Lecture 9
Linear Filtering for Image Enhancement
ECEN 5283 Computer Vision

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Goals

- To study the basic concepts about linear filtering
- To develop low-pass filters for image enhancement

Linear Filtering

- **Objective:** Transform the image intensities to:
  - Enhance or extract certain desirable image features
  - Suppress undesirable image attributes, such as noise
- **Implementations**
  - *Shift-Invariant & Linearity*
    - Separable convolution
    - Frequency-domain representation
    - Linear phase filtering
  - *Low-pass filtering*
    - Average smoothing
    - Gaussian filter

Shift-Invariant Linear System

- Most imaging or filtering systems have, to a good approximation, three significant properties:
  - Superposition (additivity):

$$R(f+g) = R(f) + R(g)$$

  - Scaling (homogeneity):

Linearity

$$R(kf) = kR(f)$$

  - Shift Invariance (SI) or time-invariance (TI):

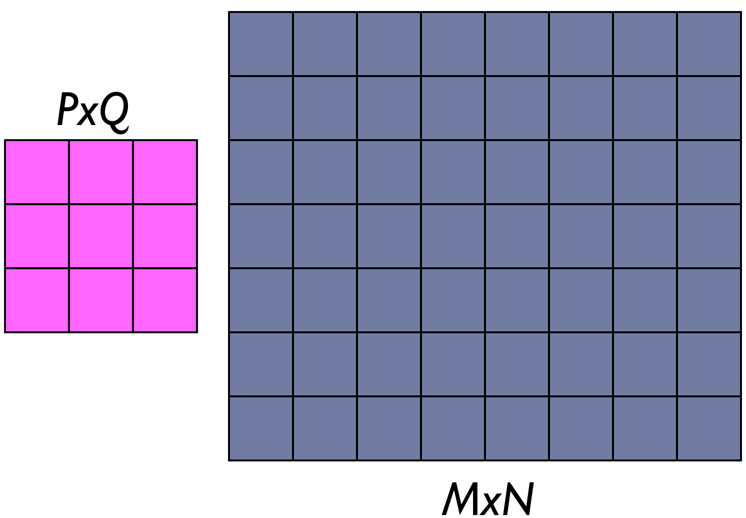
$$x(t) \rightarrow y(t) \Rightarrow x(t-t_0) \rightarrow y(t-t_0)$$

2-D Discrete Convolution

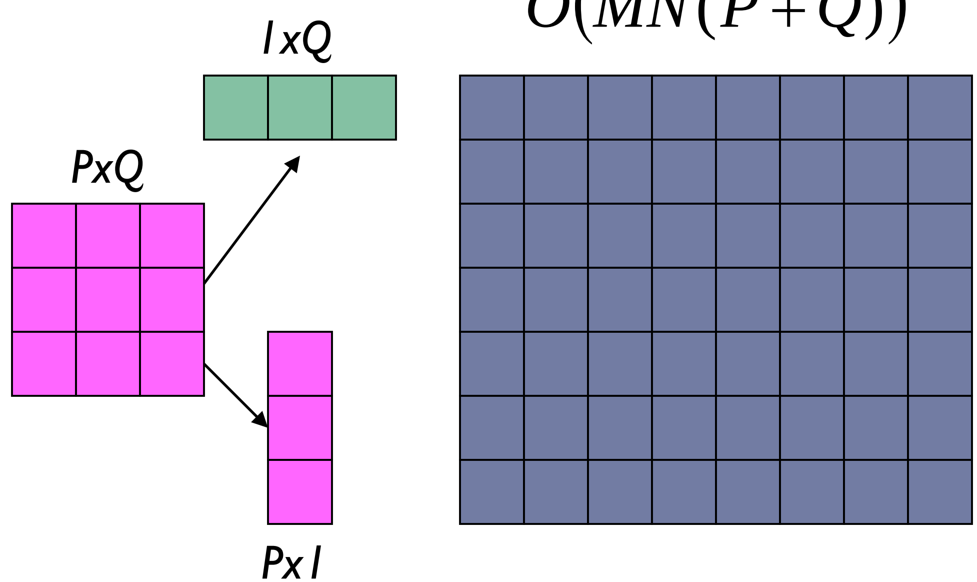
- Definition $$r[i,j] = \sum_{u,v} h[i-u, j-v]\, f[u,v]$$

