Jeff Shen
1004911526

Assignment 3

a

The luminosity of a star is given by the Stefan-Boltzmann law:

$$L = 4\pi R^2 \sigma T_{eff}^4.$$

We can consider the ratio of luminosity of some star to the luminosity of the sun:

$$\frac{L}{L_\odot} = \frac{4\pi R^2 \sigma T_{eff}^4}{4\pi R_\odot^2 \sigma T{\odot}^4} = \frac{R^2 T_{eff}^4}{R_\odot^2 T{\odot}^4}.$$

But we also know, given the mass-radius relation, that $R \propto M$. Then again, we can express the ratio of the radii of two stars (with the sun as one of the stars) in terms of the ratio of their masses:

$$\frac{R}{R_\odot} = \frac{M}{M_\odot}.$$

Substituting this in to the previous equation, we obtain an expression for L that does not depend on R:

$$\frac{L}{L_\odot} = \frac{M^2 T_{eff}^4}{M_\odot^2 T{\odot}^4}.$$

Given that $L \propto M^4$, we know that $M^2 \propto L^{1/2}$. Then

$$\frac{M^2}{M_\odot^2} = \frac{L^{1/2}}{L_\odot^{1/2}} = \sqrt{\frac{L}{L_\odot}}.$$

So

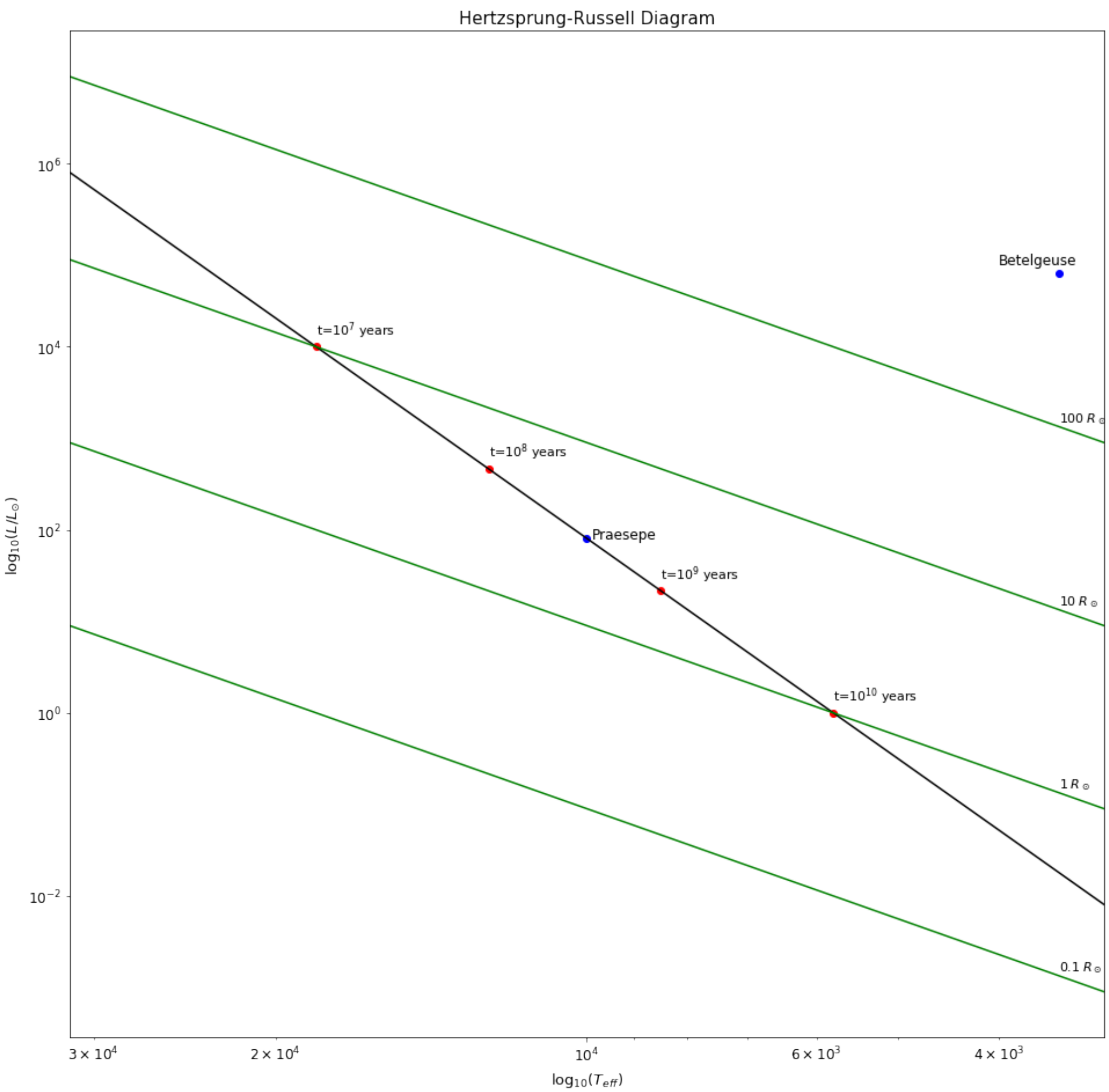
$$\frac{L}{L_\odot} = \sqrt{\frac{L}{L_\odot}} \frac{T_{eff}^4}{T{\odot}^4} = \frac{T_{eff}^8}{T{\odot}^8}.$$

