UNIVERSITY OF TORONTO

**Faculty of Arts and Science**

**DECEMBER 2016 EXAMINATIONS**

**AST 221H1F**

**Duration - 3 hours**

**Examination Aids: Hand-held calculator**

**Student Name:** **Student number:**

*There are 20 multiple-choice questions, each worth 2 marks. There are 6 long questions, worth 10 marks each – some conceptual and some calculational – be sure to explain your answers clearly.*
**Write you answers directly on these exam pages**

| | | |
|---|---|---|
| Solar mass | $1\,M_\odot$ | $= 1.989 \times 10^{33}\,g$ |
| Solar luminosity | $1\,L_\odot$ | $= 3.826 \times 10^{33}\,erg\ s^{-1}$ |
| Solar radius | $1\,R_\odot$ | $= 6.9599 \times 10^{10}\,cm$ |
| Solar effective temperature | $T_\odot$ | $= 5770\,K$ |
| Earth mass | $1\,M_\oplus$ | $= 5.974 \times 10^{27}\,g$ |
| Earth radius | $1\,R_\oplus$ | $= 6.378 \times 10^{8}\,cm$ |
| Parsec | $1\,pc$ | $= 3.0857 \times 10^{18}\,cm$ |
| Astronomical Unit | $1\,AU$ | $= 1.4960 \times 10^{13}\,cm$ |
| | | |
| Gravitational constant | $G$ | $= 6.67259 \times 10^{-8}\,dyne\ cm^2\ g^{-2}$ |
| Speed of light | $c$ | $= 2.998 \times 10^{10}\,cm\ s^{-1}$ |
| Planck's constant | $h$ | $= 6.626 \times 10^{-27}\,erg\ s$ |
| | $\hbar$ | $= h/2\pi = 1.055 \times 10^{-27}\,erg\ s$ |
| Boltzmann's constant | $k$ | $= 1.38 \times 10^{-16}\,erg\ K^{-1}$ |
| Stefan-Boltzmann constant | $\sigma$ | $= 5.67 \times 10^{-5}\,erg\ cm^{-2}\,s^{-1}\,K^{-4}$ |
| Radiation constant | $a$ | $= 4\sigma/c$ |
| Proton mass | $m_p$ | $= 1.6726231 \times 10^{-24}\,g$ |
| Neutron mass | $m_n$ | $= 1.674929 \times 10^{-24}\,g$ |
| atmoic mass unit (amu) | $u$ | $= 1.660540 \times 10^{-24}\,g$ |
| Electron mass | $m_e$ | $= 9.1093897 \times 10^{-28}\,g$ |
| Hydrogen mass | $m_H$ | $= 1.673534 \times 10^{-24}\,g$ |
| Electron volt | $1\,eV$ | $= 1.6022 \times 10^{-12}\,erg$ |
| Bohr radius | $a_0$ | $= \hbar^2/m_e e^2 = 5.292 \times 10^{-9}\,cm$ |

## 1 Multiple-choice questions

*In the square-bracket before each question, write down (clearly) the letter for the single choice you consider most reasonable. Each question is worth 2 marks.*

[ ] **1.** If all hydrogen in the center of the Sun is suddenly replaced by helium, on what timescale would the entire Sun notice the change?

a) In dynamical timescale. b) In nuclear timescale.

c) In thermal diffusion time because it takes this long for photons, which support the Sun against gravity, to leak out.

d) In a cooling time because the gas pressure, which supports the Sun against gravity, depends on gas temperature.

[ ] **2.** If $KE$ is the thermal kinetic energy stored in a gravitationally bound cloud, and $U$ its gravitational potential energy. Which of the following is a sufficient condition for this bound cloud to undergo gravitational collapse?

a) $U < 0$ b) $KE + U < 0$. c) $KE + U/2 < 0$.

d) The proper condition depends on the mass distribution inside the cloud.

[ ] **3.** Compare the four following elements, ${}^{1}_{1}H$, ${}^{4}_{2}He$, ${}^{56}_{26}Fe$, ${}^{238}_{92}U$. Which element has the lowest ratio of nucleus mass divided by number of nucleons?

a) Hydrogen. b) Helium. c) Iron. d) Uranium.

