Project 1:
Linear Approach to Camera Calibration

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Project Objective

Objectives

1. Geometric camera calibration using linear approach
2. Predict the 2D locations from static and moving 3D points
3. Calculate projection matrix **M**
4. Using M to calculate *intrinsic* and *extrinsic* parameters
5. For verification visualizing the calibrated images and videos

Tools, Input & Output

1. **Python**, PyCharm IDE, Matplotlib, Numpy
2. **Latex Beamer**, Sublime Text
3. **Input** were “model.dat” and “obesrve.dat” files.
4. Generated input by program.

Technical Background

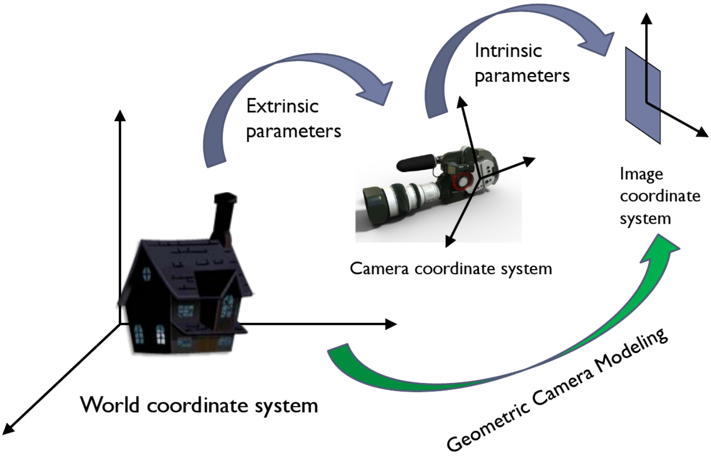


Figure: Extrinsic parameters are related to world coordinate. i.e. Rotation and Translation. Intrinsic parameters are related to camera coordinate system and hardware properties like pixel size, orientation.

Intrinsic Parameters

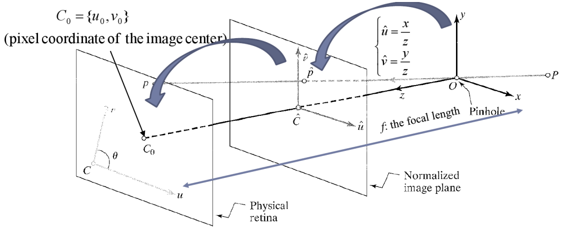
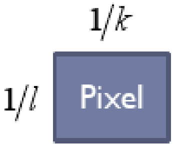


Figure 2.8 Physical and normalized image coordinate systems.

Figure: Intrinsic parameters relates the physical image, p, to a normalized image plane $\hat{p}$ by solving equations for affine and perspective projection. Parameter α, β are the product of focal length and pixel resolution. θ is the angle between horizontal and vertical axis. u_0, v_0 are the center of the physical image. First pinhole coordinate in projected on $\hat{p}$ using perspective projection. $\hat{p}$ is projected on p using affine projection.



$$\alpha = kf \quad \text{and} \quad \beta = lf$$

$$\hat{\mathbf{p}} = \begin{pmatrix} \hat{u} \\ \hat{v} \\ 1 \end{pmatrix} = \frac{1}{z}(\mathbf{I} \quad 0)\mathbf{P}$$

(3D to 2D perspective projection on the normalized image plane)

$$\mathbf{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \Rightarrow \begin{cases} u = \alpha\hat{u} - \alpha\cot\theta\hat{v} + u_0 \\ v = \dfrac{\beta}{\sin\theta}\hat{v} + v_0 \end{cases}$$

Converting the coordinates in homogeneous for makes these equation computationally suitable for optimization. All the intrinsic parameters can be encoded into a matrix K.

Intrinsic Parameters

$$\mathbf{K} = \begin{pmatrix} \alpha & -\alpha \cot\theta & u_0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_0 \\ & 0 & 1 \end{pmatrix}$$

All the intrinsic parameters are encoded in the projection matrix K. ($\hat{p}$'s projection on p).

$$\mathbf{p} = K\hat{\mathbf{p}}$$

Can rewritten as,

$$\mathbf{p} = K\frac{1}{z}(\mathbf{I} \quad 0)\mathbf{P}$$

$$\mathbf{p} = \frac{1}{z}M\mathbf{P}$$

Extrinsic Parameters

Real physical world coordinate W is not P. Extrinsic parameters are used to relate W to P.

