Concept	1-Categorical construction	∞-Categorical construction	Intuition
Accessible category	$\mathcal{C}$ is locally small, admits κ-filtered colimits, and there is a set of κ-compact objects which generates $\mathcal{C}$ under κ-filtered colimits. ([nLa25a], Def 2.1)	$\mathcal{C}$ is locally small, admits κ-filtered colimits, and there is an essentially small subcategory of κ-compact objects which generates $\mathcal{C}$ under κ-filtered colimits[1]. ([nLa25b], Def 2.1)[2]	-
F-Cartesian edge	$F : X \to S$ a functor, $f : x \to y$ a morphism in X is F-cartesian if the induced map $$X_{/f} \to X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$$ is an isomorphism of categories. ([nLa25c], Prop 2.4)	$F : X \to S$ an inner fibration of simplicial sets, $f : x \to y$ an edge in X is F-cartesian if the induced map $$X_{/f} \to X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1)	In the model structure on **sSet**, the fibrations are Kan fibrations and the weak equivalences are weak homotopy equivalences ([Lur09], A.2.7). A trivial fibration in a model category is a map which is both a fibration and a weak equivalence, which in **sSet** is equivalent to the definition given in this table. Thus, being related by a Kan fibration is a higher categorical notion of 'sameness'. Why not a categorical equivalence? [Lur09] Rem 1.2.5.5 implies this is stronger, which would match more with the fact that the 1-categorical version is defined in terms of an isomorphism (not equivalence) of categories.
Category	Collection of objects $\mathcal{C}$, set $\mathrm{Hom}(X, Y)$ for every $X, Y \in \mathcal{C}$, associative composition and identity morphisms	Simplicial set $\mathcal{C} : \Delta^{\mathrm{op}} \to \mathbf{Set}$ satisfying the inner horn filling condition. ([Lur09], Def 1.1.2.4)	Category with objects $\mathcal{C}_0$, morphisms $\mathcal{C}_1$, morphisms between morphisms $\mathcal{C}_2$, etc. Inner horn filling defines a non-unique composition.
F-Cocartesian edge	$F : X \to S$ a functor, $f : x \to y$ a morphism in X is F-cocartesian if the induced map $$X_{f/} \to X_{x/} \times_{S_{F(y)/}} S_{F(f)/}$$ is an isomorphism of categories. ([nLa25c], Prop 2.4)	$F : X \to S$ an inner fibration of simplicial sets, $f : x \to y$ an edge in X is F-cartesian if the induced map $$X_{f/} \to X_{x/} \times_{S_{F(y)/}} S_{F(f)/}$$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1 / Prop 2.4.1.8)	Note that the definitions of an inner fibration and a Kan fibration are invariant under taking opposites. For other intuition, see: F-cartesian edge.
Colimit	A colimit for $F : J \to \mathcal{C}$ is an initial cocone on F.	A colimit for $F : X \to \mathcal{C}$ (X a simplicial set, $\mathcal{C}$ an ∞-category) is an initial object of $\mathcal{C}_{F/}$. ([Lur09], Def 1.2.13.4)	The obvious extension of the definition of the undercategory $\mathcal{C}_{C/}$ for $C : \{*\} \to \mathcal{C}$ to $\mathcal{C}_{/F}$ for an arbitrary functor $F : J \to \mathcal{C}$ ends up being exactly $\mathbf{Cocone}(F)$.
κ-Compact object	Let $C \in \mathcal{C}$, and let $j_C : \mathcal{C} \to \mathbf{Set}$ denote the functor represented by C. If $\mathcal{C}$ admits κ-filtered colimits, then C is κ-compact if j_C commutes with filtered colimits. ([Lur09], 5.3.4)	Let $C \in \mathcal{C}$, and let $j_C : \mathcal{C} \to \hat{\mathcal{S}}$ denote the functor represented by C. If $\mathcal{C}$ admits κ-filtered colimits, then C is κ-compact if j_C preserves κ-filtered colimits.[3] ([Lur09], Def 5.3.4.5)	[todo]
Essentially small category	$\mathcal{C}$ equivalent to a small category.	$\mathcal{C}$ categorically equivalent to a small ∞-category. [Lur25, Def 03SJ]	-
Essentially surjective functor	$F : \mathcal{C} \to \mathcal{D}$ is essentially surjective if for every $D \in \mathcal{D}$, there exists some $C \in \mathcal{C}$ with $FC \cong D$.	$F : \mathcal{C} \to \mathcal{D}$ is essentially surjective if $\mathrm{h}F : \mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is essentially surjective. ([Lur09], Def 1.2.10.1)	Essentially surjective up to homotopy.

[1] I.e. for any full subcategory $\mathcal{C}' \subseteq \mathcal{C}$ containing every element of $\mathcal{C}^\kappa$, if $\mathcal{C}'$ is stable under small κ-filtered colimits, then $\mathcal{C}' = \mathcal{C}$. ([Lur09], p328)

[2] (Should reconcile with [Lur09], 5.4.3).

