Lecture 11
Edge Detection Theory
ECEN 5283 Computer Vision

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Goals

- To introduce the edge detection theory that includes the following three basic issues:
  - Noise modeling in an image
  - Estimating derivatives under noise
  - Optimal smoothing filter selection

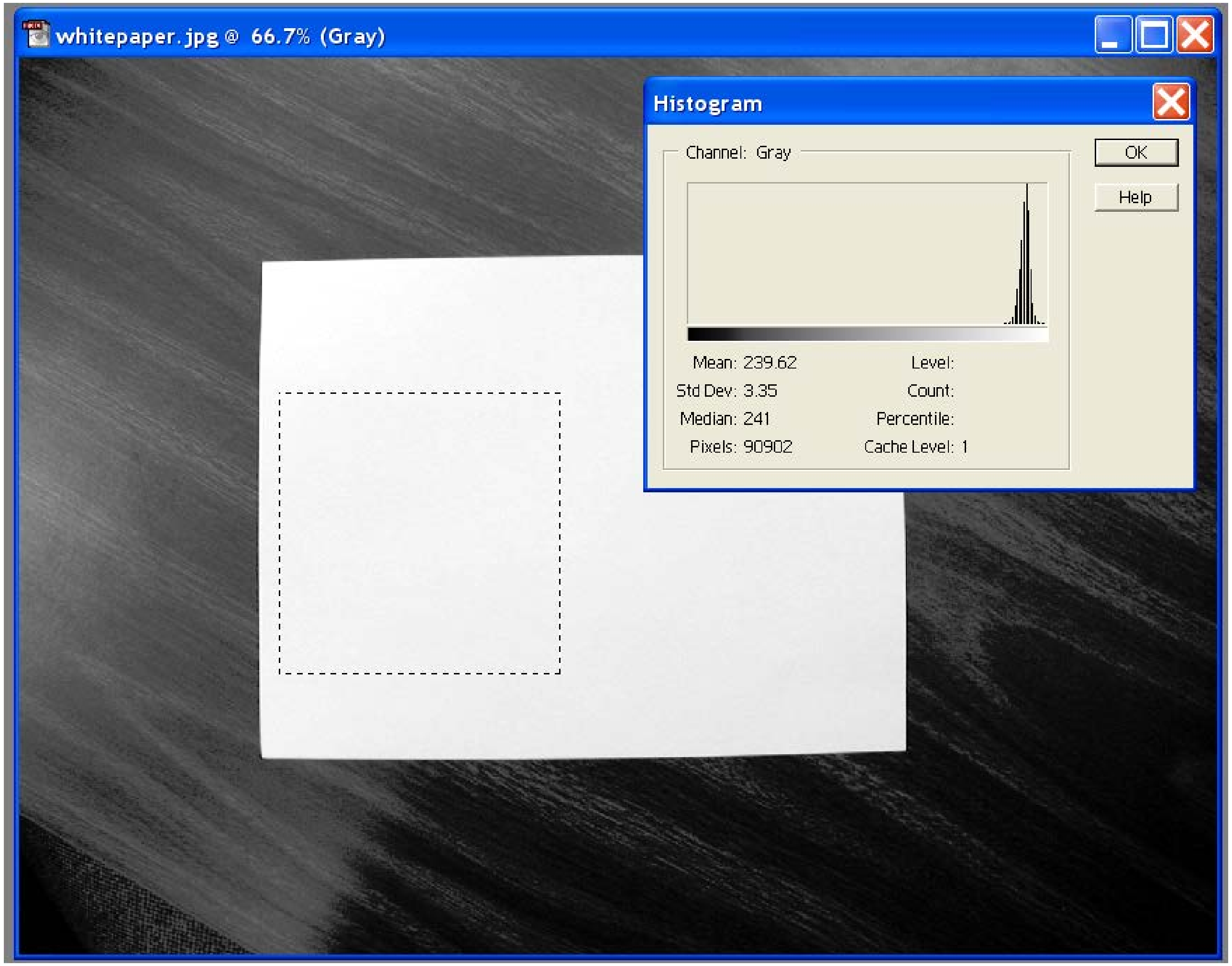
Edge detection WWW

- **Why:** Edge detection is a fundamental early stage in most computer vision tasks.

