Lecture 33
PCA-based Face Recognition
ECEN 5283 Computer Vision

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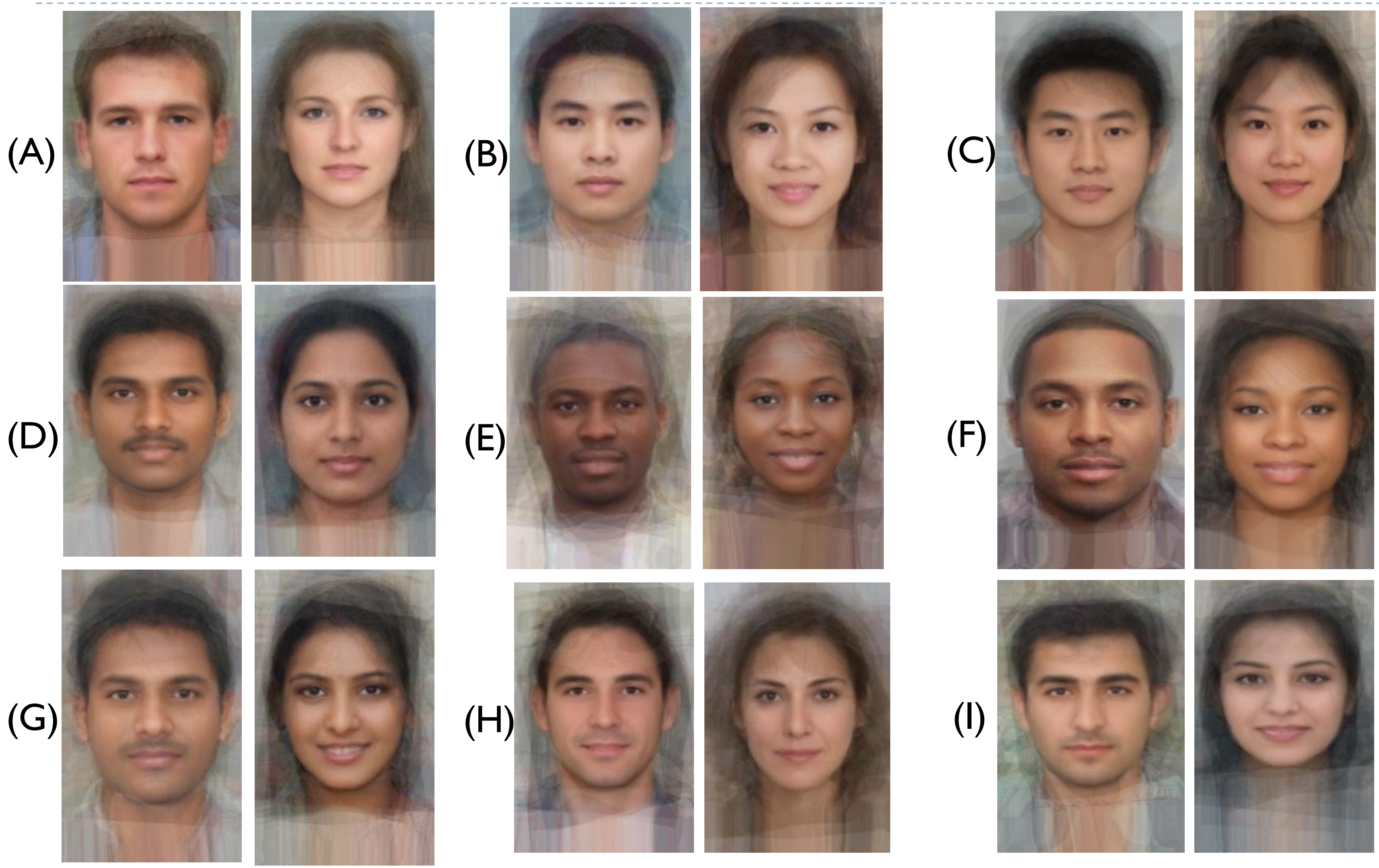
Goals

