Lecture 10
Multiscale Image Representation
ECEN 5283 Computer Vision

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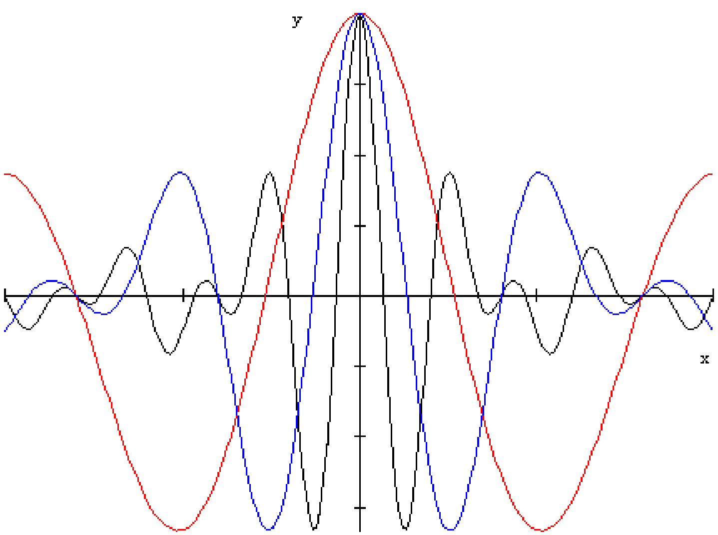
Goals

- To introduce the concepts of multiscale image analysis
- To creeate the Gaussian pyramid that supports efficient multiscale image processing
- To discuss the Laplacian pyramid that plays a complementary role to the Gaussian pyramid

Image Scales

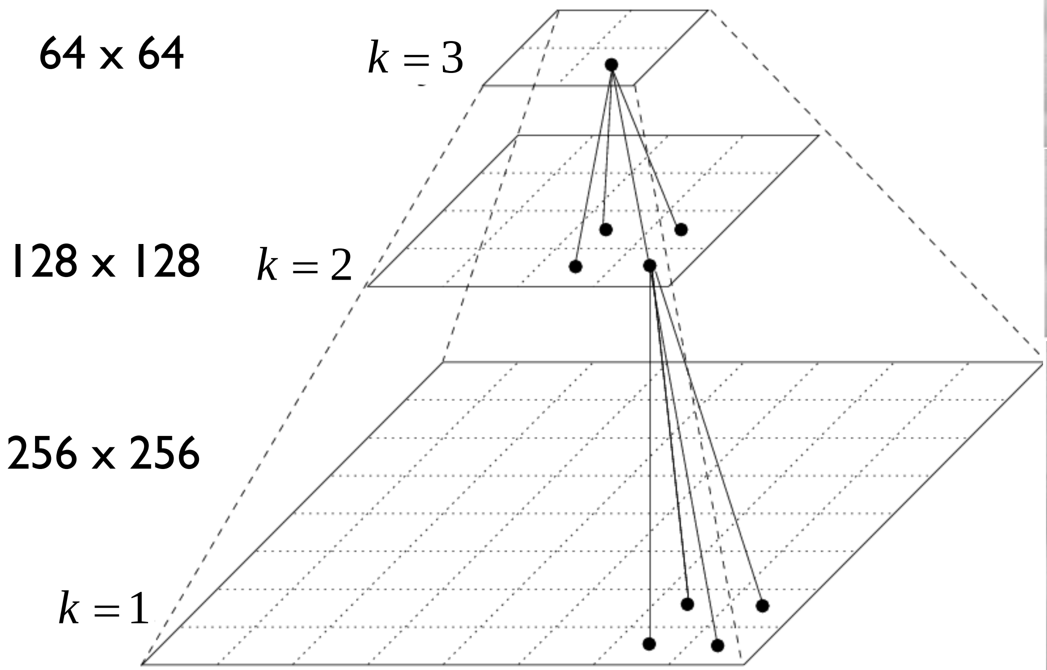
- Images look quite different at different scales.
  - Different scales are coded in terms of the response of a set of filters that operate at different number of pixels.
  - A small scale filter is used to find details while a large scale filter can find the major structures.



