Lecture 7
Linear Approach to Camera Calibration
ECEN 5283 Computer Vision

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Goal

- To use a linear approach to calibrate a camera.
- To be ready for Project #1.

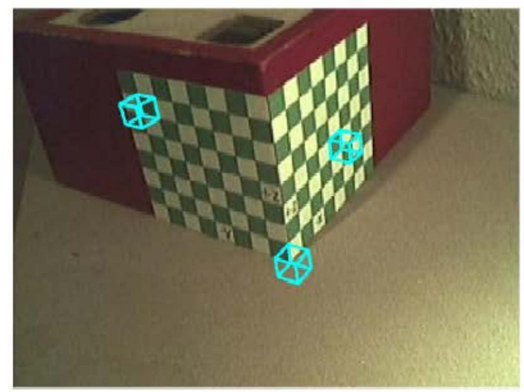


Figure 1: An image of a calibration rig, left with calibration points, right with cubes to check the calibration.

Camera Calibration

- We decompose the calibration process into two steps.
  - The computation of perspective projection matrix *M*.
  - The estimation of the intrinsic and extrinsic parameters.
    - Five intrinsic parameters $\alpha,\ \beta,\ u_0,\ v_0, \theta$
    - Six extrinsic ones (three angles and three coordinates of t).

$$\mathbf{p} = \frac{1}{z} M\mathbf{P} \Longrightarrow \mathbf{M} = \mathbf{K}\begin{pmatrix}\mathbf{R} & \mathbf{t}\end{pmatrix}$$

- There are 11 (5+6) parameters, and n>5 points generally do not admit a common root (why?), and we have to use linear least squares error solution.

Camera Projection Matrix

- It is important to understand the depth z is not independent of M and P.

$$\mathbf{p} = \frac{1}{z} M\mathbf{P} \quad \text{where } \mathbf{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \text{ and } M = \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \frac{1}{z} \begin{pmatrix} \mathbf{m}_1^T \mathbf{P} \\ \mathbf{m}_2^T \mathbf{P} \\ \mathbf{m}_3^T \mathbf{P} \end{pmatrix} = \frac{1}{z} \begin{pmatrix} \mathbf{m}_1 \cdot \mathbf{P} \\ \mathbf{m}_2 \cdot \mathbf{P} \\ \mathbf{m}_3 \cdot \mathbf{P} \end{pmatrix} \Rightarrow \begin{cases} z = \mathbf{m}_3 \cdot \mathbf{P} \\ u = \dfrac{\mathbf{m}_1 \cdot \mathbf{P}}{\mathbf{m}_3 \cdot \mathbf{P}} \\ v = \dfrac{\mathbf{m}_2 \cdot \mathbf{P}}{\mathbf{m}_3 \cdot \mathbf{P}} \end{cases}$$

Estimation of the Projection Matrix

- From previous slide, we have

$$\begin{cases} u = \dfrac{\mathbf{m}_1 \cdot \mathbf{P}}{\mathbf{m}_3 \cdot \mathbf{P}} \\ v = \dfrac{\mathbf{m}_2 \cdot \mathbf{P}}{\mathbf{m}_3 \cdot \mathbf{P}} \end{cases}$$

Given a 3D - 2D point pair, $\mathbf{P}_i \rightarrow \begin{pmatrix} u_i \\ v_i \end{pmatrix}$

Then, we have $\begin{cases} (\mathbf{m}_1 - u_i \mathbf{m}_3) \cdot \mathbf{P}_i = 0 \\ (\mathbf{m}_2 - v_i \mathbf{m}_3) \cdot \mathbf{P}_i = 0 \end{cases}$

$$\begin{pmatrix} \mathbf{P}_i^{\mathrm{T}} & \mathbf{0}^{\mathrm{T}} & -u_i \mathbf{P}_i^{\mathrm{T}} \\ \mathbf{0}^{\mathrm{T}} & \mathbf{P}_i^{\mathrm{T}} & -v_i \mathbf{P}_i^{\mathrm{T}} \end{pmatrix}_{2 \times 12} \begin{pmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{pmatrix}_{12 \times 1} = \mathbf{0}$$

Estimation of the Projection Matrix (Cont'd)

