Project 4: Image Segmentation Using K-means and EM

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Project Objective

Objectives

1. Implement K-means and Expectation Maximization(EM) algorithms
2. Apply them for image segmentation
3. Analyze the result with different initialization
4. Apply them on Gray and Colored image

K-means

K-means clustering algorithm

0. Start with initial guesses for cluster centers (centroids)
1. For each data point, find closest cluster center (partitioning step)
2. Replace each centroid by average of data points in its partition
3. Iterate 1+2 until convergence

Given a set of observations y into a K clusters.

$$x_i^{(t)} = argmin_i\{y_i : \left\|y_i - \mu_k^{(t)}\right\|^2 \forall_{k,i}, 1 \leq j \leq K, 1 \leq i \leq N\}$$

Afterwards, the center-points are repositioned by calculating the mean of the assigned observations to the respective center-points

$$\mu_k^{(t+1)} = \frac{1}{N_k} \sum_{x_i \in x_k^{(t)}} x_i$$

K-means

$y_i = (y_{i1}, ...y_{id})$:
If centroids are $m_1, m_2, ...m_k$, and partitions are $c_1, c_2, ...c_k$, then one can show that K-means converges to a *local* minimum of

$$\sum_{k=1}^{K} \sum_{i \in c_k} ||x_i - m_k||^2$$

EM algorithm for Gaussian mixtures

- EM can be thought of a probabilistic version of K-means
- Instead of hard assigning a data point to a cluster it assigns a probability
- The mean an covariance for Gaussian often initialized using fast K-means clustering
- EM is a two step algorithm

EM algorithm

E-Step:

$$\gamma(z_k) = p(z_k = 1|y) = \frac{p(y|z_k = 1)p(z_k = 1)}{p(y)}$$
$$= \frac{\alpha_k \mathcal{N}(y|\mu_k, \Sigma_k)}{\sum_{j=1}^{K} \pi_j \mathcal{N}(y|\mu_j, \Sigma_j)}$$

M-step:

$$\mu_k = \frac{\sum_{n=1}^{N} \gamma(z_{nk}) y_n}{\sum_{n=1}^{N} \gamma(z_{nk})}$$

$$\Sigma_k = \frac{1}{N_k} \gamma_k(z_{nk})(y_n - \mu_k)(y_n - \mu_k)^T$$

The objective function for EM is

$$lnp(X|\alpha, \mu, \Sigma) = \Sigma_{n=1}^{N} ln\{\Sigma_{k=1}^{K} \alpha_k \mathcal{N}(x_n|\mu_k, \Sigma_k)\}$$

Test Image Mosaic A

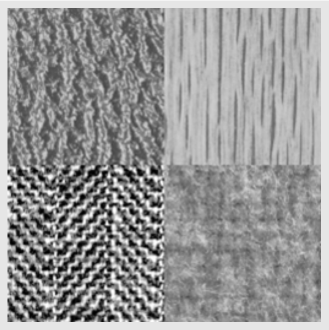


Figure: Test Image

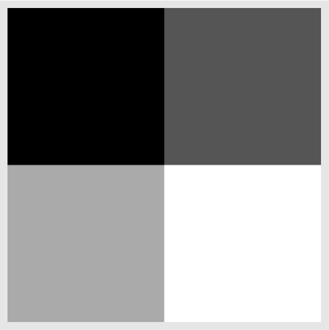


Figure: Ground Truth

K-means Initializations for A

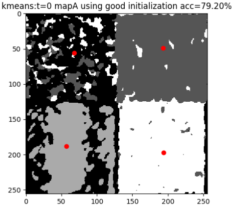


Figure: Good

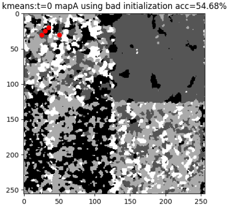


Figure: Bad

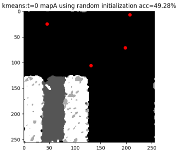
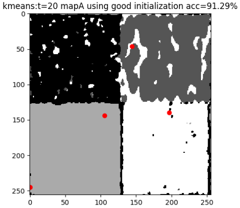


