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The Relation Between Viscosity, Radius, and Terminal Velocity

Introduction:

This experiment aims to show how the viscosity of a fluid and the size of an object falling/sinking in the fluid relate to the terminal velocity of the object. In this lab, balls of varying sizes are dropped into liquids of different viscosities, and the time and position as they sink to the bottom are recorded. Using this data, we find the terminal velocities for each situation. We investigate the wall effect and apply corrections to account for it. Using Python, we visualize the relationship between object radius and terminal velocity, and fit theoretical curves to the data.

Method:

Materials:

- Two vertical long tanks, one filled with water and the other with glycerine
- Nylon beads (5 different sizes)
- Teflon beads (5 different sizes)
- Tracking device/program to record ball position
- Calipers

Experiment:

Set up one tank such that the balls that will be submerged can be tracked and recorded. For the tank with water, use the nylon beads. Take a bead and measure its diameter using the calipers. Submerge the bead under the surface (to avoid water tension) then release and record the bead sinking in the water using the tracking device/program. Keep track of the displacement of the bead from the top, as well as the time that has elapsed since the ball was released. Repeat four more times for the same size of bead. The same procedure is repeated for the other four sizes of bead.

For the tank with glycerine, use the nylon beads. As before, measure the diameter of one bead, then submerge it under the surface then release and record the bead sinking to the bottom. Repeat two more times for the same size bead. The same procedure is done for the other four sizes of bead.

Results:

Table One (Water)

Radius (mm)	Mean Velocity (cm/s)	Corrected Velocity (cm/s)	Reynolds Number
1.175	3.59458+/-0.00002	3.78683+/-0.00002	88.990+/-0.001
1.550	4.62218+/-0.00002	4.95109+/-0.00003	153.484+/-0.001
1.975	5.51626+/-0.00003	6.02127+/-0.00003	237.840+/-0.001
2.375	6.07499+/-0.00004	6.74982+/-0.00004	320.617+/-0.002
3.150	7.17553+/-0.00004	8.25097+/-0.00004	519.811+/-0.003

Table Two (Glycerine)

Radius (mm) +/- 0.003 mm	Mean velocity (cm/s)	Corrected velocity (cm/s)	Reynolds number
0.750	0.199540+/-0.000006	0.206286+/-0.000006	0.0041743+/-0.0000001
1.150	0.432088+/-0.000005	0.454693+/-0.000006	0.0141081+/-0.0000002
1.575	0.737714+/-0.000008	0.791087+/-0.000009	0.0336169+/-0.0000004
2.350	1.551198+/-0.000009	1.72160+/-0.00001	0.1091576+/-0.0000006
3.175	2.57820+/-0.00001	2.96790+/-0.00002	0.254241+/-0.000001

Analysis:

An important source of uncertainty comes from the camera and/or the tracking software not pinpointing the ball correctly (i.e. not staying in the middle of each ball). We accounted for this by adding an uncertainty of 0.002 cm to the position of the ball, and an uncertainty of 0.0005 seconds to the time (as suggested by TA Alex). This served as the initial uncertainty that was propagated throughout the measurements.

We performed two sessions of data collection, and that means we had 10 data points for each ball size for the balls dropped in water, and 6 data points for the ones dropped in glycerine. Since the initial uncertainties are rather small, and we average the velocities over many trials, the resulting uncertainties are very small. This is the reason why so many significant figures are given in the table above.

The results for our calculations of the Reynolds number are as expected. In water, where the viscosity is much lower, the Reynolds number is high. In glycerine, the viscous forces are much more relevant, and the Reynolds number (ratio of intertial forces and viscous forces) is lower.

In Figures 1 and 2, we plot the velocities over time for each of the trials we performed. Each line represents a single drop, and the lines are grouped by colour according to the radius of the size of the ball. Note that the first few seconds of data for both water and glycerine were cut off intentionally, as during that time, the velocity of the ball is unstable, leading to plots like Figure 3.

