Lecture 4.
Camera Models: Intrinsic Parameters
ECEN 5283 Computer Vision

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Goals

- To review the geometric transforms for coordinate changes.
- To introduce the analytical machinery necessary to establish quantitative constraints between 2D image measurements and 3D real-world objects.
- To define physical parameters of camera, i.e., intrinsic parameters and extrinsic parameters, and to study the role of intrinsic parameters for geometric camera modeling.

Coordinate System Changes

- When several different coordinate systems are considered at the same time, we denote the coordinate vector of the point *P* in the frame *F* as

$$^{F}P = {}^{F}\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Leftrightarrow \overrightarrow{OP} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

- Let us consider two coordinate systems (two frames)

$$(A) = (O_A, \mathbf{i}_A, \mathbf{j}_A, \mathbf{k}_A)$$

$$(B) = (O_B, \mathbf{i}_B, \mathbf{j}_B, \mathbf{k}_B)$$

  - **Question**: How to express ^{B}P as a function of ^{A}P .

Four Cases of Coordinate Changes

- Case I: 3D Translation

$$O_A \neq O_B, \mathbf{i}_A = \mathbf{i}_B, \mathbf{j}_A = \mathbf{j}_B, \mathbf{k}_A = \mathbf{k}_B.$$

$$^{B}P = {}^{B}O_A + {}^{A}P$$

- Case II: 3D Rotation

$$O_A = O_B = O, \mathbf{i}_A \neq \mathbf{i}_B, \mathbf{j}_A \neq \mathbf{j}_B, \mathbf{k}_A \neq \mathbf{k}_B.$$

$$^{B}P = {}^{B}_{A}R\,{}^{A}P$$

$${}^{B}_{A}R = \begin{pmatrix} {}^{B}\mathbf{i}_A & {}^{B}\mathbf{j}_A & {}^{B}\mathbf{k}_A \end{pmatrix}$$

- Case III: 2D Rotation

$$O_A = O_B = O, \mathbf{k}_A = \mathbf{k}_B = \mathbf{k}.$$

$${}^{B}_{A}R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Case IV: 3D Rigid Transformation

$$O_A \neq O_B, \mathbf{i}_A \neq \mathbf{i}_B, \mathbf{j}_A \neq \mathbf{j}_B, \mathbf{k}_A \neq \mathbf{k}_B.$$

