AST 222 Problem Set 1

Due on Wed. Jan 22 at the start of class

Please staple all of your work together

Student name: *Student number:*

Problem 1: (2 points)
The very first "image" of a black hole, at the centre of galaxy M87, was recently taken by the Event Horizon Telescope (EHT). More accurately, EHT imaged radio emission from the disc of gas that orbits the black hole with a lack of emission from the centre being attributed to the black hole. This image was only possible because EHT is not a single radio telescope, but is in fact a network of telescopes from around the world that take advantage of something known as interferometry. Interferometry is a method for combining the light from multiple telescopes to create images with an angular resolution equal to the distance between the telescopes—referred to as the "baseline"—rather than the size of each individual telescope. EHT in particular combines observations from several *Very Long Baseline Interferometry* (VLBI) stations in order to achieve a high angular resolution.
(a) Given that the "baseline" of EHT is effectively the diameter of the Earth, compute its angular resolution when observing the 21 cm line of hydrogen.
(b) If one could install a radio telescope on the Moon, by what factor could EHT's resolution be increased.

Problem 2: (6 points)
The `gaia` telescope is currently measuring the positions and velocities of billions of stars in the Milky Way. It is able to view stars down to a limiting magnitude of 21. If a fictional star, named Delorean, orbiting in the plane of the Milky Way's disc has an apparent magnitude of 21 and has had its distance measured via parallax to be 3 kpc:
(a) What would you conclude Delorean's absolute magnitude and stellar type to be if reddening is ignored?
(b) How would your conclusions change for reddening values of 1 mag/pc, 2 mag/pc and 3 mag/pc? More specifically, what absolute magnitudes would you calculate and is this enough of a difference to change your estimate of Delorean's stellar type?
(c) Using our definition of the dynamical time, $t_{\rm dyn} \approx 3\,(G\,\bar{\rho})^{-1/2}$, and assuming Delorean is located directly between the Sun and the centre of the galaxy, compute its dynamical time. Assume that the Milky Way can be modelled as an exponential disc with a radial scale length of 3.2 kpc.
(d) How would $t_{\rm dyn}$ change if instead the Sun was directly between Delorean and the centre of the galaxy?

Problem 3: (4 points)
As mentioned briefly in class, in order to view the most distant galaxies possible the Hubble Space Telescope was purposefully pointed at a patch of sky that is very empty and has low extinction. After completing several long exposures, the images were combined to reveal

over 10,000 galaxies within an area of 5.76 square arcminutes. Some of the galaxies visible in the image are nearly 13 billion light years away. Ignoring extragalactic sources, discuss why each of the following pointings would be more or less favourable for viewing distant galaxies than the coordinates of the Hubble Deep Field (Ra = 3 h, 32 m, 39.0 s, Dec = -27^o, 47', 29.1 ").
(1)$(l, b) = (35°, 80°)$
(2)$(l, b) = (45°, 2°)$
(3) $(l, b) = (165°, 0°)$
(4) $(l, b) = (3°, -60°)$

Problem 4: (2 points)
A galaxy's rotation curve is a measure of the orbital speed of stars as a function of distance from the galaxy's centre. The fact that rotation curves are primarily flat at large galactocentric distances ($v_{rot}(r) \sim$ constant) is the most common example of why astronomer's believe dark matter exists. Lets work out why!

Assuming that each star in a given galaxy has a circular orbit, we know that the acceleration due to gravity felt by each star is due to the mass enclosed within its orbital radius r must be equal to v_c^2/r. Here, v_c is the circular orbit velocity of the star. (a) Show that the expected relationship between v and r due to the stellar halo ($\rho(r) \propto r^{-3.5}$) does not produce a flat rotation curve. (b) Show that a $\rho(r) \propto r^{-2}$ density profile successfully produces a flat rotation curve and must therefore be the general profile that dark matter follows in our galaxy.

Problem 5: (3 points)
Another commonly calculated velocity in galactic dynamics is the escape velocity v_{esc}, that is the minimum velocity a star must have in order to escape the gravitational field of the galaxy.
(a) Starting from the work required to move a body over a distance dr against the force of gravity show that the escape velocity from a point mass galaxy is $v_{esc}^2 = 2GM/r$ where r is your initial distance.
(b) Since we know galaxy's aren't actually point-masses, also show that v_{esc} from r for a galaxy with a $\rho(r) \propto r^{-2}$ density profile is $v_{esc}^2 = 2v_c^2(1 + \ln(R/r)$. Here you must assume that R is a cutoff radius at which the mass density is zero.
(c) The largest velocity measured for any star in the solar neighbourhood, at r=8kpc, is 440 km/s. Assuming that this star is still bound to the galaxy, find the lower limit (in kiloparsecs), to the cutoff radius R and a lower limit (in solar units) to the mass of the galaxy. Note the solar rotation velocity is 220 km/s.

Problem 6: (3 points)
Globular clusters are old star clusters orbiting in the halo of the Milky Way and are some of most distant objects for which proper motions are measurable. Globular Cluster Pal 5 has proper motions components of $(-2.736, -2.646)\,\mathrm{mas\,yr^{-1}}$, a line-of-sight velocity of $-58.6\,\mathrm{km\,s^{-1}}$, and a distance of 23.2 kpc (proper motion and distance from the recently released *Gaia* catalogue).
(a) What is its total space velocity with respect to the Sun?

(b) What is its total space velocity with respect to the centre of the galaxy?
(c) Assuming Pal 5 has a near-circular orbit (which in reality it doesn't), estimate the Milky Way's mass within 23.2 kpc.

Problem 7: (5 points) We discussed the description of the local differential-rotation kinematics of the Milky Way's disk in terms of the Oort constants in class. In this problem we will understand the $\sin 2l$ and $\cos 2l$ behavior of the Oort constants.
(a) Starting from Equations (CO 24.37-38) and referring to CO Fig. 24.22 for the geometry, substitute the local expansion of $\Omega(R)$

$$\Omega(R) = \Omega_0(R_0) + \left.\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right|_{R_0} (R - R_0)\,, \tag{1}$$

and work out the expressions for v_r and v_t at the bottom of page CO 910.
(b) Using the geometry of CO Fig 24.22 and the fact that the angle β is small, derive Equations (CO 24.41-42) in terms of A and B in Equations (CO 24.39-40). (You will likely need the relation $\sin 2x = 2\sin(x)\cos(x)$)