AST 222 Problem Set 4

Due on Fri. Mar. 20 at the start of class

Please staple all of your work together

Student name: *Student number:*

Problem 1: (2 points)
Given the (Reunified) unified model for AGN presented in class, explain why some AGN spectra have broad and narrow emission lines while others just have narrow emission lines.

Problem 2: (2 points)
Given the (Reunified) unified model for AGN presented in class, explain why some AGN are radio-loud while others are radio-quiet.

Problem 3: (2 points)
We have seen in class that accretion of matter onto a black hole is a powerful source of energy. Let's see how this works for neutron stars. A neutron star with mass $M_{ns} = 1.4\,M_{\odot}$ has a radius of 10 km. Suppose that a parcel of mass m free-falls towards the surface of such a neutron star starting from rest at a large radius. What fraction of the rest-mass energy mc^2 of this parcel of mass is released when it hits the surface of the neutron star, at which time any kinetic energy that it gained during its fall gets converted to radiation (note that this *not* exactly the same problem that we solved in class for accretion discs around black holes)?

Problem 4: (4 points)
Due to your outstanding performance in AST 222, you have been selected to be the first person to perform observations with the James Webb Space Telescope (no relation to Professor Webb). After its successful launch in 2021, you elect to go quasar hunting and take infrared spectroscopic measurements of point-like sources that were previously identified photometrically with the Hubble Space Telescope. When your first spectrum is available to analyze, you see the source has a strong emission line at $4\mu m$ that has a width of approximately 0.025 μm. Based on the shape of the overall spectrum and the existence of nearby my emission lines you conclude that this must be the $H\alpha$ emission line. i) Given this revelation and the fact that redshift z is defined as $z = \frac{\delta\lambda}{\lambda_{rest}}$, what is the redshift of your source ? ii) Is the width of the line consistent with active galactic nuclei having central velocity dispersions of ~ 2000 km/s?

Problem 5: (2 points)
The Event Horizon Telescope is a project to directly image the black hole at the center of the Milky Way, Sgr A*. To do this, it has to resolve Sgr A* with a telescope. What angular resolution would be required to resolve the Schwarzschild radius of the Milky Way's central black hole? When using *Very Long Baseline Interferometry* (VLBI; see assignment 1), what wavelength would one have to observe at to obtain this resolution?

Problem 6: (2 points)
The largest globular clusters in the Milky Way have a mass of about $5 \times 10^6\,\mathrm{M}_\odot$ and are found at about 5 kpc from the center. Using the simple model for the decay of orbits due to dynamical friction that assumes a $\rho(r) \propto 1/r^2$ density and that the cluster always stays on a circular orbit (see slides and C&O chapter 26), estimate the time for the most massive globular clusters to sink to the centre (use $C = 75$). Is dynamical friction relevant for Milky Way globular clusters?

Problem 7: (10 points)
(i) Section 3.7 of PS[1] derives the metallicity $Z(t)$ as a function of time in terms of the evolution of the gas fraction for the closed-box model: $Z(t) = -y\ln(g(t)/M_b)$. Use this equation to derive the present mass of stars with $Z < Z_x$ in terms of M_b and Z_x. (Hint: for a closed box $M_b = g(t) + s(t)$, where $g(t)$ and $s(t)$ is the mass in gas and stars, respectively).
(ii) Using the result from (a), we can express the G-dwarf problem. Use the result from (a) for the mass in stars with metallicity with $Z < Z_x$ to write down the relative fraction of (mass in stars with $Z < Z_\odot/3$) to (mass in stars with $Z < Z_\odot$) in terms of the present-day gas fraction and assuming that the ISM has a metallicity of $Z_\odot$ currently ($Z_\odot$ is solar metallicity, about 0.02). For a present-day gas fraction of $g/M_b = 0.1$, what is this fraction?
(iii) The leaky box. The closed-box model assumes that no gas leaves the system. Now assume that gas is expelled at a rate that is proportional to the star-formation rate. This causes the entire supply of baryons to change as

$$\frac{\mathrm{d}M_b}{\mathrm{d}t} = -\eta\,\alpha\,\Psi\,,$$

with η a new "outflow" parameter, and thus causes M_b to become a function of time. What is $M_b(t)$ in terms of $s(t)$ and $M_b(0)$?
(iv) The gas that gets expelled at time t has the metallicity of the interstellar medium at that time: $Z(t)$. Adjust the equation below Eqn. (3.40) in PS to account for the loss of metals in the ISM due to the fact that gas gets expelled. Then work through the rest of the derivation to derive the equivalent of Eqn. (3.42) for the leaky box (that is, to account for non-zero η). Show that η can be incorporated into the expressions for the closed box by a simple redefinition of y.
(v) The accreting box. Rather than losing gas, assume that the box that we are considering is accreting gas with $Z = 0$ at a rate that is such that the amount of gas $g(t) =$ constant. This does not change the equations at the bottom of the left column of PS p. 143, but it does mean that we cannot go from (3.41) to (3.42) like we did for the closed box. How does equation (3.41) change? Derive an expression for $Z(t)$ as a function of $s(t)$ and g, assuming that the ISM starts out with $Z = 0$.
(vi) For the accreting box, what is the fraction of stars with $Z < Z_\odot/3$ with respect to the number of stars with $Z < Z_\odot$? Does the accreting box solve the G-dwarf problem?

[1] PS = Peter Schneider "Extragalactic Astronomy and Cosmology".