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Assignment 4

Problem 1: Electron degeneracy pressure

not sure if im calculating z/a correctly for part (a).

- Equation 16.12: $P = \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left[\left(\frac{Z}{A}\right) \frac{\rho}{m_H} \right]^{5/3}$
- $\hbar = 1.055 \times 10^{-34}\ \mathrm{kg\,m^2\,s^{-1}}$
- $m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$
- $m_H = 1.674 \times 10^{-27}\ \mathrm{kg}$
- $M_\odot = 1.99 \times 10^{30}\ \mathrm{kg}$
- $R_\odot = 6.96 \times 10^{8}\ \mathrm{m}$

(a) We can calculate the core density using the given equation:

$$\rho_c = \frac{3M_c}{4\pi R_c^3} = \frac{3 \times (0.1 \times 1.99 \times 10^{30}\ \mathrm{kg})}{4\pi \times (0.1 \times 6.96 \times 10^8\ \mathrm{m})^3} = 1.41 \times 10^5\ \mathrm{kg\,m^{-3}}$$

For hydrogen, $Z = 1$, $A = 1$, and for helium, $Z = 2$, $A = 4$, so assuming a 50-50 composition of hydrogen and helium, $\frac{Z}{A} = \frac{1+2}{1+4} = \frac{3}{5}$. Using this and the other information we have, we can calculate the electron degeneracy pressure using equation 16.12:

$$\begin{aligned}
P &= \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left[\left(\frac{Z}{A}\right) \frac{\rho_c}{m_H} \right]^{5/3} \\
&= \frac{(3\pi^2)^{2/3}}{5} \frac{(1.055 \times 10^{-34}\ \mathrm{kg\,m^2\,s^{-1}})^2}{9.11 \times 10^{-31}\ \mathrm{kg}} \left[\left(\frac{3}{5}\right) \frac{1.41 \times 10^5\ \mathrm{kg\,m^{-3}}}{1.674 \times 10^{-27}\ \mathrm{kg}} \right]^{5/3} \\
&= 1.62 \times 10^{15}\ \mathrm{N\,m^{-2}}
\end{aligned}$$

The central pressure given above can be calculated as follows:

$$\begin{aligned}
P_c &= \frac{3GM_c^2}{8\pi R_c^4} \\
&= \frac{3 \times 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}} \times (0.1 \times 1.99 \times 10^{30}\ \mathrm{kg})^2}{8\pi \times (0.1 \times 6.96 \times 10^8\ \mathrm{m})^4} \\
&= 1.34 \times 10^{16}\ \mathrm{N\,m^{-2}}.
\end{aligned}$$

Electron degeneracy pressure accounts for approximately 10% of the pressure in the core. This is not an insignificant amount.

(b) Assume that the core mass remains the same. Then for core density, we have

$$\rho_c' = \frac{3M_c}{4\pi R_c'^3} = \frac{3M_c}{4\pi \times (0.1 \times R_c)^3} = \frac{1}{0.1^3}\frac{3M_c}{4\pi R_c^3} = 1000 \times \rho_c = 1.41 \times 10^8 \text{ kg m}^{-3},$$

and for central pressure, we have

$$P_c' = \frac{3GM_c^2}{8\pi R_c'^4} = \frac{3GM_c^2}{8\pi \times (0.1 \times R_c)^4} = \frac{1}{0.1^4}\frac{3GM_c^2}{8\pi R_c^4} = 10000 \times P_c = 1.34 \times 10^{20} \text{ N m}^{-2}.$$

(c) The degeneracy pressure, using $\rho = \rho_c'$ and $\frac{Z}{A} = \frac{2}{4} = 0.5$ for a pure helium core, is

$$P = \frac{(3\pi^2)^{2/3}}{5}\frac{(1.055 \times 10^{-34} \text{ kg m}^2\text{ s}^{-1})^2}{9.11 \times 10^{-31} \text{ kg}}\left[(0.5)\,\frac{1.41 \times 10^8 \text{ kg m}^{-3}}{1.674 \times 10^{-27} \text{ kg}}\right]^{5/3} = 1.19 \times 10^{20} \text{ N m}^{-2}$$

At this point, electron degeneracy accounts for most of the central pressure in the star. Yes, it is important.

