NORTH SOUTH UNIVERSITY

Introduction to Robotics

CSE495A

Home Work 4

AL IMRAN

2212602642

Ans to the Que No 1

$$I = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix} \qquad \bar{I} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 & 0 \\ 0 & 8 & 5 & 2 & 0 \\ 0 & 9 & 6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We know that,

$$G(i,j) = \sum_{u=0}^{K-1} \sum_{v=0}^{L-1} F(u,v) \cdot \bar{I}(i+u, j+v)$$

(a) $G(0,0) = \sum_{u=0}^{K-1} \sum_{v=0}^{L-1} F(u,v) \cdot \bar{I}(u,v)$ since $i=0$, $j=0$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

$= 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 7 + 0 \cdot 4 + 0 \cdot 0 + 0 \cdot 8 + 0 \cdot 5$

$= 7$

$$G(0,1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 7 & 4 & 1 \\ 8 & 5 & 2 \end{bmatrix}$$ since $i=0$, $j=1$

$= 4$

$$G(0,2) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 0 \end{bmatrix}$$ since $i=0$, $j=2$

$= 1$

$$G(1,0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 7 & 4 \\ 0 & 8 & 5 \\ 0 & 9 & 6 \end{bmatrix}$$ since $i=1$, $j=0$

$= 8$

$$G(1,1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$ since $i=1$, $j=1$

$= 5$

$$G(1,2) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 0 \\ 5 & 2 & 0 \\ 6 & 3 & 0 \end{bmatrix}$$ since $i=1$, $j=2$

$= 2$

$$G(2,0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & 5 \\ 0 & 9 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$ since $i=2$, $j=0$

$= 9$

$$G(2,1) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 8 & 5 & 2 \\ 9 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$ since $i=2$, $j=1$

$= 6$

$$G(2,2) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ $i=2$, $j=2$

$= 3$

$$G = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

(Ans).

(b) $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$G(0,0) = \sum_{u=0}^{k-1}\sum_{v=0}^{L-1} F(u,v)\cdot I(u\cancel{+i}, \cancel{j+}v) \text{ since } i=0,\ j=0$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix} \text{ since } i=0,\ j=0$$

$$= 1.0+0.0+0.0+0.0+0.7+0.4+0.0+0.8+0.5$$
$$=0$$

$$G(0,1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 7 & 4 & 1 \\ 8 & 5 & 2 \end{bmatrix} \text{ since } i=0,\ j=1$$

$$= 0$$

$$G(0,2) = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 0 \end{bmatrix} \text{ since } i=0,\ j=2$$

$$= 0$$

$$G(1,0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 7 & 4 \\ 0 & 8 & 5 \\ 0 & 9 & 6 \end{bmatrix} \text{ since } i=1,\ j=0$$

$$= 0$$

$$G(1,1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix} \text{ since } i=1,\ j=1$$

$$= 7$$

$$G(1,2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 0 \\ 5 & 2 & 0 \\ 6 & 3 & 0 \end{bmatrix} \text{ since } i=1,\ j=2$$

$$= 4$$

$$G(2,0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & 5 \\ 0 & 9 & 6 \\ 0 & 0 & 0 \end{bmatrix} \text{ since } i=2,\ j=0$$

$$= 0$$

$$G(2,1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 8 & 5 & 2 \\ 3 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix} \text{ since } i=2,\ j=1$$

$$= 8$$

$$G(2,2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ since } i=2,\ j=2$$

$$= 5$$

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

(Ans).