- Separable 2D convolution $$h[i,j] = f[i]h[j]$$

$$O(MNPQ)$$



$$O(MN(P+Q))$$



2-D Discrete Convolution: Example

- Example 1: Local average smoothing (Separable?)

$$r[i, j] = \frac{1}{(2k+1)^2} \sum_{u=i-k}^{i+k} \sum_{v=j-k}^{j+k} f[u, v]$$

Usually, we construct a (2k+1)x(2k+1) kernel for a smoothing filter.

$$\begin{pmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{pmatrix} \rightarrow \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} * \begin{pmatrix} 1/3 & 1/3 & 1/3 \end{pmatrix}$$

Magnitude and Phase Spectra

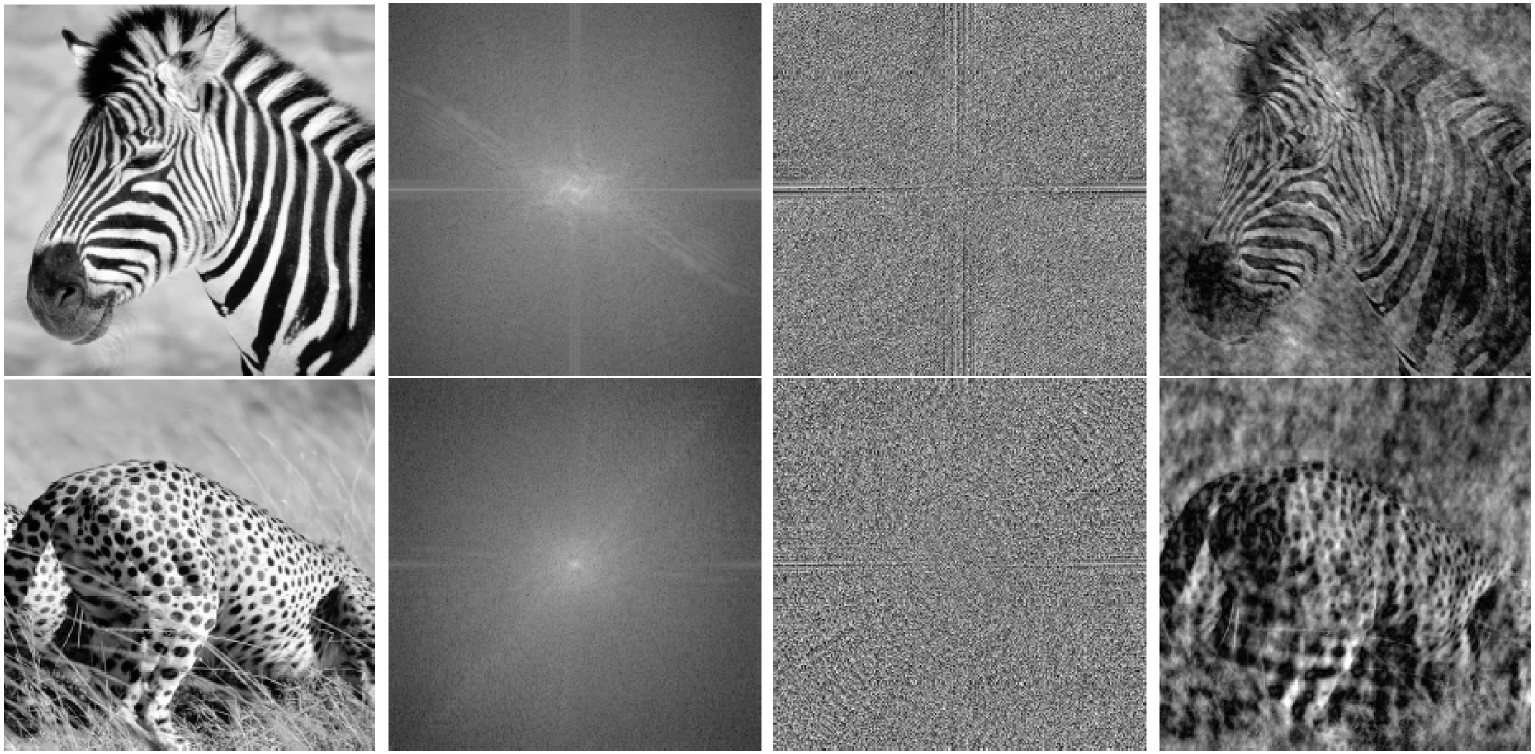
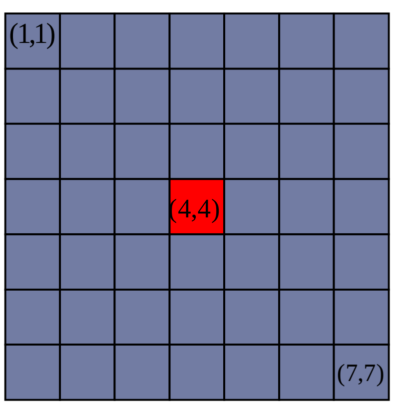
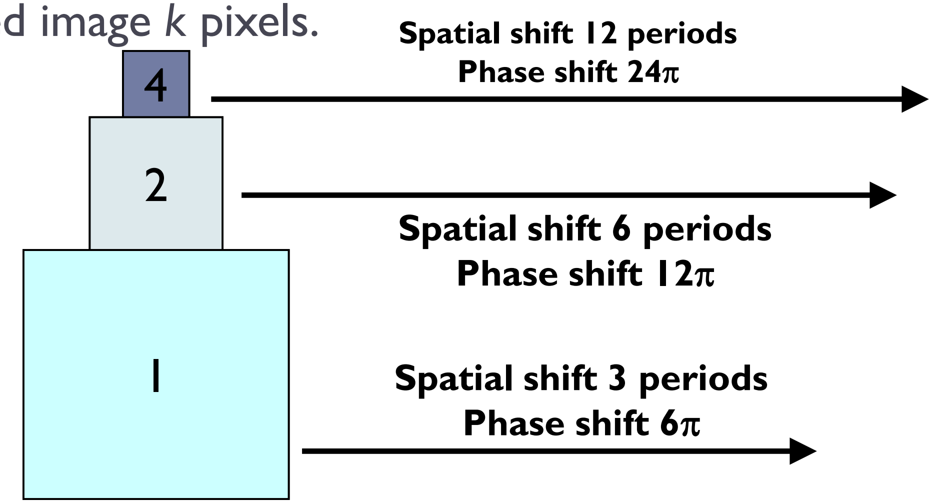


