Lecture 26
Monte Carlo Methods
ECEN 5283 Computer Vision

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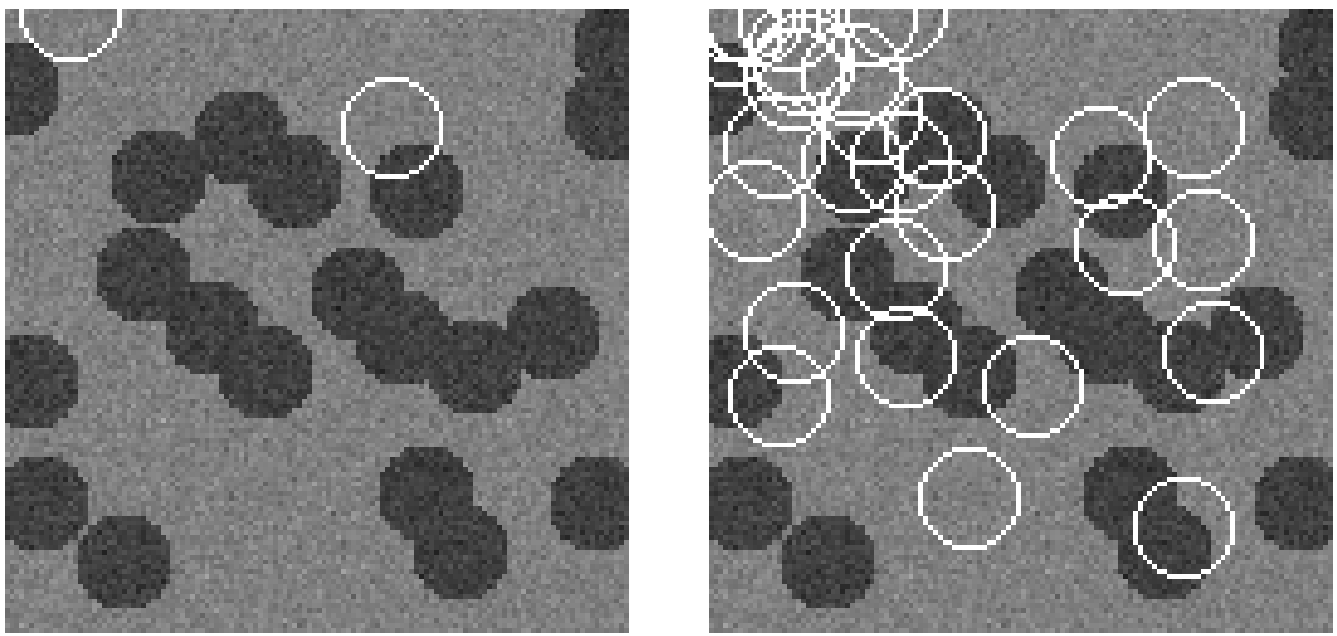
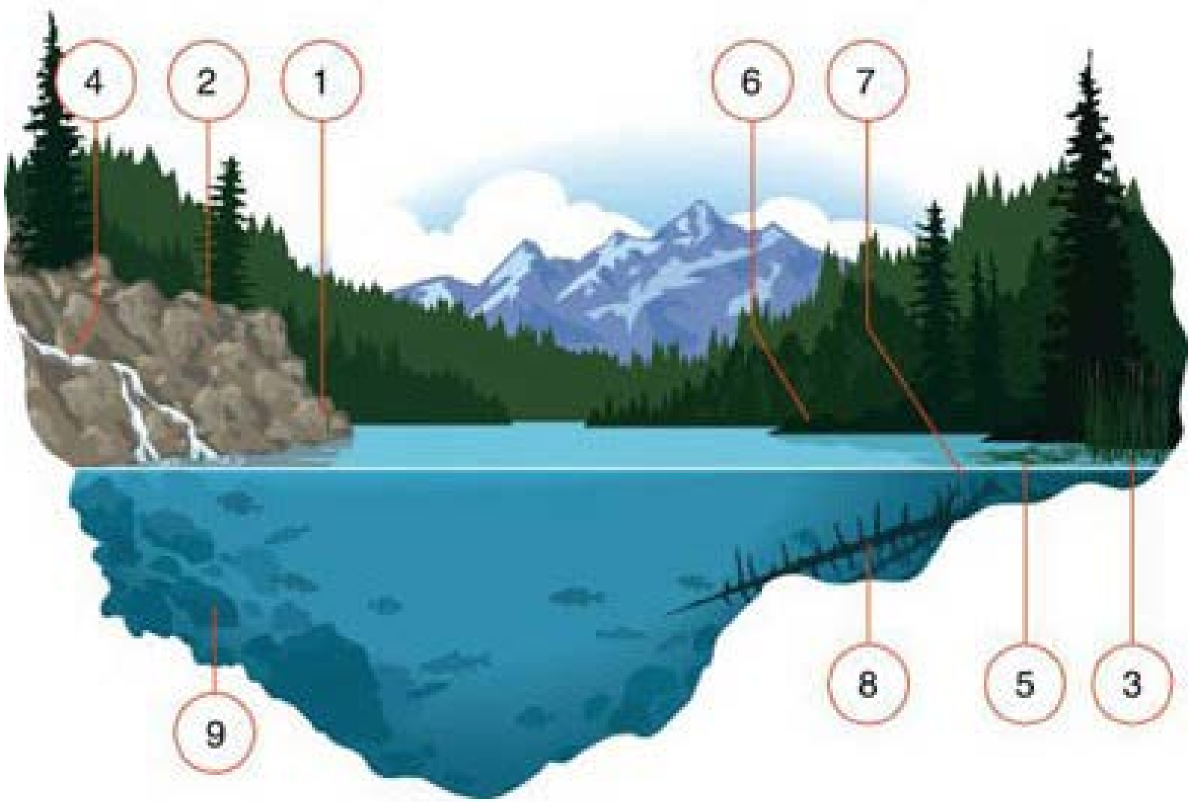
Goal