(e)

$$F = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$G(0,0) = \sum_{u=0}^{k-1} \sum_{v=0}^{L-1} F(u,v) \cdot I(u,v) \text{ since } \begin{matrix} i=0 \\ j=0 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

$= 1 \cdot 0 + 1 \cdot 0 + 1 \cdot 0 + 0 0 + 0 \cdot 7 . 0 \cdot 4 + (-1 \cdot 0) + (-1 \cdot 8) + (-1 \cdot 5) = -13$

$$G(0,1) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 7 & 4 & 1 \\ 8 & 5 & 2 \end{bmatrix}$$

$= -15$

$$G(0,2) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 0 \end{bmatrix}$$

$= -7$

$$G(1,0) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 7 & 4 \\ 0 & 8 & 5 \\ 0 & 9 & 6 \end{bmatrix}$$

$= -4$

$$G(1,1) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

$= -6$

$$G(1,2) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 1 & 0 \\ 5 & 2 & 0 \\ 6 & 3 & 0 \end{bmatrix}$$

$= -4$

$$G(2,0) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & 5 \\ 0 & 9 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 13$

$$G(2,1) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 8 & 5 & 2 \\ 3 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 15$

$$G(2,2) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 7$

$$G = \begin{bmatrix} -13 & -15 & -7 \\ -4 & -6 & -4 \\ 13 & 15 & 7 \end{bmatrix}$$

(Ans).

This filter approximate vartical Gradiant which work as a "Horizontal edge" Detector.

(d) $F' = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$G(0,0) = \sum_{u=0}^{K-1} \sum_{v=0}^{L-1} F(u,v) \cdot I(u,v)$ since $i=0$, $j=0$

$= \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 9 & 5 \end{bmatrix}$

$= (-1 \cdot 0) + 0 \cdot 0 + 0 \cdot 1 + (-1) \cdot 0 + 0 \cdot 7 + 1 \cdot 4 + (-1) \cdot 0 + 0 \cdot 8 + 1 \cdot 5 = 9$

$G(0,1) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 7 & 4 & 1 \\ 8 & 5 & 2 \end{bmatrix}$

$= -12$

$G(0,2) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 0 \end{bmatrix}$

$= -9$

$G(1,0) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 7 & 4 \\ 0 & 8 & 5 \\ 0 & 9 & 6 \end{bmatrix}$

$= 15$

$G(1,1) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$

$= -18$

$G(1,2) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 0 \\ 5 & 2 & 0 \\ 6 & 3 & 0 \end{bmatrix}$

$= -15$

$G(2,0) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & 5 \\ 0 & 9 & 6 \\ 0 & 0 & 0 \end{bmatrix}$

$= 11$

$G(2,1) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 & 5 & 2 \\ 3 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$= -12$

$G(2,2) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$= -11$

$G = \begin{bmatrix} 9 & -12 & -9 \\ 15 & -18 & -15 \\ 11 & -12 & -11 \end{bmatrix}$ (Ans).

This filter F' is a vertical edge detector where els a Horizontal edge detector.

(e) $F = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

$$G(0,0) = \sum_{u=0}^{k-1}\sum_{v=0}^{l-1} F(u,v) \cdot \bar{I}(u,v) \text{ since } i=0, j=0$$

$$= \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

$$= \frac{1}{16}(1 \cdot 0 + 2 \cdot 0 + 1 \cdot 0 + 2 \cdot 0 + 4 \cdot 7 + 2 \cdot 4 + 1 \cdot 0 + 2 \cdot 8 + 1 \cdot 5)$$

$= 3.5625$

$$G(0,1) = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 7 & 4 & 1 \\ 8 & 5 & 2 \end{bmatrix}$$

$= 3.2$

$$G(0,2) = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 0 \end{bmatrix}$$

$= 1.31$

$$G(1,0) = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 7 & 4 \\ 0 & 8 & 5 \\ 0 & 9 & 6 \end{bmatrix}$$

$= 5.25$

$$G(1,1) = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

$= 5$

$$G(1,2) = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 0 \\ 5 & 2 & 0 \\ 6 & 3 & 0 \end{bmatrix}$$

$= 2.25$

$$G(2,0) = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & 5 \\ 0 & 9 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 4.31$

$$G(2,1) = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 8 & 5 & 2 \\ 3 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 4.25$

$$G(2,2) = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 & 0 \\ 6 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 2.06$

$$G = \begin{bmatrix} 3.5625 & 3.2 & 1.31 \\ 5.25 & 5 & 2.25 \\ 4.31 & 4.25 & 2.06 \end{bmatrix}$$

(Ans).

