The Signalling Channel of Negative Interest Rates*

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Abstract

Negative interest rates remain a controversial policy for central banks. We study a novel signalling channel and ask under what conditions negative rates should exist in an optimal policymaker's toolkit. We prove two necessary conditions for the optimality of negative rates: a time-consistent policy setting and a preference for policy smoothing. These conditions allow negative rates to signal policy easing, even with deposit rates constrained at zero. In an estimated model, the signalling channel dominates the costly interest margin channel. However, the effectiveness of negative rates depends sensitively on the degree of policy inertia, level of reserves, and ZLB duration.

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1 Introduction

In the aftermath of the Great Recession, negative interest rates have become an additional policy tool for several central banks around the world while others have kept interest rates in positive territory, despite a need for further monetary accommodation. In the euro area, both the European Central Bank's (ECB) deposit facility rate—paid on bank reserves held at the ECB—and the overnight interbank market rate (EONIA) have been negative since June 2014 (Figure 1(a)). Since September 2019, the deposit facility rate has stood at -0.5%.[1] At the same time, aggregate time-series of household deposit rates have declined but remain positive, subject to a binding zero lower bound (ZLB) on deposit rates as the cross-sectional distribution reveals. Figure 1(b) plots the histogram of household deposit rates across individual euro area banks in June 2014 and December 2017. Across this period, the fraction of deposits that earn a zero interest rate has risen from 27% to 69%, with virtually no banks passing on the negative reserve rate to household depositors. Moreover, banks earning a negative interest rate on reserves did not prevent the accumulation of excess reserves, with total reserves rising to over 20% of deposits by 2018 (Figure 1(c)).[2]

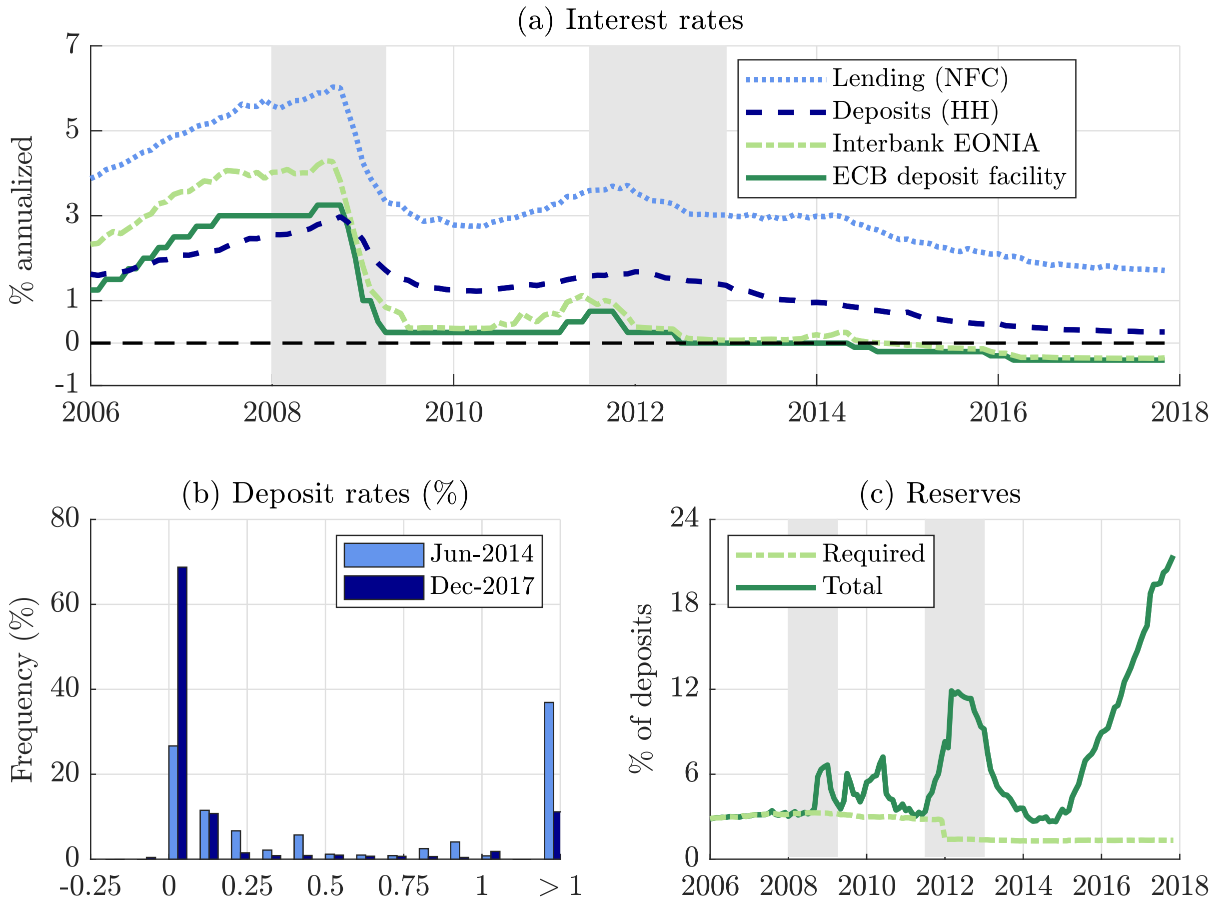
This raises four important questions. One, given that banks do not (or cannot) pass on negative interests rates to households, what is the transmission channel through which they operate? Two, given that a large fraction of the banking systems' assets are reserves that earn a negative interest rate, what are the consequences for the health of the banking system and its ability to create credit? Three, when the various effects of negative rates are taken into account, are they an effective policy tool to support output and inflation? Four, under what conditions should negative rates be in the toolkit of an optimal policymaker? This paper studies the interplay of a contractionary bank interest margin channel and a novel expansionary *signalling* channel to address all four of these questions.

The first contribution of this paper is to analytically explore the signalling channel by which negative interest rates on reserves can be expansionary, even when deposit rates are bound by zero. We begin by building a stylized financial-friction new-Keynesian model in which banks hold reserves at the central bank and monetary policy can set a negative reserve rate, but—in line with empirical evidence—household deposit rates have a ZLB.

[1] The ECB is not unique in having adopted a negative interest rate policy. In Switzerland, the SNB set its target rate for the 3-month LIBOR CHF at -0.75% to 0.25% in December 2014, and lowered it to -1.25% to -0.25% in January 2015. In Sweden, the Riksbank set its deposit rate to -0.5% in July 2014. In February 2015, it set its deposit and repo rate to -0.85% and -0.1%, respectively, and in February 2016 both rates reached their lowest point of -1.25% and -0.5%. In Denmark, the Danmarks Nationalbank set its certificates of deposit rate at -0.05% in September 2014, reaching its lowest point of -0.75% in February 2015. In Japan, the BOJ set the short-term policy interest rate at -0.1% in January 2016.

[2] The overall rise in reserves has been the result of the ECB's liquidity and asset purchase programs. However, deposit creation and lending decisions of the banking system determine the amount of reserves as a ratio of deposits and the split between required and excess reserves.

Figure 1: Interest rates and reserves in the euro area



Note: In (a) NFC and HH denote non-financial corporation and household composites, respectively; EONIA is the euro area overnight interbank market rate. In (b) deposit rates are on outstanding amounts as reported by individual banks, plotted as a fraction of total deposits in each bucket. In (c) deposits are HH and NFC deposits; excess reserves are given by total reserves minus required reserves. Source: ECB.

In the model, when the economy is away from the ZLB, an arbitrage condition ensures the interest rates on reserves and deposits move in tandem, and the behavior of the model is observationally equivalent to a model without reserve holdings. However, when the central bank introduces a negative reserve rate, the deposit rate (the rate that enters households' intertemporal Euler equation) is bound by zero, creating a wedge between the return on reserve assets and the ultimate funding source of banks—deposits. Thus, all else equal, a negative reserve rate acts like a contractionary bank net worth shock (henceforth, the costly "interest margin" channel of negative interest rates), where the size of the shock is scaled by the amount of reserves in the system. As a result, credit spreads widen (raising lending rates vis-a-vis deposit rates) and depress investment demand. This direct, or *intra*temporal, effect of negative interest rates has been a key criticism of banks to negative rate policies in Europe and elsewhere.[3]

[3] For example, Financial Times, Aug 30, 2017: "Poll exposes tensions between ECB and Germany's small banks".

Our stylized model shows that, in addition to this channel, there can be substantial positive general equilibrium effects of negative interest rates that are less directly ascribable to the policy. In particular, we emphasize the role negative rates can play in terms of signalling future policy (henceforth, the signalling channel of negative interest rates).[4]

Given this trade-off, the stylized nature of the model allows us to study *optimal* policy and analytically prove conditions under which an optimal policymaker would include negative interest rates as part of its policy toolkit. In particular, we prove that an optimal policymaker uses negative rates if the following two conditions simultaneously hold: i) it sets time-consistent discretionary policy (i.e. it cannot commit to future promises), and ii) it has an intrinsic preference for policy smoothing. Under these conditions, lowering the interest rate on reserves into negative territory can act as a tangible signal of maintaining lower deposit rates in the future. In contrast, a policymaker that can fully commit to future promises does not use negative rates. It can generate a credible future path of deposit rates without incurring the cost of a negative reserve rate via the interest margin channel. Equally, a discretionary policymaker without a preference for smoothing has no ability to signal and thus negative rates generate a direct cost to banks without benefits.[5]

The second contribution of our paper is to study the trade-off between the signalling and costly interest margin channels quantitatively. We do this by developing a medium-scale version of the stylized model, substituting optimal policy for an inertial Taylor-type rule, and carefully estimating the key structural parameters of the quantitative model.

When monetary policy is described by an inertial rule, decreasing the policy rate into negative territory allows the central bank to signal lower-for-longer deposit rates, both depressing post-ZLB deposit rates and, potentially, extending the overall ZLB duration. Even with current deposit rates unchanged, this negative interest rate policy generates an expansionary *inter*temporal aggregate demand effect. To support this analysis, we estimate the degree of policy smoothing from the data and present extensive international evidence for the robust empirical finding that central banks adjust policy gradually.

[4] Note that our use of the term "signalling" is different from the literature on imperfect information and the dispersion of information through central bank decisions such as in, for example, Melosi (2017). Instead, our usage of the term signalling captures a central bank's ability to give tangible signals about future policy, akin to Bhattarai et al. (2019)'s signalling theory of quantitative easing.

[5] It is well-established in the literature that with deficient aggregate demand and nominal interest rates constrained by the ZLB, optimal monetary policy should commit to keep rates low and allow inflation to overshoot its target, thus lowering real interest rates (Eggertsson and Woodford, 2003). However, in the absence of commitment, policymakers' ability to influence expectations about future policy by means of forward guidance is limited. In this paper, we argue that a negative rate policy allows policy to credibly keep rates lower for longer—like the optimal commitment prescription—in an environment without commitment by means of what we—in the spirit of Bhattarai et al. (2019)—call the signalling channel of negative interest rates. This signalling channel is akin to forward guidance, except forward guidance is an "open mouth" operation while negative interest rates provide tangible guidance about the future precisely because policy smoothing is a well documented characteristic of monetary policy.

In our estimated model, we show quantitatively that the contractionary intratemporal effect of negative interest rates via the interest margin channel is more than offset by the expansionary intertemporal aggregate demand effect via the signalling channel. Furthermore, the intertemporal demand effect raises asset values and banks experience capital gains. This reverses the fall in net worth through the costly interest margin channel, compresses credit spreads (lowering lending rates), and boosts investment demand. We show this through a novel decomposition of bank profits in the model. However, we also find that the effectiveness of monetary policy in negative territory relative to standard monetary policy depends crucially on three factors: i) a higher degree of policy inertia strengthens the expansionary signalling channel, ii) a larger level of reserves in the banking system magnifies the costly interest margin channel, and iii) a longer expected ZLB duration both depresses the expansionary signalling channel and magnifies the costly interest margin channel. To demonstrate the robustness of all of these findings, we conclude by showing that the signalling channel and our quantitative results are not a product of the forward guidance puzzle that plagues new-Keynesian models.

Literature There is a growing empirical literature assessing the transmission and impact of negative interest rates. Jobst and Lin (2016) and Eisenschmidt and Smets (2018) provide overviews on the early use of negative rates across countries.[6] For the euro area, and from an institutional point of view, Rostagno et al. (2021) illustrate the considerations around the introduction of negative rates and their interaction with alternative policy instruments. Regarding the transmission of negative rates, Eisenschmidt and Smets (2018) document the empirical regularity—consistent with our model—that banks have not lowered household deposits rates below zero. However, they do observe that there is a higher (yet still small) prevalence of negative rates charged on firm deposits. They conclude that negative rates have been broadly successful in easing financial conditions and creating modest credit growth, despite some adverse effects on bank balance sheets. This is broadly in line with more recent evidence by Altavilla et al. (2021) who argue that the transmission of monetary policy to firms is not inhibited by negative rates in the euro area. Regarding bank profitability, Altavilla et al. (2018) estimate the impact of a range of unconventional monetary policy measures, including negative rates, on bank balance sheets, and—in line with our paper—identify a costly interest margin channel but find that overall negative rates caused a substantial rise in banks' asset and equity values. Demiralp et al. (2019) find evidence for significant bank portfolio rebalancing in response to negative interest rates. Heider et al. (2019) show that banks adjust both lending quantity and risk profile in response to negative rates.

[6] Although negative nominal interest rates as a policy instrument are novel to recent years, Cecchetti (1988) gives a first account of negative nominal yields during the Great Depression.

The theoretical literature on negative rates is more limited. Eggertsson et al. (2019) build a new-Keynesian model but, in contrast to this paper, find that negative rates are, at best, ineffective, and at worst contractionary for output. This is a result of an intermediation cost term that becomes activated at negative rates—an effect that is microfounded in our model. Brunnermeier and Koby (2019) build a model with limited deposit rate pass-through and show that reducing the policy rate below a (time-varying) "reversal rate" can be contractionary, even at positive rates. Pariès et al. (2021) find a similar result with a model calibrated for the euro area and draw conclusions for macroprudential policy. Ulate (2021a,b) build models with a monopolistic banking sector and non-unitary interest rate pass-through and study the impact of negative rates on bank profitability and lending, finding a substantially expansionary effect. Sims and Wu (2021a,b) analyse the impact of several unconventional monetary policy measures—including negative interest rate policies—in a tractable framework similar to our stylized model.[7]

Compared to this literature, our paper is the first to explicitly characterize optimal policy in the negative interest rate environment.[8] Further, while most of this literature focuses on the contractionary impact of negative interest rate policies on bank profitability—an effect we also capture through our costly interest margin channel—and allows for an expansionary effect of negative rates via (limited) interest rate pass-through, we abstract from any pass-through to household deposit rates but explicitly model policy smoothing which results in an expansionary signalling channel of negative interest rates.

Our paper also relates to two other literatures, one on signalling and another on policy smoothing. In terms of signalling, our paper is close in spirit to Bhattarai et al. (2019), who present a signalling theory of quantitative easing. In their model, quantitative easing is effective because the government commits to honour outstanding debt obligations, enabling the discretionary policymaker to generate a credible signal of low future interest rates. Technically, the discretionary policymaker needs a state variable to signal; in our model, the state variable is the lagged reserve rate, in their model it is the debt stock. In terms of policy smoothing, our paper is closely related to Nakata and Schmidt (2019), who show that delegating policy to a policymaker with a preference for smoothing increases welfare in an economy subject to occasional ZLB episodes.[9] Our results take this one step further. We show that delegating to a policymaker with a smoothing preference opens up the possibility of an additional (welfare improving) policy instrument—negative rates.

[7] We also connect to the finance literature on the theory of costly signalling. In Bhattacharya (1979), for example, firms pay out dividends, despite it being costly, to signal to investors strong future cash flows.

[8] Rognlie (2016) studies optimal policy in a model very different to ours—without a banking sector, abstracting from bank profitability and interest margins, and no policy smoothing—where negative rates can raise aggregate demand but also inefficiently subsidize paper currency.

[9] Bonciani and Oh (2021) show how policy smoothing on balance sheet adjustments in the context of quantitative easing can further eliminate a range of new-Keynesian policy paradoxes at the ZLB.

The remainder of this paper proceeds as follows: Section 2 presents a stylized model and establishes conditions under which a negative interest rate policy is optimal. Section 3 develops a richly-specified model and presents quantitative results on the strength of the signalling channel and the effectiveness of negative interest rates. Section 4 concludes.

2 Stylized model and optimal policy

This section qualitatively illustrates the signalling channel of negative interests using a stylized (yet microfounded) model. The model can be reduced to the three equations of the canonical new-Keynesian model with the only addition of an endogenous aggregate demand shifter in the IS equation resulting from negative interest rates. Section 2.1 sets up of the model. Section 2.2 documents the log-linear equilibrium and highlights the costly interest margin channel. Section 2.3 derives key analytical insights into the optimality of negative rates and, in doing so, illuminates the signalling channel. Section 2.4 illustrates the optimal use of negative rates with numerical examples and comparative statics.

2.1 Set up

The model economy consists of households, banks, firms, and a monetary authority. Households are differentiated into two types—savers and borrowers—who transact through financial intermediaries—banks—subject to lending frictions. Monopolistic intermediate goods firms produce and set prices subject to nominal rigidities. The monetary authority sets its policy instrument—the interest rate on reserves—optimally.

Households Two types of households—savers and borrowers—exist and are distinguished by their relative patience. In particular, savers and borrowers have subjective discount factors of β and β_b, respectively, where $0 < \beta_b < \beta < 1$.

A representative saver household is composed of a fraction f workers and $1 - f$ bankers with perfect consumption insurance. Workers and bankers switch with probability $1 - \theta$ and when they do, bankers transfer retained profits back to the household. The household likes consumption, $C_{s,t}$, and dislikes labor, $L_{s,t}$. Financial markets are incomplete. Households can save in cash, M_t, which has a zero nominal return, or in bank deposits, D_t, that earn the gross nominal return $R_{d,t}$. The saver household problem is given by

$$V_{s,t} = \max_{\{C_{s,t}, L_{s,t}, D_t\}} \left(\frac{{C_{s,t}}^{1-\sigma}}{1-\sigma} - \chi \frac{{L_{s,t}}^{1+\varphi}}{1+\varphi} \right) + \beta \exp(s_t) \mathbb{E}_t V_{s,t+1}, \tag{1}$$

subject to

$$P_t C_{s,t} + M_t + D_t = P_t W_{s,t} L_{s,t} + M_{t-1} + R_{d,t-1} D_{t-1} + \Omega_{1,t} - \Omega_{2,t}, \tag{2}$$

where $s_{t+1} = \rho_s s_t + \sigma_s \varepsilon_{s,t+1}, \ \ \varepsilon_{s,t+1} \sim (0,1)$ is a time-preference shock that generates exogenous movements in the natural real rate, P_t is the aggregate price level, $W_{s,t}$ is the real wage, $\Omega_{1,t}$ are firm and bank profits, and $\Omega_{2,t}$ is a lump-sum transfer from savers to borrowers that both households take as given. While rather contrived, this transfer facilitates a clean set of equilibrium conditions that maintain focus on the key features of the model related to negative interest rate policies. The functional form of the transfer is given below. Note though, that such transfers will be dispensed with in the quantitative model. The saver household first-order conditions are given by

$$1 = \mathbb{E}_t \Lambda_{t,t+1} R_{d,t} / \Pi_{t+1}, \tag{3}$$

$$\chi L_{s,t}^{\varphi} = C_{s,t}^{-\sigma} W_{s,t}, \tag{4}$$

$$R_{d,t} \geq 1, \tag{5}$$

where $\Lambda_{t-1,t} \equiv \beta \exp(s_t) \left(C_{s,t} / C_{s,t-1} \right)^{-\sigma}$ is the household's real stochastic discount factor and $\Pi_t \equiv P_t / P_{t-1}$ is the gross rate of inflation. The ZLB constraint on nominal deposit rates—given by (5)—arises because of the existence of cash with a zero nominal net return.

The representative borrower household only consists of workers. Its problem is given by

$$V_{b,t} = \max_{\left\{C_{b,t}, L_{b,t}, B_t\right\}} \left(\frac{{C_{b,t}}^{1-\sigma}}{1-\sigma} - \chi \frac{{L_{b,t}}^{1+\varphi}}{1+\varphi} \right) + \beta_b \exp(s_t) \mathbb{E}_t V_{b,t+1}, \tag{6}$$

subject to

$$P_t C_{b,t} + R_{b,t-1} P_{t-1} B_{t-1} = P_t W_{b,t} L_{b,t} + P_t B_t + \Omega_{2,t}, \tag{7}$$

where borrower variables are denoted with subscript b. Bank loans are given by B_t and come with a gross nominal interest rate $R_{b,t}$. The transfer from savers to borrowers is given by $\Omega_{2,t} = R_{b,t-1} P_{t-1} B_{t-1} - P_t W_{b,t} L_{b,t}$. The first-order conditions are given by

$$C_{b,t}^{-\sigma} = \beta_b e^{s_t} \mathbb{E}_t C_{b,t+1}^{-\sigma} \frac{R_{b,t}}{\Pi_{t+1}}, \tag{8}$$

$$\chi L_{b,t}^{\varphi} = C_{b,t}^{-\sigma} W_{b,t}. \tag{9}$$

Banks The balance sheet of banker j is given by

$$B_t(j) + A_t(j) = D_t(j) + N_t(j), \tag{10}$$

where $N_t(j)$ is net worth and $A_t(j)$ are central bank reserves that earn the gross nominal return R_t. We assume a banker's demand for central bank reserves is given by

$$A_t(j) = \alpha(x_t) D_t(j), \tag{11}$$

where $x_t \equiv R_t/R_{d,t}$, $\alpha(x_t) > 0$, $\alpha(1) = \alpha$, $\alpha'(x_t) > 0$, $\alpha''(x_t) < 0$, and $\lim_{x_t \to 0} \alpha(x_t) = 0$. This demand schedule captures the trade-off between banks' preference for holding reserves to self-insure against idiosyncratic liquidity risk and the cost of holding reserves.[10]

Within a period, the timing is as follows: i) Bankers receive loan payments and repay depositors. ii) Bankers exit with probability $1-\theta$. An exiting banker is replaced by a worker with a fixed initial endowment of net worth given by $\bar{N}$. iii) Bankers accept new deposits and demand central bank reserves. v) A banker can divert a fraction λ of its assets (net of central bank reserves) to its household. In this case, the banker's depositors force bankruptcy and recover the remaining assets.