- Given *n* 3D-2D point pairs for calibration, then we have

$$\mathbf{Q} = \begin{pmatrix} \mathbf{P}_1^T & \mathbf{0}^T & -u_1\mathbf{P}_1^T \\ \mathbf{0}^T & \mathbf{P}_1^T & -v_1\mathbf{P}_1^T \\ \cdots & \cdots & \cdots \\ \mathbf{P}_n^T & \mathbf{0}^T & -u_n\mathbf{P}_n^T \\ \mathbf{0}^T & \mathbf{P}_n^T & -v_n\mathbf{P}_n^T \end{pmatrix}_{2n\times 12} \quad \text{and} \quad \mathbf{m} = \begin{pmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{pmatrix}_{12\times 1}$$

Q is composed the 3D (homogeneous) and 2D coordinates of the given points.

$$\boxed{\mathbf{Qm} = \mathbf{0}} \Rightarrow \boxed{\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \left|\mathbf{Qm}\right|^2}$$

The solution can be achieved by solving the eigenvalue problem of $\mathbf{Q}^T\mathbf{Q}$.

Estimation of the Intrinsic and Extrinsic Parameters

- Once the project matrix M, its expression in terms of the camera intrinsic and extrinsic parameters can be used to recover these parameters as follows.

$$\mathbf{M} = \begin{pmatrix}\mathbf{A} & \mathbf{b}\end{pmatrix} \text{ with } \mathbf{a}_1^T, \mathbf{a}_2^T, \text{ and } \mathbf{a}_3^T \text{ denoting the rows of } \mathbf{A}.$$

$$\mathbf{K} = \begin{pmatrix} \alpha & -\alpha\cot\theta & u_0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_0 \\ & 0 & 1 \end{pmatrix} \qquad \rho\begin{pmatrix}\mathbf{A} & \mathbf{b}\end{pmatrix} = \mathbf{K}\begin{pmatrix}\mathbf{R} & \mathbf{t}\end{pmatrix} = \begin{pmatrix}\mathbf{K}\mathbf{R} & \mathbf{K}\mathbf{t}\end{pmatrix}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\theta = \frac{\pi}{2}$$

$$\mathbf{K} = \begin{pmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ & 0 & 1 \end{pmatrix}$$

$$R = \begin{pmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{pmatrix} \text{ and } t = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

Estimation of the Intrinsic Parameters

$$\mathbf{K} = \begin{pmatrix} \alpha & -\alpha\cot\theta & u_0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R = \begin{pmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{pmatrix}$$

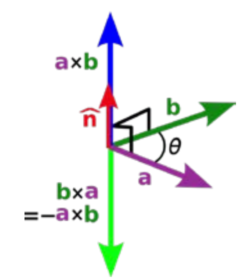
$$\rho\mathbf{A} = \mathbf{K}\cdot\mathbf{R} \Leftrightarrow \rho\begin{pmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{pmatrix} = \begin{pmatrix} \alpha\mathbf{r}_1^T - \cot\theta\mathbf{r}_2^T + u_0\mathbf{r}_3^T \\ \dfrac{\beta}{\sin\theta}\mathbf{r}_2^T + v_0\mathbf{r}_3^T \\ \mathbf{r}_3^T \end{pmatrix}$$

$$\begin{cases} \rho = \varepsilon / |\mathbf{a}_3| \\ u_0 = \rho^2(\mathbf{a}_1\cdot\mathbf{a}_3) \\ v_0 = \rho^2(\mathbf{a}_2\cdot\mathbf{a}_3) \end{cases}$$

($\varepsilon = 1$ or -1: image plan and scene are on the same or different sides)

$$\begin{cases} \cos\theta = -\dfrac{(\mathbf{a}_1\times\mathbf{a}_3)\cdot(\mathbf{a}_2\times\mathbf{a}_3)}{|\mathbf{a}_1\times\mathbf{a}_3||\mathbf{a}_2\times\mathbf{a}_3|} \\ \alpha = \rho^2|\mathbf{a}_1\times\mathbf{a}_3|\sin\theta \\ \beta = \rho^2|\mathbf{a}_2\times\mathbf{a}_3|\sin\theta \end{cases}$$

$$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$



$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ (inner product)

(outer product/cross product)

