Liquidity and Safety over the Business Cycle [*]

Andrea Ferrero [†] Alexander Haas [‡]

October 2023

Please click here for the latest version.

Abstract

This paper builds on two empirical observations: (i) financial conditions are relevant drivers of the business cycle, and (ii) early stages of the 2007/08 financial crisis, in particular, were driven by an erosion of safety and a dry-up of liquidity. Liquidity and safety are broad and interlinked notions, difficult to disentangle empirically. Their distinction is crucial for monetary and fiscal policy design though. In this paper, we endogenize the liquidity and safety of private assets in a medium-scale new-Keynesian model with heterogeneous firms and two financial frictions (on resaleability and quality paired with asymmetric information). Using U.S. macro and financial data, we estimate this model to (i) identify the structural drivers of liquidity and safety premia, (ii) study the role of both types of financial shocks over the business cycle, (iii) revisit the 2007/08 financial crisis in detail, and (iv) provide several further results on policy, fiscal multipliers, and the so called liquidity puzzle.

Keywords: financial frictions, asymmetric information, convenience yield, structural decomposition, liquidity, safe assets, monetary policy

JEL Classifications: E10, E44, E50, G01, G10

---

[*] This is a preliminary draft, please do not cite without the authors' permission.
We are grateful to Matthias Rottner for an excellent discussion. For helpful comments and suggestions, we thank numerous participants at seminars and conferences at FU Berlin, Oxford, Bank of England, Royal Economic Society, IAAE, MMF, Peter Sinclair Memorial Conference, T2M, CEF, Dynare Conference, Qatar Centre for Global Banking and Finance, ICEA, and King's College London.

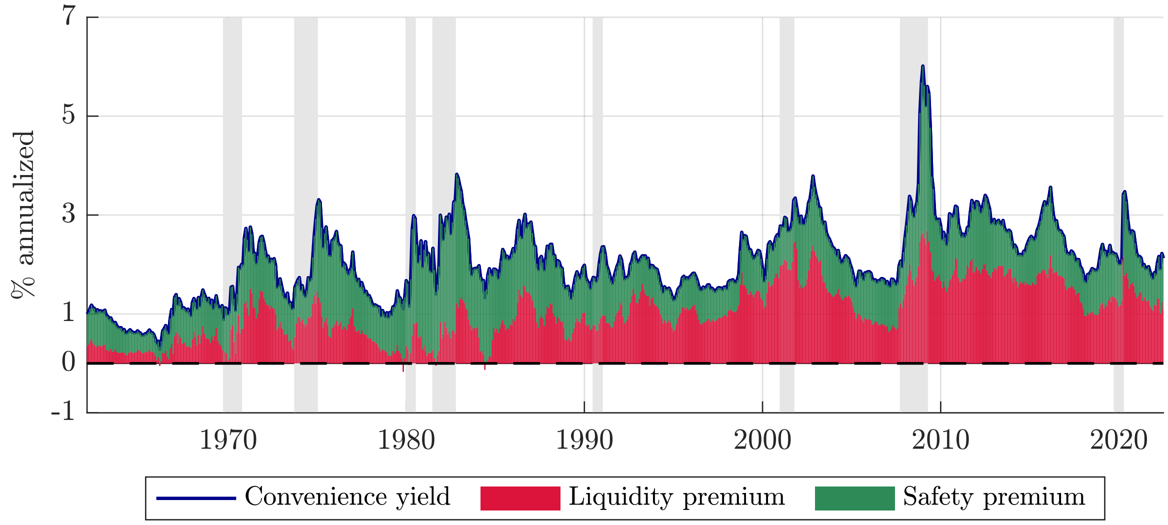
[†] Andrea Ferrero, University of Oxford (andrea.ferrero@economics.ox.ac.uk)

[‡] Alexander Haas, University of Oxford (alexander.haas@economics.ox.ac.uk)

1 Introduction

Both the Great Financial Crisis of 2007/2008 and the more recent Covid crisis have highlighted the importance of asset liquidity. A common narrative of the Great Financial Crisis attributes a central role to the sudden dry-up of liquidity in secondary asset markets (Brunnermeier and Pedersen, 2009). The Federal Reserve responded to the market turmoil by setting up lending facilities with the objective of easing credit conditions. During the pandemic several central banks around the world reopened these facilities to preempt distress in financial markets and avoid more severe consequences for the real economy.

Figure 1: BAA-Treasury convenience yield and its decomposition.



Note: This Figure plots the BAA-Treasury convenience yield, calculated as the spread between the BAA corporate bond index and the 10-year Treasury at constant maturity (in blue). It is decomposed into a "liquidity premium," defined as the spread between the AAA corporate bond index and the 10-year Treasury (in red), and a "safety premium," defined as the spread between the BAA and AAA corporate bond index (in green).

At the aggregate level, financial spreads are the most commonly used indicators of liquidity shocks. Yet, the mapping between spreads and liquidity is typically imperfect. The reason is that these measures of "market liquidity," often referred to as convenience yields, combine the premium due only to "fundamental liquidity" with a component related to the "safety" of the assets in question. To illustrate this point, consider the convenience yield that BAA corporate bonds pay relative to 10-year Treasuries (Figure 1). Krishnamurthy and Vissing-Jorgensen (2012) decompose this convenience yield in two parts: The first is a liquidity premium, computed as the spread between the yield on the AAA corporate bond index and the yield on the 10-year Treasury (AAA-10yT). The second is a safety premium, calculated as the spread between the yield on the BAA and the AAA corporate bond indexes (BAA-AAA).

Despite being positively correlated, these two components of the BAA-Treasury convenience yield display different dynamics.[1] For example, both spiked during the 2008 financial crisis. However, while the BAA-AAA spread quickly returned back to its pre-crisis level, the AAA-10yT spread remained elevated for several years. In the 1981-82 recession, the BAA-AAA spread went up much higher than the AAA-10yT spread. Even more importantly though, while this empirical decomposition of the convenience yield is useful and easy to compute, the BAA-AAA and the AAA-10yT spreads themselves are not clean measures of safety and fundamental liquidity but in practice reflect elements of both.

In this paper, we seek to understand the fundamental drivers of these spreads in terms of the underlying frictions that characterize asset markets. Building on Dong and Wen (2017), we estimate a medium-scale DSGE model with two types of financial frictions in addition to the standard nominal and real rigidities common in the literature starting with Christiano et al. (2005) and Smets and Wouters (2007). In the model, heterogeneous intermediate goods producers invest in new capital and trade existing capital subject to idiosyncratic investment efficiency shocks. Trade in the market for existing capital is subject to two frictions: First, firms can only sell an exogenous and time-varying fraction of the existing capital stock. Second, only the owner of the existing capital being traded exactly knows its quality, which also changes exogenously over time. The first friction (resaleability) captures the idea of fundamental asset liquidity. The second (quality) is related to the notion of safety. Assuming that the government issues a perfectly liquid and safe bond, the model allows for a tractable decomposition of the convenience yield of private asset into a liquidity and a safety premium. Taking the model to the data, we compute the importance of resaleability/ fundamental liquidity and quality/ safety in driving spreads and macroeconomic variables over the business cycle.

We find that at business cycle frequency, financial shocks are the almost sole driver of the AAA-10yT spread, and explain about 15% of the variability of real GDP and 25% of the variability of investment. Conversely, shocks to the supply of safe and liquid government bonds are key to understand the evolution of the BAA-AAA spread. Delving deeper into the two underlying financial frictions, it turns out resaleability shocks almost entirely account for the evolution of the AAA-10yT spread and also for a small fraction of the BAA-AAA spread. Despite this, quality shocks are substantially more important than resaleability shocks for both output and investment. The picture that emerges from our analysis suggests that policies aimed at easing liquidity dry-ups are likely to have a large impact on spreads but may not be too effective in terms of sustaining economic activity. In fact, there might be conditions in which pure liquidity interventions might be

[1] The correlation coefficient over the full sample (1962-2021) is 0.22, going up to 0.55 for the sub-sample used in the estimation (1985-2019) further below.

counterproductive allowing agents to circumvent markets characterized by concerns about asset quality, and thus worsening the asymmetric information problem in these markets.

Literature The decomposition of market liquidity in fundamental liquidity and safety relates our work to several strands of the literature, both empirical and theoretical.

Gorton and Metrick (2010) and Gorton and Metrick (2012) discuss the dynamics of the early stages of the 2008 financial crisis in light of "haircuts," this is, the difference between the face value and collateral value of an asset. Ashcraft et al. (2010) demonstrate the effectiveness of the Federal Reserve's Term Asset-Backed Securities Loan Facility (TALF) in reducing haircuts on existing securities. Since the face and collateral value of a fully liquid and safe asset coincide, haircuts are just another way to think about spreads.

The distinction between liquidity and safety is also central in the debate on the nature of the financial crisis and the role of the Term Auction Facility (TAF) in mitigating market distress. Taylor and Williams (2009) attribute the rise of the Libor-OIS spread to increased counterparty risk. Their empirical evidence suggests that TAF had no significant effect on spreads while McAndrews et al. (2017) reach the opposite conclusion.

Our estimated DSGE model complements this reduced-form evidence focusing on the consequences of shocks to both safety and liquidity for spreads and macroeconomic variables. Liquidity shocks arise through a resaleability friction as in Kiyotaki and Moore (2019). Like us, Ajello (2016) and Del Negro et al. (2017) embed this type of liquidity frictions in medium-scale DSGE models with nominal and real rigidities to study the 2008 financial crisis and the ensuing policy response. Our contribution is to ask more generally to which extent liquidity shocks are an important driver of business cycle fluctuations.

In an estimated DSGE model with a financial accelerator *à la* Bernanke et al. (1999), Christiano et al. (2014) find that risk shocks—that is, shocks to the volatility of idiosyncratic productivity in the cross section—are the single most important driver of business cycle fluctuations. The model in our paper introduces a different notion of risk (safety), tightly linked to assets trading in secondary markets, through a quality friction similar to the one in Kurlat (2013), Bigio (2015), and most recently, in Bierdel et al. (2023). Nevertheless, our findings support the conclusion that risk shocks are highly important in accounting for macroeconomic dynamics.

Finally, He et al. (2016), He et al. (2019), and Bayer et al. (2023) have developed frameworks in which government-issued assets provide liquidity services to investors. Our finding that the supply of government bonds is important in accounting for the evolution of the BAA-AAA premium is consistent with the idea in Caballero et al. (2017) and Caballero and Farhi (2018) that safety is a critical characteristic of publicly-issued assets.

2 The Model

Our model builds upon Dong and Wen (2017) and Del Negro et al. (2017). Five types of agents populate the economy: households, labor unions, investment goods firms, producers, and final goods firms. Households, labor unions, and investment and final goods firms are standard. At the core of the model are heterogeneous producers. These are subject to idiosyncratic investment efficiency and two types of financial frictions: (i) a liquidity constraint on the quantity of capital that can be sold in secondary markets in each period; (ii) a safety constraint (due to asymmetric information) on the quality of capital that is traded in each period. A government that conducts monetary and fiscal policy closes the model. The rest of this Section describes the problem of each type of agent in detail.

2.1 Households

A representative household chooses consumption, c_t, labor supply, ℓ_{ht}, and savings in shares of producer $i \in [0,1]$, s_{it}, to maximize

$$\mathbb{V}_t^H = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta_t^s \left[\ln(c_{t+s} - \hbar c_{t+s-1}) - \frac{\chi_t}{1+\xi} \ell_{ht+s}^{1+\xi} \right] \right\}$$

subject to

$$(1+\tau_p) P_t c_t + \int_{i\in[0,1]} V_{it} s_{it} \, di$$
$$= (1-\tau_w) \left(W_{ht}\ell_{ht} + \Omega_t^L \right) + \int_{i\in[0,1]} (V_{it} + D_{it}) s_{it-1} \, di + \Omega_t^I + \Omega_t^F - T_t,$$

where $\beta_t \in (0,1)$ is the individual discount factor, $\hbar \in (0,1)$ is the habits parameter, $\xi > 0$ is the inverse Frisch elasticity of labor supply, and χ pins down the steady state level of hours worked. In the budget constraint, P_t is the aggregate price level, and τ_p and τ_w are constant consumption and labor income taxes, respectively. Labor income derives from the wage households receive for their supply of the homogeneous labor good, W_{ht}, and profits of labor unions, Ω_t^L, that sell the labor good at a markup and are also owned by households. V_{it} denotes the market value of shares and D_{it} the dividend from ownership of producer i. Ω_t^I and Ω_t^F are the profits from ownership of investment goods firms and final goods firms, and T_t denotes lump sum taxes, all expressed in nominal terms. Finally, time variation in the discount factor captures exogenous shocks to preferences such that

$$\hat{\beta}_t = \rho_\beta \hat{\beta}_{t-1} + \varepsilon_{\beta t}$$

where $\hat{\beta}_t \equiv \ln(\beta_t/\beta)$, $\rho_\beta \in (0,1)$, and $\varepsilon_{\beta t} \sim \; i.i.d. \; \mathcal{N}(0, \sigma_\beta^2)$.

Denoting with Λ_t the Lagrange multiplier on the budget constraint, the three first order conditions that—with the budget constraint—characterize the household problem are:

1. The expression for the marginal utility of consumption

$$(1+\tau_p)\,\lambda_t = \frac{1}{c_t - \hbar c_{t-1}} - \beta_t \hbar \mathbb{E}_t \left(\frac{1}{c_{t+1} - \hbar c_t} \right),$$

   where $\lambda_t \equiv \Lambda_t P_t$.

2. The labor-leisure trade off

$$w_{ht} = \frac{\chi \ell_{ht}^{\xi}}{(1-\tau_w)\,\lambda_t},$$

   where $w_{ht} \equiv W_{ht}/P_t$ is the real wage households receive.

3. The Euler equations (one for each i)

$$1 = \mathbb{E}_t \left[\mathcal{M}_{t+1} \left(\frac{V_{it+1} + D_{it+1}}{V_{it}} \right) \right],$$

   where the first term is the stochastic discount factor for nominal assets

$$\mathcal{M}_{t+1} = \beta_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}},$$

   with $\Pi_t \equiv P_t/P_{t-1}$, and the second term is the nominal return from holding shares.

2.2 Labour Unions

Labor unions differentiate the homogeneous labor supplied by households. Labor used in the production process, ℓ_t, is a CES aggregator of these differentiated labor inputs, ℓ_{lt},

$$\ell_t = \left(\int_0^1 \ell_{lt}^{\frac{\theta_w - 1}{\theta_w}} \, dl \right)^{\frac{\theta_w}{\theta_w - 1}},$$

where $\theta_w > 1$ is the elasticity of substitution among differentiated inputs.

Cost minimization implies that the demand for each variety is

$$\ell_{lt} = \left(\frac{W_{lt}}{W_t} \right)^{-\theta_w} \ell_t,$$

and that the aggregate wage index is

$$W_t = \left(\int_0^1 W_{lt}^{1-\theta_w} \, dl \right)^{\frac{1}{1-\theta_w}}.$$

Labor unions operate in monopolistic competition and set wages on a staggered basis (Erceg et al., 2000). Each period, a union is able to adjust its wage with probability $1-\iota_w$. If a union cannot adjust its price, it partially updates its wage following two geometric averages: First, a geometric average of last period's growth rate and the trend growth rate, where $\mu_{z^*t} = z_t^*/z_{t-1}^*$ denotes the stochastic growth rate in the model and $\gamma_\mu \in (0,1)$ measures the degree of indexation to lagged growth. Second, a geometric average of last period's inflation and a time-varying inflation target Π_t^*, where $\gamma_w \in (0,1)$ measures the degree of indexation to lagged inflation. The problem for a labor union that can readjust at time t then is

$$\max_{\widetilde{W}_{lt}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \iota_w^s \mathcal{M}_{t+s} \left[\widetilde{W}_{lt} X_{t,t+s}^W - \chi_{wt+s} W_{ht+s} \right] \ell_{lt,t+s} \right\},$$

subject to the demand for its labor input conditional on no further price changes

$$\ell_{lt,t+s} = \left(\frac{\widetilde{W}_{lt} X_{t,t+s}^W}{W_{t+s}} \right)^{-\theta_w} \ell_{t+s},$$

where

$$X_{t,t+s}^W = \begin{cases} \prod_{k=0}^{s-1} (\mu_{z^*t+k+1})^{\gamma_\mu} (\mu_{z^*})^{1-\gamma_\mu} (\Pi_{t+k})^{\gamma_w} (\Pi_{t+k+1}^*)^{1-\gamma_w} & \text{if } s > 0 \\ 1 & \text{if } s = 0, \end{cases}$$

and χ_{wt} is a cost-push shock, introduced to capture distortions in markups, which follows

$$\hat{\chi}_{wt} = \rho_{\chi w} \hat{\chi}_{wt-1} + \varepsilon_{\chi wt},$$

where $\hat{\chi}_{wt} \equiv \ln(\chi_{wt}/\chi_w)$, $\rho_{\chi w} \in (0,1)$, and $\varepsilon_{\chi wt} \sim$ $i.i.d.$ $\mathcal{N}(0, \sigma_{\chi w}^2)$.

