Student name: AST221H
Student #: **Assignment 3** Due: Oct. 18, 2019

You may work together with other students to solve these problem sets, but all solutions must be written and submitted independently. ***Part of the mark for each question will be given for showing your work****, including intermediate steps and diagrams if necessary! Check the syllabus for reading recommendations. Careful with units!*

Problem 1: The Hertzsprung-Russell Diagram

In this problem you will construct an H-R diagram based on observational relationships between the mass, luminosity, and radius for stars on the main sequence, along with your knowledge of how a star's luminosity relates to its temperature. You can make your H-R diagram by hand or computer, whichever you are most comfortable with. You should submit all calculations (including computer code if used) for full marks. Label your plot clearly.

The luminosity L of a star with effective temperature T_{eff} is given by

$$L = 4\pi R^2 \sigma T_{eff}^4 \tag{1}$$

where σ is the Stefan-Boltzmann constant.

On the main sequence, careful measurements of binary stars show that lower-mass stars ($\sim 1 - 10\,\mathrm{M}_\odot$) follow an approximate mass-luminosity relation, $L \propto M^4$, and the mass-radius relation, $R \propto M$ (we've rounded the exponents here to make the problem below tractable).

a We observe stars' flux and if we know their distances, can convert this to luminosity. We can also determine their effective temperature by observing their flux at different wavelengths and identifying at which wavelength the emission peaks. We often don't have direct measurements of their radii, however. Using the relations for L and M given above, derive an expression for L in terms of T_{eff} that does not depend on R. *Hint: think in terms of solar properties $L_\odot$ and $M_\odot$.*

b Using the relationship between L and T_{eff} you derived in a), plot a theoretical H-R diagram relating $\log_{10} L/L_\odot$ to $\log_{10} T_{eff}$. Your H-R diagram should cover the effective temperature range $\log_{10} T_{eff} = 3.5 - 4.5$, and an appropriate range in $\log_{10} L/L_\odot$.

c On your H-R diagram, draw (in a different colour or line style) lines that show where stars with radii $R = 0.1\,R_\odot, 1\,R_\odot, 10\,R_\odot$, and $100\,R_\odot$ would lie. Show your calculations.

d The star Betelgeuse has $T_{eff} = 3,500\,\mathrm{K}$ and $L = 63,000\,L_\odot$. Mark and label its position on your H-R diagram, and explain why it is classified as a red supergiant.

e Derive an expression for the main-sequence (core hydrogen burning) lifetime t of a star in terms of its luminosity (and constants). You should assume i) that the luminosity remains constant throughout the star's main sequence lifetime, following the mass-luminosity relation given above; ii) that the star converts 10% of its mass from hydrogen

into helium during the main-sequence phase. It will help to recall our discussion on the p-p chain in stars.

f Using the expression for t, mark and label on your H-R diagram the points on the main sequence where stars have lifetimes $t = 10, 100, 1000,$ and 10000 Myr (10^6 years). Show your calculations.

g The Praesepe (Beehive) open cluster of stars has no stars on the main sequence above $T_{eff} \sim 10,000\,\mathrm{K}$. From your H-R diagram and your expression for t derived in g), can you estimate the age of the cluster?