Estimation of the Extrinsic Parameters

$$\begin{cases} \mathbf{r}_1 = \dfrac{1}{|\mathbf{a}_2 \times \mathbf{a}_3|}(\mathbf{a}_2 \times \mathbf{a}_3) \\ \mathbf{r}_3 = \rho \mathbf{a}_3 \qquad\qquad \text{(rotation vector)} \\ \mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \end{cases}$$

$$\rho(\mathbf{A} \quad \mathbf{b}) = \mathbf{K}(\mathbf{R} \quad \mathbf{t}) \to \mathbf{K}\mathbf{t} = \rho\mathbf{b} \Rightarrow \mathbf{t} = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} = \rho \mathbf{K}^{-1}\mathbf{b} \quad \text{(shift vector).}$$

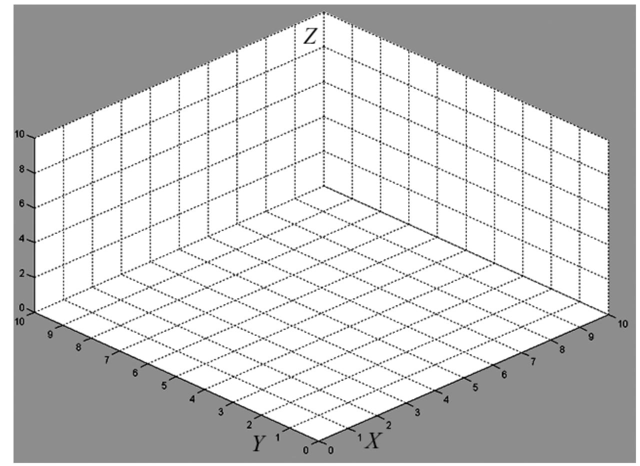
$$\mathbf{K} = \begin{pmatrix} \alpha & -\alpha \cot\theta & u_0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_0 \\ & 0 & 1 \end{pmatrix}$$

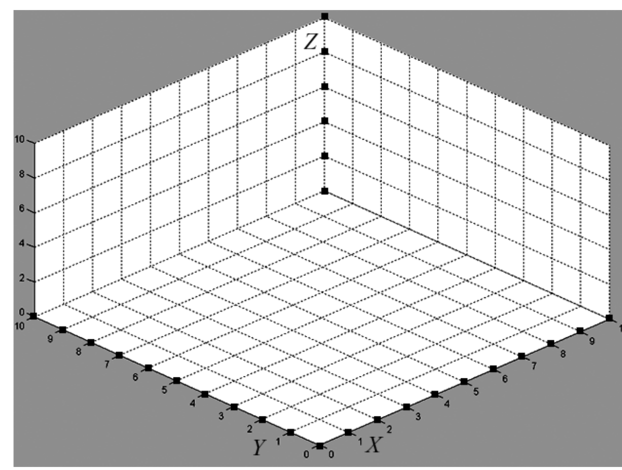
Project 1 (Due: Feb. 9)

- ***Observe.dat*** contains a set of 2D pixel coordinates in image "*test_image.bmp*"

- ***Model.dat includes*** a set of 3D coordinates in a world coordinate system

- It is suggested to first show these points on the image to ensure the correctness of the coordinate system in your program.