The first-order condition for the labor union's problem is

$$\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \iota_w^s \mathcal{M}_{t+s} \left[\widetilde{W}_{lt} X_{t,t+s}^W - \chi_{wt+s} \frac{\theta_w}{\theta_w - 1} W_{ht+s} \right] \ell_{lt,t+s} \right\} = 0.$$

Since marginal cost—the wage the representative household demands to supply the labor input—are the same across unions, in a symmetric equilibrium all labor unions that reset their price choose the same strategy. We can rewrite the optimal relative reset wage as

$$\frac{\widetilde{W}_t}{W_t} = \left(\frac{\theta_w}{\theta_w - 1} \right) \frac{\mathcal{D}_{wt}}{\mathcal{F}_{wt}},$$

where the numerator of the right-hand side is the present discounted value of total costs

$$\begin{aligned}\mathcal{D}_{wt} &\equiv \mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta_t\iota_w)^s\lambda_{t+s}\chi_{wt+s}w_{ht+s}\left(\frac{(w_{t+s}/w_t)(P_{t+s}/P_t)}{X^W_{t,t+s}}\right)^{\theta_w}\ell_{t+s}\right]\\ &= \lambda_t\chi_{wt}w_{ht}\ell_t + \beta_t\iota_w\mathbb{E}_t\left\{\left[\frac{\pi_{wt+1}\Pi_{t+1}}{(\mu_{z^*t+1})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_t)^{\gamma_w}(\Pi^*_{t+1})^{1-\gamma_w}}\right]^{\theta_w}\mathcal{D}_{wt+1}\right\},\end{aligned}$$

and the denominator is the present discounted value of total net revenues

$$\begin{aligned}\mathcal{F}_{wt} &\equiv \mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta_t\iota_w)^s\lambda_{t+s}w_{t+s}\left(\frac{(w_{t+s}/w_t)(P_{t+s}/P_t)}{X^W_{t,t+s}}\right)^{\theta_w-1}\ell_{t+s}\right]\\ &= \lambda_t w_t\ell_t + \beta_t\iota_w\mathbb{E}_t\left\{\left[\frac{\pi_{wt+1}\Pi_{t+1}}{(\mu_{z^*t+1})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_t)^{\gamma_w}(\Pi^*_{t+1})^{1-\gamma_w}}\right]^{\theta_w-1}\mathcal{F}_{wt+1}\right\},\end{aligned}$$

where we define wage inflation $\pi_{wt} \equiv w_t/w_{t-1}$. Further, we can rewrite the aggregate wage index in terms of the optimal relative reset wage and inflation as

$$\frac{\widetilde{W}_t}{W_t} = \left\{\frac{1-\iota_w\left[\frac{\pi_{wt}\Pi_t}{(\mu_{z^*t})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_{t-1})^{\gamma_w}(\Pi^*_t)^{1-\gamma_w}}\right]^{\theta_w-1}}{1-\iota_w}\right\}^{\frac{1}{1-\theta_w}}.$$

Substituting the expression for the optimal relative reset wage into the last equation, we obtain the non-linear wage Phillips curve

$$\left(\frac{\theta_w}{\theta_w-1}\right)\frac{\mathcal{D}_{wt}}{\mathcal{F}_{wt}} = \left\{\frac{1-\iota_w\left[\frac{\pi_{wt}\Pi_t}{(\mu_{z^*t})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_{t-1})^{\gamma_w}(\Pi^*_t)^{1-\gamma_w}}\right]^{\theta_w-1}}{1-\iota_w}\right\}^{\frac{1}{1-\theta_w}}.$$

Finally, staggered wage-setting introduces wage dispersion as not all labor unions will be able to adjust their prices in every period. Integrating the demand for labor inputs over the continuum of varieties, we obtain

$$\ell_{ht} \equiv \int_0^1 \ell_{lt}\,dl = \int_0^1\left(\frac{W_{lt}}{W_t}\right)^{-\theta_w}dl\ \ell_t = \Delta_{wt}\ell_t,$$

where $\Delta_{wt} \equiv \int_0^1\left(\frac{W_{lt}}{W_t}\right)^{-\theta_w}dl$ gives a measure of labor supply lost due to the inefficient allocation of labor inputs caused by wage dispersion. Rewriting this measure in terms of the optimal relative reset wage and inflation yields a recursive result for wage dispersion,

$$\Delta_{wt} = (1-\iota_w)\left[\left(\frac{\theta_w}{\theta_w-1}\right)\frac{\mathcal{D}_{wt}}{\mathcal{F}_{wt}}\right]^{-\theta_w} + \iota_w\left[\frac{\pi_{wt}\Pi_t}{(\mu_{z^*t})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_{t-1})^{\gamma_w}(\Pi^*_t)^{1-\gamma_w}}\right]^{\theta_w}\Delta_{wt-1}.$$

2.3 Investment Producers

Perfectly competitive investment goods producers transform final goods into investment goods, which they sell to producers at price P_{It}. Taking the price of investment goods as given, investment goods firms choose investment, i_t, to maximize,

$$\mathbb{V}_t^I = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \mathcal{M}_{t+s} \left\{ P_{It+s} - \left[1 + f\left(\frac{i_{t+s}}{i_{t+s-1}} \right) \right] \frac{P_{t+s}}{\Upsilon^{t+s}} \right\} i_{t+s} \right\},$$

where $f\left(i_t/i_{t-1}\right)$ is an adjustment cost function that depends on the deviation of this period's investment to last period's investment. We adopt the following functional form,

$$f(x_t) \equiv \frac{1}{2} \left\{ \exp\left[\sqrt{f''}\left(x_t - x\right) \right] + \exp\left[-\sqrt{f''}\left(x_t - x\right) \right] - 2 \right\},$$

where $x_t = (i_t/i_{t-1})$ and along a non-stochastic steady state growth path $f(x) = f'(x) = 0$, while $f'' = f''(x) > 0$. The first source of growth in the model is introduced through $\Upsilon > 1$, which captures a deterministic rise in efficiency in turning output into investment goods.

The joint occurrence of trend growth and adjustment costs in the investment goods producers' problem implies that the price of investment goods will fluctuate strictly above a declining deterministic path, $p_{It} = 1/\Upsilon^t$, as demonstrated by the first order condition,

$$p_{It} = \left(\Upsilon^t\right)^{-1} \left[1 + f\left(\frac{i_t}{i_{t-1}} \right) + f'\left(\frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] - \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} \left(\Upsilon^{t+1}\right)^{-1} f'\left(\frac{i_{t+1}}{i_t} \right) \left[\frac{i_{t+1}}{i_t} \right]^2 \right\}.$$

2.4 Intermediate Goods Producers

A continuum of heterogeneous firms, indexed by $i \in [0, 1]$, produces homogeneous intermediate goods and invests to build new capital conditional on the realization of idiosyncratic shocks to investment efficiency and subject to two types of financial frictions.

2.4.1 Production

The production process is standard. Firms combine their capital stock, k_{it-1}, with hired labor, ℓ_{it}, and set a level of capital utilization, u_{it}, to produce intermediate output, y_{mit}, according to a Cobb-Douglas technology

$$y_{mit} = a_t \left(u_{it} k_{it-1}\right)^{\alpha} \left(z_t \ell_{it}\right)^{1-\alpha},$$

where $\alpha \in [0,1]$ is the capital share, and a_t and z_t capture the stationary and non-stationary components, respectively, of aggregate total factor productivity.

We assume that the stationary component of productivity follows

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{at},$$

where $\hat{a}_t \equiv \ln(a_t/a)$, $\rho_a \in (0,1)$, and $\varepsilon_{at} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_a^2)$; the non-stationary component

$$\hat{\mu}_{zt} = \rho_z \hat{\mu}_{zt-1} + \varepsilon_{zt},$$

where $\mu_{zt} \equiv z_t/z_{t-1}$, the trend rise in productivity with $\mu_z = \exp(\Gamma)$, will be the second source of growth in the model, and $\hat{\mu}_{zt} \equiv \ln(\mu_{zt}/\mu_z)$, $\rho_z \in (0,1)$, and $\varepsilon_{zt} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_z^2)$.

With both the trend rise in the efficiency of producing investment goods and the trend increase in total factor productivity, the real growth rate in the model is captured by $z_t^* \equiv z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}$. Appendix A.2 states the full system of detrended equilibrium equations.

Producers operate in perfect competition and take the price of intermediate output, P_{mt}, and the wage as given. Since firms own capital, we can write their production problem as maximizing the internal cash flow, which corresponds to the return to capital,

$$R_{Kt} k_{it-1} \equiv \max_{\{\ell_{it}, u_{it}\}} \; P_{mt} a_t \left(u_{it} k_{it-1}\right)^{\alpha} \left(z_t \ell_{it}\right)^{1-\alpha} - W_t \ell_{it} - s(u_{it}) k_{it-1},$$

where $s(u_t)$ denotes the cost of capital utilization. We adopt the following function form,

$$s(u_t) \equiv r_K \left\{ \exp\left[\sigma_s (u_t - 1)\right] - 1 \right\} / \sigma_s,$$

where $\sigma_s > 1$ will determine the convexity of the cost function, r_K is the steady state level of the real return on capital from production, $r_{Kt} \equiv R_{Kt}/P_t$, and $s(u) = 0$, $s'(u) = r_K$, and $s''(u) = \sigma_s r_K$. The function is designed to yield a steady state utilization rate of 1.

The first order conditions for this problem give the demand for labor

$$w_t = (1-\alpha) \frac{p_{mt} y_{mit}}{\ell_{it}},$$

where $p_{mt} \equiv P_{mt}/P_t$ is the relative price of intermediate output, and the utilization rate

$$s'(u_{it}) = \alpha \frac{p_{mt} y_{mit}}{u_{it} k_{it-1}}.$$

Plugging back into the expression for the internal cash flow, we obtain

$$r_{Kt}k_{it-1} = \alpha p_{mt} y_{mit} - s(u_{it})k_{it-1}.$$

With the labor-output and capital services-output ratios the same across firms, the real return on capital from production is uniform, and aggregation is straightforward.

2.4.2 Investment

Idiosyncratic shocks and financial frictions affect the investment process. After production takes place, producers adjust their capital stock for production next period, by investing in new capital formation and by trading existing capital in secondary markets. Before the asset market opens, a fixed fraction of used capital, $\gamma \in [0, 1]$, depreciates. In addition, a fraction $1 - \bar{\psi}_t$ of the remaining used capital turns into "bad-quality" capital that will become unproductive at the end of the period. Therefore, the survival probability of capital is given by $1 - \delta_t \equiv (1 - \gamma)\,\bar{\psi}_t$. We assume that the fraction of good-quality capital evolves exogenously according to

$$\hat{\psi}_t = \rho_\psi \hat{\psi}_{t-1} + \varepsilon_{\psi t},$$

where $\hat{\psi}_t \equiv \ln(\bar{\psi}_t/\bar{\psi})$, $\rho_\psi \in (0, 1)$, and $\varepsilon_{\psi t} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_\psi^2)$. While the aggregate fraction of bad-quality capital is public knowledge, the quality of a specific unit of capital is private information. Due to this asymmetric information problem, bad-quality capital can (and will) be traded in the market. The shock $\bar{\psi}_t$ thus refers to the riskiness of traded capital.

The creation of new capital is subject to idiosyncratic investment efficiency shocks, ϵ_t, which occur before the asset market opens for trade. Specifically, one unit of investment, i_{it}, becomes $\epsilon_t i_{it}$ units of capital, where $\epsilon_t \sim F(\epsilon_t)$ with support $\mathcal{E} = [\epsilon_{\min}, \epsilon_{\max}]$ and $\mathbb{E}(\epsilon_t) = 1$. Since capital is both a productive factor and a financial asset, producers trade in the asset market to: (i) sell used bad-quality capital $k_{it}^{s,b}$; (ii) sell used good-quality capital $k_{it}^{s,g}$ to finance new investment in case the realized investment efficiency is high enough; (iii) buy used capital k_{it}^{a} to use as a store of value for future investment opportunities. In addition, producers also choose how much to hold in liquid assets (nominal government bonds), which we denote with B_{it}. Accordingly, they maximize

$$\widetilde{\mathbb{V}}_{it}^{P} = \mathbb{E}_t \left(\sum_{s=0}^{\infty} \mathcal{M}_{t+s} D_{it+s} \right),$$

where nominal dividends are given by

$$D_{it} = (1-\tau_r)\left[R_{Kt}k_{it-1} - P_{It}i_{it} + P_{Kt}(k_{it}^{s,g} + k_{it}^{s,b} - k_{it}^{a}) - B_{it} + R_{t-1}B_{it-1}\right],$$

where τ_r denotes a constant capital income tax—which we set equal to zero for now—, P_{Kt} is the market price of used capital, and R_t denotes the gross return on nominal bonds. The dividend maximization problem is subject to three constraints:

1. The law of motion of capital

$$k_{it} = (1-\delta_t)k_{it-1} + \psi_t^* k_{it}^a - k_{it}^{s,g} + \epsilon_t i_{it}, \tag{1}$$

where ψ_t^* denotes the equilibrium fraction of good-quality capital in the market.

2. Resaleability constraints on good- and bad-quality capital

$$k_{it}^{s,g} \leq \bar{\omega}_t\bar{\psi}_t(1-\gamma)k_{it-1}, \qquad k_{it}^{s,b} \leq \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)k_{it-1},$$

where $\bar{\omega}_t$ denotes the exogenous maximum amount of capital a firm can sell as a fraction of aggregate capital. We will assume that the quantity of capital that can be traded in secondary markets follows

$$\hat{\omega}_t = \rho_\omega\hat{\omega}_{t-1} + \varepsilon_{\omega t},$$

where $\hat{\omega}_t \equiv \ln(\bar{\omega}_t/\bar{\omega})$, $\rho_\omega \in (0,1)$, and $\varepsilon_{\omega t} \sim$ *i.i.d.* $\mathcal{N}(0,\sigma_\omega^2)$. The shock $\bar{\omega}_t$ thus refers to the liquidity of traded capital.

3. A non-negativity constraint on the firm's choice variables

$$\left\{D_{it}, i_{it}, k_{it}, k_{it}^{s,g}, k_{it}^{s,b}, k_{it}^a, B_{it}\right\}_{t=0}^{\infty} \geq 0.$$

For convenience, we rewrite the objective of the producers' problem in real terms as

$$\mathbb{V}^P(k_{it-1}, b_{it-1}, \epsilon_t) \equiv \mathbb{V}_{it}^P = \mathbb{E}_t\left(\sum_{s=0}^{\infty}\beta_t^s\frac{\lambda_{t+s}}{\lambda_t}d_{it+s}\right),$$

where $\mathbb{V}_{it}^P \equiv \widetilde{\mathbb{V}}_{it}^P/P_t$ and real dividends are

$$d_{it} = r_{Kt}k_{it-1} - p_{It}i_{it} + p_{Kt}(k_{it}^{s,g} + k_{it}^{s,b} - k_{it}^a) - b_{it} + (R_{t-1}/\Pi_t)b_{it-1}. \tag{2}$$

Consistent with our previous notation, the relative market price of capital is $p_{Kt} \equiv P_{Kt}/P_t$ and the real quantity of nominal bond holdings is $b_{it} \equiv B_{it}/P_t$. The remaining constraints of the problem are unchanged since all are expressed in physical units.

We can write the producers' objective in recursive form as

$$\mathbb{V}_{it}^P = d_{it} + \mathbb{E}_t \left(\beta_t \frac{\lambda_{t+1}}{\lambda_t} \mathbb{V}_{it+1}^P \right).$$

Following Dong and Wen (2017), we guess the value function of the producers' problem,

$$\mathbb{V}_{it}^P = \phi_t^K(k_{it-1}) + \phi_t^B(b_{it-1}), \tag{3}$$

which implies the discounted continuation value is

$$\mathbb{E}_t \left(\beta_t \frac{\lambda_{t+1}}{\lambda_t} \mathbb{V}_{it+1}^P \right) = q_t k_{it} + q_{Bt} b_{it}, \tag{4}$$

where

$$q_t = \mathbb{E}_t \left[\beta_t \frac{\lambda_{t+1}}{\lambda_t} \phi_{t+1}^K(\epsilon_{t+1}) \right],$$

and

$$q_{Bt} = \mathbb{E}_t \left[\beta_t \frac{\lambda_{t+1}}{\lambda_t} \phi_{t+1}^B(\epsilon_{t+1}) \right].$$

Substituting (2) and (4) into the recursive formulation of producers' problem, we obtain

$$\mathbb{V}_{it}^P = r_{Kt} k_{it-1} - p_{It} i_{it} + p_{Kt}(k_{it}^{s,g} + k_{it}^{s,b} - k_{it}^a) - b_{it} + (R_{t-1}/\Pi_t) b_{it-1} + q_t k_{it} + q_{Bt} b_{it}.$$

Next, we substitute the law of motion of capital (1) and collect terms to get

$$\mathbb{V}_{it}^P = [r_{Kt} + (1-\delta_t) q_t] k_{it-1} + (p_{Kt} - q_t) k_{it}^{s,g} + p_{Kt} k_{it}^{s,b} + (\psi_t^* q_t - p_{Kt}) k_{it}^a \\ + (\epsilon_t q_t - p_{It}) i_{it} + (q_{Bt} - 1) b_{it} + (R_{t-1}/\Pi_t) b_{it-1}.$$

From this expression, working towards verifying our guess, we set $q_{Bt} = 1$ and $\psi_t^* = p_{Kt}/q_t$. Therefore, the equation above simplifies to

$$\mathbb{V}_{it}^P = [r_{Kt} + (1-\delta_t) q_t] k_{it-1} + (p_{Kt} - q_t) k_{it}^{s,g} + p_{Kt} k_{it}^{s,b} + (\epsilon_t q_t - p_{It}) i_{it} + (R_{t-1}/\Pi_t) b_{it-1}.$$

The next step is to characterize the threshold for investment. For this purpose, we define

$$\epsilon_t^* \equiv \frac{p_{It}}{q_t} \qquad \text{and} \qquad \epsilon_t^{**} \equiv \frac{p_{It}}{p_{Kt}}.$$

Note that these thresholds are independent of idiosyncratic characteristics, which greatly simplifies aggregation below. Because of adverse selection, the market value of capital is lower than its internal value ($p_{Kt} < q_t$), which implies $\epsilon_t^{**} > \epsilon_t^*$. Furthermore, because of asymmetric information, all firms find it optimal to sell their bad-quality capital in its entirety. Therefore, the resaleability constraint on bad-quality capital holds with equality.

We then have two cases depending on the realization of the idiosyncratic efficiency shock:

Case I: $\epsilon_t > \epsilon_t^*$

If the realization of the idiosyncratic investment efficiency shock is high enough, the producer will pay no dividends ($D_{it} = 0$) because investing as much as possible is more convenient. From (2), we obtain that the investment level in this case is

$$i_{it} = \left[r_{Kt} k_{it-1} + p_{Kt}(k_{it}^{s,g} + k_{it}^{s,b} - k_{it}^{a}) - b_{it} + (R_{t-1}/\Pi_t) b_{it-1} \right] / p_{It}.$$

Substituting into the guess for the value function and collecting terms, we obtain

$$\begin{aligned} \mathbb{V}_{it}^P|_{\epsilon_t > \epsilon_t^*} &= [(\epsilon_t q_t / p_{It}) r_{Kt} + (1-\delta_t) q_t] \, k_{it-1} + (\epsilon_t p_{Kt}/p_{It} - 1) q_t k_{it}^{s,g} \\ &+ \bar{\omega}_t (1-\bar{\psi}_t)(1-\gamma)(\epsilon_t p_{Kt}/p_{It}) q_t k_{it-1} - (\epsilon_t q_t / p_{It} - 1)(p_{Kt} k_{it}^a + b_{it}) + (\epsilon_t q_t/p_{It})(R_{t-1}/\Pi_t) b_{it-1}, \end{aligned}$$

where we also substituted the resaleability constraint on bad-quality capital at equality. We propose to interpret $\epsilon_t q_t / p_{It}$ as the shadow value of investment in terms of investment costs, $\epsilon_t p_{Kt}/p_{It}$ is the market value of investment in terms of investment costs.

Since $\epsilon_t > \epsilon_t^*$, $\epsilon_t q_t / p_{It} = \epsilon_t / \epsilon_t^* > 1$. Therefore, the firm will find it optimal not to acquire any used capital and not to save in liquid assets ($k_{it}^a = b_{it} = 0$). Intuitively, investment is too attractive to save and not take advantage of the opportunity.