- **HoW:** Edge detection is essentially involving *spatial differentiation* that is sensitive to fast intensity changes.

- **What:** We need to address the following issues
  - How to model the noise in an image?
  - How to estimate derivatives under noise?
  - What is the optimal smoothing filter?

Issue 1: Noise and Smoothing



Additive Gaussian White Noise (AWGN) Model

- In the AWGN model, each pixel is *added* to a value chosen *independently* from the same *Gaussian* probability distribution.
- Almost always the mean of this distribution is zero. The standard deviation is a parameter of the model.
- The model is intended to describe thermal noise in cameras.

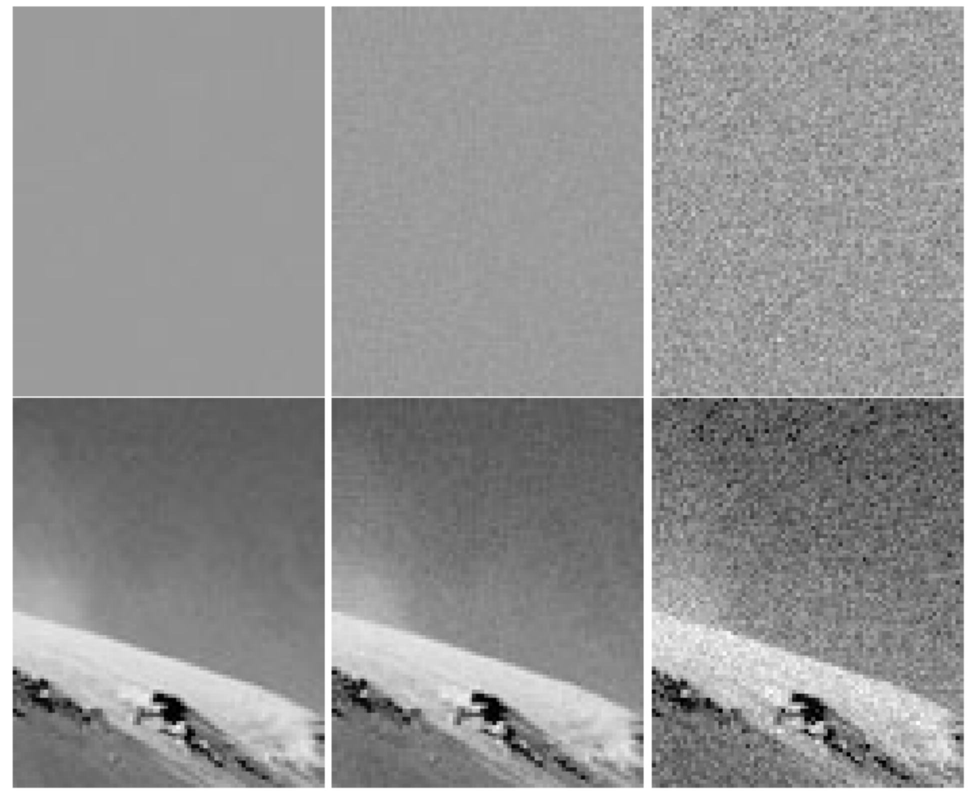
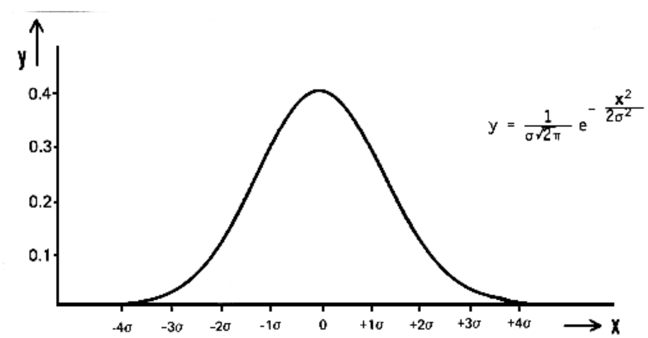


Figure 9.1. The top row shows three realisations of a stationary additive Gaussian noise process. We have added half the range of brightnesses to these images, so as to show both negative and positive values of noise. From left to right, the noise has standard deviation 1/256, 4/256 and 16/256 of the full range of brightness respectively. This corresponds roughly to bits zero, two and five of a camera that has an output range of eight bits per pixel. The lower row shows this noise added to an image. In each case, values below zero or above the full range have been adjusted to zero or the maximum value accordingly.