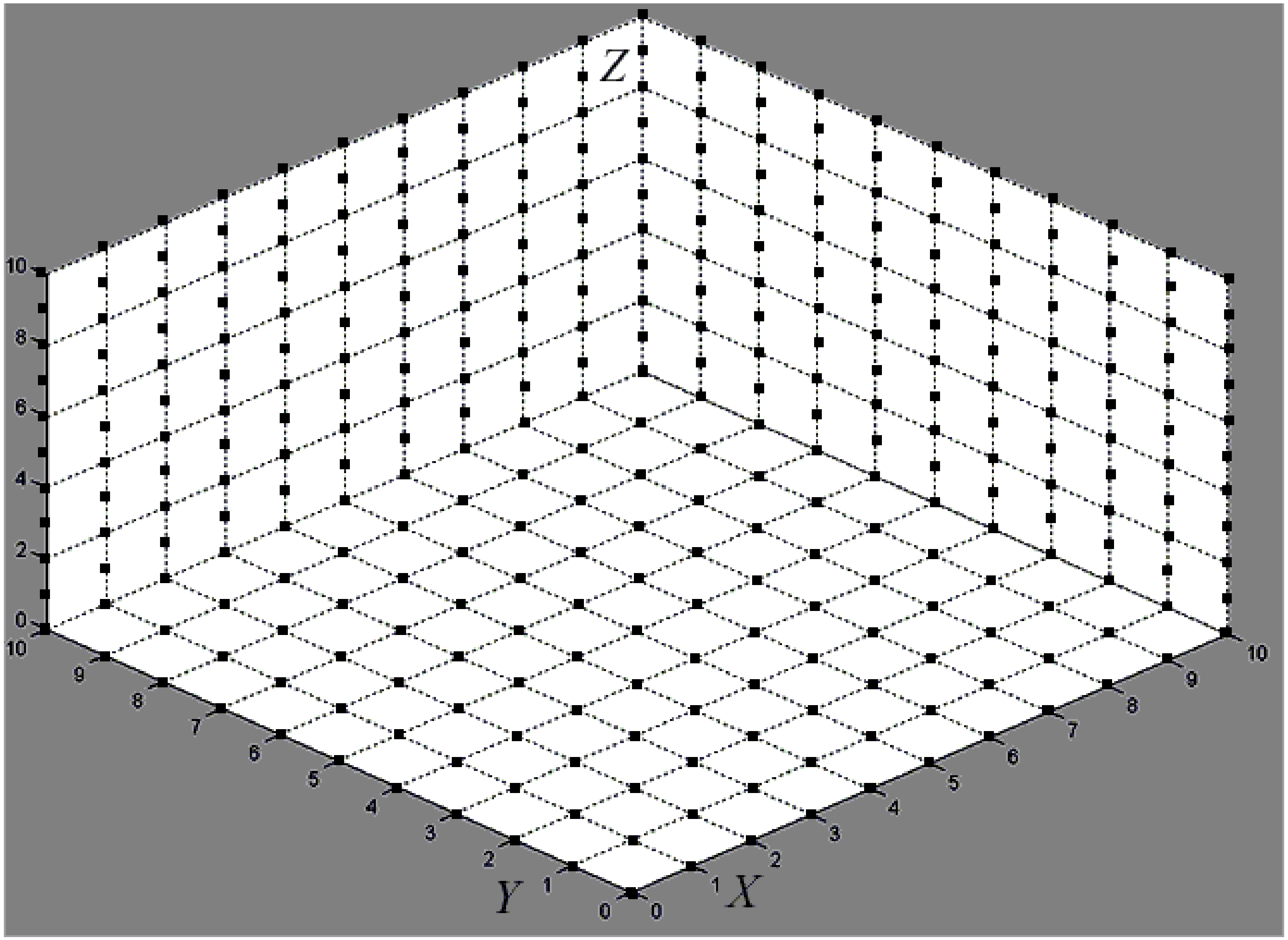




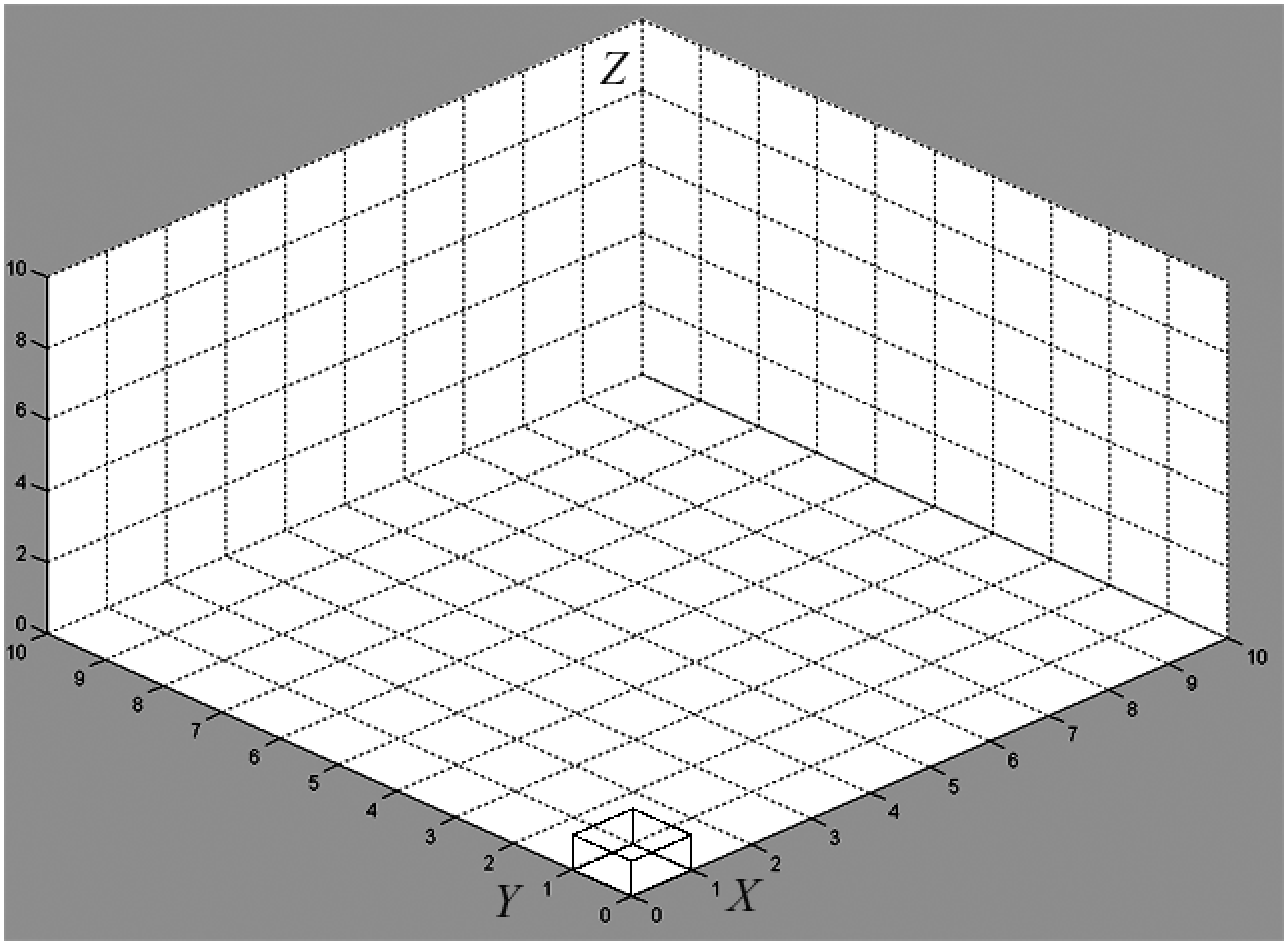
How to write a PPT report?

- **Project Objective** (1 page)
  - What is the objective of this project?
  - What kind of test data and tools have been given?
- **Technical Background and Implementation** (~5 pages)
  - What is the basic theory involved in this project?
  - Any major equations or any useful illustrations?