At the same time, the firm will sell good-quality capital only when the realization of idiosyncratic investment efficiency is high enough (that is, $k_{it}^{s,g} > 0$ if and only if $\epsilon_t > \epsilon_t^{**}$ which implies $\epsilon_t p_{Kt}/p_{It} > 1$). In this case, the firm will go all in and the resaleability constraint on good-quality capital will also bind.

Combining these considerations, we rewrite the value function as

$$\begin{aligned} \mathbb{V}_{it}^P|_{\epsilon_t > \epsilon_t^*} = [&\epsilon_t r_{Kt}/p_{It} + (1-\delta_t) + \bar{\omega}_t (1-\delta_t) \max(\epsilon_t p_{Kt}/p_{It} - 1, 0) \\ &+ \bar{\omega}_t (1-\bar{\psi}_t)(1-\gamma) \epsilon_t p_{Kt}/p_{It}] q_t k_{it-1} + (\epsilon_t q_t/p_{It})(R_{t-1}/\Pi_t) b_{it-1}, \end{aligned}$$

which implies

$$\begin{aligned} \phi_t^K(\epsilon_t)|_{\epsilon_t > \epsilon_t^*} = [&\epsilon_t r_{Kt}/p_{It} + (1-\delta_t) + \bar{\omega}_t (1-\delta_t) \max(\epsilon_t p_{Kt}/p_{It} - 1, 0) \\ &+ \bar{\omega}_t (1-\bar{\psi}_t)(1-\gamma) \epsilon_t p_{Kt}/p_{It}] q_t, \end{aligned}$$

and

$$\phi_t^B(\epsilon_t)|_{\epsilon_t > \epsilon_t^*} = (\epsilon_t q_t / p_{It})(R_{t-1}/\Pi_t).$$

Case II: $\epsilon_t \leq \epsilon_t^*$

In this case, firms will not find it convenient to invest ($i_{it} = 0$) and instead will pay out dividends. The value function becomes

$$\mathbb{V}_{it}^P|_{\epsilon_t \leq \epsilon_t^*} = [r_{Kt} + (1-\delta_t)q_t]k_{it-1} + (p_{Kt} - q_t)k_{it}^{s,g} + \\ \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)p_{Kt}k_{it-1} + (R_{t-1}/\Pi_t)b_{it-1},$$

where we have already substituted the resaleability constraint for bad-quality capital. Since $p_{Kt} < q_t$, these firms choose to keep their good-quality capital ($k_{it}^{s,g} = 0$). Therefore, the value function becomes

$$\mathbb{V}_{it}^P|_{\epsilon_t \leq \epsilon_t^*} = [r_{Kt} + (1-\delta_t)q_t + \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)p_{Kt}]k_{it-1} + (R_{t-1}/\Pi_t)b_{it-1},$$

which implies

$$\phi_t^K(\epsilon_t)|_{\epsilon_t \leq \epsilon_t^*} = r_{Kt} + (1-\delta_t)q_t + \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)p_{Kt},$$

and

$$\phi_t^B(\epsilon_t)|_{\epsilon_t \leq \epsilon_t^*} = R_{t-1}/\Pi_t.$$

2.4.3 Policy Functions

We can therefore summarize the policy functions for the producers' problem as follows:

- Dividends

$$d_{it} = \begin{cases} 0 & \text{if } \epsilon_t > \epsilon_t^* \\ r_{Kt}k_{it-1} + p_{Kt}(k_{it}^{s,g} + k_{it}^{s,b} - k_{it}^a) - b_{it} + (R_{t-1}/\Pi_t)b_{it-1} & \text{if } \epsilon_t \leq \epsilon_t^*. \end{cases}$$

- Investment

$$i_{it} = \begin{cases} \left[r_{Kt}k_{it-1} + p_{Kt}(k_{it}^{s,g} + k_{it}^{s,b} - k_{it}^a) - b_{it} + (R_{t-1}/\Pi_t)b_{it-1}\right]/p_{It} & \text{if } \epsilon_t > \epsilon_t^* \\ 0 & \text{if } \epsilon_t \leq \epsilon_t^*. \end{cases}$$

- Acquired capital

$$k_{it}^a = \begin{cases} 0 & \text{if } \epsilon_t > \epsilon_t^* \\ \text{indeterminate} & \text{if } \epsilon_t \leq \epsilon_t^*. \end{cases}$$

- Good-quality capital sold

$$k_{it}^{s,g} = \begin{cases} \bar{\omega}_t(1-\delta_t)k_{it-1} & \text{if } \epsilon_t > \epsilon_t^{**} \\ 0 & \text{if } \epsilon_t \leq \epsilon_t^{**}. \end{cases}$$

- Bad-quality capital sold

$$k_{it}^{s,b} = \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)k_{it-1} \ \forall\, \epsilon_t.$$

- Bonds

$$b_{it+1} = \begin{cases} 0 & \text{if } \epsilon_t > \epsilon_t^{*} \\ \text{indeterminate} & \text{if } \epsilon_t \leq \epsilon_t^{*}. \end{cases}$$

2.4.4 Aggregation

The capital and bonds loadings in the value function can compactly be written as

$$\begin{aligned} \phi_t^K(\epsilon_t) = &\left[1+\max\left(\frac{\epsilon_t}{\epsilon_t^*}-1,0\right)\right]\left[r_{Kt}+\bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)p_{Kt}\right] \\ &+\left[1+\max\left(\frac{\epsilon_t}{\epsilon_t^{**}}-1,0\right)\right]\bar{\omega}_t(1-\delta_t)q_t+(1-\bar{\omega}_t)(1-\delta_t)q_t, \end{aligned}$$

and

$$\phi_t^B(\epsilon_t) = (R_{t-1}/\Pi_t)\left[1+\max\left(\frac{\epsilon_t}{\epsilon_t^*}-1,0\right)\right].$$

Therefore, the Euler equation for capital becomes

$$\begin{aligned} q_t = &\,\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*}-1\right)dF(\epsilon)\right]\left[r_{Kt+1}+\bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}\right]\right\} \\ &+\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon_{t+1}^{**}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^{**}}-1\right)dF(\epsilon)\right]\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}\right\} \\ &+\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}\right\}. \end{aligned} \tag{5}$$

Similarly, the Euler equation for bonds is

$$q_{Bt} = \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\Pi_{t+1}}\left[1+\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*}-1\right)dF(\epsilon)\right]\right\}.$$

Given $q_{Bt} = 1$, we can rewrite the Euler equation for bonds as

$$R_t = \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \left[1 + \int_{\epsilon_{t+1}^*}^{\epsilon_{\max}} \left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1 \right) dF(\epsilon) \right] \right\}^{-1}. \tag{6}$$

Aggregate investment is the sum of investment by all firms with high enough realizations of the idiosyncratic efficiency shock

$$\begin{aligned} i_t &= \int_{\epsilon_t > \epsilon_t^*} i_{it} \; dF(\epsilon) \\ &= \int_{\epsilon_t > \epsilon_t^*} \frac{r_{Kt} k_{it-1} + p_{Kt}(k_{it}^{s,g} + k_{it}^{s,b} - k_{it}^a) - b_{it} + (R_{t-1}/\Pi_t) b_{it-1}}{p_{It}} \; dF(\epsilon) \\ &= \frac{(R_{t-1}/\Pi_t) b_{t-1} + [r_{Kt} + \bar{\omega}_t (1 - \bar{\psi}_t)(1-\gamma) p_{Kt}] k_{t-1}}{p_{It}} \left(1 - F(\epsilon_t^*)\right) \\ &\qquad + \frac{\bar{\omega}_t (1 - \delta_t) p_{Kt} k_{t-1}}{p_{It}} \left(1 - F(\epsilon_t^{**})\right). \end{aligned}$$

These firms finance investment with: (i) the proceeds from liquidating their bond holdings; (ii) the return on their existing capital; (iii) the sales of bad capital; (iv) the sales of good capital (if the efficiency shock is above the high threshold).

In the aggregate, acquired capital corresponds to aggregate good-quality capital sold. Therefore, the aggregate law of motion of capital is

$$\begin{aligned} k_t &= \int_0^1 k_{it} \; dF(\epsilon) \\ &= (1 - \delta_t) k_{t-1} + \frac{(R_{t-1}/\Pi_t) b_{t-1} + [r_{Kt} + \bar{\omega}_t (1 - \bar{\psi}_t)(1-\gamma) p_{Kt}] k_{t-1}}{p_{It}} \int_{\epsilon_t^*}^{\epsilon_{\max}} \epsilon_t dF(\epsilon) \\ &\qquad + \frac{\bar{\omega}_t (1 - \delta_t) p_{Kt} k_{t-1}}{p_{It}} \int_{\epsilon_t^{**}}^{\epsilon_{\max}} \epsilon_t dF(\epsilon). \end{aligned}$$

The last step consists of characterizing the equilibrium fraction of good-quality capital in the market. Recall that we have obtained $p_{Kt} = \psi_t^* q_t$. The equilibrium fraction of good-quality capital is the ratio between good-quality capital and total capital traded

$$\begin{aligned} \psi_t^* &= \frac{\int_0^1 k_{it}^{s,g} dF(\epsilon)}{\int_0^1 (k_{it}^{s,g} + k_{it}^{s,b}) dF(\epsilon)} = \frac{\int_{\epsilon_t^{**}}^{\epsilon_{\max}} k_{it}^{s,g} dF(\epsilon)}{\int_{\epsilon_t^{**}}^{\epsilon_{\max}} k_{it}^{s,g} dF(\epsilon) + k_t^{s,b}} \\ &= \frac{\bar{\omega}_t (1 - \delta_t) k_{t-1} [1 - F(\epsilon_t^{**})]}{\bar{\omega}_t (1 - \delta_t) k_{t-1} [1 - F(\epsilon_t^{**})] + \bar{\omega}_t (1 - \bar{\psi}_t)(1-\gamma) k_{t-1}} = \frac{(1 - \delta_t)[1 - F(\epsilon_t^{**})]}{(1 - \delta_t)[1 - F(\epsilon_t^{**})] + (1 - \bar{\psi}_t)(1 - \gamma)}. \end{aligned}$$

2.4.5 Convenience Yield

Since bonds are perfectly safe and liquid, in equilibrium capital must pay a premium over bonds. This premium (the "convenience yield") consists of two parts, one related to liquidity and one to safety, capturing the two key financial frictions in the model.

The demand for perfectly safe and liquid bonds is increasing in response to a negative shock to $\bar{\omega}_t$ ("flight to liquidity") and to a $\bar{\psi}_t$ ("flight to safety").

We define the convenience yield as the risk-adjusted difference between the real return on capital and bonds

$$cy_t \equiv \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} \left[1 + \int_{\epsilon^*_{t+1}}^{\epsilon_{t+1}} \left(\frac{\epsilon_{t+1}}{\epsilon^*_{t+1}} - 1 \right) dF(\epsilon) \right] \left(r^*_{Kt+1} - \frac{R_t}{\Pi_{t+1}} \right) \right\},$$

where the real return on capital is

$$r^*_{Kt} \equiv \frac{r_{Kt} + (1 - \delta_t) q_t}{q_{t-1}}.$$

Notice that the effective discount factor is higher than the individual discount factor. This term captures the option value of one real dollar (shadow value of a unit of output). If the realization of the efficiency shock is low, a producer can decide not to invest, in which case one real dollar remains such. However, if the realization is good (above the threshold ϵ^*_t), the producer can invest in capital, which is worth $\epsilon_t q_t > 1$. Therefore, the option value of one unit of output at time t is

$$\int_{\epsilon_t \leq \epsilon^*_t} dF(\epsilon) + \int_{\epsilon_t > \epsilon^*_t} \frac{\epsilon_t}{\epsilon^*_t} \, dF(\epsilon) = 1 + \int_{\epsilon_t > \epsilon^*_t} \left(\frac{\epsilon_t}{\epsilon^*_t} - 1 \right) dF(\epsilon).$$

Indeed, this additional term in the discount factor applies to both capital and bond returns, as we can see from equation (5) and (6).[2]

[2] Along similar lines, we can interpret the second term in square bracket in equation (5) as the option value of one unit of high-quality capital.

We divide through (5) by q_t and add and subtract $(1-\delta_{t+1})q_{t+1}/q_t$ to obtain

$$1 = \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon^*_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^*_{t+1}}-1\right)dF(\epsilon)\right]\right.$$
$$\left.\left[r^*_{Kt+1}-\frac{(1-\delta_{t+1})q_{t+1}+\bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}}{q_t}\right]\right\}$$
$$+\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon^{**}_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^{**}_{t+1}}-1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}}{q_t}\right\}$$
$$+\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\frac{(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}}{q_t}\right\}.$$

Next, we can subtract from this equation the Euler equation for bonds (6) to write

$$0 = \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon^*_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^*_{t+1}}-1\right)dF(\epsilon)\right]\left(r^*_{Kt+1}-\frac{R_t}{\Pi_{t+1}}\right)\right\}$$
$$-\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon^*_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^*_{t+1}}-1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}-(1-\delta_{t+1})q_{t+1}}{q_t}\right\}$$
$$+\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon^{**}_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^{**}_{t+1}}-1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}}{q_t}\right\}$$
$$+\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\frac{(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}}{q_t}\right\}.$$

The first term of the last expression is the convenience yield. We can thus readjust to get

$$cy_t = \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon^*_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^*_{t+1}}-1\right)dF(\epsilon)\right]\frac{(1-\delta_{t+1})q_{t+1}-\bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}}{q_t}\right\}$$
$$-\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon^{**}_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^{**}_{t+1}}-1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}}{q_t}\right\}$$
$$-\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\frac{(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}}{q_t}\right\}.$$

We add and subtract $\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}/q_t$ to the first term and combine terms to obtain

$$cy_t = \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[\int_{\epsilon^*_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^*_{t+1}}-1\right)dF(\epsilon)\right]\frac{(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}}{q_t}\right\}$$
$$+\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[\int_{\epsilon^*_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^*_{t+1}}-1\right)dF(\epsilon)-\int_{\epsilon^{**}_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^{**}_{t+1}}-1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}}{q_t}\right\}$$
$$-\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1+\int_{\epsilon^*_{t+1}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon^*_{t+1}}-1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}}{q_t}\right\}.$$

More compactly, we can rewrite the last expression as

$$cy_t = r_t^{\omega} + r_t^{\psi},$$

where

$$r_t^{\omega} \equiv \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} \left[\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}} \left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1 \right) dF(\epsilon) \right] \frac{(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}}{q_t} \right\},$$

and

$$\begin{aligned} r_t^{\psi} \equiv \mathbb{E}_t & \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} \left[\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}} \left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1 \right) dF(\epsilon) - \int_{\epsilon_{t+1}^{**}}^{\epsilon_{\max}} \left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^{**}} - 1 \right) dF(\epsilon) \right] \frac{\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}}{q_t} \right\} \\ & - \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\lambda_t} \left[1 + \int_{\epsilon_{t+1}^*}^{\epsilon_{\max}} \left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1 \right) dF(\epsilon) \right] \frac{\bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}}{q_t} \right\}. \end{aligned}$$

In the decomposition of the convenience yield, the r_t^{ω} captures the liquidity component and r_t^{ψ} captures the safety component. As $\bar{\omega}_{t+1} \to 1$, that is, as all capital becomes tradable in the secondary market, the liquidity premium goes to zero and disappears. Similarly, as $\bar{\psi}_{t+1} \to 1$, that is, as all capital becomes of good-quality and the proportion of bad-quality capital goes to zero, the safety premium disappears.[3]

2.5 Final Goods Producers

Final goods firms purchase homogeneous intermediate inputs from producers and differentiate them. Final output, y_t, is a CES aggregator of these differentiated goods, y_{jt},

$$y_t = \left(\int_0^1 y_{jt}^{\frac{\theta_p - 1}{\theta_p}} \, dj \right)^{\frac{\theta_p}{\theta_p - 1}},$$

where $\theta_p > 1$ is the elasticity of substitution among differentiated inputs.

Cost minimization implies that the demand for each variety is

$$y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta_p} y_t,$$

[3]The second term of r_t^{ψ} clearly goes to zero as $\bar{\psi}_{t+1}$ approaches one. To see why the first term does too, notice that $\epsilon_{t+1}^{**} \to \epsilon_{t+1}^*$ in this limit.

and that the aggregate price index is

$$P_t = \left(\int_0^1 P_{jt}^{1-\theta_p} \, dj \right)^{\frac{1}{1-\theta_p}} .$$

Final goods firms operate in monopolistic competition and set prices on a staggered basis (Calvo, 1983). Each period, a firm is able to adjust its price with probability $1 - \iota_p$. If it cannot adjust its price, the firm partially updates its price to the geometric average of last period's inflation and the time-varying inflation target Π_t^*, where $\gamma_p \in (0, 1)$ measures the degree of indexation to lagged inflation. In the absence of firm entry and exit, a fixed cost, φz_t^*, proportional to the stochastic growth rate in the model, is introduced and calibrated to ensure profits are zero in state. The problem for a final goods firm that can readjust at time t then is

$$\max_{\widetilde{P}_{jt}} \; \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \iota_p^s \mathcal{M}_{t+s} \left[\widetilde{P}_{jt} X_{t,t+s}^P - \chi_{pt+s} P_{mt+s} \right] y_{jt,t+s} - \varphi z_t^* \; , \; 0 \right\} ,$$

subject to the demand for its product conditional on no further price changes

$$y_{jt,t+s} = \left(\frac{\widetilde{P}_{jt} X_{t,t+s}^P}{P_{t+s}} \right)^{-\theta_p} y_{t+s},$$

where

$$X_{t,t+s}^P = \begin{cases} \prod_{k=0}^{s-1} (\Pi_{t+k})^{\gamma_p} (\Pi_{t+k+1}^*)^{1-\gamma_p} & \text{if } s > 0 \\ 1 & \text{if } s = 0, \end{cases}$$

and χ_{pt} is a cost-push shock, introduced to capture distortions in markups, which follows

$$\hat{\chi}_{pt} = \rho_{\chi p} \hat{\chi}_{pt-1} + \varepsilon_{\chi pt},$$

where $\hat{\chi}_{pt} \equiv \ln(\chi_{pt}/\chi_p)$, $\rho_{\chi p} \in (0, 1)$, and $\varepsilon_{\chi pt} \sim \; i.i.d. \; \mathcal{N}(0, \sigma_{\chi p}^2)$.