$$\mathbf{M} = \mathbf{K}\begin{pmatrix}\mathbf{R} & \mathbf{t}\end{pmatrix}$$

Basic Theory of the Project

z is not independent of M and P. Thus

$$M = \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \frac{1}{z} \begin{pmatrix} \mathbf{m}_1^T \mathbf{P} \\ \mathbf{m}_2^T \mathbf{P} \\ \mathbf{m}_3^T \mathbf{P} \end{pmatrix} = \frac{1}{z} \begin{pmatrix} \mathbf{m}_1 \cdot \mathbf{P} \\ \mathbf{m}_2 \cdot \mathbf{P} \\ \mathbf{m}_3 \cdot \mathbf{P} \end{pmatrix} \Rightarrow \begin{cases} z = \mathbf{m}_3 \cdot \mathbf{P} \\ u = \dfrac{\mathbf{m}_1 \cdot \mathbf{P}}{\mathbf{m}_3 \cdot \mathbf{P}} \\ v = \dfrac{\mathbf{m}_2 \cdot \mathbf{P}}{\mathbf{m}_3 \cdot \mathbf{P}} \end{cases}$$

Basic Theory of the Project

For n given World coordinates the parameter can be solved using the following linear system

$$\mathbf{Qm} = \mathbf{0} \Rightarrow \hat{\mathbf{m}} = \arg\min_{\mathbf{m}} |\mathbf{Qm}|^2$$

$$\mathbf{Q} = \begin{pmatrix} \mathbf{P}_1^T & \mathbf{0}^T & -u_1\mathbf{P}_1^T \\ \mathbf{0}^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ \cdots & \cdots & \cdots \\ \mathbf{P}_n^T & \mathbf{0}^T & -u_n\mathbf{P}_n^T \\ \mathbf{0}^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{pmatrix}_{2n\times 12} \quad \text{and} \quad \mathbf{m} = \begin{pmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{pmatrix}_{12\times 1}$$

In the project the "model.dat" & "observe.dat" file contained all information to build P and u, v respectively to calculate Q. The first task was to estimate projection matrix M. Other tasks involved manipulating M to calculate extrinsic and intrinsic parameters and 2D locations of given 3D locations.

Estimation of Intrinsic and Extrinsic Parameters

When projection matrix M is calculated it can divided into blocks

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{b} \end{pmatrix}$$

Using a scaling factor ρ we can define

$$\rho \begin{pmatrix} \mathbf{A} & \mathbf{b} \end{pmatrix} = \mathbf{K} \begin{pmatrix} \mathbf{R} & \mathbf{t} \end{pmatrix} = \begin{pmatrix} \mathbf{K}\mathbf{R} & \mathbf{K}\mathbf{t} \end{pmatrix}$$

Estimation of Intrinsic Parameters

Intrinsic parameters can be calculated using the constraint that ρa_3 is always ± 1 because rows of a rotation matrix is has unit length

$$\rho = \varepsilon / |\mathbf{a}_3|$$
$$u_0 = \rho^2 (\mathbf{a}_1 \cdot \mathbf{a}_3)$$
$$v_0 = \rho^2 (\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos\theta = -\frac{(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| |\mathbf{a}_2 \times \mathbf{a}_3|}$$
$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin\theta$$
$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin\theta$$

Estimation of Extrinsic Parameters

Extrinsic parameters can be calculated using

$$\mathbf{r}_1 = \frac{1}{|\mathbf{a}_2 \times \mathbf{a}_3|}(\mathbf{a}_2 \times \mathbf{a}_3)$$

$$\mathbf{r}_3 = \rho \mathbf{a}_3$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1$$

Where the key term is to calculate r_3 having calculated ρ. The cross products follow that right hand rule.

$$\mathbf{t} = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} = \rho \mathbf{K}^{-1}\mathbf{b}$$

Result

$5.483 \cdot 10^{-2}$	$-5.345 \cdot 10^{-2}$	$-3.88 \cdot 10^{-4}$	0.574
$-2.338 \cdot 10^{-2}$	$-2.399 \cdot 10^{-2}$	$-3.275 \cdot 10^{-2}$	0.814
$1.08 \cdot 10^{-6}$	$6.736 \cdot 10^{-8}$	$-1.246 \cdot 10^{-6}$	$1.825 \cdot 10^{-3}$

Table: Projection Matrix M

41,408.886	−3,602.885	20,591.909
0.000	27,915.848	5,116.874
0.000	0.000	1.000

Table: K

Extrinsic Parameter

0.422	−0.848	0.320
−0.627	−0.528	−0.572
0.654	$4.081 \cdot 10^{-2}$	−0.755

Table: Rotation Matrix R

−557.324
−184.923
1,105.260

Table: Translation Vector t

Intrinsic Parameter

$$\theta = 85.027358933^\circ$$
$$\alpha = 41408.8860516$$
$$\beta = 27810.7780862$$
$$u_0 = 20591.9094866$$
$$v_0 = 5116.87372168$$

