NORTH SOUTH UNIVERSITY

Introduction to Robotics

CSE495A

Home Work 1

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Answer to the Que No 1

(i) There are different types of robot system or model available. In this question I will describe 4 particular systems.

(a) Unicycle model: An Unicycle model represents a robot that have one wheel which moves forword with one velocity and one anguler velocity.

In this system we calculate the state variables from the sensors. Based on mission the processing unit calculates the controls.

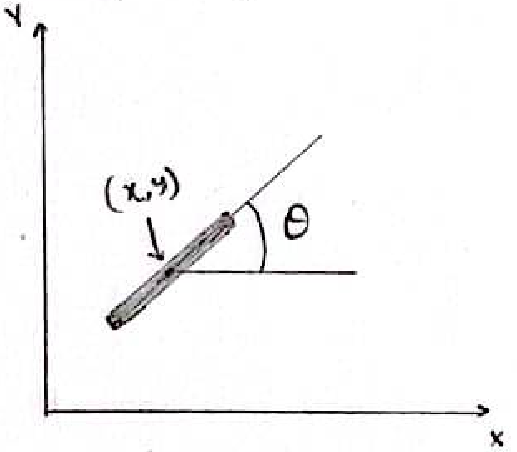

fig: Unicycle model

State space equation:

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{pmatrix} = \begin{pmatrix}\cos\theta\\ \sin\theta\\ 0\end{pmatrix} v + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix} \omega$$

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{pmatrix}_{3\times1} = \begin{pmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{pmatrix}_{3\times2} \begin{pmatrix}v\\ \omega\end{pmatrix}_{2\times1}$$

$$\therefore \dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \omega$$

Here states variables are x, y and θ

controls variables are v, ω

states and controls both are function of time. the state variables x, y, θ represents the current condition or position of the robot at a perticular time t. and the controls are the command that at time t how will be the velocity and angular velocity.

(b) Differential drive robot: A differential drive robot has two independently driven wheels which will allow the reobot moves left, right and forword and backword.

In this system we calculate the state variables using the sencors data and processor will calculate the controles besed on mission.

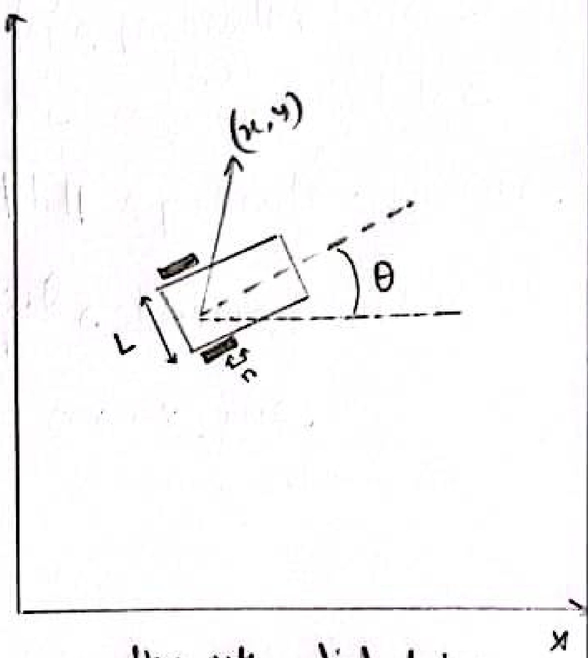

fig: Differential drive model

State space equation:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}_{(3\times1)} = \begin{pmatrix} \frac{r}{2}\cos\theta & \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta & \frac{r}{2}\sin\theta \\ \frac{r}{L} & -\frac{r}{L} \end{pmatrix}_{3\times2} \begin{pmatrix} W_r \\ W_L \end{pmatrix}_{(2\times1)}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}_{(3\times1)} = \begin{pmatrix} \frac{r}{2}(W_r + W_L)\cos\theta \\ \frac{r}{2}(W_r + W_L)\sin\theta \\ \frac{r}{L}(W_r - W_L) \end{pmatrix}_{(3\times1)}$$

Here, states are x, y, θ

controls are W_r, W_L

constant variables are L and r (L is the length between wheel and r is the radious of the wheel).