- To have a brief overview of Monte Carlo methods.
- To discuss two major issues for Monte Carlo methods.

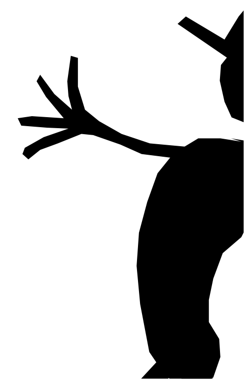


Why Monte Carlo methods?

- We want to develop a depth map for a big lake!
- The only thing we have is a boat and a sonar distance meter.



That is impossible!



Why Monte Carlo …

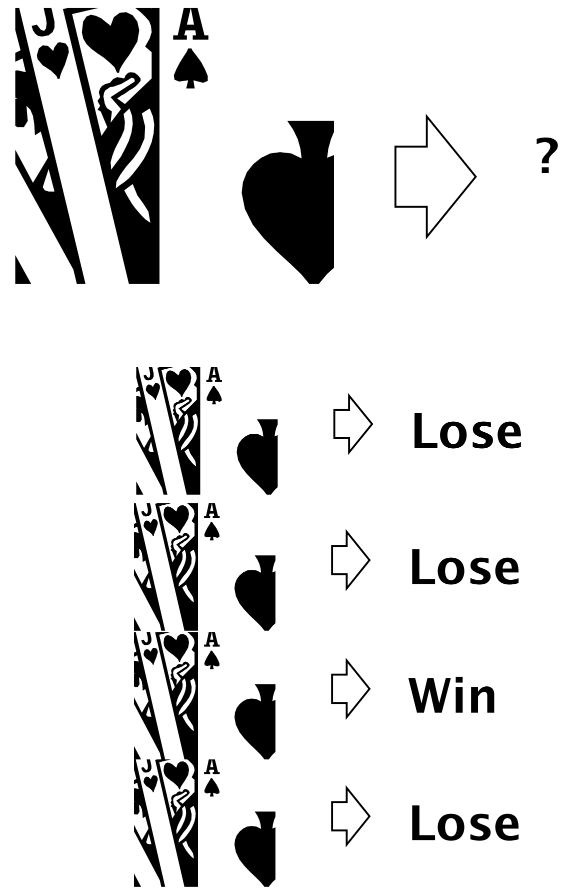
- **Monte Carlo** is a small hillside town in Monaco (near Italy) with casino since 1865 like Los Vegas in the US. It was picked by a physicist Fermi (Italian born American) who was among the first using the sampling techniques in his effort building the first manmade nuclear reactors in 1942.