AWGN Advantages and Disadvantages

- Advantages:
  - It is analytical tractable.
  - It is easy to estimate the response of filters to the AWGN model.
  - The center-limit-theorem suggests that it is a good model.
- Disadvantages:
  - The AWGN model allows both positive and negative pixel values of arbitrary magnitude. It may not represent the real problem.
  - Noise values are completely independent. It cannot capture inter-pixel dependencies.
  - The AWGN model does not describe dead pixels (pixels that consistently report no incoming light or are consistently saturated)

(The salt-and-pepper noise model is needed)

Issue 2: Finite Differences Under Noise

- Edge detection involves spatial differentiation with following kernels to estimate the first/second order derivates:

$$\frac{df}{dx} = f(x) - f(x-1)$$

$$\frac{d^2 f}{dx^2} = f(x) - 2f(x-1) + f(x-2)$$

$$\begin{bmatrix} 0 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

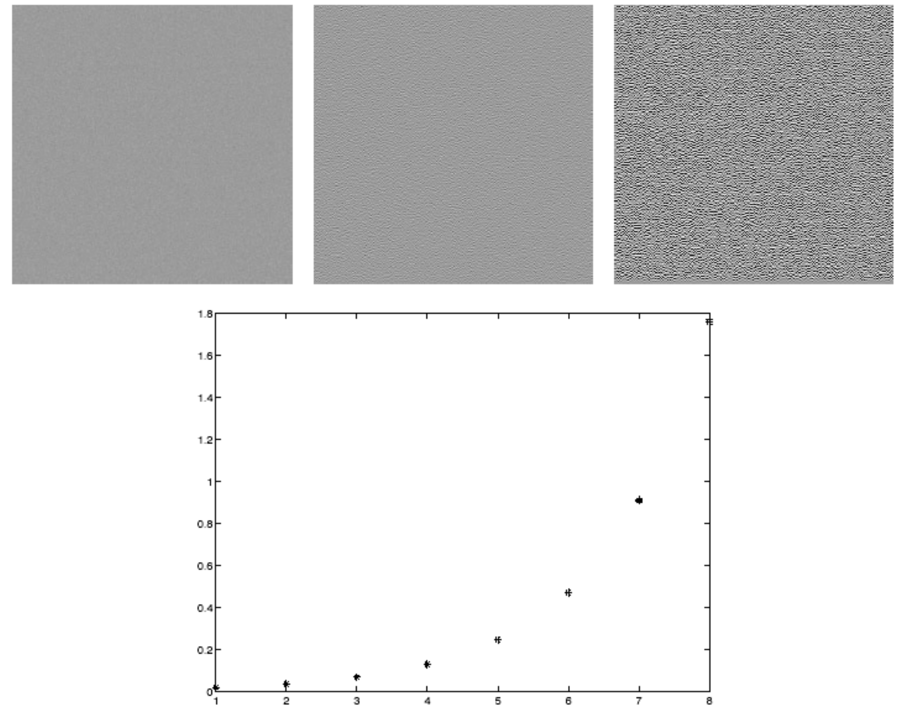
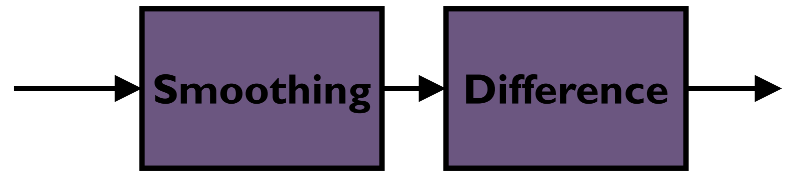