$$^{B}P = {}^{B}_{A}R\,{}^{A}P + {}^{B}O_A$$

Coordinate System Changes: Rigid Transform

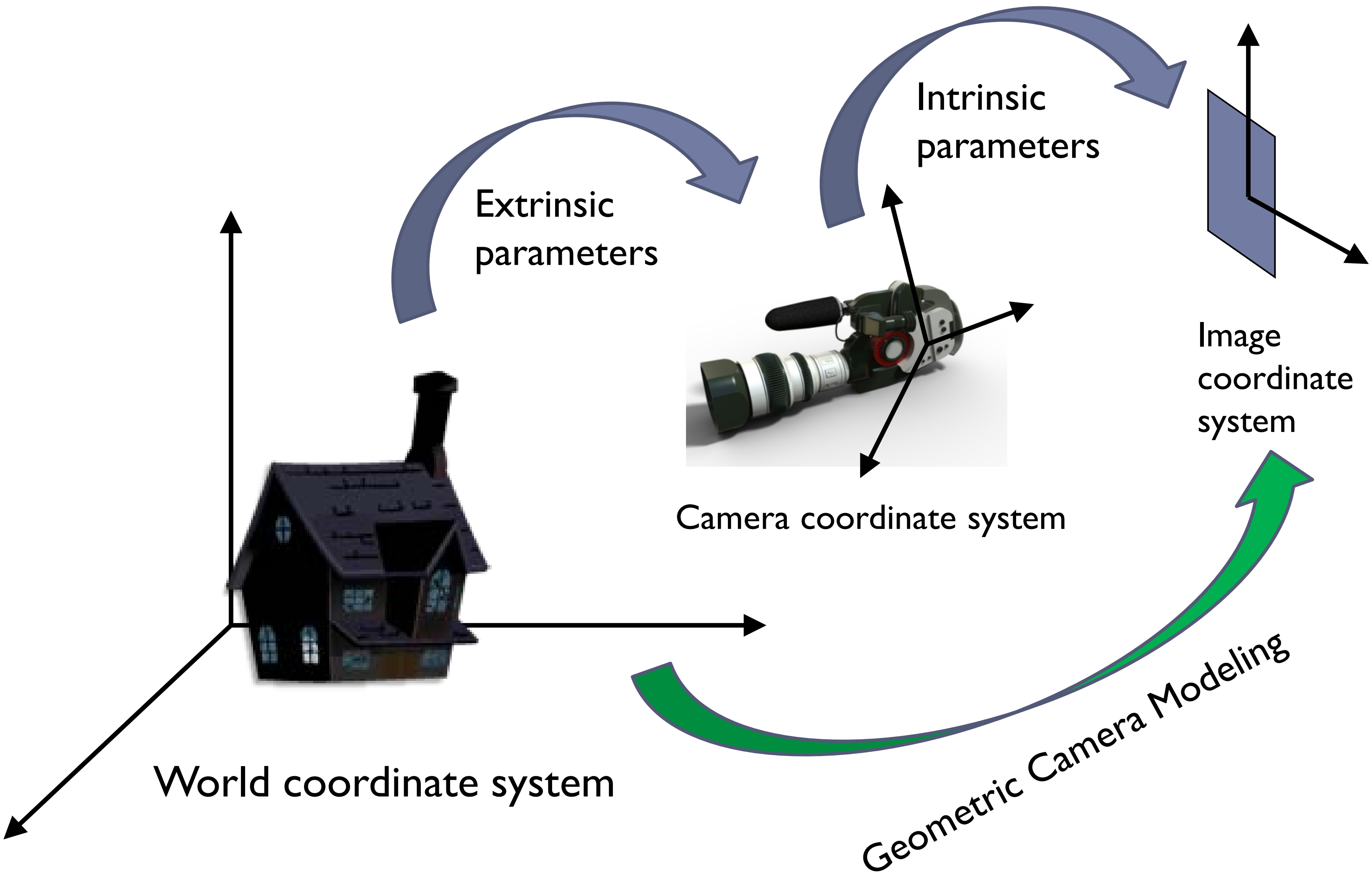
- Rigid transformation using homogeneous coordinates

$$ {}^{B}P = {}^{B}_{A}R\,{}^{A}P + {}^{B}O_{A} $$

$$ \begin{pmatrix} {}^{B}P \\ 1 \end{pmatrix} = \begin{pmatrix} {}^{B}_{A}R & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{pmatrix} \begin{pmatrix} {}^{A}P \\ 1 \end{pmatrix} $$

$$ \begin{pmatrix} {}^{B}P \\ 1 \end{pmatrix} = {}^{B}_{A}T \begin{pmatrix} {}^{A}P \\ 1 \end{pmatrix}, \quad \text{where } {}^{B}_{A}T = \begin{pmatrix} {}^{B}_{A}R & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{pmatrix}_{4\times 4} $$

Geometric Camera Modeling: Intrinsic and Extrinsic Parameters



Camera Parameters and the Perspective Projection

- In reality, the perspective equation is only valid when all distances are measured in the camera's reference frame, and image coordinates have their own origin.
- In practice, the world and camera coordinate systems are related by a set of parameters.
  - Intrinsic parameters relates the camera's coordinate system to the image pixel coordinate system.
  - Extrinsic parameters relate the camera's coordinate system to a fixed world coordinate system and specify its position and orientation in space.
- A process to estimate intrinsic & extrinsic parameters is known as *geometric camera calibration*.

Intrinsic Parameters

- We want to associate with a camera with a normalized image plane parallel to its physical retina but located at a unit distance from the pinhole.

