NORTH SOUTH UNIVERSITY

Introduction to Robotics

CSE495A

Home Work 2

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Ans to the Que No 1

(a) Given that,

$$\dot{x}_1 = u_1 \quad \text{--- (i)}$$
$$\dot{x}_2 = u_2 \quad \text{--- (ii)}$$
$$\dot{x}_3 = x_2 u_1 \quad \text{--- (iii)}$$

and

$$z = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

Here, $z = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$\therefore x_1 = z_1$ so that, $\dot{x}_1 = \dot{z}_1$

$x_3 = z_2$ $\dot{x}_3 = \dot{z}_2$

From (i)

$$u_1 = \dot{x}_1 = \dot{z}_1$$
$$\therefore u_1 = \dot{z}_1$$

From (iii)

$$\dot{x}_3 = x_2 u_1$$
$$x_2 = \frac{\dot{x}_3}{u_1}$$
$$\therefore x_2 = \frac{\dot{z}_2}{\dot{z}_1}$$

From (ii)

$$u_2 = \dot{x}_2$$
$$u_2 = \left(\frac{\dot{z}_2}{\dot{z}_1}\right)^{\cdot}$$
$$u_2 = \frac{d}{dt}\left(\frac{\dot{z}_2}{\dot{z}_1}\right)$$
$$u_2 = \frac{1}{\dot{z}_1}\ddot{z}_2 + \dot{z}_2\left(-\frac{1}{\dot{z}_1^2}\right)\ddot{z}_1$$
$$u_2 = \frac{\ddot{z}_2}{\dot{z}_1} - \frac{\dot{z}_2 \ddot{z}_1}{\dot{z}_1^2}$$

we can see that every states (x_1, x_2, x_3) and every control (u_1, u_2) can be represent by $z(z_1, z_2)$ or its derivative $(\dot{z}, \ddot{z})$ so that this system is differentially flat.

(6)

Given that,

initial conditions are:

At $t=0$; $x_1(0), x_2(0), x_3(0), \dot{x}_1(0) = 1$

At $t=T$; $x_1(T), x_2(T), x_3(T), \dot{x}_1(T) = 1$

Basis functions are:

$\psi_1 = 1$

$\psi_2 = t$

$\psi_3 = t^2$

$\psi_4 = t^3$

We know that,

$$z_1 = \sum_{i=1}^{N} \alpha_{1i} \psi_i(t) \quad \text{— (I)}$$

[N is the number of basis function]

$$z_2 = \sum_{i=1}^{N} \alpha_{2i} \psi_i(t) \quad \text{— (II)}$$

Here $N = 4$

Now, Using (I)

$$z_1 = \alpha_{11}\psi_1(t) + \alpha_{12}\psi_2(t) + \alpha_{13}\psi_3(t) + \alpha_{14}\psi_4(t)$$

$$\therefore z_1 = \alpha_{11} + \alpha_{12}t + \alpha_{13}t^2 + \alpha_{14}t^3$$

$$\therefore \dot{z}_1 = \alpha_{12} + 2\alpha_{13}t + 3\alpha_{14}t^2$$

Using (II)

$$z_2 = \alpha_{21}\psi_1(t) + \alpha_{22}\psi_2(t) + \alpha_{23}\psi_3(t) + \alpha_{24}\psi_4(t)$$

$$\therefore z_2 = \alpha_{21} + \alpha_{22}t + \alpha_{23}t^2 + \alpha_{24}t^3$$

$$\therefore \dot{z}_2 = \alpha_{22} + 2\alpha_{23}t + 3\alpha_{24}t^2.$$

our actual gole is to get $Ax = B$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & T & T^2 & T^3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2T & 3T^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & T^2 & T^3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2T & 3T^2 \end{bmatrix}}_{A \; (8\times 8)} \underbrace{\begin{bmatrix} \alpha_{11} \\ \alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ \alpha_{21} \\ \alpha_{22} \\ \alpha_{23} \\ \alpha_{24} \end{bmatrix}}_{x \; (8\times 1)} = \underbrace{\begin{bmatrix} z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_2(T) \\ z_2(T) \\ \dot{z}_2(T) \end{bmatrix}}_{B \; (8\times 1)}$$

∴ This is the matrix vector representation.

(c) Using the same method used in (b)

we know that,

$$z_1 = \sum_{i=1}^{N} \alpha_{1i} \psi_i(t)$$

$$z_2 = \sum_{i=1}^{N} \alpha_{2i} \psi_i(t)$$

new Basis functions are,

$\psi_1 = 1$

$\psi_2 = t$

$\psi_3 = t^2$

$\psi_4 = t^3$

$\psi_5 = t^4$

$\psi_6 = t^5$

$$z_1 = \alpha_{11}\psi_1(t) + \alpha_{12}\psi_2(t) + \alpha_{13}\psi_3(t) + \alpha_{14}\psi_4(t) + \alpha_{15}\psi_5(t) + \alpha_{16}\psi_6(t)$$

$$\therefore z_1 = \alpha_{11} + \alpha_{12} t + \alpha_{13} t^2 + \alpha_{14} t^3 + \alpha_{15} t^4 + \alpha_{16} t^5$$