WWW of Monte Carlo Methods

- **What?**
  - A Monte Carlo method is a computational algorithm that relies on repeated random sampling to compute its results.
- **When?**
  - Monte Carlo methods are often used when simulating physical and mathematical systems (which are principled) with random samples.
- **Why?**
  - Monte Carlo methods are most suited to calculation by a computer.
  - Monte Carlo methods tend to be used when it is infeasible or impossible to compute an exact result with a deterministic algorithm.

Monte Carlo Example

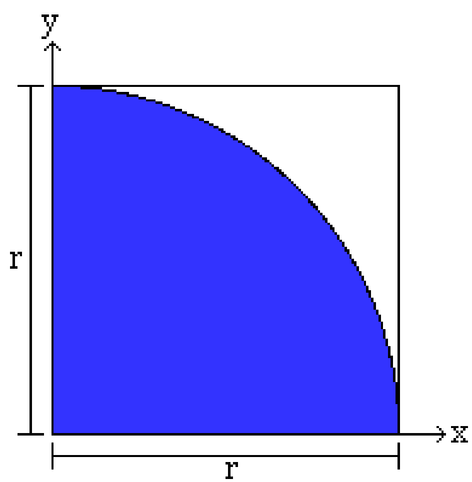
- Consider the game of solitaire: what's the chance of winning with a properly shuffled deck?
  - Problem: Hard to compute analytically because winning or losing depends on a complex procedure of reorganizing cards
  - Insight: Why not just *play a few hands*, and see empirically how many do in fact win?
- More generally, it can approximate a probability density function using only samples from that density.



Chance of winning is 1 in 4!

Steps of Monte Carlo Methods

- There is no single Monte Carlo method; instead, the term describes a large and widely-used class of approaches that tend to follow a particular pattern:
  - Define a domain of possible inputs.
  - Generate inputs randomly from the domain, and perform a deterministic computation on them.
  - Aggregate the results of the individual computations into the final result.



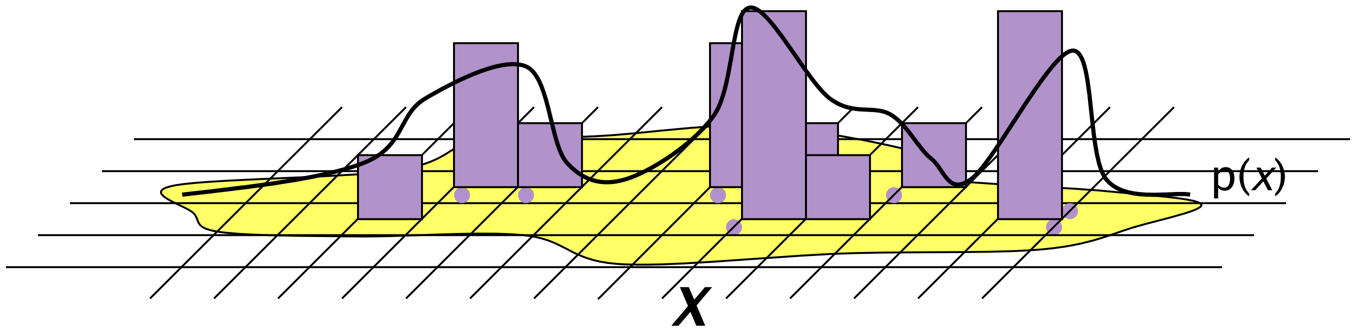
$$\frac{\#\text{ darts hitting shaded area}}{\#\text{ darts hitting inside square}} = \frac{\frac{1}{4}\pi r^2}{r^2} = \frac{1}{4}\pi$$

or

$$\pi = 4\,\frac{\#\text{ darts hitting shaded area}}{\#\text{ darts hitting inside square}}$$