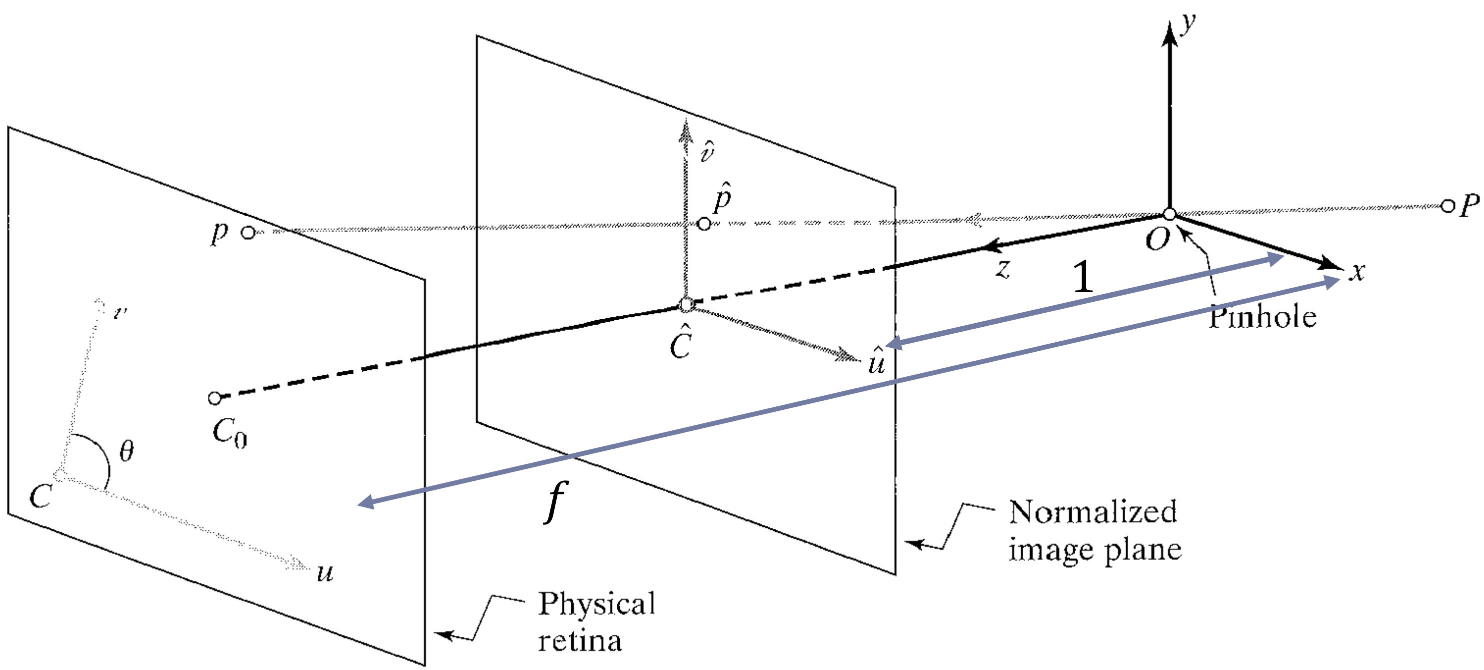


Figure 2.8 Physical and normalized image coordinate systems.