The first-order condition for the firm's problem is

$$\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \iota_p^s \mathcal{M}_{t+s} \left[\widetilde{P}_{jt} X_{t,t+s}^P + \chi_{pt+s} \frac{\theta_p}{\theta_p - 1} P_{mt+s} \right] y_{jt,t+s} \right\} = 0.$$

Since the marginal cost—the price of intermediate inputs—is the same across firms, in a symmetric equilibrium all firms that reset their price choose the same strategy. We can rewrite the optimal relative reset price as

$$\frac{\widetilde{P}_t}{P_t} = \left(\frac{\theta_p}{\theta_p - 1}\right)\frac{\mathcal{D}_{pt}}{\mathcal{F}_{pt}},$$

where the numerator of the right-hand side is the present discounted value of total costs

$$\mathcal{D}_{pt} \equiv \mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta_t\iota_p)^s\lambda_{t+s}\chi_{pt+s}p_{mt+s}\left(\frac{(P_{t+s}/P_t)}{X^P_{t,t+s}}\right)^{\theta_p}y_{t+s}\right]$$
$$= \lambda_t\chi_{pt}p_{mt}y_t + \beta_t\iota_p\mathbb{E}_t\left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\gamma_p}(\Pi^*_{t+1})^{1-\gamma_p}}\right]^{\theta_p}\mathcal{D}_{pt+1}\right\},$$

and the denominator is the present discounted value of total net revenues

$$\mathcal{F}_{pt} \equiv \mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta_t\iota_p)^s\lambda_{t+s}\left(\frac{(P_{t+s}/P_t)}{X^P_{t,t+s}}\right)^{\theta_p-1}y_{t+s}\right]$$
$$= \lambda_t y_t + \beta_t\iota_p\mathbb{E}_t\left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\gamma_p}(\Pi^*_{t+1})^{1-\gamma_p}}\right]^{\theta_p-1}\mathcal{F}_{pt+1}\right\}.$$

Further, we can rewrite the aggregate price index in terms of the optimal relative reset price and inflation as

$$\frac{\widetilde{P}_t}{P_t} = \left\{\frac{1-\iota_p\left[\frac{\Pi_t}{(\Pi_{t-1})^{\gamma_p}(\Pi^*_t)^{1-\gamma_p}}\right]^{\theta_p-1}}{1-\iota_p}\right\}^{\frac{1}{1-\theta_p}}.$$

Substituting the expression for the optimal relative reset price into the last equation, we obtain the non-linear Phillips curve

$$\left(\frac{\theta_p}{\theta_p - 1}\right)\frac{\mathcal{D}_{pt}}{\mathcal{F}_{pt}} = \left\{\frac{1-\iota_p\left[\frac{\Pi_t}{(\Pi_{t-1})^{\gamma_p}(\Pi^*_t)^{1-\gamma_p}}\right]^{\theta_p-1}}{1-\iota_p}\right\}^{\frac{1}{1-\theta_p}}.$$

Finally, staggered price-setting introduces price dispersion as not all final goods firms will be able to adjust their prices in every period. Integrating the demand for intermediate inputs over the continuum of varieties, we obtain

$$y_{mt} \equiv \int_0^1 y_{jt}\ dj = \int_0^1\left(\frac{P_{jt}}{P_t}\right)^{-\theta_p}dj\ y_t = \Delta_{pt}y_t,$$

where $\Delta_{pt} \equiv \int_0^1\left(\frac{P_{jt}}{P_t}\right)^{-\theta_p}dj$ gives a measure of output lost due to the inefficient allocation of resources caused by price dispersion. Rewriting this measure in terms of the optimal relative reset price and inflation yields the familiar recursive result for price dispersion,

$$\Delta_{pt} = (1-\iota_p)\left[\left(\frac{\theta_p}{\theta_p-1}\right)\frac{\mathcal{D}_{pt}}{\mathcal{F}_{pt}}\right]^{-\theta_p} + \iota_p\left[\frac{\Pi_t}{(\Pi_{t-1})^{\gamma_p}(\Pi_t^*)^{1-\gamma_p}}\right]^{\theta_p}\Delta_{pt-1}. \tag{7}$$

2.6 Monetary and Fiscal Policy

The government consists of a monetary and a fiscal authority. The central bank sets interest rates following a standard interest rate feedback rule subject to the constraint of the lower bound, which we assume to be zero

$$R_t = \max\left\{R_{t-1}^{\rho_m}\left[R\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\left(\frac{y_t/y_{t-1}}{\mu_{z^*}}\right)^{\phi_y}\right]^{1-\rho_m}\exp\left(\varepsilon_{mt}\right), 1\right\},$$

where $\rho_m \in (0,1)$ is the interest rate smoothing parameter, $\phi_\pi > 1$ is the feedback coefficient on inflation in deviations from its target, and $\phi_y > 0$ is the feedback coefficient on output growth in deviations from trend growth, Π^* is the non-zero inflation target, and orthogonal monetary policy shocks follow $\varepsilon_{mt} \sim$ *i.i.d.* $\mathcal{N}(0,\sigma_m^2)$.

Fiscal policy in the model is non-Ricardian as the supply of perfectly safe and liquid government debt directly affects spreads and investment. We assume the treasury issues one-period nominal debt, B_t, and levies lump-sum taxes on households, T_t, to fund real government spending, g_t, according to

$$B_t = R_{t-1}B_{t-1} + P_t g_t - T_t.$$

Government spending, in real terms and as a fraction of output, is exogenous and follows

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{gt},$$

where $\hat{g}_t \equiv \ln[(g_t/y_t)/(g/y)]$, $\rho_g \in (0,1)$, and $\varepsilon_{gt} \sim$ *i.i.d.* $\mathcal{N}(0,\sigma_g^2)$. Likewise, we make the assumption that government debt is exogenous and follows

$$\hat{b}_t = \rho_b \hat{b}_{t-1} + \varepsilon_{bt},$$

where $\hat{b}_t \equiv \ln[(b_t/y_t)/(b/y)]$, $\rho_b \in (0,1)$, and $\varepsilon_{bt} \sim$ *i.i.d.* $\mathcal{N}(0,\sigma_b^2)$.

As a consequence, the government budget constraint residually pins down lump-sum taxes.

2.7 Equilibrium

Labor market clearing implies labor supply by the representative household must equal the aggregate stock of labor employed by the continuum of producers, $\ell_t = \int_{i\in[0,1]} \ell_{it}\ di$.

Equally, general equilibrium requires the aggregate stock of capital to be equal to the amount of capital owned by the continuum of producers, $k_t = \int_{i\in[0,1]} k_{it}\ di$.

With linear homogeneity of the Cobb-Douglas technology and equal factor ratios across producers, final output is then given by the following aggregate production function,

$$y_t = y_{mt}/\Delta_{pt} = \left[a_t\left(u_t k_{t-1}\right)^{\alpha}\left(z_t\ell_t\right)^{1-\alpha}\right]/\Delta_{pt}.$$

Equally, aggregate labor used in the production process can be related to the stock of homogeneous labor supplied by the representative households according to

$$\ell_t = \ell_{ht}/\Delta_{wt}.$$

Finally, the representative household's savings in shares of producers i must be equal to 1. In equilibrium, the representative household owns all producers, $s_{it} = 1\ \forall\ i \in [0,1]$.

Also, market clearing in the market for government bonds requires producers' holdings to be equal to the aggregate supply of government debt, $b_t = \int_{i\in[0,1]} b_{it}\ di$.

Thus, we can rewrite the household's budget constraint to yield the economy's aggregate resource constraint in real terms,[4]

$$\begin{aligned}(1+\tau_p)c_t &= (1-\tau_w)(w_{ht}\ell_{ht} + \Omega_t^L) + \int_{i\in[0,1]} d_{it}\ di + \left(\Omega_t^I + \Omega_t^F - T_t\right)/P_t \\ &= (1-\tau_w)\left[w_{ht}\ell_{ht} + \left(w_t - w_{ht}\chi_{wt}\Delta_{wt}\right)\ell_t\right] + \left(r_{Kt}k_{t-1} - p_{It}i_t + (R_{t-1}/\Pi_t)b_{t-1} - b_t\right) \\ &\quad + \left[p_{It} - \left(1 + f\left(\frac{i_t}{i_{t-1}}\right)\right)\right] i_t + \left[\left(1 - p_{mt}\chi_{pt}\Delta_{pt}\right)y_t - \varphi z_t^*\right] \\ &\quad - \left[(R_{t-1}/\Pi_t)b_{t-1} - b_t + g_t - \tau_w w_t\ell_t - \tau_p c_t - \tau_r r_{Kt}k_{t-1}\right] \\ &= y_t - g_t - i_t\left[1 + f\left(\frac{i_t}{i_{t-1}}\right)\right] - s(u_t)k_{t-1} - \varphi z_t^*.\end{aligned}$$

[4] Where we assume the time-varying markups induced through cost-push shocks to wages, χ_{wt}, and intermediate input prices, χ_{pt}, are rebated to households in a lump-sum fashion.

3 Estimation

3.1 Overview

We estimate the model at quarterly frequency using U.S. macroeconomic and financial data covering the period 1985Q1 to 2019Q2. In total, we use nine time series, seven of which are standard macroeconomic variables—GDP, consumption, investment, real wages, hours worked, inflation, and the federal funds rate (substituted by Wu and Xia (2016)'s shadow rate from 2009Q3).[5] The remaining two time series are market-implied liquidity and safety premia which we construct following Krishnamurthy and Vissing-Jorgensen (2012) (as displayed in Figure 1).[6] As the combination of the two financial frictions at the core of the model gives rise to an endogenous convenience yield that can be decomposed into a liquidity and safety premium, we crucially include these two spread measures in the estimation to get a quantitative sense of the stringency of the two financial frictions and the importance of the structural shocks to asset liquidity and safety over the business cycle. Prior to the estimation, we take logarithmic first-differences of trending variables (GDP, consumption, investment, and real wages), and demean all variables to prevent low frequency movements in the data that are not accounted for in the model to interfere with our inference at business cycle frequency.

The structural model as derived in Section 2 comes with ten exogenous shock processes. For a consistent estimation of the model, we ensure all transformations applied to the data are are also applied to the variables in the model. In particular, we detrend the structural model to account for the two non-stationary processes in investment specific technology and labor-augmenting productivity, and specify demeaned measurement equations.

For a detailed overview, Appendix A.1 gives a complete list of the non-linear equilibrium conditions as derived in the paper plus all exogenous processes. Appendix A.2 restates the list of equilibrium conditions after detrending. Appendix A.3 describes the algorithm we use to solve for the deterministic steady state of the model calibrating selected parameters. The results of the full estimation of the model are documented in Table 1 and Table 2.

[5] GDP, consumption (purchases of non-durable goods and services), investment (gross private domestic investment and purchases of durable goods), and wages (hourly compensation for employees in the non-farm business) are deflated using the implicit GDP price deflator. Hours worked are constructed as aggregate non-farm business hours of all persons. Inflation is the log first-differenced GDP price deflator. From 1985Q1 to 2019Q2, the 3-month average of the daily effective federal funds rate is used as a proxy for the short-run save nominal interest rate in the model. From 2009Q3, we use Wu and Xia (2016)'s shadow rate to proxy for unconventional monetary policy. All quantities are measured in per-capita terms, dividing aggregate quantities by hp-filtered data for U.S. civilian population above 16 years.

[6] We compute the liquidity premium as the spread between the yield on the AAA corporate bond index and the yield on the 10-year Treasury (AAA-10yT). The safety premium is computed as the spread between the yield on the BAA and the AAA corporate bond indexes (BAA-AAA).

3.2 Calibrated Parameters

We partition the structural parameters and steady state values of exogenous processes into a first set we fix a priori and a second set that is estimated using Bayesian methods.

The first set of parameters is fixed based on a combination of established results in the literature and a range of carefully selected calibration targets matched at the posterior mode for estimated parameters. Appendix A.3 describes the algorithm employed and provides details beyond the scope of the short summary below. Table 1 states the results.

Table 1: Calibrated parameters.

Parameter		Value	Parameter		Value
Households					
σ	Risk aversion	1.0000	β	Discount factor	0.9901
χ	Disutility weight on labor	0.8011	ξ	Curvature of labor disutility	1.0000
Labor unions			**Investment goods firms**		
θ_p	Elasticity of labor substitution	11.000	Υ	Trend in inv specific technology	1.0025
Producers					
α	Capital share	0.4000	γ	Depreciation rate	0.0216
ν	Inv efficiency: Pareto param	5.7661	$\varepsilon_{\min}$	Inv efficiency: Pareto bound	0.8266
a	Steady state cyclical productivity	1.0000	μ_{z^*}	Trend growth rate of economy	1.0038
$\bar{\omega}$	Steady state capital resaleability	0.7740	$\bar{\psi}$	Steady state capital quality	0.9966
Final goods firms			**Government**		
θ_w	Elasticity of goods substitution	6.0000	Π^*	Steady state inflation	1.0050
g/y	Steady state govt spending/GDP	0.2000	b/y	Steady state govt debt/GDP	1.6772

We normalize households' risk aversion σ and the curvature of labor disutility ξ to 1. The steady state discount factor β is set to 0.9901, which pins down the annualized steady state federal funds rate R to 3.31%, the mean in our sample. Households' disutility weight on labor χ is set to 0.8011 to normalize steady labor supply l to 1. The elasticity of labor substitution θ_w is set to 11, the elasticity of goods substitution θ_p to 6, yielding state state wage and price mark-ups λ_w and λ_p of 1.10 and 1.2, respectively.

We fix the trend in investment specific technology Υ at 1.0023 and the trend growth rate in the economy μ_z^* at 1.0038 to match the annualized mean decline in the price of investment goods of -0.92% and the annualized real growth rate of 1.52% in the sample. The steady state value of cyclical productivity a is normalized to 1. The capital share α is set to 0.4 and the depreciate rate γ to 0.0216, calibrated to yield an annualized steady state depreciation rate δ of 10% (accounting for imperfect capital quality $\bar{\psi}$). We further fix steady state inflation Π^* at 1.0050% to match the annualized mean inflation rate of approximately 2% in the sample. Steady state government spending and government debt to GDP, g/y and b/y, are set to 0.2 and 1.677, respectively. In targeting an annualized steady state debt to GDP ratio of 41.93%, the mean of public debt minus FED holdings, we use a 'liquid assets in the hands of the public' concept as in Del Negro et al. (2017).

Finally, at the core of the model are shocks to idiosyncratic investment efficiency and two types of financial market imperfections: limited resaleability and shocks to capital quality paired with asymmetric information. We jointly calibrate the remaining four parameters directly associated with this, $\{\epsilon_{\text{min}}, \nu, \bar{\omega}, \bar{\psi}\}$, as follows: One, we assume the level of investment efficiency ϵ_t is drawn by firm i on a period-by-period basis from a Pareto distribution described by $F(\epsilon) = 1-(\epsilon/\epsilon_{\text{min}})^{-\nu}$, where $\epsilon > \epsilon_{\text{min}}$ and $\nu > 1$. Normalizing the expected realisation of investment efficiency $\mathbb{E}(\epsilon) = \frac{\nu}{\nu-1}\epsilon_{\text{min}}$ to 1 and targeting a quarterly steady state investment frequency $1 - F(\epsilon^*) = (\epsilon^*/\epsilon_{\text{min}})^{-\nu}$ of 5%—an average of values used in the literature following Dong and Wen (2017)—pins down ϵ_{min} at 0.8266 and ν at 5.7661. Two, rather than matching difficult-to-measure empirical notions of resaleability and quality in asset markets, we back out the steady state capital resaleability $\bar{\omega}$ and steady state capital quality $\bar{\psi}$ targeting the mean liquidity premium and safety premia in the sample. The combination of a mean annualized liquidity premium r_ω and safety premium r_ψ of 91bp and 99bp, respectively, yields an exogenous steady state capital resaleability and capital quality of 0.7740 and 0.9966, respectively.

Encouragingly—and as indicative evidence on the validity of our parameterization—the above calibration yields at least two further untargeted moments, a steady state investment to GDP ratio i/y of 22% and a market resaleability ω^* of 30%, both of which are very close to values commonly used in the literature (e.g. in Del Negro et al., 2017).

3.3 Estimated Parameters

We estimate the second set of parameters using Bayesian methods. Table 2 displays the results. All of the economic parameters are standard in the new-Keynesian literature and we follow Smets and Wouters (2007), Justiniano et al. (2010), Christiano et al. (2014), and Becard and Gauthier (2022) in our selection of priors. With the exception of the wage stickiness parameter ι_w, which is rather small, all of the estimated posterior modes are close to values found in the above cited literature. The posterior standard deviations are typically much smaller than the specified standard deviations of the prior distributions, indicating a certain degree of information about the estimated parameters in the data.

Turning to the nine exogenous processes estimated, a similar picture emerges. While we manually set the autocorrelation of the monetary policy shock to zero, a range of exogenous processes display a high degree of persistence, in particularly wage-up markups, stationary and non-stationary productivity, and government spending. The standard deviation of the government debt to GDP innovation is very large—an indication of the interaction of the supply of save and liquid assets and volatile spreads. Section 4 will shed more light on this and the importance of the two types of financial shocks in the model.

Table 2: Estimated parameters.

Parameter		Prior			Posterior	
		Distr	Mean	SD	Mode	SD
`(A) Economic parameters`						
Households						
$\hbar$	Habit parameter	beta	0.5000	0.1500	0.6589	0.0544
Labor unions						
ι_w	Calvo wage stickiness	beta	0.7500	0.1500	0.4423	0.0643
γ_w	Wage indexing weight on π_{t-1}	beta	0.5000	0.1500	0.6298	0.1553
Investment goods firms						
f''	Curvature of inv adj costs	normal	5.0000	3.0000	1.0333	0.1506
Producers						
σ_s	Curvature of cap util costs	normal	1.0000	1.0000	5.1870	0.6348
Final goods firms						
ι_p	Calvo price stickiness	beta	0.7500	0.1500	0.7975	0.0235
γ_p	Price indexing weight on π_{t-1}	beta	0.5000	0.1500	0.1256	0.0531
Government						
ϕ_π	Policy weight on inflation	gamma	1.5000	0.2500	2.6894	0.2337
ϕ_y	Policy weight on output	gamma	0.2500	0.1000	0.1442	0.0557
ρ_m	Policy inertia parameter	beta	0.8000	0.1000	0.8383	0.0161
`(B) Exogenous processes`						
ρ_β	AC preference shock	beta	0.5000	0.2000	0.9496	0.0215
σ_β	SD preference innovation	invg2	0.0100	1.0000	0.0010	0.0004
$\rho_{\chi w}$	AC wage mark-up shock	beta	0.5000	0.2000	0.9839	0.0117
$\sigma_{\chi w}$	SD wage mark-up innovation	invg2	0.0100	1.0000	0.0235	0.0035
ρ_a	AC cyclical productivity shock	beta	0.5000	0.2000	0.2089	0.1519
σ_a	SD cyclical productivity innovation	invg2	0.0100	1.0000	0.0020	0.0004
ρ_z	AC trend growth rate shock	beta	0.5000	0.2000	0.3781	0.1279
σ_z	SD trend growth rate innovation	invg2	0.0100	1.0000	0.0060	0.0009
ρ_ψ	AC capital quality shock	beta	0.5000	0.2000	0.8938	0.0316
σ_ψ	SD capital quality innovation	invg2	0.0100	1.0000	0.0023	0.0002
ρ_ω	AC capital resaleability shock	beta	0.5000	0.2000	0.7996	0.0426
σ_ω	SD capital resaleability innovation	invg2	0.0100	1.0000	0.0647	0.0052
$\rho_{\chi p}$	AC price mark-up shock	beta	0.5000	0.2000	0.8997	0.0379
$\sigma_{\chi p}$	SD price mark-up innovation	invg2	0.0100	1.0000	0.0188	0.0032
σ_m	SD monetary policy innovation	invg2	0.0100	1.0000	0.0013	0.0001
ρ_g	AC govt spending/GDP shock	beta	0.5000	0.2000	0.9674	0.0142
σ_g	SD govt spending/GDP innovation	invg2	0.0100	1.0000	0.0156	0.0009
ρ_b	AC govt debt/GDP shock	beta	0.5000	0.2000	0.7605	0.0283
σ_b	SD govt debt/GDP innovation	invg2	0.0100	1.0000	0.5050	0.0320

4 Results

4.1 Financial Shocks over the Business Cycle

This Section discusses the result of the Bayesian estimation of the model focusing on the importance of financial shocks for the dynamics of macroeconomic and financial variables.