It smooths the image (reduce noise/softens edge), giving more weight to the center pixels.

(f) $F = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

$G(0,0) = \sum_{u=0}^{k-1}\sum_{v=0}^{l-1} F(u,v)\cdot I(u,v)$ since $i=0$, $j=0$

$= \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}0&0&0\\0&7&4\\0&8&5\end{bmatrix}$

$= \frac{1}{9}(1\cdot0+1\cdot0+1\cdot0+1\cdot0+1\cdot7+1\cdot4+1\cdot0+1\cdot8+1\cdot5)$

$= 2.67$

$G(0,1) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}0&0&0\\7&4&1\\8&5&2\end{bmatrix}$

$= 3$

$G(0,2) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}0&0&0\\4&1&0\\5&2&0\end{bmatrix}$

$= 1.33$

$G(1,0) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}0&7&4\\0&8&5\\0&9&6\end{bmatrix}$

$= 4.33$

$G(1,1) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}7&4&1\\8&5&2\\9&6&3\end{bmatrix}$

$= 5$

$G(1,2) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}4&1&0\\5&2&0\\6&3&0\end{bmatrix}$

$= 2.33$

$G(2,0) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}0&8&5\\0&9&6\\0&0&0\end{bmatrix}$

$= 3.11$

$G(2,1) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}8&5&2\\9&6&3\\0&0&0\end{bmatrix}$

$= 3.67$

$G(2,2) = \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}5&2&0\\6&3&0\\0&0&0\end{bmatrix}$

$= 1.78$

$G = \begin{bmatrix}2.67&3&1.33\\4.33&5&2.33\\3.11&3.67&1.78\end{bmatrix}$ (Ans)

e is Gaussian blur, weighted average, smoother and more natural. On the other hand f is box blur (moving average) Uniform average, can look blocky

Ans to the Que No 2 (a)

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \qquad \bar{I} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$

let, $f = \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}$ $\qquad t_{ij} = \begin{bmatrix} I_{11} \\ I_{12} \\ I_{13} \\ I_{21} \\ I_{22} \\ I_{23} \\ I_{31} \\ I_{32} \\ I_{33} \end{bmatrix}$

We need to prove $F \circ \bar{I} = f^T \cdot t_{ij}$.

<u>L.H.S</u> $F \circ \bar{I}$

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \circ \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$

$$= f_{11}I_{11} + f_{12}I_{12} + f_{13}I_{13} + f_{21}I_{21} + f_{22}I_{22} + f_{23}I_{23} + f_{31}I_{31} + f_{32}I_{32} + f_{33}I_{33}$$

R.H.S $f^T \cdot t_{ij}$

$$= \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}^T \cdot \begin{bmatrix} I_{11} \\ I_{12} \\ I_{13} \\ I_{21} \\ I_{22} \\ I_{23} \\ I_{31} \\ I_{32} \\ I_{33} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{21} & f_{22} & f_{23} & f_{31} & f_{32} & f_{33} \end{bmatrix} \cdot \begin{bmatrix} I_{12} \\ I_{12} \\ I_{13} \\ I_{21} \\ I_{22} \\ I_{23} \\ I_{31} \\ I_{32} \\ I_{33} \end{bmatrix}$$

$$= f_{11}I_{11} + f_{12}I_{12} + f_{13}I_{13} + f_{21}I_{21} + f_{22}I_{22} + f_{23}I_{23} + f_{31}I_{31} + f_{32}I_{32} + f_{33}I_{33}.$$

L.H.S = R.H.S .