Multi-scale filters

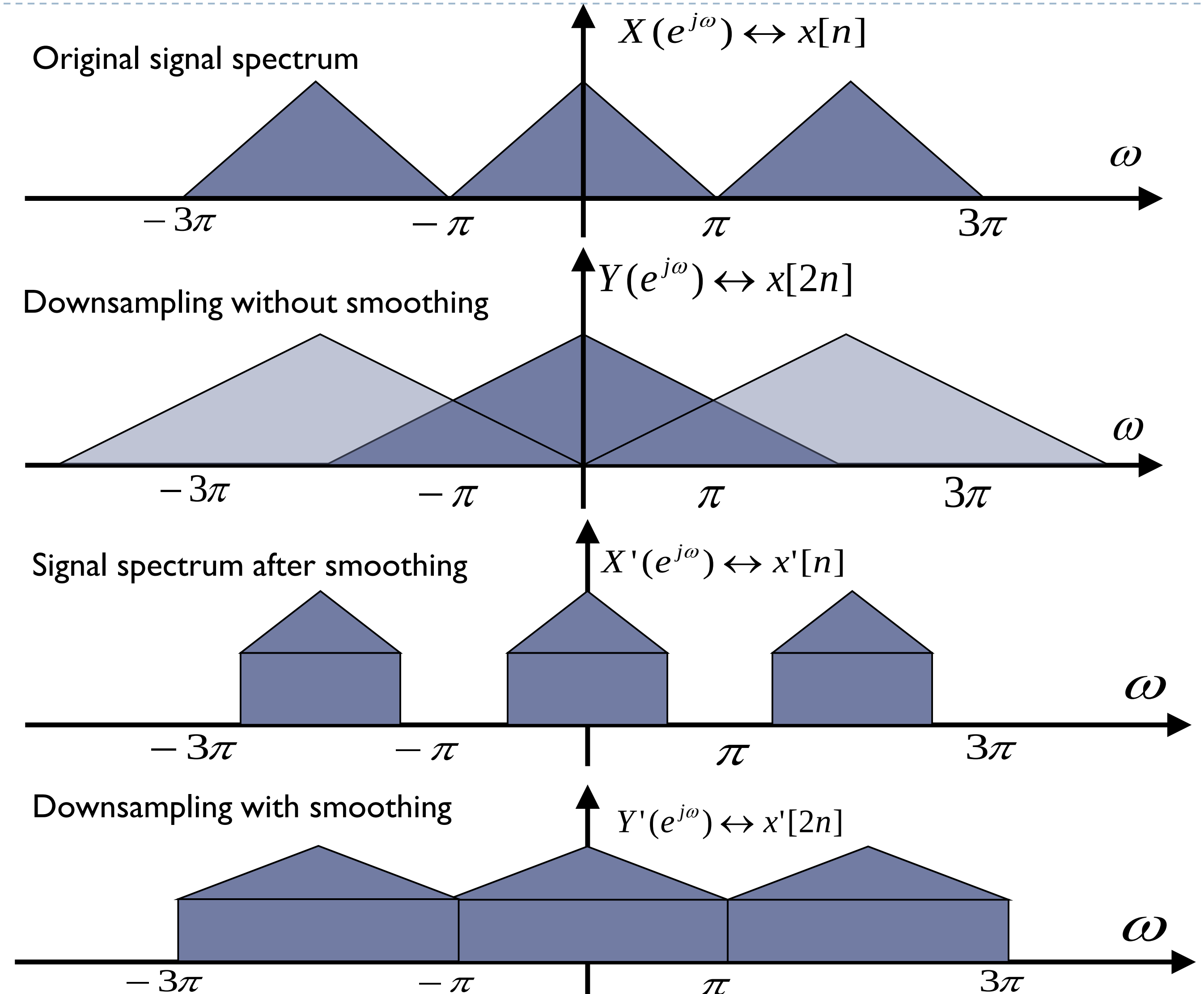
Image Pyramid: Why?

- Instead of using a large scale filter, we can use a single small scale filter to a smoothed and re-sampled version of an image.



