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AST221: Stars and Planets

University of Toronto — Fall 2019

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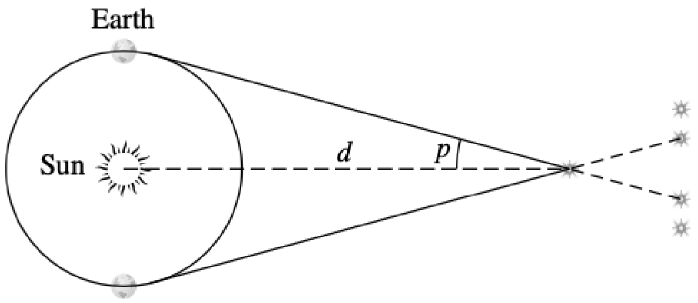
1 Week 1

1.1 Stellar Parallax

Trigonometric parallax: using a known distance as a baseline, the distance to an object can be determined by observing it from different locations. Measurements of distances to a star can be made on Earth six months apart, when the Sun will have moved a distance of 2 AU (orbital diameter).



The parallax angle p is half of the maximum change in position. From this, we can calculate distance as follows:

where for small angles $\tan p \simeq p$ (small-angle approximation).
Convert this into arcseconds:

$$1\,\text{rad} = 57.3^\circ = 206264.8''$$

Defining a new unit called a **parsec** (parallax-second) as

$$1\,\text{pc} = 2.062648 \times 10^5\,\text{AU} = 3.0856776 \times 10^{16}\,\text{m},$$

we get

$$d \simeq \frac{1}{p['']}\ \text{pc}.$$

In particular, when $p = 1''$, $d = 1\,\text{pc}$.
Light year: the distance travelled by light through a vacuum in a Julian year: $1\,\text{ly} = 9.460735 \times 10^{15}\,\text{m} = \frac{1}{3.26}\ \text{pc}$.

1.2 The Magnitude Scale, Luminosity, and Flux

Apparent magnitude (m): a logarithmic measure of relative brightness of objects. Brighter objects have a lower m value. Ranges from $m = -26.83$ for the Sun to $m = 30$ for the faintest objects in the sky. A 1 mag increase corresponds to a brightness increase of $100^{1/5} \simeq 2.512$. Dimensionless.
Luminosity (L]): total amount of energy radiated (across all wavelengths) per unit time. Is an intrinsic property which, for stars, depends on the rate of fusion. $\text{erg}\,\text{s}^{-1}$.
Radiant flux (F): luminosity incident on a unit area oriented perpendicular to the direction of the light. Determines how bright an object is perceived to be. Flux (and radiation in general) follows the **inverse-square law**, meaning that it is inversely proportional to the square of the distance. Given that stars radiate energy in spheres, and that the area of a sphere is $4\pi r^2$, the flux as measured at some distance r away from a source is:

$$F = \frac{L}{4\pi r^2}\ [\text{erg}\,\text{s}^{-1}\,\text{cm}^{-2}].$$

Absolute magnitude (M): apparent magnitude of an object, measured at a distance of $10\,\text{pc}$.
Define the **flux ratio** as

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}\ [\text{dimensionless}].$$

Alternatively, taking the logarithm of both sides and rearranging,

$$m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right) \text{ [dimensionless]}.$$

Knowing both the apparent and the absolute magnitudes of a star gives us the distance:

$$d = 10^{(m-M+5)/5} \text{ [pc]}.$$

Alternatively, taking the logarithm of both sides and rearranging, we get the **distance modulus**:

$$m - M = 5\log(d) - 5 = 5\log\left(\frac{d}{10\,\text{pc}}\right) \text{ [mag]}.$$

Constants for the Sun are well known:

- $M_\odot = +4.74$
- $L_\odot = 3.839 \times 10^{33}\,\text{erg}\,\text{s}^{-1}$

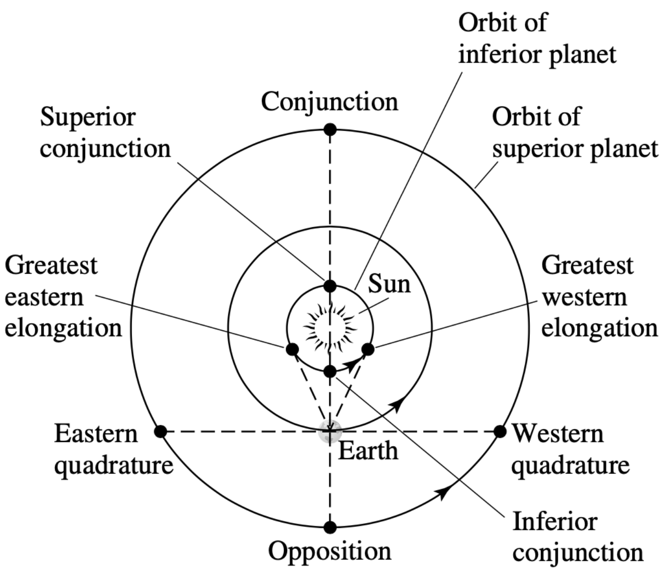
Thus, the Sun can be used as a reference star in the ratio formulae:

$$M = M_\odot - 2.5\log\left(\frac{L}{L_\odot}\right) \text{ [mag], and}$$
$$m = M_\odot - 2.5\log\left(\frac{F}{F_{10,\odot}}\right) \text{ [mag],}$$

where $F_{10,\odot}$ is the flux received from the Sun at a distance of 10 pc.

1.3 The Copernican Revolution

Heliocentric model: model of the Solar System proposed by Copernicus. The Sun, rather than the Earth, is placed at the center of the universe, and all other bodies orbit it. Allowed retrograde motion to be better explained.
Inferior planets: planets orbiting between the Sun and Earth. Mecury, Venus.
Superior planets: planets with orbits further from the Sun than Earth's. Mars, Jupiter, Saturn, Uranus, Neptune.



Synodic period (S): time between successive oppositions or conjunctions
Sidereal period (P): time to complete one complete orbit as measured relative to background stars

1.4 Equations

Parallax $$d = \frac{1}{\tan p\,[\mathrm{rad}]}\ [\mathrm{AU}] \simeq \frac{1}{p\,[\mathrm{rad}]}\ \mathrm{AU} \simeq \frac{206264}{p['']}\ [\mathrm{AU}] = \frac{1}{p['']}\ [\mathrm{pc}]$$

Flux and luminosity $$F = \frac{L}{4\pi r^2}\ [\mathrm{erg\,s^{-1}\,cm^{-2}}]$$

Flux ratio $$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}\ [\mathrm{dimensionless}]$$

$$m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right)\ [\mathrm{dimensionless}]$$

$$M = M_\odot - 2.5 \log\left(\frac{L}{L_\odot}\right)\ [\mathrm{mag}]$$

$$m = M_\odot - 2.5 \log\left(\frac{F}{F_{10,\odot}}\right)\ [\mathrm{mag}]$$

Distance $$d = 10^{(m - M + 5)/5}\ [\mathrm{pc}]$$

Distance modulus $$m - M = 5 \log(d) - 5 = 5 \log\left(\frac{d}{10\,\mathrm{pc}}\right)\ [\mathrm{mag}]$$

$$m - M = 5 \log(d) - 5 = 5 \log\left(\frac{d}{10\,\mathrm{pc}}\right)\ [\mathrm{mag}]$$

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mfp

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6.2 Stellar Evolution: MS

mass, size, brightness relations

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