  - *(Note: It is OK to copy equations/illustrations from the handouts, but you will need to add some personal understanding and analysis to all technical details.)*
- **Experimental Results** (~5 pages)
  - What do see in your experiment and what do they mean?
  - Try different parameter settings and what is the optimal one, why?
- **Discussion and Conclusion** (1 page)
  - What you have learned? Any question you may have?
- **Appendix**
  - Matlab source code with comments.

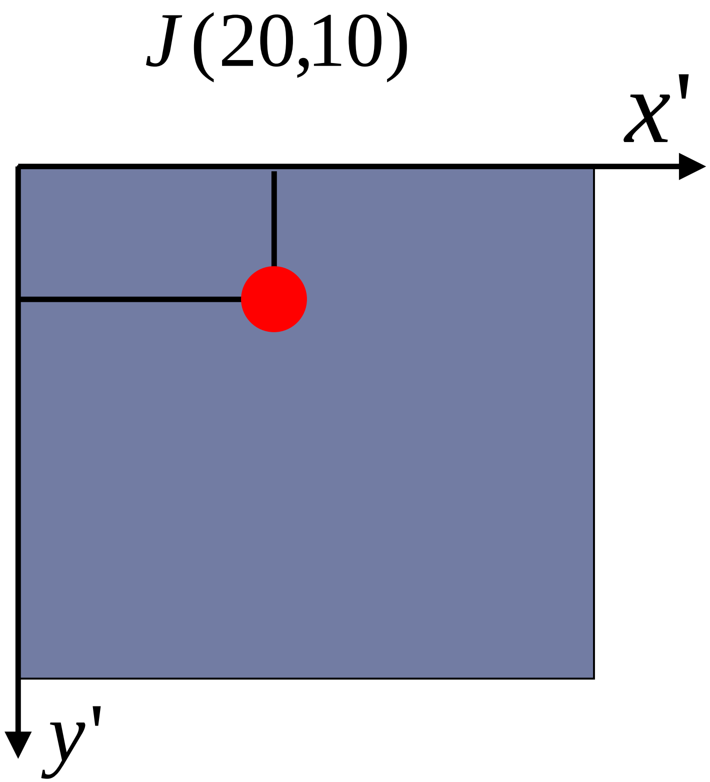
Expected Result (1)