Table 3: Variance decomposition.

	$\varepsilon_{\beta t}$	$\varepsilon_{\chi wt}$	ε_{at}	ε_{zt}	$\varepsilon_{\psi t}$	$\varepsilon_{\omega t}$	$\varepsilon_{\chi pt}$	ε_{mt}	ε_{gt}	ε_{bt}
y_t	5.11	27.26	0.03	8.13	**13.53**	**0.48**	33.93	4.74	4.23	2.56
i_t	30.83	11.85	0.01	0.86	**24.08**	**0.57**	25.45	2.30	0.34	3.72
c_t	44.71	11.41	0.00	30.43	7.51	0.11	0.95	0.58	3.62	0.68
Π_t	10.19	5.43	0.73	2.00	35.66	1.04	15.77	21.46	1.29	6.43
R_t	6.53	2.16	0.05	1.29	59.11	1.39	4.62	13.37	1.07	10.41
cy_t	0.99	0.29	0.00	0.13	**51.55**	**7.30**	0.66	0.06	0.02	**39.00**
r_t^{ψ}	1.40	0.40	0.00	0.18	**34.93**	**7.40**	0.87	0.07	0.03	**54.71**
r_t^{ω}	0.10	0.02	0.00	0.01	**5.36**	**90.71**	0.04	0.00	0.00	**3.75**

NOTE: This table displays the percent of the variance of the endogenous variables (rows) explained by the structural shocks in the model (columns) at business cycle frequency (HP-filtered variables with parameter $\lambda = 1600$).

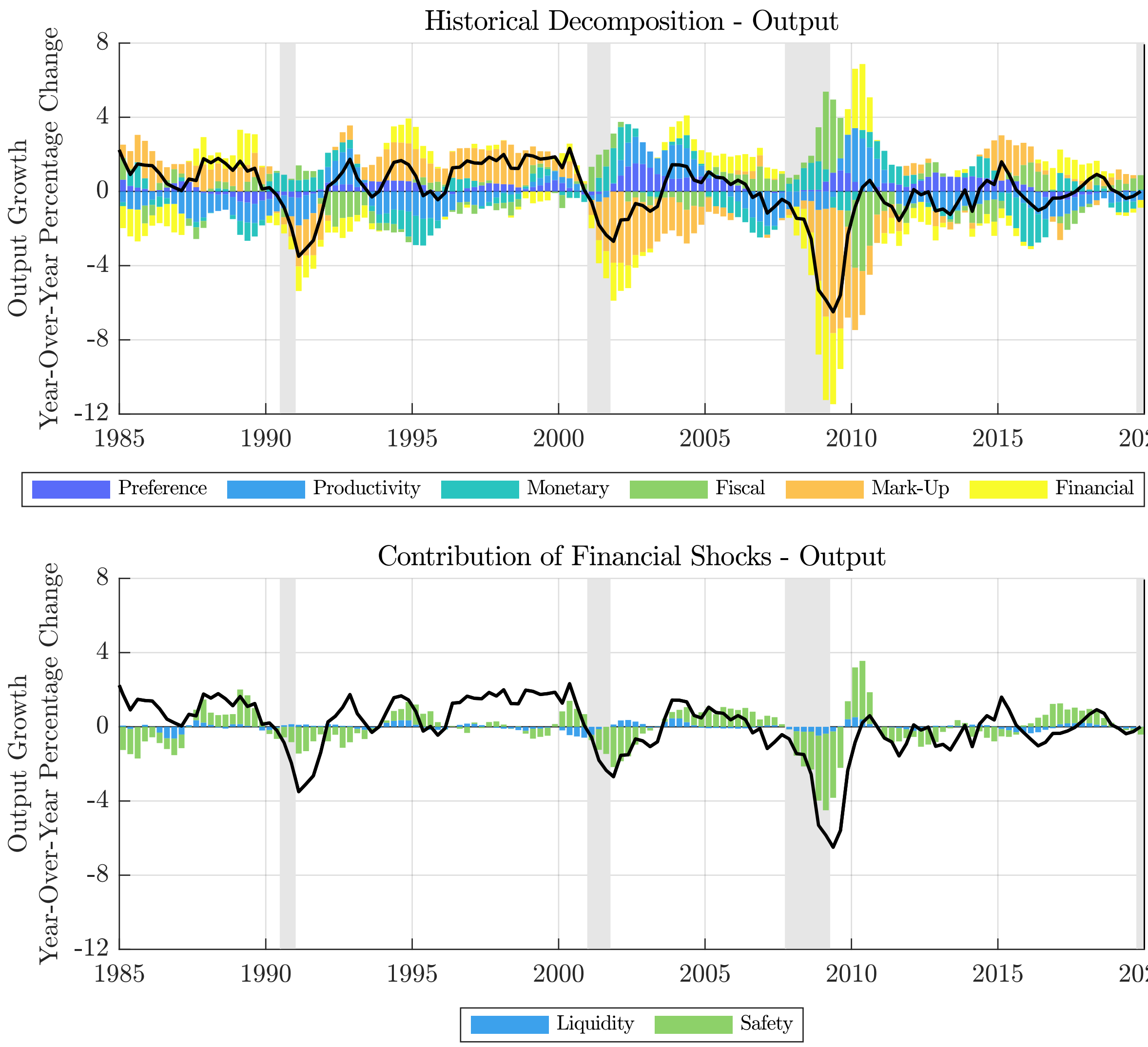
The rows of Table 3 report the variance decomposition of the main macroeconomic and financial variables in the model at business cycle frequency. Financial shocks (safety, $\varepsilon_{\psi t}$, and liquidity, $\varepsilon_{\omega t}$) account for about 15% of the variability of output and 25% of the variability of investment. In both cases, the safety shock is the dominant force while the contribution of pure liquidity shocks is almost negligible.

Perhaps not surprisingly, financial shocks explain a large fraction of the variance of spreads. For these spreads, liquidity shocks play a more important role compared to their contribution to real variables. For the convenience yield as a whole, safety shocks explain slightly more than 50% of the variance, liquidity shocks account for about 8%, and the remainder is largely explained by shock to the supply of public debt. Liquidity shocks are practically the sole driver of the AAA-10yT spread (r_t^{ω}), which Krishnamurthy and Vissing-Jorgensen (2012) indeed consider to be a measure of liquidity attributes. However, for the BAA-AAA spread (r_t^{ψ}) our decomposition shows that it is driven by a combination of shocks to the supply of public debt (around 40%), safety shocks (35%), and liquidity shocks (7.5%). This illustrates the relevance of a structural decomposition, as the BAA-AAA spread is typically considered to be a direct measure of safety attributes.

Finally, financial shocks strongly matter for the nominal interest rate (about 60%) and inflation (37%), but much less for consumption (only about 8%). This is in line with the primary role frictional financial intermediation plays in the funding of investment projects.

A historical decomposition of output, investment, and the convenience yield confirms the results of Table 3 and provides a range of further insights as Figures 2 - 4 illustrate. The top panel of Figure 2 plots the historical decomposition of real GDP growth in six shock groups: preference (dark blue), productivity (cyclical and trend, light blue), monetary policy (cyan), fiscal policy (spending and debt issuance, green), mark-ups (price and wage, orange), and financial (safety and liquidity, yellow). Not surprisingly, productivity and mark-up shocks are important drivers of real GDP growth over the cycle. Financial shocks are especially important during recessions and their aftermaths. The model attributes a large fraction of the persistent effects of recessions to financial disturbances.

Figure 2: Historical decomposition of real GDP.

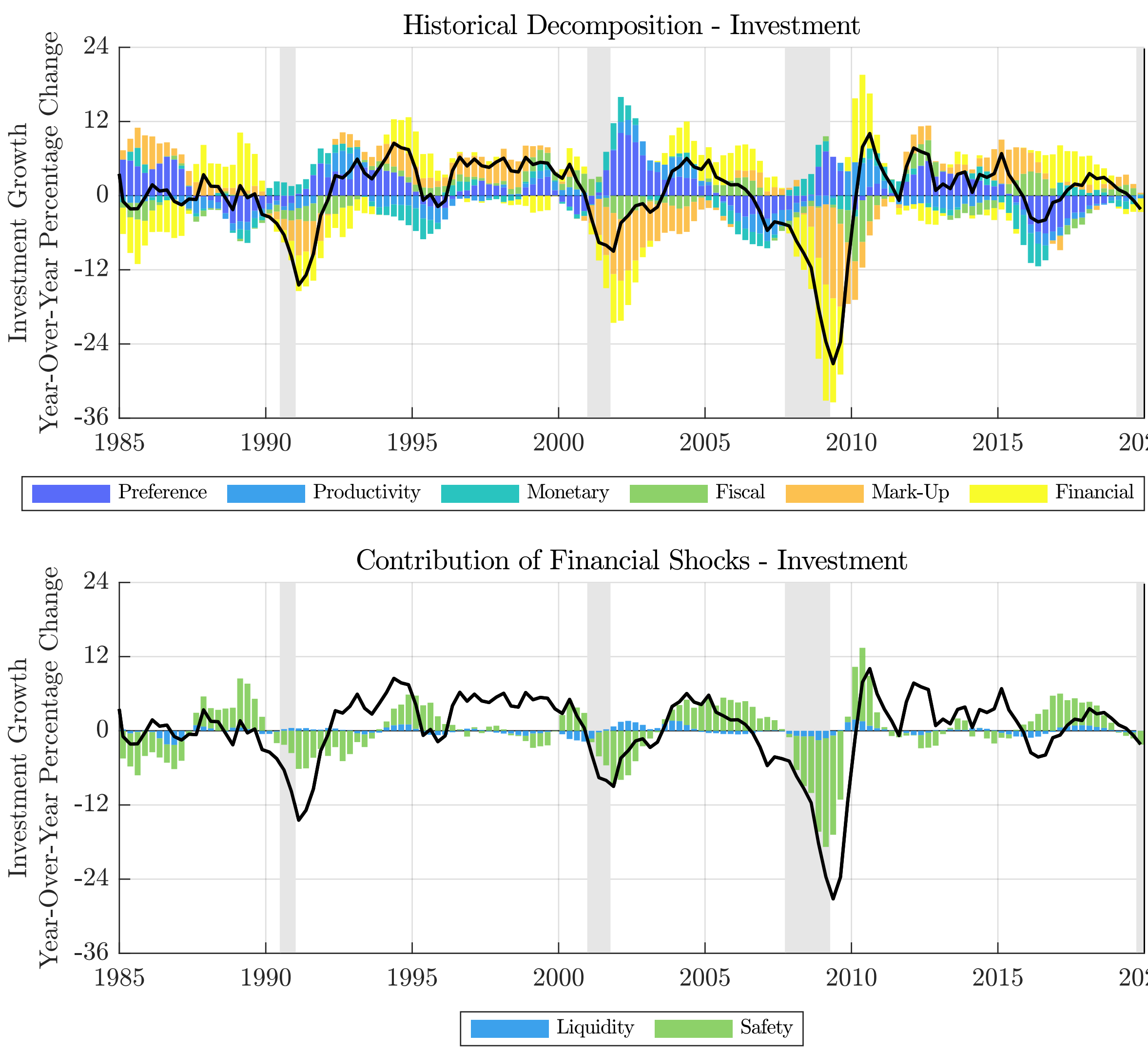


Note: The top panel plots the historical decomposition of real GDP growth. The bottom panel decomposes the contribution of financial shocks to real GDP growth in liquidity and safety shocks.

The bottom panel of Figure 2 decomposes financial shocks in its two components: liquidity (in blue) and safety (in green). It clearly suggests that safety shocks are the main source of volatility between the two, once again confirming some of the ideas seen above.

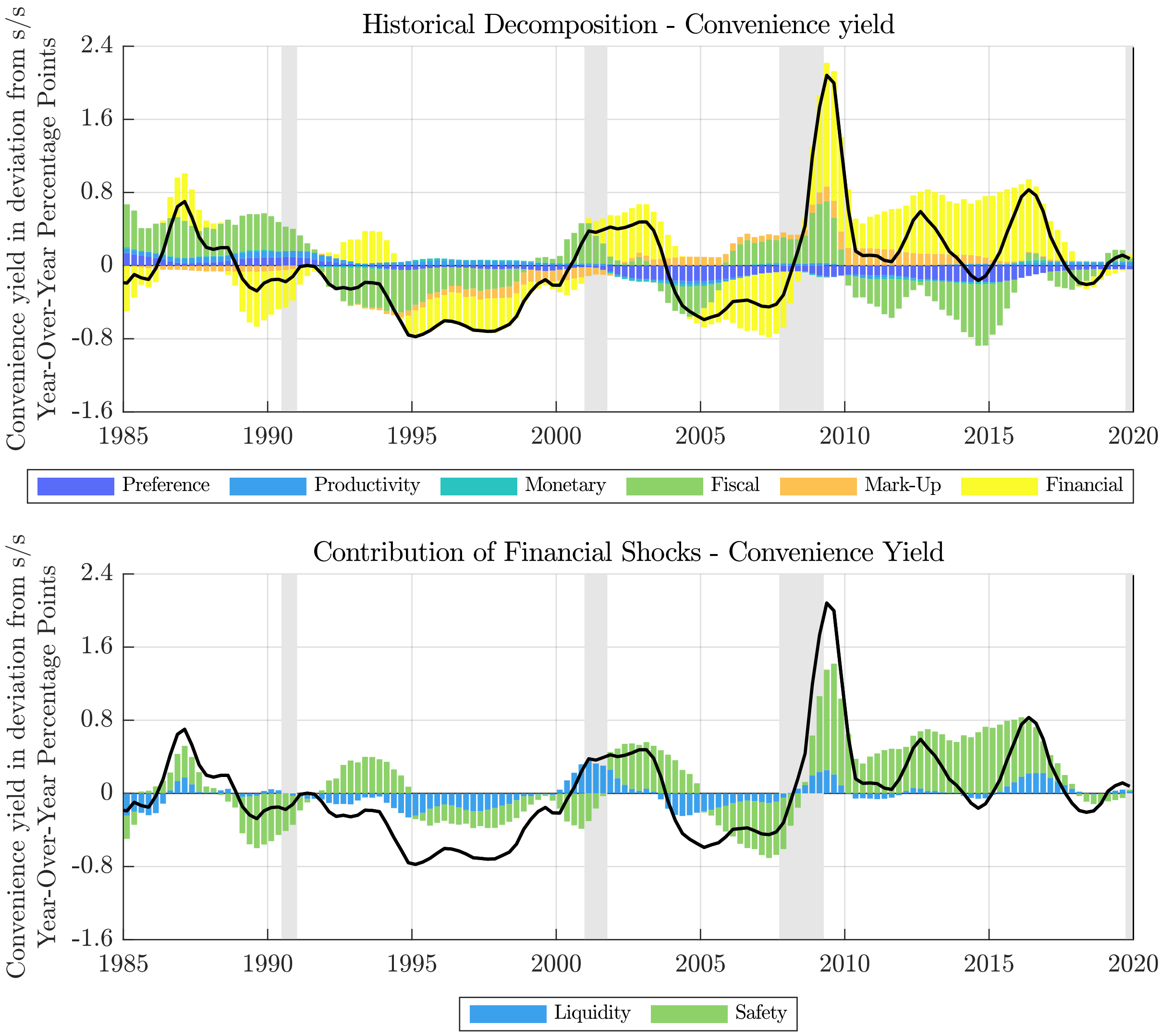
Investment is much more volatile than output. Figure 3 shows that financial shocks, and safety shocks in particular, affect real GDP primarily through investment during downturns. It is in these downturns, the 2007/08 Great Financial Crisis is a particularly stark example, that shocks to asset quality paired with asymmetric information are key in explaining fluctuations in investment and thereby output. In fact, our historical decomposition suggests that safety shocks might have—through an increase in the perceived quality of financial assets followed by an abrupt and stark deterioration—played a very important role both in the run-up to and at the height of the Great Financial Crisis.

Figure 3: Historical decomposition of real investment.



Note: The top panel plots the historical decomposition of real investment growth. The bottom panel decomposes the contribution of financial shocks to real investment growth in liquidity and safety shocks.

Figure 4: Historical decomposition of the BAA-10yT convenience yield.



