Student name: AST221H
Student #: **Assignment 2** Due: Oct. 7, 2019

You may work together with other students to solve these problem sets, but all solutions must be written and submitted independently. ***Part of the mark for each question will be given for showing your work****, including intermediate steps and diagrams if necessary! Check the syllabus for reading recommendations. Careful with units!*

Problem 1: Tides

a Tides on the Earth are dominated by the gravitational effect of the Moon, but the Sun also plays a role. Following our discussion in class, show that the relative tidal force on the Earth due to the Sun and the Moon depends only on M_{Sun}, M_{moon}, the distance from the Earth to the Moon, and the distance from the Earth to the Sun. Given the values of these parameters from Appendix C in your textbook, how much stronger is the tidal force due to the Moon than the Sun?

b Io is one of Jupiter's moons, and orbits the planet with a semimajor axis of $a = 421.6 \times 10^3$ km. Derive an expression for the relative strength of the tidal force at Io's surface compared to the Io's gravitational force at a point on the surface directly facing Jupiter. Given the masses and radii of Jupiter and Io from Appendix C in your textbook, calculate the ratio of forces. Compare this to what we discussed in class, where $F_{\text{tide}}/F_{\text{grav}} \sim 10^{-7}$ for tides on Earth due to the Moon.

c Calculate the tidal acceleration between your head and your toes due to the Moon. Compare this value with the tidal acceleration between your head and your toes due to the Earth, and the overall acceleration due to the Earth's gravity.

d In the movie *Interstellar*, an astronaut crosses the event horizon of a 100 million solar mass black hole - and survives! Is this reasonable? Base your answer on calculating the tidal acceleration on an astronaut at the black hole's event horizon (see C&O section 17.3, particularly Equation 17.27), and compare your result with your calculations from part c. What would happen instead if the astronaut crossed the event horizon of a smaller black hole with a mass of only 10 solar masses?

Problem 2: Virial theorem

a For a star composed of a classical, nonrelativistic, ideal gas, $E_{\text{total}} = E_{\text{th}}^{\text{tot}} + E_{\text{grav}} = -E_{\text{th}}^{\text{tot}}$, and therefore the star is bound. In hydrostatic equilibrium, you can also integrate $\mathrm{d}P/\mathrm{d}r$ over the star and show that the mean pressure over the entire star, $\bar{P}$, is related to the gravitational potential energy as $\bar{P} = -\frac{1}{3}E_{\text{grav}}/V$, where V is the volume of the star. (This is similar to a derivation we did in class, but you don't need to do this here). This is another form of the virial theorem.

Show that for a classical **relativistic** gas of particles, $E_{\rm grav} = -E_{\rm th}^{\rm tot}$, and hence $E_{\rm total} = E_{\rm th}^{\rm tot} + E_{\rm grav} = 0$; i.e., the star is marginally bound. You will need to use the equation of state for a relativistic gas, $P = \frac{1}{3}E_{\rm th}/V$.

b In the next few parts, we will estimate the mass of the most massive stars. Because of the destabilizing influence of radiation pressure (as shown in part a), the most massive stars that can form are those in which the radiation pressure and the nonrelativistic kinetic pressure are approximately equal. First, use the virial theorem to show that

$$P \sim \left(\frac{4\pi}{3^4}\right)^{1/3} G M^{2/3} \rho^{4/3} \tag{1}$$

where ρ is the typical density.

c Show that if the radiation pressure, $P_{\rm rad} = \frac{1}{3}aT^4$, equals the kinetic pressure, then the total pressure is

$$P = 2\left(\frac{3}{a}\right)^{1/3} \left(\frac{k\rho}{\bar{m}}\right)^{4/3} \tag{2}$$

where $\bar{m}$ is the mean particle mass.

d Equate the expressions for the pressure in the previous parts to obtain an expression for the maximum mass of the star. Find its value, in solar masses, assuming a fully ionized hydrogen composition.

Problem 3: The Sun and neutrinos

a Most of the Sun's energy comes from the fusion of hydrogen in the Sun's core via the proton-proton chain (p-p chain) of nuclear reactions. One cycle of the p-p chain of reactions 'converts' mass into about 26.2 MeV of net energy that heats the Sun (1 eV $= 1.602 \times 10^{-12}$ erg $= 1.602 \times 10^{-19}$ J). Estimate how many times per second the p-p chain cycle is being completed in the Sun.

b The mass of the Eiffel Tower in Paris is about 7300 metric tonnes (or 7.3×10^9 g). How many Eiffel Towers worth of mass is 'converted' into energy every second in the Sun?

c Neutrinos are elementary particles that have a really, really tiny cross section for interacting with normal matter (i.e. the probability that a neutrino produced in the core is absorbed as it flies out of the Sun is $\sim 10^{-9}$!). Two neutrinos, $2\nu_e$, are produced for every cycle of the p-p chain. Estimate the particle flux of neutrinos at the distance of the Earth (neutrinos/s/cm^2) and how many are flying through your brain each second.