For example:

$$F \circ \bar{I} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 3 \\ 0 & 4 & 2 \end{bmatrix}$$

$$= 4$$

$$f^T \cdot t_{ij} = [0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0] \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 3 \\ 0 \\ 4 \\ 2 \end{bmatrix}$$

$$= 4.$$

So it is proven that $F \circ \bar{I} = f^T t_{ij}$

Ans to the Que No. 3

ⓐ We know that, P in (x,y) frame,

$$x = f\frac{x_c}{z_c}$$

$$y = f\frac{y_c}{z_c}$$

Given,

$f = 35\text{ mm} = 35\times10^{-3}\text{ m}$

$x_c = 6\text{ m}$

$y_c = 8\text{ m}$

$z_c = -2\text{ m}$

$$\therefore x = 35\times10^{-3}\times\frac{6}{-2}$$

$$= -0.105\text{ m}$$

$$y = 35\times10^{-3}\times\frac{8}{-2}$$

$$= -0.14$$

$\therefore (x,y) = (-0.105, 0.14)$ (Ans).

ⓑ We know that,

For image coordinate in $(\tilde{x},\tilde{y})$ frame'

$$\tilde{x} = f\frac{x_c}{z_c} + \tilde{x}_0 , \quad \tilde{y} = f\frac{y_c}{z_c} + \tilde{y}_0$$

$$\tilde{x} = \left(35\times10^{-3}\times\frac{6}{-2}\right) + 2$$

$$= 1.895\text{ m}$$

$$\tilde{y} = \left(35\times10^{-3}\times\frac{8}{-2}\right) + 2$$

$$= 1.86\text{ m}$$

$\therefore$ in $(\tilde{x},\tilde{y})$ P is $(1.895, 1.86)$ (Ans).

(c) Given, $K_x = \cancel{20}\ 250$ pixels/meter in x direction

$K_y = 250$ pixels/meter in y direction

From b we can get $(\tilde{x}, \tilde{y})$ of p is $(1.895, 1.86)$

We know that,

Pixel coordinate (u, v)

$u = K_x \tilde{x}$

$v = K_y \tilde{y}$

$u = 250 \times 1.895$

$= 473.75$

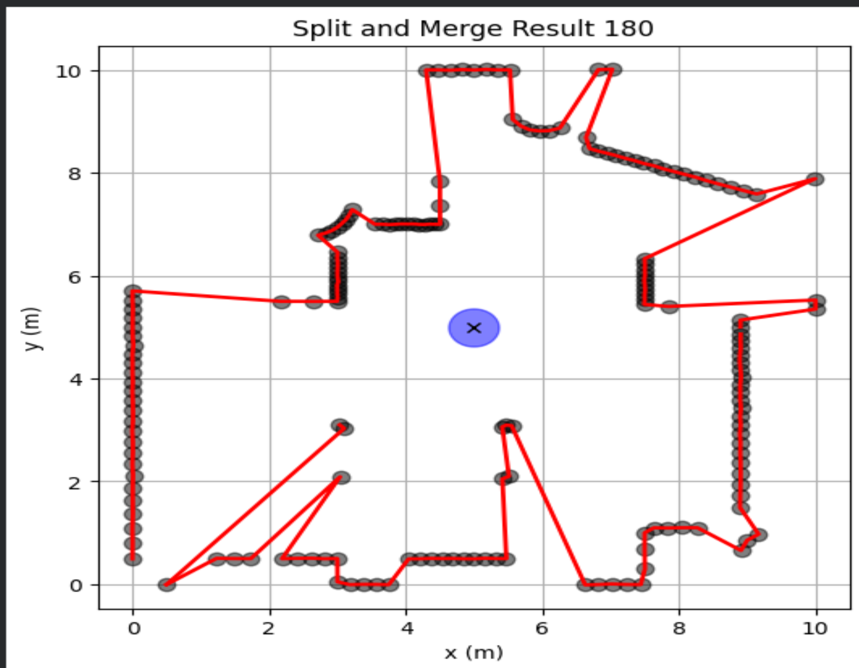
$v = 250 \times 1.86$

$= 465$

$\therefore (u, v) = (473.75, 465)$ (Ans)

