Lecture 16.
Gabor Filters for Texture Analysis
ECEN 5283 Computer Vision

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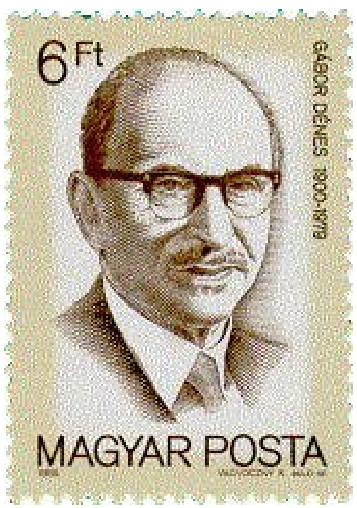
Goals

- To review two issues of texture analysis using a filter bank.
- To interpret texture analysis in the frequency domain.
- To apply Gabor filters for texture analysis.

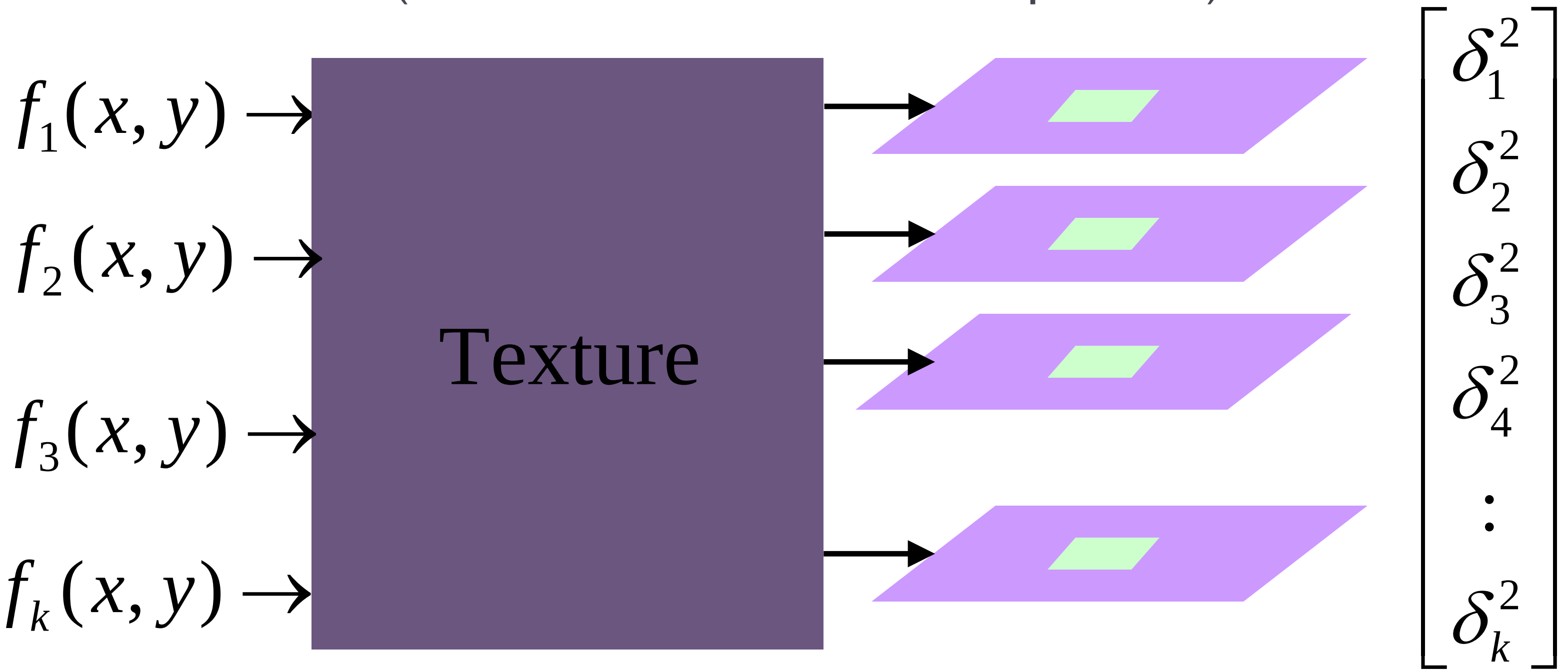


Dennis Gabor (original Hungarian name: Gábor Dénes), FRS, (June 5, 1900, Budapest – February 9, 1979, London) was a Hungarian physicist and inventor, most notable for inventing holography, for which he later received the Nobel Prize in Physics.