$$\therefore \dot{z}_1 = \alpha_{12} + 2\alpha_{13} t + 3\alpha_{14} t^2 + 4\alpha_{15} t^3 + 5\alpha_{16} t^4$$

$$z_2 = \alpha_{21}\psi_1(t) + \alpha_{22}\psi_2(t) + \alpha_{23}\psi_3(t) + \alpha_{24}\psi_4(t) + \alpha_{25}\psi_5(t) + \alpha_{26}\psi_6(t).$$

$$\therefore z_2 = \alpha_{21} + \alpha_{22} t + \alpha_{23} t^2 + \alpha_{24} t^3 + \alpha_{25} t^4 + \alpha_{26} t^5$$

$$\therefore \dot{z}_2 = \alpha_{22} + 2\alpha_{23} t + 3\alpha_{24} t^2 + 4\alpha_{25} t^3 + 5\alpha_{26} t^4$$

matrixvector Representation $Ax = B$

$$
\underbrace{\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & T & T^2 & T^3 & T^4 & T^5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2T & 3T^2 & 4T^3 & 5T^4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & T & T^2 & T^3 & T^4 & T^5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2T & 3T^2 & 4T^3 & 5T^4
\end{bmatrix}}_{A,\ 8\times 12}
\underbrace{\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{15} \\ a_{16} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{25} \\ a_{26} \end{bmatrix}}_{x,\ 12\times 1}
=
\underbrace{\begin{bmatrix} z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T) \end{bmatrix}}_{B,\ 8\times 1}
$$

Ans to the Que No 1 d code

Ans to the Que No 2

(a) Given,

$$\dot{x}(t) = \overset{v(t)}{\cos\theta(t)}$$
$$\dot{y}(t) = v(t)\sin\theta(t)$$
$$\dot{v}(t) = a(t)$$
$$\dot{\theta}(t) = w(t)$$

We know that every thing is function of time so we can write this like:

$$\dot{x} = v\cos\theta \quad \text{—(i)}$$
$$\dot{y} = v\sin\theta \quad \text{—(ii)}$$
$$\dot{v} = a \quad \text{—(iii)}$$
$$\dot{\theta} = w \quad \text{—(iv)}$$

Here,

$$z = \begin{bmatrix} x \\ y \end{bmatrix}$$

$\therefore z_1 = x$ and $\dot{x} = \dot{z}_1$

$z_2 = y$ $\quad \dot{y} = \dot{z}_2$

From 1 and 2

$$\frac{\dot{y}}{\dot{x}} = \frac{v\sin\theta}{v\cos\theta}$$

$$\frac{\dot{y}}{\dot{x}} = \tan\theta$$

$$\therefore \theta = \tan^{-1}\frac{\dot{y}}{\dot{x}}$$

$$\therefore \theta = \tan^{-1}\frac{\dot{z}_2}{\dot{z}_1}$$

$$\therefore \dot{\theta} = \frac{d}{dt}\left[\tan^{-1}\frac{\dot{z}_2}{\dot{z}_1}\right]$$

From 1^2 and 2^2

$$v^2\cos^2\theta + v^2\sin^2\theta = \dot{x}^2 + \dot{y}^2$$

$$v^2(\cos^2\theta + \sin^2\theta) = \dot{x}^2 + \dot{y}^2$$

$$v^2 = \dot{x}^2 + \dot{y}^2 \quad [\cos^2\theta + \sin^2\theta = 1]$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$\therefore v = \sqrt{\dot{z}_1^2 + \dot{z}_2^2}$$

Now $\frac{dv}{dt} = \frac{d}{dt}\sqrt{\dot{z}_1^2 + \dot{z}_2^2}$

$$\dot{\theta} = \frac{1}{1+\left(\frac{\dot{z}_2}{\dot{z}_1}\right)^2} \cdot \frac{d}{dt}\left[\frac{\dot{z}_2}{\dot{z}_1}\right]$$

$$\dot{\theta} = \frac{1}{1+\left(\frac{\dot{z}_2}{\dot{z}_1}\right)^2}\left[\frac{1}{\dot{z}_1}\ddot{z}_2 + \dot{z}_2\left(-\frac{1}{\dot{z}_1^2}\right)\ddot{z}_1\right]$$

$$\dot{\theta} = \frac{1}{1+\left(\frac{\dot{z}_2}{\dot{z}_1}\right)^2}\left(\frac{\ddot{z}_2}{\dot{z}_1} - \frac{\dot{z}_2\ddot{z}_1}{\dot{z}_1^2}\right)$$

$\therefore\ \omega = \dot{\theta}$

$$\omega = \frac{1}{1+\left(\frac{\dot{z}_2}{\dot{z}_1}\right)^2}\left(\frac{\ddot{z}_2}{\dot{z}_1} - \frac{\dot{z}_2\ddot{z}_1}{\dot{z}_1^2}\right)$$

every state variable and control variable can be represent ~~in~~ with $z\ (z_1, z_2)$. So that this system is differentially flat.