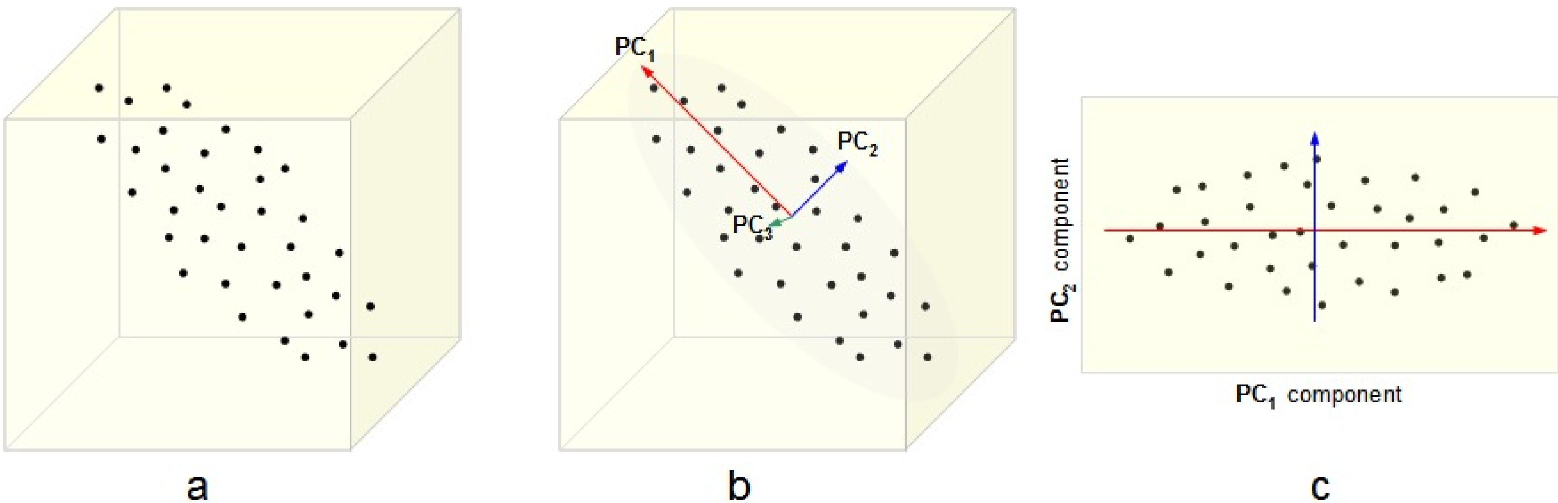
- To review the linear dimension reduction technique, Principal Component Analysis (PCA).
- To apply PCA for face recognition.

Average Male and Female Faces from Different Races (pmsol3.wordpress.com)



Principal Component Analysis (PCA)

- **PCA** *provides compact data representation*
  - PCA constructs a lower dimensional linear subspace that "best explains"(in the MSE sense) the variation of data points from their mean.
  - All data will be represented in this low-dimension feature space where high-level vision tasks can be efficiently accomplished.