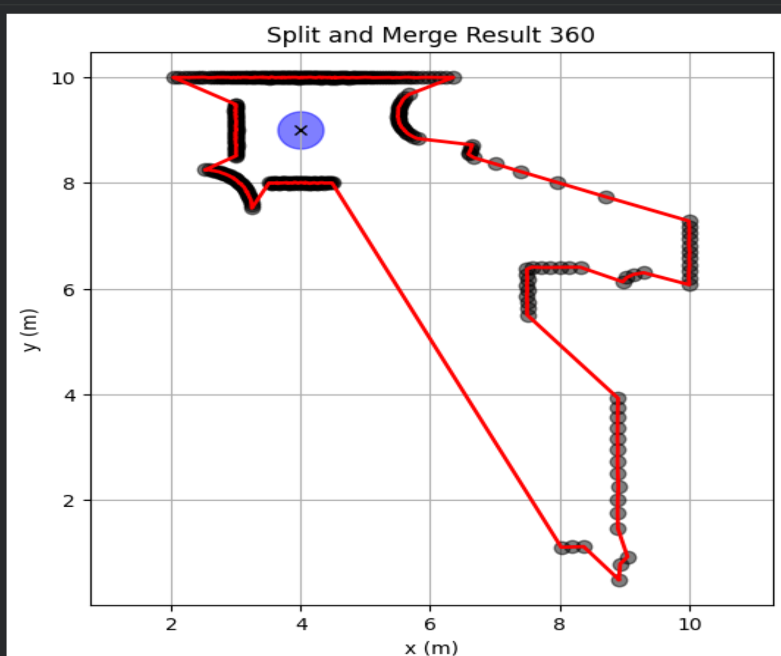
Code 4 A and B :

colab

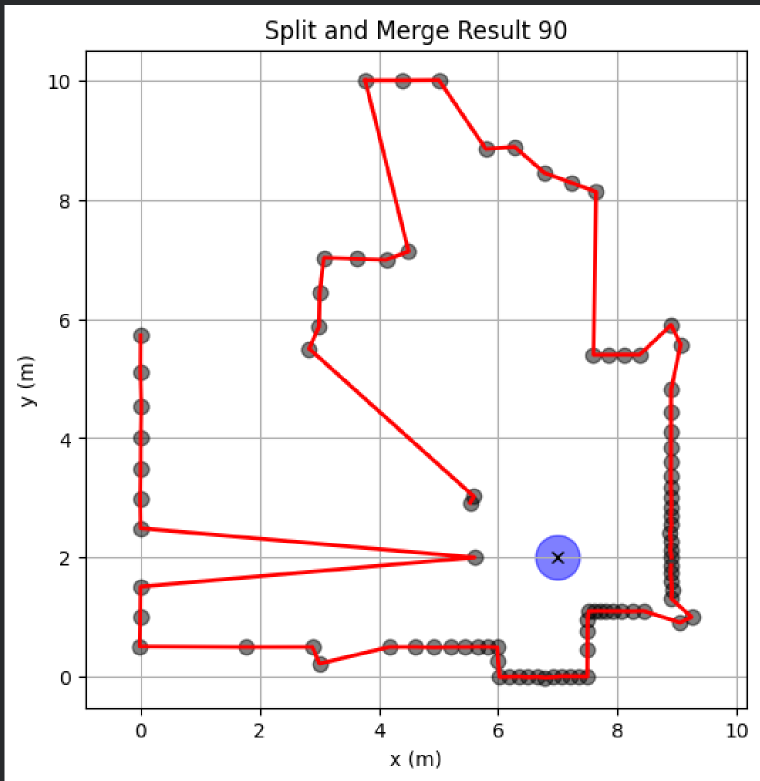



```
Total points: 180
Number of segments: 3
```

```
print("Number of segments:", len(segments))
```




```
Total points: 360
Number of segments: 2
```




```
Total points: 90
Number of segments: 3
```

