UNIVERSITY OF TORONTO

Faculty of Arts and Science

DECEMBER 2017 EXAMINATIONS

AST221H1 (Fall 2017)

Duration: 3 hours; Aids: Calculator only; Answers in exam booklets

Marks: All four questions have equal weight, with the weight distributed equally over the subitems.

Solar mass	$1\,M_\odot$	$= 1.989 \times 10^{30}\,\mathrm{kg}$
Solar luminosity	$1\,L_\odot$	$= 3.839 \times 10^{26}\,\mathrm{W}$
Solar radius	$1\,R_\odot$	$= 6.955 \times 10^{8}\,\mathrm{m}$
Solar effective temperature	$T_\odot$	$= 5777\,\mathrm{K}$
Earth mass	$1\,M_\oplus$	$= 5.974 \times 10^{24}\,\mathrm{kg}$
Earth radius	$1\,R_\oplus$	$= 6.378 \times 10^{6}\,\mathrm{m}$
Astronomical Unit	1 AU	$= 1.4960 \times 10^{11}\,\mathrm{m}$
Parsec	1 pc	$= 1\,\mathrm{AU}/\tan 1'' = 3.0857 \times 10^{16}\,\mathrm{m}$
Gravitational constant	G	$= 6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Speed of light	c	$= 2.9979 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck's constant	h	$= 6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar$	$= h/2\pi = 1.055 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann's constant	k	$= 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	$= 5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Radiation constant	a	$= 4\sigma/c = 7.5657 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K}$
Proton mass	m_p	$= 1.67262 \times 10^{-27}\,\mathrm{kg}$
Neutron mass	m_n	$= 1.67493 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit (amu)	u	$= 1.66054 \times 10^{-27}\,\mathrm{kg}$
Electron mass	m_e	$= 9.10939 \times 10^{-31}\,\mathrm{kg}$
Hydrogen mass	m_H	$= 1.67353 \times 10^{-27}\,\mathrm{kg}$
Electron volt	1 eV	$= 1.6022 \times 10^{-19}\,\mathrm{J}$
Bohr radius	a_0	$= \hbar^2/m_e e^2 = 5.292 \times 10^{-11}\,\mathrm{m}$

Planet	Mass ($M_\oplus$)	Equatorial radius ($R_\oplus$)	Semi-major axis (AU)	Albedo
Mercury	0.05528	0.3825	0.3871	0.119
Venus	0.81500	0.9488	0.7233	0.750
Earth	1.00000	1.0000	1.0000	0.306
Mars	0.10745	0.5326	1.5236	0.250
Jupiter	317.83	11.209	5.2044	0.343
Saturn	95.159	9.4492	9.5826	0.342
Uranus	14.536	4.0073	19.2012	0.300
Neptune	17.147	3.8926	30.0476	0.290

1. We consider two stars (masses M_1, M_2; radii R_1, R_2) that approach each other with relative velocity v_∞ at an impact parameter b (i.e., absent gravity, their closest approach would be b).

   (a) Make a sketch of the orbit the stars would have for the case that closest approach is at $a = R_1 + R_2$. Use a frame fixed to one of the stars.

   (b) Show that for given closest approach a, one has $b = a\sqrt{1+2GM/av_\infty^2}$ (where $M = M_1 + M_2$ is the total mass). *Hint: compare angular momentum and total energy at infinity and closest approach, writing in terms of total mass and reduced mass.*

   (c) Insert numbers to estimate the factor by which the cross section for interaction is increased over the estimate that ignores gravity for conditions in a globular cluster: solar-like stars ($M_1 = M_2 = 1\,M_\odot$, $R_1 = R_2 = 1\,R_\odot$) and typical velocities of $v_\infty = 10\,\mathrm{km/s}$.

2. Properties of main-sequence stars with a range of mass.

   (a) Derive how central temperature T_c scales with mass M and radius R for a self-gravitating ball of ideal gas. For main sequence stars, observations of binaries show that $R \propto M^{3/4}$. Show that this implies that $T_c \propto M^{1/4}$.

   (b) Observations of binaries also show that $L \propto M^4$. Given that $T_c \propto M^{1/4}$, what does this imply for nuclear reactions, are they (very) sensitive to temperature or (very) insensitive? Explain your answer based on the physics of these reactions (i.e., what is required for particles to succeed in fusing, and why is that (in)sensitive to temperature?)

   (c) Explain why brown dwarfs, with masses below $\sim 0.08\,M_\odot$, never become able to fuse hydrogen. To help your explanation, sketch the evolution of central density and temperature during the contraction phase of a proto-star and a proto-brown dwarf.

3. The temperature-luminosity or Hertzsprung-Russell diagram (HRD) and the evolution of stars like the Sun.

   (a) Draw a HRD, and sketch the zero-age-main-sequence. Mark the rough locations for 10, 1, and $0.1\,M_\odot$ main-sequence stars. Be sure to add representative values to the axes.

   (b) Draw the evolutionary track for a $1\,M_\odot$ star, starting at the main sequence. Label the various stages.

   (c) For each stage, write down the source of pressure *in the core* as well as the source(s) of energy and their location(s) (e.g., on the pre-main-sequence, it would be ideal gas pressure and gravitational contraction throughout the star).

4. Temperatures on the Earth.

   (a) Show that, if the Earth's surface radiated exactly in equilibrium with the amount of sunlight absorbed, the average temperature at the surface would be 255 K.

   (b) The true average temperature near the ground on Earth is about $15^\circ\,\mathrm{C} = 288\,\mathrm{K}$. Why is this different from your estimate above? Be sure to give a physical explanation.

   (c) The total mass of the atmosphere is about $5 \times 10^{18}\,\mathrm{kg}$. Estimate the amount of thermal kinetic energy it has. Given this and the luminosity emitted by the Earth's atmosphere, what is the thermal time? How does this time compare to the delay between Summer solstice and the time of year in which the tempatures are highest?