Problem Set 5

Problem 1

Using the distance modulus formula, this distance is (ignoring extinction)

$$d = 10^{(m-M+5)/5} = 10^{(25+19.5+5)/5} = 7.94 \text{ Gpc}.$$

Problem 2

Density for matter is given by $\rho_{m,0} = \Omega_{m,0}(1+z)^3$, and for radiation by $\rho_{r,0} = \Omega_{r,0}(1+z)^4$. Equate the two and solve for z:

$$\begin{aligned} \rho_{m,0} &= \rho_{r,0} \\ \implies \Omega_{m,0}(1+z)^3 &= \Omega_{r,0}(1+z)^4 \\ \implies \qquad z &= \frac{\Omega_{m,0}}{\Omega_{r,0}} - 1 = \frac{0.317}{10^{-4}} - 1 = 3169. \end{aligned}$$

Problem 3

The Schwarzchild radius of the black hole is

$$R_S = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \times 2 \times 10^9 \text{ M}_\odot}{(2.99 \times 10^8 \text{ m s}^{-1})^2} = 5.91 \times 10^{12} \text{ m}.$$

Given that the distance to M87 is 53.5 Mly[1] $= 5.06 \times 10^{23}$ m, the angular radius is

$$\theta \simeq \tan\theta = \frac{R_S}{d} = \frac{5.91 \times 10^{12} \text{ m}}{5.06 \times 10^{23} \text{ m}} = 1.17 \times 10^{-11} \text{ rad} = 2.41 \times 10^{-6} \text{ arcsec}.$$

velocity dispersion aprt? observable? something about not being observable because seeing limits the resolution of ground telescopes. velocity dispersion has something to do with full width half maximum which relates to the seeing resolution.

Problem 4

what? if the parallax error is the same as the distance error then isnt the distance error just

Problem 5

```python
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
from astropy import units as u
from astropy import constants as const
```

```python
dists, vels = np.loadtxt('ShenJeff.dat', unpack=True)
```

```python
# fit a linear model to the data. the slope should be the hubble parameter H_0
slope, offset = np.polyfit(dists, vels, 1)
slope, offset
```

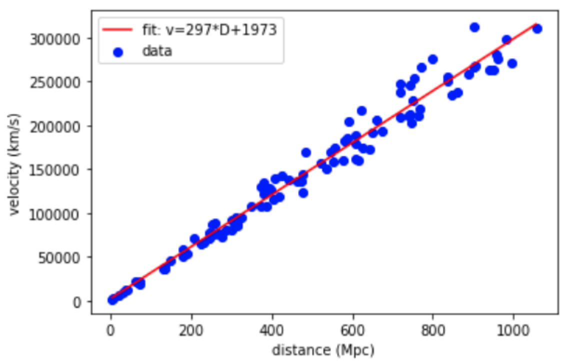
```
(296.7834527807584, 1972.9606705987662)
```

The present-day value of the Hubble parameter of this universe is $H_0 = 297$ km/s/Mpc.

```python
xvals = np.linspace(min(dists), max(dists))
```

```python
# plot data and fit
plt.scatter(dists, vels, c='b', label='data')
plt.plot(xvals, offset + slope * xvals, c='r', label=f'fit: v={m:.0f}*D+{b:.0f}'
plt.xlabel('distance (Mpc)')
plt.ylabel('velocity (km/s)')
plt.legend()
```

```
<matplotlib.legend.Legend at 0x1239e8350>
```




```python
# assuming constant expansion, the age of the universe is given by 1/H_0
h0 = m * (u.km / u.s) / u.Mpc
(1/h0).to(u.Gyr)
```

3.2946319 Gyr

The age of this universe is 3.3 Gyr.

```python
# use the friedmann equations to find the critical density:
((3 * h0**2) / (8 * np.pi * const.G)).to(u.kg / u.m**3)
```

$1.6544511 \times 10^{-25} \ \frac{\mathrm{kg}}{\mathrm{m}^3}$

The critical density of the universe is $\rho_c = 1.65 \times 10^{-25}$ kg/m^3. From week 10's lecture slides, we know that $\rho_{m,0} = 2.56 \times 10^{-27}$ kg/m^3 and $\rho_{b,0} = 4.17 \times 10^{-28}$ kg/m^3. This means that if the density of matter in this alternate universe is the same as the density of our universe, that universe has a density parameter $\Omega = \rho/\rho_c$ less than 1, so the universe is open.

Problem 6

(i) C&O Eq. 29.10:

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^2 = -kc^2.$$

Multiplying this by R, we get

$$\left(\frac{dR}{dt}\right)^2 R - \frac{8}{3}\pi G\rho R^3 = -kc^2 R$$

Taking the time derivative and applying the product and chain rules to the first term, we get

$$2\frac{dR}{dt}\frac{d^2R}{dt}R + \left(\frac{dR}{dt}\right)^3 - \frac{8}{3}\pi G\frac{d}{dt}(\rho R^3) = \frac{d}{dt}(-kc^2R).$$

We use C&O Eq. 29.50 to replace the third term on the left side, then expand the derivative using the chain rule and cancel out a dR/dt term:

$$\begin{aligned}
2\frac{dR}{dt}\frac{d^2R}{dt^2}R + \left(\frac{dR}{dt}\right)^3 + \frac{8}{3}\pi G\frac{P}{c^2}\frac{d(R^3)}{dt} &= -kc^2\frac{dR}{dt} \\
\implies 2\frac{dR}{dt}\frac{d^2R}{dt^2}R + \left(\frac{dR}{dt}\right)^3 + \frac{8}{3}\pi G\frac{P}{c^2}3R^2\frac{dR}{dt} &= -kc^2\frac{dR}{dt} \\
\implies \qquad 2\frac{d^2R}{dt^2}R + \left(\frac{dR}{dt}\right)^2 + 8\pi G\frac{P}{c^2}R^2 &= -kc^2.
\end{aligned}$$

We can use Eq. 29.10 again to replace the $-kc^2$ term on the right side:

$$2\frac{d^2R}{dt^2}R + \left(\frac{dR}{dt}\right)^2 + 8\pi G\frac{P}{c^2}R^2 = \left(\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho R^2.$$

Cancelling terms and rearranging, we get

$$2\frac{d^2R}{dt^2}R = -8\pi GR^2\left(\frac{\rho}{3} + \frac{P}{c^2}\right).$$

Dividing both sides by $2R$ and then factoring out $1/3$ from the right side, we arrive at C&O Eq. 29.51:

$$\frac{d^2R}{dt^2} = -\frac{4}{3}GR\left(\rho + \frac{3P}{c^2}\right).$$

(ii)

(iii)