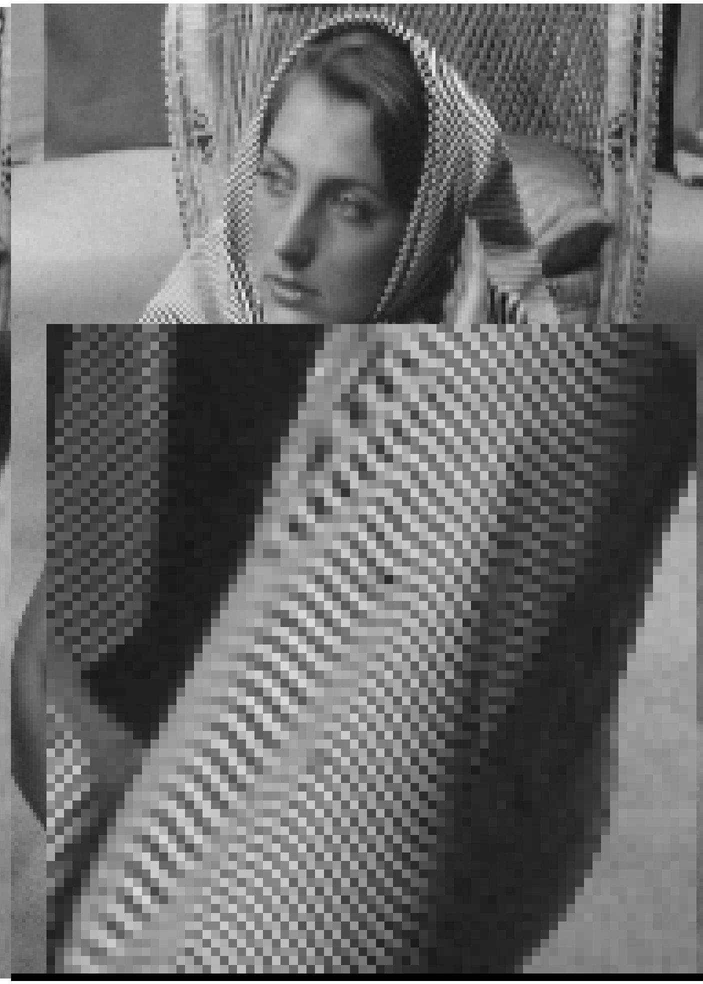
$$I_{k+1} = S\downarrow(G**I_k) = S\downarrow I_k^G \rightarrow I_{k+1}(i,j) = I_k^G(2i,2j)$$

Why smoothing before resampling?



Why smoothing before resampling?

Original Image



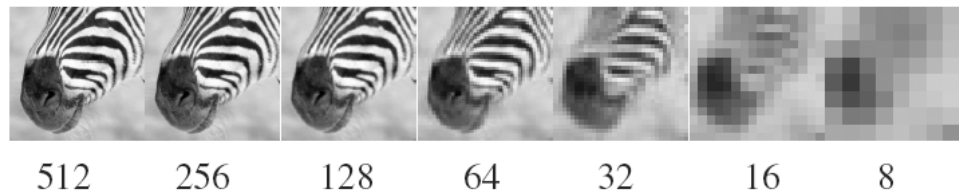
Subsampled image

Sampling with anti-aliasing

Gaussian Pyramid

- Gaussian pyramid is a collection of representation of an image at different scales or resolutions.

$$P_{\text{Gaussian}}(I)_1 = I \quad \text{(finest scale)}$$



$$P_{\text{Gaussian}}(I)_{n+1} = S\downarrow(G_{\delta} ** P_{\text{Gaussian}}(I)_n)$$

