

You may work together with other students to solve these problem sets, but all solutions must be written and submitted independently. Show your work, including intermediate steps and diagrams if necessary! Check the syllabus for reading recommendations. Careful with units!

Problem 1: Magnitudes in the Solar System

Here we will examine the Moon's brightness relative to the Sun. You will need to look up the Moon's radius and mean distance to the Earth.

- a Above its atmosphere, the Earth receives a radiant flux $F = L_{\odot}/4\pi r^2 = 1.360 \times 10^6$ erg s⁻¹ cm⁻² from the Sun (also known as the solar constant). Assuming this is the same value as for the Moon, calculate the total solar power absorbed by the Moon. Remember that power has the units erg s⁻¹.
- b The moon's albedo is ~ 0.12 , which means that it reflects roughly 12% of the solar power incident on its surface. What is the luminosity of the Moon due to reflected sunlight? Assuming this light is reflected equally in all directions from the sunlit hemisphere of the Moon, and assuming the Moon is currently viewed as full from Earth, what is the flux of moonlight on the Earth?
- c Compare the result from b) with the solar constant. Explain why it makes sense from an evolutionary perspective that humans (and many other animals) have eyes that work as logarithmic light detectors.
- d Some of the most distant objects in our Solar System are Kuiper belt objects, which are small planetoids that orbit the Sun at distances of ~ 1000 au. Given the Sun's apparent magnitude as viewed on Earth, $m = -26.81$, calculate the Sun's apparent magnitude at the Kuiper belt. How does this compare with the apparent magnitude of the Moon as viewed from Earth, determined from b)? Is the Sun still the brightest star in the sky when viewed from the Kuiper belt?

Problem 2: The Alpha Centauri system

- a The low-mass star, Proxima Centauri, is the closest known star to the Sun, and has a parallax of $0.77''$ and an apparent magnitude $m = 11.05$. How much brighter or fainter in luminosity is the star Proxima Centauri compared to the Sun?
- b Given your result in a), how close to Proxima Centauri would a planet need to orbit to receive the same amount of flux as the Earth does from the Sun? What would the period be for such a planet? Remember that the Sun has an absolute magnitude $M_{\odot} = 4.76$.

- c In the above, you calculated the luminosity of Proxima Centauri based on its absolute visual magnitude. Proxima Centauri is a red dwarf star, and is cooler than the Sun. As we will learn more later, this means that it emits more light at longer wavelengths. The total, or bolometric luminosity of Proxima Centauri is actually $L_{\text{bol}} = 0.0017$ times the luminosity of the Sun. Given this value, at what distance would a planet orbit to obtain the same amount of flux as the Earth does from the Sun? In 2016, it was discovered that there was an Earth-sized planet orbiting Proxima Centauri at a distance of approximately 0.05 au. Given your calculations, do you think this Earth-sized planet is Earth-like? Why or why not?
- d Proxima Centauri is the faintest member of a triple star system. The other two stars, Alpha Centauri A and Alpha Centauri B, have apparent magnitudes $m = 0.01$ and $m = 1.33$, respectively. To the naked eye, however, they cannot be resolved into two separate stars. If you had a telescope with the same resolution as your eyes, what would the apparent magnitude of this unresolved binary star be? Remember you can't simply add magnitudes!

Problem 3: The Pluto-Charon system

The dwarf planet Pluto was originally discovered in 1930. It has a relatively large moon, Charon.

- a Charon and Pluto orbit each other with a period of 6.3870 days, and have a separation of 19,570 km. Calculate the total mass of the system. What other observational information could you use to determine the individual masses of Pluto and Charon?
- b Show how the mass ratio of the two objects, like Pluto and Charon, relate to the ratio of their orbital velocities, assuming they are in circular orbits around the centre of mass. If Charon was observed to orbit with a velocity of 0.2 km s^{-1} , what is the distance from Charon to the barycenter of the Pluto-Charon system? What is the distance of the barycenter from Pluto? How does this compare with Pluto's radius?