State and Control both are function of time. The state variables represent the current condition or position of the robot at a perticular time and the controls are the command at time t ~~how~~ that will represent the particular angular velocity of the two wheels (left and Right wheel).

(c) Simplified car model: A simplified car model represents ~~above~~ a four wheeled car using a single virtual front and rear wheel. the vehicle turns by steering the front wheel, which forms the steering angle (ϕ).

state space equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}_{(3\times1)} = \begin{pmatrix} v\cos\theta \\ v\sin\theta \\ \frac{v}{L}\tan\phi \end{pmatrix}_{(3\times1)}$$

Here, states are x, y, θ

controls are v, ϕ

constant variable is L

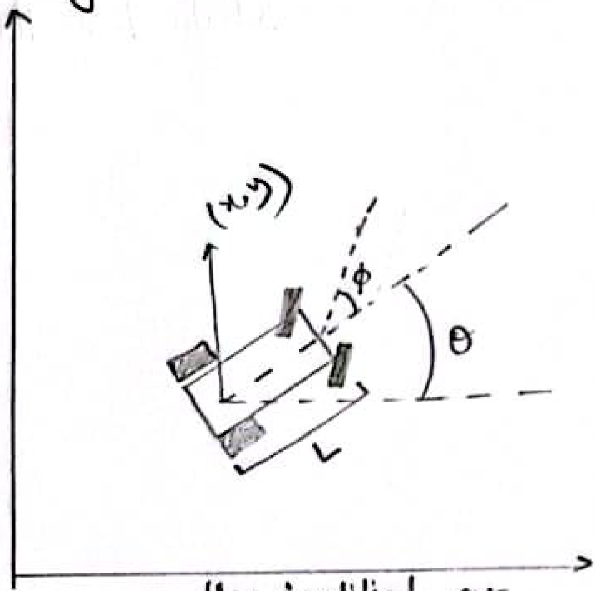

fig: simplified car

states and controls are function of time. the state variables gives car's position for a particular time on the other hand controls give the command of the robot ~~to~~ how it will move to fulfill a particular mission.

~~In~~ Here the ϕ is the angle of the stearing. It is compariable to our daily used car or bus.

① Planar Quadrotor : A planar Quadrotor is a 2D version of a regular Quadrotor. It has 2 motor and propeler. It creat thrust to move in 2D space.

state space equations:

$$\begin{pmatrix} \dot{x} \\ \dot{v}_x \\ \dot{y} \\ \dot{v}_y \\ \dot{\phi} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} v_x \\ \frac{-(T_1+T_2)\sin\phi}{m} \\ v_y \\ \frac{(T_1+T_2)\cos\phi}{m} - g \\ \omega \\ \frac{(T_2-T_1)L}{I_{zz}} \end{pmatrix}$$

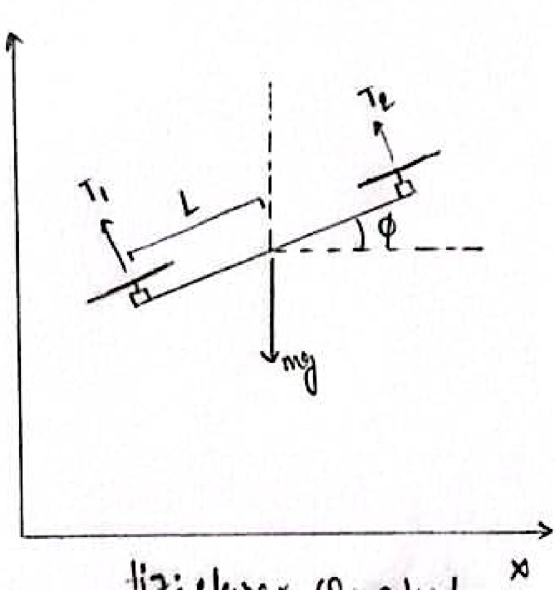

fig: planar Quadrotor

state variables are $x, y, v_x, v_y, \phi, \omega$

Control variables are T_1, T_2

Constant variables are L, m, g

other variables I_{zz}

Here all state variables and Control variables are function of time, T_1 and T_2 is the trust (force) of the Quadrotor from the two motor, m is the mass of the Quadrotor. I_{zz} is a variable which hold the value of moment of inertia.