[3] Lurie introduces the term *κ-continuous* for such functors, but in ordinary category theory this generally means a functor which preserves κ-small limits; a functor preserving κ-filtered colimits is called *κ-finitary*. I have thus steered clear of this term.

Faithful functor	$F : \mathcal{C} \to \mathcal{D}$ is faithful if $\mathrm{Hom}(X,Y) \to \mathrm{Hom}(FX,FY)$ is injective for all $X, Y \in \mathcal{C}$.	$F : \mathcal{C} \to \mathcal{D}$ is faithful if $\mathrm{h}F : \mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is faithful. ([Lur09], Def 1.2.10.1)	Faithful up to homotopy.
κ-Filtered category	For a regular cardinal κ, $\mathcal{C}$ is κ-filtered if, for every κ-small category J and every functor $F : J \to \mathcal{C}$, there exists a cocone on F.	For a regular cardinal κ, $\mathcal{C}$ is κ-filtered if, for every κ-small simplicial set X and every map $f : X \to \mathcal{C}$, there exists a map $\overline{f} : K^{\triangleright} \to \mathcal{C}$ extending f. ([Lur09], Def 5.3.1.7)	A cocone on F is a collection of compatible maps $(\lambda_j : F(j) \to C)$. Define $\overline{F} : J \star [0] \to \mathcal{C}$ to be F on J, send the cone point to C, and send the unique morphisms $*_j$ from $j \in J$ to the cone point to the λ_j. Conversely, if you have some $\overline{F}$ extending F, define $\lambda_j := F(*_j)$.
Final object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C' \to C$.	Object $C \in \mathcal{C}$ such that C is final in $\mathrm{h}\mathcal{C}$, regarded as an enriched category over $\mathcal{H}$. ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C' \to C$.
Full functor	$F : \mathcal{C} \to \mathcal{D}$ is full if $\mathrm{Hom}(X,Y) \to \mathrm{Hom}(FX,FY)$ is surjective for all $X, Y \in \mathcal{C}$.	$F : \mathcal{C} \to \mathcal{D}$ is full if $\mathrm{h}F : \mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is full. ([Lur09], Def 1.2.10.1)	Full up to homotopy.
Functor	Functor.	Natural transformation of simplicial sets. ([Lur09], 1.2.7)	-
Groupoid	Category whose morphisms are all invertible.	Kan complex.	Not only can you find (non-unique) 'composites', but you can also fill in diagrams like $\begin{tikzcd} C \arrow[r, "\mathrm{id}"] \arrow[d, "f"'] & C \\ D \arrow[ur, dashed] & \end{tikzcd}$ $\begin{tikzcd} C \arrow[r, "\mathrm{id}"] \arrow[d, dashed] & D \\ C \arrow[ur, "f"'] & \end{tikzcd}$
Initial object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C \to C'$.	Object $C \in \mathcal{C}$ such that C is initial in $\mathrm{h}\mathcal{C}$, regarded as an enriched category over $\mathcal{H}$. ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C \to C'$.
Join	$\mathcal{C} \star \mathcal{D}$ has objects $\mathrm{ob}\,\mathcal{C} \sqcup \mathrm{ob}\,\mathcal{D}$, and $\mathrm{Hom}_{\mathcal{C}\star\mathcal{D}}(X,Y)$ is given by: $\begin{cases} \mathrm{Hom}_{\mathcal{C}}(X,Y) & X, Y \in \mathcal{C}, \\ \mathrm{Hom}_{\mathcal{D}}(X,Y) & X, Y \in \mathcal{D}, \\ \emptyset & X \in \mathcal{D}, Y \in \mathcal{C}, \\ * & X \in \mathcal{C}, Y \in \mathcal{D}. \end{cases}$ ([Lur09], 1.2.8)	$\mathcal{C} \star \mathcal{D}$ has n-simplicies $(\mathcal{C} \star \mathcal{D}) = \mathcal{C}_n \cup \mathcal{D}_n \cup \bigcup_{i+j=n-1} \mathcal{C}_i \times \mathcal{D}_j$. The ith boundary map $d_i : (\mathcal{C} \star \mathcal{D})_n \to (\mathcal{C} \star \mathcal{D})_{n-1}$ is defined on $\mathcal{C}_n$ and $\mathcal{D}_n$ using the ith boundary map on $\mathcal{C}$ and $\mathcal{D}$. Given $\sigma \in S_j, \tau \in T_k$, $d_i(\sigma, \tau)$ is given by $$\begin{cases} (d_i\sigma, \tau) & i \leq j,\ j \neq 0, \\ (\sigma, d_{i-j-1}\tau) & i > j,\ k \neq 0. \end{cases}$$ If $j = 0$, then $d_0(\sigma, \tau) = \tau$, and if $k = 0$, then $d_n(\sigma, \tau) = \sigma$. ([Lur09], Def 1.2.8.1 / [nLa25d])	Objects are in both cases disjoint unions of objects from the two categories being joined. Morphisms are also exactly the same in both cases (you get all the morphisms from $\mathcal{C}$ and $\mathcal{D}$, plus a morphism from $c \to d$ for every pair $(c, d) \in \mathcal{C}_0 \times \mathcal{D}_0$). Whenever you have an n-simplex in $\mathcal{C}$ and an m-simplex in $\mathcal{D}$, you get an $(m+n+1)$-simplex in $\mathcal{C} \star \mathcal{D}$, so in particular $\Delta^n \star \Delta^m \cong \Delta^{m+n+1}$.
Left cone	$\mathcal{C}^{\triangleleft} := [0] \star \mathcal{C}$.	$\mathcal{C}^{\triangleleft} := \Delta^0 \star \mathcal{C}$. ([Lur09], Not 1.2.8.4)	$\mathcal{C}$ with extra vertex (cone point) added, as well as a map from that cone point to every other vertex in $\mathcal{C}$ (plus obligatory degenerate simplicies).