This agency problem creates a financial friction and makes bankers' net worth a crucial determinant of equilibrium outcomes in the model. The banker problem is given by

$$V_{n,t}(j) = \max_{\{B_t(j), A_t(j), D_t(j), N_t(j)\}} \mathbb{E}_t \Lambda_{t,t+1} \left((1-\theta) N_{t+1}(j) + \theta V_{n,t+1}(j)\right), \tag{12}$$

subject to the banker's balance sheet, (10), the reserve demand, (11), and

$$V_{n,t}(j) \geq \lambda B_t(j), \tag{13}$$

$$N_t(j) = \frac{R_{b,t-1}}{\Pi_t}(1-\tau) B_{t-1}(j) + \frac{R_{t-1}}{\Pi_t} A_{t-1}(j) - \frac{R_{d,t-1}}{\Pi_t} D_{t-1}(j), \tag{14}$$

where (13) and (14) are the incentive compatibility constraint and net worth accumulation equation, respectively. The steady state tax $\tau \equiv 1 - \beta_b/\beta$ ensures that the steady state is not distorted by the financial friction.

The central bank sets the interest rate on reserves and supplies reserves elastically. Since banks are competitive, arbitrage ensures $R_t = R_{d,t}$ when $R_{d,t} > 0$. In a symmetric equilibrium, bankers have a common leverage ratio, denoted $\Phi_t \equiv B_t/N_t = B_t(j)/N_t(j)$. This implies the solution to the bankers' problem can be summarized in two equations.[11]

[10] For a rigorously microfounded model of idiosyncratic liquidity risk that is consistent with this functional form, see, amongst others, Güntner (2015) and Bianchi and Bigio (2021).

[11] Appendix B.1 documents the full derivation of the banker's problem for the quantitative model.

Aggregate net worth, N_{t+1}, is given by

$$N_{t+1} = \theta \left(\frac{R_{b,t}}{\Pi_{t+1}} (1-\tau) \Phi_t - \frac{R_{d,t} - \alpha(x_t) R_t}{(1-\alpha(x_t)) \Pi_{t+1}} (\Phi_t - 1) \right) N_t + (1-\theta) \bar{N}, \tag{15}$$

and, if the incentive constraint is binding,

$$\lambda \Phi_t = \mathbb{E}_t \Lambda_{t,t+1} \frac{1-\theta+\theta\lambda\Phi_{t+1}}{\Pi_{t+1}} \left(R_{b,t} (1-\tau) \Phi_t - \frac{R_{d,t} - \alpha(x_t) R_t}{1-\alpha(x_t)} (\Phi_t - 1) \right). \tag{16}$$

Alternatively, if the incentive constraint does not bind, then arbitrage ensures

$$R_{b,t} (1-\tau) = \frac{R_{d,t} - \alpha(x_t) R_t}{1-\alpha(x_t)}. \tag{17}$$

Production Production consists of a continuum of intermediate and retail goods firms. Intermediate goods firm $i \in [0,1]$ produces differentiated output, $X_t(i)$, according to $X_t(i) = L_{s,t}(i)^{\omega} L_{b,t}(i)^{1-\omega}$ by hiring workers in a competitive labor market. Retail goods firms repackage intermediate output one-for-one, $Y_t(i) = X_t(i)$. Final output, Y_t, is a CES aggregate of differentiated retail firms' output given by $Y_t = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}$, where ϵ is the elasticity of substitution between goods. Cost minimization ensures the demand for final good i is given by $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$, where the aggregate price index, P_t, is defined as $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}$. Retail firms face nominal price rigidities of the type presented in Calvo (1983). Each period, a firm is able to adjust its price with probability $1-\iota$. A retail firm's objective is to maximize

$$\max_{P_t(i)} \mathbb{E}_t \sum_{\tau=0}^{\infty} \iota^{\tau} \Lambda_{t,t+\tau} \left(\frac{P_t(i)}{P_{t+\tau}} - \mathcal{M}_{t+\tau} \right) Y_{t+\tau}(i), \tag{18}$$

subject to the demand for good i, where $\mathcal{M}_t = W_{s,t}^{\omega} W_{b,t}^{1-\omega} / \left(\omega^{\omega} (1-\omega)^{1-\omega} \right)$ constitutes retail firms' marginal cost. The first-order condition is given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \iota^{\tau} \Lambda_{t,t+\tau} \left(\frac{P_{*,t}}{P_{t+\tau}} - \frac{\epsilon}{\epsilon-1} \mathcal{M}_{t+\tau} \right) Y_t(i) = 0, \tag{19}$$

where $P_{*,t}$ is the optimal reset price and the evolution of the aggregate price index is

$$P_t = \left((1-\iota) P_{*,t}^{1-\epsilon} + \iota P_{t-1}^{1-\epsilon} \right)^{1/(1-\epsilon)}. \tag{20}$$

The aggregate resource constraint of the economy is given by $C_{s,t} + C_{b,t} = Y_t$. The model is closed by the monetary authority setting the interest rate on reserves, R_t. The exact behaviour of the monetary authority, however, will be described in Section 2.3.

2.2 Log-linear private-sector equilibrium conditions

The beauty of this stylized model is that in its log-linear form, it is very similar to the canonical three-equation new-Keynesian model (see Appendix A.1 for the full derivation). For now, we focus on the case in which $\theta = 0$, such that bankers survive for only a single period. In this case, when the financial sector incentive compatibility constraint binds, the private sector equilibrium conditions can be condensed as follows,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t, \tag{21}$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-\mathsf{c}}{\sigma}\left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) - \mathsf{c}\left(\mathbb{E}_t \phi_{t+1} - \phi_t\right), \tag{22}$$

$$\phi_t = \tau_1 \mathbb{E}_t \phi_{t+1} - \tau_2 \left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) - \tau_3 \left(r_{d,t} - r_t\right), \tag{23}$$

where lower-case letters are the log-levels of their upper-case counterparts and c is the steady state consumption share of borrowers. The other parameters are given by

$$\kappa = \frac{(1-\iota\beta)(1-\iota)(\varphi+\sigma)}{\iota}, \ \tau_1 = \frac{\Phi\sigma}{1+\Phi\sigma}, \ \tau_2 = \frac{\Phi}{1+\Phi\sigma}, \ \tau_3 = \frac{\Phi-1}{1+\Phi\sigma}\frac{\alpha}{1-\alpha}.$$

Equation (21) is the standard new-Keynesian Phillips Curve. Since we consider only time-preference disturbances, output and output gap coincide. Equation (22) is the IS curve. When $\mathsf{c} = 0$, it reduces to the standard IS curve. Two more points are worth noting: First, the deposit rate, $r_{d,t}$, rather than the policy rate, r_t, enters the IS curve. Second, the IS curve features an endogenous aggregate demand shifter, $\mathsf{c}\left(\mathbb{E}_t \phi_{t+1} - \phi_t\right)$, resulting from fluctuations in leverage. Leverage is determined by Equation (23) which is derived from banks' incentive constraint. The final term in (23) directly captures the costly interest margin channel: When the reserve and deposit rates deviate, this generates inefficient fluctuations in bank leverage that feed through into aggregate demand fluctuations.

The costly interest margin channel even operates in a financially frictionless environment because the banks, operating in a competitive environment, still need to break-even.[12] When the incentive constraint does not bind, Equation (23) disappears and (22) can be conveniently rewritten as follows,

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) - \phi\left(r_{d,t} - r_t\right), \tag{24}$$

where $\phi \equiv (\mathsf{c}/\sigma)\,\alpha/(1-\alpha)$. In this case, endogenous aggregate demand shifts in the IS curve only occur as a result of the reserve rate deviating from the deposit rate. All else equal, when the reserve rate, r_t, turns negative and the deposit rate, $r_{d,t}$, is bound

[12] While in the quantitative model in Section 3, we ensure the incentive constrain binds and the financial accelerator operates, in this section we pursue the frictionless case as it provides clean analytical insights.

by zero, this pushes down output, y_t. To see why, we can also write the credit spread between bank lending and borrowing, $r_{b,t} - r_{d,t}$, as

$$r_{b,t} - r_{d,t} = \frac{\alpha}{1-\alpha}\left(r_{d,t} - r_t\right). \tag{25}$$

When $r_t < r_{d,t}$, banks pass on the cost of negative interest rates into higher borrowing rates, $r_{b,t}$, resulting in a reduction in consumption demand by borrowers. This pass-through from negative rates to the credit spread—the *costly interest margin channel*—is increasing in the quantity of reserves in the banking system, α (see Proposition 1). One can think of this result in terms of tax theory, with the reserve rate as a tax on reserves and the quantity of reserves as the tax base.

Proposition 1 *The costly interest margin channel of negative interest rates is exacerbated by an increase in the quantity of reserves, α, in the banking system.*

Having identified why a negative interest rates may be contractionary, the next section introduces our theory of the signalling channel and the conditions under which it exists.

2.3 Analytical results

This section studies the conditions for negative interest rates to be an instrument in an *optimal* policymaker's toolkit, and in so doing, illuminates the signalling channel of negative rates.[13] Our stylized model allows us to derive clear analytical results.

Optimal policy To study optimal policy in a tractable way, we assume social welfare can be approximated by a quadratic function in inflation and the output gap,[14]

$$V_t^{SW} = -\frac{1}{2}\left(\pi_t^2 + \lambda y_t^2\right) + \beta \mathbb{E}_t V_{t+1}^{SW}. \tag{26}$$

The policymaker maximizes this welfare function—setting the reserve rate, r_t—subject to the private sector equilibrium conditions, and three further constraints given by

$$r_{d,t} \geq 0, \qquad r_{d,t} - r_t \geq 0, \qquad r_{d,t}\left(r_{d,t} - r_t\right) = 0. \tag{27}$$

[13] In Appendix A.2 we show the behaviour of the model under a simple Taylor-type rule.

[14] Since our stylized model features both savers and borrowers, the social welfare function consistent with our model would depend on arbitrary welfare weights. Instead, we use a policy-relevant welfare function that i) is motivated by the microfounded loss function of the canonical 3-equation new-Keynesian model, and ii) is consistent with the dual mandate of many central banks.

The first constraint in (27) is the ZLB on the deposit rate. The second constraint states that the deposit rate faced by households cannot be below the central bank's reserve rate. The third constraint ensures that the reserve and deposit rate can only diverge when the deposit rate is at zero. While the interest rate on reserves can turn negative, away from the ZLB on the deposit rate, arbitrage equates the deposit and reserve rate.

First, we consider an optimal policymaker that maximizes (26) under full commitment.

Proposition 2 *Under commitment—when the policymaker solves for a state-contingent plan* $\{\pi_t, y_t, r_t, r_{d,t}\}_{t=0}^{\infty}$ *by maximizing* (26) *subject to the sequence of constraints* (21), (22), (27)*—it follows that* $r_t \geq 0 \; \forall \; s_t$.

Proof See Appendix A.3. ■

Proposition 2 states that with full commitment, a policymaker will never use negative interest rates. The intuition is relatively simple. Under commitment, the central bank can credibly promise to hold the deposit rate lower-for-longer in the future in order to compensate, in part, for the presence of the ZLB. Thus, setting a negative reserve rate results in a cost via the interest margin channel without any further benefit.

Next, we consider the optimal time-consistent (discretionary) policy.

Proposition 3 *Under discretion—when the policymaker solves for* $\{\pi_t, y_t, r_t, r_{d,t}\}$ *re-optimizing* (26) *every period subject to* (21), (22), (27) *and the actions of future policymakers—it follows that* $r_t \geq 0 \; \forall \; s_t$.

Proof See Appendix A.3. ■

Proposition 3 states that negative interest rates are also not part of the optimal time-consistent policymaker's toolkit. Under discretion, the policymaker cannot commit to future actions and so a negative interest rate has no ability to signal lower rates in the future. Once again, setting a negative reserve rate results in a cost via the interest margin channel without any further benefit.

Propositions 2 and 3 suggest that negative interest rates are never optimal. However, society can sometimes make itself better off by appointing a central banker whose preferences do not coincide with the social welfare function (Rogoff, 1985). In the following, we explore this idea and show that delegating policy to a central banker that places a weight on smoothing policy will—under certain conditions—use negative interest rates and can increase welfare. By lowering policy rates today, a policymaker with a preference for smoothing interest rates is also signalling lower policy rates in the future. This is the essence of the expansionary *signalling channel of negative interest rates.*

Technically, smoothing gives the policymaker an endogenous state variable that allows it to signal. However, this is not to say that any endogenous state variable will do the job. In Appendix A.3 we also show that Propositions 2 and 3 still hold when we introduce, for example, lagged inflation, into the model by augmenting the Phillips Curve as follows,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \gamma \pi_{t-1} + \kappa y_t. \tag{28}$$

Optimal policy with delegation Woodford (2003) shows that under discretion—even in the absence of the ZLB—delegating monetary policy to a policymaker with a preference for smoothing is desirable. More recently, Nakata and Schmidt (2019) demonstrate that the benefit of delegating policy to a policymaker with a preference for smoothing is even greater when the ZLB occasionally binds. We consider the same delegated central bank loss function (that deviates from the social welfare function) given by

$$V_t = -\frac{1}{2}\left((1-\psi)\left(\pi_t^2 + \lambda y_t^2\right) + \psi\left(r_t - r_{t-1}\right)^2\right) + \beta \mathbb{E}_t V_{t+1}, \tag{29}$$

with an explicit preference for interest rate smoothing weighted by $\psi \in (0, 1)$. A set of necessary conditions under which negative interest rates are a welfare improving tool of the optimal policy toolkit are given in Proposition 4:

PROPOSITION 4 *Two necessary conditions for the optimality of negative interest rates are i) a discretionary policy setting, and ii) the delegation of policy to a policymaker with a preference for smoothing interest rates* ($\psi > 0$).

The first necessary condition prevents the policymaker from exploiting "open-mouth" forward guidance to ease policy at the ZLB. The second enables the policymaker to use a change in the current level of the policy rate, r_t, to signal a change in future deposit rates.

Table 1: Optimality of negative interest rates

	Commitment	**Discretion**
Smoothing	×	✓
No Smoothing	×	×

Table 1 summarizes Propositions 2-4. The intuition for Proposition 4 is as follows. The discretionary policymaker reoptimizes every period, taking the policy functions of future policymakers as given. When $\psi > 0$, r_{t-1} becomes an endogenous state variable making negative rates a tangible signal of future rates in a time-consistent equilibrium.[15] To be more precise, when maximizing (29) subject to (21), (22) and (27) the first-order conditions can be written as follows (conditional on the "regime" the reserve rate r_t is in):

`Regime I`: $(r_t > 0)$

$$0 = \psi (1+\beta) r_t - \psi r_{t-1} - \psi \beta \mathbb{E}_t r_{t+1} + (1-\psi) \beta \mathbb{E}_t \frac{\partial \pi (r_t, s_{t+1})}{\partial r_t} + (1-\psi) \left(\mathbb{E}_t \frac{\partial y (r_t, s_{t+1})}{\partial r_t} + \sigma^{-1} \mathbb{E}_t \frac{\partial \pi (r_t, s_{t+1})}{\partial r_t} - \sigma^{-1} \right) (\lambda y_t + \kappa \pi_t), \qquad r_{d,t} = r_t.$$

`Regime II`: $(r_t < 0)$

$$0 = \psi (1+\beta) r_t - \psi r_{t-1} - \psi \beta \mathbb{E}_t r_{t+1} + (1-\psi) \beta \mathbb{E}_t \frac{\partial \pi (r_t, s_{t+1})}{\partial r_t} + (1-\psi) \left(\mathbb{E}_t \frac{\partial y (r_t, s_{t+1})}{\partial r_t} + \sigma^{-1} \mathbb{E}_t \frac{\partial \pi (r_t, s_{t+1})}{\partial r_t} + \phi \right) (\lambda y_t + \kappa \pi_t), \qquad r_{d,t} = 0.$$

`Regime III`: $(r_t = 0)$

$$r_t = r_{d,t} = 0,$$

where $y_t = y(r_{t-1}, s_t)$, for example, denotes the policy function for the output gap as a function of the state vector, (r_{t-1}, s_t). For a given state vector, the economy can be in three possible regimes: `I`: The ZLB is not binding, `II`: The ZLB on the deposit rate is binding and the reserve rate is set negative, or `III`: The ZLB is binding and the reserve rate is also set to zero. `Regime III` allows for the possibility that, even though negative interest rates are feasible, the policymaker may choose not to make use of them. For example, we will see that if ψ is sufficiently small or ϕ is sufficiently large, then `Regime II` is never visited and at the ZLB, the reserve rate is always set to zero. The first-order condition clarifies the role of policy smoothing in generating the signalling channel. When $\psi = 0$, it reduces to a static condition: $0 = \lambda y_t + \kappa \pi_t$. When $\psi > 0$, the policymaker takes account of the actions of future policymakers and past actions influence current decisions.

[15] Following Bianchi and Mendoza (2018) a discretionary equilibrium is a set of Markov stationary policy rules that are expressed as functions of the payoff-relevant state variables (r_{t-1}, s_t). Since the policymaker cannot commit to future policy rules, it chooses its policy rules at any given period taking as given the policy rules that represent future policymakers' decisions. A Markov perfect equilibrium is characterized by a fixed point at which the policy rules of future policymakers that the policymaker takes as given to solve its optimization match those that the current policymaker finds optimal. Hence, there is no incentive to deviate from other policymakers' policy rules, thereby making these rules time-consistent.

2.4 Numerical results and comparative statics

The previous section showed that negative rates can be optimal when policy is set under discretion and delegated to a policymaker with a preference for policy smoothing. In this section, we illustrate the optimal use with a numerical example and comparative statics.

A numerical example We solve the model using the Endogenous Grid Method of Carroll (2006).[16] The parameterization follows Nakata and Schmidt (2019) as specified in Table 2. In addition, we set the consumption share of borrowers, c, to 0.4 and the reserve-to-deposit ratio, α, to 0.2 (consistent with the empirical evidence), implying $\phi = 0.2$. All else equal, a one-period 25 basis point gap between the deposit and reserve rate widens the output gap by 5 basis points. Regarding the exogenous disturbance, we assume the natural real rate, s_t, follows an AR(1) process with persistence 0.85 and standard deviation 0.04. We approximate it using Tauchen and Hussey (1991)'s quadrature algorithm with 21 grid points. Details of the solution algorithm are described in Appendix A.4.

Table 2: Parameters

σ	Risk aversion	0.500	β	Discount factor	0.990
κ	Phillips curve slope	0.008	ϕ	Cost of negative rates	0.200
λ	Weight on output gap	7.85×10^{-4}	ψ	Weight on policy smoothing	0.029

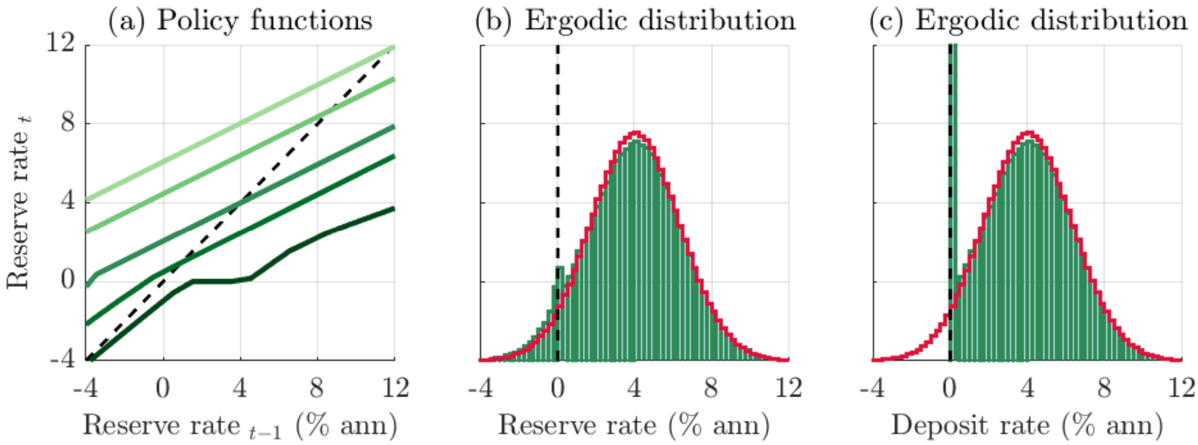
In the following, the strength of the costly interest margin channel of negative interest rates will depend sensitively on the quantity of central bank reserves in the banking system. Equally, the weight on policy smoothing, $\psi = 0.029$—which we set equal to the value maximizing the social welfare function in the absence of negative rates as a policy tool (see Appendix A.5)—will be crucial for the strength of the signalling channel of negative interest rates. The parameterization of the stylized model is only suggestive. Figure 4 further below provides the comparative statics of changing both parameters. In Section 3.2 we document and justify the parameterization of our quantitative model in detail.

Figure 2 provides several useful insights into the optimal discretionary policy solution with smoothing. Panel (a) plots policy functions for the reserve rate, r_t, as a function of the endogenous state variable, r_{t-1}, for selected values of s_t. The shape of the policy functions are notable for two reasons. First, the policy functions turn negative, suggesting the optimal policymaker, under this parameterization, is willing to use negative rates under

[16] Blake and Kirsanova (2012) warn that optimal discretionary policy in a linear-quadratic rational expectations model can yield multiple equilibria. In extensive numerical testing we have not come across multiple equilibria for our model, but we cannot rule out their existence.

certain conditions. This proves Proposition 4. Second, there are regions of “inaction” where the policy functions are horizontal. That is, there is a region of the state variable, r_{t-1}, where for a given fall in s_t, the policymaker initially drops the reserve rate to zero and only in subsequent periods lowers it into negative territory. Furthermore, to the left of the inaction region, the slope of the policy function is steeper than to the right of it. That is to say, once the policymaker passes the threshold into negative territory, it will continue cutting the reserve rate more aggressively than if unconstrained by the ZLB.