Figure 9.2. Finite differences can accentuate additive Gaussian noise substantially, following the argument in section 9.1.2. On the **top left**, an image of zero mean Gaussian noise with standard deviation 4/256 of the full range. The **top center** figure shows a finite difference estimate of the third derivative in the x direction, and the **top right** shows the sixth derivative in the x direction. In each case, the image has been centered by adding half the full range to show both positive and negative deviations. The images are shown using the same grey level scale; in the case of the sixth derivative, some values exceed the range of this scale. The graph on the **bottom** shows the standard deviations of these noise images for the first eight derivatives (estimated using the argument based around Pascal's triangle).

Derivates of Smoothing Filters

- *Important Question:*
  - *Is differentiation* is LTI/LSI operation?
  - If yes, we define a kernel that implements differentiation as a linear filter

$$\frac{\partial I}{\partial x} = K_{(\partial/\partial x)} ** I.$$



- Now we want the derivative of a smoothing function *S*

$$(K_{(\partial/\partial x)} ** (S ** I)) = (K_{(\partial/\partial x)} ** S) ** I = \left(\frac{\partial S}{\partial x} \right) ** I.$$

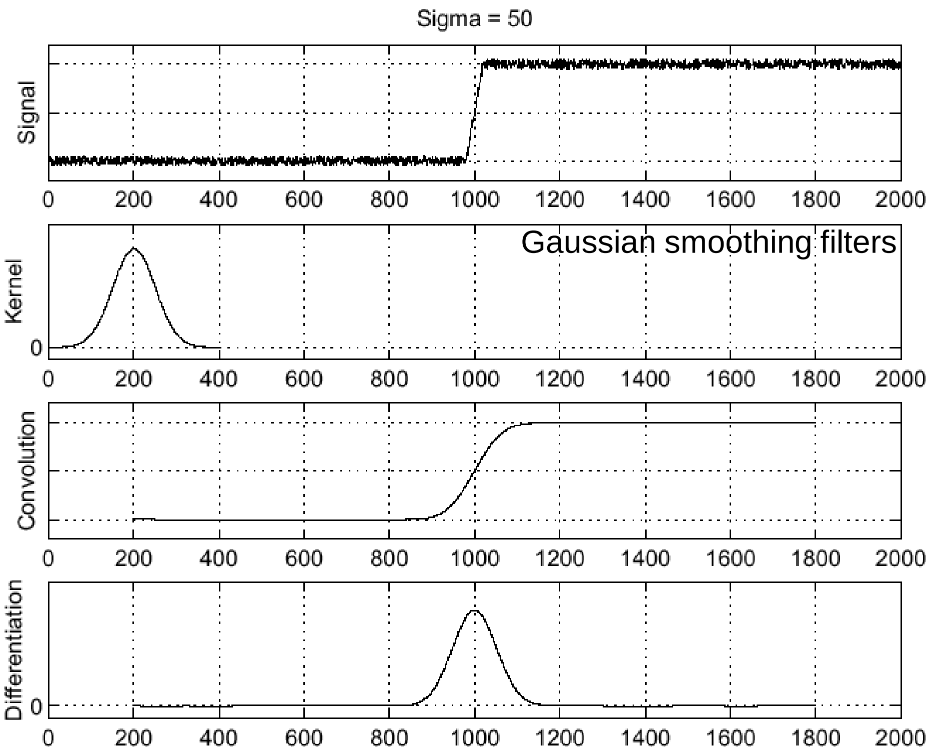
- This fact appears in its most commonly used form when the smoothing function is a Gaussian

$$\left(\frac{\partial G_\delta ** I}{\partial x} \right) = \left(\frac{\partial G_\delta}{\partial x} \right) ** I.$$