Figure 8.5. The second image in each row shows the log of the magnitude spectrum for the first image in the row; the third image shows the phase spectrum, scaled so that $-\pi$ is dark and π is light. The final images are obtained by swapping the magnitude spectra. While this swap leads to substantial image noise, it doesn't substantially affect the interpretation of the image, suggesting that the phase spectrum is more important for perception than the magnitude spectrum.

Linear Phase Filtering

- <u>Linear phase filtering</u>
  - Why? It can keep the frequency integrity of a signal.
  - What? The *relative location* of different frequency components are shifted linearly proportional to their frequency.
  - How? A symmetric filter will have the linear phase filtering.
- The *filter length is odd (2k+1) and symmetric about the center (k+1)*
  - The filter is designed to be symmetric about the center (k+1,k+1).
  - The image is shifted by *k* pixels, and we can perform zero-phase filtering by shifting back the filtered image *k* pixels.





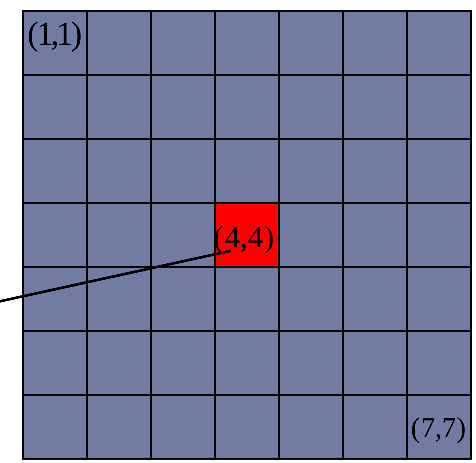
2-D Discrete Convolution: Example

- Example 2: Gaussian smoothing (Separable?)

$$g(x,y)=\frac{1}{2\pi\delta^2}\exp\left(-\frac{x^2+y^2}{2\delta^2}\right)\rightarrow f(x)h(y)=\frac{\exp\left(-\frac{x^2}{2\delta^2}\right)}{\sqrt{2\pi}\delta}\frac{\exp\left(-\frac{y^2}{2\delta^2}\right)}{\sqrt{2\pi}\delta}$$

Usually, we construct a (2k+1)x(2k+1) kernel for a smoothing filter.

$$h[i,j]=\frac{1}{2\pi\delta^2}\exp\left(-\frac{(i-k-1)^2+(j-k-1)^2}{2\delta^2}\right)$$



$(k+1, k+1)$

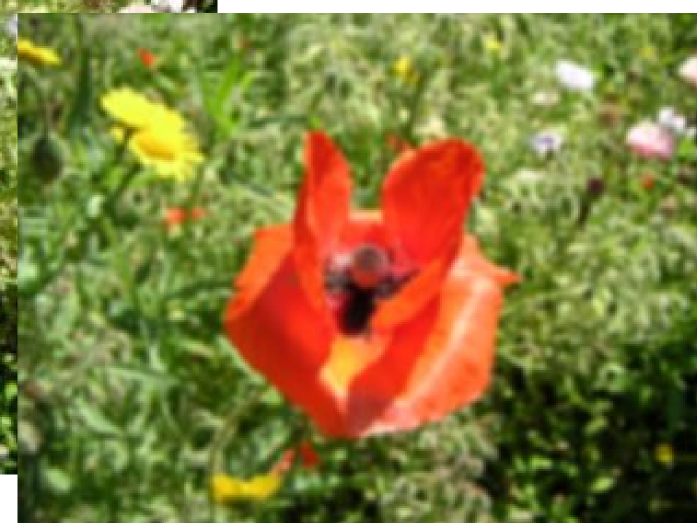
$k=3$

Low-pass Filtering

- **Objective:**
  - To maintain smoothly varying image intensities
  - To suppress high-frequency noise

Original Image

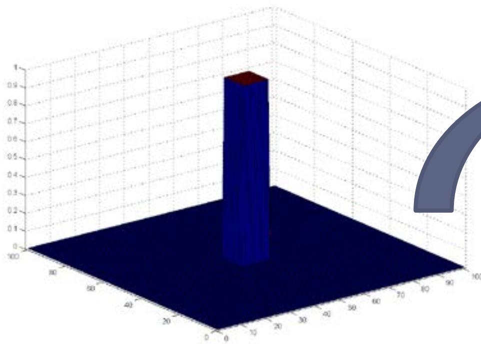
Noisy Image



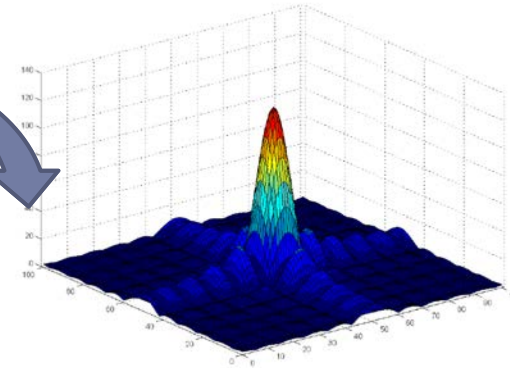
Denoised Image

Smoothing Operator

- Average smoothing:

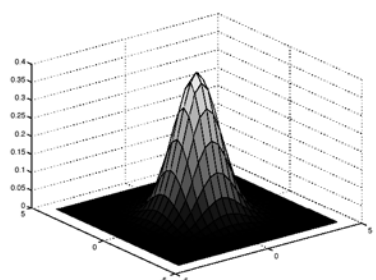


Frequency representation

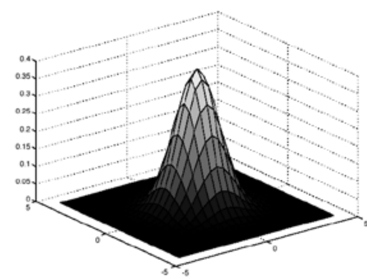


$$f(i,j)=\frac{1}{(2k+1)^2}(i,j=1,\ldots,(2k+1))$$

- Gaussian smoothing:



Frequency representation



$$g(x,y)=\frac{1}{2\pi\delta^2}\exp\left(-\frac{x^2+y^2}{2\delta^2}\right)$$

$$G(u,v)=\frac{1}{2\pi}\exp\left(-\frac{(u^2+v^2)}{2(1/\delta)^2}\right)$$

Average and Gaussian Smoothing

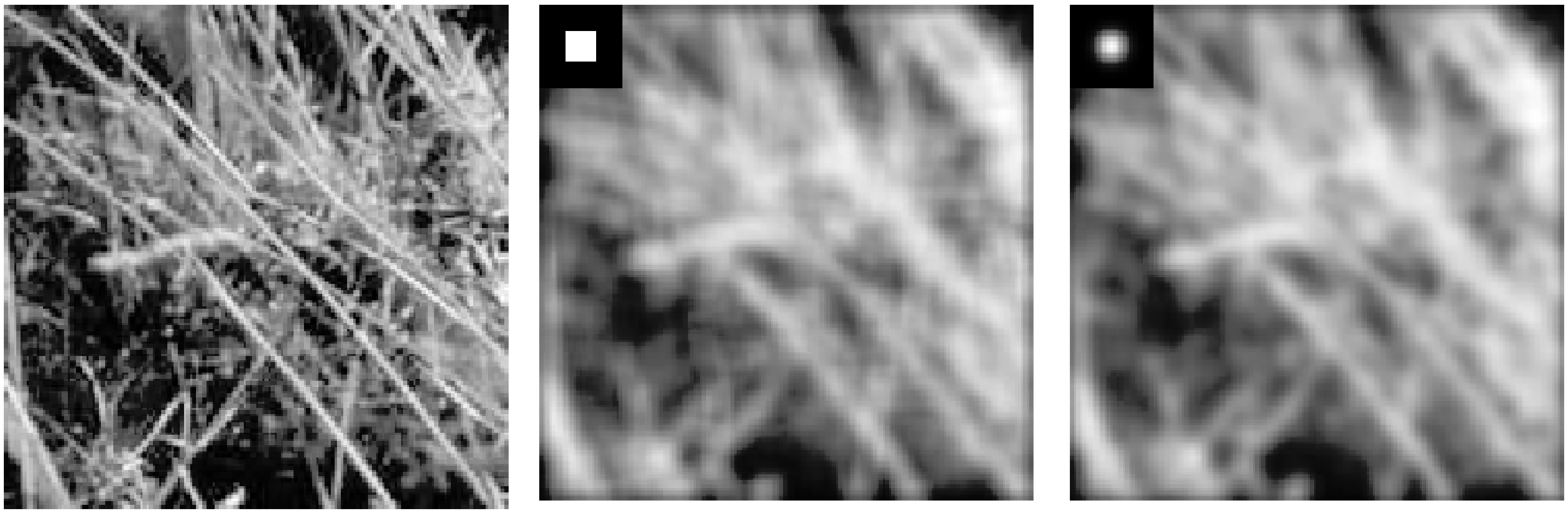


Figure 8.1. Although a uniform local average may seem to give a good blurring model, it generates effects that are not usually seen in defocussing a lens. The images above compare the effects of a uniform local average with weighted average. The image at the top shows a view of grass. On the left in the second row, the result of blurring this image using a uniform local model and on the right, the result of blurring this image using a set of Gaussian weights. The degree of blurring in each case is about the same, but the uniform average produces a set of narrow vertical and horizontal bars — an effect often known as **ringing**.