Figure 2: Optimal policy solution



Note: (a) plots policy functions for five different s_t values. The black-dash is the 45-degree line. (b) and (c) plot ergodic distributions generated from simulations of length 10^6 with a burn-in of length 10^3. The filled-green plots the distribution with negative rates, the red line the distribution without a ZLB constraint.

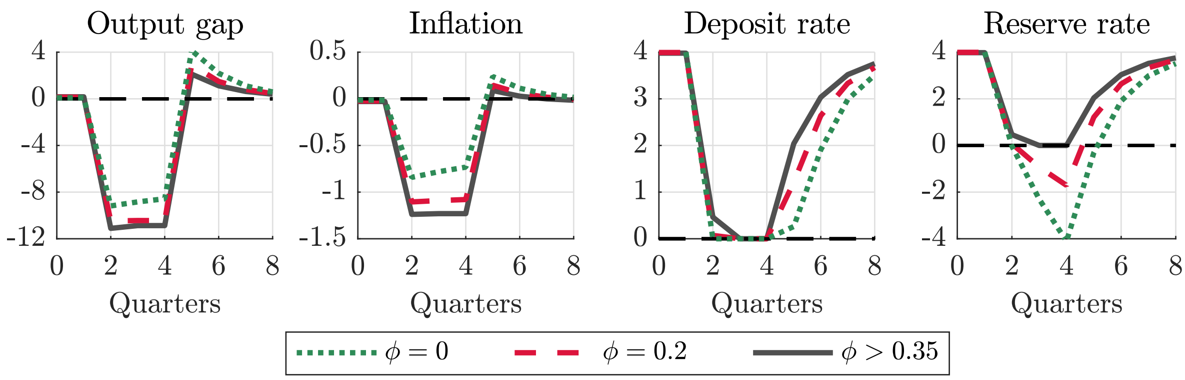
Panels (b) and (c) display the ergodic distributions (in green) for r_t and $r_{d,t}$, respectively, to highlight the effect of this inaction. The ergodic distribution of the deposit rate is naturally truncated by the ZLB. The ergodic distribution of the reserve rate has a non-trivial mass below zero. However, in line with the observed inaction, the ergodic distribution is not symmetric. First, there is additional mass around $r_t = 0$. Second, there is additional mass for $r_t < 0$ relative to the distribution without the ZLB constraint (red line).

Comparing ergodic distributions with and without negative interest rate policies, we further find that without the use of negative rates, the ZLB on deposit rates is expected to bind 3.7% of the time compared to the higher frequency of 4.4% when negative rates are used. The benefits of this increased frequency at the ZLB becomes clear in the next exercise. In terms of welfare, in the absence of negative interest rates, the household would forgo 2.57% of consumption per period to avoid uncertainty. Allowing for a negative interest rate policy reduces this value to 2.33%. Thus, the addition of negative rates into the policymaker's toolkit can generate a small but meaningful improvement in welfare.[17]

[17] Appendix A.5 derives the consumption equivalent welfare measure and plots welfare against different values of the smoothing parameter, ψ. Welfare is hump-shaped and concave in ψ, suggesting the optimal policymaker to whom to delegate monetary policy is one with a positive but finite desire for smoothing.

Figure 3 shows an experiment in which the natural real rate, s_t, drops into negative territory and remains at that level for 3 quarters before returning to steady state. The red-dash line is our baseline parameterization. The black-solid line is the equilibrium outcome when the policymaker is not able to set a negative reserve rate (or, equivalently, when the cost of negative interest rates is sufficiently high—in this case $\phi > 0.35$—such that the policymaker chooses not to use negative interest rates). The green-dotted line plots an extreme scenario where there is no cost of negative interest rates ($\phi = 0$).

Figure 3: Optimal policy scenarios



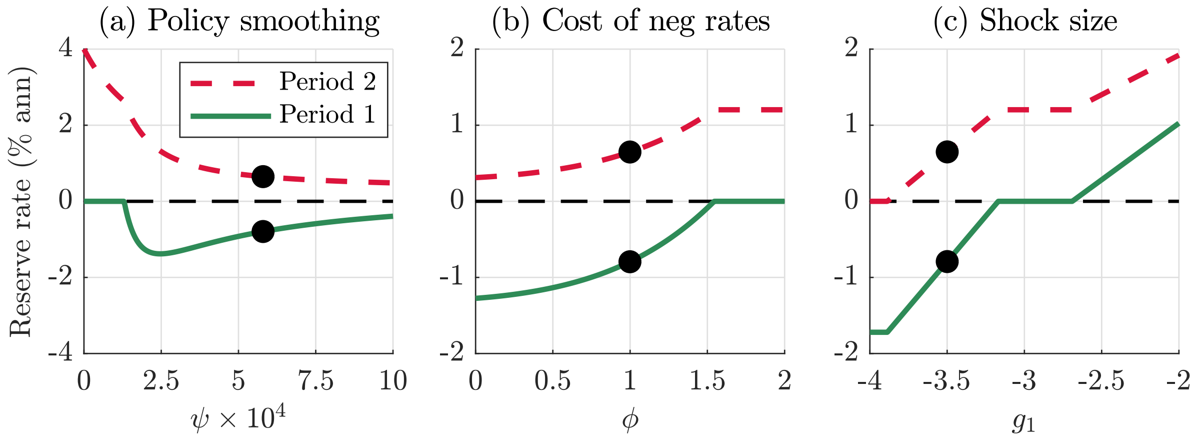
Note: Impulse responses to a drop in s_t into negative territory for 3 quarters before jumping back to its steady state value. The output gap is measured in percent. Inflation is in annualized percent deviation from steady state. The deposit and reserve rates are in levels, annualized.

When $\phi > 0.35$, the policymaker behaves as if there was a ZLB on the reserve rate. The nominal reserve rate is lowered to the ZLB, but this easing does not generate a sufficient fall in the real deposit rate, $r_{d,t} - \mathbb{E}_t \pi_{t+1}$, to offset the fall in s_t. As a result, inflation falls and the output gap opens. In contrast, when $\phi = 0.2$ the policymaker gradually lowers the reserve rate into negative territory, reaching -1.2% in period 4. Although the deposit rate remains bounded by zero, this negative reserve rate ensures that the deposit rate is lower after period 4 than without negative interest rates. This lower path for the deposit rate allows inflation to overshoot after s_t is back at steady state, also lowering the expected real deposit rate in early periods. As a consequence the drop in inflation and the widening of the output gap is less severe. The scenario without the cost of negative rates ($\phi = 0$) shows the maximum impact of negative interest rates. In this case, the reserve rate reaches -3.8% in period 2 and the deposit rate is a full 1 percentage point lower in period 6 than in the case without negative rates. The drop in the output gap and inflation is much less pronounced than in the other two scenarios.[18]

[18] This exercise illustrates that the increased frequency at the ZLB arises for two reasons: First, signalling with negative rates keeps the deposit rate "lower for longer" in response to a contractionary shock. Second, on impact the policymaker with access to negative rates is willing to cut the policy rate faster. Observe that, due to smoothing, the black-solid line does not reach the ZLB until period 3 as the benefit of cutting the period-2 policy rate further is outweighed by the cost in terms of smoothing rates. In contrast, the red-dash and green-dot lines (negative rate scenarios) already reach the ZLB in period 2.

Comparative statics We conclude our analysis of the stylized model with an investigation of how negative the policymaker is willing to set the reserve rate across the parameter range. The aim is again for qualitative insights rather than quantitative predictions. We do this in the following way: First, we set the natural rate shock, s_t, to be iid. Second, we presume that the policymaker disregards the output gap ($\lambda = 0$) and only cares about smoothing interest rates between periods 2 and 1. These two assumptions effectively reduce the model to a 2-period problem since $\{\pi_t, y_t\} = \{0, 0\}$ for $t \geq 3$, allowing for closed-form solutions, given in Appendix A.6. To highlight the trade-offs at play, we start from an extreme parameterization with ψ scaled down by 50 and ϕ scaled up by 5.

Figure 4: Optimal policy sensitivity analysis



Note: The black-dot refers to the baseline parameterization across the three panels, where, relative to Table 2, $\psi = \psi/50$ and $\phi = \phi \times 5$. The natural real rate, s_1, is set to -3.5.

Figure 4 illustrates the comparative statics effects of varying ψ and ϕ on r_1 and r_2 when the natural real rate in period 1 is -3.5% (Panel (c) varies the severity of the scenario by varying the natural real rate, s_1). In Panel (a), we vary the smoothing parameter, ψ. The value of ψ has a non-monotonic effect on the optimal period-1 reserve rate. When $\psi = 0$, the policymaker is unable to signal and thus does not use negative interest rates. Also for small positive smoothing values, the signalling benefit is outweighed by the cost of negative rates and the reserve rate remains at zero. Once the smoothing parameter becomes sufficiently large, however, the signalling channel of negative interest rates dominates the cost channel and a negative interest rate policy becomes optimal. In this simplified model, with $r_{d,1}$ constrained at zero, the only benefit of lowering r_1 for period-1 inflation, π_1, is to lower r_2 and thus raise period-2 inflation, π_2, which lowers the period-1 real interest rate. When ψ is small the policymaker sets a very negative interest rate in order to induce a lowering of r_2. However, as ψ rises, the signalling channel becomes more powerful and the policymaker need not set such a negative rate to achieve the same fall in r_2. Thus, we end up with a non-monotonic result in which both policymakers with very low and very high smoothing preferences make very little use of negative interest rates, while policymakers with an intermediate smoothing preference optimally set a very negative reserve rate.

In Panel (b), we vary the cost parameter, ϕ. In this case, r_1 is increasing in ϕ, which is not a surprise. However, the relationship is nonlinear and convex. Starting from $\phi = 0$, a marginal increase in the cost parameter has only a small effect on the equilibrium decision of the policymaker, but as ϕ increases, the policymaker rapidly reduces how negative it is willing to set the reserve rate. As we increase ϕ further, there comes a point at which the cost of setting a negative interest rate outweighs the benefit in terms of signalling. At this point negative interest rates are no longer optimal, and the policymaker sets $r_1 = 0$.

Finally, in Panel (c) we vary the size of the natural real rate shock, s_1. Starting from the right, and looking left as we increase the size of the shock, the policymaker naturally lowers the policy rate in order to accommodate the shock. However, we again observe a region of inaction in which, for a natural real rate between -2.8% to -3.4%, the policymaker does not engage in setting a negative rate. However, when the shock is sufficiently large the policymaker begins using negative interest rates and with a slope $(\partial r_1/\partial s_1)$ that is steeper than to the right of the inaction region, similar to our finding in Figure 2.

3 Quantitative model

The stylized model of Section 2 provides valuable insights into the optimality of negative rates. To quantitatively assess the effectiveness of a negative rate policy, this section develops a richly specified and carefully estimated medium-scale model. Section 3.1 sets up of the model. Section 3.2 documents our estimation strategy and provides further empirical evidence on key parameters. Section 3.3 presents our main results regarding the effectiveness of negative interest rates and illustrates the transmission mechanism using a novel decomposition of bank net worth. Section 3.4 conducts a sensitivity analysis and shows that the results are not a manifestation of the forward guidance puzzle.

3.1 Set up

The basis of the quantitative model is the financial-friction new-Keynesian model of the type developed by Gertler and Karadi (2011). In contrast to the stylized model, we dispense with borrower households and instead have firms borrowing from banks in order to finance the rental of physical capital. In addition, we introduce endogenous capital formation and investment adjustment costs, allow for consumption habits, and, finally, instead of studying optimal policy, we endow the monetary policymaker with an inertial Taylor-type rule to set the reserve rate. For compactness, rather than specifying the model in its entirety, in the following we only focus on features that differ markedly from the stylized model in Section 2. Appendix B.1 documents how to condense the bankers' problem in two equations, the full set of equilibrium conditions is given in Appendix B.2.

Households Three changes have been made to the household sector. One, only a representative (saver) household exists. Two, household preferences exhibit habits in consumption, $\tilde{C}_t \equiv C_t - \hbar C_{t-1}$. Three, we introduce a Smets and Wouters (2007) risk premium shock, $\zeta_{t+1} = \rho_\zeta \zeta_t + \sigma_\zeta \varepsilon_{\zeta,t+1}$, $\varepsilon_{\zeta,t+1} \sim (0,1)$, that will be used to generate the ZLB scenario.[19] As a result, the household problem is given by

$$V_t = \max_{\{C_t, L_t, M_t, D_t\}} \left(\log \tilde{C}_t - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right) + \beta \mathbb{E}_t V_{t+1}, \tag{30}$$

subject to

$$P_t C_t + M_t + D_t = P_t W_t L_t + M_{t-1} + \exp\left(\zeta_{t-1}\right) R_{d,t-1} D_{t-1} + \Omega_t. \tag{31}$$

Bankers Three changes have been made to the banking sector. One, banker j buys $S_t(j)$ units of firm equity at price Q_t (rather than lending to borrower households). As a result, firm equity pays a stochastic real return, $R_{k,t+1}$. The banker solves

$$V_{n,t}(j) = \max_{\{S_t(j), A_t(j), D_t(j), N_t(j)\}} \mathbb{E}_t \Lambda_{t,t+1} \left((1-\theta) N_{t+1}(j) + \theta V_{n,t+1}(j) \right), \tag{32}$$

subject to

$$Q_t S_t(j) + A_t(j) = D_t(j) + N_t(j), \tag{33}$$

$$V_{n,t}(j) \geq \lambda Q_t S_t(j), \tag{34}$$

$$A_t(j) = \alpha\left(x_t\right) D_t(j), \tag{35}$$

$$N_t(j) = R_{k,t} Q_{t-1} S_{t-1}(j) + \left(R_{t-1}/\Pi_t\right) A_{t-1}(j) - \left(R_{d,t-1}/\Pi_t\right) D_{t-1}(j). \tag{36}$$

The estimated parameterization of the model will ensure that the incentive constraint is always binding. Two, in equilibrium, $S_t = K_t$, where $S_t = \int_j S_t(j) dj$ and K_t is the aggregate capital stock in the economy. Three, an exiting banker is replaced by a worker with an initial endowment of net worth equal to a fraction ω of total firm equity in the previous period. As a consequence, the evolution of aggregate net worth is given by

$$N_t = \theta \left(R_{k,t} \Phi_{t-1} - \frac{R_{d,t-1} - \alpha\left(x_t\right) R_{t-1}}{\left(1 - \alpha\left(x_t\right)\right) \Pi_t} \left(\Phi_{t-1} - 1\right) \right) N_{t-1} + \omega Q_t K_{t-1}. \tag{37}$$

[19] While both risk premium and discount factor shocks are common in the literature to induce a demand-driven ZLB scenario, the risk premium shock is preferable in a model with endogenous capital formation as it induces a positive co-movement of consumption and investment.

Capital goods firms Capital goods firms are new to the model and repair depreciated capital and produce new capital. Existing capital depreciates at rate δ and is refurbished at unit cost. New capital, $K_{n,t}$, is produced using technology $K_{n,t} = f\left(I_{n,t}, I_{n,t-1}\right)$, where $I_{n,t}$ is investment in new capital formation. The capital goods firm solves

$$V_{k,t} = \max_{I_{n,t}} \left(Q_t K_{n,t} - I_{n,t}\right) + \mathbb{E}_t \Lambda_{t,t+1} V_{k,t+1}. \tag{38}$$

The first-order condition is given by $1 = Q_t f_{1,t} + \mathbb{E}_t \Lambda_{t,t+1} Q_{t+1} f_{2,t+1}$, where $f_{i,t}$ is the derivative of f with respect to the i-th argument. With quadratic flow adjustment costs, $f\left(\cdot\right) \equiv \left(1 - \left(\eta/2\right)\left(\left(I_{n,t} + I\right)/\left(I_{n,t-1} + I\right) - 1\right)^2\right) I_{n,t}$, where $I = \delta K$ is defined as steady state investment pertaining to gross investment given by $I_t = f\left(I_{n,t}, I_{n,t-1}\right) + \delta K_{t-1}$. Capital accumulation follows $K_t = K_{t-1} + f\left(I_{n,t}, I_{n,t-1}\right)$.

Intermediate goods firms Intermediate goods firms produce and sell intermediate output, Y_t, using technology $K_{t-1}^{\gamma} L_t^{1-\gamma}$, at price $P_{m,t}$. Profits per unit of capital are $P_{m,t} \gamma Y_t / K_{t-1}$. Labor demand is $W_t = P_{m,t}\left(1-\gamma\right) Y_t / L_t$. Firms need external finance to purchase capital. At the beginning of the period, they issue S_t units of equity to bankers at price Q_t. In return, the banker receives the realized return per unit of capital next period,

$$R_{k,t} = \frac{P_{m,t} \gamma Y_t / K_{t-1} + Q_t - \delta}{Q_{t-1}}. \tag{39}$$

Retail firms Retail firms are unchanged from the stylized model except we introduce a cost-push shock by making the elasticity of substitution between goods time-varying. In particular, we define $\epsilon_{t+1} = \rho_\epsilon \epsilon_t + \sigma_\epsilon \varepsilon_{\epsilon,t+1}$, $\varepsilon_{\epsilon,t+1} \sim (0,1)$. The aggregate resource constraint is $Y_t = C_t + I_t + G$, where G is exogenous government spending, $G/Y = 0.2$.

Monetary policy The central bank's policy instrument is the nominal interest rate on reserves, which when unconstrained follows a Taylor-type inertial policy rule given by

$$R_{T,t} = \left(R \Pi_t^{\phi_\pi} \left(\frac{X_t}{X}\right)^{\phi_x}\right)^{1-\rho} R_{t-1}^{\rho} \exp\left(\varepsilon_{m,t}\right), \tag{40}$$

where $R_{T,t}$ is the rate implied by the policy rule, $X_t = 1/\mathcal{M}_t$ is the mark-up and proxies the output gap, and $\varepsilon_{m,t}$ is a mean-zero *i.i.d.* monetary policy shock. We implicitly assume zero steady-state inflation. The degree of inertia is given by ρ and the inertial term is the lagged reserve rate. The policy rule is not inertial when the policy rate is bounded at.[20]

[20] Other studies have considered policy rules in which the inertial term is on the Taylor-rule implied rate, $R_{T,t}$, rather than the actual policy rate, R_t. To the extent that such a rule is credible ($R_{T,t}$ is a latent variable), it also increases the effectiveness of monetary policy in a standard ZLB scenario. Thus, this latter formulation is more akin to explicit forward guidance, whereas in our specification inertia is a structural feature of monetary policy that is orthogonal to whether the economy is at the ZLB or not.

In what follows, we compare three scenarios for monetary policy:

I. The unconstrained ("UNC") scenario, in which both the reserve and deposit rate are unconstrained and can turn negative, is given by

$$R_t = R_{d,t} = R_{T,t}. \tag{41}$$

II. The deposit rate-only ZLB ("ZLB: R_d only") scenario is given by

$$R_t = R_{T,t} \quad \text{and} \quad R_{d,t} = \max\{1, R_{T,t}\}. \tag{42}$$

In this scenario—our baseline to study the effects of negative interest rates—the deposit rate is bounded by zero, but the interest rate on reserves can turn negative. Thus, lowering the policy rate below zero has no contemporaneous effect on nominal deposit rates.

III. The standard ZLB scenario ("ZLB: R_d & R"), in which both the reserve and deposit rate are constrained by zero, is given by

$$R_t = R_{d,t} = \max\{1, R_{T,t}\}. \tag{43}$$

3.2 Parameterization

Table 3 presents the baseline parameterization of the quantitative model. The parameters are grouped into three blocks. Block A contains structural parameters that are assigned standard values from the literature. Block B is calibrated using steady state relationships. The parameters in Block C are estimated using a simulated method of moments procedure. Appendix B.3 documents the data sources and transformations used.

Time in the model is quarterly. Based on standard values in the literature, the discount factor is $\beta = 0.99$, the capital share of income is $\gamma = 0.33$, and the depreciation rate of capital is $\delta = 0.025$. Following Primiceri et al. (2006) and Gertler and Karadi (2011), consumption habits are $\hbar = 0.815$ and for the inverse Frisch labor supply elasticity we set $\varphi = 0.276$. The elasticity of substitution between goods is $\epsilon = 4.167$, resulting in a steady state mark up over marginal cost of approximately 30%. For the Calvo parameter we set $\iota = 0.9$ which means firms can adjust prices on average every 10 quarters. This represents a relatively high degree of price stickiness but is the consequence of assuming a CES aggregator rather than a Kimball aggregator as in Smets and Wouters (2007). For the survival probability of bankers we set $\theta = 0.975$, implying an average tenure of 10 years. Finally, the Taylor rule coefficients are standard with $\phi_\pi = 1.5$ and $\phi_x = 0.125$ for the response to inflation and our proxy for the output gap, respectively.

Block B contains the parameter values that are calibrated to match steady state values in the data. First, the utility weight on labor is $\chi = 3.411$, which ensures that steady state labor supply is normalized to 1/3. Second, the two financial sector parameters are $\lambda = 0.411$ and $\omega = 0.001$, and are calibrated to match a steady state leverage ratio of 4 and a steady state credit spread, $400(R_k/R_d - 1)$, equal to 1% annualized. A good data counterpart to aggregate leverage in the model is hard to come by. For the period from 2009 to 2019, leverage of the non-financial corporate business sector in the US was 1.9.[21] For the same period, the commercial banking sector had a much higher leverage of 9.4. However, this measure excludes non-bank financial institutions such as hedge funds and broker dealers that are typically even more leveraged. In 2021, estimates for the total assets of the non-bank financial sector were 1.86 times larger than the total assets of commercial banks. Aggregating across these highly heterogeneous sectors, and assuming that leverage in the non-bank financial sector is twice that of the commercial bank sector, we end up with a conservative estimate of aggregate leverage of 3.6. Given the uncertainty in these calculations, we opt to calibrate the model to a leverage of 4.