Figure: Random

K-means Output for A



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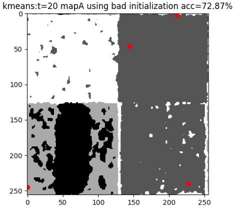

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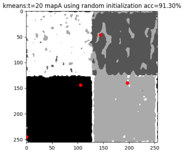


Figure: Random

K-means Performance for A

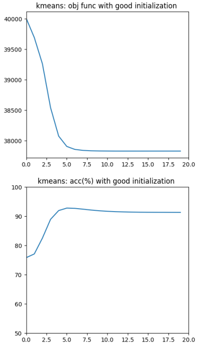


Figure: Good

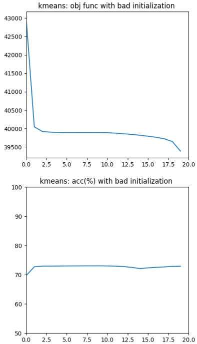


Figure: Bad

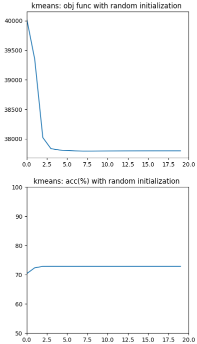


Figure: Random

Test Image Mosaic B

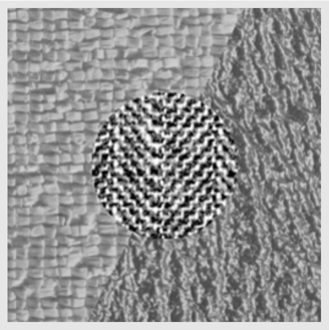


Figure: Test Image

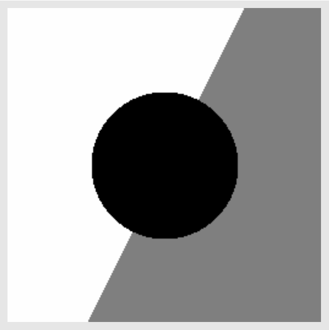


Figure: Ground Truth

K-means Initializations for B

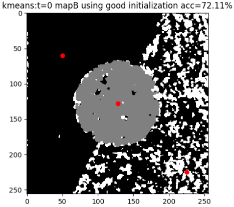
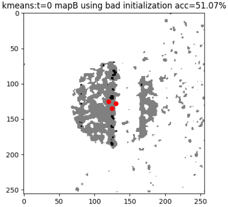


Figure: Good



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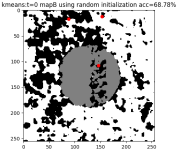
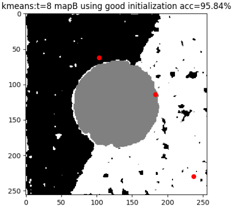


Figure: Random

K-means Output for B



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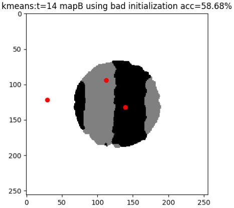

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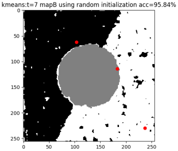
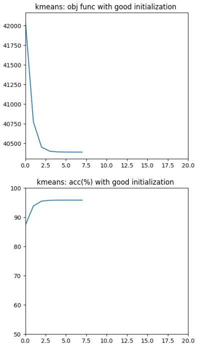


Figure: Random

K-means Performance for B



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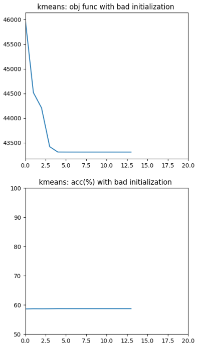

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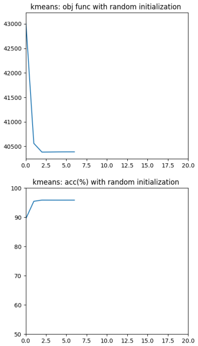


Figure: Random

N.B. EM initialized using the optimized output of the K-means. In this context, bad initialization for EM means the K-means was started with bad initialization and EM initialized with the output of that K-means