b

[2]:
```python
# imports
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
%matplotlib inline

# graph setup
fig, ax = plt.subplots(figsize=(16, 16))
ax.set_xlabel(r'$\log_{10}(T_{eff})$', fontsize=13)
ax.set_ylabel(r'$\log_{10}(L/L_{\odot})$', fontsize=13)
ax.set_title('Hertzsprung-Russell Diagram', fontsize=15)
ax.set_xlim(left=10**4.5, right=10**3.5)
ax.yaxis.set_tick_params(labelsize=12, which='both')
ax.xaxis.set_tick_params(labelsize=12, which='both')
```

```
# logarithmic axes
ax.set_xscale('log')
ax.set_yscale('log')

# setup
teff = np.linspace(10**4.5, 10**3.5, 100) # generate x-coordinates covering␣
↪appropriate range of temperatures
tsun = 5778 # temperature of sun in kelvin
ltsun = 10**10 # lifetime of sun in years


########################## part b
# calculate, using formula from part a, luminosity of star (in terms of solar␣
↪luminosities) given its temperature
def f_b(t):
    return (t**8) / (tsun**8)

# plot line
plt.plot(teff, f_b(teff), c='k')


########################## part c
# calculate lines for the various radii according to equation
def f_c(t):
    return (R**2) / (tsun**4) * (t)**4

# plot and label lines
for R in [0.1, 1, 10, 100]:
    plt.plot(teff, f_c(teff), c='g')
    plt.annotate(r'$%s\;R_\odot$' % R, (3500, f_c(3600)), fontsize=11)


########################## part d
plt.scatter(3500, 63000, c='b')
plt.annotate('Betelgeuse', (4000, 80000), fontsize=12)


########################## part f
# calculate luminosity (in terms of luminosity of sun) given lifetime of star
def f_f(t):
    return (ltsun / t) ** (4/3)

# plot and label points
for t in np.arange(7, 11):
    plt.scatter(f_f(10**t)**(1/8) * tsun, f_f(10**t), c='r')
    plt.annotate(r't=10$^{%s}$ years' % t, (f_f(10**t)**(1/8) * tsun,␣
↪f_f(10**(t-0.1))), fontsize=11)
```

```
########################## part g
# plt.scatter(10000, f_b(10000)) # luminosity directly from temperature
plt.scatter(10000, f_f((1/f_b(10000))**(3/4)*ltsun), c='b') # converting to␣
 ↪lifetimes, then to luminosity, does the same as above
plt.annotate('Praesepe', (9900, f_b(10000)), fontsize=12);
```



c

Given that $L = 4\pi R^2 \sigma T^4$, we can rearrange for R and express it in terms of solar radii:

$$\frac{R^2}{R_\odot^2} = \frac{\frac{L}{4\pi\sigma T_{eff}^4}}{\frac{L_\odot}{4\pi\sigma T_\odot^4}} \implies \frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot}} \left(\frac{T_\odot}{T_{eff}}\right)^2.$$

We can rearrange to obtain the equation of a line, where $\frac{L}{L_\odot}$ is the dependent variable, and T_{eff} is the independent variable:

$$\frac{L}{L_\odot} = \frac{R^2}{R_\odot^2} \frac{1}{T_\odot^4} T_{eff}^4.$$

For $R = 0.1R_\odot$:

$$\frac{L}{L_\odot} = \left(\frac{0.1R_\odot}{R_\odot}\right)^2 \left(\frac{1}{T_\odot^4}\right) T_{eff}^4 = (0.01) \left(\frac{1}{5776^4}\right) T_{eff}^4.$$

For $R = 1R_\odot$:

$$\frac{L}{L_\odot} = \left(\frac{0.1R_\odot}{R_\odot}\right)^2 \left(\frac{1}{T_\odot^4}\right) T_{eff}^4 = (1) \left(\frac{1}{5776^4}\right) T_{eff}^4.$$