Table 3: Structural parameter values

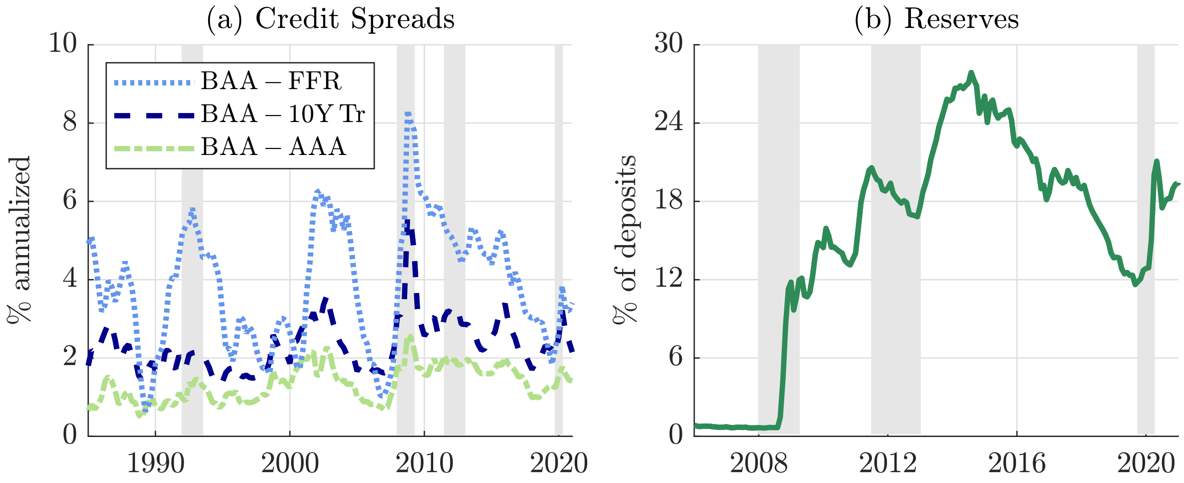
Block A. Standard parameters					
β	Discount factor	0.990	$\hbar$	Habit parameter	0.815
φ	Inverse Frisch elasticity	0.276	γ	Capital share	0.330
δ	Depreciation rate	0.025	ϵ	Elasticity of substitution	4.167
ι	Probability of fixed prices	0.900	θ	Survival probability of bankers	0.975
ϕ_π	Policy rule inflation response	1.500	ϕ_x	Policy rule output response	0.125
ρ_ζ	Persistence of risk premium shocks	0.800	ρ_ϵ	Persistence of cost-push shocks	0.800
Block B. Steady state calibrated parameters					
χ	Utility weight on labor	3.411	α	Reserve-to-deposit ratio	0.200
λ	Fraction of divertible assets	0.411	ω	Transfer to new bankers	0.001
Block C. Estimated parameters					
η	Inverse investment elasticity	1.617	ρ	Policy rule inertia	0.856
σ_ζ	S.d. of risk premium innovations	0.002	σ_ϵ	S.d. of cost-push innovations	0.033

Calibrating the steady state credit spread is equally tricky. In Figure 5(a) we plot three alternative spread measures commonly used in the literature. The first is the spread between the BAA corporate bond yield and the federal funds rate (light blue-dot). The two component interest rates that compromise the spread are a reasonable match for the expected return on capital and the short-term policy rate in the model, respectively. We

[21] Consistent with the model, we measure leverage as $A/(A-L)$, where A is total assets and L is total liabilities. See Appendix B.3 for more details.

thus use the cyclical properties of these series in the estimation stage below. However, for matching the steady state credit spread, this measure is not ideal because it contains a maturity mismatch. The corporate bonds yields are based on long-term bonds with a maturity of 20 years and above whereas the federal funds rate is a short-term rate. Thus, this series is likely to contain both a liquidity and term premium in addition to a pure risk premium. To get a sense of these various premia, we plot the spread between the BAA corporate bond yield and the 10 year Treasury yield (dark blue-dash) and between the BAA and AAA corporate bond yields (green dot-dash), respectively. For the credit spread in the model, we match its steady state to 1% annualized which corresponds to the mean of the "BAA-AAA" series over the sample period. This series is generally perceived to be a good empirical measure of the safety or quality premium that we capture with the financial friction in our model (see Krishnamurthy and Vissing-Jorgensen, 2012).

Figure 5: Credit spreads and reserves in the US



Note: (a) AAA and BAA are Moody's Seasoned AAA and BAA Corporate Bond Yields, respectively; FFR is the Effective Federal Funds Rate; and 10Y Tr is the Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity. (b) Total reserves of depository institutions over total deposits of commercial banks. Sources: Federal Reserve Bank of St Louis.

Finally, we set the reserve-to-deposit rate in the model to $\alpha = 0.2$. This value is broadly in line with data for both the euro area—as displayed in Figure 1—and the United States. Figure 5(b) shows the evolution of the US reserve-to-deposit ratio. In the aftermath of the 2007/08 financial crisis, total reserve holdings strongly increased, reflecting banks' desire to hedge against heightened liquidity risk and the Federal Reserve's willingness to supply extensive additional reserves to the banking system via a range of liquidity and quantitative easing programs. Accordingly, the reserve-to-deposit ratio rose from a pre-crisis level of around 1% to a peak of 27.9% in August 2014. The banking system's demand for liquidity spiked again during the Covid-19 crisis when the Federal Reserve once more sharply increased the provision of reserves to meet this additional demand. Overall, we find a value of 18.9% for the average reserve-to-deposit ratio over the postfinancial crisis period in the US. As the strength of the costly interest margin channel of negative interest rates will depend sensitively on the quantity of reserves in the banking system, in Section 3.4 we conduct a sensitivity analysis where we vary this quantity and show the implications on the effectiveness of a negative interest rate policy.

Block C contains the structural parameters that we estimate. We do this following the method of simulated moments in Basu and Bundick (2017). In particular, the parameter values are chosen to minimize the distance between the model implied moments and their data counterparts. Formally, the vector of estimated parameters, Θ, is the solution to

$$\min_{\Theta} \left(\mathcal{H}^D - \mathcal{H}\left(\Theta\right)\right)' \mathcal{W}^{-1} \left(\mathcal{H}^D - \mathcal{H}\left(\Theta\right)\right), \tag{44}$$

where $\mathcal{H}^D$ is a vector of data moments, $\mathcal{H}\left(\Theta\right)$ denotes its model counterpart, and $\mathcal{W}$ is a diagonal weighting matrix containing the standard errors of the estimated data moments.

The estimation targets ten moments from aggregate US time-series data and five yield curve moments. The first ten moments are the standard deviations and autocorrelations of output, consumption, inflation, the federal funds rate, and the credit spread, respectively. The remaining five moments are the movements in the 6-month, 1-, 2-, 5-, and 10-year risk-free rates, respectively, relative to the movement in the 3-month risk-free rate in response to a monetary shock. Empirical estimates are taken from Altavilla et al. (2019). The risk-free yield curve can be extracted from the model using the following set of equations:

$$\begin{aligned} P_{2,t} &= \mathbb{E}_t \Lambda_{t,t+1} P_{1,t+1}, \\ &\vdots \\ P_{40,t} &= \mathbb{E}_t \Lambda_{t,t+1} P_{39,t+1}, \end{aligned}$$

where $P_{1,t} = 1/R_t$ is the price of a one-period risk-free bond that pays 1 unit in period $t+1$. The annualized yield on the 10-year risk-free bond is therefore given by $R_{40,t} = P_{40,t}^{-1/10}$.

With 15 moments, we estimate four parameters $\theta = \{\eta, \rho, \sigma_\zeta, \sigma_\epsilon\}$, the inverse investment elasticity, the policy rule inertia coefficient, and the standard deviations of risk premium and cost-push innovations. The estimation is thus over-identified. We choose to estimate the investment elasticity parameter because its value is not well-informed by the literature and its value has implications for the strength of the financial accelerator and the dynamics of credit spreads and net worth. The estimation delivers an inverse investment elasticity of $\eta = 1.617$. We also choose to estimate the policy rule inertia coefficient because it is crucial for the strength of the signalling channel of negative interest rates. The estimation delivers a value of $\rho = 0.856$, which suggests a significant amount of policy smoothing.

Having completed the parameterization of the model, Table 4 compares the model implied moments with those from the data. The table also includes the 95% confidence interval around the data estimates. Despite only estimating a small number of parameters, the model does a reasonable job of matching the data. The model implied moments are within the confidence interval for the yield curve moments. In terms of the business cycle moments, the model does well in terms of matching most of the standard deviations but generates too much persistence relative to the data (the exception is the credit spread, in which the data is more persistent than the model). The table also presents untargeted moments that were not included in the estimation. The estimated model does well on these in terms of investment dynamics, but performs less well in terms of cross-correlations. In particular, the cross-correlations suggest that the cost-push shock is dominating the dynamics of inflation in the model while demand-side shocks are important in the data.

Table 4: Simulated method of moments results

	Data	Model		Data	Model		Data	Model
Targeted moments								
std(y)	1.014 (0.76-1.27)	0.877	ac(y)	0.874 (0.82-0.93)	0.973	mp(r_{6m})	0.843 (0.80-0.89)	0.839
std(c)	0.714 (0.54-0.89)	0.641	ac(c)	0.831 (0.77-0.89)	0.990	mp(r_{1y})	0.677 (0.55-0.81)	0.587
std(π)	0.175 (0.14-0.21)	0.196	ac(π)	0.330 (0.14-0.52)	0.760	mp(r_{2y})	0.503 (0.29-0.72)	0.301
std(r)	0.265 (0.20-0.33)	0.144	ac(r)	0.935 (0.89-0.98)	0.961	mp(r_{5y})	0.324 (0.11-0.54)	0.135
std(cs)	0.279 (0.20-0.36)	0.345	ac(cs)	0.895 (0.83-0.95)	0.745	mp(r_{10y})	0.092 (-0.08-0.26)	0.101
Untargeted moments								
std(i)	4.470 (2.92-6.02)	4.272	ac(i)	0.914 (0.84-0.99)	0.972	cr(y, c)	0.807 (0.72-0.89)	0.599
cr(y, i)	0.906 (0.86-0.95)	0.890	cr(y, π)	0.362 (0.14-0.58)	-0.539	cr(y, r)	0.689 (0.56-0.82)	-0.644
cr(y, cs)	-0.690 (-0.84–0.54)	-0.539						

Note: Construction of moments given in Appendix B.3. y, c, i, π, r, and cs refer to GDP, consumption, investment, inflation, the federal funds rate, and the credit spread, respectively. sd($\cdot$), ac($\cdot$), and cr($\cdot$) refer to the standard deviation, first-order autocorrelation, and cross-correlation, respectively. $r_{6m}, r_{1y}, r_{2y}, r_{5y}$, and r_{10y} refers to the OIS 6 month, 1, 2, 5, and 10 year rate, respectively. mp($\cdot$) refers to the relative response of the relevant OIS rate to the 3 month OIS rate in response to a monetary policy shock. Estimates are taken from Altavilla et al. (2019).

Further evidence on policy smoothing In the estimation, we find a policy inertia coefficient of $\rho = 0.856$, suggesting that policy smoothing is an important feature of the data. As the strength of the signalling channel of negative interest rates will depend sensitively on the degree of policy inertia, we support the results of this estimation with further evidence, and—as for the reserve-to-deposit ratio—show sensitivity results in Section 3.4.

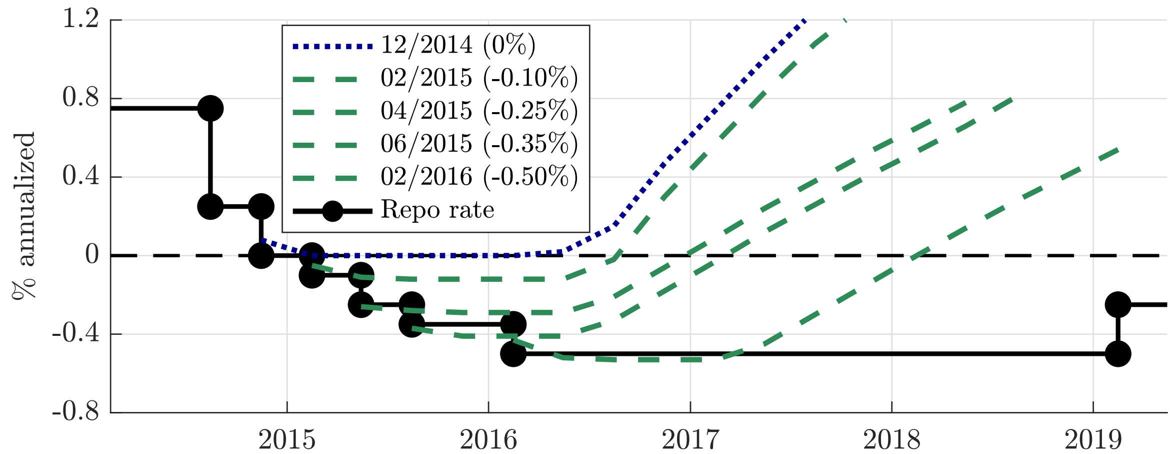
Figure 6: Monetary policy inertia in the literature and in practice

(a) Estimates of policy rule inertia

United States		**Euro area**	
Primiceri et al. (2006)	0.80	Smets and Wouters (2003)	0.96
Smets and Wouters (2007)	0.81	Christiano et al. (2010)	0.84
Coibion and Gorodnichenko (2012)	0.83	Darracq Pariès et al. (2011)	0.84
Brayton et al. (2014)	0.85	Coenen et al. (2018)	0.93
Christiano et al. (2014)	0.85	**Japan**	
United Kingdom		Sugo and Ueda (2007)	0.84
Burgess et al. (2013)	0.83	**Sweden**	
Switzerland		Adolfson et al. (2008)	0.88
Rudolf and Zurlinden (2014)	0.90	Christiano et al. (2011)	0.82

Note: Estimates of ρ for a selection of papers and central bank policy models. Brayton et al. (2014) is the Federal Reserve's FRB/US model, Burgess et al. (2013) is the Bank of England's COMPASS model, and Coenen et al. (2018) is the ECB's New Area Wide Model II.

(b) Riksbank repo rate forecasts during negative interest rates



Note: The blue-dot and green-dash lines show the Riksbank's own repo rate forecasts around monetary policy meetings in which they lowered the repo rate, based on quarterly averages. The actual repo rate (black-solid line) is based on daily data. Source: Riksbank monetary policy reports.

Figure 6(a) documents estimates of policy inertia from the literature for the US, euro area, and four additional countries. Two key messages emerge. First, there is robust evidence for a large inertial component of monetary policy, irrespective of estimation technique or country considered. Second, estimates range from 0.80 (Primiceri et al., 2006, US) to 0.96 (Smets and Wouters, 2003, euro area). Thus, our baseline value of $\rho = 0.856$ is, if anything, on the more conservative side of possible parameterizations in terms of quantifying the strength of the signalling channel.[22]

[22] Rudebusch (2002, 2006) argues that observed policy inertia may, in fact, reflect persistent shocks rather than interest rate smoothing. However, recent work by Coibion and Gorodnichenko (2012) finds strong evidence in favour of the interest rate smoothing explanation.

One concern though might be that these estimates are limited to periods in which policy rates were in positive territory. Figure 6(b) provides suggestive evidence from Sweden that monetary policy inertia extends to negative rate episodes as well. Between February 2015 and February 2016, the Swedish Riksbank lowered the repo rate, its key policy rate, in four steps from 0% to -0.5%. Repo rate forecasts published by the Riksbank around the respective monetary policy decisions show that every negative rate decision also came with a substantial downward revision of the forecasted path of the future policy rate, both extending the expected ZLB duration and lowering the expected future policy rate. This is consistent with inertial policy-setting behaviour documented above.

3.3 Main results

This section presents our main results on the effectiveness of negative interest rates and illustrates the transmission mechanism using a novel decomposition of bank net worth.

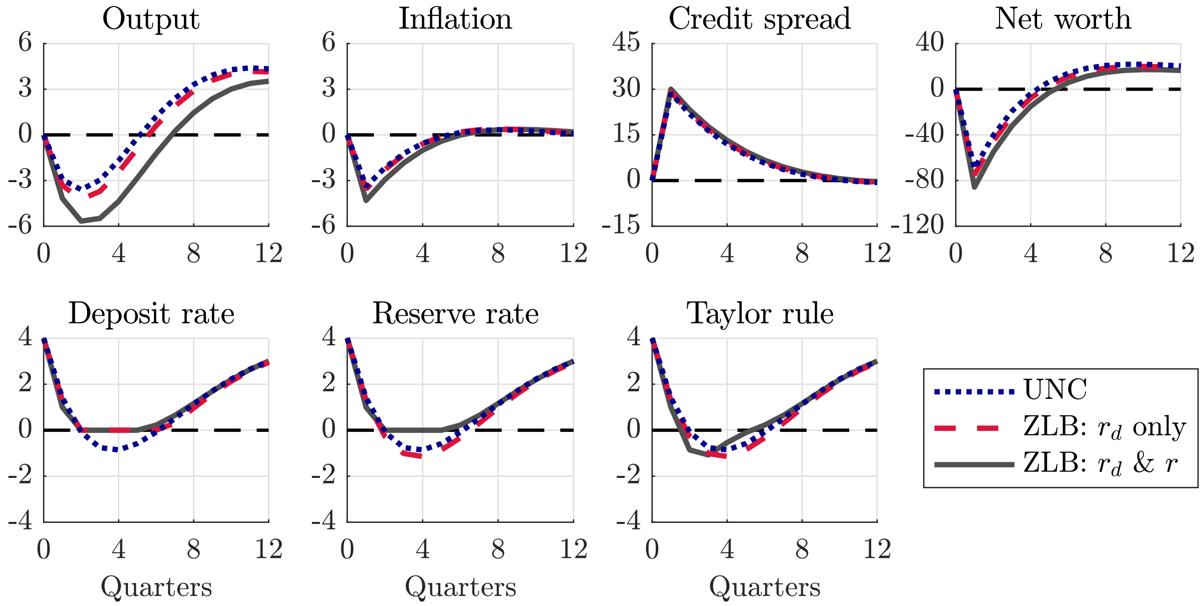
Main results In our baseline experiment we consider a risk premium shock that drives the economy to the ZLB and makes it remain there for 4 quarters when neither the nominal reserve rate nor deposit rates can turn negative (scenario III).[23] Figure 7 shows impulse responses for our three monetary policy scenarios.[24] In response to the exogenous increase in the risk premium, households save more and reduce their consumption. Bank net worth falls, raising credit spreads and lowering investment demand. Thus, the risk premium shock acts as an aggregate demand shock, depressing both output and inflation.

The fall in output (and inflation) in the deposit rate-only ZLB scenario (II, red-dash) as compared to the standard ZLB scenario (III, black-solid) indicates that a negative interest rate policy—at least under our baseline calibration—is expansionary. The unconstrained scenario (I, blue-dot) results in the smallest fall in output. This is when both the deposit and reserve rate mirror the Taylor-type rule implied rate and turn negative in order to partly offset the contraction in output and inflation. When policy is constrained by the ZLB and unable to fully react to the drop in aggregate demand, the fall in output is largest. However, when the central bank can decrease the policy rate into negative territory—despite the deposit rate being bounded by zero—it is able to extend the ZLB duration by 1 additional quarter (to 5 quarters in total) and lower the post-ZLB deposit rate path (in line with the empirical evidence presented in Figure 6(b)) thus providing additional stimulus. A negative interest rate policy is expansionary even when the deposit rate which is relevant for households' intertemporal substitution decision is constrained.

[23] In using a single large shock (a 7% drop), we are trading off realism for expositional clarity.

[24] For comparability, Appendix A.2 replicates the experiments presented in this section using the stylized model from Section 2. Despite the additional features, the qualitative findings are largely unchanged.

Figure 7: Risk premium shock **with inertia** in the policy rule

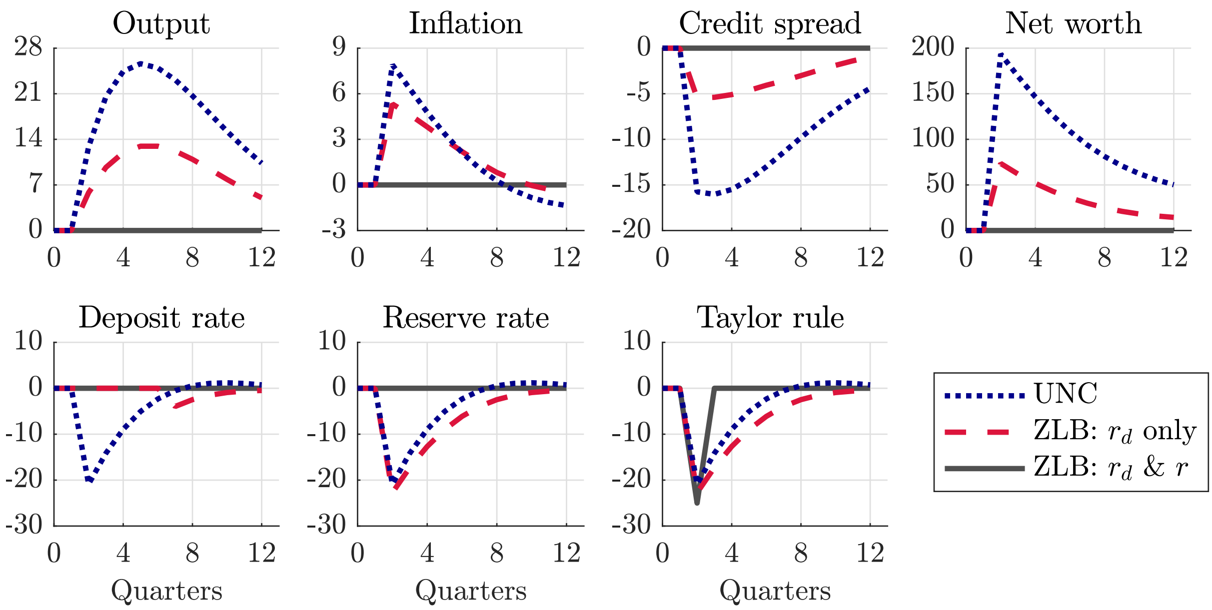


Note: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a risk premium shock that brings the economy to the ZLB for 4 quarters. All interest rates displayed are in annualized percent. All other variables are in 100×log-deviation from steady state. Inflation is annualized. Log-deviations are a good approximation of percent deviations when the deviation is small. For net worth, the -80 log-deviation, however, translates to a more modest 55 percent drop.