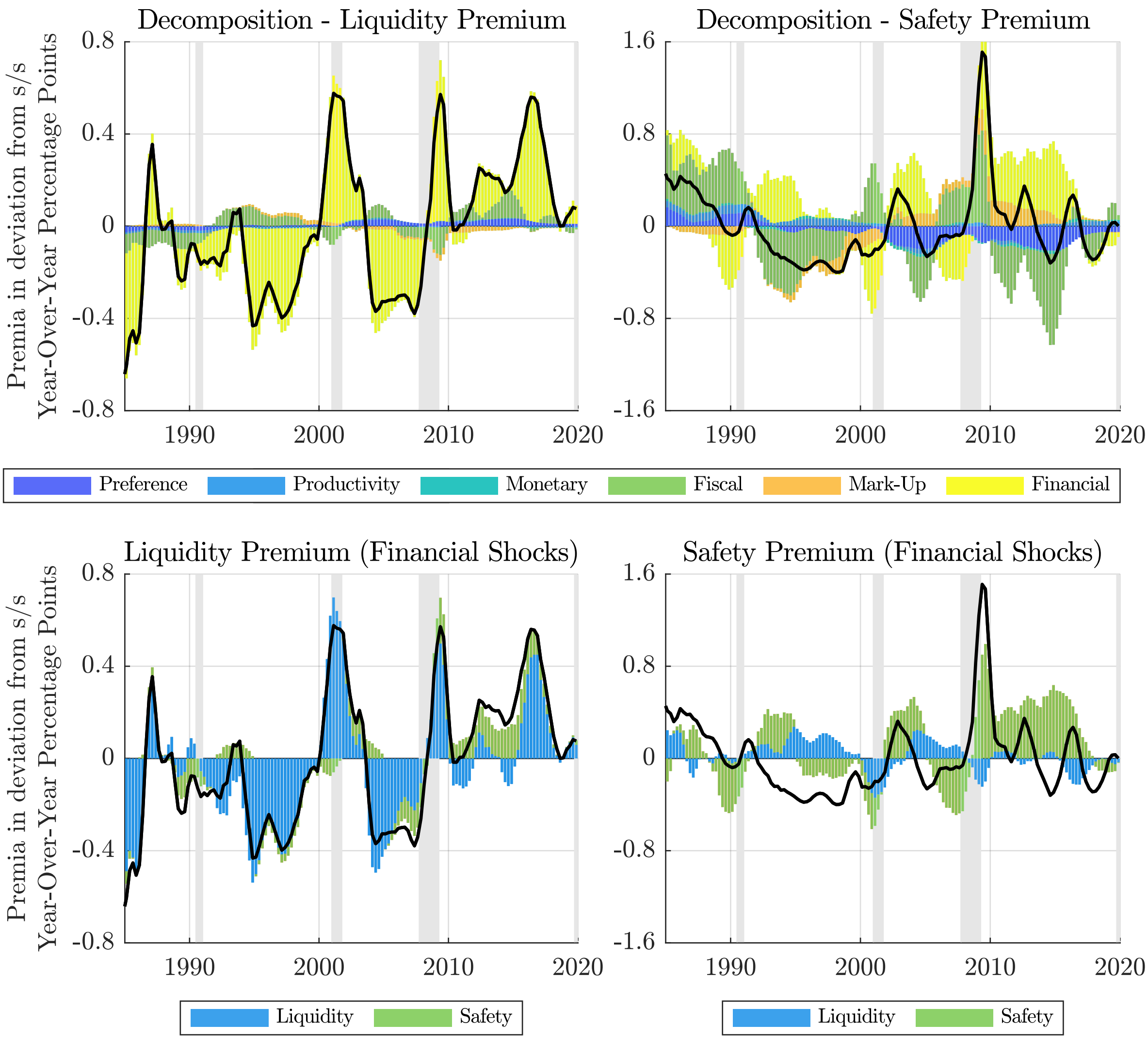
NOTE: The top panel plots the historical decomposition of the BAA-10yT convenience yield. The bottom panel decomposes the contribution of financial shocks to the convenience yield in liquidity and safety shocks.

The top panel of Figure 4 shows the historical decomposition of the BAA-10yT convenience yield. Financial and fiscal shocks are the most important in the decomposition. A tightening of financial conditions increases the convenience yield of safe relative to risky assets. Similarly, the convenience yield increases because of the relative scarcity of safe assets, especially in times of high demand such as recessions. Perhaps more surprising is the result in the bottom panel of Figure 4 (even though it is in line with previous findings). While safety shocks are almost solely responsible for the financial disturbances affecting real GDP and investment, liquidity shocks are relevant for the convenience yield.

The top-left panel of Figure 5 shows that financial shocks almost entirely account for the AAA-10yT spread (the liquidity premium). Conversely, fiscal shocks, particularly shocks to the supply of government bonds, drive the BAA-AAA spread (the safety premium).

The bottom panel of Figure 5 reinforces the earlier point about the importance of liquidity shocks for the convenience yield. Liquidity shocks are historically the sole driver of the AAA-10yT spread and also account for a significant fraction of the variation in the financial component of the safety premium.

Figure 5: Historical decomposition of the liquidity and safety premia.



NOTE: The top panel plots the historical decomposition of the AAA-10yT spread (left) and of the BAA-AAA spread (right). The bottom panel decomposes the contribution of financial shocks to the two spreads in liquidity and safety shocks.

Overall, the message that emerges from the variance and historical decomposition exercises is nuanced. Financial shocks are important for business cycle fluctuations of output and investment as well as of the convenience yield. However, while safety shocks are the key financial disturbances for real activity, directly affecting investment and propagating to output—suggesting an important for quality frictions in financial intermediation paired with asymmetric information—liquidity shocks are relevant for financial spreads.

4.2 Public liquidity, Fiscal Multipliers, and Liquidity Puzzle

To be finalized.

5 Conclusion

While there is a consensus in the literature on financial conditions being relevant drivers of the business cycle and early stages of 2007/08 Great Financial Crisis, in particular, being associated with an erosion of safety and a dry-up of liquidity, liquidity and safety are broad and interlinked concepts that are difficult to disentangle in practice. Yet, their distinction is critical for policy design, as we show. In this paper, we endogenize both the liquidity and safety of private assets in a medium-scale new-Keynesian with heterogeneous firms and two financial frictions on asset resaleability and asset quality paired with asymmetric information. Using U.S. macroeconomic and financial data, we estimate the model matching empirical liquidity and safety premia and identify their structural drivers in terms of shocks to the supply of safe assets, asset resaleability, and asset quality. Shocks to asset quality paired with asymmetric information turn out to explain a large share of fluctuations in investment and financial spreads over the business cycle. Further, we provide insights on the so-called liquidity puzzle and highlight fiscal-monetary interactions in the spirit of a financial channel of government debt issuance.

References

Ajello, A. (2016): "Financial Intermediation, Investment Dynamics,and Business Cycle Fluctuations," *American Economic Review*, 106, 2256–2303.

Ashcraft, A., N. Gârleanu, and L. H. Pedersen (2010): "Two Monetary Tools: Interest Rates and Haircuts," in *Macroeconomics Annual*, University of Chicago Press, vol. 25, 143–180.

Bayer, C., B. Born, and R. Luetticke (2023): "The Liquidity Channel of Fiscal Policy," *Journal of Monetary Economics*, 134, 86–117.

Becard, Y. and D. Gauthier (2022): "Collateral Shocks," *American Economic Journal: Macroeconomics*, 14, 83–103.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, 1, 1341–1393.

Bierdel, A., A. Drenik, J. Herreño, and P. Ottonello (2023): "Illiquid Lemon Markets and the Macroeconomy," *NBER Working Paper Series*, 31711 (September).

Bigio, S. (2015): "Endogenous Liquidity and the Business Cycle," *American Economic Review*, 105, 1883–1927.

Brunnermeier, M. K. and L. H. Pedersen (2009): "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, 22, 2201–2238.

Caballero, R. J. and E. Farhi (2018): "The Safety Trap," *Review of Economic Studies*, 85, 223–274.

Caballero, R. J., E. Farhi, and P. O. Gourinchas (2017): "The Safe Assets Shortage Conundrum," *Journal of Economic Perspectives*, 31, 29–46.

Calvo, G. A. (1983): "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.

Christiano, L. J., R. Motto, and M. Rostagno (2014): "Risk Shocks," *American Economic Review*, 104, 27–65.

Del Negro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki (2017): "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities," *American Economic Review*, 107, 824–857.

Dong, F. and Y. Wen (2017): "Flight to What? - Dissecting Liquidity Shortages in the Financial Crisis," *Unpublished Manuscript.*

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, 281–313.

Gorton, G. B. and A. Metrick (2010): "Haircuts," *Federal Reserve Bank of St. Louis Review*, 92, 507–519.

——— (2012): "Securitized banking and the run on repo," *Journal of Financial Economics*, 104, 425–451.

He, Z., A. Krishnamurthy, and K. Milbradt (2016): "What Makes US Government Bonds Safe Assets?" *American Economic Review*, 106, 519–523.

——— (2019): "A Model of Safe Asset Determination," *American Economic Review*, 109, 1230–1262.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010): "Investment Shocks and Business Cycles," *Journal of Monetary Economics*, 57, 132–145.

Kiyotaki, N. and J. Moore (2019): "Liquidity, Business Cycles, and Monetary Policy," *Journal of Political Economy*, 127, 2926–2966.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

Kurlat, P. (2013): "Lemons Markets and the Transmission of Aggregate Shocks," *American Economic Review*, 103, 1463–1489.

McAndrews, J., A. Sarkar, and Z. Wang (2017): "The Effect of the Term Auction Facility on the London Interbank Offered Rate," *Journal of Banking and Finance*, 83, 135–152.

Smets, F. and R. Wouters (2007): "Shocks and Frictions In US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

Taylor, J. B. and J. C. Williams (2009): "A Black Swan in the Money Market," *American Economic Journal: Macroeconomics*, 1, 58–83.

Wu, J. C. and F. D. Xia (2016): "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit, and Banking*, 48, 253–291.

A The Model: Additional Material

This appendix is divided into three sections. Section A.1 gives a complete list of the non-linear equilibrium conditions as derived in the paper plus all exogenous processes. Section A.2 restates the list of equilibrium conditions after detrending. Section A.3 describes the algorithm we use to solve for the deterministic steady state of the model calibrating selected parameters.

A.1 Equilibrium conditions and exogenous processes

Our model builds upon Dong and Wen (2017) and Del Negro et al. (2017). The features we introduce to the core model that comes with a twofold financial friction—highlighted in blue—include (i) Calvo-type nominal price and wage rigidities and monetary policy—in red—, (ii) consumption habits, investment adjustment costs, fixed cost in production, variable capital utilization, and fiscal policy—in green—, and (iii) two sources of stochastic trend growth and further exogenous processes used in the estimation of the model—in orange. With some additional definitions, the equilibrium can be summarized by 33 equations in 33 endogenous variables, $\{\ell_{ht}$, $\mathcal{M}_{t+1}$, λ_t, w_{ht}, Δ_{wt}, π_{wt}, p_{It}, y_{mt}, w_t, u_t, r_{Kt}, i_t, q_t, ϵ_t^*, ϵ_t^{**}, ψ_t^*, p_{Kt}, Π_t, p_{mt}, Δ_{pt}, R_t, τ_t, y_t, ℓ_t, c_t, k_t, δ_t, ω_t^*, r_{Kt}^*, r_t, cy_t, r_t^{ω}, $r_t^{\psi}\}$, and 10 exogenous processes, $\{\hat{\beta}_t, \hat{\chi}_{wt}, \hat{a}_t, \hat{\mu}_{zt}, \hat{\psi}_t, \hat{\omega}_t, \hat{\chi}_{pt}, \epsilon_{mt}, \hat{g}_t, \hat{b}_t\}$. Lowercase letters denote quantities, real prices, and real interest rates. Uppercase letters denote nominal prices and nominal interest rates.

Households $\{D_{it}, \ell_{ht}, \mathcal{M}_{t+1}, \lambda_t\}$

- Euler equations (one for each i)

$$1 = \mathbb{E}_t \left[\mathcal{M}_{t+1} \left(\frac{V_{it+1} + D_{it+1}}{V_{it}} \right) \right] \tag{$\square$}$$

- Labor supply

$$(1 - \tau_w) w_{ht} \lambda_t = \chi_t \ell_{ht}^{\xi} \tag{A.1}$$

- Stochastic discount factor (for nominal assets)

$$\mathcal{M}_{t+1} = \beta_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \tag{A.2}$$

- Marginal utility of consumption

$$(1 + \tau_p)\, \lambda_t = (c_t - \hbar c_{t-1})^{-1} - \beta_t \hbar \mathbb{E}_t\, (c_{t+1} - \hbar c_t)^{-1} \tag{A.3}$$

Labor unions $\{w_{ht}, \Delta_{wt}, \pi_{wt}\}$

- Wage Phillips curve

$$\left(\frac{\theta_w}{\theta_w - 1}\right)\frac{\mathcal{D}_{wt}}{\mathcal{F}_{wt}} = \left\{\frac{1 - \iota_w\left[\frac{\pi_{wt}\Pi_t}{(\mu_{z^*t-1})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_{t-1})^{\gamma_w}(\Pi_t^*)^{1-\gamma_w}}\right]^{\theta_w - 1}}{1 - \iota_w}\right\}^{\frac{1}{1-\theta_w}} \tag{A.4}$$

$$\text{where}\quad \mathcal{D}_{wt} \equiv \lambda_t \chi_{wt} w_{ht} \ell_t + \beta_t \iota_w \mathbb{E}_t\left\{\left[\frac{\pi_{wt+1}\Pi_{t+1}}{(\mu_{z^*t})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_t)^{\gamma_w}(\Pi_{t+1}^*)^{1-\gamma_w}}\right]^{\theta_w}\mathcal{D}_{wt+1}\right\},$$

$$\mathcal{F}_{wt} \equiv \lambda_t w_t \ell_t + \beta_t \iota_w \mathbb{E}_t\left\{\left[\frac{\pi_{wt+1}\Pi_{t+1}}{(\mu_{z^*t})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_t)^{\gamma_w}(\Pi_{t+1}^*)^{1-\gamma_w}}\right]^{\theta_w - 1}\mathcal{F}_{wt+1}\right\}.$$

- Wage dispersion

$$\Delta_{wt} = (1 - \iota_w)\left[\left(\frac{\theta_w}{\theta_w - 1}\right)\frac{\mathcal{D}_{wt}}{\mathcal{F}_{wt}}\right]^{-\theta_w} + \iota_w\left[\frac{\pi_{wt}\Pi_t}{(\mu_{z^*t-1})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_{t-1})^{\gamma_w}(\Pi_t^*)^{1-\gamma_w}}\right]^{\theta_w}\Delta_{wt-1} \tag{A.5}$$

- Wage inflation

$$\pi_{wt} = w_t / w_{t-1} \tag{A.6}$$

Investment goods firms $\{p_{It}\}$

- Price of investment goods

$$p_{It} = \left(\Upsilon^t \mu_{\Upsilon t}\right)^{-1}\left[1 + f\left(\frac{i_t}{i_{t-1}}\right) + f'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right] - \mathbb{E}_t\left\{\beta_t \frac{\lambda_{t+1}}{\lambda_t}\left(\Upsilon^{t+1}\mu_{\Upsilon t+1}\right)^{-1} f'\left(\frac{i_{t+1}}{i_t}\right)\left[\frac{i_{t+1}}{i_t}\right]^2\right\}, \tag{A.7}$$

$$\text{where}\quad f(x_t) \equiv \frac{1}{2}\left\{\exp\left[\sqrt{f''}(x_t - x)\right] + \exp\left[-\sqrt{f''}(x_t - x)\right] - 2\right\}.$$

Producers $\{y_{mt}, w_t, u_t, r_{Kt}, i_t, k_t, q_t, \epsilon_t^*, \epsilon_t^{**}, \psi_t^*, p_{K,t}, \Pi_t\}$

- Production

$$y_{mt} = a_t (u_t k_{t-1})^\alpha (z_t \ell_t)^{1-\alpha} \tag{A.8}$$

- Labor demand

$$w_t = (1 - \alpha)\, p_{mt} y_{mt} / \ell_t \tag{A.9}$$

- Capital utilization

$$s'(u_t) = \alpha p_{mt} y_{mt} / (u_t k_{t-1}), \tag{A.10}$$

$$\text{where}\quad s(u_t) \equiv r_K \{\exp[\sigma_s(u_t - 1)] - 1\} / \sigma_s.$$

- Return on capital

$$r_{Kt} = \alpha p_{mt} y_{mt}/k_{t-1} - s(u_t) \tag{A.11}$$

- Investment

$$i_t = \left\{(R_{t-1}/\Pi_t)b_{t-1} + [r_{Kt} + \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)p_{Kt}]k_{t-1}\right\} \frac{1-F(\epsilon_t^*)}{p_{It}} \\ + \bar{\omega}_t(1-\delta_t)p_{Kt}k_{t-1}\frac{1-F(\epsilon_t^{**})}{p_{It}} \tag{A.12}$$

- Capital stock

$$k_t = (1-\delta_t)k_{t-1} + \frac{(R_{t-1}/\Pi_t)b_{t-1} + [r_{Kt} + \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)p_{Kt}]k_{t-1}}{p_{It}} \\ \int_{\epsilon_t^*}^{\epsilon_{\max}} \epsilon_t dF(\epsilon) + \frac{\bar{\omega}_t(1-\delta_t)p_{Kt}k_{t-1}}{p_{It}} \int_{\epsilon_t^{**}}^{\epsilon_{\max}} \epsilon_t dF(\epsilon) \tag{A.13}$$

- Shadow value of capital

$$q_t = \mathbb{E}_t\left\{\beta_t \frac{\lambda_{t+1}}{\lambda_t}\left[1 + \int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1\right) dF(\epsilon)\right]\left[r_{Kt+1} + \bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}\right]\right\} \\ + \mathbb{E}_t\left\{\beta_t \frac{\lambda_{t+1}}{\lambda_t}\left[1 + \int_{\epsilon_{t+1}^{**}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^{**}} - 1\right) dF(\epsilon)\right]\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}\right\} \\ + \mathbb{E}_t\left\{\beta_t \frac{\lambda_{t+1}}{\lambda_t}(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}\right\} \tag{A.14}$$

- Investment cut-off

$$\epsilon_t^* = \frac{p_{It}}{q_t} \tag{A.15}$$

- Good-quality capital sale cut-off

$$\epsilon_t^{**} = \frac{p_{It}}{p_{Kt}} \tag{A.16}$$

- Endogenous asset quality in the market

$$\psi_t^* = \frac{(1-\delta_t)[1-F(\epsilon_t^{**})]}{(1-\delta_t)[1-F(\epsilon_t^{**})] + (1-\bar{\psi}_t)(1-\gamma)} \tag{A.17}$$

- Market price of capital

$$p_{Kt} = \psi_t^* q_t \tag{A.18}$$

- Return on government bonds

$$R_t = \mathbb{E}_t\left\{\beta_t \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\Pi_{t+1}}\left[1 + \int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1\right) dF(\epsilon)\right]\right\}^{-1} \tag{A.19}$$

Final goods firms $\{p_{mt}, \Delta_{pt}\}$

- Price Phillips curve

$$\left(\frac{\theta_p}{\theta_p-1}\right)\frac{\mathcal{D}_{pt}}{\mathcal{F}_{pt}} = \left\{\frac{1-\iota_p\left[\frac{\Pi_t}{(\Pi_{t-1})^{\gamma_p}(\Pi_t^*)^{1-\gamma_p}}\right]^{\theta_p-1}}{1-\iota_p}\right\}^{\frac{1}{1-\theta_p}}. \tag{A.20}$$

$$\text{where} \quad \mathcal{D}_{pt} \equiv \lambda_t \chi_{pt} p_{mt} y_t + \beta_t \iota_p \mathbb{E}_t \left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\gamma_p}(\Pi_{t+1}^*)^{1-\gamma_p}}\right]^{\theta_p} \mathcal{D}_{pt+1}\right\},$$

$$\mathcal{F}_{pt} \equiv \lambda_t y_t + \beta_t \iota_p \mathbb{E}_t \left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\gamma_p}(\Pi_{t+1}^*)^{1-\gamma_p}}\right]^{\theta_p-1} \mathcal{F}_{pt+1}\right\}.$$