PCA: Solution

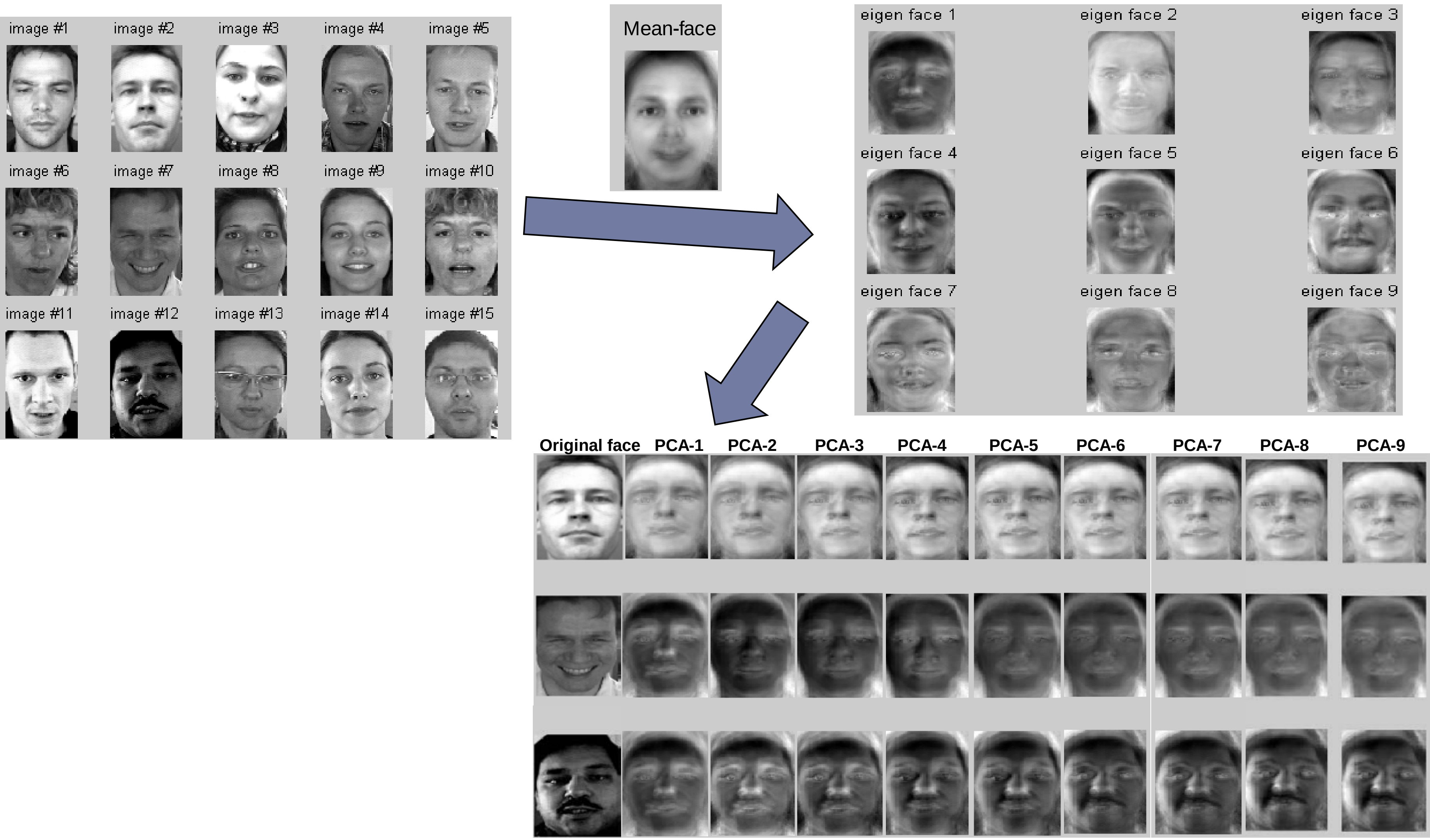
- We would like to maximize $\mathbf{v}^T \sum \mathbf{v}$ subject to $\mathbf{v}^T \mathbf{v} = 1$.

$$\Sigma = \mathbf{Q}\begin{bmatrix} \lambda_1^2 & 0 & 0 & 0 \\ 0 & \lambda_2^2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \lambda_q^2 \end{bmatrix}\mathbf{Q}^T \quad \text{with } 0 \le \lambda_1^2 \le \cdots \le \lambda_q^2 \text{ and } \mathbf{Q} = \begin{pmatrix} \mathbf{e}_1^T \\ : \\ \mathbf{e}_q^T \end{pmatrix}$$

- This is an eigenvalue problem, and the eigenvector corresponding to the largest eigenvalue $\mathbf{e}_q$ is the solution.
- The eigenvectors associated large eigenvalues reveals the underlying data distribution.
- The accuracy of PCA is determined by the ratio between the sum of top largest eigenvalues and that of all eigenvalues.

$$\frac{\sum_{k=p}^{q} \lambda_k^2}{\sum_{k=1}^{q} \lambda_k^2}$$

Eigen-face for Face Recognition



Face Recognition: Off-line Training

- Step 1: Collect a set of images of m persons, reflecting *n* variations in expression, pose and lighting

$$\{\mathbf{I}_j^k \mid j = 1,\ldots m, k = 1,\ldots,n\};$$

- Step 2: Compute the mean μ and eigenfaces $\{\mathbf{u}_1,\ldots,\mathbf{u}_p\}$ via PCA that construct a low-dimensional subspace V_P .