http://en.wikipedia.org/wiki/Dennis_Gabor

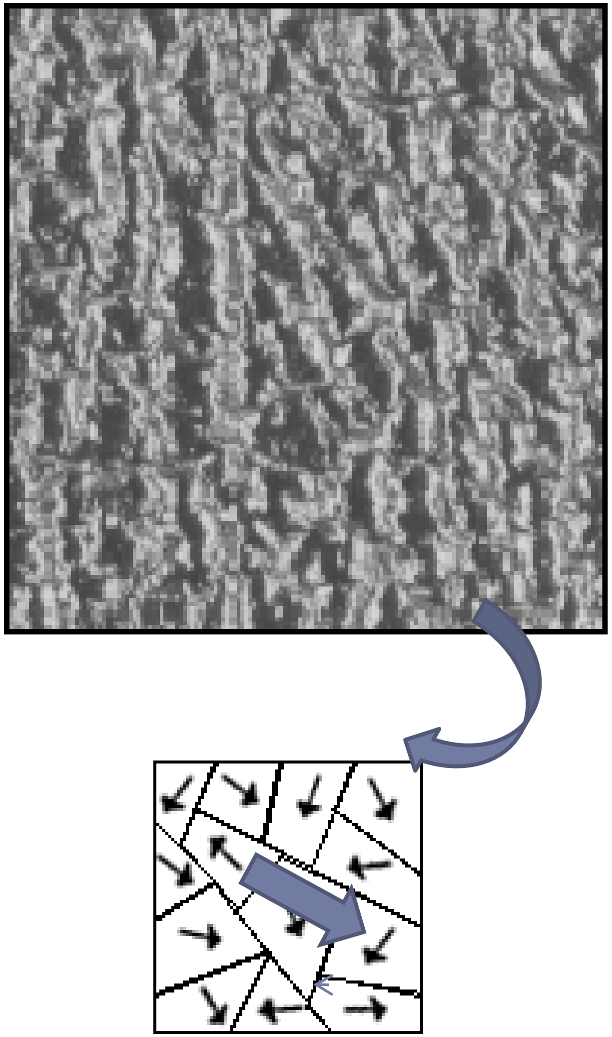
Filter Bank-based Texture Analysis

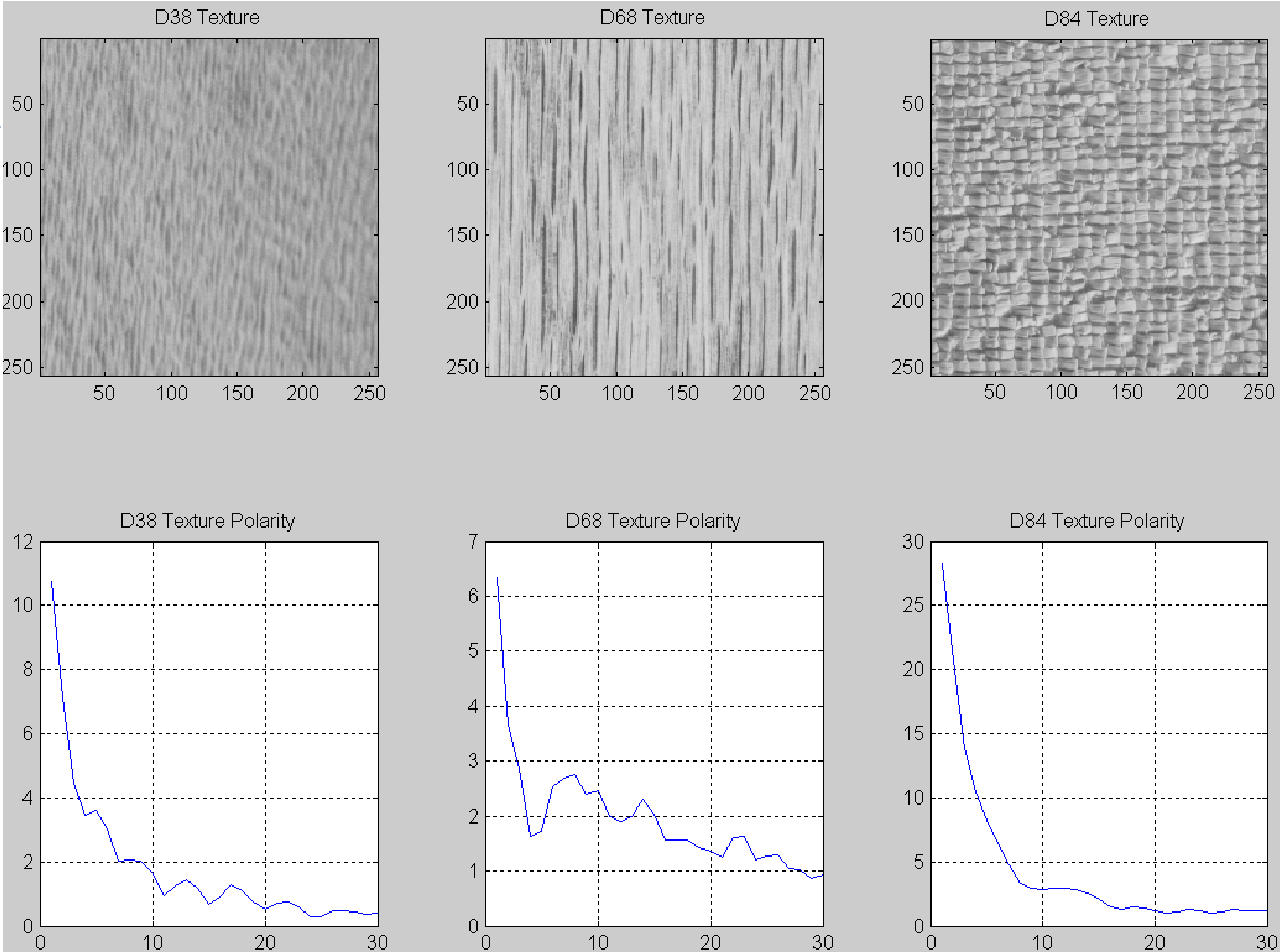
- Texture analysis is referred to as the process of convolving an image with a range of oriented filters.
- Two issues are involved for texture analysis using a filter bank.
  - Statistics (mean, variance, skewness, kurtosis)
  - Choice of scale (window-based statistics computation)



Scale Determination via Polarity

- Step 1: We first determine the *dominant orientation* in a window (given a size and randomly selected).
- Step 2: For each gradient vector, we form the *dot product* between the gradient vector and the dominant orientation.
- Step 3: We then form a smoothed average of the *positive dot products* and a smoothed average of the magnitude of the negative dot product, and take the difference of the two.
- Step 4: This polarity measures *the extent to which gradient points along the dominant orientations vs. ones against the dominant orientations*.
- Step 5: We measure polarity for a range of window sizes, and then start at the finest scale and look at increasingly large windows until the polarity has not changed when the scale changed.