Normalized Image Plane

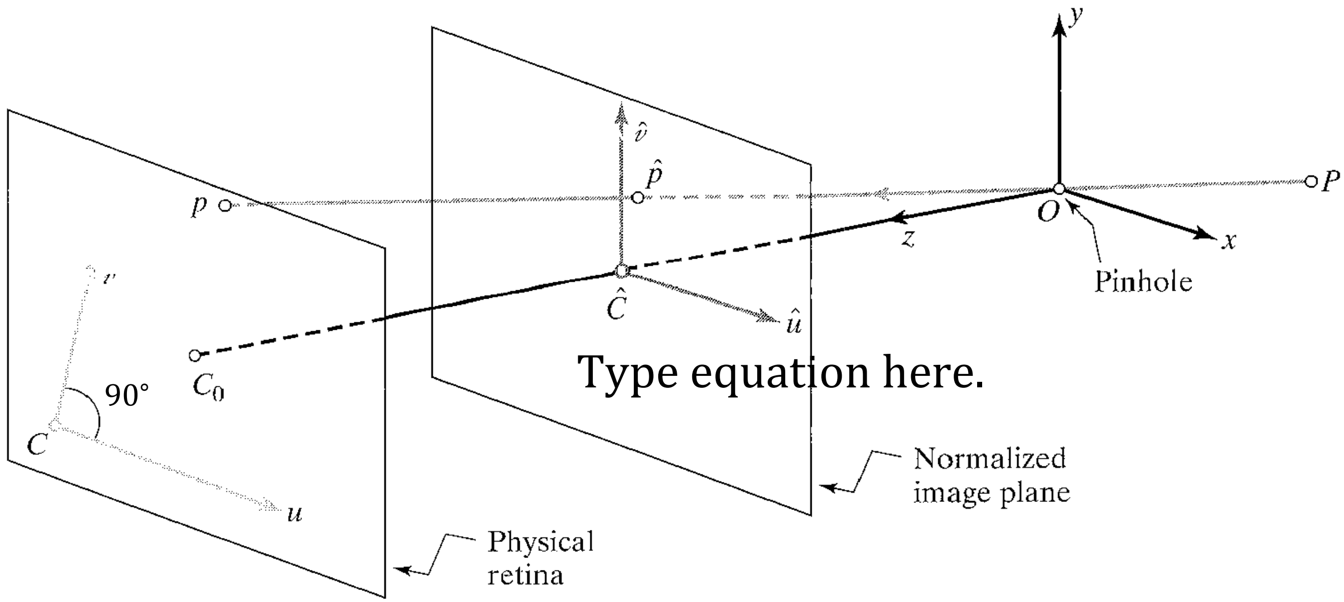


Figure 2.8 Physical and normalized image coordinate systems.

$$\begin{cases} \hat{u} = \dfrac{x}{z} \\ \hat{v} = \dfrac{y}{z} \end{cases} \Leftrightarrow \hat{\mathbf{p}} = \frac{1}{z}(\mathbf{I} \quad 0)\mathbf{P} \qquad \text{where } \begin{cases} \mathbf{P} = (x \quad y \quad z \quad 1)^T \\ \hat{\mathbf{p}} = (\hat{u} \quad \hat{v} \quad 1)^T \end{cases}$$

Perspective projection

Physical Retina of the Camera

$C_0 = \{u_0, v_0\}$

(pixel coordinate of the image center)

$$\begin{cases} \hat{u} = \dfrac{x}{z} \\ \hat{v} = \dfrac{y}{z} \end{cases}$$

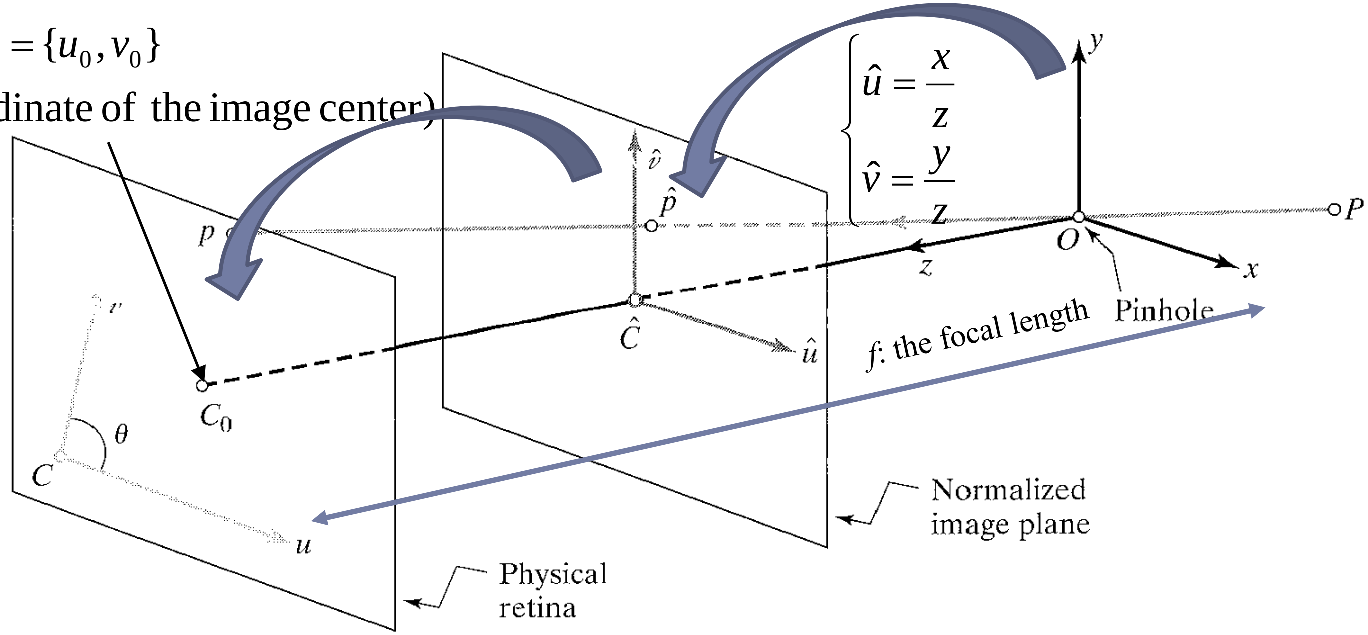


Figure 2.8 Physical and normalized image coordinate systems.