- Step 3: For the j*th* person in the database, calculate the corresponding representative vector in the subspace V_P spanned by the eigenfaces

$$\alpha_j^i = \frac{1}{n}\sum_{k=1}^{n}\left((\mathbf{I}_j^k - \mu), \mathbf{u}_i\right) \rightarrow \mathbf{w}_j = \left(\alpha_j^1 \quad \alpha_j^2 \quad \ldots\ldots \quad \alpha_j^p\right)$$

p-dimensional representation of the jth person's face

  - The representative image for the j*th* person is $\hat{\mathbf{I}}_j = \left(\sum_{i=1}^{p}\alpha_j^i\mathbf{u}_i\right) + \boldsymbol{\mu}$

Face Recognition: Online Recognition

- Step 4: Compute the projection of an new image $\mathbf{I}_t$ on to V_p.

$$\beta_t^i = \left((\mathbf{I}_t - \mu), \mathbf{u}_i\right) \rightarrow \mathbf{w}_t = \begin{pmatrix}\beta_t^1 & \beta_t^2 & \ldots\ldots & \beta_t^p\end{pmatrix} \rightarrow \hat{\mathbf{I}}_t = \left(\sum_{i=1}^{p} \beta_t^i \mathbf{u}_i\right) + \boldsymbol{\mu}$$

- Step 5. If the distance $\left|\hat{\mathbf{I}}_t - \mathbf{I}_t\right|$ is greater than a pre-set threshold, classify the image as "non-face".

- Step 6. Otherwise, if the minimum distance $d_k = \left|\mathbf{w}_t - \mathbf{w}_k\right|$ between the projection of the new image and the known representative is smaller than some pre-set threshold, classify the image as "person number k"

$$k = \arg_k \min d_k = \arg_k \min\left|\mathbf{w}_t - \mathbf{w}_k\right|$$

- Step 7. In the remaining case, classify the image as "unknown".

More discussion

Reconstructed face image of the given unknown person

Representative face image of person k

$$e_k = \left| \hat{\mathbf{I}}_t - \hat{\mathbf{I}}_k \right|$$

$$\hat{\mathbf{I}}_t = \left(\sum_{i=1}^{p} \beta_t^i \mathbf{u}_i \right) + \boldsymbol{\mu} \rightarrow \mathbf{w}_t = \begin{pmatrix} \beta_t^1 & \beta_t^2 & \ldots\ldots & \beta_t^p \end{pmatrix}$$

$$\hat{\mathbf{I}}_j = \left(\sum_{i=1}^{p} \alpha_j^i \mathbf{u}_i \right) + \boldsymbol{\mu} \rightarrow \mathbf{w}_j = \begin{pmatrix} \alpha_j^1 & \alpha_j^2 & \ldots\ldots & \alpha_j^p \end{pmatrix}$$

$$= \left| \sum_{i=1}^{p} (\beta_t^i - \alpha_k^i) \mathbf{u}_i \right| \qquad (\mathbf{u}_i, \mathbf{u}_j) = \begin{cases} 1 & i = j \\ 0 & \text{Otherwise} \end{cases}$$

$$= \left(\sum_{i=1}^{p} (\beta_t^i - \alpha_k^i)^2 \right)^{1/2} = \left| \mathbf{w}_k - \mathbf{w}_t \right| = d_k$$

Recognition by Rank

- Usually, face recognition is done by a rank. Given the top K candidates with the best match, if a face of the correct identity is included, then the recognition is a success.

View-based PCA

View-1 PCA subspace

View-2 PCA subspace

View-3 PCA subspace

View-4 PCA subspace

View-5 PCA subspace

Pentland, B. Moghaddam, and T. Starner. View-based and modular eigenspaces for face recognition. In Proc. of IEEE CVPR, 1994

How to estimate the view?

View-1 PCA subspace

View-2 PCA subspace

Test image

View-3 PCA subspace

View-4 PCA subspace

View-5 PCA subspace

Just to find out which subspace provides the smallest reconstruction error.