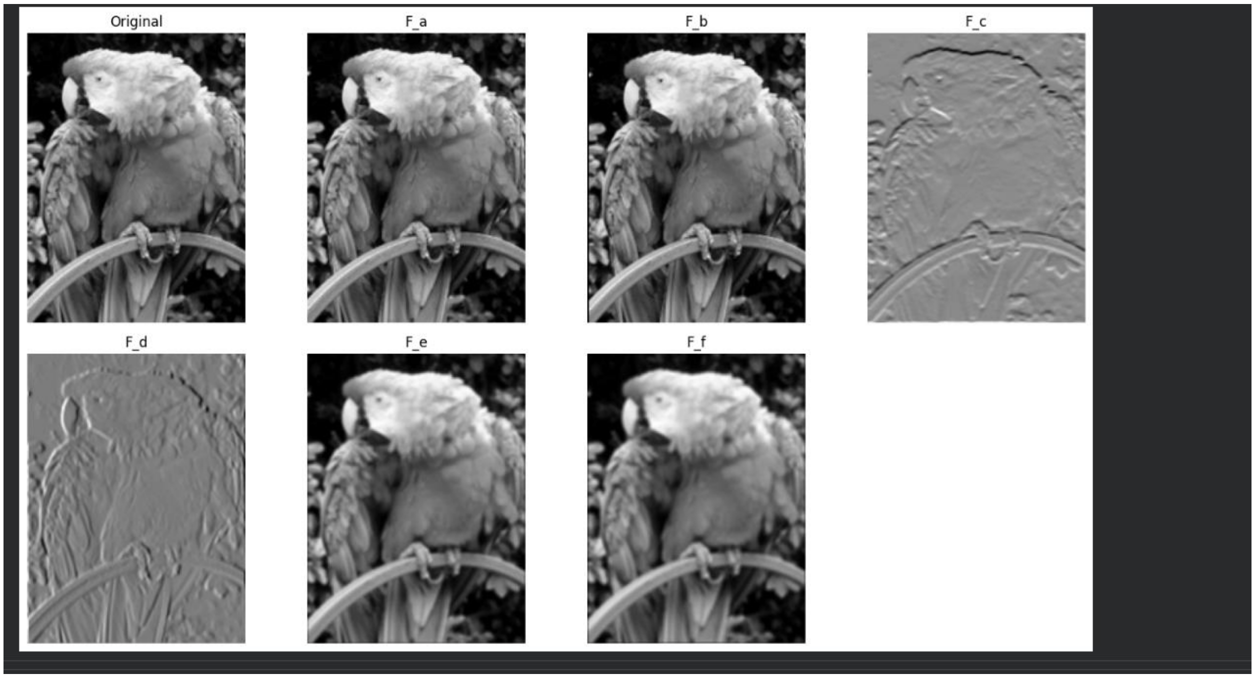
```
data = pd.read_csv('/content/rangeData_4_9_360 - rangeData_4_9_360.csv.csv',header=None).values

# Robot position
xr, yr = data[0]

# Lidar scan (theta, rho)
scan = data[1:]



# Threshold

LINE_POINT_DIST_THRESHOLD = 0.7
MIN_POINTS_PER_SEGMENT = 3
MIN_SEG_LENGTH = 0.1
MAX_P2P_DIST = 0.2
```

```
data = pd.read_csv('/content/rangeData_5_5_180 - rangeData_5_5_180.csv.csv',header=None).values

# Robot position
xr, yr = data[0]

# Lidar scan (theta, rho)
scan = data[1:]



# Threshold

LINE_POINT_DIST_THRESHOLD = 0.5
MIN_POINTS_PER_SEGMENT = 3
MIN_SEG_LENGTH = 0.55
MAX_P2P_DIST = 0.5
```

```
data = pd.read_csv('/content/rangeData_7_2_90 - rangeData_7_2_90.csv.csv',header=None).values

# Robot position
xr, yr = data[0]

# Lidar scan (theta, rho)
scan = data[1:]



# Threshold

LINE_POINT_DIST_THRESHOLD = 0.6
MIN_POINTS_PER_SEGMENT = 3
MIN_SEG_LENGTH = 0.5
MAX_P2P_DIST = 0.4
```

Code 2 B

[COlab]()


Original
F_a
F_b
F_c
F_d
F_e
F_f