To isolate the quantitative implications of a negative interest rate response to the crisis scenario, we introduce an additional -25 basis point iid monetary policy shock to the economy in period 2 when the economy is at the ZLB for 5 periods. Figure 8 shows the impulse responses to the pure monetary policy shock by stripping out the effect of the underlying risk premium shock. When both the deposit rate and reserve rate are constrained by zero and cannot turn negative, a shock to the Taylor-type rule implied rate has no effect on equilibrium outcomes (III, black-solid). However, allowing for a negative reserve rate (II, red-dash) the monetary policy shock is expansionary and the peak output effect is 51% of an unconstrained monetary policy shock (I, blue-dot). In terms of inflation, negative interest rates are even more effective, with a peak inflation response of 67% of the unconstrained response. Again, the path of the deposit rate is key to understanding these outcomes. Despite the fact that the monetary policy shock occurs in period 2, the deposit rate remains unchanged until period 7 when it drops by 4 basis points and remains persistently below the baseline thereafter.

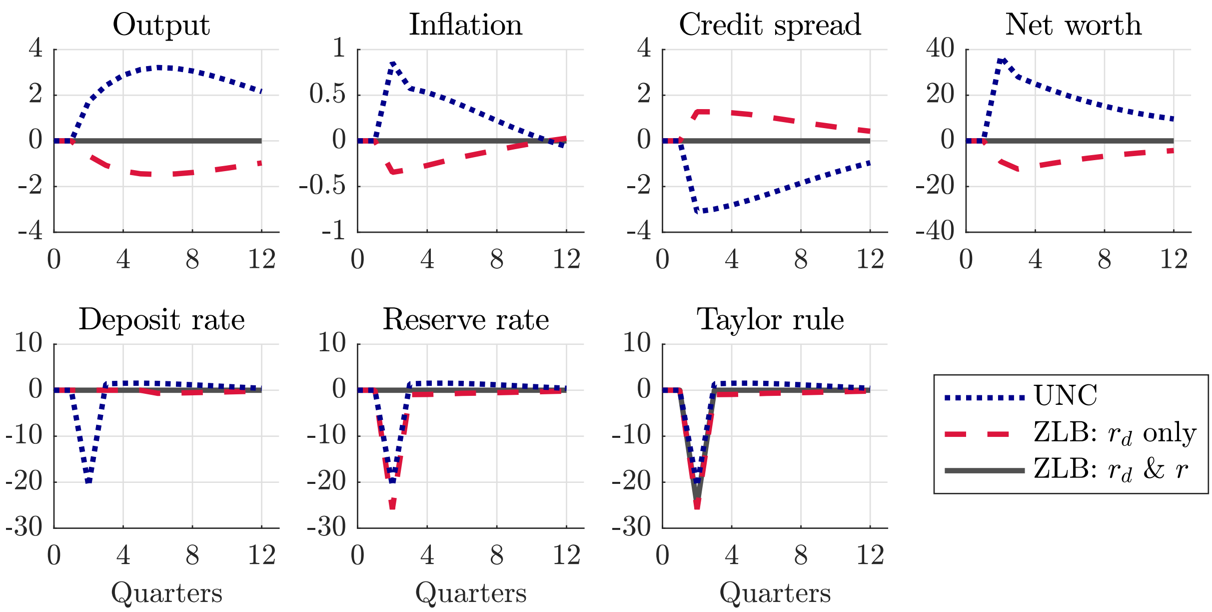
To explicitly identify the role of the signalling and interest margin channels to which we have eluded frequently, we remove policy inertia and re-run the previous experiment. Figure 9 shows impulse responses to the monetary policy shock at the ZLB with $\rho = 0$. The most striking difference (relative to Figure 8) is that under the deposit rate-only ZLB scenario (II, red-dash), negative rates are now contractionary rather than expansionary.

Figure 8: Monetary policy shock **with inertia** in the policy rule



Note: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a -25 basis point iid monetary policy shock at the ZLB. All interest rates displayed are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Figure 9: Monetary policy shock **without inertia** in the policy rule

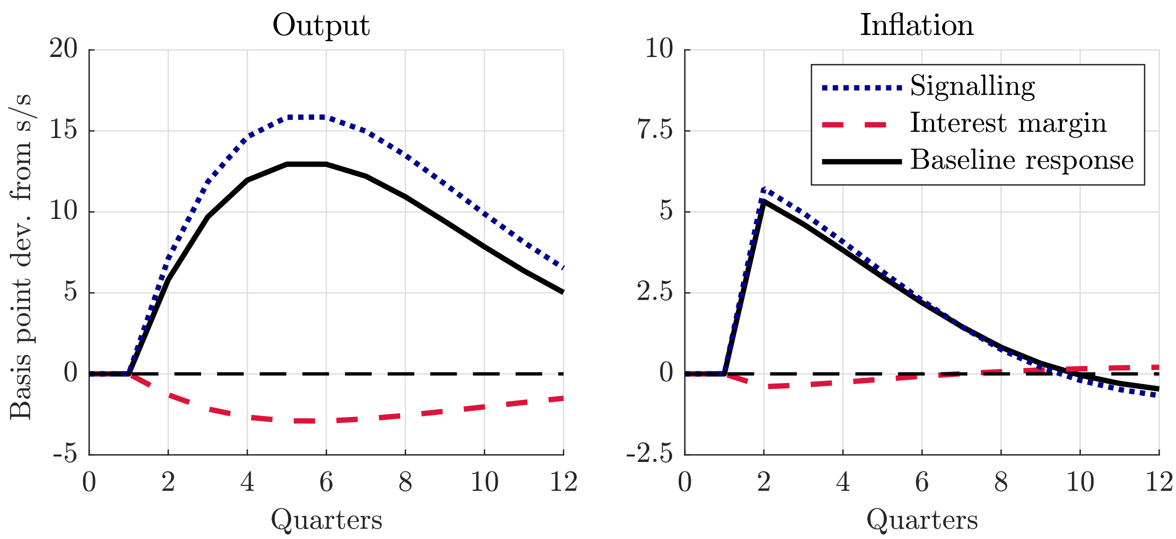


Note: $\alpha = 0.2$, $\rho = 0$. Impulse responses to a -25 basis point iid monetary policy shock at the ZLB. All interest rates displayed are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

The monetary policy easing into negative territory results in a fall in output and inflation. There are two reasons for this. First, by setting $\rho = 0$ we have switched off the signalling channel. The fall in the reserve rate has no effect on the path of the deposit rate. Second, the costly interest margin channel results in bank net worth falling. This tightens banks' incentive constraint and causes credit spreads to rise. With the deposit rate constrained, a rise in credit spreads implies a higher lending rate for firms which depresses investment demand. The negative interest rate policy becomes contractionary when the deposit rate is at the ZLB and the drop in the reserve rate is not transmitted via the signalling channel.

Figure 10 puts these results together and decomposes the output and inflation response to negative interest rates into the signalling channel and costly interest margin channel. The baseline response (black-solid) is equivalent to the deposit rate-only ZLB scenario (II) in Figure 8. Setting $\alpha = 0$ and $\rho = 0$, respectively, we document impulse responses for a pure signalling (blue-dot) and a pure interest margin channel (red-dash). In the baseline, the peak output and inflation responses are 13 and 5 basis points, respectively. For output, this effect can be decomposed into a 16 basis point contribution of the expansionary signalling channel, and a -3 basis point contribution of the costly interest margin channel.

Figure 10: Contribution of signalling and interest margin channels



Note: Impulse responses to a -25 basis point iid monetary policy shock at the ZLB. Inflation is annualized. "Signalling" and "Interest margin" plot $\alpha = 0$ and $\rho = 0$, respectively.

Decomposition of bank profits As we have seen comparing Figures 8 and 9 regarding the effectiveness of negative interest rates with and without policy inertia, the response of bank net worth is a key determinant in the transmission of negative rates. In the following, we investigate this further examining a novel decomposition of bank net worth.

We begin by defining bank profits, $\texttt{prof}_t$ (the gross growth rate of an individual banker's nominal net worth, conditional on not exiting)—building on Section 3.1—as follows:

$$\texttt{prof}_t = \left(\Pi_t R_{k,t} - R_{d,t-1}\right)\Phi_{t-1} + R_{d,t-1} - \frac{\alpha\left(x_t\right)}{1-\alpha\left(x_t\right)}\left(R_{d,t-1} - R_{t-1}\right)\left(\Phi_{t-1} - 1\right). \quad (45)$$

Next, we log-linearize profits and decompose them into 7 distinct terms given by

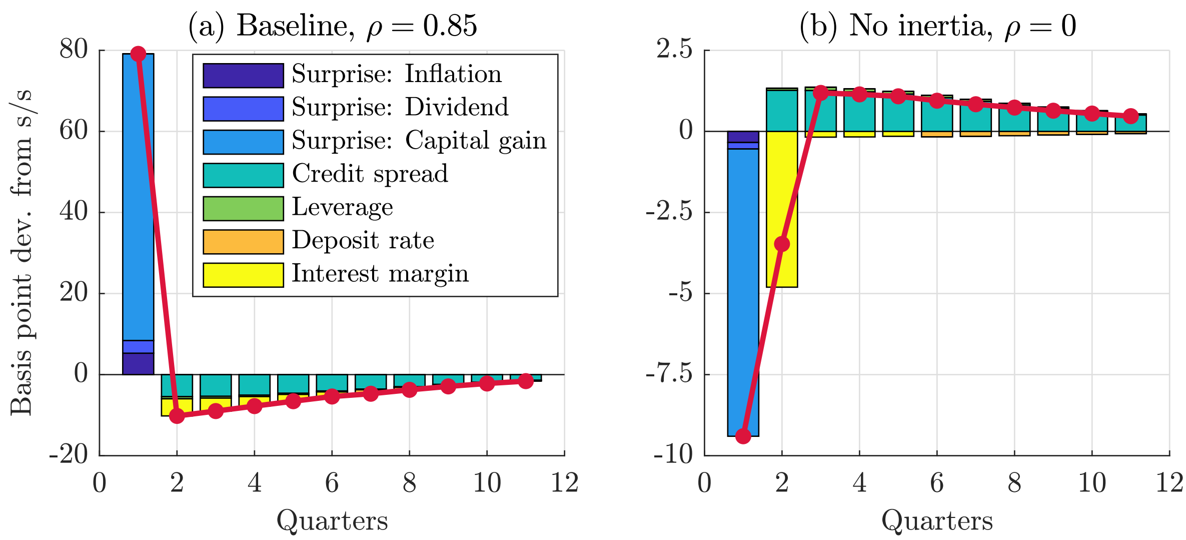
$$\begin{aligned} \hat{\texttt{prof}}_t = & \underbrace{\frac{R_k\Phi}{\texttt{prof}}\left(\hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t\right)}_{\text{Surprise: Inflation}} + \underbrace{\frac{\texttt{mpk}\Phi}{\texttt{prof}}\left(\hat{\texttt{mpk}}_t - \mathbb{E}_{t-1}\hat{\texttt{mpk}}_t\right)}_{\text{Surprise: Dividend}} + \underbrace{\frac{\texttt{mpk}\Phi}{\texttt{prof}}\left(\hat{q}_t - \mathbb{E}_{t-1}\hat{q}_t\right)}_{\text{Surprise: Capital gain}} \\ & + \underbrace{\frac{R_k\Phi}{\texttt{prof}}\hat{\texttt{cs}}_{t-1}}_{\text{Credit spread}} + \underbrace{\frac{\texttt{cs}\Phi}{\texttt{prof}}\hat{\phi}_{t-1}}_{\text{Leverage}} + \underbrace{\frac{\texttt{cs}\Phi}{\texttt{prof}}\hat{r}_{d,t-1}}_{\text{Deposit rate}} - \underbrace{\frac{\alpha}{1-\alpha}\frac{R_d\left(\Phi-1\right)}{\texttt{prof}}\left(\hat{r}_{d,t-1} - \hat{r}_{t-1}\right)}_{\text{Interest margin channel}}, \end{aligned} \quad (46)$$

where hats denote log-deviations from steady state, variables without subscripts are steady states, $\texttt{cs}_t \equiv \mathbb{E}_t\Pi_{t+1}R_{k,t+1} - R_{d,t}$ is the nominal credit spread, and $\texttt{mpk}_t \equiv P_{m,t}\gamma Y_t/K_{t-1}$ is the marginal product of capital. As Equation (46) shows, bank profits can be decomposed into three windfall (or "surprise") components and four predetermined components. In general, the return on an asset can be split into a dividend payment and a capital gain, so that, for banks' assets, we term the surprise change in the marginal product of capital as the "dividend" and the surprise change in the price of the asset as the "capital gain". The third surprise component is inflation since we report nominal profits. The four predetermined terms are the evolution of (1) the credit spread, (2) leverage, (3) the risk-free rate, and (4) the partial equilibrium effect of negative rates on interest margins (i.e. the costly interest marginal channel of a negative interest rate policy).

Figure 11 plots the decomposition of bank profits in response to a -25 basis point iid monetary policy shock at the ZLB with and without policy inertia. In Panel (a), with the signalling channel of negative interest rates switched on, in period 1 we observe a sharp increase in bank profits driven by the three surprise terms from the decomposition. The largest effect on bank profits comes from the capital gain term—that is, a revaluation of the banks' assets in response to the monetary policy shock. With the signalling channel of negative rates in play, a drop of the reserve rate into negative territory depresses the future expected path of deposit rates. Households adjust their intertemporal consumption decision and bring forward consumption demand, aggregate production and the price of capital increase instantaneously, driving up bank profits. From period 2 on, tighter credit spreads (the revaluation of bank assets raises net worth, slackens the banks' incentive compatibility constraint, contracting credit spreads) and the costly interest margin channel reduce bank profits, slowly bringing bank net worth back to steady state. This decomposition of how negative interest rates affect different parts of banks' balance sheets is consistent with empirical evidence in, for example, Altavilla et al. (2018).

In Panel (b), with the signalling channel of negative interest rates switched off, bank profits fall in reaction to the negative interest rate policy. Without policy inertia, negative interest rates do not come with an expansionary aggregate demand effect but reduce bank net worth via the costly interest margin channel. Lower bank net worth implies rising credit spreads that affect the decomposition of bank profits in two ways: One, on impact in period 1, higher expected credit spreads depress the investment demand of firms and induce capital losses. Two, from period 2 on, higher realized credit spreads generate additional profits slowly bringing bank net worth back to steady state. Note that compared to Panel (a), most of the partial equilibrium terms have switched sign. The only term which has a consistently negative sign is the interest margin channel. This term reduces bank profits irrespective of the value of policy inertia ρ.

Figure 11: Decomposition of bank profits



Note: The red-dot line plot the impulse response of bank profits to a -25 basis point iid monetary policy shock at the ZLB. Stacked bars decompose the impulse response for every period.

3.4 Sensitivity and the forward guidance puzzle

The previous section showed that negative interest rates can be both expansionary and contractionary when deposit rates are constrained. In this section, we investigate the factors that determine their effectiveness more thoroughly. In particular, we conduct a sensitivity analysis with respect to the degree of policy inertia (ρ), banks' reserve-to-deposit ratio (α), and the ZLB duration. Further, we show that the signalling channel is not a reflection of the "forward guidance puzzle" critique of new-Keynesian models.

Sensitivity analysis In Figure 12(a) we plot the *absolute* peak response of output to the 25 basis point iid monetary policy shock at the ZLB for different combinations of policy inertia and sizes of the initial risk premium shock.[25] The x-axis scales with the size of the initial shock and proxies the severity of the crisis, plotting the number of quarters the ZLB is expected to bind when the monetary policy shock is introduced. The y-axis reports the effectiveness of the policy easing as a percentage of the effect of an unconstrained monetary policy shock.

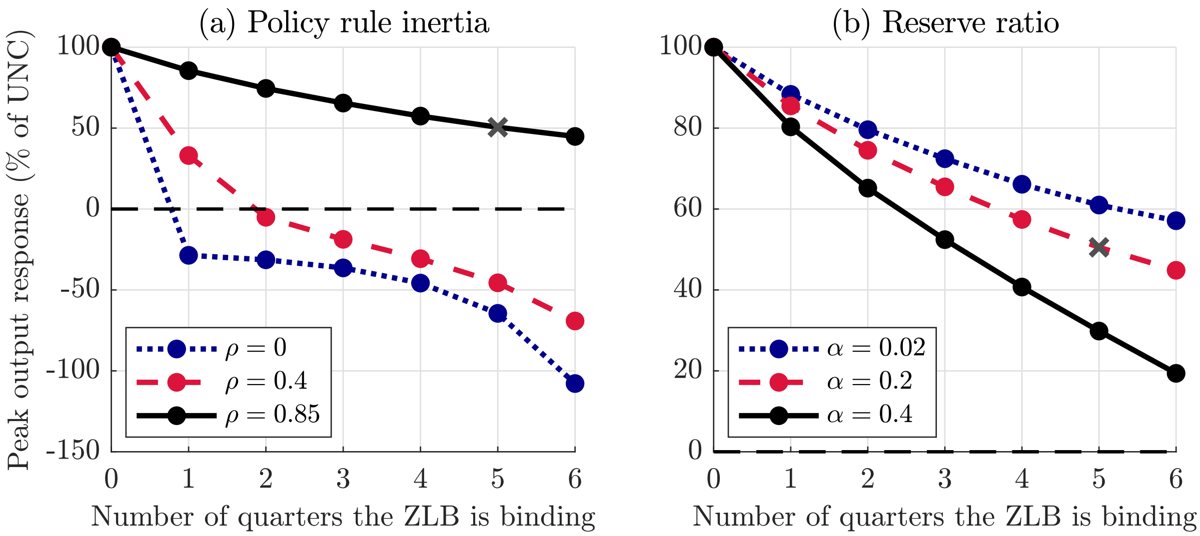
This normalization is important because it strips out the effect of parameter changes on the effectiveness of "conventional" monetary policy in the model. Figures 8 and 9 illustrate the need for this. The effect of monetary policy on output in the unconstrained scenario (I, blue-dot) is heavily dependent on the degree of policy inertia: Changing $\rho = 0.85$ to $\rho = 0$, the peak output response decreases from 26 to 3 basis points, respectively. To strip out this effect, for the purpose of this sensitivity exercise we do not report the absolute effectiveness of negative rates but their effectiveness relative to unconstrained monetary policy with the same value of ρ. The same normalization is used in Figure 12(b), where we shift the focus to the reserve-to-deposit ratio. In both cases, when the ZLB binds for zero quarters (the model is unconstrained) the value reported is always 100%.

Figure 12(a) offers two important insights. One, negative interest rates are less effective if the ZLB is expected to bind for longer. This is because, if the deposit rate is likely to be constrained at zero for a long period of time, then the effect of lowering the reserve rate by 25 basis points today will have very little effect on the path of the deposit rate. By increasing the severity of the initial risk premium shock (with the ZLB binding for 6 rather than 4 quarters), the effectiveness of negative interest rates drop from 57% to 45% of an unconstrained monetary policy easing. Two, a central bank with a lower degree of inertia will find negative interest rates to be less effective. In our model, negative interest rates are only expansionary as a result of signalling a lower expected path of future deposit rates. In fact, once we reduce the degree of inertia to $\rho = 0.4$, negative interest rates are only effective in a 1-period ZLB scenario. If instead the ZLB is expected to last for 2 periods, then negative interest rates become contractionary. Without inertia ($\rho = 0$), negative interest rates are contractionary even for a 1-period ZLB scenario.

Figure 12(b) conducts a similar sensitivity analysis for the size of the reserve-to-deposit ratio. When banks want to hold a higher reserve-to-deposit ratio, this diminishes the positive impact of a negative interest rate policy as the costly interest margin channel is amplified. With inertia in the policy rule held constant at $\rho = 0.85$, doubling the amount of reserves banks want to hold to an extreme value of $\alpha = 0.4$ results in an only marginally expansionary effect of negative rates if the ZLB is expected to bind for many quarters. In this case, the signalling channel only slightly dominates the interest margin channel.

[25] To be precise, for an impulse response vector denoted by $\boldsymbol{y}$, where y_t is the value of output in period t after the shock, we find t^* that is the maximum of $|\boldsymbol{y}|$. We then plot y_{t^*} as a percentage of $y_{t^*}^{\mathrm{UNC}}$.

Figure 12: Policy inertia, reserve-to-deposit ratio, and the ZLB



Note: The x-axis scales with the size of the initial risk premium shock. The y-axis reports the absolute peak response of output to a -25 basis point iid monetary policy shock for the corresponding ZLB duration relative to the effect of an unconstrained monetary policy shock. The $\times$ denotes the baseline experiment.

Overall, we conclude that our main finding that negative interest rates are effective is robust. Even with $\rho = 0.8$ (the lowest degree of policy inertia in Figure 6), $\alpha = 0.27$ (the largest reserve-to-deposit ratio in Figure 5), and a severe economic downturn with a ZLB duration of 6 quarters, negative interest rates are expansionary in our model.

Signalling and the forward guidance puzzle A criticism of new-Keynesian models is that agents in the model are too forward-looking and sensitive to changes in future interest rates. In the following, we show that our main results on the signalling channel are robust to the "forward guidance puzzle" critique (Del Negro et al., 2012).