Figure 1:

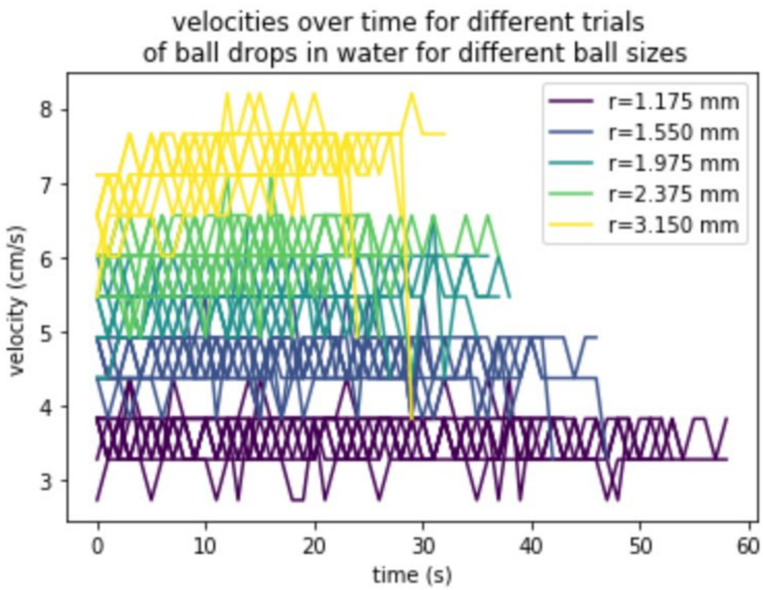


Figure 2

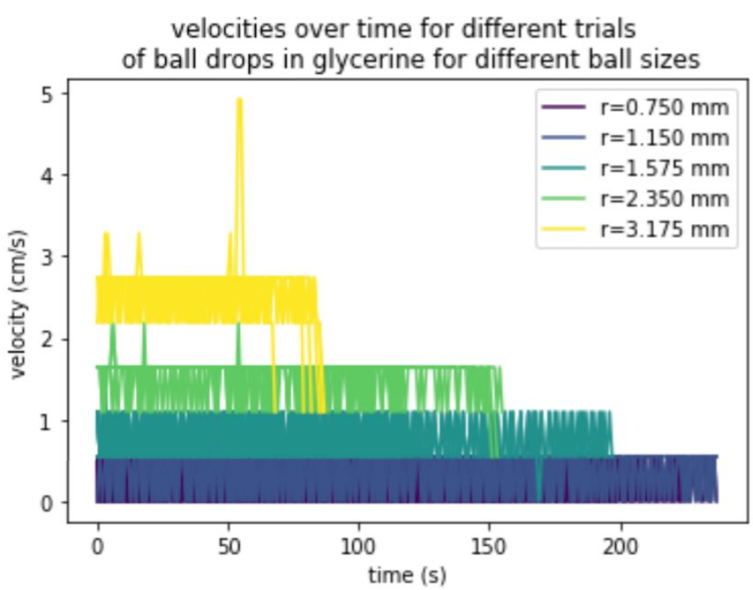
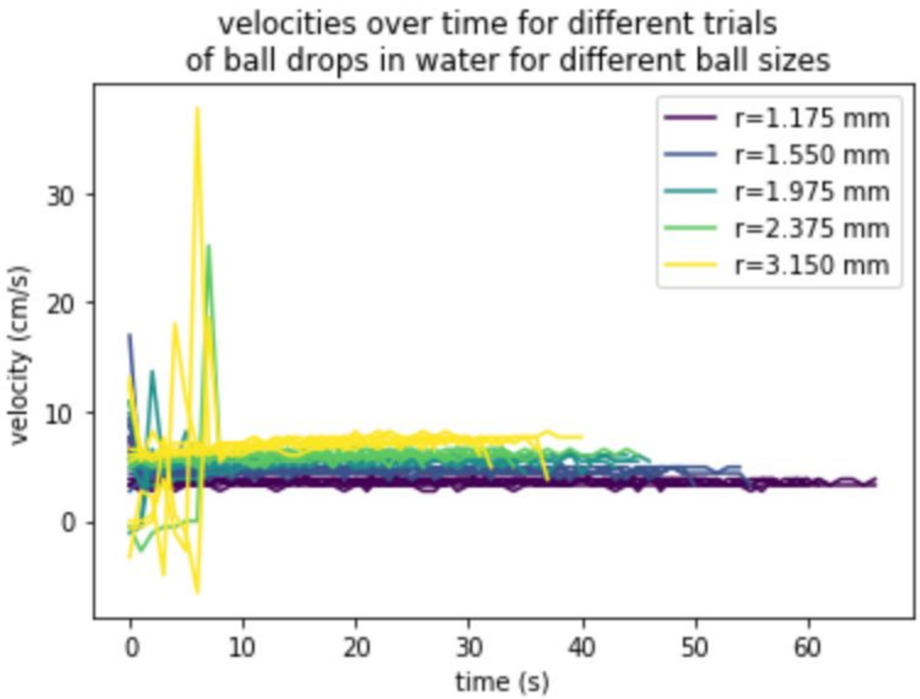


Figure 3



The velocity in both fluids reaches terminal velocity quickly and stabilizes after a few seconds. In Figures 1 and 2 this is accounted for by removing the first few seconds of data.

