Student name: AST221H
Student #: **Assignment 5** Due: Nov. 29, 2019

You may work together with other students to solve these problem sets, but all solutions must be written and submitted independently. ***Part of the mark for each question will be given for showing your work****, including intermediate steps and diagrams if necessary! Check the syllabus for reading recommendations. Careful with units!*

Problem 1: The greenhouse effect and global warming

One of the main worries of our time is global warming and climate change. Here, we make an overly simple model of the greenhouse effect to get an idea of how numbers scale.

The Sun heats the Earth. For an air-less Earth, one can derive an equilibrium temperature $T_p = 255$ K (see lectures, and C&O Eq. 19.5), but this has to be modified where an atmosphere exists. Earth's atmosphere is optically thin (nearly transparent) at visible wavelengths, so the solar radiation hits the ground directly. The atmosphere is optically thick (opaque), however, at infrared wavelengths and absorbs the ground's infrared blackbody radiation. The ground is now hotter, as its heat cannot escape directly to space. Instead, it has to be diffused toward the (infrared) photosphere which radiates with a temperature $T = T_p$ (think why; hint: energy conservation).

a (3 points) Imagine the atmosphere as a single opaque layer with a uniform temperature T_p. It is receiving heat from the ground (at temperature T_g) and radiates as much energy towards the ground as it radiates towards space. First ignoring the gradual warming of the atmosphere, use energy conservation to show that $T_g = 2^{1/4} T_p$. Is the current ground temperature (288 K) colder or hotter than this? Why?
Hint: Recall from C&O the energy flux emitted by a blackbody.

b A more sophisticated approach is to allow different layers in the atmosphere to have different temperatures, each emitting both upwards and downwards, with a constant net flux passing through. From this, one can derive (e.g., C&O, Eq. 9.53) that the temperature will follow

$$T_g^4 = T_p^4 \left[1 + \frac{3}{4}\left(\tau - \frac{2}{3}\right)\right] \tag{1}$$

where τ is the infrared optical depth from the point being considered to the top of the atmosphere. (Opacity is discussed in C&O 9.2. The atmosphere emits an an effective optical depth $\tau = 2/3$.)

(i) (1 point) Given $T_g = 288$ K and $T_p = 255$ K, what is the atmospheric optical depth τ_g to the ground?

(ii) (1 point) Also calculate τ_g on Venus, given its no-atmosphere and actual ground temperatures (see lectures).

(iii) (2 points) On Earth, water vapour is more abundant and thus more important than CO_2 in absorbing infrared photons. But let us make the simplifying assumption that CO_2 is the only greenhouse gas around. Supposing, simplistically, that the greenhouse effect scales linearly with CO_2, what is the expected rise in temperature on Earth as CO_2 is doubled from the current abundance? A more accurate calculation, resulting from simulations that include both positive and negative feedbacks, as well as more sophisticated radiative transfer effects, predicts a rise of 3-5 degrees globally (still very bad news).

Problem 2: Comet impacts

Comets are mainly composed of frozen water and CO_2. Here we will investigate some of the impacts of comets on the early Earth.

a (3 points) Calculate the kinetic energy of a spherical comet of radius 4 km, composed of water ice, which arrives from far away to the region of the Earth's orbit around the Sun. Assume that the density of water ice $\rho = 0.92$ g/cm^3.

b (4 points) Estimate the radius of the cylindrically shaped crater that such a comet creates when it strikes the Moon. Assume that the crater, of depth 10 km, is formed by heating to 3,500 K, and thus vaporizing, a cylindrical volume of moon rocks. Moon rocks are made of silicates, which have molecular weights of ~ 30 (i.e., $\bar{m} \sim 30\, m_{\rm H}$), and mean solid densities $\rho \sim 2$ g/cm^3. Ignore the latent heat required to melt and vapourize the rocks, and the energy involved in vapourizing the comet itself.
Hint: equate the kinetic energy of the comet with the thermal energy of the vapourized rocks.

c (3 points) The number of craters per unit area in the relatively smooth 'mare' regions of the Moon, which trace the impact history over the past ~ 3 Gyr, indicate a total of about 10 impacts, leaving 50 km radius craters, during this period. Based on the assumptions in b), these would be impacts of objects with radii > 4 km. From geometrical considerations alone (i.e., the relative target sizes posed by the Earth and by the Moon, and ignoring gravity), estimate how many such objects have struck the Earth.
Bonus (1 point): why can we assume the visible craters track accurately the impact history of the Moon?

d (2 points) What is the mean time interval between impacts? How does the impact you found compare to ~ 60 Myr, the typical interval between large extinctions of species on Earth? (The most recent large extinction, 65 Myr ago, eliminated the dinosaurs, and marked the rise of the mammals.) Explain why you might find a different value.

Problem 3: Rayleigh scattering

(6 points) Gas molecules in our atmosphere have sizes (~ 10 Å) much smaller than the wavelength of optical photons. As a result, bound electrons within the molecules preferentially scatter shorter wavelength photons (Rayleigh scattering). The photon cross-section σ for Rayleigh scattering goes as λ^{-4}, where λ is the photon wavelength. Obtain an expression for the typical travel time of a red photon ($\lambda = 8000$ Å) across a layer of atmosphere with thickness d, knowing that a blue photon ($\lambda = 4000$ Å) has a mean-free-path length $l_{mfp} = d/3$ in the same atmosphere. Express your answer using d and c, where c is the speed of light.

Problem 4: Temperatures on Earth

a (3 points) The total mass of the Earth's atmosphere is about 5×10^{21} g. Assuming a mean molecular mass $\bar{m} = 29\ m_{\rm H}$ (for an atmospheric composition of 71% N_2 and 28% O_2), estimate the amount of thermal (kinetic) energy it has. Use the appropriate temperature from Problem 1.

b (2 points) Calculate the luminosity emitted by the Earth's atmosphere.

c (3 points) The thermal time for the Earth's atmosphere can be expressed as the ratio between the thermal energy calculated in a) and the luminosity calculated in b). What is this value? How does this time compare to the delay between summer solstice and the time of year in which the temperatures are highest?