(b) we kn that,

$$z_1 = \sum_{i=1}^{N} \alpha_{1i} \psi_i(t)$$

$$z_2 = \sum_{i=1}^{N} \alpha_{2i} \psi_i(t)$$

Given,

$\psi_1 = 1$

$\psi_2 = t$

$\psi_3 = t^2$

$\psi_4 = t^3$

using (1)

$$z_1 = \alpha_{11}\psi_1(t) + \alpha_{12}\psi_2(t) + \alpha_{13}\psi_3(t) + \alpha_{14}\psi_4(t)$$

$$z_1 = \alpha_{11} + \alpha_{12}t + \alpha_{13}t^2 + \alpha_{14}t^3$$

$$\dot{z}_1 = \alpha_{12} + 2\alpha_{13}t + 3\alpha_{14}t^2$$

$$z_2 = \alpha_{21}\psi_1(t) + \alpha_{22}\psi_2(t) + \alpha_{23}\psi_3(t) + \alpha_{24}\psi_4(t)$$

$$z_2 = \alpha_{21} + \alpha_{22}t + \alpha_{23}t^2 + \alpha_{24}t^3$$

$$\dot{z}_2 = \alpha_{22} + 2\alpha_{23}t + 3\alpha_{24}t^2$$

Matrix vector representation

$$
\begin{array}{ccc}
A & x & B \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & T & T^2 & T^3 & 0 & 0 & 0 & 0 \\
0 & 1 & 2T & 3T^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & T & T^2 & T^3 \\
0 & 0 & 0 & 0 & 0 & 1 & 2T & 3T^2
\end{bmatrix}
&
\begin{bmatrix}
\alpha_{11} \\ \alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ \alpha_{21} \\ \alpha_{22} \\ \alpha_{23} \\ \alpha_{24}
\end{bmatrix}
=
&
\begin{bmatrix}
z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T)
\end{bmatrix}_{(8\times 1)}
\end{array}
$$

Now for initial condition.

$x(0) = 0$ $\quad$ $y(0) = 0$

$x(15) = 5$ $\quad$ $y(15) = 5$

$\dot{x}(0) = 0.5\cos\left(-\frac{\pi}{2}\right) = 0$ $\quad$ $\dot{y}(0) = 0.5\sin\left(-\frac{\pi}{2}\right) = -0.5$

$\dot{x}(15) = 0.5\cos\left(-\frac{\pi}{2}\right) = 0$ $\quad$ $\dot{y}(15) = 0.5\sin\left(-\frac{\pi}{2}\right) = 0.5$

Ans to the Que No 2C code

Ans to the Que No C

Given

$\dot{x} = v\cos\theta$

$\dot{y} = v\sin\theta$

$\dot{v} = a$

$\dot{\theta} = \omega$

$$\overset{a}{\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix}} = \overset{b}{\begin{bmatrix}\cos\theta & -v\sin\theta\\ \sin\theta & v\cos\theta\end{bmatrix}} \overset{c}{\begin{bmatrix}a\\ \omega\end{bmatrix}}$$

$\therefore \ddot{x} = a\cos\theta - v\sin\theta\,\omega$

$\ddot{y} = a\sin\theta - v\cos\theta\,\omega$

We need to find a and ω (c)

$a = b \times c$

$c = b^{-1} \times a.$

For b^{-1}

$$\det|b| = \cos\theta.(v\cos\theta) - (-v\sin\theta).\sin\theta$$

$$= v(\cos^2\theta + \sin^2\theta)$$

$$= v$$

$$\therefore b^{-1} = \frac{1}{v}\begin{bmatrix}v\cos\theta & v\sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$\therefore c = \frac{1}{v}\begin{bmatrix}v\cos\theta & v\sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix}$$