D38 Texture
D68 Texture
D84 Texture
50
100
150
200
250
D38 Texture Polarity
D68 Texture Polarity
D84 Texture Polarity
0
10
20
30

Frequency-domain Representation

- The frequency-domain representation can give us some insights to a systematic design of a filter bank for texture analysis.

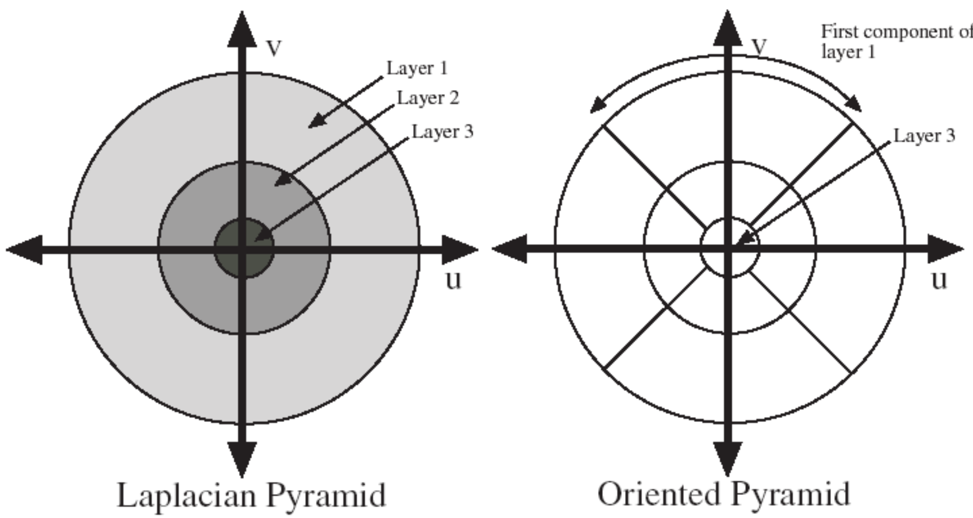
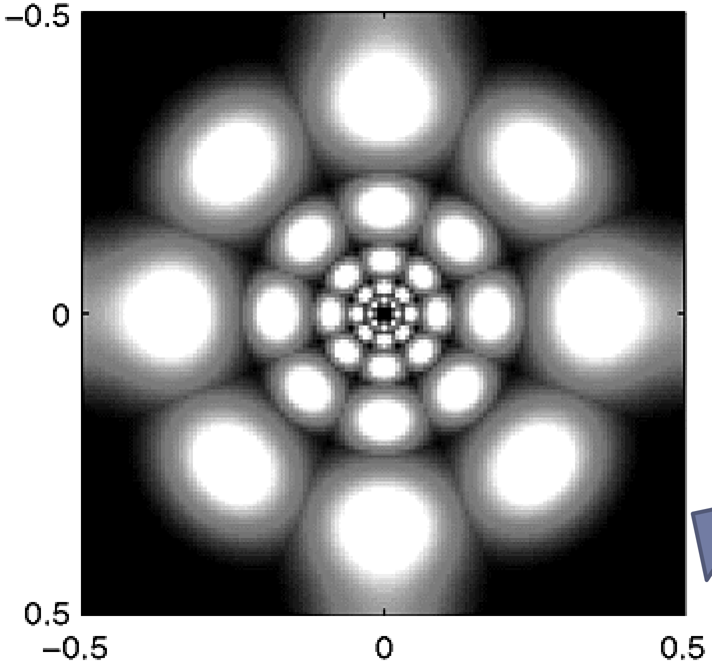


Figure 11.8. Each layer of the Laplacian pyramid consists the elements of a smoothed and resampled image that are not represented by the next smoother layer. Assuming that a Gaussian is a sufficiently good smoothing filter, each layer can be thought of as representing the image components within a range of spatial frequencies — this means that the Fourier transform of each layer is an annulus of values from the Fourier transform space (u, v) space (recall that the magnitude of (u, v) gives the spatial frequency). The sum of these annuluses is the Fourier transform of the image, so that each layer cuts an annulus out of the image's Fourier transform. An oriented pyramid cuts each annulus into a set of wedges. If (u, v) space is represented in polar coordinates, each wedge corresponds to an interval of radius values and an interval of angle values (recall that $\arctan(u/v)$ gives the orientation of the Fourier basis element).

