Box number: _____ (name on back)
Math 60: Section _____
Assignment 2
Due Date: 9/19/18

Section 2.3: R17, R23, 24, R31, 33, R38, 40, 42
Section 2.4: R3, 5, 17**, R22, 23, 29a
Section 2.5: R2, R4, 7, R7, R13, 14, R21, 24, 34, R43
Section 2.6: R5, 6, R11, 12, R15, 18, R21

Section 2.3:
Find the gradient $\nabla f(\mathbf{a})$, where f and $\mathbf{a}$ are given in Exercises 18-25.
24. $f(x,y,z) = \cos z \ln\left(x + y^2\right), \mathbf{a} = (e, 0, \pi/4)$

In Exercises 26-33, find the matrix $D\mathbf{f}(\mathbf{a})$ of partial derivatives, where $\mathbf{f}$ and $\mathbf{a}$ are as indicated.
33. $\mathbf{f}(s,t) = (s^2, st, t^2)$, $\mathbf{a} = (-1, 1)$

40. Find the equations for the planes tangent to $z = x^2 - 6x + y^3$ that are parallel to the plane $4x - 12y + z = 7$.

42. Suppose that you have the following information concerning a differentiable function f:
$f(2,3) = 12$, $f(1.98,3) = 12.1$, $f(2,3.01) = 12.2$.

(a) Give an approximation for the plane tangent to the graph of f at $(2,3,12)$.

(b) Use the result of part (a) to estimate $f(1.98,2.98)$.

Section 2.4:

Verify the product and quotient rules (Proposition 4.2) for the pairs of functions given in Exercises 5-8.

5. $f(x, y) = x^2y + y^3$, $g(x, y) = \frac{x}{y}$

For the functions given in Exercises 9-17 determine all second-order partial derivatives (including mixed partials).
17. $f(x, y, z) = x^2 e^y + e^{2z}$

23. Let $f(x, y) = ye^{3x}$. Give general formulas for $\partial^n f/\partial x^n$ and $\partial^n f/\partial y^n$, where $n \geq 2$.

29a. The three-dimensional **heat equation** is the partial differential equation

$$k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}) = \frac{\partial T}{\partial t} \tag{1}$$

where k is a positive constant. It models the temperature $T(x, y, z, t)$ at the point (x, y, z) and time t of a body in space.

(a) We examine a simplified version of the heat equation. Consider a straight wire "coordinatized" by x. Then the temperature $T(x, t)$ at time t and position x along the wire is modeled by the one-dimensional heat equation

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \tag{2}$$

Show that the function $T(x, t) = e^{-kt}cos(x)$ satisfies this equation. Note that if t is held constant at value t_0, then $T(x, t_0)$ shows how the temperature varies along the wire at time t_0. Graph the curves $z = T(x, T_0)$ for $t_0 = 0, 1, 10$, and use them to understand the graph of the surface $z = T(x, t)$ for $t \geq 0$. Explain what happens to the temperature of the wire after a long period of time.

Section 2.5:

7. Suppose that the following function is used to model the monthly demand for bicycles:

$$P(x,y) = 200 + 20\sqrt{0.1x + 10} - 12\sqrt[3]{y}. \tag{3}$$

In this formula, x represents the price (in dollars per gallon) of automobile gasoline and y represents the selling price (in dollars) of each bicycle. Furthermore, suppose that the price of gasoline t months from now will be

$$x = 1 + 0.1t - \cos\left(\frac{\pi t}{6}\right) \tag{4}$$

and the price of each bicycle will be

$$y = 200 + 2t\sin\left(\frac{\pi t}{6}\right) \tag{5}$$

At what rate will the monthly demand for bicycles be changing six months from now?

14. Suppose that $z = f(x+y, x-y)$ has continuous partial derivatives with respect to $u = x+y$ and $v = x-y$. Show that

$$\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = \left(\frac{\partial z}{\partial u}\right)^2 - \left(\frac{\partial z}{\partial v}\right)^2. \qquad (6)$$

In Exercises 19-27, calculate $D(\mathbf{f} \circ \mathbf{g})$ in two ways: (a) by first evaluating $\mathbf{f} \circ \mathbf{g}$ and (b) by using the chain rule and the derivative matrices $D\boldsymbol{f}$ and $D\boldsymbol{g}$.

24. $\mathbf{f}(x, y, z) = (x^2y + y^2z, xyz, e^z)$, $\mathbf{g}(t) = (t - 2, 3t + 7, t^3)$

34. Suppose that y is defined implicitly as a function $y(x)$ by an equation of the form

$$F(x,y) = 0. \tag{7}$$

(For example, the equation $x^3 - y^2 = 0$ defines y as two functions of x, namely, $y = x^{3/2}$ and $y = -x^{3/2}$. The equation $\sin(xy) - x^2y^7 + e^y = 0$, on the other hand, cannot readily be solved for y in terms of x. See the end of section 2.6 for more about implicit functions.)

(a) Show that if F and $y(x)$ are both assumed to be differentiable functions, then

$$\frac{dy}{dx} = -\frac{F_x(x,y)}{F_y(x,y)} \tag{8}$$

provided $F_y(x,y) \neq 0$.

(b) Use the result of part (a) to find dy/dx when y is defined implicitly in terms of x by the equation $x^3 - y^2 = 0$. Check your result by explicitly solving for y and differentiating.

Section 2.6:
*In Exercises 2-8, calculate the directional derivative of the given function f at the point **a** in the direction parallel to the vector **u**.*
6. $f(x,y,z) = xyz, \mathbf{a} = (-1,0,2), \mathbf{u} = \frac{2\mathbf{k}-\mathbf{i}}{\sqrt{5}}$

12. A ladybug (who is very sensitive to temperature) is crawling on graph paper. She is at the point $(3,7)$ and notices that if she moves in the $\mathbf{i}$-direction, the temperature increases at a rate of 3 deg/cm. If she moves in the $\mathbf{j}$-direction, she finds that her temperature decreases at a rate of2 deg/cm. In what direction should the ladybug move if

(a) she wants to warm up most rapidly?

(a) she wants to cool off most rapidly?

(a) she desires her temperature *not* to change?

In Exercises 16-19, find an equation for the tangent plane to the surface given by the equation at the indicated point (x_0, y_0, z_0).

18. $2xz + yz - x^2y + 10 = 0, (x_0, y_0, z_0) = (1, -5, 5)$