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How Change in Gravity Determines the Radius of the Earth

Introduction:
This experiment aims to show how precise changes in gravity and in height (e.g. from one floor of a building to the next) can be used to deduce the radius of the Earth. In this lab, a gravimeter is used to measure precise changes in the gravity. The absolute value of the reading is not of particular importance—rather, we want to measure changes. We can relate these changes to the radius of the Earth. We also consider various sources of uncertainty, and whether they are significant enough to warrant corrections.

Method:
Materials:
- W. Sodin Ltd. Model 410 gravimeter

Experiment:
The gravimeter is placed on the ground. The light switch is pulled up, and the gravimeter is leveled by adjusting the knobs at the bottom while looking through the levelling hole (the one without an eyepiece). Once it is leveled, the vertical parallel black lines are moved (which changes the given number) by turning the reading dial until they are centered on the x-axis—the y-axis should be in between the two parallel lines. Record the reading on the gravimeter. Also note the meter constant on the gravimeter. The steps are repeated for 12 - 14 floors with the gravimeter being in roughly the same spot on each floor.

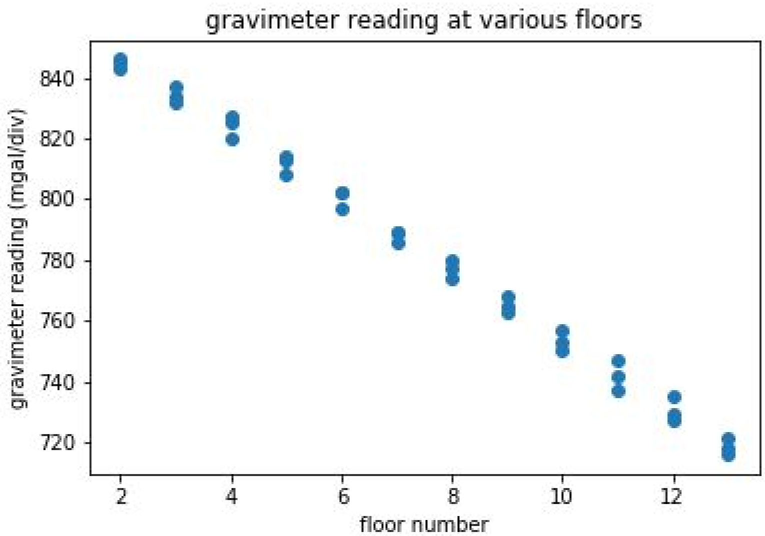
Results:

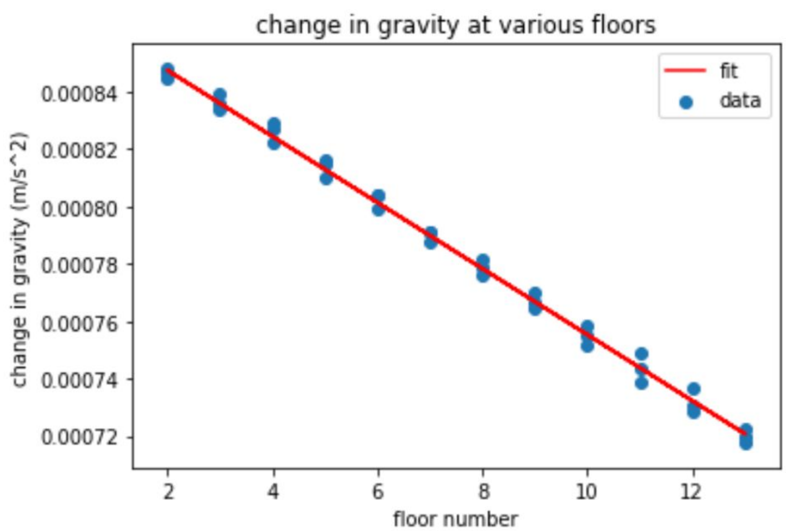
Meter constant: 0.10023 milligals/div

Floor	Reading	Day, time
2	843	28 Feb 2020, 10:12
3	832	28 Feb 2020, 10:18
4	820	28 Feb 2020, 10:23
5	808	28 Feb 2020, 10:26
6	797	28 Feb 2020, 10:30

7	786	28 Feb 2020, 10:33
8	774	28 Feb 2020, 10:35
9	763	28 Feb 2020, 10:38
10	750	28 Feb 2020, 10:41
11	737	28 Feb 2020, 10:44
12	727	28 Feb 2020, 10:47
13	718	28 Feb 2020, 10:50
14	701	28 Feb 2020, 9:59
2	846	2 Mar 2020, 15:10
3	837	2 Mar 2020, 15:23
4	827	2 Mar 2020, 15:28
5	813	2 Mar 2020, 15:33
6	802	2 Mar 2020, 15:36
7	789	2 Mar 2020, 15:39
8	780	2 Mar 2020, 15:42
9	768	2 Mar 2020, 15:45
10	757	2 Mar 2020, 15:48
11	747	2 Mar 2020, 15:51
12	735	2 Mar 2020, 15:54
13	721	2 Mar 2020, 15:56
basement	877	9 Mar 2020, 9:20
1	865	9 Mar 2020, 9:22
2	845	9 Mar 2020, 9:16
3	834	9 Mar 2020, 9:25
4	825	9 Mar 2020, 9:28
5	814	9 Mar 2020, 9:31
6	802	9 Mar 2020, 9:34
7	789	9 Mar 2020, 9:38
8	777	9 Mar 2020, 9:44
9	765	9 Mar 2020, 9:48
10	753	9 Mar 2020, 9:51
11	742	9 Mar 2020, 9:54
12	729	9 Mar 2020, 10:00
13	716	9 Mar 2020, 10:03
14	705	9 Mar 2020, 10:06