Monte Carlo Principle

- Given a very large set X and a distribution $p(x)$ over it
- We draw i.i.d. a set of N samples
- Samples are rejected or accepted (or weighted) after evaluation.
- We can then approximate the distribution using accepted or weighted samples.



$$p_N(x) = \frac{1}{N}\sum_{i=1}^{N} 1(x^{(i)} = x) \underset{N\to\infty}{\longrightarrow} p(x)$$

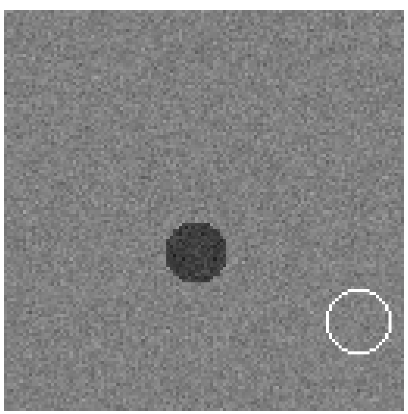
Why do we care a distribution?

- Given measurement $\boldsymbol{y}$, $\boldsymbol{x}(\boldsymbol{y})$ represents a function of $\boldsymbol{y}$. The minimum mean square estimator (MMSE) of X is the conditional mean $\widehat{\boldsymbol{x}}_{MMSE} = E[x|y]$.

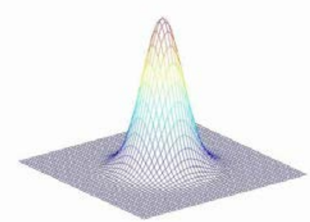
$$\hat{\mathbf{x}}_{MMSE} = E(\mathbf{x} \mid \mathbf{y}) = \int \mathbf{x} \cdot p(\mathbf{x} \mid \mathbf{y}) d\mathbf{x}$$

