Givensove rotacije

Primer:

$$A = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \xrightarrow{R_{12}^T} \begin{bmatrix} x & x & x \\ 0 & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \xrightarrow{R_{13}^T} \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & x & x \\ x & x & x \end{bmatrix} \xrightarrow{R_{14}^T} \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & x & x \\ 0 & x & x \end{bmatrix} \xrightarrow{R_{23}^T}$$

$$\begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \\ 0 & x & x \end{bmatrix} \xrightarrow{R_{24}^T} \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \\ 0 & 0 & x \end{bmatrix} \xrightarrow{R_{34}^T} \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \\ 0 & 0 & 0 \end{bmatrix} = \tilde{R}.$$

Velja torej:

$$R_{34}^T \cdot R_{24}^T \cdot R_{23}^T \cdot R_{14}^T \cdot R_{13}^T \cdot R_{12}^T \cdot A = \tilde{R}.$$

Sledi:

$$\tilde{Q}^T = R_{34}^T \cdot R_{24}^T \cdot R_{23}^T \cdot R_{14}^T \cdot R_{13}^T \cdot R_{12}^T$$

oz.

$$\tilde{Q} = R_{12} \cdot R_{13} \cdot R_{14} \cdot R_{23} \cdot R_{24} \cdot R_{34}.$$

Velja $A = \tilde{Q}\tilde{R}$, kjer je A $m \times n$, $\tilde{Q}$ $m \times m$ ortogonalna, $\tilde{R}$ pa $m \times n$ zgornja trapezna matrika.
Predoločen sistem $Ax = b$ rešujemo tako, da rešimo sistem $\tilde{R}x = \tilde{Q}^T b$.

Algoritem za Givensovo rotacijo:

$\tilde{Q} = I_m$
for $i = 1 : n$
  for $k = i + 1 : m$
    if $a_{ki} \neq 0$
      $r = (a_{ii}^2 + a_{ki}^2)^{\frac{1}{2}}, c = \frac{a_{ii}}{r}, s = \frac{a_{ki}}{r}$
      A([i k], i:n) $= \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \cdot$ A([i k], i:n)
      $\tilde{Q}$([i k], 1:m) $= \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \cdot \tilde{Q}$([i k], 1:m)
    end
  end
end
$\tilde{Q} = \tilde{Q}^T$