Following McKay et al. (2016) and Bhattarai et al. (2019) we resolve the forward guidance puzzle with the introduction of additional discounting, $\partial \leq 1$, into the consumption Euler equation, dampening households' sensitivity to changes in expected future interest rates. The augmented consumption Euler equation is given by

$$1 = \mathbb{E}_t \beta \frac{\mu_{t+1}^{\partial}}{\mu^{\partial - 1} \mu_t} \frac{\exp(\zeta_t) R_{d,t}}{\Pi_{t+1}}, \tag{47}$$

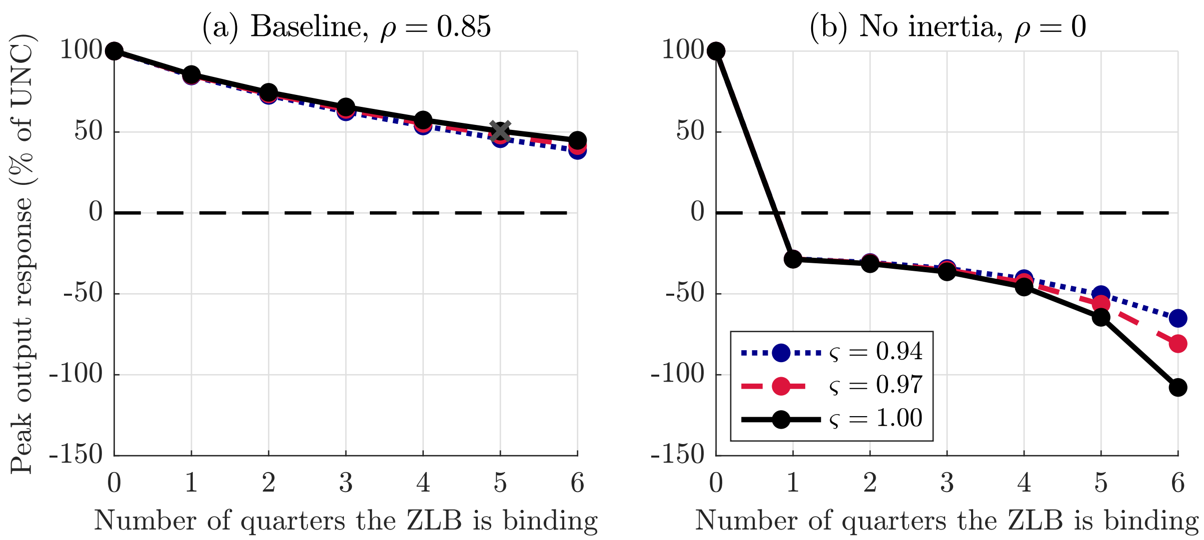
where μ_t is the marginal utility of consumption. The discounting parameter, ∂, is introduced in the specified form so as not to distort the steady state. A first-order approximation of (47) yields the linearized discounted Euler equation from McKay et al. (2017).

Figure 13 reports the effectiveness of negative interest rates when the discounting parameter, ∂, is set to 1, 0.97, and 0.94, respectively. When $\partial = 1$, there is no discounting, and the results are as above. The value of $\partial = 0.97$ is the standard value prescribed by

McKay et al. (2017) for resolving the forward guidance puzzle and so $\partial = 0.94$ can be considered an extreme parameterization for the sake of robustness. Figure 13 shows that our results regarding the effectiveness of negative interest rates are both qualitatively and quantitatively robust to variation in the discounting parameter.

For example, in our baseline experiment with 5 periods at the ZLB, correcting for the forward guidance puzzle only reduces the effectiveness of a negative rates from 51% to 48% and 46%, respectively. The reason is that we report the effectiveness of a negative rate policy *relative* to an unconstrained policy easing, so we control for the effects of the forward guidance puzzle in the denominator, even in the baseline without discounting. When we add discounting, it reduces both the power of the negative rate shock and the unconstrained monetary policy shock, and so the relative effect is left largely unchanged.

Figure 13: Correcting for the forward guidance puzzle



Note: The x-axis scales with the size of the initial risk premium shock. The y-axis reports the absolute peak response of output to a -25 basis point iid monetary policy shock for the corresponding ZLB duration relative to the effect of an unconstrained monetary policy shock. x denotes the baseline experiment.

Focusing on the small quantitative differences that can be observed in Figure 13, we make two further observations: One, correcting for the forward guidance puzzle dampens both the positive and negative effects of negative interest rates. As Panel (b) highlights, discounting that reduces households' sensitivity to future interest rates diminishes not only the power of the expansionary signalling channel but also moderates the contractionary interest margin channel (the only channel active in the case without policy inertia). This again sheds light on why our results are not an artefact of the forward guidance puzzle. Two, correcting for the forward guidance puzzle has more effect on the results when the expected ZLB duration is long. It is only when negative rates are expected to be used for longer and their impact in future periods becomes more relevant that introducing additional discounting has a small impact.

4 Conclusion

Negative interest rates are a relatively new, albeit controversial, monetary policy tool. This paper studies a novel signalling channel and asks 1) under what conditions are negative interest rates an optimal monetary policy tool and 2) are negative interest rates effective? To answer the latter, we provide evidence that in a carefully estimated medium-scale new-Keynesian model, the answer is likely yes. For the majority of the parameter space, the signalling channel dominates the costly interest margin channel. This exemplifies the importance of taking into account general equilibrium effects and cautions against partial equilibrium views of policy actions. In countries in which the central bank has adopted a negative interest rate policy, many commercial banks have been vocally critical about the contractionary effects on their interest margins and profits. However, as we demonstrate, negative interest rates—via policy signalling—have potentially large beneficial general equilibrium effects for banks' asset values and balance sheet health not obviously attributable to the actions of the central bank.

One may be concerned that this quantitative result relies on inertia in a non-optimized, estimated policy rule. In the first part of the paper, we take an optimal policy approach and prove conditions under which negative interest rates are (not) part of an optimal policymaker's toolkit. We prove that negative rates are redundant when the policymaker has full commitment. Full commitment, however, is generally not regarded as a reasonable description of the reality. We show that under more realistic conditions, in which central banks do not have full commitment but have a preference for policy smoothing, negative interest rates can be a welfare improving policy tool.

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APPENDIX: FOR ONLINE PUBLICATION

A Stylized model and optimal policy

Appendix A relates to Section 2 on optimal policy in the stylized model. Section A.1 documents the full derivation of the stylized model. Section A.2 shows that the stylized model captures key features of the quantitative model if a Taylor-type policy rule is added. Section A.3 derives the first-order conditions under commitment and discretion and proves Propositions 2 and 3, respectively. Section A.4 describes the non-linear solution algorithm used for the numerical example. Section A.5 derives the consumption equivalent measure of welfare and provides welfare results. Finally, Section A.6 provides analytical results for a simplified version of the model that effectively reduces to a two-period problem.[26]

A.1 Derivation of the log-linear form [Section 2.2]

New-Keynesian IS equation The household problems and first-order conditions are given in the main text. In steady state, $R_d = 1/\beta$. The log-linear form of the first-order conditions for the saver household are given by

$$c_{s,t} = \mathbb{E}_t c_{s,t+1} - \frac{1}{\sigma}\left(r_{d,t} - \mathbb{E}_t\pi_{t+1} - s_t\right), \tag{A.1.1}$$

$$\varphi l_{s,t} = -\sigma c_{s,t} + w_{s,t}, \tag{A.1.2}$$

where lower case letters refer to log-levels. Equally, the log-linear first-order conditions for the borrower household are given by

$$c_{b,t} = \mathbb{E}_t c_{b,t+1} - \frac{1}{\sigma}\left(r_{b,t} - \mathbb{E}_t\pi_{t+1} - s_t\right), \tag{A.1.3}$$

$$\varphi l_{b,t} = -\sigma c_{b,t} + w_{b,t}, \tag{A.1.4}$$

where, in steady state, $R_b = 1/\beta_b$. The log-linear aggregate resource constraint is given by $y_t = (1-\mathsf{c})\, c_{s,t} + \mathsf{c} c_{b,t}$, where $\mathsf{c} \equiv C_b/Y$. Combining this definition with the two individual Euler equations gives the aggregate Euler equation:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-\mathsf{c}}{\sigma}\left(r_{d,t} - \mathbb{E}_t\pi_{t+1} - s_t\right) - \frac{\mathsf{c}}{\sigma}\left(\mathbb{E}_t r_{b,t+1} - \mathbb{E}_t\pi_{t+1} - s_t\right). \tag{A.1.5}$$

[26] For expositional clarity, we simplify the notation compared to Section 2. In particular, we drop time subscripts and replace them with recursive notation. y denotes the output gap.

Next, substituting the transfer from savers to borrowers into the borrower household's budget constraint gives the following simple borrower household consumption function: $C_{b,t} = B_t$. Using the definition for leverage, $\Phi_t = B_t/N_t$, the log-linear form of the borrower household consumption function is given by $c_{b,t} = \phi_t + n_t$. Rearranging the borrower household's Euler condition, $\frac{1}{\sigma}\left(r_{b,t} - \mathbb{E}_t\pi_{t+1} - s_t\right) = \mathbb{E}_t c_{b,t+1} - c_{b,t}$, and combining it with the consumption function above, we can rewrite the aggregate Euler equation as

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-\mathsf{c}}{\sigma}\left(r_{d,t} - \mathbb{E}_t\pi_{t+1} - s_t\right) - \mathsf{c}\left(\mathbb{E}_t\phi_{t+1} - \phi_t + \mathbb{E}_t n_{t+1} - n_t\right). \tag{A.1.6}$$

New-Keynesian Phillips curve Log-linearizing the production sector's first-order conditions yields the textbook new-Keynesian Phillips curve in terms of marginal cost,

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \frac{(1-\iota\beta)(1-\iota)}{\iota} mc_t. \tag{A.1.7}$$

Log-linear marginal cost and aggregate output are given by $mc_t = \omega w_{s,t} + (1-\omega) w_{b,t}$ and $y_t = \omega l_{s,t} + (1-\omega) l_{b,t}$, respectively. Using the two labour-supply first-order conditions from the household problem, we can rewrite marginal cost as follows:

$$mc_t = (\varphi + \sigma) y_t, \tag{A.1.8}$$

and the Phillips curve as

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \frac{(1-\iota\beta)(1-\iota)(\varphi+\sigma)}{\iota} y_t. \tag{A.1.9}$$

Note that since we only consider disturbances to households' subjective discount factors, the output gap coincides with output and hence y_t can be relabeled as the output gap.

Financial sector equilibrium conditions Steady state leverage is given by $\bar{N}$. The log-linear net worth evolution equation is given by

$$n_{t+1} = \theta R\left(n_t + \Phi\left(r_{b,t} - \pi_{t+1}\right) - (\Phi - 1)\left(\frac{r_{d,t} - \alpha r_t}{1-\alpha} - \pi_{t+1}\right)\right). \tag{A.1.10}$$

When $\theta = 0$, then $n_{t+1} = 0$. The steady state tax on banks ensures that in steady state $R_b(1-\tau) = R_d$. The log-linear incentive compatibility constraint is given by

$$\phi_t = \left(\mathbb{E}_t m_{t,t+1} - \pi_{t+1}\right) + \theta\mathbb{E}_t\phi_{t+1} + \left(\Phi r_{b,t} - (\Phi - 1)\frac{r_{d,t} - \alpha r_t}{1-\alpha}\right). \tag{A.1.11}$$

where $m_{t,t+1}$ is the log-linear stochastic discount factor of the saver household.

Substituting for $r_{b,t}$ using the borrower household's Euler equation gives

$$\phi_t = r_{d,t} + \theta \mathbb{E}_t \phi_{t+1} + \Phi\sigma \left(\mathbb{E}_t \phi_{t+1} - \phi_t + \mathbb{E}_t n_{t+1} - n_t\right) + \Phi \left(\mathbb{E}_t \pi_{t+1} + s_t\right) - (\Phi - 1) \frac{r_{d,t} - \alpha r_t}{1 - \alpha}. \tag{A.1.12}$$

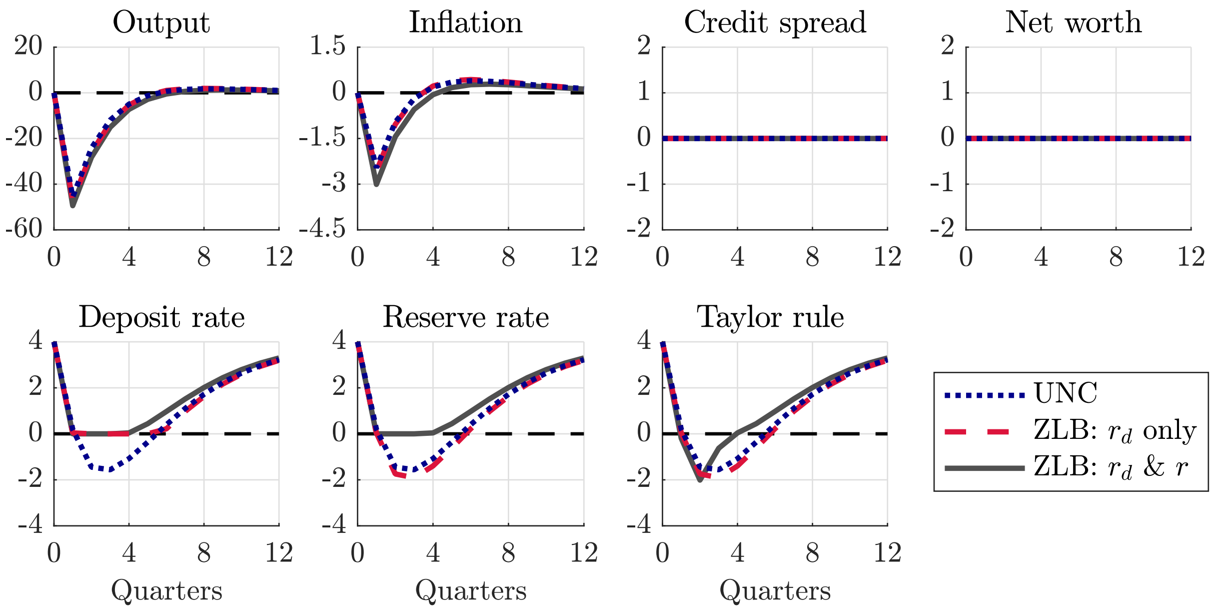
Rearranging and setting $\theta = 0$ such that $n_t = 0$ gives (23). To conclude, when $\theta > 0$, the model is described by five endogenous variables $\{\pi_t, y_t, \phi_t, n_t, r_{d,t}\}$ and four private sector conditions: (A.1.6), (A.1.9), (A.1.10), and (A.1.12).

A.2 The stylized model with a Taylor-type rule [Section 2.3]

In order to study optimal policy, we make use of the stylized model derived above. In the following, we show this model captures the key features of the quantitative model from Section 3 rather well by replicating several of the figures from Section 3.3.

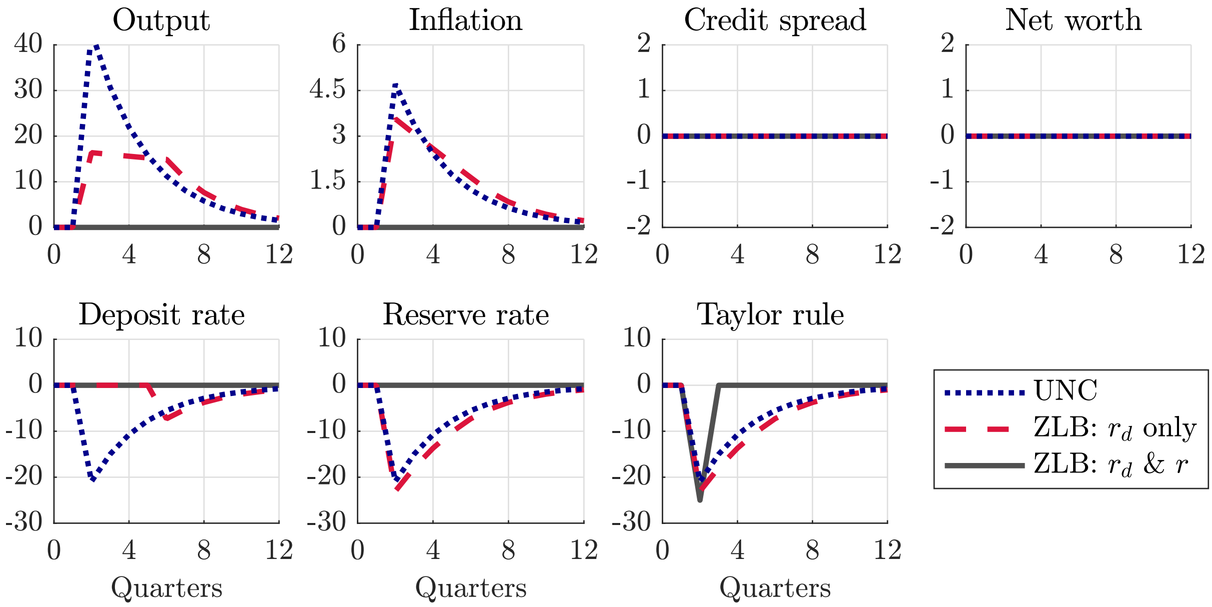
The experiments are conducted combining the IS and Phillips curve of the stylized model, Equations (21) and (24), respectively, as parameterized in Table 2, and the Taylor-type rule of the quantitative model, Equation (40), as parameterized in Table 3.

Figure A.1: Natural real rate shock **with inertia** in the policy rule



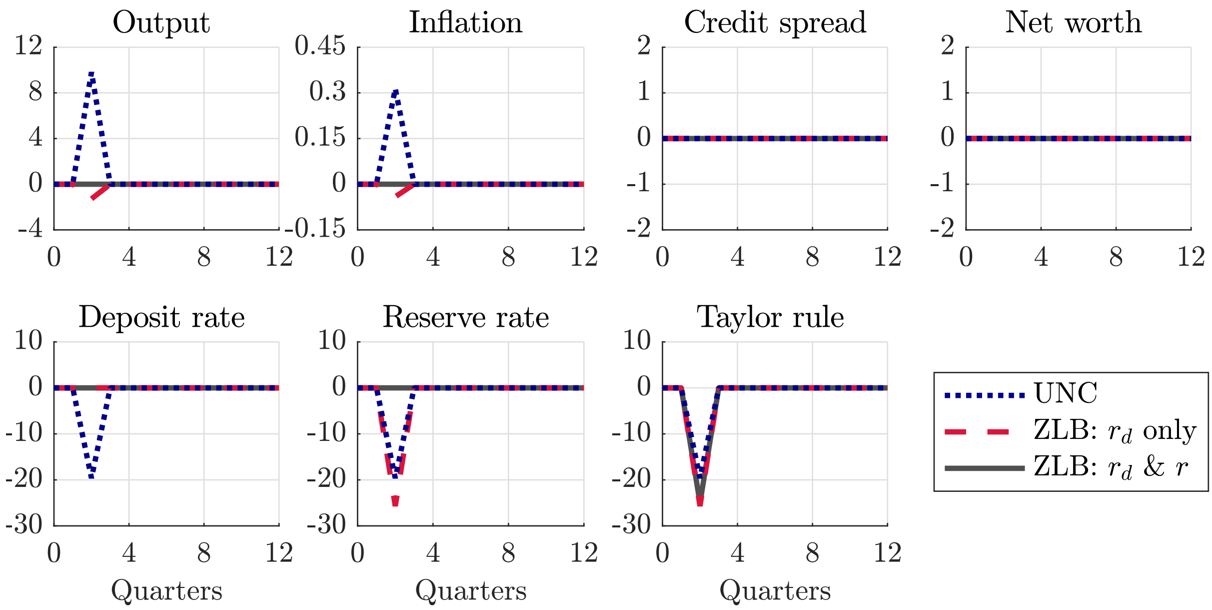
Note: Stylized NK model with a policy rule. $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a natural rate shock that brings the economy to the ZLB for 4 quarters. All interest rates displayed are in annualized percent. All other variables are in 100×log-deviation from steady state. Inflation is annualized.

Figure A.2: Monetary policy shock **with inertia** in the policy rule



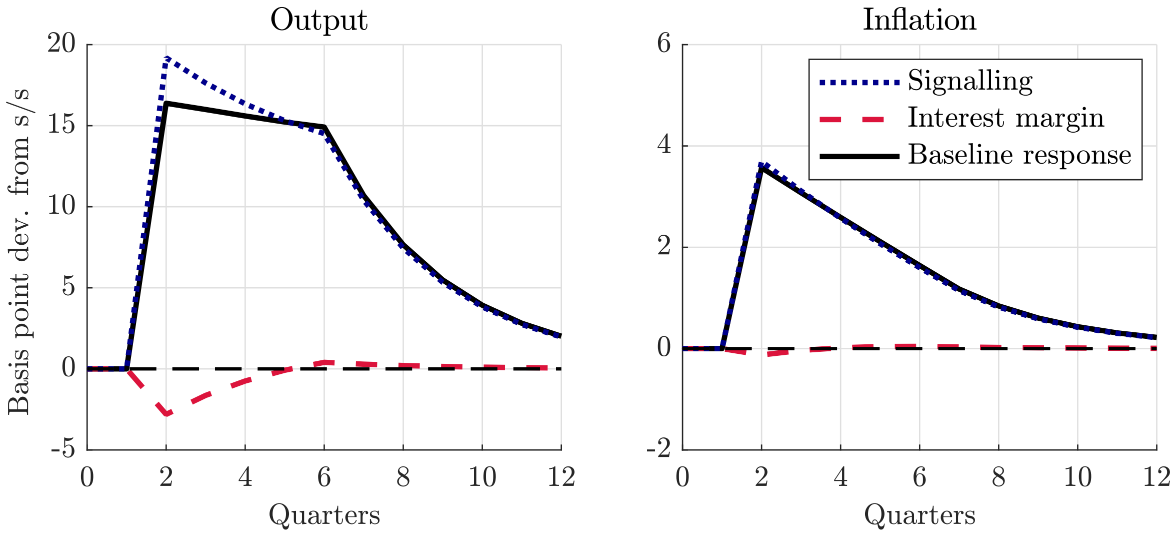
Note: Stylized NK model with a policy rule. $\phi = 0.2$, $\rho = 0.85$. Impulse responses to a -25 basis point iid monetary policy shock at the ZLB. All interest rates displayed are in annualized basis points. Output and inflation are in basis point deviation from steady state. Inflation is annualized.