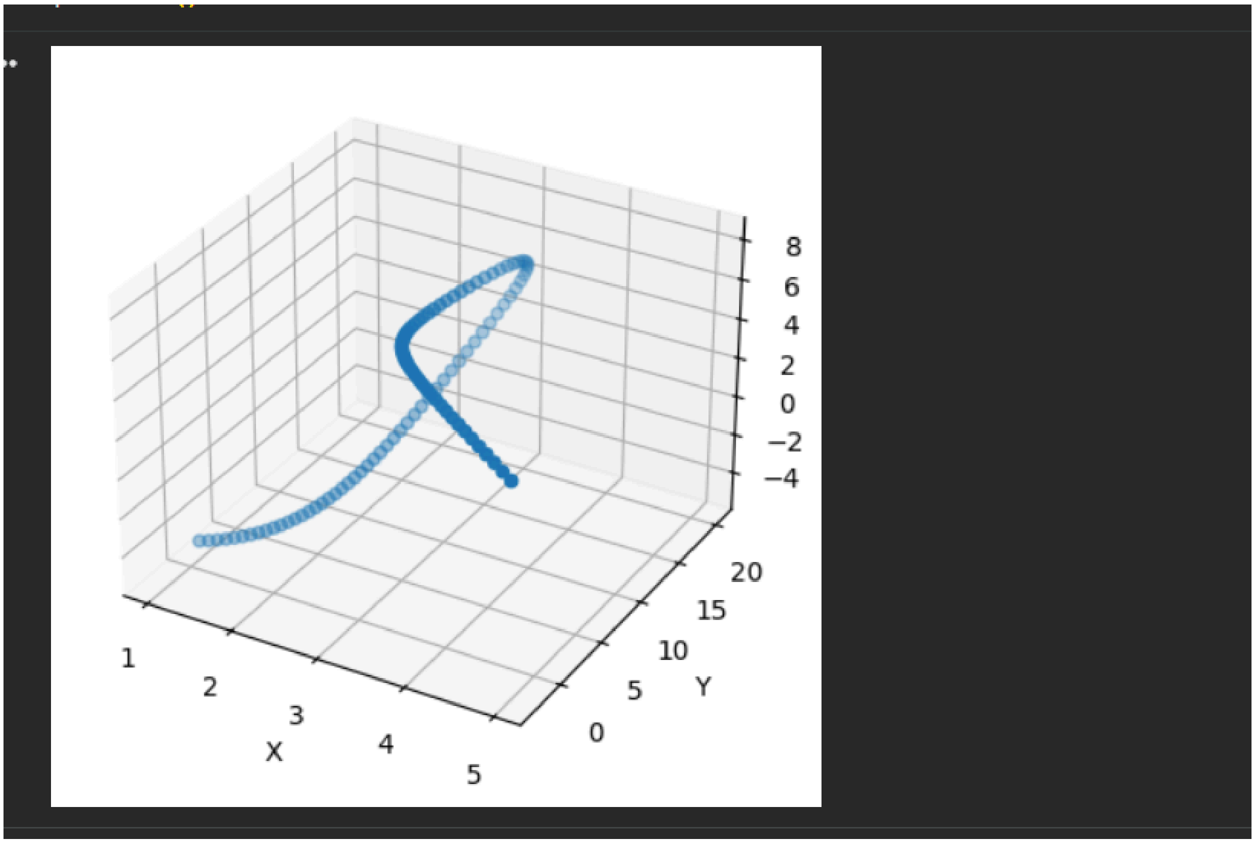
$$\therefore \begin{bmatrix}a\\ \omega\end{bmatrix} = \begin{bmatrix}\ddot{x}\cos\theta + \ddot{y}\sin\theta\\ \dfrac{-\sin\theta\,\ddot{x}}{v} + \dfrac{\cos\theta\,\ddot{y}}{v}\end{bmatrix}$$

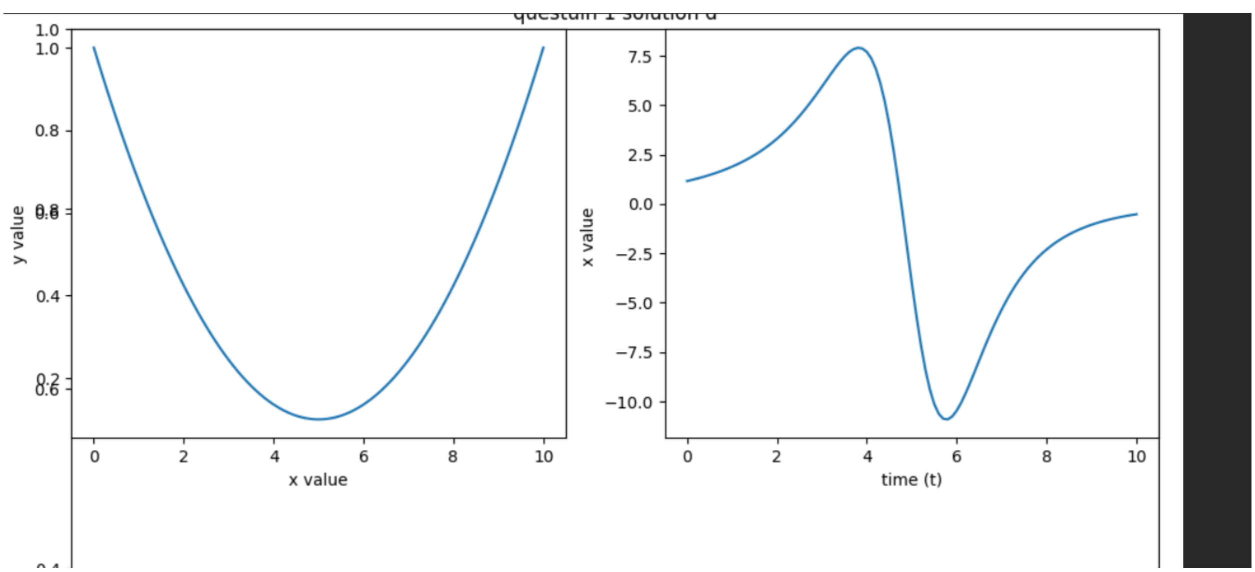
As to the Issue. NO B code

Code links

Ans 1 d part 1

https://colab.research.google.com/drive/1_9WYRjOiObx9eakb9dig2XvhUGasGAdb?usp=sharing