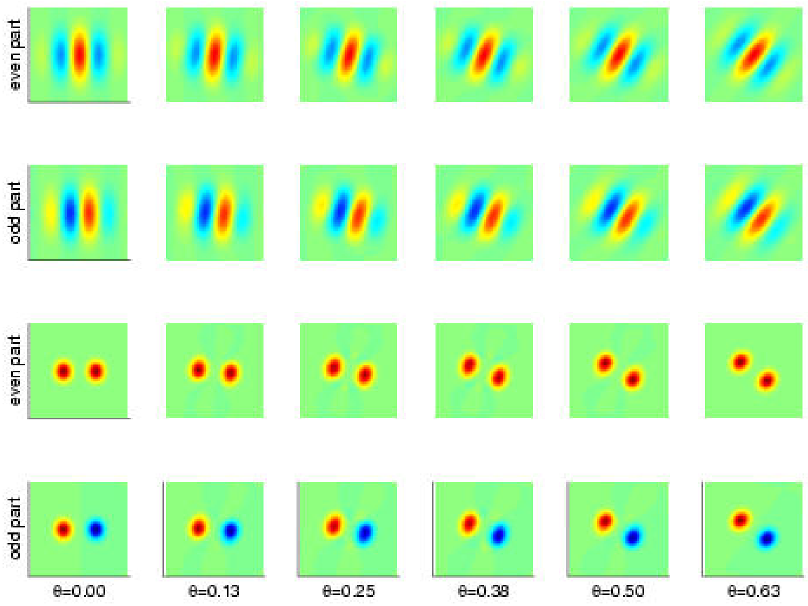
Oriented filters can reveal more distinct frequency characteristics between different textures.

Motivation of Gabor Filters



Frequency-domain partition

Gabor filter bank in the spatial domain



Fourier Basis vs. Gabor Filters

- The Fourier basis has not spatial selectivity but provides the best frequency selectivity.

$$f(x, y \mid k_x, k_y) = \exp^{j(k_x x + k_y y)} \quad \text{(Fourier basis)}$$

$$\delta(k_x, k_y) \xleftarrow{\text{Fourier transform}}$$

- Gabor filters can achieve ***localized frequency characterization*** by multiplying the Fourier basis elements with Gaussians.

$$g(x, y \mid \omega_x, \omega_y) = \exp^{j(\omega_x x + \omega_y y)} \exp^{-\left\{\frac{x^2 + y^2}{2\sigma^2}\right\}} \quad \text{(Gabor filter)}$$

- **Uncertainty principle:** spatial resolution and frequency resolution cannot be enhanced at the same time.

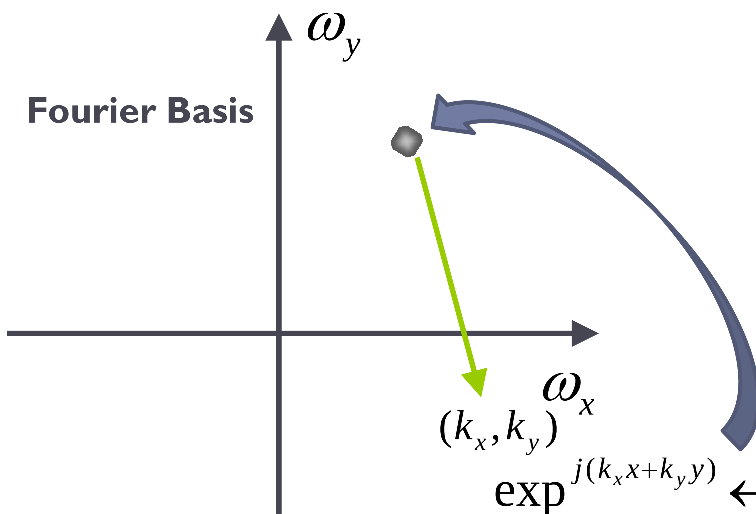
Frequency Representation of Gabor Filters

$$g(x,y)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\xleftrightarrow{\text{Fourier}}G(\omega_x,\omega_y)=\frac{1}{2\pi}\exp\left(-\frac{(\omega_x^2+\omega_y^2)}{2/\sigma^2}\right)$$

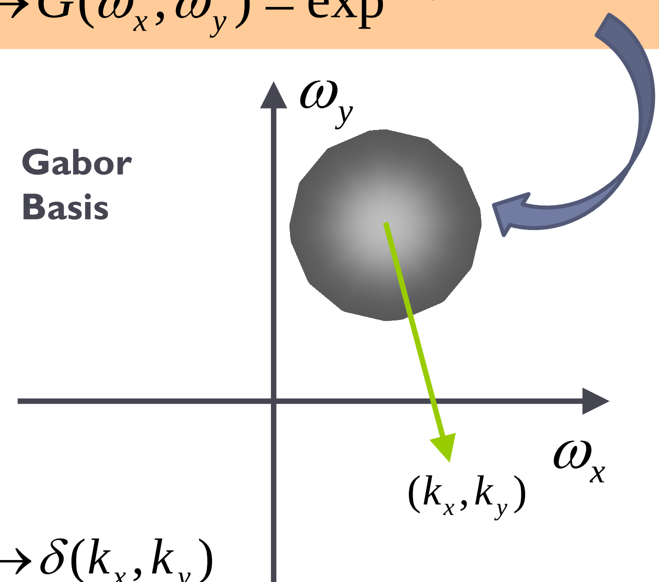
$$e^{j\omega_0 t}x(t)\xleftrightarrow{\text{Fourier}}X(j(\omega-\omega_0))$$

Fourier Transform Modulation Property

$$g(x,y)=\exp^{j(k_x x+k_y y)}\exp^{-\left\{\frac{x^2+y^2}{2\sigma^2}\right\}}\xleftrightarrow{\text{Fourier}}G(\omega_x,\omega_y)=\exp^{-\left\{\frac{(\omega_x-k_x)^2+(\omega_y-k_y)^2}{2/\sigma^2}\right\}}$$