EM Initializations by K-means for A

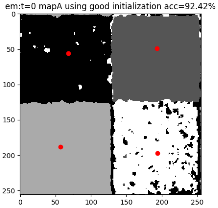


Figure: Good

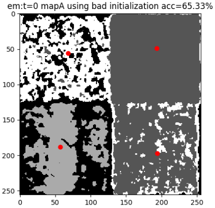


Figure: Bad

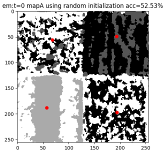


Figure: Random

EM Output for A

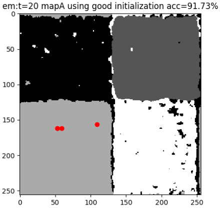


Figure: Good

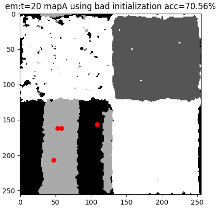


Figure: Bad

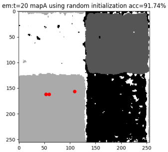


Figure: Random

EM Performance for A

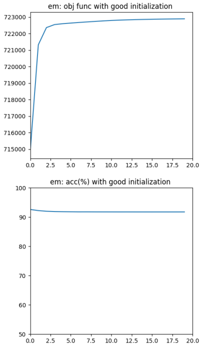


Figure: Good

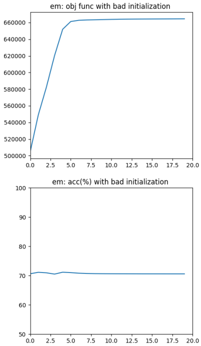


Figure: Bad

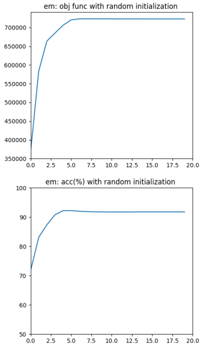


Figure: Random

EM Initialized by K-means for B

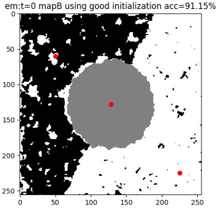


Figure: Good

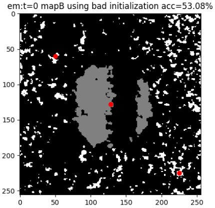


Figure: Bad

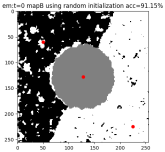


Figure: Random

EM Output for B

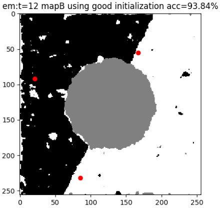
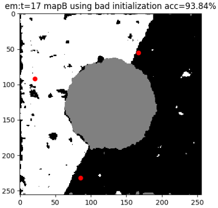


Figure: Good



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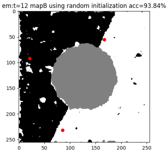