[ ] **4.** Do we expect that more metal-rich stars are more likely to have giant planets? why or why not?

a) Yes. Although giant planets are not largely made up of metals, forming their cores require a metal-rich environment.

b) Yes. Because giant planets are largely made up of metals. They can grow more massive around metal-rich stars.

c) No. Giant planets are mostly made up of hydrogen and helium with metals being an insignificant part of mass.

d) No. More metal-rich stars are born later in the universe so have less time to form giant planets.

[ ] **5.** In the Solar system, the Sun has 99.9% of the mass but 0.1% of the angular momentum. What is the underlying explanation for this observed fact?

a) The planets need the angular momentum. Without it, they would have fallen into the Sun.

b) The Sun has to shed almost all of its initial angular momentum in order to collapse and to start fusion.

c) Material that formed the Sun must have initially very little angular momentum.

d) The Sun spins off little gas clouds (called planets) on its way to collpase. This is why the planets contain 99.9% of the angular momentum.

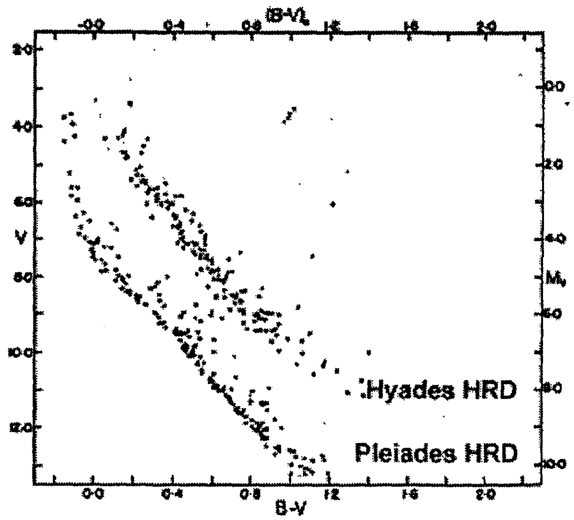


Figure 1: Hurtzsprung-Russell diagram for two open clusters, Hyades (top group of points) and Pleiades (the bottom group). The horizontal axis stands for B-V color and the vertical axis is the apparent V-magnitude.

[ ] **6.** Figure 1 shows the HR diagram for two open clusters, the Hyades cluster and the Pleiades cluster. Which conclusion is correct regarding their relative ages and distances?

a) Pleiades is younger and Hyades is further away.

b) Pleiades is younger and Hyades is closer to us.

c) Pleiades is older and Hyades is further away.

d) Pleiades is older and Hyades is closer to us.

[ ] **7.** Which of the following wavelength group can **NOT** be observed on the ground?

a) X-ray, UV, optical

b) X-ray, UV, far-infrared

c) optical, far-infrared, microwave

d) near-infrared, microwave, radio

[ ] **8.** Why do stars twinkle?

a) Rayleigh scattering in the Earth's atmosphere.

b) Molecular absorption in the Earth's atmosphere.

c) Interstellar absorption along the star's line of sight.

d) Atmospheric turbulence on the stellar surface.

[ ] **9.** Which of the following statement is **NOT** a reason for core-collapse at the end of a massive star's life?

a) Core collapse happens because no more nuclear fusion is possible.

b) Core collapse occurs after electron degeneracy pressure is lost, as all electrons are absorbed into neucleons.

c) Core collapse occurs as heat in the core is rapidly removed by free-streaming neutrinos.

d) Core collapse occurs after the stellar envelope has cooled sufficiently. The loss of pressure causes the envelope to cave in onto the core.

[ ] **10.** How do we know if a planet has a substantial self-luminosity?

a) If its interior is much hotter than the surface.

b) If it has a strong green-house effect.

c) If its interior is hot enough for nuclear fusion.

d) If its surface temperature is much hotter than expected from passive heating.

[ ] **11.** On the way to forming a star, a contracting cloud

a) gains gravitational energy and loses kinetic energy.

b) heats up. It gains as much thermal kinetic energy as it loses in gravitational energy.

c) heats up but its gain in kinetic energy is at most half of the loss in gravitational energy.

d) cools down as it loses energy.

[ ] **12.** What fundamentally determines the luminosity of the Sun?

a) The surface area and surface temperature of the Sun. b) The rate of nuclear fusion.

c) Maximum gas pressure support before degeneracy sets in. d) The rate of photon diffusion.

[ ] **13.** Why did Venus experience run-away green-house effect to become a living hell, while Earth avoided this fate?

a) Venus is formed out of material that contains more carbon dioxide. Its outgassing lead to the run-away green-house effect.

b) Venus is slightly closer to the Sun. This causes the run-away greenhouse effect.

c) Earth has life. Life modifies the atmospheric content and prevents the run-away.

d) Earth has ocean. This absorbs much of the carbon dioxide and prevents the run-away.