Result

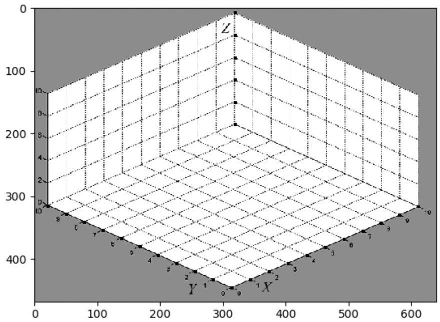


Figure: Plot of input data in model file using window size 4. The plot shows the input matches with the projection. It implies two things; the input data and the calculated projection matrix both are correct. The same window size has been used throughout the project. Here window size is the number of pixel which has been blacken out to draw a dot.

Result

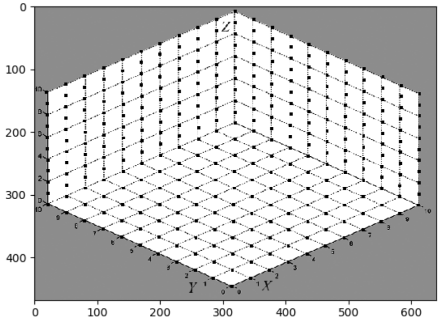


Figure: In the project generated 3D coordinate data and projected them on 2D image. The dots matches the coordinate.

Result

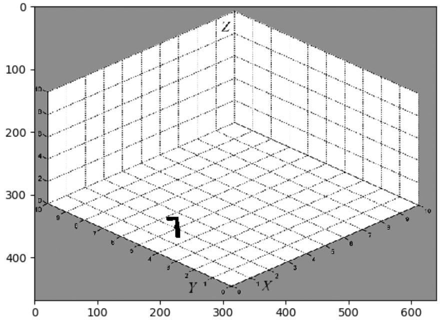


Figure: Table lamp projected from a public 3D data set[1].

[1] http://pointclouds.org/media/

Result

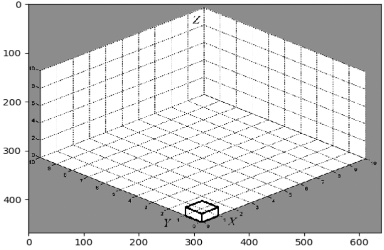


Figure: A 3D box has been created using 7 vertices and 8 lines. 100 sample 3D points has been used to represent the line. The animation emulates the movement of the box by moving the points along X axis

2

[2] N.B. The videos and the presentation pdf file should be in the same directory. phonon-backend-vlc or updated should be installed for Okular pdf reader. For adobe reader real time player plugin should be installed

Result

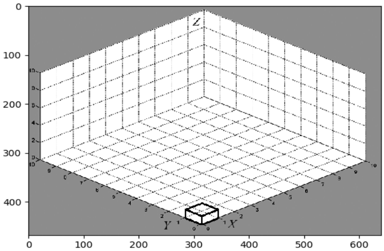


Figure: Similar to previous video but now it shows my implementation can move the box diagonally.

3

[3] N.B. The videos and the presentation pdf file should be in the same directory. phonon-backend-vlc or updated should be installed for Okular pdf reader. For adobe reader real time player plugin should be installed

Result

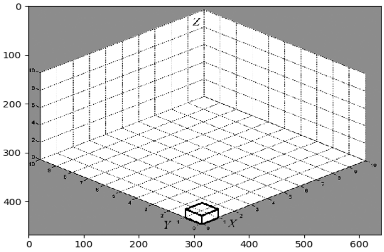


Figure: Similar to previous two videos but now it shows my implementation can have different original position and arbitrary goals in the space.

[4]

[4] N.B. The videos and the presentation pdf file should be in the same directory. phonon-backend-vlc or updated should be installed for Okular pdf reader. For adobe reader real time player plugin should be installed

Experiment With Different Parameter

- We were give 27 world coordinates and image pixel coordinate in input
- I have manually added $(5, 10, 0), (10, 5, 0)$ in model and corresponding pixel $(171, 250), (465, 251)$ in observation using Gimp Image editor and then run the program.
- I have found that adding more observation increases the accuracy of projection
- I have defined the accuracy as the difference between the projected 2D observation and actual observation $E = \frac{||A - B||}{N}$. Where N is the number of observation.

Table: Effect of Changing Parameters

N	E	theta (in deg)
27	0.097990	85.027358
28	0.094491	85.033215
29	0.091232	91.722549

Conclusion

- In this project, we have studied the linear method for geometric camera calibration
- We have implemented in Python
- We have verified our result by projecting and visualizing 3D points from the world coordinate to 2D image plane.

Conclusion

- **Thank You**
- **Questions ?**