Figure: Random

EM Performance for B

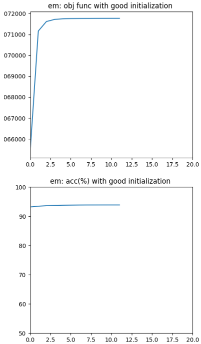


Figure: Good

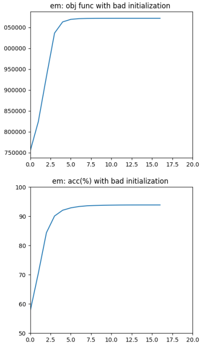


Figure: Bad

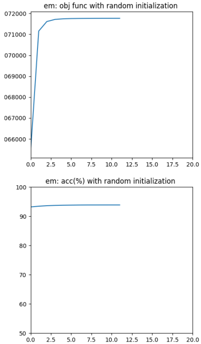


Figure: Random

K-means Vs EM on A

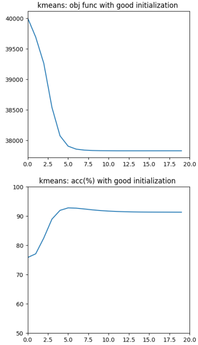


Figure: K-means Good

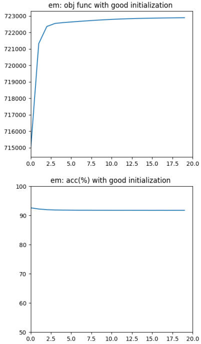


Figure: EM Good

K-means Vs EM on B

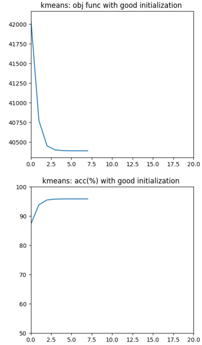


Figure: K-means Good

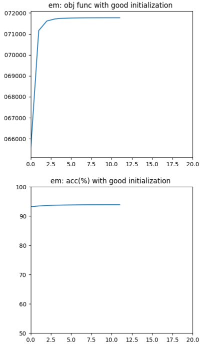


Figure: EM Good

K-means Vs EM on A

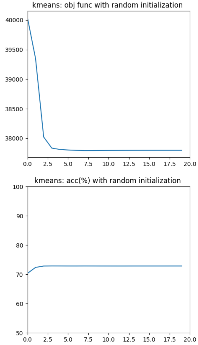


Figure: K-means Random

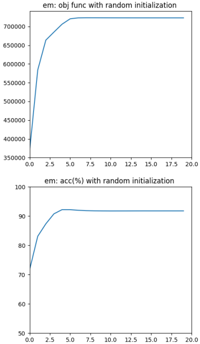


Figure: EM random

K-means Vs EM on B

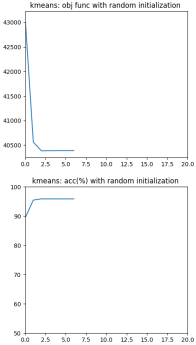


Figure: K-means Random

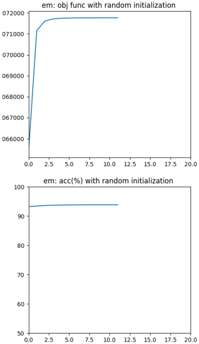
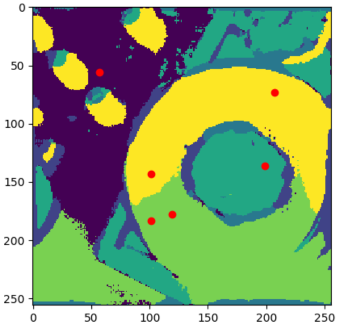


Figure: EM Random

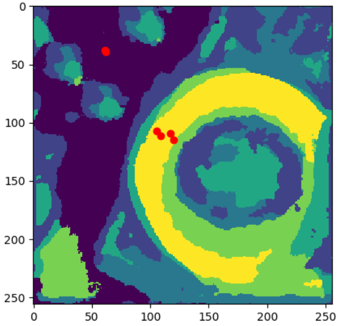
Analysis

- Experiments reaffirms the fact that both K-means and EM are sensitive to initialization; good initializations have better accuracy
- EM is more robust than K-means because in all cases it performs almost as good as K-means or much better than it.
- For example in random case K-means suffer 70% accuracy whereas EM has 90% accuracy
- The robustness is more visible in more complex segmentation for example in more complex real world image
- However K-means converges faster than EM

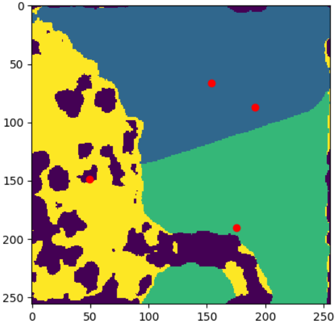
K-means on Real Image K=6



EM on Real Image

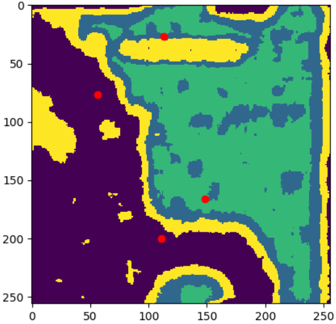


K-means on Real Image K=4



view of Boomer Lake. K means is confused by Google's watermark. It also did not segmented the bank.

EM on Real Image



segmented the watermark in a different cluster. Also segmented the lake, land, bank and trees.