- Applications
  - Coarse-to-fine search
  - Multiscale image segmentation

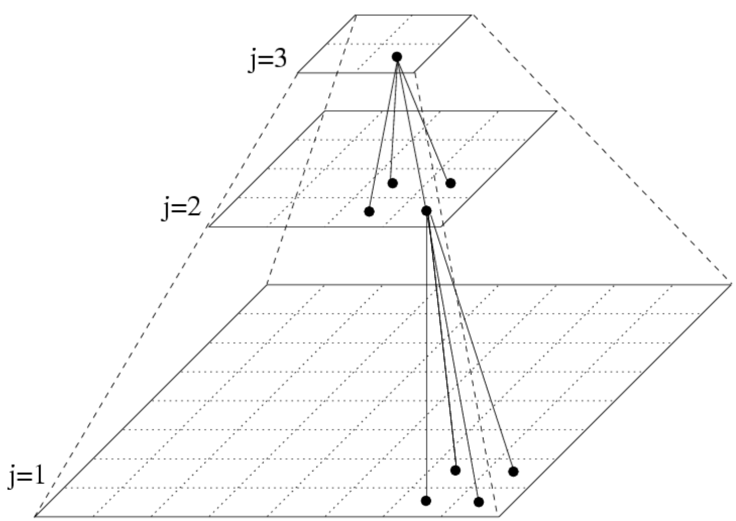
Gaussian Pyramid Applications: Hierarchical Pattern Matching