(ii) Differences in Dynamics between Unicycle model, Differential drive robot, Simplified car:

Topic	Unicycle Model	Differential Drive	Simplified Car
(i) Type of model	Pure Kinematic	Kinematic	Kinematic (Ackermann steering)
(ii) State variable	x, y, θ	x, y, θ	x, y, θ, ϕ
(iii) Control variable	v, ω	ω_l, ω_r	v, ϕ
(iv) Constant variable	none	L, r	L
(v) Turning mechanism	By Directly applying angular speed	By Difference in wheel speeds	By turning the steering wheel.
(vi) Rotate in place	Yes	yes	no
(vii) minimum turning radius	None (can turn on spot)	None	It depends on ϕ steering angle
(viii) usage	simple path planning in theory	Real mobile robots which two wheels	cars, rovers etc.

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Text(0, 0.5, 'Theta value')
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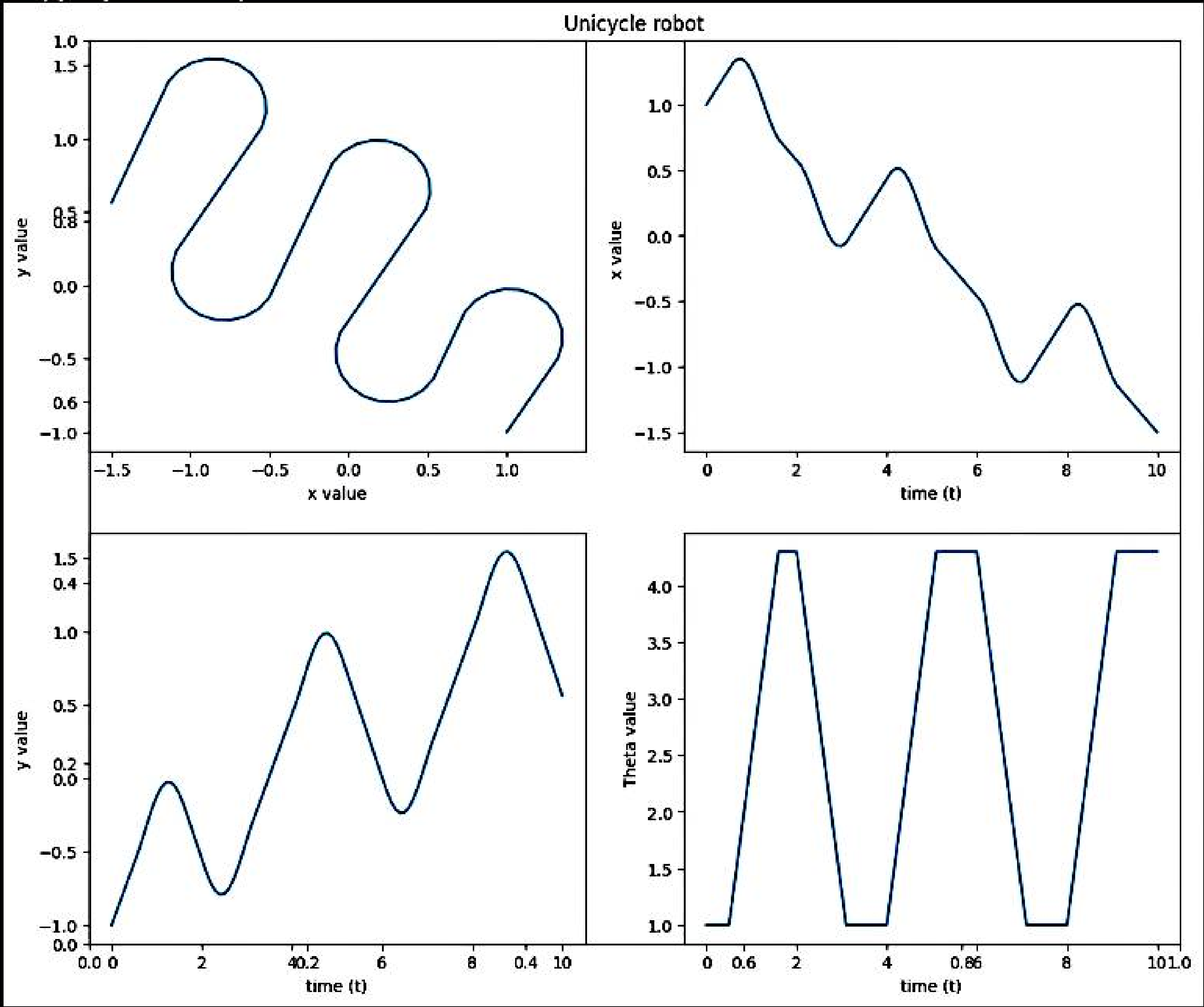