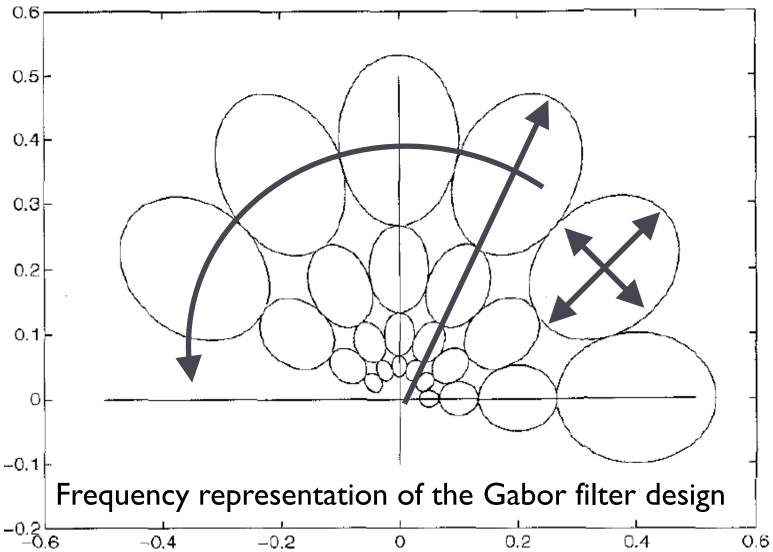
$$\exp^{j(k_x x+k_y y)}\xleftrightarrow{\text{Fourier}}\delta(k_x,k_y)$$



Gabor Filter Bank Design

- Four main factors to be considered
  - The number of free parameters should be small.
  - The whole spectrum should be covered
  - The overlap between neighboring channels should be minimized.
  - The characteristics of visual perception should be taken into account.
- There are some parameters to determine a Gabor filter bank.
  - Scales and orientations
  - Scaling factor between successive filters.
  - The standard deviations of the Gaussian in each scale and orientation.

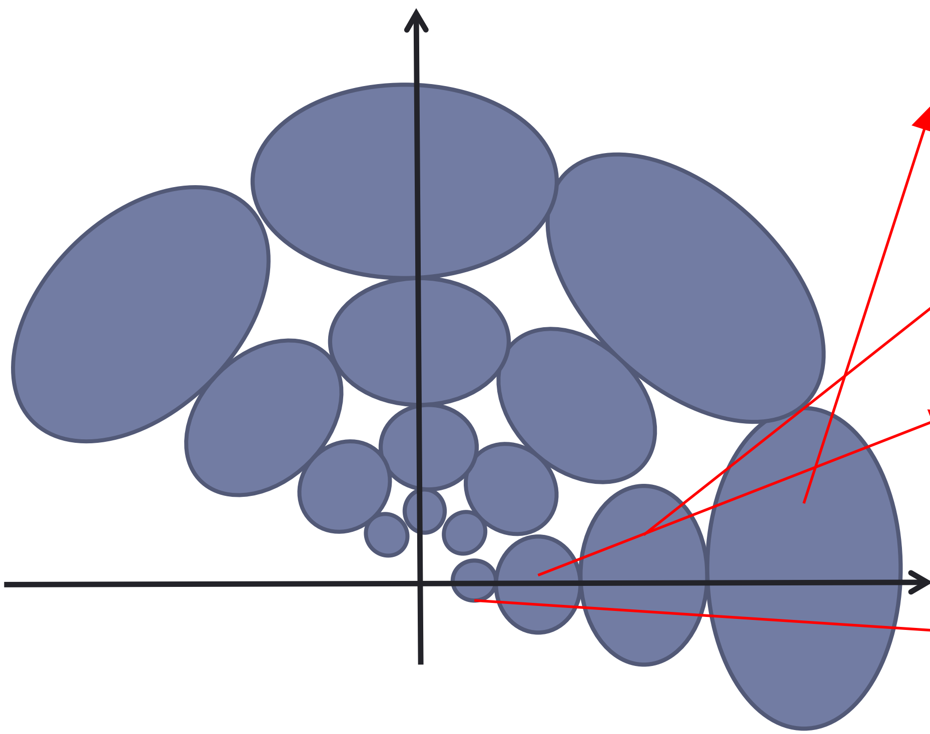
Why not like this?



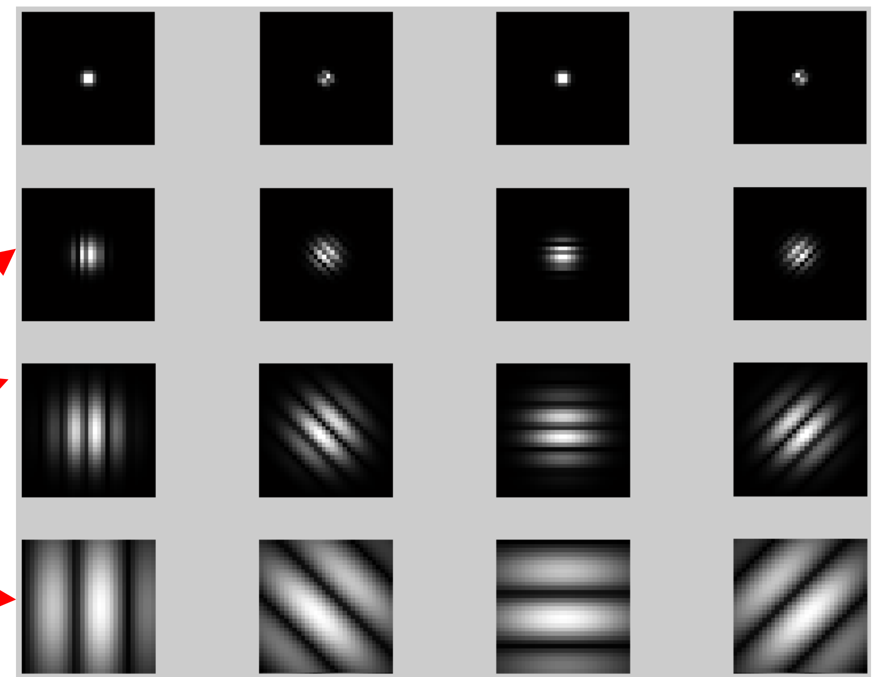
Frequency representation of the Gabor filter design

