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Problem Set 5

Problem 1

Using the distance modulus formula, this distance is (ignoring extinction)

$$d = 10^{(m-M+5)/5} = 10^{(25+19.5+5)/5} = 7.94 \text{ Gpc}.$$

Problem 2

Density for matter is given by $\rho_{m,0} = \Omega_{m,0}(1+z)^3$, and for radiation by $\rho_{r,0} = \Omega_{r,0}(1+z)^4$. Equate the two and solve for z:

$$\begin{aligned} \rho_{m,0} &= \rho_{r,0} \\ \implies \Omega_{m,0}(1+z)^3 &= \Omega_{r,0}(1+z)^4 \\ \implies \qquad z &= \frac{\Omega_{m,0}}{\Omega_{r,0}} - 1 = \frac{0.317}{10^{-4}} - 1 = 3169. \end{aligned}$$

Problem 3

The radius of the sphere of influence of the black hole is

$$r = \frac{GM}{\sigma^2} = \frac{6.67 \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1} \times 2 \times 10^9\ \text{M}_\odot}{4 \times 10^5\ \text{m}\,\text{s}^{-1}} = 1.66 \times 10^{18}\ \text{m}.$$

Given that the distance to M87 is 53.5 Mly[1] $= 5.06 \times 10^{23}$ m, the angular radius is

$$\theta \simeq \tan\theta = \frac{r}{d} = \frac{1.66 \times 10^{18}\ \text{m}}{5.06 \times 10^{23}\ \text{m}} = 3.28 \times 10^{-6}\ \text{rad} = 0.67\ \text{arcsec}.$$

Given that the limit of seeing is

Problem 4

The parallax formula is

$$\begin{aligned} \theta \simeq \tan\theta &= \frac{r}{d} \\ \implies \qquad d &= \frac{r}{\theta}, \end{aligned}$$

where d is the distance we want, θ is the parallax angle, and r is the distance from Gaia to the Sun. Gaia is at the Earth-Sun L2 Lagrangian point, which has a radius of orbit of approximately 1.01 au. Plugging these values in, we find that d is

$$d = \frac{1.01\ \text{au}}{0.05\ \text{arcsec}} = 20.22\ \text{pc}.$$

At this distance,

Problem 5

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
from astropy import units as u
from astropy import constants as const
```

```
dists, vels = np.loadtxt('ShenJeff.dat', unpack=True)
```

```
# fit a linear model to the data. the slope should be the hubble parameter H_0
slope, offset = np.polyfit(dists, vels, 1)
slope, offset
```

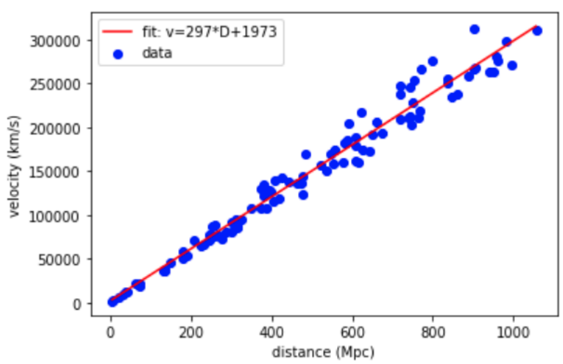
```
(296.7834527807584, 1972.9606705987662)
```

The present-day value of the Hubble parameter of this universe is $H_0 = 297$ km/s/Mpc.

```
xvals = np.linspace(min(dists), max(dists))
```

```
# plot data and fit
plt.scatter(dists, vels, c='b', label='data')
plt.plot(xvals, offset + slope * xvals, c='r', label=f'fit: v={m:.0f}*D+{b:.0f}'
plt.xlabel('distance (Mpc)')
plt.ylabel('velocity (km/s)')
plt.legend()
```

```
<matplotlib.legend.Legend at 0x1239e8350>
```




```
# assuming constant expansion, the age of the universe is given by 1/H_0
h0 = m * (u.km / u.s) / u.Mpc
(1/h0).to(u.Gyr)
```

3.2946319 Gyr

The age of this universe is 3.3 Gyr.

```
# use the friedmann equations to find the critical density:
((3 * h0**2) / (8 * np.pi * const.G)).to(u.kg / u.m**3)
```

$1.6544511 \times 10^{-25} \ \frac{\mathrm{kg}}{\mathrm{m}^3}$

The critical density of the universe is $\rho_c = 1.65 \times 10^{-25}$ kg/m^3. From week 10's lecture slides, we know that $\rho_{m,0} = 2.56 \times 10^{-27}$ kg/m^3 and $\rho_{b,0} = 4.17 \times 10^{-28}$ kg/m^3. This means that if the density of matter in this alternate universe is the same as the density of our universe, that universe has a density parameter $\Omega = \rho/\rho_c$ less than 1, so the universe is open.

Problem 6

(i) C&O Eq. 29.10:

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right] R^2 = -kc^2.$$

Multiplying this by R, we get

$$\left(\frac{dR}{dt}\right)^2 R - \frac{8}{3}\pi G\rho R^3 = -kc^2 R$$

Taking the time derivative and applying the product and chain rules to the first term, we get

$$2\frac{dR}{dt}\frac{d^2R}{dt}R + \left(\frac{dR}{dt}\right)^3 - \frac{8}{3}\pi G\frac{d}{dt}(\rho R^3) = \frac{d}{dt}(-kc^2R).$$

We use C&O Eq. 29.50 to replace the third term on the left side, then expand the derivative using the chain rule and cancel out a dR/dt term:

$$\begin{aligned}
2\frac{dR}{dt}\frac{d^2R}{dt^2}R + \left(\frac{dR}{dt}\right)^3 + \frac{8}{3}\pi G\frac{P}{c^2}\frac{d(R^3)}{dt} &= -kc^2\frac{dR}{dt} \\
\implies 2\frac{dR}{dt}\frac{d^2R}{dt^2}R + \left(\frac{dR}{dt}\right)^3 + \frac{8}{3}\pi G\frac{P}{c^2}3R^2\frac{dR}{dt} &= -kc^2\frac{dR}{dt} \\
\implies \qquad 2\frac{d^2R}{dt^2}R + \left(\frac{dR}{dt}\right)^2 + 8\pi G\frac{P}{c^2}R^2 &= -kc^2.
\end{aligned}$$

We can use Eq. 29.10 again to replace the $-kc^2$ term on the right side:

$$2\frac{d^2R}{dt^2}R + \left(\frac{dR}{dt}\right)^2 + 8\pi G\frac{P}{c^2}R^2 = \left(\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho R^2.$$

Cancelling terms and rearranging, we get

$$2\frac{d^2R}{dt^2}R = -8\pi GR^2(\frac{\rho}{3} + \frac{P}{c^2}).$$

Dividing both sides by $2R$ and then factoring out $1/3$ from the right side, we arrive at C&O Eq. 29.51:

$$\frac{d^2R}{dt^2} = -\frac{4}{3}GR(\rho + \frac{3P}{c^2}).$$

(ii) .

(iii) .