- Price dispersion

$$\Delta_{pt} = (1-\iota_p)\left[\left(\frac{\theta_p}{\theta_p-1}\right)\frac{\mathcal{D}_{pt}}{\mathcal{F}_{pt}}\right]^{-\theta_p} + \iota_p\left[\frac{\Pi_t}{(\Pi_{t-1})^{\gamma_p}(\Pi_t^*)^{1-\gamma_p}}\right]^{\theta_p}\Delta_{pt-1} \tag{A.21}$$

Monetary and fiscal policy $\{R_t, \tau_t\}$

- Interest rate rule

$$R_t = \max\left\{R_{t-1}^{\rho_m}\left[R\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\left(\frac{y_t/y_{t-1}}{\mu_{z^*}}\right)^{\phi_y}\right]^{1-\rho_m}\exp\left(\varepsilon_{mt}\right), 1\right\} \tag{A.22}$$

- Government budget constraint

$$\tau_t = (R_{t-1}/\Pi_t)b_{t-1} - b_t + g_t - \tau_w w_t \ell_t - \tau_p c_t - \tau_r r_{Kt} k_{t-1} \tag{A.23}$$

General equilibrium $\{y_t, \ell_t, c_t\}$

- Aggregate output

$$y_t = y_{mt}/\Delta_{pt} \tag{A.24}$$

- Aggregate labor

$$\ell_t = \ell_{ht}/\Delta_{wt} \tag{A.25}$$

- Aggregate resource constraint

$$c_t = y_t - g_t - i_t\left(1+f\left(\frac{i_t}{i_{t-1}}\right)\right)/\left(\Upsilon^t \mu_{\Upsilon t}\right) - s(u_t)k_{t-1} - z_t^*\varphi \tag{A.26}$$

Further definitions $\{\delta_t, \omega_t^*, r_{Kt}^*, r_t, cy_t, r_t^\omega, r_t^\psi\}$

- Average survival rate of capital

$$1 - \delta_t \equiv (1-\gamma)\,\bar{\psi}_t \tag{A.27}$$

- Endogenous asset resaleability in the market

$$\omega_t^* \equiv \left[\left(1-\bar{\psi}_t\right)(1-\gamma) + (1-\delta_t)\,\frac{1-F\left(\epsilon_t^{**}\right)}{1-F\left(\epsilon_t^{*}\right)}\right]\bar{\omega}_t \tag{A.28}$$

- Total return on capital

$$r_{Kt}^* \equiv \frac{r_{Kt} + (1-\delta_t)q_t}{q_{t-1}} \tag{A.29}$$

- Real return on government bonds

$$r_t \equiv R_{t-1}/\Pi_t \tag{A.30}$$

- Convenience yield

$$cy_t \equiv \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1 + \int_{\epsilon_{t+1}^*}^{\epsilon_{t+1}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1\right)dF(\epsilon)\right]\left(r_{Kt+1}^* - r_{t+1}\right)\right\} \tag{A.31}$$

- Liquidity premium

$$r_t^\omega \equiv \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1\right)dF(\epsilon)\right]\frac{(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}}{q_t}\right\} \tag{A.32}$$

- Safety premium

$$\begin{aligned} r_t^\psi \equiv\ & \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1\right)dF(\epsilon) - \int_{\epsilon_{t+1}^{**}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^{**}} - 1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}}{q_t}\right\} \\ & - \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\lambda_t}\left[1 + \int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}}{q_t}\right\} \end{aligned} \tag{A.33}$$

Exogenous processes $\{\hat{\beta}_t, \hat{\chi}_{wt}, \hat{a}_t, \hat{\mu}_{zt}, \hat{\psi}_t, \hat{\omega}_t, \hat{\chi}_{pt}, \epsilon_{mt}, \hat{g}_t, \hat{b}_t\}$

- Preference

$$\hat{\beta}_t = \rho_\beta \hat{\beta}_{t-1} + \varepsilon_{\beta t} \tag{S.1}$$

where $\hat{\beta}_t \equiv \ln(\beta_t/\beta)$, $\rho_\beta \in (0,1)$, and $\varepsilon_{\beta t} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_\beta^2)$.

- Wage cost-push shock

$$\hat{\chi}_{wt} = \rho_{\chi w} \hat{\chi}_{wt-1} + \varepsilon_{\chi wt}, \tag{S.2}$$

where $\hat{\chi}_{wt} \equiv \ln(\chi_{wt}/\chi_w)$, $\rho_{\chi w} \in (0,1)$, and $\varepsilon_{\chi wt} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_{\chi w}^2)$.

- Productivity, cyclical component

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{at} \tag{S.3}$$

where $\hat{a}_t \equiv \ln(a_t/a)$, $\rho_a \in (0,1)$, and $\varepsilon_{at} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_a^2)$.

- Productivity, trend component

$$\hat{\mu}_{zt} = \rho_z \hat{\mu}_{zt-1} + \varepsilon_{zt}, \tag{S.4}$$

where $\mu_{zt} \equiv z_t/z_{t-1}$, the trend rise in productivity with $\mu_z = \exp(\Gamma)$, will be the second source of growth in the model, and $\hat{\mu}_{zt} \equiv \ln(\mu_{zt}/\mu_z)$, $\rho_z \in (0,1)$, and $\varepsilon_{zt} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_z^2)$.

- Capital quality

$$\hat{\psi}_t = \rho_\psi \hat{\psi}_{t-1} + \varepsilon_{\psi t}, \tag{S.5}$$

where $\hat{\psi}_t \equiv \ln(\bar{\psi}_t/\bar{\psi})$, $\rho_\psi \in (0,1)$, and $\varepsilon_{\psi t} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_\psi^2)$.

- Capital resaleability

$$\hat{\omega}_t = \rho_\omega \hat{\omega}_{t-1} + \varepsilon_{\omega t}, \tag{S.6}$$

where $\hat{\omega}_t \equiv \ln(\bar{\omega}_t/\bar{\omega})$, $\rho_\omega \in (0,1)$, and $\varepsilon_{\omega t} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_\omega^2)$.

- Price cost-push shock

$$\hat{\chi}_{pt} = \rho_{\chi p} \hat{\chi}_{pt-1} + \varepsilon_{\chi pt}, \tag{S.7}$$

where $\hat{\chi}_{pt} \equiv \ln(\chi_{pt}/\chi_p)$, $\rho_{\chi p} \in (0,1)$, and $\varepsilon_{\chi pt} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_{\chi p}^2)$.

- Monetary policy shock

$$\varepsilon_{mt} \sim \textit{ i.i.d. } \mathcal{N}(0, \sigma_m^2) \tag{S.8}$$

- Government spending

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{gt}, \tag{S.9}$$

where $\hat{g}_t \equiv \ln[(g_t/y_t)/(g/y)]$, $\rho_g \in (0,1)$, and $\varepsilon_{gt} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_g^2)$.

- Government borrowing

$$\hat{b}_t = \rho_b \hat{b}_{t-1} + \varepsilon_{bt}, \tag{S.10}$$

where $\hat{b}_t \equiv \ln[(b_t/y_t)/(b/y)]$, $\rho_b \in (0,1)$, and $\varepsilon_{bt} \sim$ *i.i.d.* $\mathcal{N}(0, \sigma_b^2)$.

A.2 Stationary equilibrium conditions

To solve the model outlined above, we stationarize it. With two non-stationary exogenous process —investment specific technology and labor-augmenting productivity— the growth rate in the model can be defined $\mu_{z_t^*} = z_t/z_{t-1}$, where $z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}$.

We rescale $\{w_{ht}, y_{mt}, w_t, y_t, c_t, g_t, \tau_t, b_t\}$ such that $x_t = x_t/z_t^*$. Investment and capital holdings, $\{i_t, k_t\}$, are rescaled following $x_t = x_t/\left(z_t^* \Upsilon^t\right)$. This means the two state variables in the model, $\{k_t, b_t\}$, are rescaled with the level of productivity prevalent when the stock of capital/ debt is being determined/ issued (in contrast to when it is used/ repaid). This timing assumption is in line with our definition of real bond holdings $b_t = B_t/P_t$ and aligns with a 'time-to-build' notion for capital. Given the trend in investment specific technology, the prices associated with capital formation, $\{p_{It}, p_{Kt}, q_t, r_{Kt}\}$, are further rescaled according to $x_t = x_t \Upsilon^t$. Finally, detrended marginal utility of consumption is $\lambda_t = \lambda_t z_t^*$.

Households $\{D_{it}, \ell_{ht}, \mathcal{M}_{t+1}, \lambda_t\}$

- Euler equations (one for each i)

$$1 = \mathbb{E}_t \left[\mathcal{M}_{t+1} \left(\frac{V_{it+1} + D_{it+1}}{V_{it}} \right) \right] \tag{$\square$}$$

- Labor supply

$$(1 - \tau_w) w_{ht} \lambda_t = \chi_t \ell_{ht}^{\xi} \tag{B.1}$$

- Stochastic discount factor (for nominal assets)

$$\mathcal{M}_{t+1} = \beta_t \frac{\lambda_{t+1}}{\mu_{z^*t+1} \lambda_t} \frac{1}{\Pi_{t+1}} \tag{B.2}$$

- Marginal utility of consumption

$$(1 + \tau_p) \lambda_t = \left[c_t - \hbar \left(c_{t-1}/\mu_{z^*t} \right) \right]^{-1} - \beta_t \hbar \mathbb{E}_t \left(\mu_{z^*t+1} c_{t+1} - \hbar c_t \right)^{-1} \tag{B.3}$$

Labor unions $\{w_{ht}, \Delta_{wt}, \pi_{wt}\}$

- Wage Phillips curve

$$\left(\frac{\theta_w}{\theta_w - 1} \right) \frac{\mathcal{D}_{wt}}{\mathcal{F}_{wt}} = \left\{ \frac{1 - \iota_w \left[\frac{\pi_{wt} \Pi_t}{(\mu_{z^*t-1})^{\gamma_\mu} (\mu_{z^*})^{1-\gamma_\mu} (\Pi_{t-1})^{\gamma_w} (\Pi_t^*)^{1-\gamma_w}} \right]^{\theta_w - 1}}{1 - \iota_w} \right\}^{\frac{1}{1-\theta_w}} \tag{B.4}$$

$$\text{where} \quad \mathcal{D}_{wt} \equiv \lambda_t \chi_{wt} w_{ht} \ell_t + \beta_t \iota_w \mathbb{E}_t \left\{ \left[\frac{\pi_{wt+1} \Pi_{t+1}}{(\mu_{z^*t})^{\gamma_\mu} (\mu_{z^*})^{1-\gamma_\mu} (\Pi_t)^{\gamma_w} (\Pi_{t+1}^*)^{1-\gamma_w}} \right]^{\theta_w} \mathcal{D}_{wt+1} \right\},$$

$$\mathcal{F}_{wt} \equiv \lambda_t w_t \ell_t + \beta_t \iota_w \mathbb{E}_t \left\{ \left[\frac{\pi_{wt+1} \Pi_{t+1}}{(\mu_{z^*t})^{\gamma_\mu} (\mu_{z^*})^{1-\gamma_\mu} (\Pi_t)^{\gamma_w} (\Pi_{t+1}^*)^{1-\gamma_w}} \right]^{\theta_w - 1} \mathcal{F}_{wt+1} \right\}.$$

- Wage dispersion

$$\Delta_{wt} = (1-\iota_w)\left[\left(\frac{\theta_w}{\theta_w - 1}\right)\frac{\mathcal{D}_{wt}}{\mathcal{F}_{wt}}\right]^{-\theta_w} + \iota_w\left[\frac{\pi_{wt}\Pi_t}{(\mu_{z^*t-1})^{\gamma_\mu}(\mu_{z^*})^{1-\gamma_\mu}(\Pi_{t-1})^{\gamma_w}(\Pi_t^*)^{1-\gamma_w}}\right]^{\theta_w}\Delta_{wt-1} \quad \text{(B.5)}$$

- Wage inflation

$$\pi_{wt} = \mu_{z^*t}(w_t/w_{t-1}) \quad \text{(B.6)}$$

Investment goods firms $\{p_{It}\}$

- Price of investment goods

$$p_{It} = \mu_{\Upsilon t}^{-1}\left[1 + f\left(\frac{\mu_{z^*t}\Upsilon i_t}{i_{t-1}}\right) + f'\left(\frac{\mu_{z^*t}\Upsilon i_t}{i_{t-1}}\right)\frac{\mu_{z^*t}\Upsilon i_t}{i_{t-1}}\right] - \mathbb{E}_t\left\{\beta_t\mu_{z^*t+1}\Upsilon\mu_{\Upsilon t+1}^{-1}\frac{\lambda_{t+1}}{\lambda_t}f'\left(\frac{\mu_{z^*t+1}\Upsilon i_{t+1}}{i_t}\right)\left[\frac{i_{t+1}}{i_t}\right]^2\right\}, \quad \text{(B.7)}$$

$$\text{where} \quad f(x_t) \equiv \frac{1}{2}\left\{\exp\left[\sqrt{f''}\,(x_t - x)\right] + \exp\left[-\sqrt{f''}\,(x_t - x)\right] - 2\right\}.$$

Producers $\{y_{mt}, w_t, u_t, r_{Kt}, i_t, k_t, q_t, \epsilon_t^*, \epsilon_t^{**}, \psi_t^*, p_{K,t}, \Pi_t\}$

- Production

$$y_{mt} = a_t\left(u_t k_{t-1}/\mu_{z^*t}\Upsilon\right)^{\alpha}\ell_t^{1-\alpha} - \varphi \quad \text{(B.8)}$$

- Labor demand

$$w_t = (1-\alpha)\,p_{mt}y_{mt}/\ell_t \quad \text{(B.9)}$$

- Capital utilization

$$s'(u_t) = \alpha p_{mt}y_{mt}/\left(u_t k_{t-1}/\mu_{z^*t}\Upsilon\right), \quad \text{(B.10)}$$

$$\text{where} \quad s(u_t) \equiv r_K\left\{\exp\left[\sigma_s(u_t - 1)\right] - 1\right\}/\sigma_s.$$

- Return on capital

$$r_{Kt} = \alpha p_{mt}y_{mt}/\left(k_{t-1}/\mu_{z^*t}\Upsilon\right) - s(u_t) \quad \text{(B.11)}$$

- Investment

$$i_t = \left\{\left\{(R_{t-1}/\Pi_t)b_{t-1} + [r_{Kt} + \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)p_{Kt}]k_{t-1}\right\}\frac{1-F(\epsilon_t^*)}{p_{It}} + \bar{\omega}_t(1-\delta_t)p_{Kt}k_{t-1}\frac{1-F(\epsilon_t^{**})}{p_{It}}\right\}/\left(\mu_{z^*t}\Upsilon\right) \quad \text{(B.12)}$$

- Capital stock

$$k_t = \Bigg\{ (1-\delta_t)k_{t-1} + \frac{(R_{t-1}/\Pi_t)b_{t-1} + [r_{Kt} + \bar{\omega}_t(1-\bar{\psi}_t)(1-\gamma)p_{Kt}]k_{t-1}}{p_{It}} \int_{\epsilon_t^*}^{\epsilon_{\max}} \epsilon_t dF(\epsilon) + \frac{\bar{\omega}_t(1-\delta_t)p_{Kt}k_{t-1}}{p_{It}} \int_{\epsilon_t^{**}}^{\epsilon_{\max}} \epsilon_t dF(\epsilon) \Bigg\} / \left(\mu_{z^*t}\Upsilon\right) \quad \text{(B.13)}$$

- Shadow value of capital

$$q_t = \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\mu_{z^*t+1}\Upsilon\lambda_t} \left[1 + \int_{\epsilon_{t+1}^*}^{\epsilon_{\max}} \left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1 \right) dF(\epsilon) \right] [r_{Kt+1} + \bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}] \right\}$$
$$+ \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\mu_{z^*t+1}\Upsilon\lambda_t} \left[1 + \int_{\epsilon_{t+1}^{**}}^{\epsilon_{\max}} \left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^{**}} - 1 \right) dF(\epsilon) \right] \bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1} \right\}$$
$$+ \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\mu_{z^*t+1}\Upsilon\lambda_t} (1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1} \right\} \quad \text{(B.14)}$$

- Investment cut-off

$$\epsilon_t^* = \frac{p_{It}}{q_t} \quad \text{(B.15)}$$

- Good-quality capital sale cut-off

$$\epsilon_t^{**} = \frac{p_{It}}{p_{Kt}} \quad \text{(B.16)}$$

- Endogenous asset quality in the market

$$\psi_t^* = \frac{(1-\delta_t)[1-F(\epsilon_t^{**})]}{(1-\delta_t)[1-F(\epsilon_t^{**})] + (1-\bar{\psi}_t)(1-\gamma)} \quad \text{(B.17)}$$

- Market price of capital

$$p_{Kt} = \psi_t^* q_t \quad \text{(B.18)}$$

- Return on government bonds

$$R_t = \mathbb{E}_t \left\{ \beta_t \frac{\lambda_{t+1}}{\mu_{z^*t+1}\lambda_t} \frac{1}{\Pi_{t+1}} \left[1 + \int_{\epsilon_{t+1}^*}^{\epsilon_{\max}} \left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*} - 1 \right) dF(\epsilon) \right] \right\}^{-1} \quad \text{(B.19)}$$

Final goods firms $\{p_{mt}, \Delta_{pt}\}$

- Price Phillips curve

$$\left(\frac{\theta_p}{\theta_p-1}\right)\frac{\mathcal{D}_{pt}}{\mathcal{F}_{pt}} = \left\{\frac{1-\iota_p\left[\frac{\Pi_t}{(\Pi_{t-1})^{\gamma_p}(\Pi_t^*)^{1-\gamma_p}}\right]^{\theta_p-1}}{1-\iota_p}\right\}^{\frac{1}{1-\theta_p}}. \tag{B.20}$$

$$\text{where}\quad \mathcal{D}_{pt} \equiv \lambda_t\chi_{pt}p_{mt}y_t + \beta_t\iota_p\mathbb{E}_t\left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\gamma_p}(\Pi_{t+1}^*)^{1-\gamma_p}}\right]^{\theta_p}\mathcal{D}_{pt+1}\right\},$$

$$\mathcal{F}_{pt} \equiv \lambda_t y_t + \beta_t\iota_p\mathbb{E}_t\left\{\left[\frac{\Pi_{t+1}}{(\Pi_t)^{\gamma_p}(\Pi_{t+1}^*)^{1-\gamma_p}}\right]^{\theta_p-1}\mathcal{F}_{pt+1}\right\}.$$

