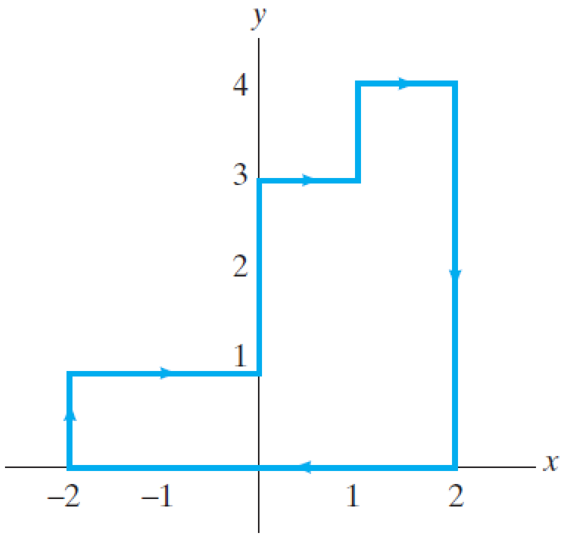

Figure 6.29 The oriented curve C of Exercise 13.

Section 6.2 #15.

(a) Sketch the curve given parametrically by $\mathbf{x}(t) = (1 - t^2, t^3 - t)$

(b) Find the area inside the closed loop of the curve.

Section 7.2 #6. Find $\iint_S (x^2 + y^2)\, dS$, where S is the lateral surface of the cylinder of radius a and height h whose axis is the z-axis.

Section 7.2 #14. In Exercises 10-18, let S denote the closed cylinder with bottom given by $z = 0$, top given by $z = 4$, and lateral surface given by the equation $x^2 + y^2 = 9$. Orient S with outward normals. Determine the indicated scalar and vector surface integrals.

$$\iint_S (x\mathbf{i} + y\mathbf{j}) \cdot d\mathbf{S}$$

Section 7.2 #22. In Exercises 19-22, find the flux of the given vector field **F** across the upper hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$. Orient the hemisphere with an upward-pointing normal.

$$\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + xz\mathbf{k}$$

Section 7.2 #28. The glass dome of a futuristic greenhouse is shaped like the surface $z = 8 - 2x^2 - 2y^2$. The greenhouse has a flat dirt floor at $z = 0$. Suppose that the temperature T, at points in and around the greenhouse, varies as

$$T(x, y, z) = x^2 + y^2 + 3(z - 2)^2.$$

Then the temperature gives rise to a **heat flux density field H** given by $\mathbf{H} = -k\nabla T$. (Here k is a positive constant that depends on the insulating properties of the particular medium.) Find the total heat flux outward across the dome and the surface of the ground if $k = 1$ on the glass and $k = 3$ on the ground.

Section 7.3 #4. In Exercises 1-4, verify Stokes's Theorem for the given surface and vector field.

$$S \text{ is defined by } x^2 + y^2 + z^2 = 4, z \leq 0, \text{ oriented by downward normal;}$$

$$\mathbf{F} = (2y - z)\mathbf{i} + (x + y^2 - z)\mathbf{j} + (4y - 3x)\mathbf{k}.$$

Section 7.3 #6. In Exercises 6-9, verify Gauss's theorem for the given three-dimensional region D and vector field $\mathbf{F}$.

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k},$$

$$D = \{(x, y, z) | 0 \leq z \leq 9 - x^2 - y^2\}$$