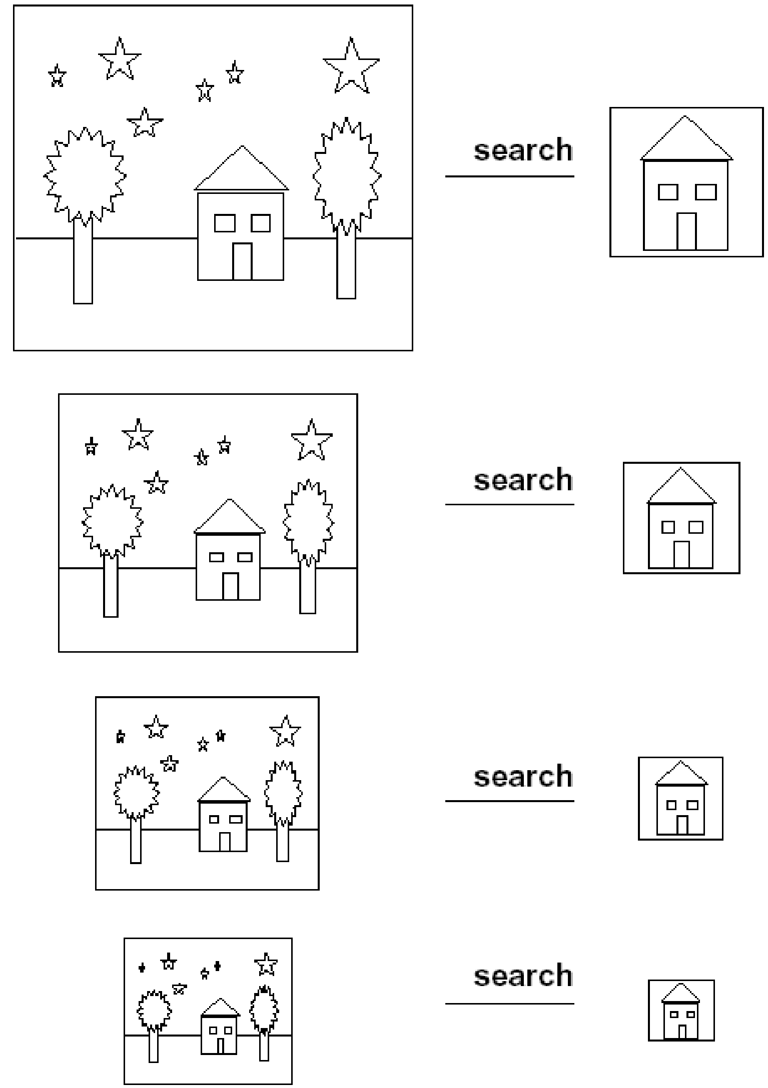
- Memory

$$M = 2^N \times 2^N (1 + 1/4 + 1/16 + \ldots)$$

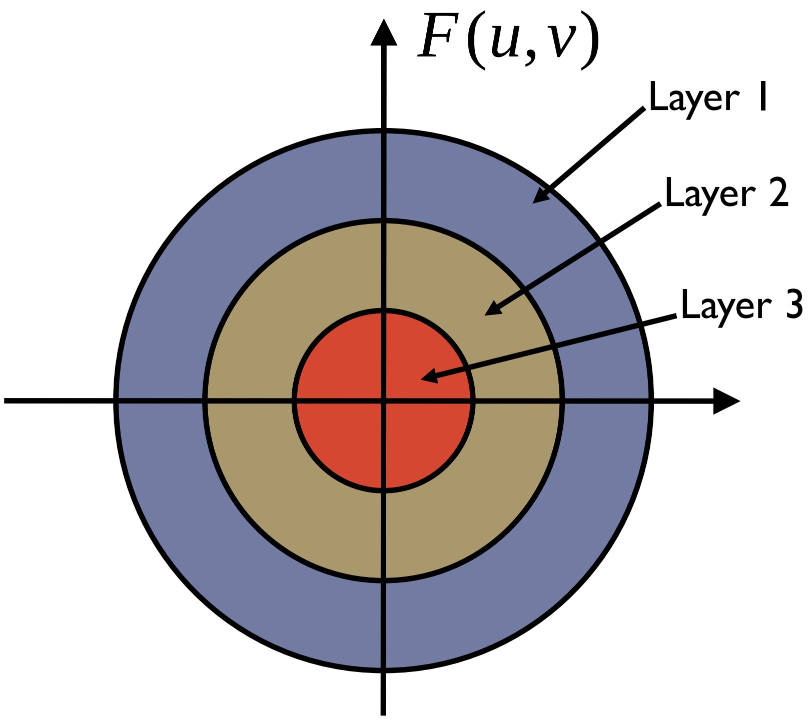
$$= 2^N \times 2^N \times 4/3$$

- Coarse-to-fine search

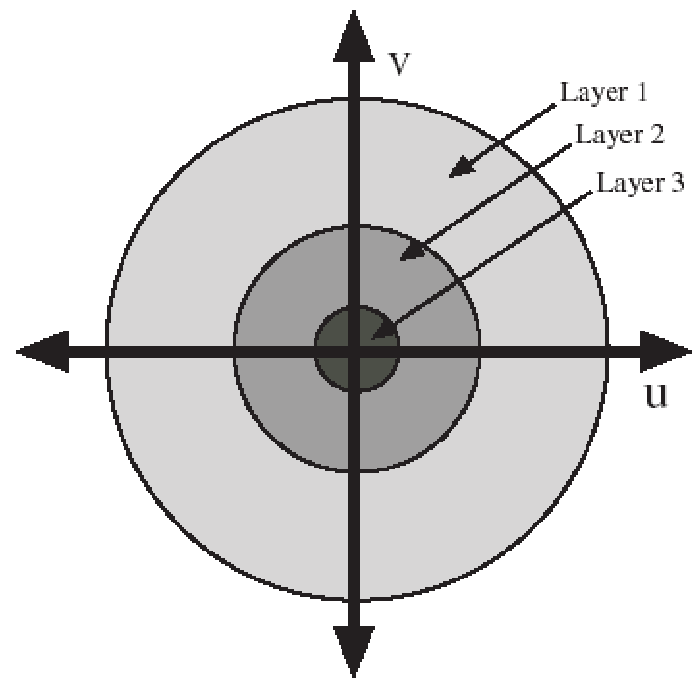




Frequency-domain Interpretation

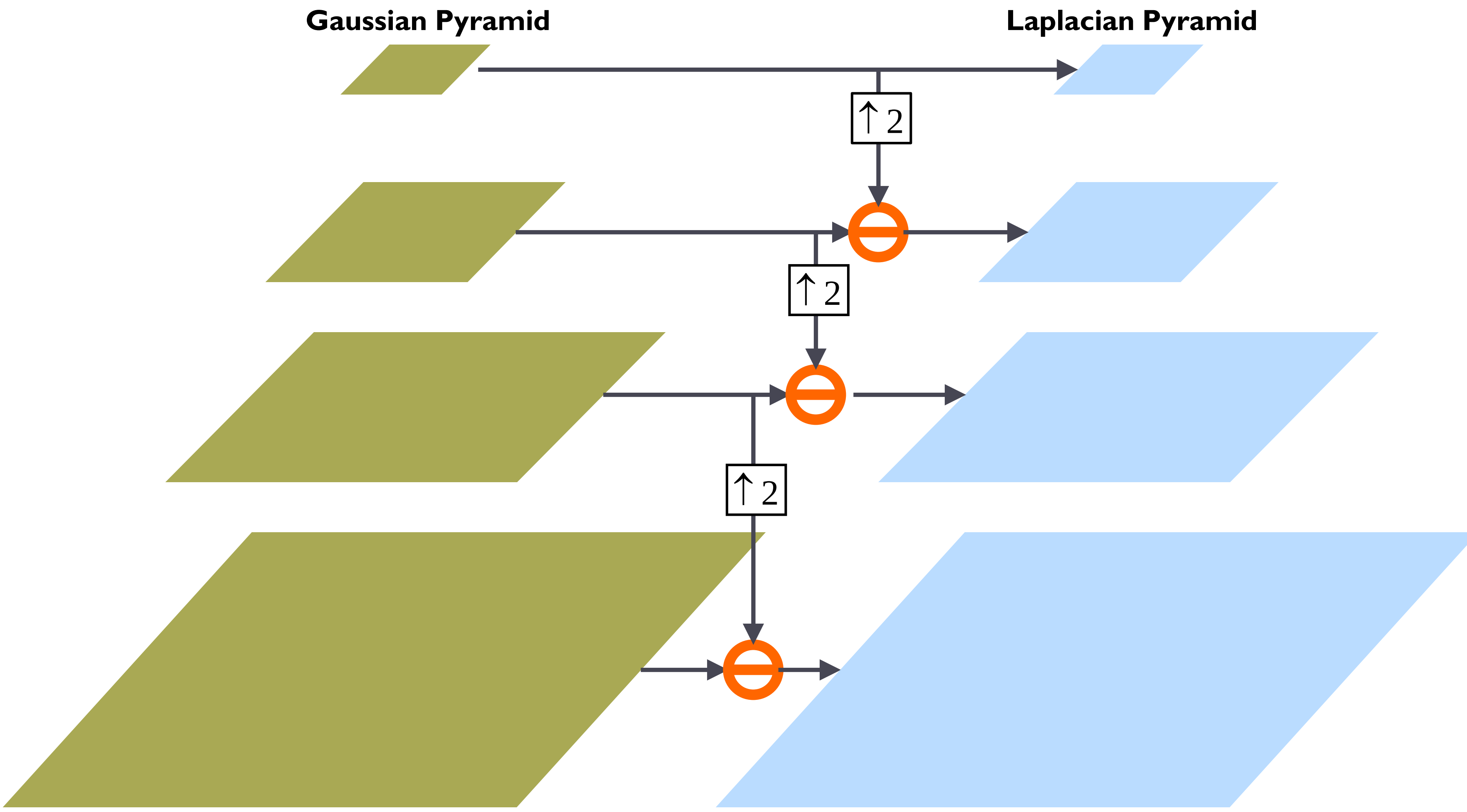


Gaussian Pyramid



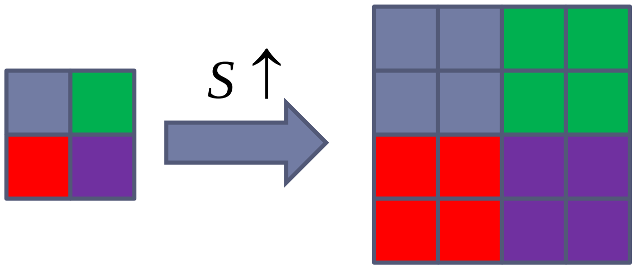
Laplacian Pyramid

Laplacian and Gaussian Pyramids



Laplacian Pyramid

- Laplacian pyramid makes use of the fact that a coarser layer of Gaussian pyramid can predict the next finer scale.
  - We can expand a coarser scale image by replicating pixels which involves an up-sampling operator S↑.
  - Compared to Gaussian pyramid, Laplacian pyramid is less redundant and has rich high-frequency information (why?)
  - We need only store the difference between this prediction and the next finer layer itself (high-frequency).