S. Manjunath and W.Y. Ma, "Texture features for browsing and retrieval of image data", IEEE Trans. on Pattern Analysis and Machine Intelligence (PAMI), vol.18, no.8, pp.837-42, Aug 1996.

Gabor Filter Kernels (Magnitude): 4 Scales and 4 Orientations

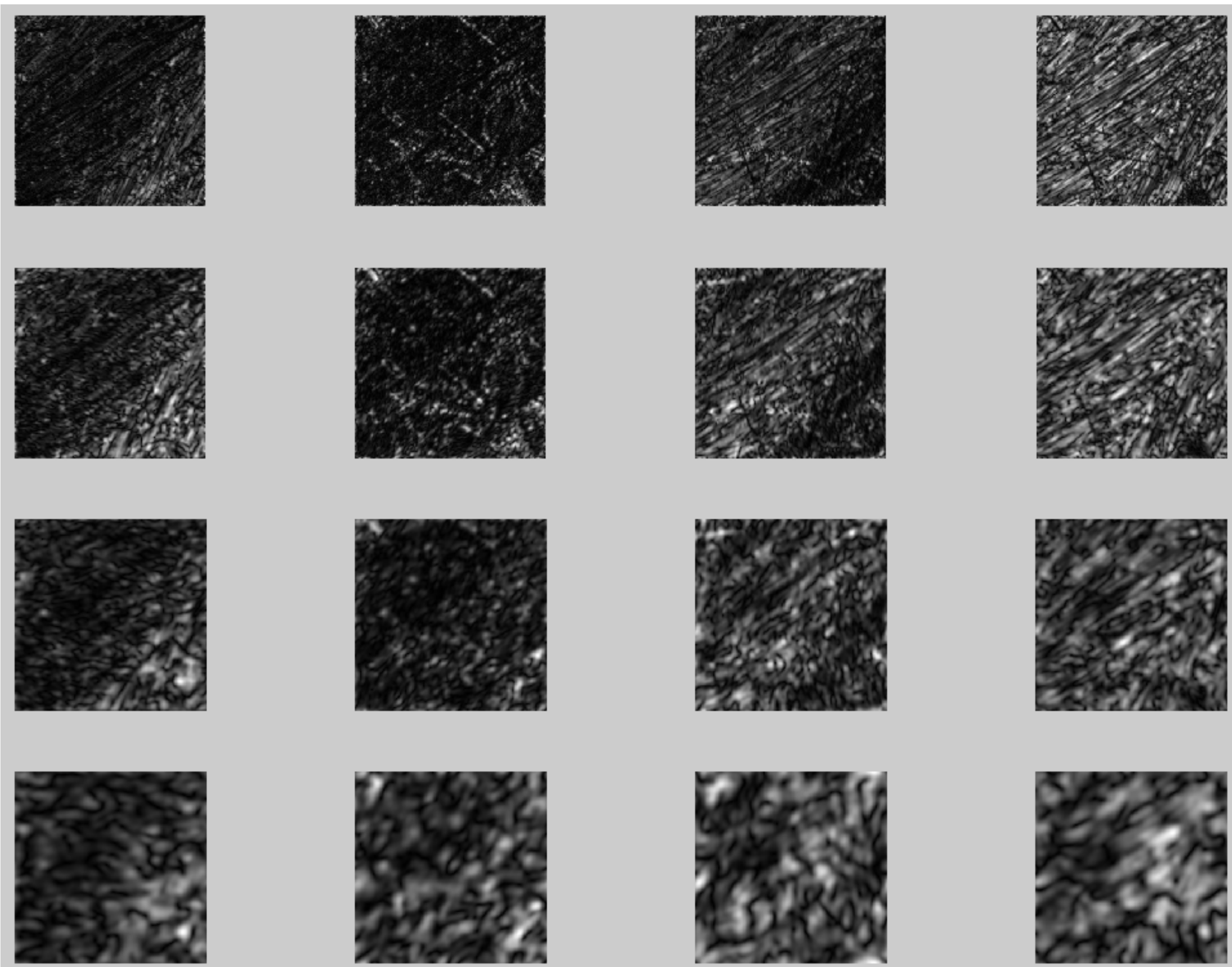


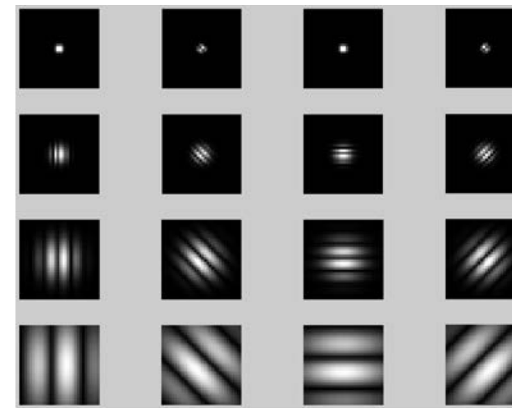
Frequency representation of the Gabor filter design