$$\begin{cases} u = kf\dfrac{x}{z} \\ v = lf\dfrac{y}{z} \end{cases}$$ where a pixel has dimension $\dfrac{1}{k} \times \dfrac{1}{l}$, f is the focal length. $\Rightarrow$ Pixel coordinates $$\begin{cases} u = \alpha\dfrac{x}{z} + u_0 \\ v = \beta\dfrac{y}{z} + v_0 \end{cases}$$

(assuming $\theta = \dfrac{\pi}{2}$)

$1/k$

$1/l$ Pixel

$$\alpha = kf \quad \text{and} \quad \beta = lf$$

Planar Similarity Transformation

$$\mathbf{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \text{ where } \begin{cases} u = \alpha \dfrac{x}{z} + u_0 \\ v = \beta \dfrac{y}{z} + v_0 \end{cases}$$

(physical retinal plane)

$$\hat{\mathbf{p}} = \begin{pmatrix} \hat{u} \\ \hat{v} \\ 1 \end{pmatrix} \text{ where } \begin{cases} \hat{u} = \dfrac{x}{z} \\ \hat{v} = \dfrac{y}{z} \end{cases}$$

(normalized image plane)

$$\hat{\mathbf{p}} = \frac{1}{z}(\mathbf{I} \quad 0)\mathbf{P}$$

(3D to 2D perspective projection on the normalized image plane)

$$\mathbf{p} = K\hat{\mathbf{p}}, \longrightarrow \mathbf{p} = K\frac{1}{z}\begin{pmatrix}\mathbf{I} & 0\end{pmatrix}\mathbf{P}$$

$$\text{where } K = \begin{pmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Intrinsic Parameters

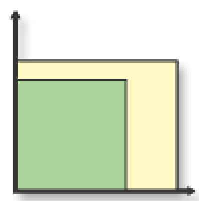
$$\mathbf{p} = \frac{1}{z} M\mathbf{P}, \quad \text{where } M = \begin{pmatrix} K & \mathbf{0} \end{pmatrix}$$

Affine Transformation Review

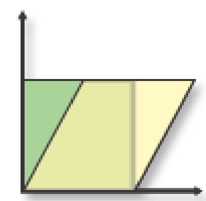
- An affine transformation can differentially scale the data, skew it, rotate it, and translate it.

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} s_u & 0 & 0 \\ 0 & s_v & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



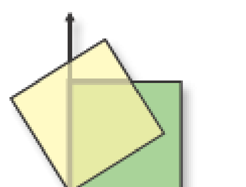
DIFFERENTIAL SCALING



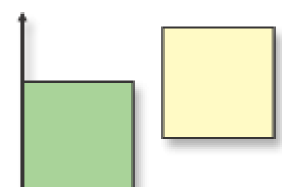
SKEW

$$\begin{pmatrix} 1 & h_x & 0 \\ h_y & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



ROTATION



TRANSLATION

$$\begin{pmatrix} 1 & 0 & u_0 \\ 0 & 1 & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

Normalized Image Plan and Physical Retina: Revisited

$C_0 = \{u_0, v_0\}$

(pixel coordinate of the image center)