$\mathbf{y}$: a given image with an object

$\mathbf{x}$: the location of the object



The goal of statistical inference is to Find the posterior density function



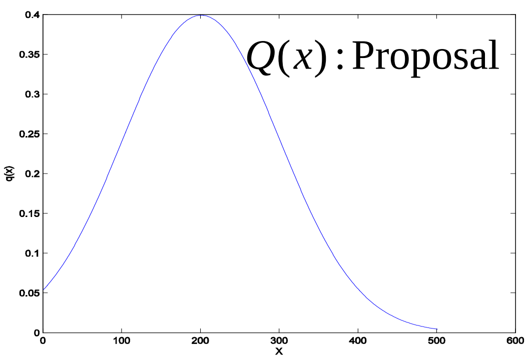
$p(\mathbf{x} \mid \mathbf{y})$

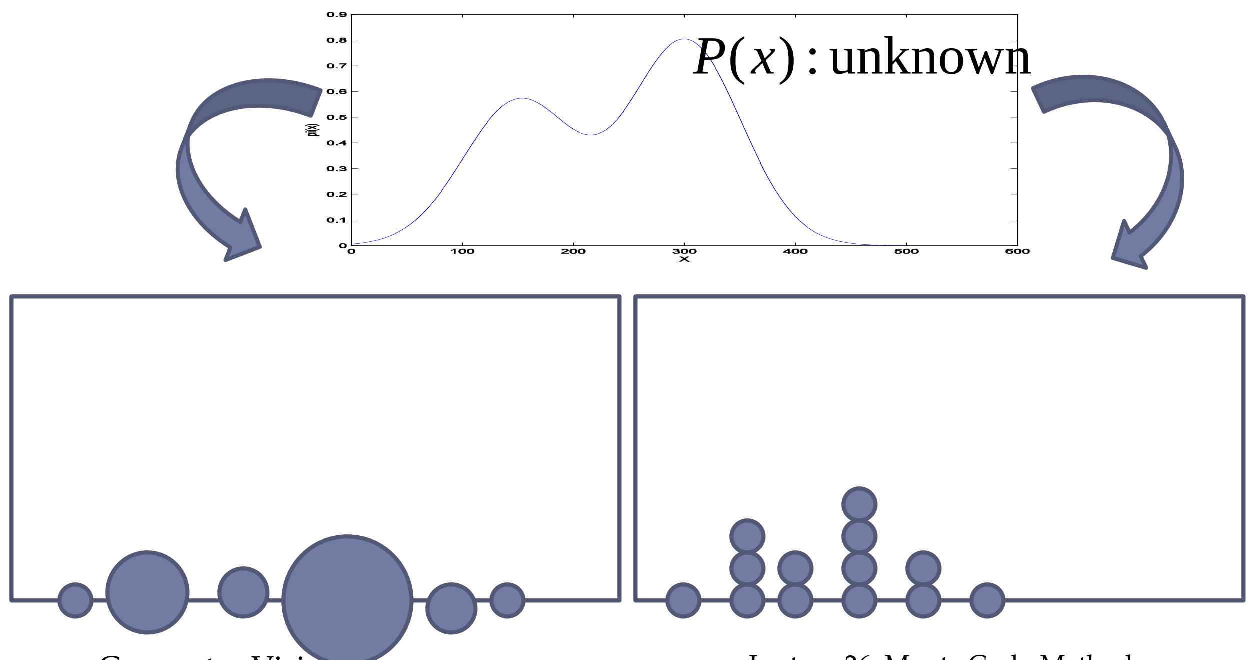
Then the MMSE of $\mathbf{x}$ can be obtained

$$\hat{\mathbf{x}}_{MMSE} = E(\mathbf{x} \mid \mathbf{y})$$

Two Major Issues of MC methods

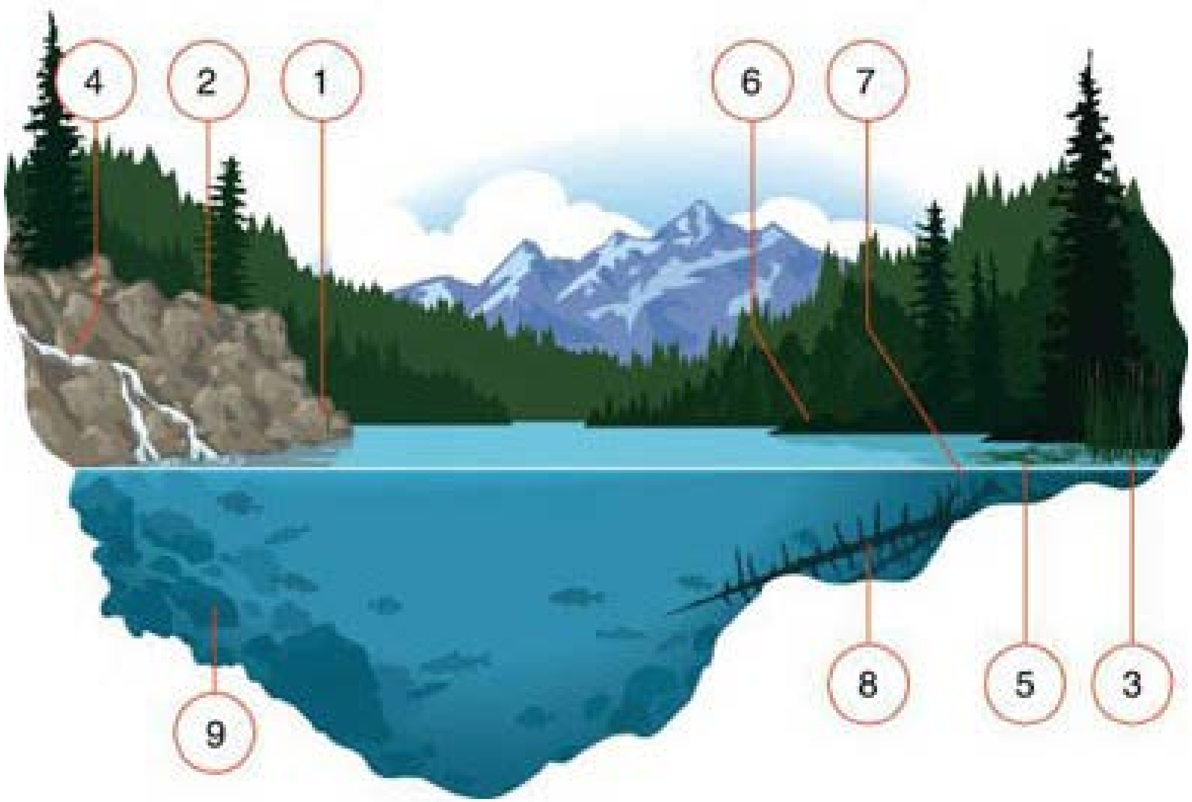
- There are two major issues of implementing a MC method:
  - Find a proposal from which we can easily draw samples to approximate the unknown distribution.
  - Evaluate the samples to compute a weight for each sample or to keep some of good samples, so that the unknown distribution can be approximated by either weighted samples or samples of a same weight.





