# The Maths Workbook

# Oxford University Department of Economics

The *Maths Workbook* has been developed for use by students preparing for the Preliminary Examination in PPE, E&M and History and Economics. It covers the mathematical techniques that students are expected to know for the exam.

The Maths Workbook was written by Margaret Stevens, with the help of: Alan Beggs, David Foster, Mary Gregory, Ben Irons, Godfrey Keller, Sujoy Mukerji, Mathan Satchi, Patrick Wallace, Tania Wilson.

©Margaret Stevens 2003

# Complementary Textbooks

As far as possible the *Workbook* is self-contained, but it should be used in conjunction with standard textbooks for a fuller coverage:

- Malcolm Pemberton and Nicholas Rao Mathematics for Economists: An Introductory Textbook 4th edition 2016, or earlier editions. (Comprehensive; appropriate level. This book is not included in the specific references in workbook chapters, but students should find it easy to identify relevant sections.)
- Ian Jacques *Mathematics for Economics and Business* 7th Edition 2013, or earlier editions. (The most elementary.)
- Martin Anthony and Norman Biggs Mathematics for Economics and Finance, 1996.
  (Useful and concise, but less suitable for students who have not previously studied mathematics to A-level.)
- Carl P. Simon and Lawrence Blume *Mathematics for Economists*, 1994 or 2010. (A good but more advanced textbook, that goes well beyond the *Workbook*.)
- Hal R. Varian *Intermediate Microeconomics: A Modern Approach* covers many of the economic applications, particularly in the Appendices to individual chapters, where calculus is used.
- In addition, students who have not studied A-level maths, or feel that their maths is weak, may find it helpful to use one of the many excellent textbooks available for A-level Pure Mathematics (particularly the first three modules).

#### How to Use the Workbook

There are ten chapters, each of which can be used as the basis for a class. It is intended that students should be able to work through each chapter alone, doing the exercises and checking their own answers. References to some of the textbooks listed above are given at the end of each section.

At the end of each chapter is a worksheet, the answers for which are available for the use of tutors only.

The first two chapters are intended mainly for students who have not done A-level maths: they assume GCSE maths only. In subsequent chapters, students who have done A-level will find both familiar and new material.

## Contents

### (1) Review of Algebra

Simplifying and factorising algebraic expressions; indices and logarithms; solving equations (linear equations, equations involving parameters, changing the subject of a formula, quadratic equations, equations involving indices and logs); simultaneous equations; inequalities and absolute value.

## (2) Lines and Graphs

The gradient of a line, drawing and sketching graphs, linear graphs (y = mx+c), quadratic graphs, solving equations and inequalities using graphs, budget constraints.

## (3) Sequences, Series and Limits; the Economics of Finance

Arithmetic and geometric sequences and series; interest rates, savings and loans; present value; limit of a sequence, perpetuities; the number e, continuous compounding of interest.

#### (4) Functions

Common functions, limits of functions; composite and inverse functions; supply and demand functions; exponential and log functions with economic applications; functions of several variables, isoquants; homogeneous functions, returns to scale.

#### (5) Differentiation

Derivative as gradient; differentiating  $y = x^n$ ; notation and interpretation of derivatives; basic rules and differentiation of polynomials; economic applications: MC, MPL, MPC; stationary points; the second derivative, concavity and convexity.

## (6) More Differentiation, and Optimisation

Sketching graphs; cost functions; profit maximisation; product, quotient and chain rule; elasticities; differentiating exponential and log functions; growth; the optimum time to sell an asset.

#### (7) Partial Differentiation

First- and second-order partial derivatives; marginal products, Euler's theorem; differentials; the gradient of an isoquant; indifference curves, MRS and MRTS; the chain rule and implicit differentiation; comparative statics.

#### (8) Unconstrained Optimisation Problems with One or More Variables

First- and second-order conditions for optimisation, Perfect competition and monopoly; strategic optimisation problems: oligopoly, externalities; optimising functions of two or more variables.

#### (9) Constrained Optimisation

Methods for solving consumer choice problems: tangency condition and Lagrangian; cost minimisation; the method of Lagrange multipliers; other economic applications; demand functions.

## (10) Integration

Integration as the reverse of differentiation; rules for integration; areas and definite integrals; producer and consumer surplus; integration by substitution and by parts; integrals and sums; the present value of an income flow.