(d) Again using the same formulas, we find that core density is

$$\rho_c = \frac{3M_c}{4\pi R_c^3} = \frac{3 \times (0.1 \times 8 \times 1.99 \times 10^{30} \text{ kg})}{4\pi \times (0.1 \times 8 \times 6.96 \times 10^8 \text{ m})^3} = 2.2 \times 10^3 \text{ kg m}^{-3},$$

central pressure is

$$P_c = \frac{3GM_c^2}{8\pi R_c^4} = \frac{3 \times 6.67 \times 10^{-11} \text{ N m}^2\text{ kg}^{-2} \times (0.1 \times 8 \times 1.99 \times 10^{30} \text{ kg})^2}{8\pi \times (0.1 \times 8 \times 6.96 \times 10^8 \text{ m})^4} = 2.1 \times 10^{14} \text{ N m}^{-2},$$

and electron degeneracy pressure is

$$P = \frac{(3\pi^2)^{2/3}}{5}\frac{(1.055 \times 10^{-34} \text{ kg m}^2\text{ s}^{-1})^2}{9.11 \times 10^{-31} \text{ kg}}\left[(0.5)\,\frac{2.2 \times 10^3 \text{ kg m}^{-3}}{1.674 \times 10^{-27} \text{ kg}}\right]^{5/3} = 1.16 \times 10^{12} \text{ N m}^{-2}.$$

Degeneracy pressure is not as important in this higher mass star as it was for the lower mass star. This implies that something about being able to get more gravitational pressure because less outward resistance, more gravitational pressure -¿ hotter -¿ can fuse heavier elements -¿ ???

Problem 2: Hot Jupiters and tidal disruption

(a) The point at which an object is tidally disrupted is the Roche limit. We can appproximate this by assuming that this is the point when the differential force is greater than the self-gravity of the planet:

$$\frac{GM_J}{R_J^2} < \frac{2GM_S R_J}{r^3}.$$

If we assume that both the Sun-like star and the Jupiter-like planet are spherical, then

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}.$$

Substituting this into the previous equation, cancelling terms, and rearranging, we find the following expression:

$$\begin{aligned} & G\left(\frac{4\pi\rho_J R_J}{3}\right) < \frac{2GR_J}{r^3}\left(\frac{4\pi\rho_S R_S^3}{3}\right) \\ \implies & \rho_J < \frac{2\rho_S R_S^3}{r^3} \\ \implies & r < \left(\frac{2\rho_S R_S^3}{\rho_J}\right)^{1/3} \\ \implies & r < \left(2\,\frac{\rho_S}{\rho_J}\right)^{1/3} R_S. \end{aligned}$$

(b) First we need to calculate the mean densities of the star and the planet. If we assume constant density everywhere, then

$$\begin{aligned} \rho_S &= \frac{3M_S}{4\pi R_S^3} \\ &= \frac{3 \times 1.9891 \times 10^{33}\ \text{g}}{4\pi \times (6.955 \times 10^{10}\ \text{cm})^3} \\ &= 1.411\ \text{g}\,\text{cm}^{-3} \end{aligned}$$

$$\begin{aligned} \rho_J &= \frac{3M_J}{4\pi R_J^3} \\ &= \frac{3 \times 1.90 \times 10^{30}\ \text{g}}{4\pi \times (7.1492 \times 10^{9}\ \text{cm})^3} \\ &= 1.241\ \text{g}\,\text{cm}^{-3} \end{aligned}$$

Then the Roche limit r is given by

$$r = 2.456\left(\frac{1.411\ \text{g}\,\text{cm}^{-3}}{1.241\ \text{g}\,\text{cm}^{-3}}\right)^{1/3} R_S = 2.56\,R_S$$

Since this is greater than the radius of the star, this means that the tidal disruption limit is outside the host star.

