

## Homework 1

Due Date: Sunday, 2 November 2025

- 1) i) Write down the state space equations (differential equations) for the following robots. For each robot identify the state, control, and control variables. For a given robot, what do the state, control and constant variables represent? Please explain.

- a) Unicycle model
- b) Differential drive robot
- c) Simplified car model
- d) Planar Quadrotor

ii) Describe the differences in dynamics between the robot models a, b and c.

- 2) Simulate the differential equations for the unicycle robot model using Euler's method for time  $t \in [0, 10]$  and time step  $\Delta t = 0.1$ . Use the following control input:

$$v = 1 \quad 0 \leq t \leq 10$$

$$\omega = \begin{cases} 3 & 0.5 \leq t \leq 1.5 \\ -3 & 2 \leq t \leq 3 \\ 3 & 4 \leq t \leq 5 \\ -3 & 6 \leq t \leq 7 \\ 3 & 8 \leq t \leq 9 \\ 0 & \text{all other } t \end{cases}$$

With initial conditions  $x(0) = 1$ ,  $y(0) = -1$ ,  $\theta(0) = 1$ .

- a) Plot  $x$  vs  $y$ ,  $x$  vs  $t$ ,  $y$  vs  $t$  and  $\theta$  vs  $t$ . Submit your code and plots.
- b) If you plot  $x$  vs  $y$  at each time step, you'll be able to see a movie of the robot moving in 2D space.

Feel free to simulate different initial conditions and control input sequences to gain a greater understanding.

- 3) Simulate the state space equations for the differential drive robot model using Euler's method for time  $t \in [0, 10]$  and time step  $\Delta t = 0.1$ . Use the following control input:

$$\omega_l = \begin{cases} 20 & 2 \leq t \leq 3 \\ 20 & 6 \leq t \leq 7.5 \\ 5 & \text{all other } t \end{cases} \quad \omega_r = \begin{cases} 20 & 4 \leq t \leq 5 \\ 20 & 8.5 \leq t \leq 9 \\ 5 & \text{all other } t \end{cases}$$

With  $r = 0.1$ ,  $L = 0.9$  and initial conditions  $x(0) = 1$ ,  $y(0) = 1$ ,  $\theta(0) = 1$ .

- a) Plot  $x$  vs  $y$ ,  $x$  vs  $t$ ,  $y$  vs  $t$  and  $\theta$  vs  $t$ . Submit your code and plots.
- b) If you plot  $x$  vs  $y$  at each time step, you'll be able to see a movie of the robot moving in 2D space.

Feel free to simulate different initial conditions and control input sequences to gain a greater understanding.