[ ] **14.** The Keck telescope has a collecting area that is $\sim 16$ times that of the Hubble space telescope. What is the best angular resolution of the Keck?

a) 16 times better than Hubble. b) 8 times better than Hubble.

c) 4 times better than Hubble. d) 2 times better than Hubble.

[ ] **15.** What will be the temperature of a Kuiper belt object at 40 AU, if the Earth is heated up to 220 K? Assume the two have the same albedos.

a) 5.5 K. b) 35 K. c) 64 K. d) 88 K.

[ ] **16.** Which of the following is **NOT** explained by Rayleigh scattering of photons?

a) Sunset is red. b) The sky is blue.

c) The ocean is blue. d) Star lights reaching us are reddened.

[ ] **17.** What is the free-fall time of a $0.1 M_\odot$ main-sequence star?

a) $\sim$ 100 hours. b) $\sim$ 10 hours. c) $\sim$ 1 hours. d) $\sim$ 0.1 hours.

[ ] **18.** What drives the strong jet streams that we observe on giant planets?

a) The internal luminosities of these planets cause fast atmospheric circulation.

b) The jet streams are the rotation of the planet atmospheres.

c) The jet streams result from both rapid rotation and un-even solar-heating.

d) The uneven solar heating pushes hot air from the day side to the night side, leading to the jet streams.

[ ] **19.** Why is the asteroid belt where it is today?

a) It is inside the ice-line. So no planets can form out of the asteroids.

b) The asteroids are constantly perturbed by Jupiter to reach Earth-crossing orbits. This determines the position of the asteroid belt.

c) The asteroids in this location can remain cold here so they are not evaporated by the Sun.

d) The asteroids are safe here because they are far from planetary perturbations.

[ ] **20.** Which of the following statements regarding blackholes is correct?

a) They are called 'black' because they radiate blackbody radiation.

b) They are black because even photons at infinity will be sucked in.

c) They are not really black. Photons can leak out freely because photons have no mass and cannot feel the gravity.

d) To form a blackhole you need an enormous amount of mass. There cannot be low-mass blackholes.

e) None of the above.

# 2 Long questions.

**1.** Gas molecular weight. What is the mean molecular weight of the gas in the Sun's interior? For your calculation, the gas is composed of 72% in ${}^1_1$H, and 28% in ${}^4_2$He, by mass fraction. Both Hydrogen and Helium can be considered to be fully ionized.

**2.** For an atmosphere at hydrostatic equilibrium, its pressure gradient resists the gravitational down-ward pull of the planet. Now imagine a planet X: it has the same total planet mass and the same total atmospheric mass as does the Earth, but a radius that is twice smaller. How much will its pressure at ground level differ from that on Earth?

**3.** Brown-dwarf limit. Main-sequence stars are expected to satisfy a mass-radius relation of $(R/R_{\odot}) \approx (M/M_{\odot})$, where $M_{\odot}, R_{\odot}$ are the mass and radius of the Sun, respectively. However, this relations fails for low mass objects where electron degeneracy starts to matter. Derive the mass threshold at which this occurs, assuming for simplicity that the star is a sphere of constant temperature and constant density and that your star is made up of fully ionized hydrogen. You do not have to evaluate your final expression for the critical mass, but your final expression should only contain constants that appear in the table of page 1.

**4.** Planet formation by gravitational collapse. Consider a spherical clump of gas (density $\rho$ and radius $R$) orbiting around the Sun (mass $M_{\odot}$) at a distance $a$. Derive the critical density above which this clump can collapse gravitationally to form a planet. Evaluate your expression for the critical density at $a = 1\text{AU}$.

**5.** The Earth-Moon system. Draw a picture of the tidal bulge on the Earth, in the Earth-Moon system, and use this as a tool to explain why the Moon is currently receding away from us. Be careful with the pointing of the tidal bulge in relation to the Earth-Moon line.

**6.** Low-mass planet. The best radial velocity instruments today still suffer from an intrinsic noise level of 0.5 m/s. Given this constraint, what is the smallest mass planets we can reliably detect (with full amplitude for the radial velocity being comparable to the above noise level), if the planets of interest are at 1 AU around Sun-like stars?

*Total Marks = 100*