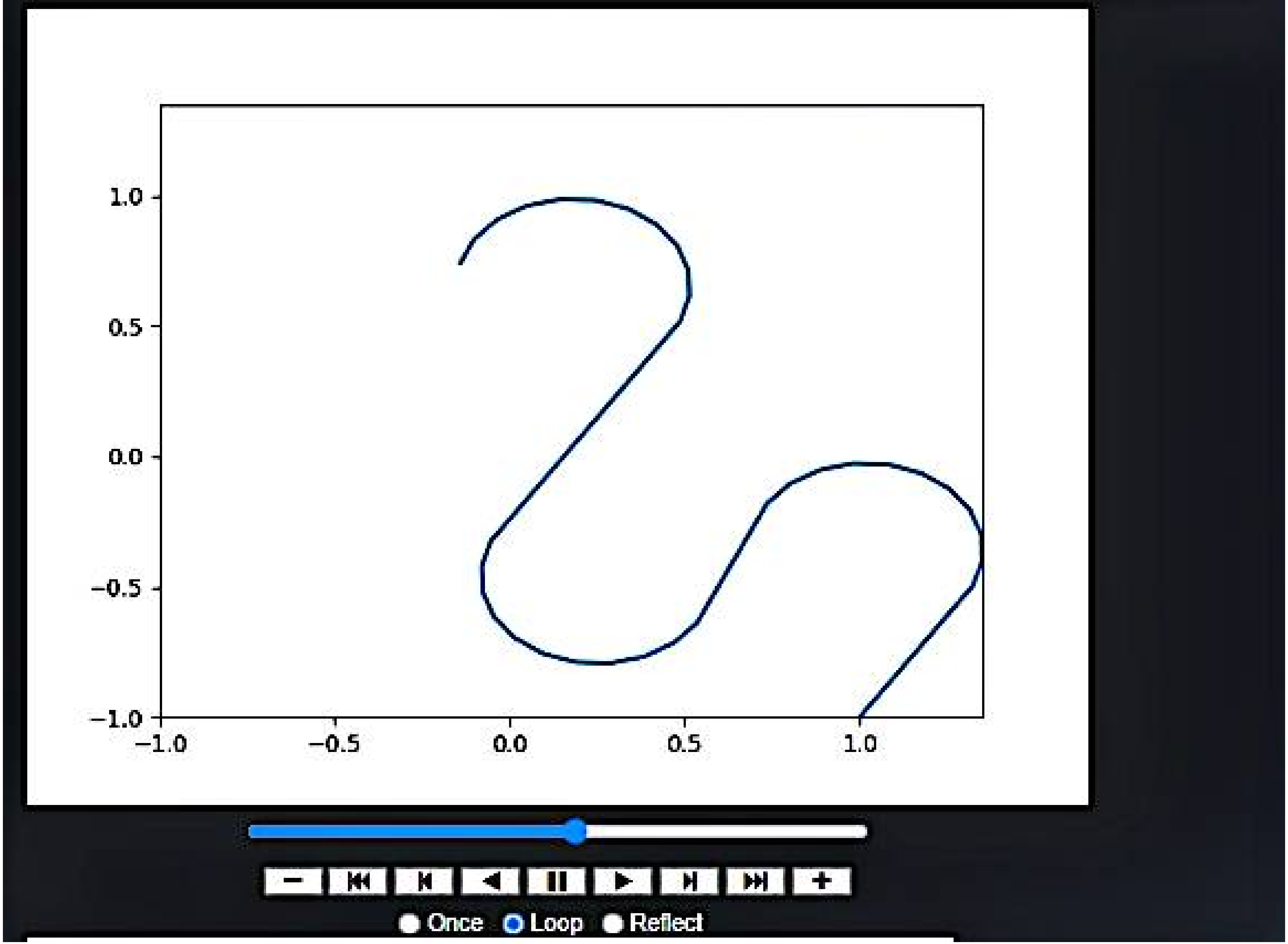
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0.5
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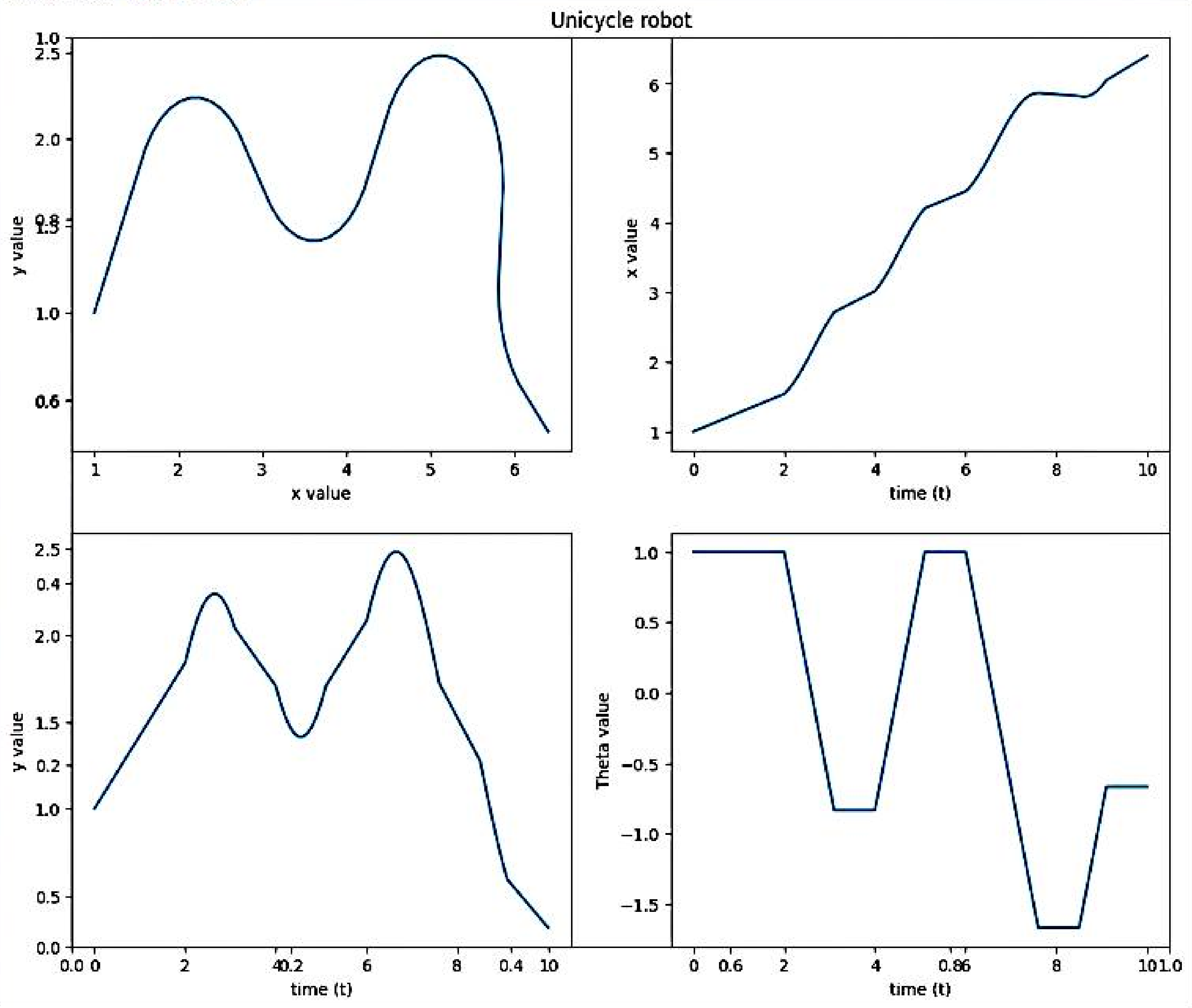