Part 2

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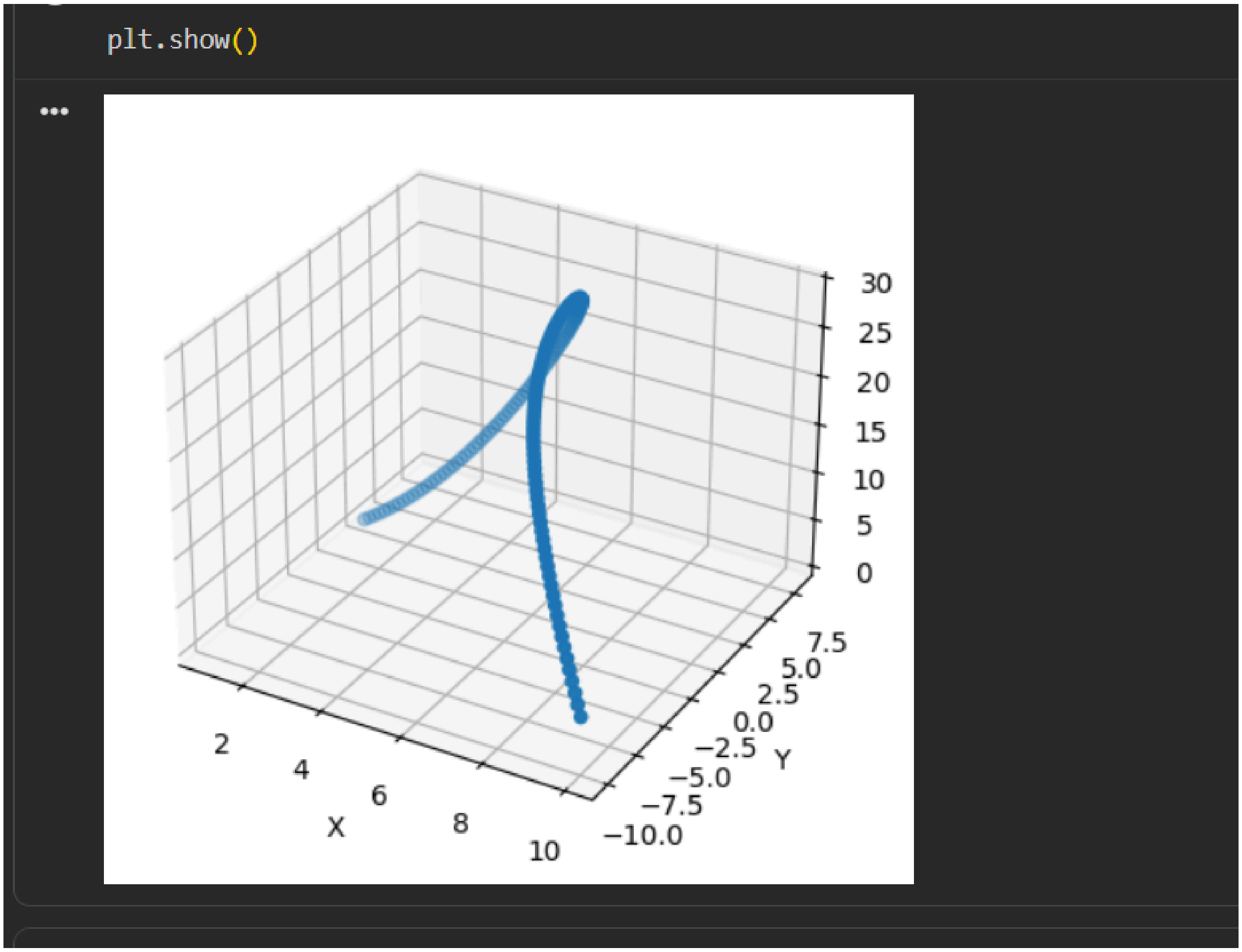

