AST 222 Problem Set 5

Due on Fri. Apr. 3 at 5 pm

Please submit electronically via Quercus

Student name: *Student number:*

Problem 1: A type Ia supernovae has a peak absolute magnitude $M_B \approx -19.5$. The *LSST* (Large Synoptic Survey Telescope) project that will start in a few years is an 8m telescope that scans the sky every few nights and can detect supernovae as faint as $m_B = 25$. How far in Gpc is this? [2 pts]

Problem 2: The density parameters for matter and radiation today are $\Omega_{m,0} = 0.317$ and $\Omega_{r,0} = 10^{-4}$. At what redshift were the density of matter and radiation equal? [2 pts]

Problem 3: M87 is a galaxy in the Virgo cluster that has a black hole with a mass of $M_\bullet = 2 \times 10^9\, M_\odot$. What is the angular radius (in arcsec) of its sphere of influence? Is this observable with optical telescopes from the ground without using adaptive optics? (Hint: The core velocity dispersion of M87 is approximately 400 km/s) [2 pts]

Problem 4: *Gaia*'s typical parallax precision is $\approx 50\,\mu$as, while studies have found that for red-clump stars a standard-candle distance estimate with a precision of 8 % can be obtained. The relative error in the parallax (parallax error over parallax) is approximately equal to the relative distance error. At what *distance* is *Gaia*'s distance error equal to 8 %? Beyond this distance, the standard-candle distance for red-clump stars is more precise than the parallax distance. [2 pts]

Problem 5: In an alternate Universe, the Canadian Space Agency builds the Friesen-Webb Cosmological Telescope, named after legendary astronomers Rachel Friesen and Jeremy Webb. The FWC Telescope is the first telescope capable of measuring the proper distance and velocity of galaxies out to 1000 Mpc to within 10%. The data has been covertly provided to you and you only on the AST 222 Quercus Website, in the Assignments Module, in the FWC Data folder. In that folder, look for a file with your name on it. The first column of data is the proper distance to each galaxy (in Mpc) and the second column is velocity away from us (in km/s). Using this data determine the present day value of this alternate Universe's Hubble parameter (in km/s/Mpc), the age of the alternate Universe (in Gyr, assuming a constant expansion rate), and its critical density (in units of kg/m^3). If the density of matter in this alternate Universe is the same as in our Universe, would you conclude that this alternative Universe is open, closed, or flat. As part of your submission, include a plot illustrating Hubble's Law and any fits to the data that you make. [6 pts]

Problem 6: The "jerk" of the Universe.

(i) We discussed the acceleration equation in class, but did not discuss its derivation in detail. Starting from C&O (29.10), derive the acceleration equation C&O (29.51) for a Universe filled with a substance with pressure that satisfies the fluid equation C&O (29.50), by multiplying by R and taking the time derivative. [2 pts]

(ii) All components of the Universe that one commonly encounters satisfy a simple equation of state given by $P = w\rho c^2$, with w a constant. The deceleration parameter is defined in C&O (29.54) as

$$q(t) = -\frac{R(t)\ [\mathrm{d}^2R/\mathrm{d}t^2]}{[\mathrm{d}R/\mathrm{d}t]^2}\,. \tag{1}$$

Combine the Friedman equation C&O (29.10) and the acceleration equation to determine the deceleration parameter for a single-component Universe with the equation of state $P = w\rho c^2$ (but do not assume that the Universe is flat, instead keep the total density parameter Ω_0).[2 pts]

(iii) The "jerk" is the time derivative of acceleration, and is defined similar to the deceleration parameter as

$$j(t) = \frac{R^2(t)\ [\mathrm{d}^3R/\mathrm{d}t^3]}{[\mathrm{d}R/\mathrm{d}t]^3}\,. \tag{2}$$

For a single-component Universe with the equation of state $P = w\rho c^2$, compute the jerk in terms of w and Ω_0. For a flat Universe that consists only of pressureless matter or only of dark energy, what is j? (Hint: Taking the time derivative of the acceleration equation directly results in a $\frac{d(\rho R)}{dt}$ term that cannot easily be dealt with. Instead, similar to the trick used in (i), try multiplying both sides of the acceleration equation by R^2 before taking the time derivative. $\frac{d(\rho R^3)}{dt}$ is much easier to deal with.) [2 pts]