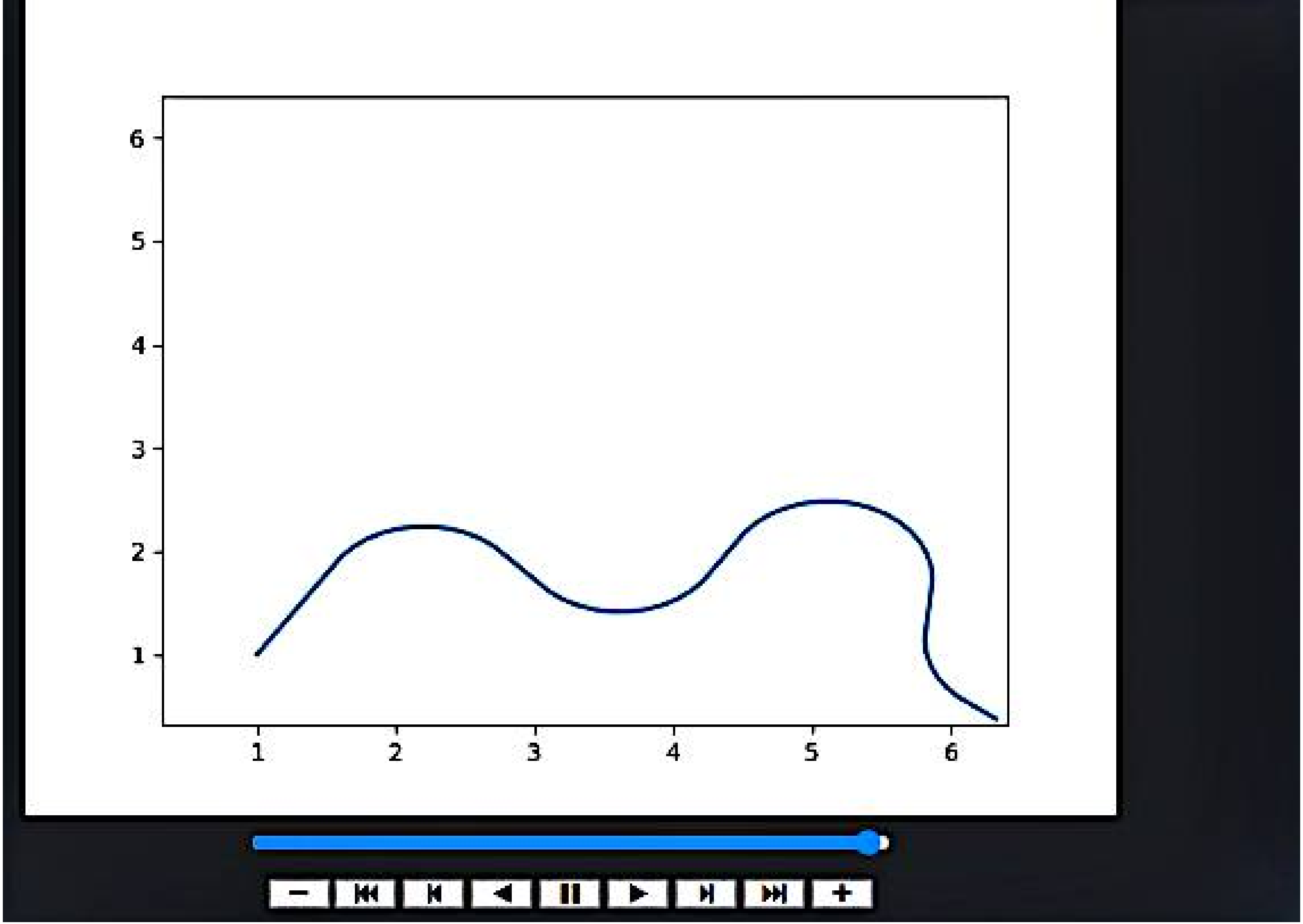
Once
Loop
Reflect

Text(0, 0.5, 'Theta value')




1
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Code Link:

HW 2 :

https://colab.research.google.com/drive/1cuxXU_dXFAnu2_r0-Nai5ERkWQydOuqa?usp=sharing

HW3:

https://colab.research.google.com/drive/1gzcFaoTGL-EWXqdfxSNcdXs5QLzGFQi2?usp=sharing