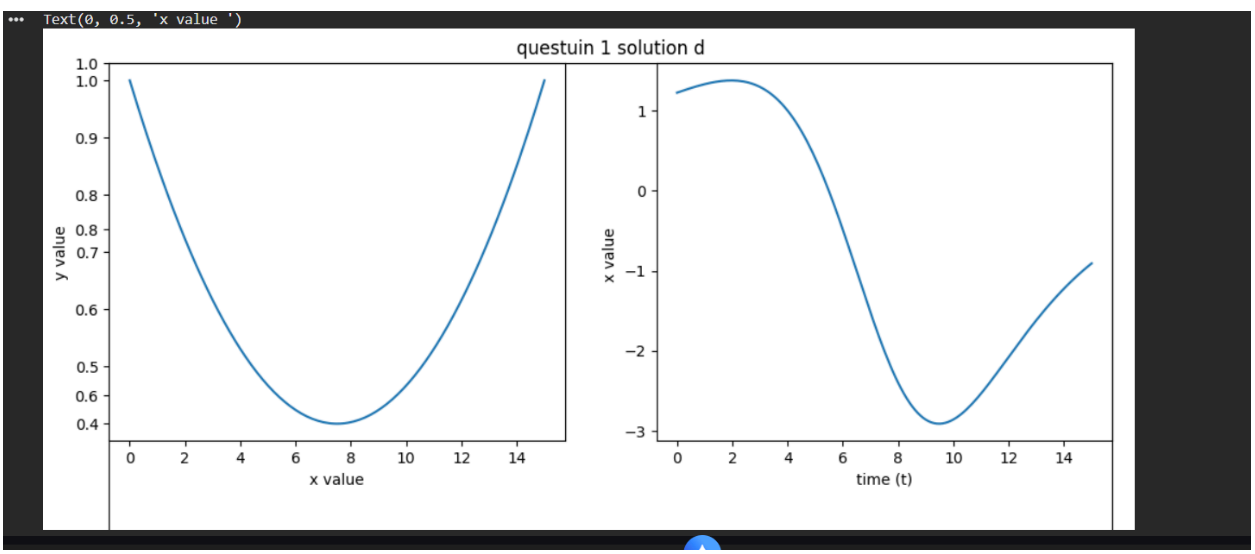

Ans 2 b
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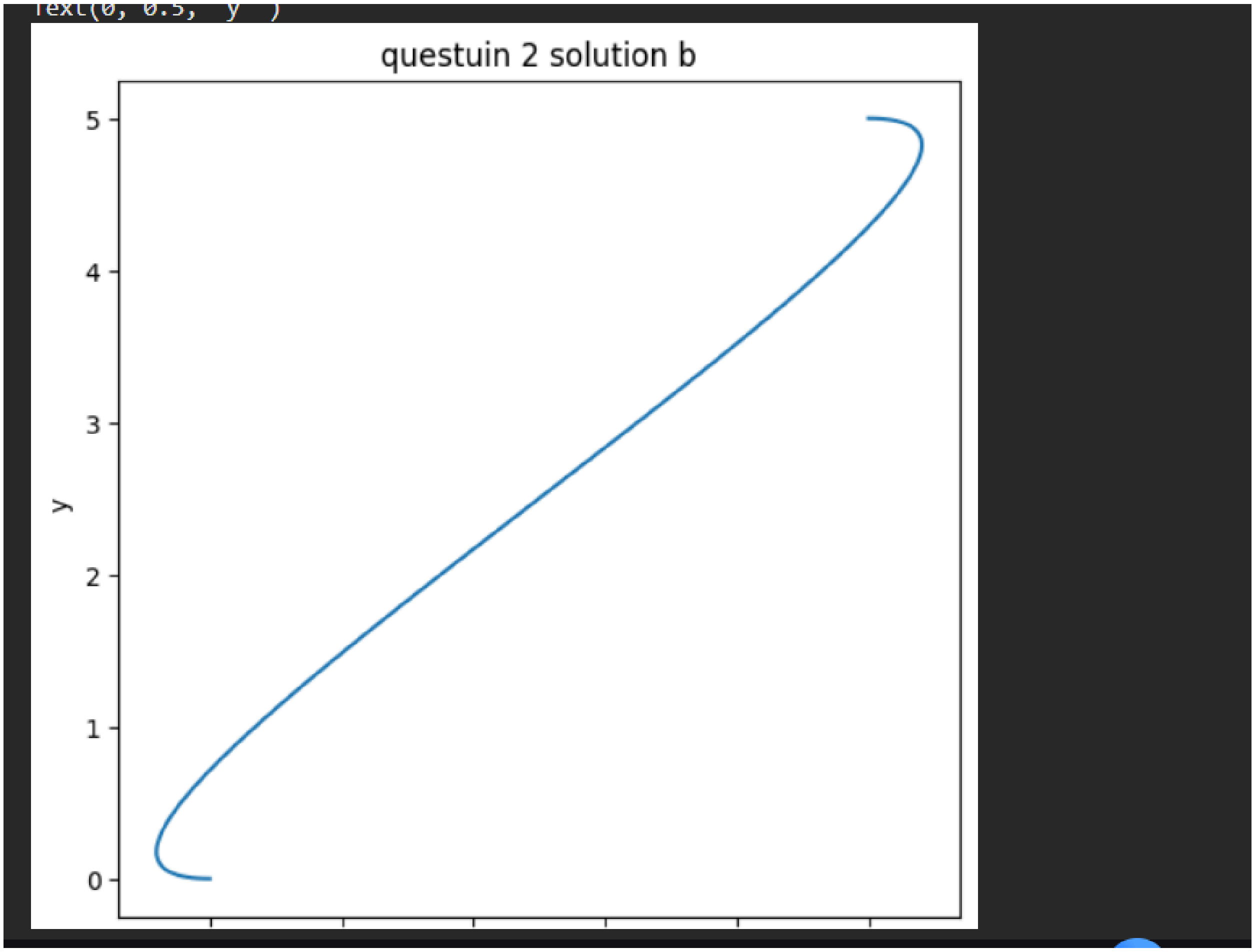