Below, we plot the radius of each ball against the mean terminal velocity (with corrections for the wall effect). The mean terminal velocity was calculated by taking the mean of the mean velocities over time for each ball (i.e. in the velocity-time plots above, take the mean of each line, then group by colour and take the mean again). The uncertainties are too small to see. The points are fitted with lines according to the instructions and equations given in the lab manual: Eq. 12 for water, and Eq. 18 for glycerine.

Figure 4

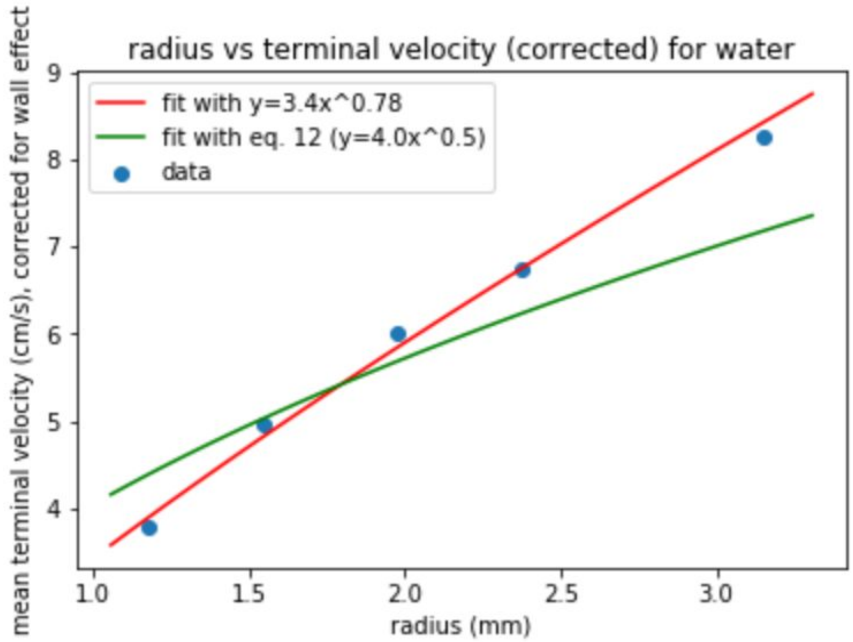
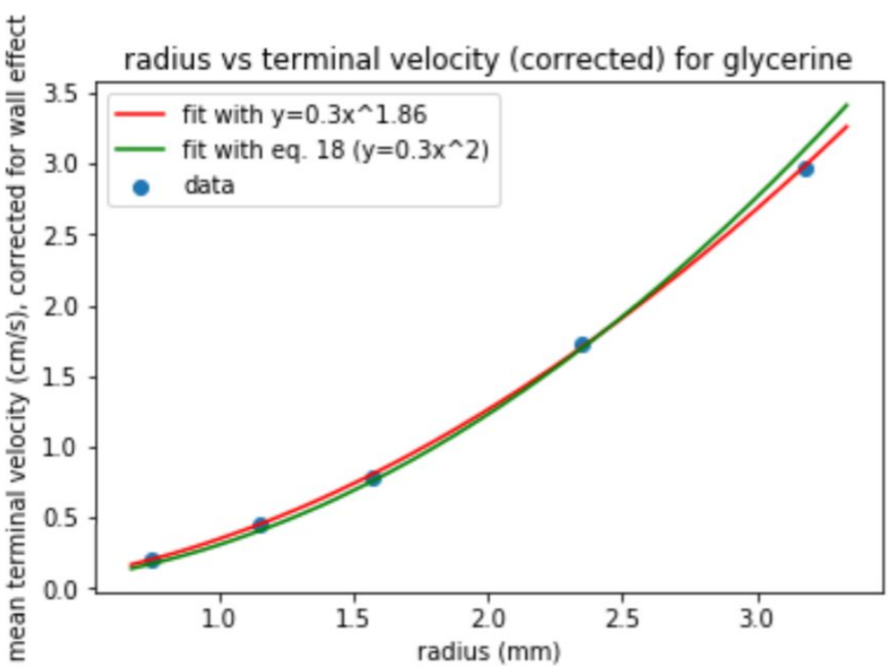