Analysis:





By using curve-fitting methods in Python, the value we obtained as an estimate for the radius of the Earth is $6.49 * 10^6 \pm 0.12 \ * 10^6$ meters. The reference value that we compared it to is $6.37 * 10^6$ meters. Our estimate is off by 120 kilometers (~1.9%), which also happens to be what our uncertainty is.

This was done by first multiplying all our readings by the meter constant and rescaling the units to meters per second squared. Then, grouping our data by floor, we fit a linear model to obtain the slope of the data (fitted to the mean in cases where we had more than one data point). This slope was used as the Δg value in the formula $\frac{\Delta g}{g} = -2\frac{\Delta R}{R}$, which we rearranged to solve for R. The ΔR value is the difference in height between each floor, which in our case was 3.95 meters. The uncertainties were calculated by putting in the standard deviation of our measurements (multiplied by the meter constant and rescaled to meters per second squared), grouped by floor, into the curve-fitting function.

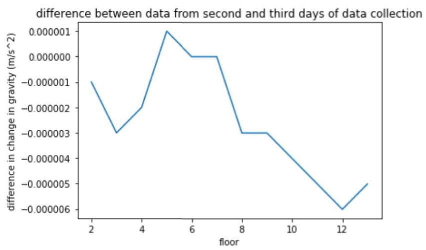
The goodness of our fit as measured by the reduced chi-squared metric is 110. As it is much larger than 1, this indicates that the model is a poor fit and does not fully capture the data.

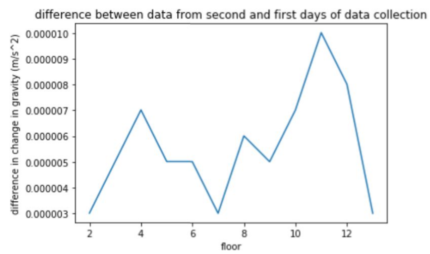
Sources of Uncertainty

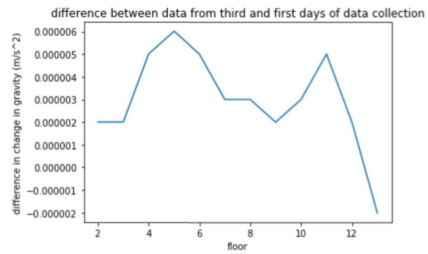
Moving floors changes the mass above and below (each floor has a mass of $>10^6$ kg). This may influence the measurements obtained from the gravimeter. However, even this is not a consistent influence: the basement is missing some mass, and the roof is lighter than the other floors. This means that the readings from the lowest floors and the highest floors will not be consistent with the other readings. This is compounded by the fact that the floor and ceiling heights of the basement and the first floor are also different from those of the other floors. Also, heavy instruments from different floors (specifically ones on the roof) could be moved in between the days the measurements were taken, which would also change results. For this reason, when performing our analysis, we took out the data from the basement, the first floor, and the fourteenth (highest) floor, and relied only on floors 2-13.

The measurements were also affected by the position of the gravimeter as it is impossible (or extremely unlikely) to have the gravimeter in the exact same space on each floor, as well as have the gravimeter perfectly leveled each time. Furthermore, the gravimeter is very sensitive, and therefore any small (unquantifiable or noticeable by the human eye) can affect its measurements.

The positions of the Sun and the Moon may also have an impact on the readings since they contain a large amount of mass, and their relative position to the Earth varied between our three days of data collection. We can take the difference between measurements for each pair of days and plot them:







Note in particular that the magnitude of the y-axis is very small. That is, the positions of the Sun and the Moon cause a 1% or smaller fluctuation in reading. For this reason, we did not correct for this uncertainty, as other sources are much larger and would have a greater impact on our analysis. However, this could be corrected for by taking periodic readings from some reference station. Along the same lines, the eccentricity of the Earth also plays an insignificant role in this lab. The current eccentricity is 0.0167, meaning that the orbit of the Earth is nearly circular, so our distance from the Sun is nearly constant.

Conclusion:

In this lab, we used a gravimeter to measure changes in gravitational acceleration as we move to different heights above the surface of the Earth. Using these changes, along with the change in height, we used curve-fitting methods in Python to deduce the radius of the Earth. Our result differed from reference values by less than 2%, the same amount as our uncertainty. Although this is not the best result, we took note of the many sources of error that were present. We also explained whether we corrected for them, and if so, how we did that.