Laplacian Pyramid

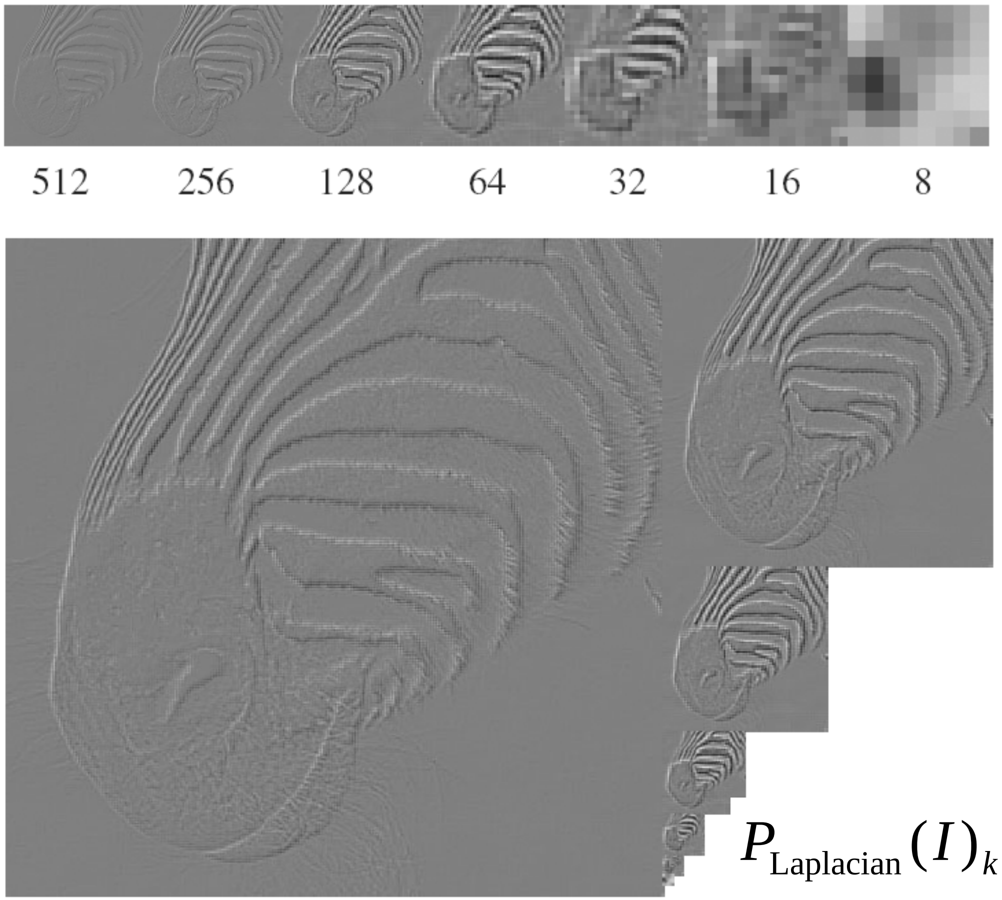


Figure 11.7. A Laplacian pyramid of images, running from 512x512 to 8x8. A zero response is coded with a mid-grey; positive values are lighter and negative values are darker. Notice that the stripes give stronger responses at particular scales, because each layer corresponds (roughly) to the output of a band-pass filter.