Figure 5



For the balls dropped in water, we operate in a high Reynolds number regime, and fit the data with $v_{term} \propto r^{1/2}$. We also fit a more general power curve to the data points to serve as a reference. We see that there is a discrepancy with the theory: $v_{term} \propto r^{1/2}$ does not fit the data very well. Visually, the line following Eq. 12 (the green one) is a very poor fit for the data. Using curve-fitting methods with a general power law equation, a value that was found to be a better fit is 0.78 for the exponent.This is because the Reynolds number in pure (or almost pure) H2O is very high and therefore not an average for a high Reynolds number.

For the balls dropped in glycerine, we operate in a low Reynolds number regime (due to the much higher viscosity of glycerine, relative to water), and fit the data with $v_{term} \propto r^2$ according to the lab manual. Again, we fit a more general power curve to the data points to serve as a reference. Visually, it seems that Eq. 18 is a fair approximation for the data we collected. However, it would seem that the exponent is slightly too high, as the reference value that we found to be a better fit is 1.86. However, it seems that the theory is more in line with the experimental data for balls dropped in glycerine.

Sources of Uncertainty

A potential source of error could come from imperfections in the beads. If they are not manufactured to tight tolerances, the variation in each bead could produce some uncertainty. However, since we conducted many trials for each bead size, this should be negligible. Another source of uncertainty might caused by the wall effect. Although we applied corrections to the mean terminal velocity to account for this effect, the correction depends on the distance between the ball and the wall. When we dropped the ball, we did not ensure that the x-y position was consistent, and so some drops might have been closer to the wall than others. This means that different trials would have required different amounts of correction. What we did was to take the average of all the trials, and then correct that single velocity, rather than apply a different correction to each of the trials, then take the average.

Since the uncertainties are incredibly small, the reduced chi squared values for the fits are incredibly high. For the fit of Eq. 12 to water, it is 2.04E+11, and for the fit of Eq. 18 to glycerine, it is 4.05E+07. Even though the general power curves fit the data better visually, the reduced chi squared values for those are also very large: 2.21E+06 for water, and 2.33E+07 for glycerine. Again, this is because the reduced chi squared metric gives the goodness of fit relative to the uncertainties, which are very small.

Questions

→ Integrate (9) and solve for υ(t). Verify that solution from (10) is correct.

Rewrite the equation:

$mv' = mg - cv^2 \;\rightarrow\; \frac{mv'}{mg - cv^2} = 1$

This is a separable equation. Integrate both sides with respect to t and evaluate, using the initial condition that v(0)=0. Then we get Eq. 10. Checked it with WolframAlpha and it's correct.

→ Integrate and solve for ν(t). Verify that solution from eq. (17) is correct.

Rewrite the equation:

$mv' = mg - bv \;\rightarrow\; \frac{mv'}{mg - bv} = 1$

This is a separable equation. Integrate both sides with respect to t and evaluate, using the initial condition that v(0)=0. Then we get Eq. 17. Checked it with WolframAlpha and it's correct.

→ Estimate τ for a 1 mm diameter aluminum sphere in glycerine, r = 0.5 x 10-3 m, η = 1500cp, ρ = 2.7 g/cm3.

m = ρV = ρ*4/3*πr^3 = 2.7 g/cm3 * 4/3 * π * (0.05cm)^3 = 0.0015 g

$\tau = \frac{m}{6\pi r\eta}$ = 0.0015g / (6π * 0.05cm * 1500cp) = 1.04e-06 s, after unit conversions

Conclusion:

In this lab, we measured the time and position of balls as they sank in fluids of different viscosities. Using these measurements, we calculated the mean terminal velocity for each ball size in each fluid. Afterwards, we corrected the velocities for the wall effect, and use that corrected velocity to calculate the Reynolds number. We found that in water, the Reynolds number was high, and the theory that $v_{term} \propto r^{1/2}$ was not a good fit for our results. On the other hand, in glycerine, the Reynolds number was low, and the theory that $v_{term} \propto r^2$ was a somewhat good fit. However, because of small initial uncertainties being averaged over a number of trials, the uncertainties we ended up with were extremely small (probably too small). We investigated other sources of uncertainty and explained how we took those into consideration.