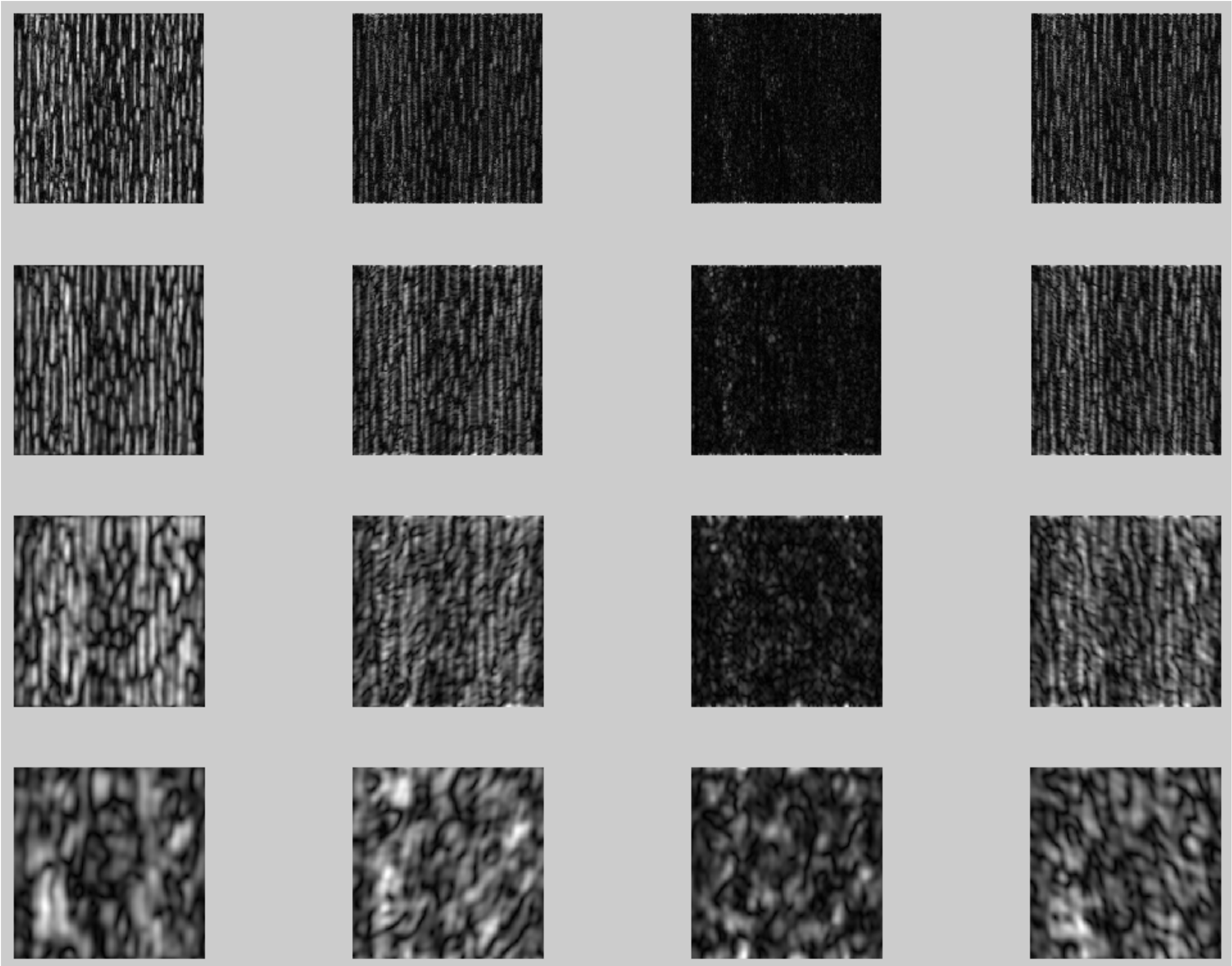
Spatial representation of of Gabor filter kernels