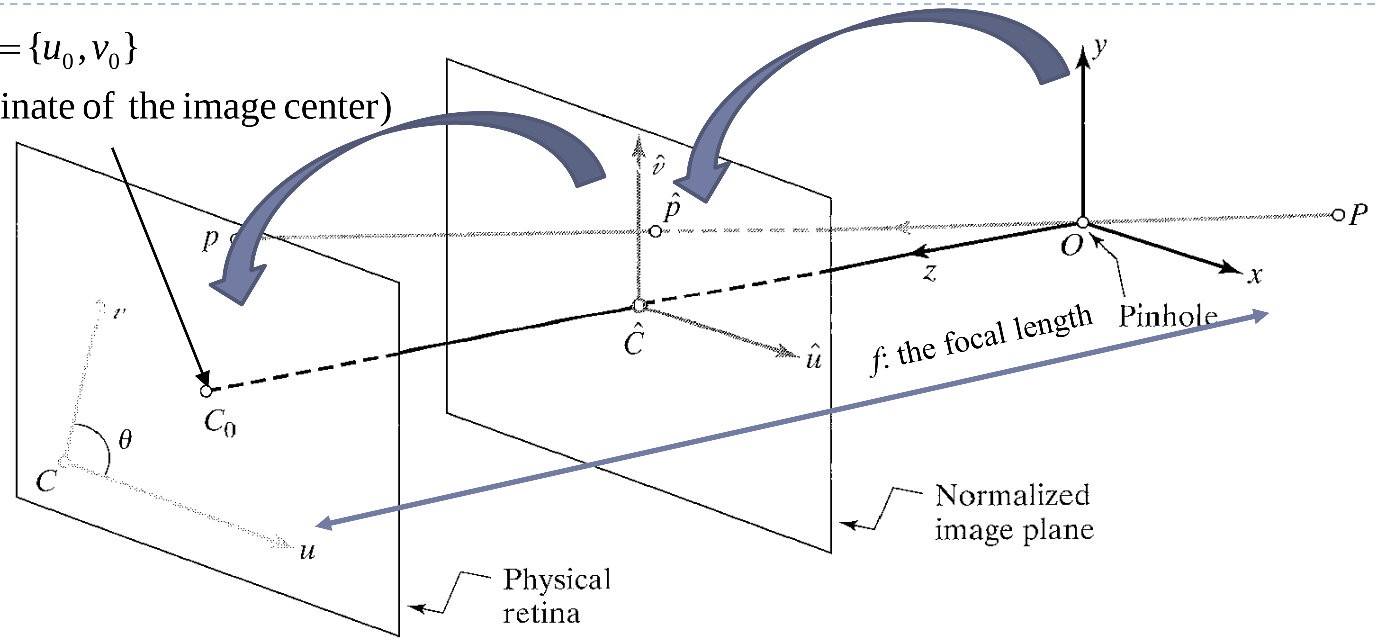


Figure 2.8 Physical and normalized image coordinate systems.

$$\begin{cases} \hat{u} = \dfrac{x}{z} \\ \hat{v} = \dfrac{y}{z} \end{cases} \xrightarrow{\text{Affine Transformation}} \begin{cases} u = \alpha\hat{u} - \alpha\cot\theta\hat{v} + u_0 \\ v = \dfrac{\beta}{\sin\theta}\hat{v} + v_0 \end{cases}$$

$$\left(\cot\theta = \frac{\cos\theta}{\sin\theta}\right)$$

$$\alpha = kf, \beta = lf$$

Planar Affine Transformation

$$\mathbf{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \Rightarrow \begin{cases} u = \alpha\hat{u} - \alpha \cot\theta \hat{v} + u_0 \\ v = \dfrac{\beta}{\sin\theta}\hat{v} + v_0 \end{cases}$$

$$\hat{\mathbf{p}} = \begin{pmatrix} \hat{u} \\ \hat{v} \\ 1 \end{pmatrix} \Rightarrow \begin{cases} \hat{u} = \dfrac{x}{z} \\ \hat{v} = \dfrac{y}{z} \end{cases}$$

(normalized image plane)

$$\hat{\mathbf{p}} = \begin{pmatrix} \hat{u} \\ \hat{v} \\ 1 \end{pmatrix} = \frac{1}{z}(\mathbf{I} \quad 0)\mathbf{P}$$

(3D to 2D perspective projection on the normalized image plane)

$$\mathbf{p} = K\hat{\mathbf{p}},$$

$$\text{where } K = \begin{pmatrix} \alpha & -\alpha\cot\theta & u_0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\mathbf{p} = K\frac{1}{z}(\mathbf{I} \quad 0)\mathbf{P}$$

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Intrinsic Parameters

$$\mathbf{p} = \frac{1}{z}M\mathbf{P}, \quad \text{where } M = (K \quad \mathbf{0})$$