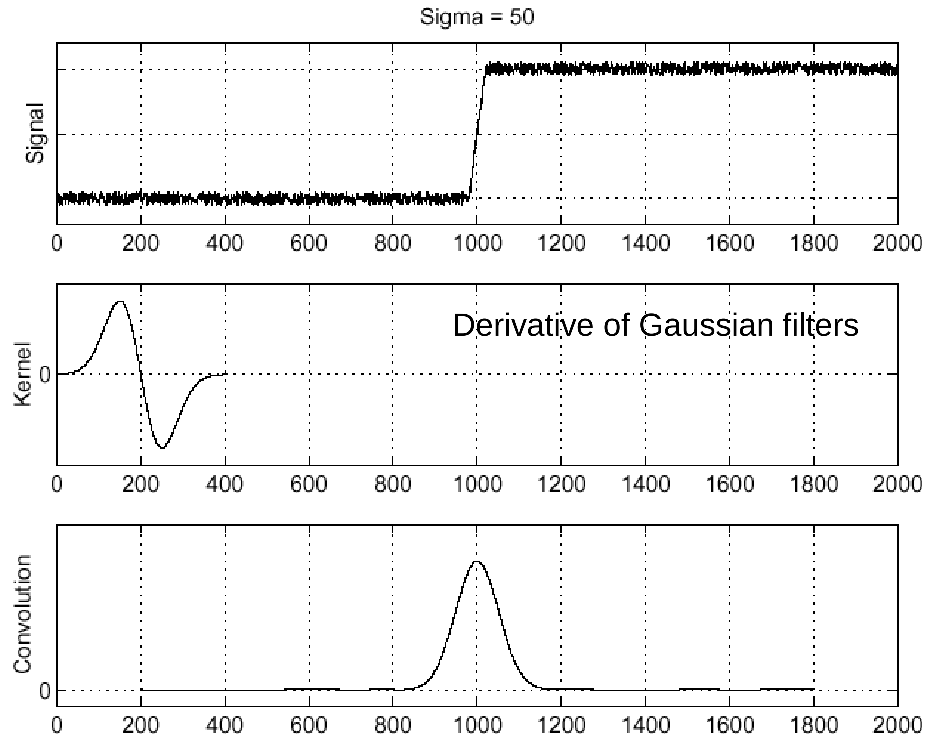
Derivative of Gaussian filters

Gaussian Smoothing for Edge Detection

Smoothing followed by difference



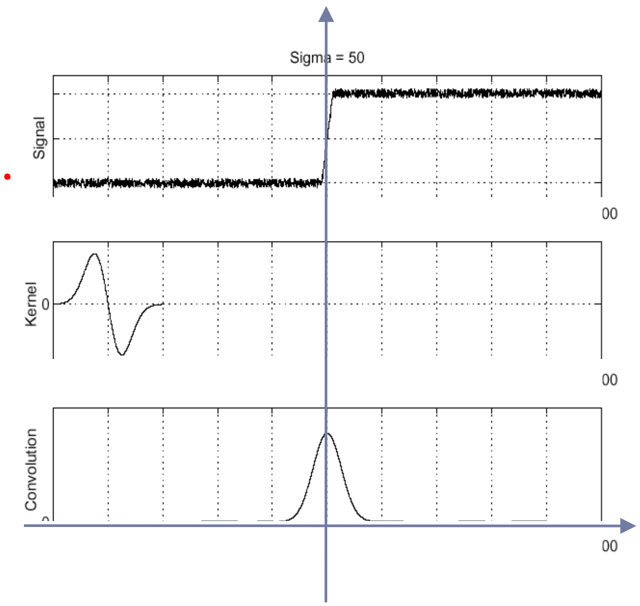
Combined smoothing and difference



Issue 3: Optimal smooth filter

- Canny (1986) established the practice of choosing a derivative estimation filter by optimizing three criteria:
  - Signal to noise ratio: the filter should respond more strongly to the edge at x=0 than to noise.
  - Edge Localization: the filter response should reach a maximum close to x=0.
  - Low false positives: there should be only one maximum of the response in a reasonable neighborhood x=0.
- It is a remarkable fact that the optimal smoothing filters tend to look a great deal like Gaussians.

$$edge(x) = AU(x) + B + n(x).$$



Canny, J., *A Computational Approach To Edge Detection*, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.