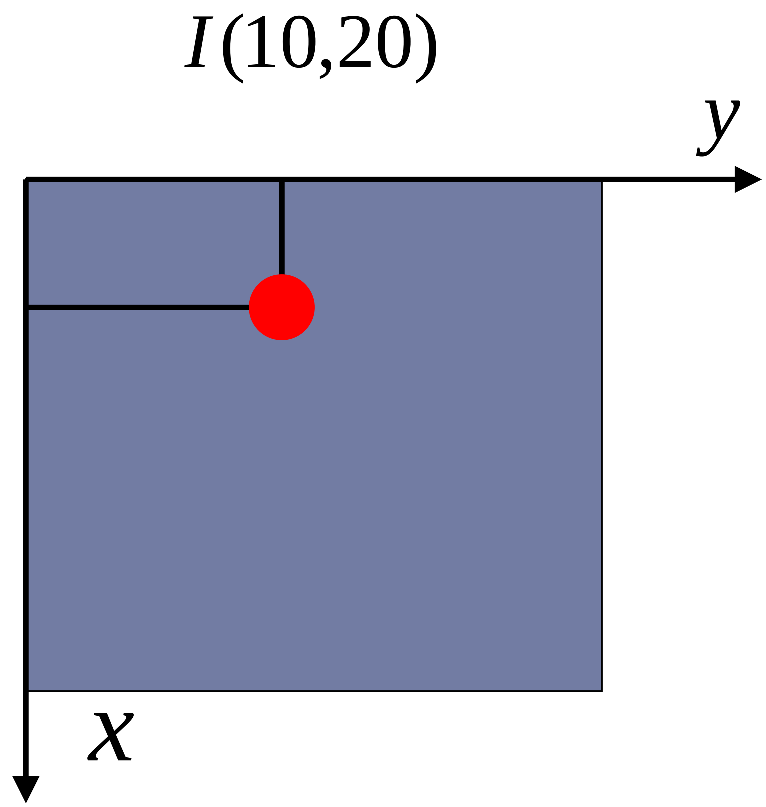


Expected Result (2)



M-file Programming:
Matlab 2-D coordinate systems





The 2D coordinate system where the 2D points ("observe.dat") are collected.

The 2D coordinate system used in Matlab for images.

M-file Programming: Show an image

```
I=imread('test_image.bmp');   % read an image into I
[Ix Iy]=size(I);              % the dimension of image I (#row, #column)
figure(1), imshow(I);         % show image I in figure 1
load observe.dat              % read the 2D observation data
[On Ot]=size(observe)         % the dimension of observe data
for i=1:On
    mx=observe(i,1);           % read the x coordinate of each point
    my=observe(i,2);           % read the y coordinate of each point
    for j=mx-2:mx+2            % Mark each point in the image
          for k=my-2:my+2
               I(k,j)=0;
          end
    end
end
figure (2), imshow(I);        % show the marked image in figure 2
```

M-file Programming: Video Creation

- T=imread('test_image.bmp');
- for i=1:frame_num
  - L=T;
  - % some processing in L
  - Frame(:,:,1)=L; % Red channel
  - Frame(:,:,2)=L; % Blue channel
  - Frame(:,:,3)=L; % Green channel
  - Mo(i)=im2frame(Frame)
- end
- Movie2avi(Mo,’filename.avi’);