Just like this

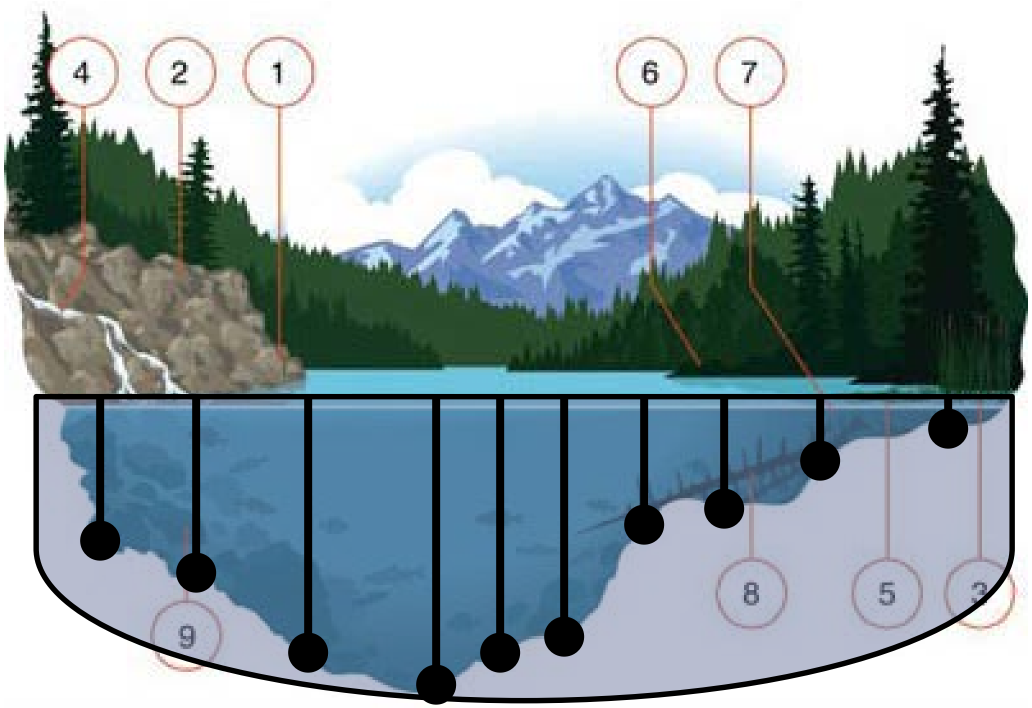
- We want to develop a depth map for a big lake!
- The only thing we have is a boat and a sonar distance meter.



That is impossible!

However, we can do this…

- Let’s find an ideal distribution where we can draw samples easily. Then we can evaluate the sample by going to the lake and measuring the depth…



However, where to go to to collect depth samples?