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AST221: Stars and Planets

University of Toronto — Fall 2019

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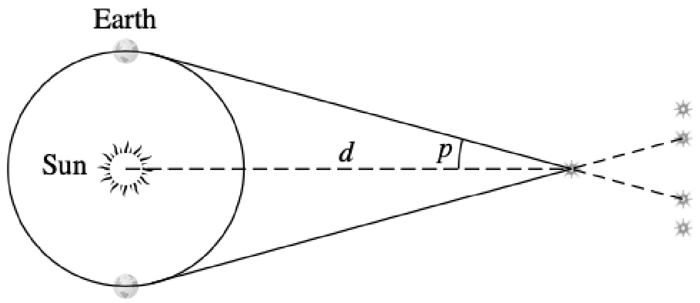
1 Week 1

1.1 Stellar Parallax

Trigonometric parallax: using a known distance as a baseline, the distance to an object can be determined by observing it from different locations. Measurements of distances to a star can be made on Earth six months apart, when the Sun will have moved a distance of 2 AU (orbital diameter).



The parallax angle p is half of the maximum change in position. From this, we can calculate distance as follows:

$$d = \frac{1\text{AU}}{\tan p\,[\text{rad}]} \simeq \frac{1}{p\,[\text{rad}]}\ \text{AU},$$

where for small angles $\tan p \simeq p$ (small-angle approximation).
Convert this into arcseconds:

$$1\,\text{rad} = 57.3^\circ = 206264.8''$$

Defining a new unit called a **parsec** (parallax-second) as

$$1\,\text{pc} = 2.062648 \times 10^5\ \text{AU} = 3.0856776 \times 10^{16}\,\text{m},$$

we get

$$d \simeq \frac{1}{p['']}\ \text{pc}.$$

In particular, when $p = 1''$, $d = 1\,\text{pc}$.
Light year: the distance travelled by light through a vacuum in a Julian year: $1\,\text{ly} = 9.460735 \times 10^{15}\,\text{m} = \frac{1}{3.26}$ pc.

1.2 The Magnitude Scale, Luminosity, and Flux

Apparent magnitude (m): a logarithmic measure of relative brightness of objects. Brighter objects have a lower m value. Ranges from $m = -26.83$ for the Sun to $m = 30$ for the faintest objects in the sky. A 1 mag increase corresponds to a brightness increase of $100^{1/5} \simeq 2.512$. Dimensionless.
Luminosity (L]): total amount of energy radiated (across all wavelengths) per unit time. Is an intrinsic property which, for stars, depends on the rate of fusion. $\text{erg}\,\text{s}^{-1}$.
Radiant flux (F): luminosity incident on a unit area oriented perpendicular to the direction of the light. Determines how bright an object is perceived to be. Flux (and radiation in general) follows the **inverse-square law**, meaning that it is inversely proportional to the square of the distance. Given that stars radiate energy in spheres, and that the area of a sphere is $4\pi r^2$, the flux as measured at some distance r away from a source is:

$$F = \frac{L}{4\pi r^2}\ [\text{erg}\,\text{s}^{-1}\,\text{cm}^{-2}].$$

Absolute magnitude (M): apparent magnitude of an object, measured at a distance of 10 pc.
Define the **flux ratio** as

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}\ [\text{dimensionless}].$$

Alternatively, taking the logarithm of both sides and rearranging,

$$m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right) \text{ [dimensionless]}.$$

Knowing both the apparent and the absolute magnitudes of a star gives us the distance (**distance modulus**):

$$d = 10^{(m-M+5)/5} \text{ [pc]}.$$

Alternatively, taking the logarithm of both sides and rearranging,

$$m - M = 5\log(d) - 5 = 5\log\left(\frac{d}{10\,\text{pc}}\right) \text{ [mag]}.$$

Constants for the Sun are well known:

- $M_\odot = +4.74$
- $L_\odot = 3.839 \times 10^{33}\,\text{erg}\,\text{s}^{-1}$

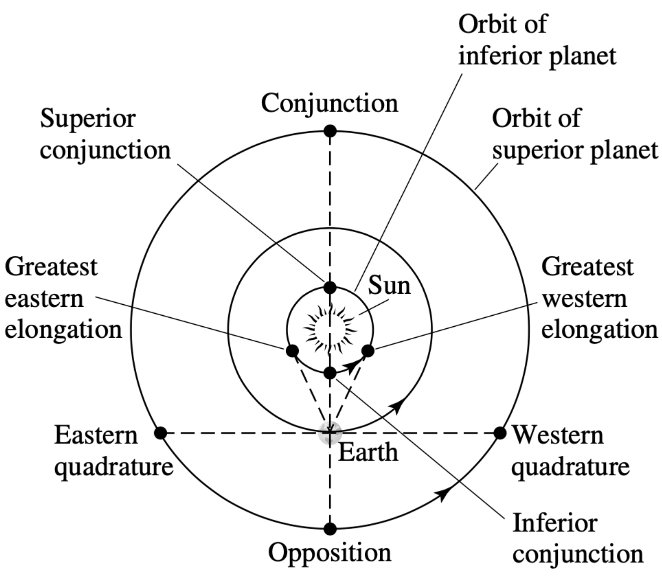
Thus, the Sun can be used as a reference star in the ratio formulae:

$$M = M_\odot - 2.5\log\left(\frac{L}{L_\odot}\right) \text{ [mag], and}$$
$$m = M_\odot - 2.5\log\left(\frac{F}{F_{10,\odot}}\right) \text{ [mag]},$$

where $F_{10,\odot}$ is the flux received from the Sun at a distance of 10 pc.

1.3 The Copernican Revolution

Heliocentric model: model of the Solar System proposed by Copernicus. The Sun, rather than the Earth, is placed at the center of the universe, and all other bodies orbit it.
Inferior planets: planets orbiting between the Sun and Earth. Mecury, Venus.
Superior planets: planets with orbits further from the Sun than Earth's. Mars, Jupiter, Saturn, Uranus, Neptune.



Synodic period (S): time between successive oppositions or conjunctions
Sidereal period (P): time to complete one complete orbit as measured relative to background stars

1.4 Equations

2 Week 2

2.1 Orbital Mechanics

2.2 Newtonian Mechanics

2.3 Kepler's Laws of Planetary Motion

derivations

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4.1 Hydrostatic Equilibrium

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4.3 Definitions and Equations

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5.3 Spectral Lines

quantization doppler

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mfp

5.6 Definitions and Equations

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6.1 Stellar Evolution: Pre-MS

6.2 Stellar Evolution: MS

mass, size, brightness relations

6.3 Timescales

6.4 Definitions and Equations

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7.2 Electron Degeneracy

7.3 Definitions and Equations

8 Week 8

8.1 Stellar Evolution: Post-MS

8.2 Neutron Stars

8.3 Black Holes

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