Figure A.3: Monetary policy shock **without inertia** in the policy rule



Note: Stylized NK model with a policy rule. $\phi = 0.2$, $\rho = 0$. Impulse responses to a -25 basis point iid monetary policy shock at the ZLB. All interest rates displayed are in annualized basis points. Output and inflation are in basis point deviation from steady state. Inflation is annualized.

Figure A.4: Contribution of signalling and interest margin channels



Note: Stylized NK model with a policy rule. Impulse responses to a -25 basis point iid monetary policy shock at the ZLB. Inflation is annualized. "Signalling" and "Interest margin" plot $\phi = 0$ and $\rho = 0$.

A.3 Propositions 2 and 3 [Section 2.3]

The recursive problem of the optimal policymaker is given by

$$V(\pi_{-1}, s) = \max_{\{\pi, y, r_d, r\}} -\frac{1}{2}\left(\pi^2 + \lambda y^2\right) + \beta \mathbb{E} V(\pi, s_{+1})$$

$$\pi = \beta \mathbb{E}\pi_{+1} + \gamma \pi_{-1} + \kappa y, \tag{PC}$$

$$y = \mathbb{E} y_{+1} - \sigma^{-1}\left(r_d - \mathbb{E}\pi_{+1} - s\right) - \phi\left(r_d - r\right), \tag{IS}$$

$$r_d \geq 0 \quad \text{(ZLB)}, \qquad r_d - r \geq 0 \quad \text{(ARB)}, \qquad r_d\left(r_d - r\right) = 0 \quad \text{(X)},$$

where the decentralized competitive equilibrium and a set of three inequality constraints on the policy instruments constrain the policymakers optimal choice.

Note that this model is a slightly more general version of the stylized model in Section 2. In particular, all proofs go through even if we add lagged inflation into the new-Keynesian Phillips Curve, resulting from, for example, price indexation.

Under **commitment**, the equilibrium can be summarized by the following equations:

$$
\begin{aligned}
& & \pi &= \beta \mathbb{E}\pi_{+1} + \gamma \pi_{-1} + \kappa y, \\
& & y &= \mathbb{E}y_{+1} - \sigma^{-1}\left(r_d - \mathbb{E}\pi_{+1} - s\right) - \phi\left(r_d - r\right), \\
\pi: & & 0 &= \pi - \beta \mathbb{E}\boldsymbol{V_1}(\pi, s_{+1}) - \zeta_{PC} + \zeta_{PC_{-1}} + \sigma^{-1}\beta^{-1}\zeta_{IS_{-1}}, \\
y: & & 0 &= \lambda y + \kappa \zeta_{PC} - \zeta_{IS} + \beta^{-1}\zeta_{IS_{-1}}, \\
r_d: & & 0 &= \zeta_{IS}\left(\sigma^{-1} + \phi\right) + \zeta_{ZLB} + \zeta_{ARB} + \zeta_X\left(2r_d - r\right), \\
r: & & 0 &= \zeta_{IS}\phi + \zeta_{ARB} + \zeta_X r_d, \\
\mathrm{KT}_1: & & 0 &= \zeta_{ZLB} r_d, \\
\mathrm{KT}_2: & & 0 &= \zeta_{ARB}\left(r_d - r\right), \\
\mathrm{EC}: & & \boldsymbol{V_1}(\pi_{-1}, s) &= -\gamma \zeta_{PC},
\end{aligned}
$$

where the ζ are Lagrange multipliers. Based on the set of three inequality constraints on the policy instruments, the following regimes can be defined: `Regime I`: $\{r_d > 0,\ r = r_d\}$, `Regime II`: $\{r_d = 0,\ r < 0\}$, and `Regime III`: $\{r_d = 0,\ r = 0\}$.

Proof of Proposition 2 Proposition 2 states that, under commitment, the reserve rate will never be set negative. This is equivalent to stating, $r \in$ `Regime II` is not optimal. We prove this by contradiction.

For a given state vector, $\boldsymbol{s} = \left\{\pi_{-1}, \zeta_{IS_{-1}}, \zeta_{PC_{-1}}, s\right\}$, define $r^{\mathrm{c,zlb}}(\boldsymbol{s})$ and $r_d^{\mathrm{c,zlb}}(\boldsymbol{s})$ as the reserve and deposit rate, respectively, that are the solution to the constrained commitment problem where negative rates are not allowed, $r \in \{$`Regime I, Regime III`$\}$, and $r^{\mathrm{c,nir}}(\boldsymbol{s})$ and $r_d^{\mathrm{c,nir}}(\boldsymbol{s})$ as the reserve and deposit rate that solve the commitment problem where negative reserve rates are allowed, i.e. $r \in \{$`Regime I, Regime II, Regime III`$\}$.

Consider $\phi > 0$. Suppose $\exists\ \boldsymbol{s} \mid V^{\mathrm{c,nir}}(\boldsymbol{s}) > V^{\mathrm{c,zlb}}(\boldsymbol{s}) \longrightarrow r^{\mathrm{c,nir}} < 0$ and $r_d^{\mathrm{c,nir}} = 0$ (`Regime II`). Then, the equilibrium allocation for $\{\pi, y\}$ is given by (PC) and (IS), where (IS) can be reduced to $y = \mathbb{E}y_{+1} + \sigma^{-1}\left(\mathbb{E}\pi_{+1} + s\right) + \phi r^{\mathrm{c,nir}}$. Yet, $r^{\mathrm{c,*}} = r_d^{\mathrm{c,*}} = -\phi\sigma r^{\mathrm{c,nir}} > 0$ (`Regime I`) generates the same equilibrium allocation, $V^{\mathrm{c,*}}(\boldsymbol{s}) = V^{\mathrm{c,nir}}(\boldsymbol{s})$. However, $r^{\mathrm{c,*}}$ and $r_d^{\mathrm{c,*}}$ are in the space of the constrained commitment problem such that $V^{\mathrm{c,*}}(\boldsymbol{s}) = V^{\mathrm{c,nir}}(\boldsymbol{s}) \leq V^{\mathrm{c,zlb}}(\boldsymbol{s})$. Thus, we have a contradiction.

Consider $\phi = 0$. The reserve rate in this case drops out of the equilibrium system that determines $\{y, \pi, r_d, \zeta_{IS}, \zeta_{PC}\}$ as $\phi\left(r_d - r\right) = 0\ \forall\ r$ in (IS). There is no role for negative interest rates. ■

To study optimal time-consistent policy with and without policy smoothing, we augment the policymaker's objective function by adding a preference for smoothing interest rates, given by ψ. This gives the following, slightly modified, recursive planner's problem:

$$\begin{aligned}
V(r_{-1},\pi_{-1},s) = \max_{\{\pi,y,r_d,r\}} & -\frac{1}{2}\left((1-\psi)\left(\pi^2+\lambda y^2\right)+\psi\left(r-r_{-1}\right)^2\right)+\beta\mathbb{E}V(r,\pi,s_{+1}) \\
& \pi = \beta\mathbb{E}\pi_{+1}+\gamma\pi_{-1}+\kappa y, && \text{(PC)} \\
& y = \mathbb{E}y_{+1}-\sigma^{-1}\left(r_d-\mathbb{E}\pi_{+1}-s\right)-\phi\left(r_d-r\right), && \text{(IS)} \\
& r_d \geq 0 \quad \text{(ZLB)}, \quad r_d - r \geq 0 \quad \text{(ARB)}, \quad r_d\left(r_d-r\right)=0 \quad \text{(X)}.
\end{aligned}$$

Under **discretion**, the equilibrium can be summarized by the following equations:

$$\begin{aligned}
& & \pi &= \beta\mathbb{E}\boldsymbol{\pi}\left(r,\pi,s_{+1}\right)+\gamma\pi_{-1}+\kappa y, \\
& & y &= \mathbb{E}\boldsymbol{y}\left(r,\pi,s_{+1}\right)-\sigma^{-1}\left(r_d-\mathbb{E}\boldsymbol{\pi}\left(r,\pi,s_{+1}\right)-s\right)-\phi\left(r_d-r\right), \\
\pi: & & 0 &= (1-\psi)\,\pi-\mathbb{E}\boldsymbol{V_2}\left(r,\pi,s_{+1}\right)-\zeta_{PC}\left(1-\beta\mathbb{E}\boldsymbol{\pi_2}\left(r,\pi,s_{+1}\right)\right) \\
& & &\quad +\zeta_{IS}\left(\mathbb{E}\boldsymbol{y_2}\left(r,\pi,s_{+1}\right)+\sigma^{-1}\mathbb{E}\boldsymbol{\pi_2}\left(r,\pi,s_{+1}\right)\right), \\
y: & & 0 &= (1-\psi)\,\lambda y-\zeta_{IS}+\kappa\zeta_{PC}, \\
r_d: & & 0 &= \zeta_{IS}\left(\sigma^{-1}+\phi\right)+\zeta_{ZLB}+\zeta_{ARB}+\zeta_X\left(2r_d-r\right), \\
r: & & 0 &= \psi\left(r-r_{-1}\right)-\beta\mathbb{E}\boldsymbol{V_1}\left(r,\pi,s_{+1}\right)+\beta\mathbb{E}\boldsymbol{\pi_1}\left(r,\pi,s_{+1}\right)\zeta_{PC} \\
& & &\quad +\zeta_{IS}\left(\mathbb{E}\boldsymbol{y_1}\left(r,\pi,s_{+1}\right)+\sigma^{-1}\mathbb{E}\boldsymbol{\pi_1}\left(r,\pi,s_{+1}\right)\right)+\zeta_{ARB}+\zeta_X r_d, \\
\text{KT}_1: & & 0 &= \zeta_{ZLB}r_d, \\
\text{KT}_2: & & 0 &= \zeta_{ARB}\left(r_d-r\right), \\
\text{EC}_1: & & \boldsymbol{V}_1\left(r_{-1},\pi_{-1},S\right) &= -\psi\left(r-r_{-1}\right), \\
\text{EC}_2: & & \boldsymbol{V}_2\left(r_{-1},\pi_{-1},S\right) &= -\zeta_{PC}\gamma,
\end{aligned}$$

where the ζ are Lagrange multipliers. Analogous to the commitment problem, based on the set of three inequality constraints, once again the following regimes can be defined: `Regime I`: $\{r_d > 0,\ r = r_d\}$, `Regime II`: $\{r_d = 0,\ r < 0\}$, and `Regime III`: $\{r_d = 0,\ r = 0\}$.

Proof of Proposition 3 Proposition 3 states that, under discretion, with $\psi = 0$, the reserve rate will never be set negative. Equivalently, $r \in$ `Regime II` is not optimal. We prove this by contradiction.

For a given state vector, $\boldsymbol{s} = \left\{r_{-1},\pi_{-1},\zeta_{IS_{-1}},\zeta_{PC_{-1}},s\right\}$, define $r^{\mathrm{d,zlb}}(\boldsymbol{s})$ and $r_d^{\mathrm{d,zlb}}(\boldsymbol{s})$ as the reserve and deposit rate, respectively, that are the solution to the constrained discretion problem where negative rates are not an option, $r \in \{$`Regime I, Regime III`$\}$, and

$r^{\mathrm{d,nir}}(\boldsymbol{s})$ and $r_d^{\mathrm{d,nir}}(\boldsymbol{s})$ as the reserve and deposit rate that solve the discretion problem where negative reserve rates are allowed, i.e. $r \in \{\texttt{Regime I, Regime II, Regime III}\}$.

With $\psi = 0$, $\boldsymbol{V_1}(r_{-1}, \pi_{-1}, s) = 0$ and r_{-1} drops out as a state variable, i.e. expectations and allocations in the discretionary equilibrium are independent of r_{-1}. Thus, redefining $\boldsymbol{s} = \left\{\pi_{-1}, \zeta_{IS_{-1}}, \zeta_{PC_{-1}}, s\right\}$ we proceed as in the commitment case.

Consider $\phi > 0$: Suppose $\exists\ \boldsymbol{s} \mid V^{\mathrm{d,nir}}(\boldsymbol{s}) > V^{\mathrm{d,zlb}}(\boldsymbol{s}) \longrightarrow r^{\mathrm{d,nir}} < 0$ and $r_d^{\mathrm{d,nir}} = 0$ (`Regime II`). Then, the equilibrium allocation for $\{\pi, y\}$ is given by (PC) and (IS), where (IS) can be reduced to $y = \mathbb{E}\boldsymbol{y}(\pi, s_{+1}) + \sigma^{-1}(\mathbb{E}\boldsymbol{\pi}(\pi, s_{+1}) + s) + \phi r^{\mathrm{d,nir}}$. Yet, $r^{\mathrm{d,*}} = r_d^{\mathrm{d,*}} = -\phi\sigma r^{\mathrm{d,nir}} > 0$ (`Regime I`) generates the same equilibrium allocation, $V^{\mathrm{d,*}}(\boldsymbol{s}) = V^{\mathrm{d,nir}}(\boldsymbol{s})$. However, $r^{\mathrm{d,*}}$ and $r_d^{\mathrm{d,*}}$ are in the space of the constrained commitment problem such that $V^{\mathrm{d,*}}(\boldsymbol{s}) = V^{\mathrm{d,nir}}(\boldsymbol{s}) \leq V^{\mathrm{d,zlb}}(\boldsymbol{s})$. Thus, we have a contradiction.

Consider $\phi = 0$: The reserve rate in this case drops out of the equilibrium system that determines $\{y, \pi, r_d, \zeta_{IS}, \zeta_{PC}\}$ as $\phi(r_d - r) = 0\ \forall\ r$ in (IS). There is no role for negative interest rates. ■

A.4 Policy function iteration [Section 2.4]

To derive a solution to the time-consistent optimal policymaker's problem, we use a policy function iteration algorithm, solving for $\pi(r, g)$, $y(r, g)$, $r'(r, g)$, $r_d(r, g)$, $\zeta_{ZLB}(r, g)$, and $\zeta_{ARB}(r, g)$. The algorithm proceeds as follows:

1. Set N_i: number of points on the interest rate grid, N_s: number of exogenous states, ϵ: tolerance limit for convergence, $\mathtt{u}$: updating parameter. Set grid points $\{i_0, \ldots, i_{N_i}\}$. The AR(1) process for the natural rate, g, is approximated using Tauchen and Hussey (1991)'s quadrature algorithm that gives a set of grid points $\{s_0, \ldots, s_{N_s}\}$ and a transmission matrix, M.

2. Start iteration j with conjectured functions for $r'^j(r, g)$ and $\pi^j(r, g)$. The initial functions are set to $r'^0(r, g) = 1/\beta - 1$ and $\pi^0(r, g) = 0$. $\pi(r, g)$ is only defined at the nodes of the grids for the policy rate and shock, but since $r'(r, g)$ is generally not going to match node grids exactly, the function $\pi(r, g)$ is interpolated over the first argument to determine its values at $\boldsymbol{\pi}^j(r'^j(r, g), g')$. Construct expectations $\mathbb{E}\boldsymbol{\pi}^j(r'^j(r, g), g')$, denoted $\mathbb{E}\boldsymbol{\pi}^j$ for short. Repeat for r', giving $\mathbb{E}\boldsymbol{r}^j$.

3. Using the Phillips curve, calculate y:

$$y^j(r,g) = \frac{1}{\kappa}\left(\pi^j(r,g) - \mathbb{E}^j\boldsymbol{\pi}\right).$$

4. Construct one-step ahead output gap expectations, $\mathbb{E}\boldsymbol{y}^j$.

5. Construct the deposit rate function $r_d(r,g) = \max(0, r'^j(r,g))$.

6. Using the IS and Phillips curve, re-calculate y and π, respectively:

$$y^*(r,g) = \mathbb{E}\boldsymbol{y}^j - \sigma^{-1}\left(r_d(r,g) - \mathbb{E}\boldsymbol{\pi}^j - g\right) - \phi\left(r_d(r,g) - r'^j(r,g)\right),$$
$$\pi^*(r,g) = \beta\mathbb{E}\boldsymbol{\pi}^j + \kappa y^*(r,g),$$

and then update expectations, $\mathbb{E}\boldsymbol{y}^*$ and $\mathbb{E}\boldsymbol{\pi}^*$.

7. Construct numerical derivatives of π as follows:

$$\pi_1(r,g) \equiv \frac{\partial \pi^*(r,g)}{\partial r} = \begin{cases} \frac{\pi^*(i_k,g)-\pi^*(i_{k-1},g)}{i_k-i_{k-1}} & \text{for} \quad k=1,...,N_i, \\ \frac{\pi^*(i_1,g)-\pi^*(i_0,g)}{i_1-i_0} & \text{for} \qquad k=0. \end{cases}$$

and denote the function π_1 for short. Calculate the one-step ahead values of these derivative functions, $\boldsymbol{\pi}_1(r'^j(r,g), g')$, and calculate expectations, denoted $\mathbb{E}\boldsymbol{\pi}_1$. Repeat for y giving $\mathbb{E}\boldsymbol{y}_1$.

8. Using the FOC equation to re-calculate r':

`for` $r'^j(r,g) > 0$,

$$r'^*(r,g) = \frac{1}{\psi(1+\beta)}\begin{pmatrix} \psi r + \psi\beta\mathbb{E}\boldsymbol{r}^j - (1-\psi)\beta\mathbb{E}\boldsymbol{\pi}_1\pi^*(r,g) + \zeta^*_{ZLB}(r,g) \\ -(1-\psi)\left(\mathbb{E}\boldsymbol{y}_1 + \sigma^{-1}\mathbb{E}\boldsymbol{\pi}_1 - \sigma^{-1}\right)(\lambda y^*(r,g) + \kappa\pi^*(r,g)) \end{pmatrix},$$

`else`

$$r'^*(r,g) = \frac{1}{\psi(1+\beta)}\begin{pmatrix} \psi r + \psi\beta\mathbb{E}\boldsymbol{r}^j - (1-\psi)\beta\mathbb{E}\boldsymbol{\pi}_1\pi^*(r,g) + \zeta^*_{ZLB}(r,g) \\ -(1-\psi)\left(\mathbb{E}\boldsymbol{y}_1 + \sigma^{-1}\mathbb{E}\boldsymbol{\pi}_1 + \phi\right)(\lambda y^*(r,g) + \kappa\pi^*(r,g)) \end{pmatrix},$$

9. `if` $\max\left((\pi^*(r,g) - \pi^j(r,g)), (r' * (r,g) - r'^j(r,g))\right) < \epsilon$, then `stop`.
`else` $j = j+1$ and update the guess as follows:

$$\pi^j(r,g) = \mathtt{u}\pi^{j-1}(r,g) + (1-\mathtt{u})\pi^*(r,g),$$
$$r'^j(r,g) = \mathtt{u}r'^{j-1}(r,g) + (1-\mathtt{u})r'^*(r,g).$$

Repeat steps 2-9.

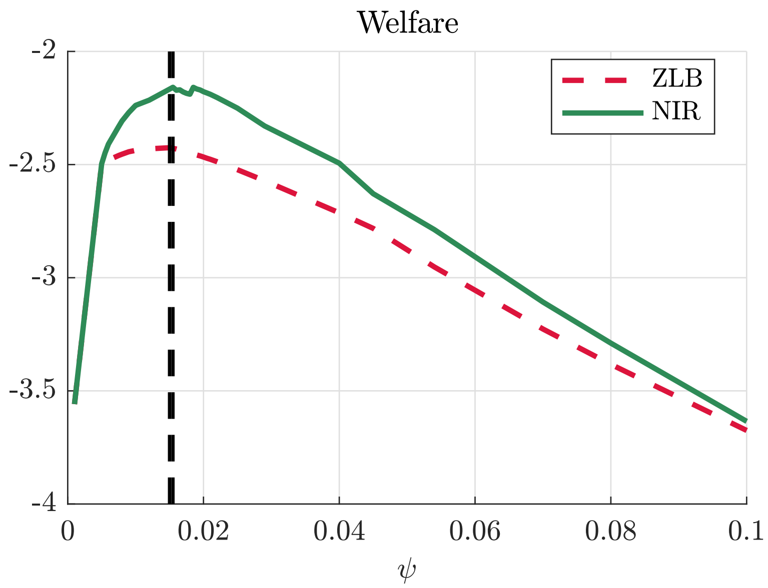
A.5 Welfare [Section 2.4]

The social welfare function can be translated into a consumption equivalent measure via

$$CE = 100 \times (1-\beta)\,\lambda^{-1}\left(\sigma^{-1}+\eta\right)\mathbb{E}\left(V^{SW}\right), \tag{A.5.1}$$

where η is the inverse labor supply elasticity, set to 0.47 in our calibration, and $\mathbb{E}\left(V^{SW}\right)$ is the unconditional mean of the social welfare function. CE is the percentage of steady state consumption that the representative household would forgo in each period to avoid uncertainty. Less negative values thus represent an improvement in welfare. Figure A.5 plots the consumption equivalent measure of welfare across a range of values for the smoothing parameter, ψ. It demonstrates three features. One, allowing for negative interest rates in the toolkit of the policymaker is weakly welfare dominant. Two, it is optimal to delegate policy to a central banker with a small but meaningful preference for smoothing. Three, the optimal value of ψ is virtually the same, irrespective of whether negative interest rates are available or not.

Figure A.5: Welfare and the optimal degree of smoothing



Note: Consumption equivalent as a percent of steady state consumption. Black-dash denotes the optimal value of ψ. ZLB denotes policy without negative interest rates. NIR denotes policy with negative rates.