C
Part 1
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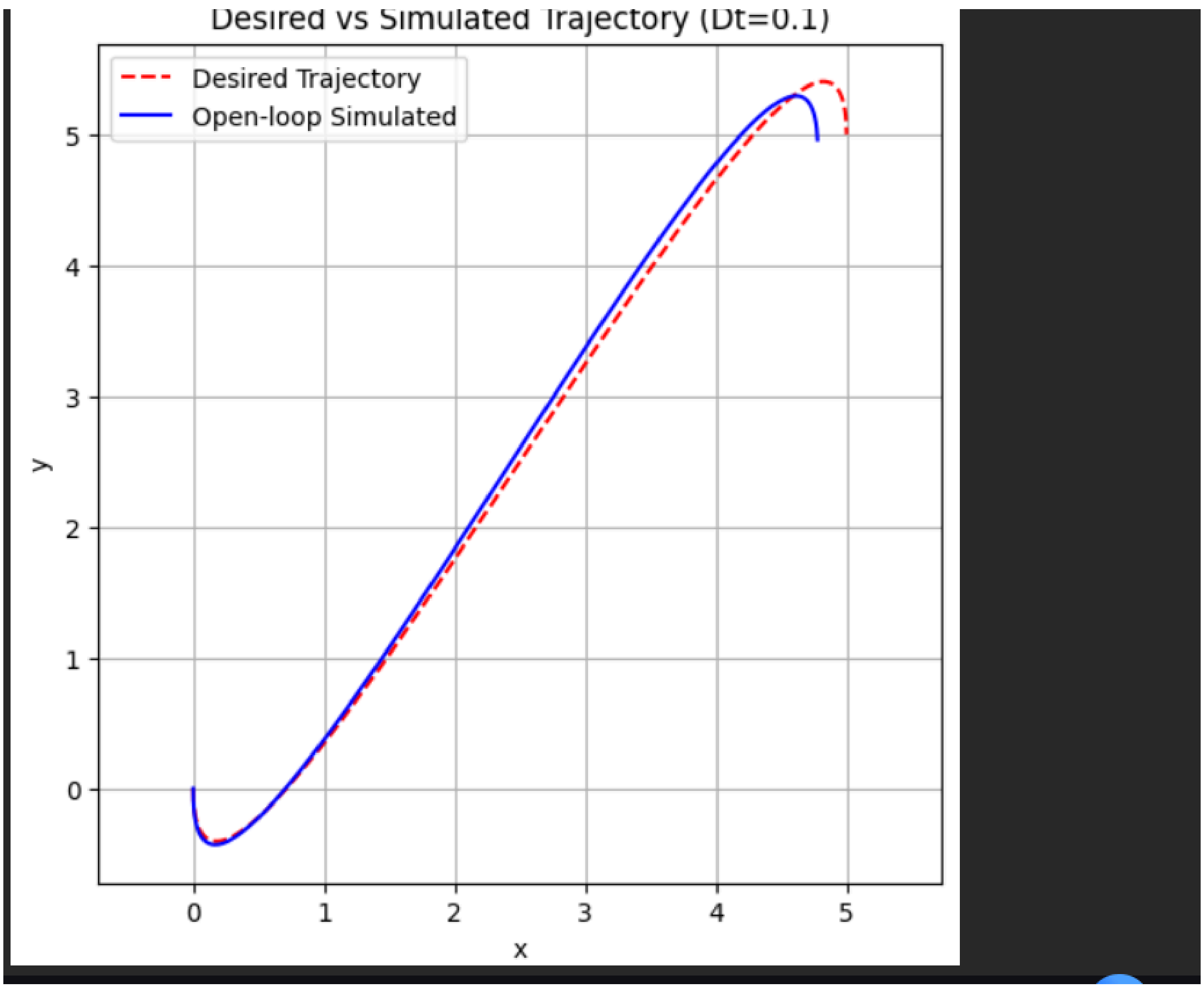

Part 2

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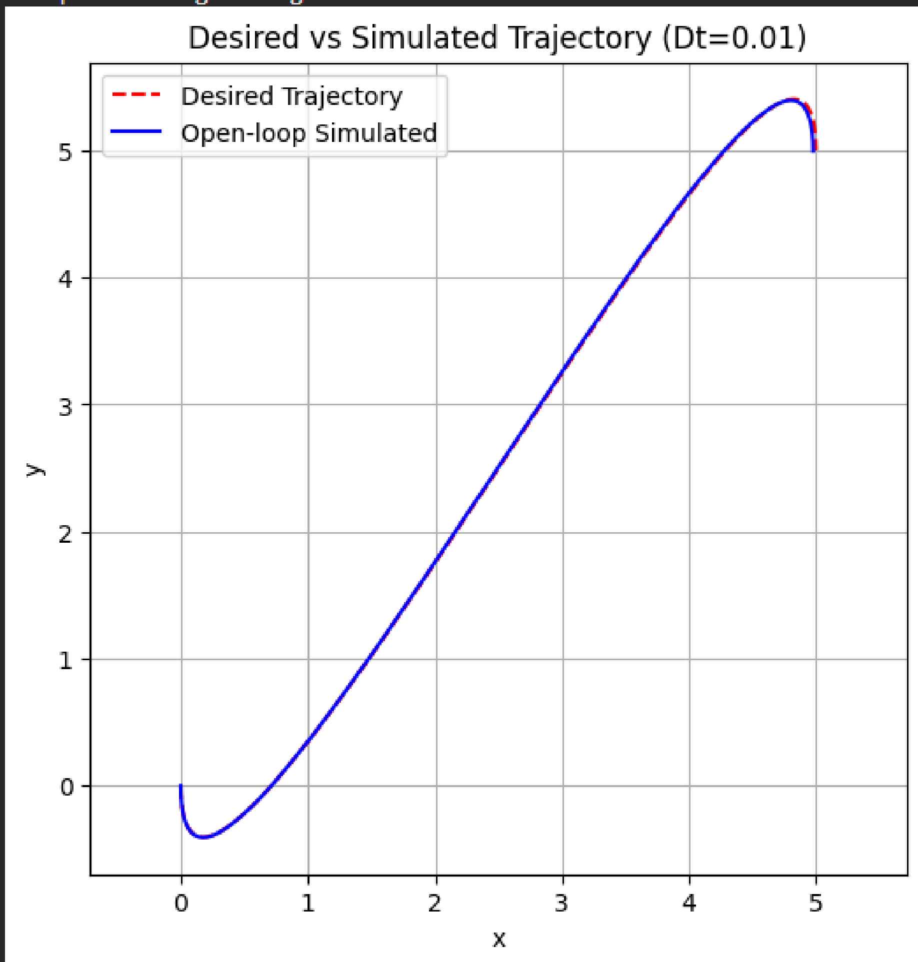

