

AST221: STARS AND PLANETS

UNIVERSITY OF TORONTO — FALL 2019

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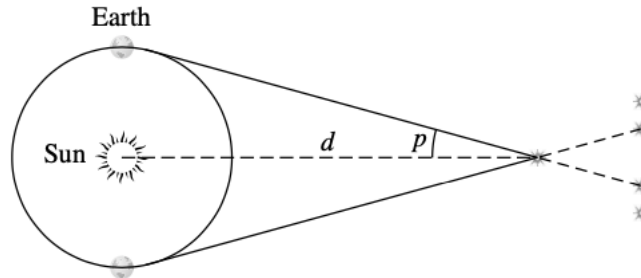
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1 Week 1

1.1 Stellar Parallax

Trigonometric parallax: using a known distance as a baseline, the distance to an object can be determined by observing it from different locations. Measurements of distances to a star can be made on Earth six months apart, when the Sun will have moved a distance of 2 AU (orbital diameter).



The parallax angle p is half of the maximum change in position. From this, we can calculate distance as follows:

$$d = \frac{1\text{AU}}{\tan p [\text{rad}]} \simeq \frac{1}{p [\text{rad}]} \text{AU},$$

where for small angles $\tan p \simeq p$ (small-angle approximation).

Convert this into arcseconds:

$$1 \text{ rad} = 57.3^\circ = 206264.8''$$

Defining a new unit called a **parsec** (parallax-second) as

$$1 \text{ pc} = 2.062648 \times 10^5 \text{ AU} = 3.0856776 \times 10^{16} \text{ m},$$

we get

$$d \simeq \frac{1}{p['']} \text{ pc}.$$

In particular, when $p = 1''$, $d = 1 \text{ pc}$.

Light year: the distance travelled by light through a vacuum in a Julian year: $1 \text{ ly} = 9.460735 \times 10^{15} \text{ m} = \frac{1}{3.26} \text{ pc}$.

1.2 The Magnitude Scale, Luminosity, and Flux

Apparent magnitude (m): a logarithmic measure of relative brightness of objects. Brighter objects have a lower m value. Ranges from $m = -26.83$ for the Sun to $m = 30$ for the faintest objects in the sky. A 1 mag increase corresponds to a brightness increase of $100^{1/5} \simeq 2.512$. Dimensionless.

Luminosity (L): total amount of energy radiated (across all wavelengths) per unit time. Is an intrinsic property which, for stars, depends on the rate of fusion. erg s^{-1} .

Radiant flux (F): luminosity incident on a unit area oriented perpendicular to the direction of the light. Determines how bright an object is perceived to be. Flux (and radiation in general) follows the **inverse-square law**, meaning that it is inversely proportional to the square of the distance. Given that stars radiate energy in spheres, and that the area of a sphere is $4\pi r^2$, the flux as measured at some distance r away from a source is:

$$F = \frac{L}{4\pi r^2} [\text{erg s}^{-1} \text{ cm}^{-2}].$$

Absolute magnitude (M): apparent magnitude of an object, measured at a distance of 10 pc.

Define the **flux ratio** as

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5} [\text{dimensionless}].$$

Alternatively, taking the logarithm of both sides and rearranging,

$$m_1 - m_2 = -2.5 \log \left(\frac{F_1}{F_2} \right) [\text{dimensionless}].$$

Knowing both the apparent and the absolute magnitudes of a star gives us the distance (**distance modulus**):

$$d = 10^{(m-M+5)/5} \text{ [pc]}.$$

Alternatively, taking the logarithm of both sides and rearranging,

$$m - M = 5 \log(d) - 5 = 5 \log\left(\frac{d}{10 \text{ pc}}\right) \text{ [mag]}.$$

1.3 The Copernican Revolution

1.4 Definitions and Equations

2 Week 2

2.1 Orbital Mechanics

2.2 Newtonian Mechanics

2.3 Kepler's Laws of Planetary Motion

derivations

2.3.1 N-Body Orbits

2.3.2 First Law

2.3.3 Second Law

2.3.4 Third Law

3 Week 3

3.1 Tides and Moons

3.2 Definitions and Equations

4 Week 3

4.1 Hydrostatic Equilibrium

4.2 The Virial Theorem

4.3 Definitions and Equations

5 Week 5

5.1 Nuclear Fusion

5.2 Blackbody Radiation

5.3 Spectral Lines

quantization doppler

5.4 Light

5.5 Photon Diffusion

mfp

5.6 Definitions and Equations

6 Week 6

6.1 Stellar Evolution: Pre-MS

6.2 Stellar Evolution: MS

mass, size, brightness relations

6.3 Timescales

6.4 Definitions and Equations

7 Week 7

7.1 White Dwarfs

7.2 Electron Degeneracy

7.3 Definitions and Equations

8 Week 8

8.1 Stellar Evolution: Post-MS

8.2 Neutron Stars

8.3 Black Holes

8.4 Definitions and Equations