# AST 222 Problem Set 2

*Due on Fri. Feb. 7 at the start of class*

**Please staple all of your work together**

*Student name:* *Student number:*

**Problem 1:** (2 points)
Compute the Oort constants $A$ and $B$ for Keplerian rotation: $\Theta(R) = \Theta_0\ (R/R_0)^{-0.5}$ in terms of $\Theta_0$ and $R_0$. If Keplerian rotation described the rotation of the Milky Way near the Sun, what would the numerical value of $A$ and $B$ (in $\mathrm{km\,s^{-1}\,kpc^{-1}}$) be and how does this compare to the observed values?

**Problem 2:** (4 points)
(a)The Sun's velocity in the vertical direction is about $7.25\,\mathrm{km\,s^{-1}}$ and the Sun is approximately 20 pc above the mid-plane. In the vertical oscillation approximation, compute the amplitude $A_z$ of the vertical oscillation of the Sun in pc. Use that the vertical period near the Sun is 85 Myr.
(b) The Sun's radial position (in cylindrical coordinates) varies between 7.92 kpc and 9.16 kpc over the course of one orbit. In the epicycle approximation, what is the Sun's current velocity in the radial direction? Use that the radial period near the Sun is 170.0 Myr and $R_0 = 8.0$ kpc.

**Problem 3:** (6 points )
Galaxy Zoo (https://www.zooniverse.org/projects/zookeeper/galaxy-zoo) uses citizen science to help classify the large number of galaxies that have been observed by astronomers. Algorithms and machine learning are still no match for the human eye (for now.....). For this question, make an account on Galaxy Zoo and "classify" three galaxies. For each galaxy, Galaxy Zoo will ask you a series of questions. For submission, take a snapshot of each galaxy you are classifying and record the answers you gave to Galaxy Zoo. Finally, estimate the Hubble type of each galaxy. (Note: Your submissions to Galaxy Zoo are being used for actual science!)

If by chance the three galaxies you look at are all the same Hubble type or not resolved well enough to classify, perform the same analysis on Figure 1 below.

**To save ink and paper, for this question please submit a .doc or .pdf containing your three screen shots and responses to Quercus.**

**Problem 4:** (10 points)
At the very centre of the Milky Way sits a *supermassive black hole* which is surrounded by a spherical cluster of stars—the central star cluster (sometimes referred to as a nuclear star cluster). We can learn much about the properties of the black hole and its effect on surrounding stars from observing the motions of stars in this region with large telescopes.

Figure 1: An unlabeled spiral galaxy.

In this problem, we will gain some understanding of the dynamics of this special place in our Galaxy.

(a) The estimated mass of the supermassive black hole is $4 \times 10^6\, M_\odot$. This mass is essentially concentrated in a point. Considering only the influence of the black hole, write down the rotation curve $\Theta(r)$—the velocity of a circular orbit at different distances—near the black hole in units of km/s as a function of distance in pc.

(b) The mass of the central star cluster within 1 pc from the black hole equals that of the supermassive black hole. Assuming that the density of the central star cluster is spherical and that it depends on radius $r$ as $\rho(r) \propto r^{-1.9}$, what is its density at 1 pc (express your result in $M_\odot/\mathrm{pc}^3$)? That is, what is $a$ in $\rho(r) = a\ (r/1\,\mathrm{pc})^{-1.9}$.

(c) How does the circular velocity of a gravitational potential with a spherically-symmetric density $\rho(r) \propto r^{-\alpha}$ ($\alpha \neq 2$) depend on the distance $r$ from the center? What is the circular velocity due to the central star cluster, again in units of km/s as a function of distance in pc?

(d) Starting from the expression for the circular velocity in terms of the enclosed mass of a spherical mass distribution, work out how to combine the circular velocities of multiple components. That is, if the black hole has a circular velocity $\Theta_{\mathrm{bh}}(r)$ and the star cluster has a circular velocity $\Theta_{\mathrm{sc}}(r)$, what is the circular velocity $\Theta(r)$ from the combination of the two mass distributions in terms of $\Theta_{\mathrm{bh}}(r)$ and $\Theta_{\mathrm{sc}}(r)$?

(e) Plot the circular velocity in km/s of (i) the black hole alone, (ii) the star cluster alone, and (iii) the combination of the central star cluster and the supermassive black hole between 0.001 pc and 50 pc; use a logarithmic $x$ axis. Make this plot *on a computer.* (Hint: The python package matplotlib is your friend, as seen in python solution to Assignment 1, Question 2a,2b on Quercus)