We can also write the distance in AU:

$$r = 2.56\,R_S = 2.56 \times 6.955 \times 10^{10}\ \text{cm} = 1.78 \times 10^{11}\ \text{cm} = 0.012\ \text{AU}.$$

(c) Since we know the tidal disruption radius, and the star is a solar mass star, we can use the simplified form (probably why we did the AU calculation?) of Kepler's Third Law:

$$P^2 = a^3,$$

where P is in years and a is in AU. Then

$$P = a^{3/2} = 0.012^{3/2} = 0.0013 \text{ years} = 11.4 \text{ hours}$$

Since we have the masses of the star and the planet, we can use the general form of Kepler's Third Law (to verify),

$$P^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)},$$

where P is in seconds, m_1, m_2 are in kilograms, and a is in meters. Then

$$\begin{aligned}(P\text{ s})^2 &= \frac{4\pi^2 r^3}{G(M_S + M_J)} \\ &= \frac{4\pi^2 \times (1.78 \times 10^9 \text{ m})^3}{6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}} \times (1.9891 \times 10^{30}\text{ kg} + 1.90 \times 10^{27}\text{ kg})} \\ &= 1.68 \times 10^9 \text{ s}^2\end{aligned}$$

Taking the square root and converting to hours, we find that the orbital period is

$$P = \sqrt{1.68 \times 10^9 \text{ s}^2} = 4.095 \times 10^4 \text{ s} = 11.4 \text{ hours},$$

as expected.

(d)

Problem 3:

- radius of Jupiter: $R_J = 69,911 \text{ km} = 6.991 \times 10^9 \text{ cm}$
- mass of Jupiter: $M_J = 1.898 \times 10^{30}$ g
- mean distance from Sun to Jupiter: $d = 778.5 \times 10^6 \text{ km} = 7.785 \times 10^{13} \text{ cm}$
- luminosity of Sun: $L_\odot = 3.84 \times 10^{33}\,\mathrm{erg\,s^{-1}}$

(a) We can find the total energy emitted in the infrared by multiplying the flux of Jupiter by its surface area:

$$L = F * 4\pi R^2 = 1.41 \times 10^4\,\mathrm{erg\,s^{-1}} \times 4\pi \times (6.991 \times 10^9 \text{ cm})^2 = 8.66 \times 10^{24}\,\mathrm{erg\,s^{-1}}.$$

(b) Since the distance from Jupiter to the Sun is very large relative to the radius of Jupiter, we can approximate the shape of Jupiter on which light is incident as a circle, which has area πR_J^2. Then the rate of energy absorbed by Jupiter from solar radiation is

$$\begin{aligned} L &= (1-A) \times \frac{L_\odot}{4\pi d^2} \times \pi R_J^2 \\ &= (1-A) \times \frac{L_\odot R_J^2}{4d^2} \\ &= (1-0.343)\frac{(3.84 \times 10^{33}\ \mathrm{erg\,s^{-1}})(6.991 \times 10^9\ \mathrm{cm})^2}{4 \times (7.785 \times 10^{13}\ \mathrm{cm})^2} \\ &= 5.09 \times 10^{24}\ \mathrm{erg\,s^{-1}}. \end{aligned}$$

(c) not sure what it means by "beyond that explained by the absorption and re-emission of solar radiation"? you want some OTHER excess energy calculation???

The excess rate of energy emitted by Jupiter is the difference between the rate of energy emission and the rate of energy absorption:

$$L_{excess} = 8.66 \times 10^{24}\ \mathrm{erg\,s^{-1}} - 5.09 \times 10^{24}\ \mathrm{erg\,s^{-1}} = 3.57 \times 10^{24}\ \mathrm{erg\,s^{-1}}$$

(d)

(e) The work done to move a cloud of mass M_J from $r = \infty$ to $r = R_J$ is the force ($\frac{GMM_J}{r^2}$) integrated over the displacement:

$$W = \int_\infty^{R_J} \frac{GMM_J}{r^2} dr = -\frac{GMM_J}{r}\Big|_\infty^{R_J}$$

Assuming that the Sun is the only object imparting a gravitational force on this cloud of mass,

$$W = -\frac{GM_\odot M_J}{r}\Big|_\infty^{R_J}$$

Since $\lim_{r\to\infty} \frac{1}{r} = 0$, we can rewrite this as

$$\begin{aligned} W &= -\frac{GM_\odot M_J}{R_J} \\ &= -\frac{(6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}) \times (1.989 \times 10^{30}\ \mathrm{kg}) \times (1.898 \times 10^{27}\ \mathrm{kg})}{6.991 \times 10^7\ \mathrm{m}} \\ &= -3.6 \times 10^{39}\ \mathrm{Nm} \end{aligned}$$

$Nm = J$. this seems really really high

(f)