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Student #: **Assignment 4** Due: Nov. 15, 2019

You may work together with other students to solve these problem sets, but all solutions must be written and submitted independently. ***Part of the mark for each question will be given for showing your work****, including intermediate steps and diagrams if necessary! Check the syllabus for reading recommendations. Careful with units!*

Problem 1: Electron degeneracy pressure

It will be helpful to go over lecture slides and C&O 16.3 (including Equation 16.12) for this problem.

For some purposes, the core of the Sun can be treated as an independent, self-gravitating object. Assume the core is homogeneous (same composition throughout), and has mass $M_c = 0.1\ \mathrm{M}_\odot$ and radius $R_c = 0.1\ \mathrm{R}_\odot$. Then the core density and central pressure are:

$$\rho_c = \frac{3M_c}{4\pi R_c^3}, \qquad P_c = \frac{3GM_c^2}{8\pi R_c^4} \tag{1}$$

a Suppose that the core contains 50 % hydrogen and 50 % helium, both fully ionized. Given the core's current density as given above, how much degeneracy pressure could its electrons produce? Compare this to the central pressure given above (calculate its value). Is degeneracy currently an important source of pressure in the Sun's core?

b When the core's hydrogen is completely used up, it will contract until it gets hot enough to ignite helium. Assume it contracts by a factor of 10, to radius $R'_c = 0.01\ \mathrm{R}_\odot$. If the core remains homogeneous, what density ρ'_c and central pressure P'_c will it have?

c At this stage the core is pure helium, fully ionized. What degeneracy pressure will the core's electrons produce? Compare this pressure to the pressure P'_c you derived in b). Is degeneracy pressure important at this point in the Sun's evolution?

d Now consider a higher mass star of 8 $\mathrm{M}_\odot$, at the same stage of evolution, with a fully ionized helium core. Assume again that the core mass is 10 % of the star's mass, and that $R \propto M$ (and $R_c \propto M_c$) - true for the main sequence, and we will extrapolate slightly here. Calculate the central density and resulting central pressure, and compare with the electron degeneracy pressure for this star. Is degeneracy pressure important at this stage for higher mass stars? What does this imply for the continued evolution of low vs. high mass stars? *Hint: fusion of heavier and heavier elements in the core requires higher and higher central temperatures.*

Problem 2: Hot Jupiters and tidal disruption

A surprising result of searches for planets around other stars is the number of 'Hot Jupiters' identified: Jupiter-mass planets found to be orbiting extremely close to their parent stars. Exoplanet detection surveys show that $\sim 1\,\%$ of Sun-like stars have a hot Jupiter! This raises the question of how close is too close for a Jupiter-like planet. Consider a Sun-like star, orbited by a Jupiter-like planet, with masses and radii given by:

$$M_S = 1.9891 \times 10^{33}\,\mathrm{g}, \qquad R_S = 6.955 \times 10^{10}\,\mathrm{cm} \tag{2}$$

$$M_J = 1.90 \times 10^{30}\,\mathrm{g}, \qquad R_J = 7.1492 \times 10^{9}\,\mathrm{cm} \tag{3}$$

a If the planet is orbiting a distance r from the host star, find an expression for the distance at which the planet would be tidally disrupted in terms of the mean densities of the star and planet (ρ_S and ρ_J), and the star's radius (R_S). You can assume this is the distance where the tidal force is greater than the planet's self-gravity.

b In reality, the effects of tidal disruption are felt at greater distances than described by the relationship you derived in a), with

$$r < 2.456 \left(\frac{\bar{\rho}_S}{\bar{\rho}_J}\right)^{1/3} R_S \tag{4}$$

for tidal effects to be important. Evaluate the mean densities of the star and planet, and the distance for tidal disruption r from this relation. For r, give your answer in AU and also in terms of the stellar radius. Is the tidal disruption distance inside or outside the host star itself?

c What is the orbital period (in hours) of the hot Jupiter orbiting at the tidal disruption radius?

d We frequently detect hot Jupiters through measurements of the variation in the radial velocity of the host star as the two bodies orbit a common centre of mass. Assume the hot Jupiter in this problem is instead orbiting at a distance of 0.03 au, so as not to be affected by tides. What is the resulting period (in days) and amplitude (in m/s) of the radial velocity of the star caused by the orbital motion of this planet? Assume circular orbits.

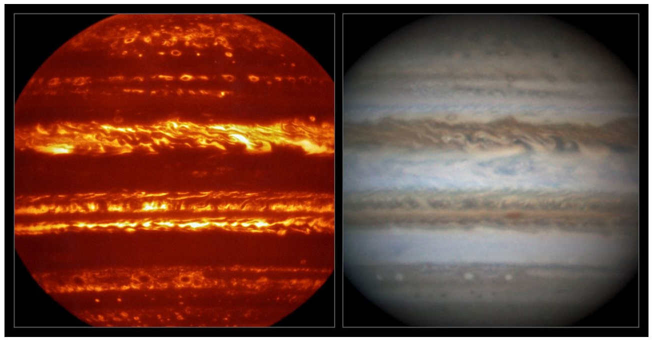

Figure 1: Jupiter observed in the infrared (left, ESO Very Large Telescope) and optical (right, from a coordinated amateur observation campaign in advance of NASA's Juno mission to Jupiter.

Problem 3:

Measurements of Jupiter show that it radiates in the infrared with a total flux of 1.41×10^4 erg/s/cm^2. Here, we will explore the origin of this emission. You will need to look up values like Jupiter's radius, mass, and mean distance from the Sun.

a What is the total rate of energy emitted by Jupiter in the infrared?

b What is the rate of energy absorbed by Jupiter from solar radiation? Assume Jupiter has an albedo of $A = 0.343$.

c Calculate the excess rate of energy emitted by Jupiter, beyond that explained by the absorption and re-emission of solar radiation.

d One possibility is that radioactive decay of elements may account for this excess energy. Using the Earth as an analogue, radioactive decay of long-lived radioactive isotopes (such as ^{40}K, ^{238}U, etc.) account for $\sim 2.2 \times 10^{20}$ erg/s in the Earth. Assuming Jupiter has the same mass fraction of radioactive elements as the Earth, what would be the radioactive heating rate for Jupiter?

e Since part d) shows that radioactive heating is far too small to account for the difference, we will next calculate the heating of a young Jupiter via gravitational contraction from its parent cloud. To do this, calculate the change in gravitational potential energy for a molecular cloud of mass M_J as it contracts from $r \to \infty$ to $r = R_J$.

f Following the virial theorem and using your result from e), how hot was proto-Jupiter? Assume that the mean molecular weight is $\bar{m} = 2.2\, m_H$. We will discuss in class how the cooling time for a planet like Jupiter is long enough that this residual, primordial heat can explain a significant fraction of the infrared emission we see from Jupiter today.