A.6 Analytical derivations [Section 2.4]

In Section 2.4 we set $\lambda = 0$, we set $\psi = 0$ except for between periods 1 and 2, and $s_t = 0$ for $t > 1$. This allows for an analytical derivation of equilibrium outcomes. In particular, $\pi_t = y_t = 0$ for $t > 2$. Thus, the central banks loss function reduces to

$$-V \propto \pi_1^2 + \beta \left((1-\psi)\, \pi_2^2 + \psi \left(r_2 - r_1 \right)^2 \right). \tag{A.6.1}$$

The policymaker is subject to the following constraints

$$\pi_1 = \beta \pi_2 + \kappa y_1, \tag{A.6.2}$$
$$y_1 = y_2 - \sigma^{-1} \left(r_{d,1} - \pi_2 \right) - \phi \left(r_{d,1} - r_1 \right) + g, \tag{A.6.3}$$
$$\pi_2 = \kappa y_2, \tag{A.6.4}$$
$$y_2 = -\sigma^{-1} r_2, \tag{A.6.5}$$
$$r_{d,1} \geq -\bar{r}, \tag{A.6.6}$$
$$r_2 \geq -\bar{r}, \tag{A.6.7}$$
$$r_{d,1} - r_1 \geq 0, \tag{A.6.8}$$
$$\left(r_{d,1} + \bar{r} \right) \left(r_{d,1} - r_1 \right) = 0, \tag{A.6.9}$$

where the expectations operator has been dropped because there is no uncertainty. In addition, there is no incentive to set a negative interest rate in period 2 so $r_{d,2} = r_2$. In contrast to the main text, we make g mean zero and set the ZLB constraint as $-\bar{r}$.

We consider optimal policy under discretion. There are 4 possible equilibrium outcomes:

$$(++): \quad r_1 > -\bar{r}, \quad r_2 > -\bar{r}, \tag{A.6.10}$$
$$(0+): \quad r_1 = -\bar{r}, \quad r_2 > -\bar{r}, \tag{A.6.11}$$
$$(-+): \quad r_1 < -\bar{r}, \quad r_2 > -\bar{r}, \tag{A.6.12}$$
$$(-0): \quad r_1 = -\bar{r}, \quad r_2 < -\bar{r}. \tag{A.6.13}$$

We solve the problem backwards. First soving for the optimal r_2 given a value for r_1.

For $(\cdot 0)$, we have

$$r_2^{*(0)} = -\bar{r}. \tag{A.6.14}$$

For $(\cdot +)$, the period 2 problem is given by

$$\min_{r_2} \quad (1-\psi)\, \pi_2^2 + \psi \left(r_2 - r_1 \right)^2 \quad \text{s.t.} \quad \pi_2 = -\kappa \sigma^{-1} r_2. \tag{A.6.15}$$

The first-order condition is given by

$$(1-\psi)\left(\kappa\sigma^{-1}\right)^2 r_2 + \psi\left(r_2 - r_1\right) = 0, \tag{A.6.16}$$

or, rearranged, as

$$r_2^{*(+)} = R_2^{(+)} r_1, \tag{A.6.17}$$

$$\pi_2^{*(+)} = \Pi_2^{(+)} r_1, \tag{A.6.18}$$

$$\text{where} \quad R_2^{(+)} \equiv \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2}, \tag{A.6.19}$$

$$\Pi_2^{(+)} \equiv -\kappa\sigma^{-1} R_2^{(+)}. \tag{A.6.20}$$

Now that we have the optimal reaction function for r_2 as a function of r_1, we can solve the period 1 problem, taking the behaviour of the policymaker in period 2 as given.

For (++), the period 1 problem is given by

$$\min_{r_1} \quad \pi_1^2 + \beta\left((1-\psi)\pi_2^2 + \psi\left(r_2 - r_1\right)^2\right) \tag{A.6.21}$$

$$\text{s.t.} \quad \pi_1 = \Pi_1^{(++)} r_1 + \kappa g, \tag{A.6.22}$$

$$\pi_2 = \Pi_2^{(+)} r_1, \tag{A.6.23}$$

$$r_2 = R_2^{(+)} r_1, \tag{A.6.24}$$

$$\text{where} \quad \Pi_1^{(++)} \equiv -\kappa\left(\left(\beta + 1 + \kappa\sigma^{-1}\right)\sigma^{-1} R_2^{(+)} + \sigma^{-1}\right), \tag{A.6.25}$$

and the first-order condition is given by

$$\left(\Pi_1^{(++)} r_1 + \kappa g\right)\Pi_1^{(++)} + \beta\left((1-\psi)\left(\Pi_2^{(+)}\right)^2 r_1 + \psi\left(R_2^{(+)} - 1\right)^2 r_1\right) = 0, \tag{A.6.26}$$

or, rearranged, as

$$r_1^{*(++)} = -\frac{\kappa\Pi_1^{(++)} g}{\left(\Pi_1^{(++)}\right)^2 + \beta\left((1-\psi)\left(\Pi_2^{(+)}\right)^2 + \psi\left(R_2^{(+)} - 1\right)^2\right)}. \tag{A.6.27}$$

For $(-+)$, the constraints are given by

$$\pi_1 = \Pi_1^{(-+)} r_1 + C\Pi_1^{(-+)} \bar{r} + \kappa g, \tag{A.6.28}$$

$$\pi_2 = \Pi_2^{(+)} r_1, \tag{A.6.29}$$

$$r_2 = R_2^{(+)} r_1, \tag{A.6.30}$$

$$\text{where} \quad \Pi_1^{(-+)} \equiv -\kappa \left(\left(\beta + 1 + \kappa\sigma^{-1} \right) \sigma^{-1} R_2^{(+)} - \phi \right), \tag{A.6.31}$$

$$C\Pi_1^{(-+)} \equiv \kappa \left(\sigma^{-1} + \phi \right), \tag{A.6.32}$$

and the solution is given by

$$r_1^{*(-+)} = -\frac{C\Pi_1^{(-+)} \Pi_1^{(-+)} \bar{r} + \kappa \Pi_1^{(-+)} g}{\left(\Pi_1^{(-+)} \right)^2 + \beta \left((1-\psi) \left(\Pi_2^{(+)} \right)^2 + \psi \left(R_2^{(+)} - 1 \right)^2 \right)}. \tag{A.6.33}$$

For $(0+)$, we have

$$r_1^{*(0+)} = -\bar{r}. \tag{A.6.34}$$

For (-0), the constraints are given by

$$\pi_1 = \Pi_1^{(-0)} r_1 + C\Pi_1^{(-0)} \bar{r} + \kappa g, \tag{A.6.35}$$

$$\pi_2 = C\Pi_2^{(0)} \bar{r}, \tag{A.6.36}$$

$$r_2 = -\bar{r}, \tag{A.6.37}$$

$$\text{where} \quad \Pi_1^{(-0)} \equiv \kappa\phi, \tag{A.6.38}$$

$$C\Pi_1^{(-0)} \equiv \kappa \left(\left(\beta + 2 + \kappa\sigma^{-1} \right) \sigma^{-1} + \phi \right), \tag{A.6.39}$$

$$C\Pi_2^{(0)} \equiv \kappa\sigma^{-1}, \tag{A.6.40}$$

and the first-order condition is given by

$$\left(\Pi_1^{(-0)} r_1 + C\Pi_1^{(-0)} \bar{r} + \kappa g \right) \Pi_1^{(-0)} + \beta\psi \left(\bar{r} + r_1 \right) = 0, \tag{A.6.41}$$

or, rearranged, as

$$r^{*(-0)} = -\frac{\Pi_1^{(-0)} C\Pi_1^{(-0)} \bar{r} + \Pi_1^{(-0)} \kappa g + \beta\psi\bar{r}}{\left(\Pi_1^{(-0)} \right)^2 + \beta\psi}. \tag{A.6.42}$$

This completes the full set of equilibrium conditions. Numerically, we solve for each possible case and throw out any solutions which violate the assumptions of that case. If multiple solutions exist, we choose the one that maximizes welfare.

B Quantitative model

Appendix B relates to Section 3 on the effectiveness of negative rates in a quantitative new-Keynesian model. Section B.1 documents the derivation of the financial sector equilibrium in just two equations. Section B.2 provides the full list of equilibrium equations. Section B.3 documents data source and treatment for all time series used in the estimation.

B.1 Derivation of the financial sector equilibrium [Section 3.1]

A banker j solves

$$V_{n,t}\left(j\right) = \max_{\{S_t(j),A_t(j),D_t(j),N_t(j)\}} \mathbb{E}_t \Lambda_{t,t+1}\left(\left(1-\theta\right)N_{t+1}(j) + \theta V_{n,t+1}\left(j\right)\right), \tag{B.1.1}$$

subject to

$$Q_t S_t(j) + A_t(j) = D_t(j) + N_t(j), \tag{B.1.2}$$

$$V_{n,t}\left(j\right) \geq \lambda Q_t S_t(j), \tag{B.1.3}$$

$$A_t(j) = \alpha\left(x_t\right) D_t(j), \tag{B.1.4}$$

$$N_t(j) = R_{k,t} Q_{t-1} S_{t-1}(j) + \frac{R_{t-1}}{\Pi_t} A_{t-1}(j) - \frac{R_{d,t-1}}{\Pi_t} D_{t-1}(j),, \tag{B.1.5}$$

where the constraints are the balance sheet constraint, incentive compatibility constraint, reserve ratio, and net worth accumulation, respectively. We calibrate the model such that the incentive constraint is always binding. Next, we simplify the system of constraints by substituting reserves, $A_t(j)$, and deposits, $D_t(j)$, making use of Equations (B.1.2) and (B.1.4). We also define $\Phi_t \equiv Q_t S_t(j)/N_t(j)$ to be the leverage ratio of a banker (and Φ_t is common across banks). Thus, the accumulation of net worth, (B.1.5), is given by

$$N_t(j) = \left(R_{k,t}\Phi_{t-1} - \frac{R_{d,t-1} - \alpha\left(x_t\right) R_{t-1}}{\left(1-\alpha\left(x_t\right)\right)\Pi_t}\left(\Phi_{t-1} - 1\right)\right) N_{t-1}(j). \tag{B.1.6}$$

Furthermore, we conjecture the value function to take the form

$$V_{n,t}(j) = \left(\zeta_{s,t}\Phi_t + \zeta_{n,t}\right) N_t(j), \tag{B.1.7}$$

where $\zeta_{s,t}$ and $\zeta_{n,t}$ are as yet undetermined.

Substituting (B.1.6) and (B.1.7), the banker's problem can be rewritten as

$$
\begin{aligned}
(\zeta_{s,t}\Phi_t + \zeta_{n,t}) = \max_{\Phi_t} \mathbb{E}_t\Lambda_{t,t+1} & ((1-\theta) + \theta(\zeta_{s,t+1}\Phi_{t+1} + \zeta_{n,t+1})) \\
& \times \left(R_{k,t+1}\Phi_t - \frac{R_{d,t} - \alpha(x_{t+1}) R_t}{(1-\alpha(x_{t+1}))\Pi_{t+1}} (\Phi_t - 1) \right), \qquad \text{(B.1.8)}
\end{aligned}
$$

subject to

$$
\zeta_{s,t}\Phi_t + \zeta_{n,t} = \lambda\Phi_t. \qquad \text{(B.1.9)}
$$

We rearrange the incentive compatibility constraint (B.1.9) and iterate one period forward to find optimal (and maximum) leverage given by

$$
\Phi_{t+1} = \frac{\zeta_{n,t+1}}{\lambda - \zeta_{s,t+1}}. \qquad \text{(B.1.10)}
$$

With (B.1.10), comparing the left and right hand side of (B.1.8), we verify the conjectured functional form of the value function. This allows us to summarize the solution to the financial intermediary's problem in the binding incentive constraint given by

$$
\lambda\Phi_t = \mathbb{E}_t\Lambda_{t,t+1}((1-\theta) + \theta\lambda\Phi_{t+1}) \left(R_{k,t+1}\Phi_t - \frac{R_{d,t} - \alpha(x_{t+1}) R_t}{(1-\alpha(x_{t+1}))\Pi_{t+1}} (\Phi_t - 1) \right). \qquad \text{(B.1.11)}
$$

Aggregate net worth in the financial sector evolves as a weighted sum of existing banks' accumulated net worth (B.1.6) and start up funds new banks receive from the household. Entering banks receive a fraction ω of the total value of intermediated assets, i.e. $\omega Q_t S_{t-1}$. In equilibrium, $S_t = K_t$. Thus, the evolution of aggregate net worth is given by

$$
N_t = \theta \left(R_{k,t}\Phi_{t-1} - \frac{R_{d,t-1} - \alpha(x_t) R_{t-1}}{(1-\alpha(x_t))\Pi_t} (\Phi_{t-1} - 1) \right) N_{t-1} + \omega Q_t K_{t-1}. \qquad \text{(B.1.12)}
$$

Equations (B.1.11) and (B.1.12) express the financial sector problem in just two equations. This completes the derivation.

B.2 List of equilibrium conditions [Section 3.1]

In equilibrium, we summarize the quantitative model in 23 equations in 23 endogenous variables, $\{Y_t,\, L_t,\, C_t,\, \tilde{C}_t,\, \Lambda_{t,t+1},\, \mu_t,\, K_t,\, I_t,\, I_{n,t},\, N_t,\, \Phi_t,\, W_t,\, \Pi_t,\, X_t,\, P_{m,t},\, P_{*,t},\, P_t,\, Q_t,\, R_{k,t},\, R_{T,t},\, R_t,\, R_{d,t},\, CS_t\}$, and 3 exogenous processes, $\{\zeta_t,\, \epsilon_t,\, \varepsilon_{m,t}\}$. Government expenditure, G, is financed via lump-sum taxes and kept constant.

Households

- Euler equation

$$1 = \mathbb{E}_t \Lambda_{t,t+1} \exp\left(\zeta_t\right) R_{d,t}/\Pi_{t+1} \tag{B.2.1}$$

- Labor supply

$$\mu_t W_t = \chi L_t^{\varphi} \tag{B.2.2}$$

- Stochastic discount factor

$$\Lambda_{t,t+1} = \beta \mu_{t+1}/\mu_t \tag{B.2.3}$$

- Marginal utility of consumption

$$\mu_t = \tilde{C}_t^{-\sigma} - \beta \hbar \mathbb{E}_t \tilde{C}_{t+1}^{-\sigma} \tag{B.2.4}$$

Financial intermediaries

- Incentive compatibility constraint

$$\lambda \Phi_t = \mathbb{E}_t \Lambda_{t,t+1} \left((1-\theta) + \theta \lambda \Phi_{t+1}\right) \left(R_{k,t+1} \Phi_t - \frac{R_{d,t} - \alpha\left(x_t\right) R_t}{\left(1-\alpha\left(x_t\right)\right) \pi_{t+1}} \left(\phi_t - 1\right) \right) \tag{B.2.5}$$

- Evolution of aggregate net worth

$$N_t = \theta \left(R_{k,t} \Phi_{t-1} - \frac{R_{d,t-1} - \alpha\left(x_t\right) R_{t-1}}{\left(1-\alpha\left(x_t\right)\right) \Pi_t} \left(\Phi_{t-1} - 1\right) \right) N_{t-1} + \omega Q_t K_{t-1} \tag{B.2.6}$$

Producers

- Price of capital

$$\begin{aligned} 1 = Q_t &\left(1 - \frac{\eta}{2} \left(\frac{I_{n,t} - I_{n,t-1}}{I_{n,t-1} + I} \right)^2 - \eta \frac{I_{n,t} - I_{n,t-1}}{\left(I_{n,t-1} + I\right)^2} I_{n,t} \right) \\ &+ \mathbb{E}_t \Lambda_{t,t+1} Q_{t+1} \left(\eta \left(I_{n,t+1} - I_{n,t}\right) \frac{I_{n,t+1} + I}{\left(I_{n,t} + I\right)^3} I_{n,t+1} \right) \end{aligned} \tag{B.2.7}$$

- Production function

$$Y_t = K_{t-1}^{\gamma} L_t^{1-\gamma} \tag{B.2.8}$$

- Labor demand

$$W_t = P_{m,t} \left(1 - \gamma\right) Y_t / L_t \tag{B.2.9}$$

- Return on capital

$$R_{k,t} = \frac{P_{m,t} \gamma Y_t / K_{t-1} + Q_t - \delta}{Q_{t-1}} \tag{B.2.10}$$

- Reset price

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \iota^{\tau} \Lambda_{t,t+\tau} \left(\frac{P_{*,t}}{P_{t+\tau}} - \frac{\epsilon_t}{\epsilon_t - 1} P_{m,t+\tau} \right) Y_{i,t+\tau} = 0 \tag{B.2.11}$$

- Price index

$$P_t = \left(\left(1 - \iota\right) P_{*,t}^{1-\epsilon_t} + \iota P_{t-1}^{1-\epsilon_t} \right)^{1/(1-\epsilon_t)} \tag{B.2.12}$$

Monetary policy

- Policy rule

$$R_{T,t} = \left(R \Pi_t^{\phi_{\pi}} \left(\frac{X_t}{X} \right)^{\phi_x} \right)^{1-\rho} R_{t-1}^{\rho} \exp\left(\varepsilon_{m,t}\right) \tag{B.2.13}$$

- No arbitrage

$$\begin{aligned} &\text{(I)} \quad R_t = R_{d,t} = R_{T,t} \ , \ \text{or} \\ &\text{(II)} \quad R_t = R_{d,t} = \max\left\{1, R_{T,t}\right\}, \ \text{or} \\ &\text{(III)} \quad R_t = R_{T,t} \quad \text{and} \quad R_{d,t} = \max\left\{1, R_{T,t}\right\}. \end{aligned} \tag{B.2.14}$$

General equilibrium

- Aggregate resource constraint

$$Y_t = C_t + I_t + G \tag{B.2.16}$$

- Capital accumulation

$$K_t = K_{t-1} + f\left(I_{n,t}, I_{n,t-1}\right), \tag{B.2.17}$$

where $f\left(I_{n,t}, I_{n.t-1}\right) \equiv \left(1 - (\eta/2)\left(\left(I_{n,t} + I_{n,t-1}\right) / \left(I_{n,t-1} + I\right)\right)^2\right) I_{n,t}$.

Further definitions

- Habit adjusted consumption

$$\tilde{C}_t = C_t - \hbar C_{t-1} \tag{B.2.18}$$

- Total investment

$$I_t = I_{n,t} + \delta K_{t-1} \tag{B.2.19}$$

- Inflation

$$\Pi_t = P_t / P_{t-1} \tag{B.2.20}$$

- Leverage

$$\Phi_t = Q_t K_t / N_t \tag{B.2.21}$$

- Marginal cost

$$X_t = P_{m,t} \tag{B.2.22}$$

- Credit spread

$$CS_t = R_{k,t+1} / \left(R_{d,t} / \Pi_{t+1} \right) \tag{B.2.23}$$

B.3 Data [Section 3.2]

Data sources For the estimation of the quantitative model we use US quarterly observations covering the period 1985:Q1 to 2019:Q1. All macroeconomic and financial time series used are extracted from the Federal Reserve Economic Data (FRED) database. Table B.1 provides a complete overview.

Data treatment We transform all nominal aggregate quantities into real per-capita terms. Inflation is defined as the quarter-on-quarter log growth rate of the GDP deflator. Nominal interest rates and spreads are divided by four to generate quarterly rates. For the estimation, all variables are stationarized using a standard HP-filter ($\lambda = 1600$). Data moments are matched with model moments for all relevant observables, where a lower case denotes the log deviation of the corresponding variable from steady state. Table B.2 documents the data transformations in detail.

Table B.1: Data Sources

Mnemonic	Description
CNP16OV	Population level
GDP	Gross domestic product
GDPDEF	Gross domestic product: implicit price deflator
GPDI	Gross private domestic investment
PCDG	Personal consumption expenditures: durable goods
PCND	Personal consumption expenditures: nondurable goods
PCESV	Personal consumption expenditures: services
FEDFUNDS	Effective federal funds rate
DGS10	10-Year Treasury constant maturity rate
AAA	Moody's seasoned Aaa corporate bond yield
BAA	Moody's seasoned Baa corporate bond yield
TOTRESNS	Total reserves of depository institutions
DPSACBM027NBOG	Deposits, all commercial banks
TABSNNCB	Total assets, nonfinancial corporate business
TLBSNNCB	Total liabilities, nonfinancial corporate business
TLAACBW027SBOG	Total assets, all commercial banks
TLBACBW027SBOG	Total liabilities, all commercial banks

Table B.2: Data treatment

Observable	Description	Construction
Steady state calibration & Figure 5		
	Spread measure I	BAA - FEDFUNDS
	Spread measure II	BAA - DGS10
	Spread measure III	BAA - AAA
	Reserve ratio	TOTRESNS/DPSACBM027NBOG
	Leverage	*see computation below**
Dynamic moment matching		
y	Output	HP-filter[GDP/(GDPDEF x NCP160V)]
c	Consumption	HP-filter[(PCND + PCESV)/(GDPDEF x NCP160V)]
π	Inflation	HP-filter[ln(GDPDEF/GDPDEF$_{-1}$)]
r	Reserve rate	HP-filter[FEDFUNDS/4]
cs	Credit spread	HP-filter[(BAA - FEDFUNDS)/4]
i	Investment	HP-filter[(PCDG + GPDI)/(GDPDEF x NCP160V)]

* Construction of the leverage series:

$$\text{Aggregate Leverage}_t = \frac{A_t^{\text{cb}}(1+s) + A_t^{\text{nfc}}}{A_t^{\text{cb}}(1+s) + A_t^{\text{nfc}} - L_t^{\text{cb}} - L_t^{\text{ncbfi}} - L_t^{\text{nfc}}}, \tag{B.3.1}$$

where A_t and L_t denote assets and liabilities and where the superscipts "cb", "nfc", and "ncbfi" refer to commercial banks, non-financial corporations, and non-commercial bank financial institutions, respectively. L_t^{ncbfi} is given by

$$L_t^{\text{ncbfi}} = sA_t^{\text{cb}}\left(1 - \frac{1}{f\left(\frac{A_t^{\text{cb}}}{A_t^{\text{cb}} - L_t^{\text{cb}}}\right)}\right) \tag{B.3.2}$$

where $s = 1.86$ and we assume $f = 2$.[27]

[27] The scaling factor s is derived from the May 2021 Federal Reserve Financial Stability Report, Chapter 3, Table 3. We calculate $s = A/B$ where A is the total assets of mutual funds, insurance companies, hedge funds, and broker-dealers and B is the total assets of banks and credit unions.