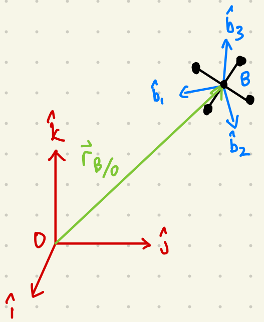
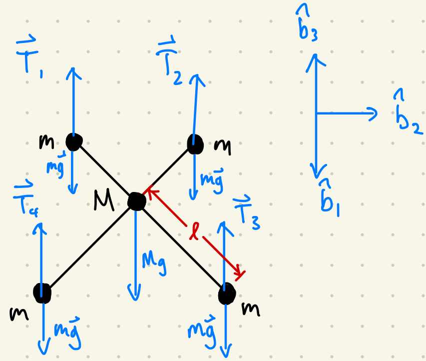
Quadrotor Drone Model





$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = [R] \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

Yaw $= \psi$

Pitch $= \theta$

Roll $= \phi$

$$[R] = R_\phi R_\theta R_\psi = R_{3-2-1}$$

$$R_\psi = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

$$[R]_{3-2-1} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = [R]^{-1}_{3\text{-}2\text{-}1} \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix}$$

$$[R]^{-1}_{3\text{-}2\text{-}1} = \begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - c\phi s\psi & s\phi s\psi + c\phi c\psi s\theta \\ c\theta s\psi & c\phi c\psi + s\theta s\phi s\psi & c\phi s\theta s\psi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

Planes of symmetry: x-z, y-z

$$I_{G11} = m\left(\left(\frac{-l}{\sqrt{2}}\right)^2 + 0^2\right) + m\left(\left(\frac{l}{\sqrt{2}}\right)^2 + 0^2\right) + m\left(\left(\frac{l}{\sqrt{2}}\right)^2 + 0^2\right)$$
$$+ m\left(\left(\frac{-l}{\sqrt{2}}\right)^2 + 0^2\right)$$
$$= 2ml^2$$

$I_{G12} = 0 \quad I_{Gxz} = 0 \quad I_{Gyz} = 0$

$$I_{G22} = m\left[\left(\left(\frac{-l}{\sqrt{2}}\right)^2 + 0^2\right) + \left(\left(\frac{-l}{\sqrt{2}}\right)^2 + 0^2\right) + \left(\left(\frac{l}{\sqrt{2}}\right)^2 + 0^2\right) + \left(\left(\frac{l}{\sqrt{2}}\right)^2 + 0^2\right)\right]$$
$$= 2ml^2$$

$$I_{G33} = m\left[\left(\left(\frac{-l}{\sqrt{2}}\right)^2 + \left(\frac{-l}{\sqrt{2}}\right)^2\right) + \left(\left(\frac{-l}{\sqrt{2}}\right)^2 + \left(\frac{l}{\sqrt{2}}\right)^2\right) + \left(\left(\frac{l}{\sqrt{2}}\right)^2 + \left(\frac{l}{\sqrt{2}}\right)^2\right)\right.$$
$$\left. + \left(\left(\frac{l}{\sqrt{2}}\right)^2 + \left(\frac{-l}{\sqrt{2}}\right)^2\right)\right]$$
$$= 4ml^2$$

$$I_G = \begin{bmatrix} 2ml^2 & 0 & 0 \\ 0 & 2ml^2 & 0 \\ 0 & 0 & 4ml^2 \end{bmatrix}$$

$$\sum \vec{M}_G = \vec{r}_1 \times \vec{T}_1 + \vec{r}_2 \times \vec{T}_2 + \vec{r}_3 \times \vec{T}_3 + \vec{r}_4 \times \vec{T}_4$$
$$+ \vec{r}_1 \times -mg\hat{k} + \vec{r}_2 \times -mg\hat{k} + \vec{r}_3 \times -mg\hat{k} + \vec{r}_4 \times -mg\hat{k}$$

$$\hat{k} = (-s\theta)\hat{b}_1 + (c\theta s\phi)\hat{b}_2 + (c\theta c\phi)\hat{b}_3$$

$$\sum \vec{M}_{G_1} = I_{11}\dot{w}_{b_1} - (I_{22} - I_{33}) w_{b_2} w_{b_3}$$
$$\sum M_{G_2} = I_{22}\dot{w}_{b_2} - (I_{33} - I_{11}) w_{b_3} w_{b_1}$$
$$\sum M_{G_3} = I_{33}\dot{w}_{b_3} - (I_{11} - I_{22}) w_{b_1} w_{b_2}$$

$$\dot{w}_{b_1} = \frac{(I_{22} - I_{33}) w_{b_2} w_{b_3} + \sum M_{G_1}}{I_{11}}$$
$$\dot{w}_{b_2} = \frac{(I_{33} - I_{11}) w_{b_3} w_{b_1} + \sum M_{G_2}}{I_{22}}$$
$$\dot{w}_{b_3} = \frac{(I_{11} - I_{22}) w_{b_1} w_{b_2} + \sum M_{G_3}}{I_{33}}$$

$$\begin{bmatrix} w_{b_1} \\ w_{b_2} \\ w_{b_3} \end{bmatrix} = \begin{bmatrix} -\sin\theta & 0 & 1 \\ \cos\theta\sin\phi & \cos\phi & 0 \\ \cos\theta\cos\phi & -\sin\phi & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \\ 0 & \cos\phi & -\sin\phi \\ 1 & \frac{\sin\theta\sin\phi}{\cos\theta} & \frac{\sin\theta\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} w_{b_1} \\ w_{b_2} \\ w_{b_3} \end{bmatrix}$$

$$\sum \vec{F} = (T_1 + T_2 + T_3 + T_4)\hat{b}_3 - (4m + M) g\hat{k}$$
$$= (4m + m)\vec{a}$$

$$\vec{b}_3 = (c\phi s\theta c\psi + s\phi s\psi)\hat{\imath} + (c\phi s\theta s\psi - s\phi c\psi)\hat{\jmath} + (c\phi c\theta)\hat{k}$$

$$\vec{a} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \dot{V}_x \\ \dot{V}_y \\ \dot{V}_z \end{bmatrix}$$

$$\begin{bmatrix} \dot{V}_x \\ \dot{V}_y \\ \dot{V}_z \end{bmatrix} = \frac{1}{(4m+M)} \left[R\right]^{-1}_{3\text{-}2\text{-}1} \begin{bmatrix} 0 \\ 0 \\ T_1+T_2+T_3+T_4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$