Ans 3

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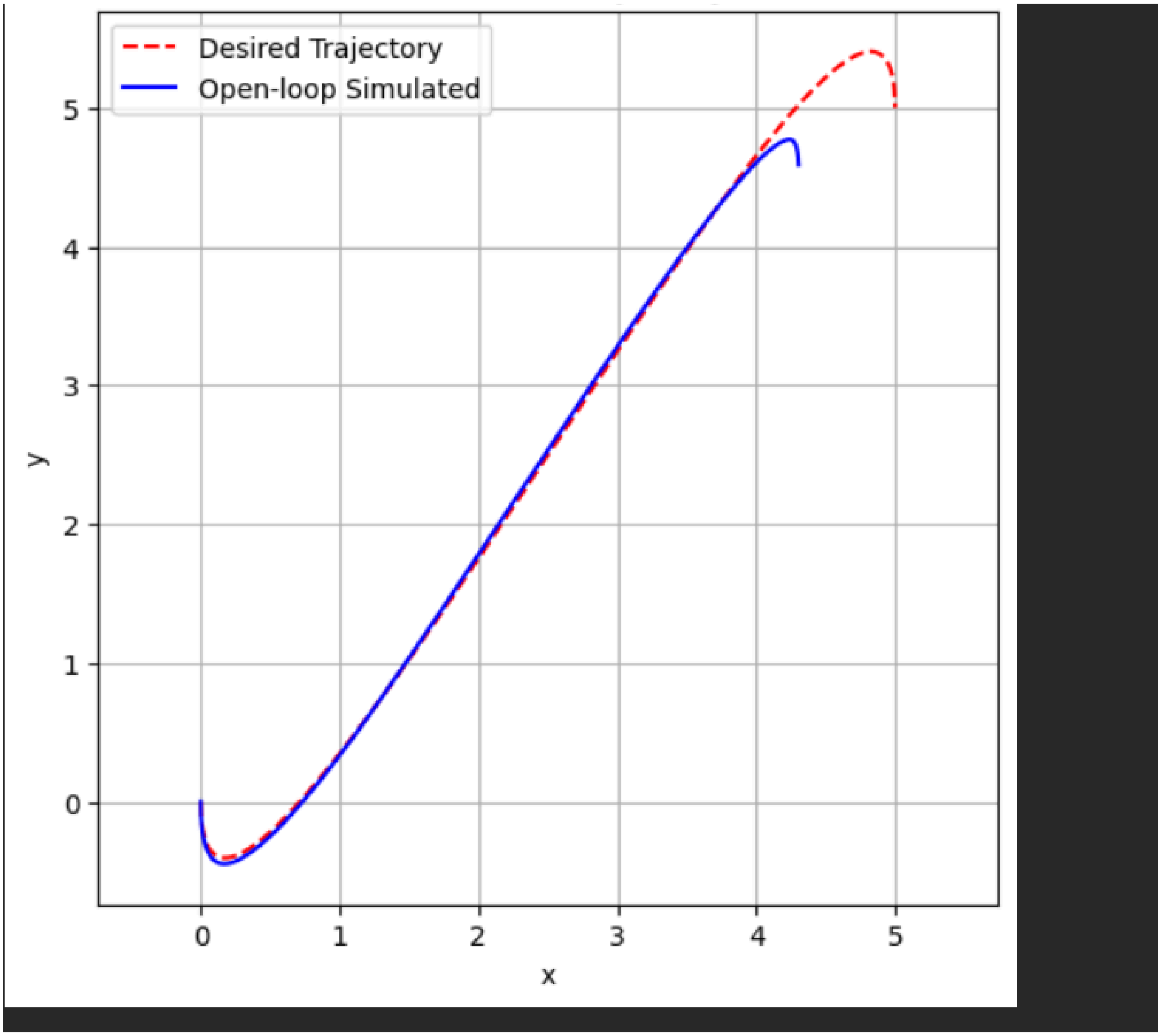
Part 1



Part 2

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