For $R = 10R_\odot$:

$$\frac{L}{L_\odot} = \left(\frac{0.1R_\odot}{R_\odot}\right)^2 \left(\frac{1}{T_\odot^4}\right) T_{eff}^4 = (100) \left(\frac{1}{5776^4}\right) T_{eff}^4.$$

For $R = 100R_\odot$:

$$\frac{L}{L_\odot} = \left(\frac{0.1R_\odot}{R_\odot}\right)^2 \left(\frac{1}{T_\odot^4}\right) T_{eff}^4 = (10000) \left(\frac{1}{5776^4}\right) T_{eff}^4.$$

d

Betelgeuse is classified as a red supergiant because it is cool and large (and thus luminous). This can be seen from its upper (high luminosity) right (low temperature) position on the HR diagram.

e

The luminosity of a star is the energy released per unit time:

$$L = \frac{E}{t},$$

where E is the total amount of energy in the star. We can use the mass-energy equivalence to relate energy to the mass of the star:

$$E = 0.1mc^2,$$

where m is the mass of the star and c is the speed of light. Combining these equations, we find that

$$L = \frac{E}{t} = \frac{0.1mc^2}{t}.$$

We can isolate t, giving

$$t = \frac{0.1mc^2}{L}.$$

We can express the lifetime of a star in terms of lifetimes of the sun:

$$\frac{t}{t_\odot} = \frac{\frac{0.1mc^2}{L}}{\frac{0.1m_\odot c^2}{L_\odot}} = \frac{L_\odot m}{L m_\odot}.$$

Again using the mass-luminosity relation $L \propto M^4 \implies M \propto L^{1/4}$, we find that

$$\frac{m}{m_\odot} = \left(\frac{L}{L_\odot}\right)^{1/4}.$$

Substituting this into the previous equation, we get

$$\frac{t}{t_\odot} = \frac{L_\odot}{L}\left(\frac{L}{L_\odot}\right)^{1/4} = \left(\frac{L_\odot}{L}\right)^{3/4}.$$

Isolating for t, we get

$$t = \left(\frac{L_\odot}{L}\right)^{3/4} t_\odot.$$

f

Knowing that the sun has a lifetime of 10^{10} years, we can arrange the previous equation to find luminosity (in terms of the luminosity of the sun) given the lifetime of a star:

$$\frac{t}{t_\odot} = \left(\frac{L_\odot}{L}\right)^{3/4} \implies \frac{L_\odot}{L} = \left(\frac{t}{t_\odot}\right)^{4/3} \implies \frac{L}{L_\odot} = \left(\frac{t_\odot}{t}\right)^{4/3}.$$

For $t = 10$ Myrs:

$$\frac{L}{L_\odot} = \left(\frac{10^{10}}{10^7}\right)^{4/3} = \left(10^3\right)^{4/3}.$$

For $t = 100$ Myrs:

$$\frac{L}{L_\odot} = \left(\frac{10^{10}}{10^8}\right)^{4/3} = \left(10^2\right)^{4/3}.$$

For $t = 1000$ Myrs:

$$\frac{L}{L_\odot} = \left(\frac{10^{10}}{10^9}\right)^{4/3} = (10)^{4/3}.$$

For $t = 10000$ Myrs:

$$\frac{L}{L_\odot} = \left(\frac{10^{10}}{10^{10}}\right)^{4/3} = (1)^{4/3}.$$

To plot this, we also need the x-coordinate (T_{eff}) of the point. Since we are plotting on the main sequence, we can use our equation from part a:

$$\frac{L}{L_\odot} = \frac{T_{eff}^8}{T_\odot^8} \implies T_{eff} = \left(\frac{L}{L_\odot}\right)^{1/8} T_\odot$$

g

Looking at the HR diagram (specifically at the point labelled Praesepe)s and the other reference lifetime points, it seems as though the oldest stars in the cluster are somewhere between 100 million and a billion years old. We could estimate that the star cluster is a few hundred million years old.

We can also use the equation from part a to find that the luminosity of a star with $T_{eff} \approx 10000\ K$:

$$\frac{L}{L_\odot} = \frac{T_{eff}^8}{T\odot^8} = \left(\frac{10000}{5778}\right)^8 \approx 80.5.$$

Plugging this into the equation from part e, we get that

$$t = \left(\frac{L_\odot}{L}\right)^{3/4} t_\odot = \left(\frac{1}{80.5}\right)^{3/4} \times 10^{10} \text{ years} \approx 372 \text{ Myrs}.$$

Google says 625~ Myrs.