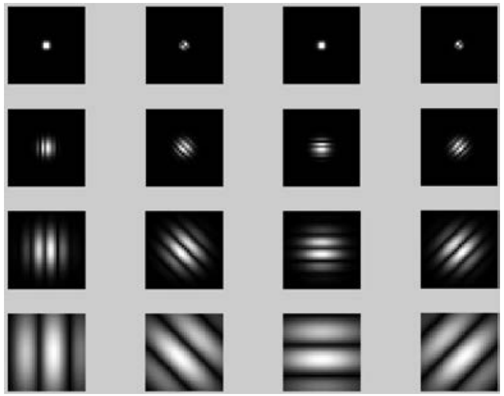
Gabor Filtering of Brodaz Texture D15

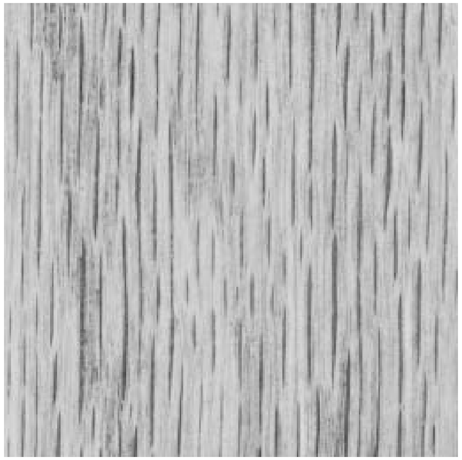




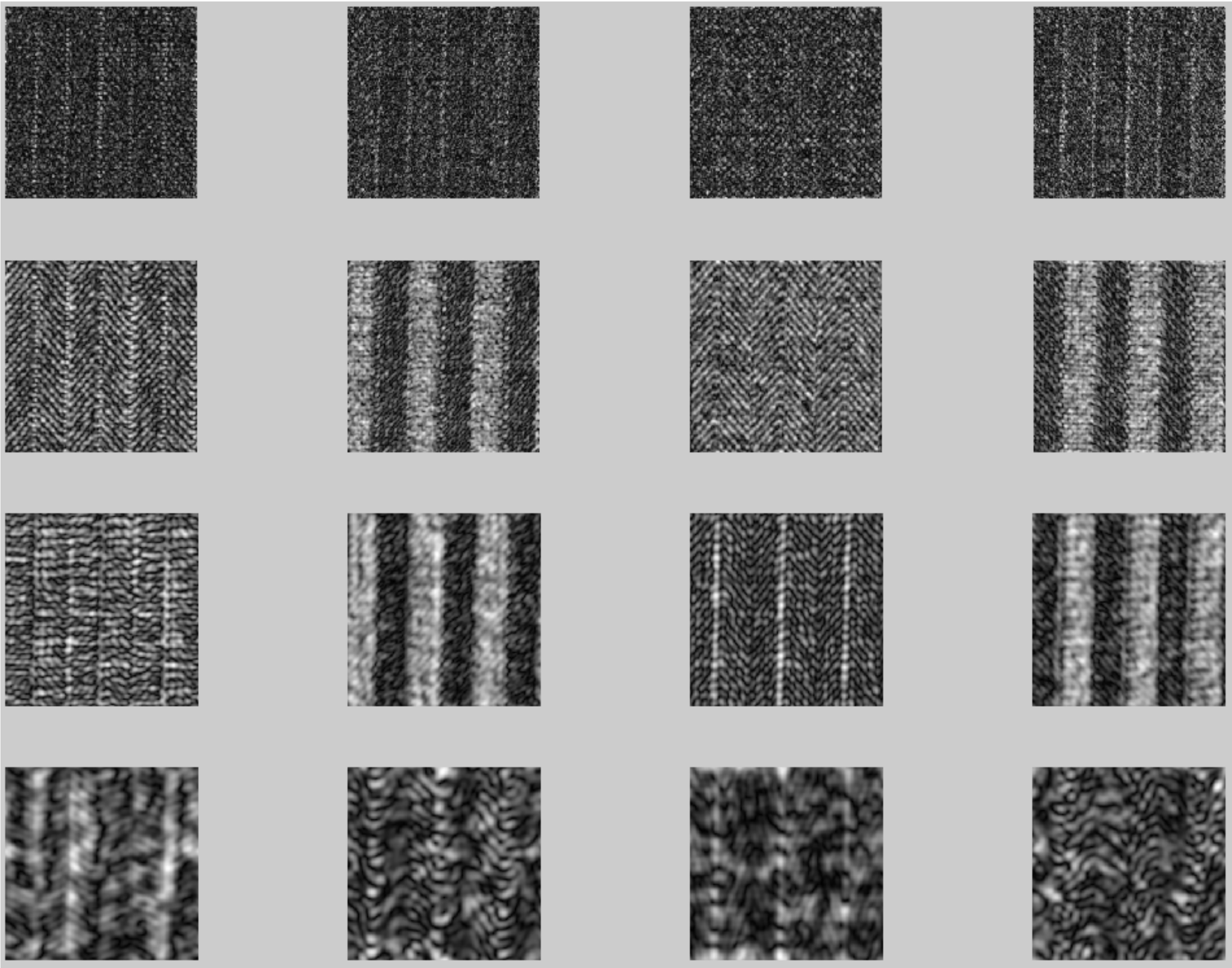
Gabor Filtering of Brodaz Texture D68

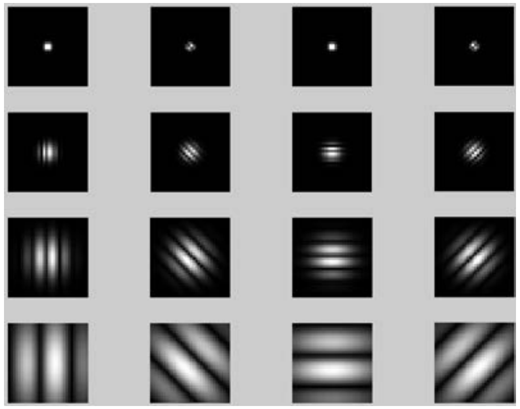


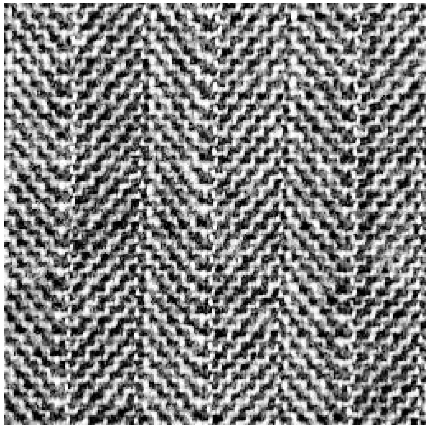




Gabor Filtering of Brodaz Texture D16







Gabor Filtering of Brodaz Texture D84