$$P_{\text{Gaussian}}(I)_m \quad (k = 1, \ldots, m)$$

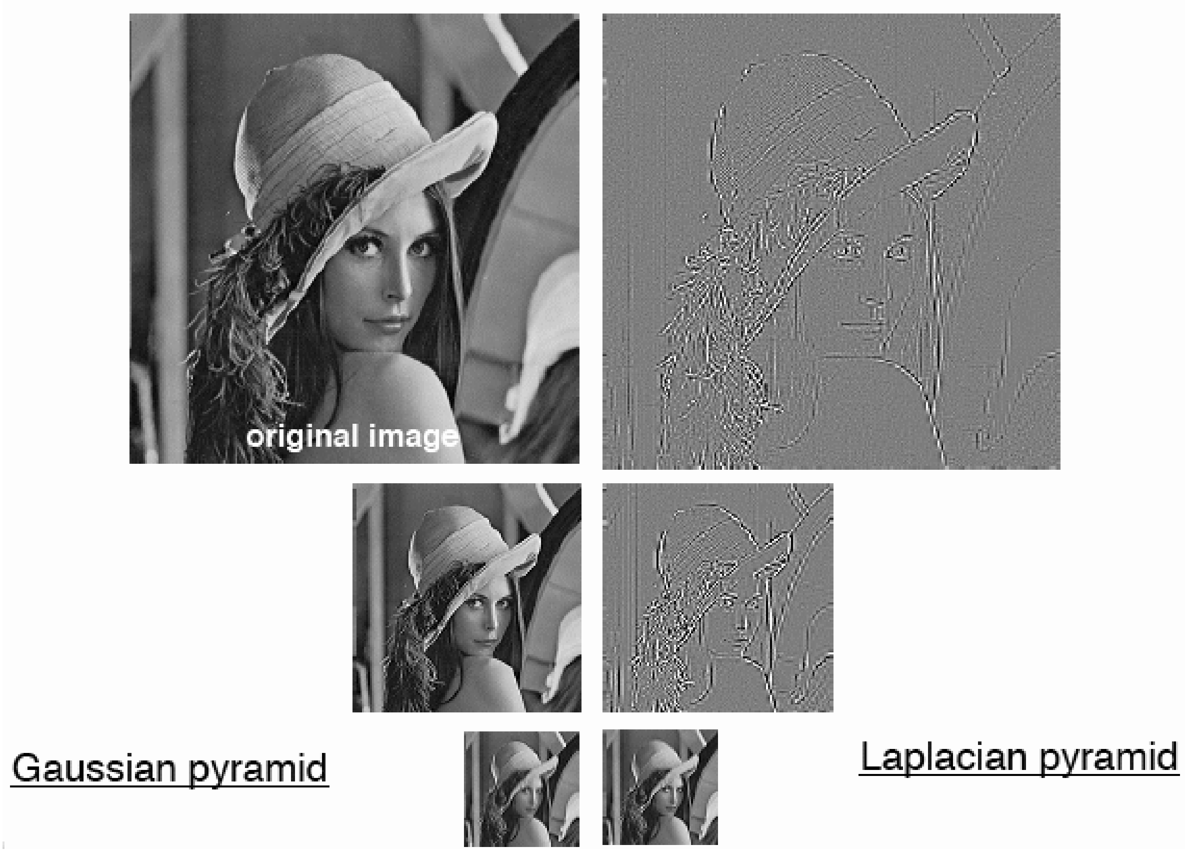
$$P_{\text{Laplacian}}(I)_m = P_{\text{Gaussian}}(I)_m$$

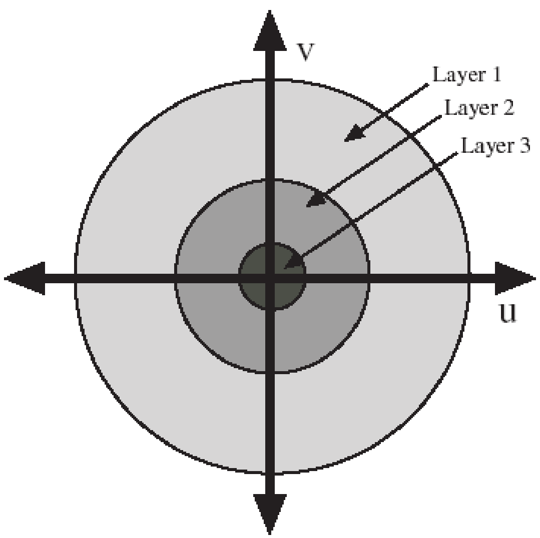
(where m is the coarest scale.)

$$P_{\text{Laplacian}}(I)_k = P_{\text{Gaussian}}(I)_k - S\uparrow\left(P_{\text{Gaussian}}(I)_{k+1}\right)$$

Laplacian Pyramid Application

- Image compression
- Texture analysis
- Edge detection





Laplacian Pyramid