- Price dispersion

$$\Delta_{pt} = (1-\iota_p)\left[\left(\frac{\theta_p}{\theta_p-1}\right)\frac{\mathcal{D}_{pt}}{\mathcal{F}_{pt}}\right]^{-\theta_p} + \iota_p\left[\frac{\Pi_t}{(\Pi_{t-1})^{\gamma_p}(\Pi_t^*)^{1-\gamma_p}}\right]^{\theta_p}\Delta_{pt-1} \tag{B.21}$$

Monetary and fiscal policy $\{R_t, \tau_t\}$

- Interest rate rule

$$R_t = \max\left\{R_{t-1}^{\rho_m}\left[R\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\left(\frac{\mu_{z^*t}(y_t/y_{t-1})}{\mu_{z^*}}\right)^{\phi_y}\right]^{1-\rho_m}\exp\left(\varepsilon_{mt}\right), 1\right\} \tag{B.22}$$

- Government budget constraint

$$\tau_t = (R_{t-1}/\Pi_t)\left(b_{t-1}/\mu_{z^*t}\right) - b_t + g_t - \tau_w w_t\ell_t - \tau_p c_t - \tau_r r_{Kt}k_{t-1}/\left(\mu_{z^*t}\Upsilon\right) \tag{B.23}$$

General equilibrium $\{y_t, \ell_t, c_t\}$

- Aggregate output

$$y_t = y_{mt}/\Delta_{pt}, \tag{B.24}$$

- Aggregate labor

$$\ell_t = \ell_{ht}/\Delta_{wt}, \tag{B.25}$$

- Aggregate resource constraint

$$c_t = y_t - g_t - (i_t/\mu_{\Upsilon t})\left(1 + f\left(\frac{\mu_{z^*t}\Upsilon i_t}{i_{t-1}}\right)\right) - s(u_t)k_{t-1}/\left(\mu_{z^*t}\Upsilon\right) - \varphi \tag{B.26}$$

Further definitions $\{\delta_t, \omega_t^*, r_{Kt}^*, r_t, cy_t, r_t^\omega, r_t^\psi\}$

- Average survival rate of capital

$$1-\delta_t \equiv (1-\gamma)\,\bar{\psi}_t \tag{B.27}$$

- Endogenous asset resaleability in the market

$$\omega_t^* \equiv \left[\left(1-\bar{\psi}_t\right)(1-\gamma)+(1-\delta_t)\,\frac{1-F\left(\epsilon_t^{**}\right)}{1-F\left(\epsilon_t^{*}\right)}\right]\bar{\omega}_t \tag{B.28}$$

- Total return on capital

$$r_{Kt}^* \equiv \frac{r_{Kt}+(1-\delta_t)q_t}{\Upsilon q_{t-1}} \tag{B.29}$$

- Real return on government bonds

$$r_t \equiv R_{t-1}/\Pi_t \tag{B.30}$$

- Convenience yield

$$cy_t \equiv \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\mu_{z^*t+1}\lambda_t}\left[1+\int_{\epsilon_{t+1}^*}^{\epsilon_{t+1}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*}-1\right)dF(\epsilon)\right]\left(r_{Kt+1}^*-r_{t+1}\right)\right\} \tag{B.31}$$

- Liquidity premium

$$r_t^\omega \equiv \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\mu_{z^*t+1}\lambda_t}\left[\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*}-1\right)dF(\epsilon)\right]\frac{(1-\bar{\omega}_{t+1})(1-\delta_{t+1})q_{t+1}}{\Upsilon q_t}\right\} \tag{B.32}$$

- Safety premium

$$\begin{aligned} r_t^\psi \equiv\ & \mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\mu_{z^*t+1}\lambda_t}\left[\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*}-1\right)dF(\epsilon)-\int_{\epsilon_{t+1}^{**}}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^{**}}-1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\delta_{t+1})q_{t+1}}{\Upsilon q_t}\right\} \\ & -\mathbb{E}_t\left\{\beta_t\frac{\lambda_{t+1}}{\mu_{z^*t+1}\lambda_t}\left[1+\int_{\epsilon_{t+1}^*}^{\epsilon_{\max}}\left(\frac{\epsilon_{t+1}}{\epsilon_{t+1}^*}-1\right)dF(\epsilon)\right]\frac{\bar{\omega}_{t+1}(1-\bar{\psi}_{t+1})(1-\gamma)p_{Kt+1}}{\Upsilon q_t}\right\} \end{aligned} \tag{B.33}$$

A.3 Deterministic steady state

This Section describes the algorithm used to solve for the deterministic steady state of the detrended model. To simplify the exposition, both the color coding and equation names align with the list of equilibrium conditions in sections A.1 and A.2.

[1] Calibration targets

Set steady state values for the five endogenous variables directly pinned down by the calibration targets. The fourth and fifth target also implicitly determine the steady state convenience yield. An additional sixth target on investment frequency, $i^{\text{freq}}_{\text{TARGET}}$, is used to pin down the parameters of the Pareto distribution of idiosyncratic investment efficiency further below.

- Target 1: labor supply (to pin down χ)

$$l = l_{\text{TARGET}} \qquad (\square)$$

- Target 2: nominal interest rate (to pin down β)

$$R = R_{\text{TARGET}} \qquad (\square)$$

- Target 3: average death rate of capital (to pin down γ)

$$\delta = \delta_{\text{TARGET}} \qquad (\square)$$

- Target 4: Liquidity premium (to pin down $\bar{\omega}$)

$$r^{\omega} = r^{\omega}_{\text{TARGET}} \qquad (\square)$$

- Target 5: Safety premium (to pin down $\bar{\psi}$)

$$r^{\psi} = r^{\psi}_{\text{TARGET}} \qquad (\square)$$

- Implicit target: Convenience yield

$$cy = r^{\omega}_{\text{TARGET}} + r^{\psi}_{\text{TARGET}} \qquad (\square)$$

[2] Exogenous processes and analytical steady state results

Set steady state values for 7 of 10 exogenous processes—all but $\{\beta_t, \bar{\psi}_t, \bar{\omega}_t\}$ calibrated further below—and compute a first range of steady state results where it can be done analytically.

$$\chi_w \equiv \bar{\chi}_w,\ a \equiv \bar{a},\ \mu_z \equiv \exp(\Gamma),\ \chi_p \equiv \bar{\chi}_p,\ g/y \equiv \bar{g}/\bar{y},\ b/y \equiv \bar{b}/\bar{y}$$

- Wage dispersion

$$\Delta_w = 1 \qquad \text{(C.1)}$$

- Wage inflation

$$\pi_w = \mu_z^* \tag{C.2}$$

- Price of investment goods

$$p_I = 1 \tag{C.3}$$

- Capital utilization

$$u = 1 \tag{C.4}$$

- Price Phillips curve

$$p_m = (\theta_p - 1)/\theta_p \tag{C.5}$$

- Price dispersion

$$\Delta_p = 1 \tag{C.6}$$

- Interest rate rule

$$\Pi = \Pi^* \tag{C.7}$$

[3] Iterative solution for a subset of endogenous variables

Set an initial guess for the investment efficiency cut-off level, $\epsilon^*_{\mathbf{0}}$, and sequentially solve the following system of equations to derive $\epsilon^*_{\mathbf{i+1}}$ given $\epsilon^*_{\mathbf{i}}$. Iterate until convergence.

$$\epsilon^* = \epsilon^*_{\mathbf{i}}$$

- Target 6: Investment frequency [and normalization of $\mathbb{E}(\epsilon) = 1$]

$$\mathbb{E}(\epsilon) = \frac{\nu}{\nu - 1}\epsilon_{\min} = 1 \qquad i^{\text{freq}}_{\text{TARGET}} = 1 - F(\epsilon^*) = (\epsilon^*/\epsilon_{\min})^{-\nu}$$

Substitute and find the root to determine $\{\nu, \epsilon_{\min}\}$: $i^{\text{freq}}_{\text{TARGET}} = [\epsilon^*/(1 - 1/\nu)]^{-\nu}$ ($\Box$)

- Return on government bonds

$$\mathcal{M} = \left\{ R \left[1 + \int_{\epsilon^*}^{\epsilon_{\max}} \left(\frac{\epsilon}{\epsilon^*} - 1 \right) dF(\epsilon) \right] \right\}^{-1} \tag{C.8}$$

- Stochastic discount factor (for nominal assets) [used for calibration]

$$\beta = \mathcal{M}(\mu_{z^*}\Pi) \tag{C.9}$$

- Liquidity premium [used for calibration]

$$\bar{\omega} = 1 - r^{\omega} / \left\{ (\beta/\mu_{z^*}) \int_{\epsilon^*}^{\epsilon_{\max}} \left(\frac{\epsilon}{\epsilon^*} - 1 \right) dF(\epsilon)(1-\delta)\Upsilon^{-1} \right\} \tag{C.10}$$

- Real return on government bonds

$$r = R/\Pi \tag{C.11}$$

- Convenience yield

$$r_K^* = cy / \left\{ (\beta/\mu_{z^*}) \left[1 + \int_{\epsilon^*}^{\epsilon_{\max}} \left(\frac{\epsilon}{\epsilon^*} - 1 \right) dF(\epsilon) \right] \right\} + r \tag{C.12}$$

- Total return on capital

$$r_K/q = r_K^* \Upsilon - (1-\delta) \tag{C.13}$$

- Shadow value of capital
  [Rearranging + sequentially substituting the following equations: (1) average survival rate of capital, (2) endogenous asset quality in the market, and (3) market price of capital]

$$\begin{aligned} q = {} & (\beta/\mu_{z^*}\Upsilon) \left[1 + \int_{\epsilon^*}^{\epsilon_{\max}} \left(\frac{\epsilon}{\epsilon^*} - 1 \right) dF(\epsilon) \right] [r_K + \bar{\omega}(1-\bar{\psi})(1-\gamma)p_K] \\ & + (\beta/\mu_{z^*}\Upsilon) \left[1 + \int_{\epsilon^{**}}^{\epsilon_{\max}} \left(\frac{\epsilon}{\epsilon^{**}} - 1 \right) dF(\epsilon) \right] \bar{\omega}(1-\delta)q + (\beta/\mu_{z^*}\Upsilon)(1-\bar{\omega})(1-\delta)q \end{aligned}$$

$$\begin{aligned} (R/\Pi)\,\Upsilon = {} & (r_K/q) + \bar{\omega}(1-\bar{\psi}) \left[(1-\delta)/\bar{\psi} \right] (p_K/q) \\ & + \mathcal{M}R \left[1 + \int_{\epsilon^{**}}^{\epsilon_{\max}} \left(\frac{\epsilon}{\epsilon^{**}} - 1 \right) dF(\epsilon) \right] \bar{\omega}(1-\delta) + \mathcal{M}R(1-\bar{\omega})(1-\delta) \end{aligned}$$

$$\begin{aligned} (R/\Pi)\,\Upsilon = {} & (r_K/q) + \bar{\omega}(1-\delta) \left\{ \{ 1 + [1 - F(\epsilon^{**})](1-\psi^*)/\psi^* \} - 1 \right\} (p_K/q) \\ & + \mathcal{M}R \left[1 + \int_{\epsilon^{**}}^{\epsilon_{\max}} \left(\frac{\epsilon}{\epsilon^{**}} - 1 \right) dF(\epsilon) \right] \bar{\omega}(1-\delta) + \mathcal{M}R(1-\bar{\omega})(1-\delta) \end{aligned}$$

$$\begin{aligned} (R/\Pi)\,\Upsilon = {} & (r_K/q) + \bar{\omega}(1-\delta)[1 - F(\epsilon^{**})](1 - \epsilon^*/\epsilon^{**}) \\ & + \mathcal{M}R \left[1 + \int_{\epsilon^{**}}^{\epsilon_{\max}} \left(\frac{\epsilon}{\epsilon^{**}} - 1 \right) dF(\epsilon) \right] \bar{\omega}(1-\delta) + \mathcal{M}R(1-\bar{\omega})(1-\delta) \end{aligned}$$

$$\text{Find the root to determine } \epsilon^{**}. \tag{C.14}$$

- Investment cut-off

$$q = \frac{p_I}{\epsilon^*} \tag{C.15}$$

Making use of this result for q, solve for the level steady state of $\{r_K\}$, then continue.

- Good-quality capital sale cut-off

$$p_K = \frac{p_I}{\epsilon^{**}} \tag{C.16}$$

- Market price of capital

$$\psi^* = p_K/q \tag{C.17}$$

- Endogenous asset quality in the market [used for calibration]

$$\psi^* = \frac{(1-\delta)[1-F(\epsilon^{**})]}{(1-\delta)[1-F(\epsilon^{**})] + (1-\bar{\psi})(1-\gamma)}$$

$$\bar{\psi} = \{1 + [1-F(\epsilon^{**})](1-\psi^*)/\psi^*\}^{-1} \tag{C.18}$$

- Average survival rate of capital [used for calibration]

$$\gamma = 1 - (1-\delta)/\bar{\psi} \tag{C.19}$$

- Return on capital

$$\frac{y_m}{k} = (r_K/\mu_{z^*}\Upsilon)/(\alpha p_m) \tag{C.20}$$

- Aggregate output

$$\frac{y}{k} = \frac{y_m}{k}/\Delta_p \tag{C.21}$$

The updated guess for the price of capital, $\epsilon^*_{\mathbf{i+1}}$, is found by solving equation (C.22) for ϵ^*.

- Capital stock

$$k = \left\{(1-\delta)k + \frac{(R/\Pi)b + [r_K + \bar{\omega}(1-\bar{\psi})(1-\gamma)p_K]k}{p_I}\int_{\epsilon^*}^{\epsilon_{\max}} \epsilon dF(\epsilon) + \frac{\bar{\omega}(1-\delta)p_K k}{p_I}\int_{\epsilon^{**}}^{\epsilon_{\max}} \epsilon dF(\epsilon)\right\} / (\mu_{z^*}\Upsilon)$$

$$\int_{\epsilon^*}^{\epsilon_{\max}} \epsilon dF(\epsilon) = \left[\mu_{z^*}\Upsilon - (1-\delta) - \frac{\bar{\omega}(1-\delta)p_K}{p_I}\int_{\epsilon^{**}}^{\epsilon_{\max}} \epsilon dF(\epsilon)\right] / \frac{(R/\Pi)(b/y)(y/k) + r_K + \bar{\omega}(1-\bar{\psi})(1-\gamma)p_K}{p_I}$$

$$\epsilon^*_{\mathbf{i+1}} = \left\{\frac{\nu-1}{\nu}\epsilon_{\min}^{-\nu}\left[\mu_{z^*}\Upsilon - (1-\delta) - \frac{\bar{\omega}(1-\delta)p_K}{p_I}\int_{\epsilon^{**}}^{\epsilon_{\max}} \epsilon dF(\epsilon)\right] / \frac{(R/\Pi)(b/y)(y/k) + r_K + \bar{\omega}(1-\bar{\psi})(1-\gamma)p_K}{p_I}\right\}^{\frac{1}{1-\nu}} \tag{C.22}$$

Specify a tolerance level ς and test for convergence.
If $|\epsilon^*_{\mathbf{i+1}} - \epsilon^*_{\mathbf{i}}| > \varsigma$, update $\epsilon^*_{\mathbf{i}}$ and iterate. If $|\epsilon^*_{\mathbf{i+1}} - \epsilon^*_{\mathbf{i}}| \leq \varsigma$, stop and set $\epsilon^* = \epsilon^*_{\mathbf{i+1}}$.

[4] Remaining steady state results

With ϵ^* determined, solve for the remaining steady state results one by one.

- Investment

$$\frac{i}{k} = \left\{ \{(R/\Pi)(b/y)(y/k) + [r_K + \bar{\omega}(1-\bar{\psi})(1-\gamma)p_K]\} \frac{1-F(\epsilon^*)}{p_I} \right.$$
$$\left. +\bar{\omega}(1-\delta)p_K \frac{1-F(\epsilon^{**})}{p_I} \right\} / \left(\mu_{z^*}\Upsilon\right) \quad \text{(C.23)}$$

- Aggregate resource constraint

$$\frac{c}{k} = \frac{y}{k} - \frac{g}{y}\frac{y}{k} - \frac{i}{k} \quad \text{(C.24)}$$

- Marginal utility of consumption

$$(\lambda k) = [1 - (\beta\hbar/\mu_{z^*})] \left\{ [1 - (\hbar/\mu_{z^*})] \frac{c}{k} \right\}^{-1} / (1 + \tau_p) \quad \text{(C.25)}$$

- Labor supply (& Wage PC & Labor demand & Aggregate labor) [used for calibration]

$$(1-\tau_w)w_h\lambda = \chi\ell_h^{\xi} \;\; ; \;\; w_h/w = (1-\tau_w)(\theta_w - 1)/\theta_w \;\; ; \;\; w = (1-\alpha)\, p_m y_m/\ell \;\; ; \;\; \ell = \ell_h$$

$$\chi\ell^{\xi} = (1-\tau_w)\left[(\theta_w - 1)/\theta_w\right]\left[(1-\alpha)\, p_m y_m/\ell\right]\lambda$$

$$\chi = (1-\tau_w)\left[(\theta_w - 1)/\theta_w\right] \frac{(1-\alpha)\, p_m}{\ell^{(1+\xi)}} \frac{y_m}{k}(\lambda k) \quad \text{(C.26)}$$

- Production

$$k = \ell\left((1/a)\,(\mu_{z^*}\Upsilon)^{\alpha}\,\frac{y_m}{k}\right)^{1/(\alpha-1)} \quad \text{(C.27)}$$

Making use of this result for k, solve for level steady states of $\{i, c, y, y_m\lambda\}$, then continue:

- Labor demand

$$w = (1-\alpha)\, p_m y_m/\ell \quad \text{(C.28)}$$

- Wage Phillips curve

$$w_h = \left[(\theta_w - 1)/\theta_w\right] w \quad \text{(C.29)}$$

- Aggregate labor

$$\ell_h = \ell\Delta_w \quad \text{(C.30)}$$

- Endogenous asset resaleability in the market

$$\omega^* = \left[\left(1-\bar{\psi}\right)(1-\gamma) + (1-\delta)\frac{1-F\left(\epsilon^{**}\right)}{1-F\left(\epsilon^{*}\right)}\right]\bar{\omega} \quad \text{(C.31)}$$

- Government budget constraint

$$\tau = \left[(R/\Pi)/\mu_{z^*} - 1\right] b + g - \tau_w w\ell - \tau_p c - \tau_r r_K k/\left(\mu_{z^*t}\Upsilon\right) \quad \text{(C.32)}$$