Left dualisable object	Object $C \in \mathcal{C}$ such that there exists some $C^* \in \mathcal{C}$ and maps $e : C \otimes C^* \to \mathbb{1}$, $c : \mathbb{1} \to C^* \otimes C$ such that the composites $C \to C \otimes C^* \otimes C \to C$ and $C^* \to C^* \otimes C \otimes C^* \to C^*$ are equal to the identity.[4]	Object $C \in \mathcal{C}$ such that there exists some $C^* \in \mathcal{C}$ and maps $e : C \otimes C^* \to \mathbb{1}$, $c : \mathbb{1} \to C^* \otimes C$ such that the composites $C \to C \otimes C^* \otimes C \to C$ and $C^* \to C^* \otimes C \otimes C^* \to C^*$ are homotopic to the identity.	C has a left dual up to homotopy.
Left Kan extension (along the inclusion of a full subcategory)	Given a commutative diagram $\begin{tikzcd} \mathcal{C}^0 \arrow[r, "F_0"] \arrow[d, hook, "\iota"'] & \mathcal{D} \\ \mathcal{C} \arrow[ur, dashed, "F"'] \end{tikzcd}$, F is a left Kan extension of F_0 along ι if there is a natural transformation $\eta : F_0 \to F\iota$ such that for any other pair $(G : \mathcal{C} \to \mathcal{D}, \gamma : F_0 \to G\iota)$, there exists a unique natural transformation $\alpha : F \to G$ such that $\gamma = (\alpha * \iota) \circ \eta$. ([Rie16], Def 6.1.1)	Given a commutative diagram $\begin{tikzcd} \mathcal{C}^0 \arrow[r, "F_0"] \arrow[d, hook, "\iota"'] & \mathcal{D} \\ \mathcal{C} \arrow[ur, dashed, "F"'] \end{tikzcd}$, F is a left Kan extension of F_0 along ι if for all $C \in \mathcal{C}$, the induced diagram $\begin{tikzcd} \mathcal{C}^0_{/C} \arrow[r, "F_C"] \arrow[d, hook] & \mathcal{D} \\ (\mathcal{C}^0_{/C})^{\triangleright} \arrow[ur, dashed] \end{tikzcd}$ exhibits FC as a colimit of F_C. ([Lur09], Def 4.3.2.2)	In the 1-categorical case, the colimits of $\mathcal{C}^0_{/C} \to \mathcal{C}^0 \xrightarrow{F_0} \mathcal{D}$ for each $C \in \mathcal{C}$ (if they all exist) define the left Kan extension of F_0 along ι ([Rie16], Thm 6.2.1)[5]. This is the case if and only if F is a *pointwise* Kan extension ([Rie16], Thm 6.3.7), so really the higher categorical version generalises pointwise left Kan extensions (along the inclusion of a full subcategory).
Limit	A limit for $F : J \to \mathcal{C}$ is a terminal cone on F.	A limit for $F : X \to \mathcal{C}$ (X a simplicial set, $\mathcal{C}$ an ∞-category) is a final object of $\mathcal{C}_{/F}$. ([Lur09], Def 1.2.13.4)	The obvious extension of the definition of the overcategory $\mathcal{C}_{/C}$ for $C : \{*\} \to \mathcal{C}$ to $\mathcal{C}_{/F}$ for an arbitrary functor $F : J \to \mathcal{C}$ ends up being exactly $\mathbf{Cone}(F)$.
Locally small category	For every $X, Y \in \mathcal{C}$, $\mathrm{Hom}(X, Y)$ is a set.	For every $X, Y \in \mathcal{C}$, the space $\mathrm{Hom}(X, Y)$ is essentially small. ([Lur09], Prop 5.4.1.7)	-

[4]The opposite convention (i.e. that this defines a *right* dualisable object) is taken in [Eti+15], and by many other authors. I'm attempting to follow Lurie's conventions as far as possible.
[5]I think Lurie is saying exactly the same thing in the ∞-categorical case, just in a slightly confusing way.

<table>
<tr>
<td>Monoidal category</td>
<td>Opfibration of categories $p : \mathcal{C}^{\otimes} \to \Delta^{\mathrm{op}}$ such that, for each $n \geq 0$, the associated functors $\mathcal{C}^{\otimes}_{[n]} \to \mathcal{C}^{\otimes}_{\{i,i+1\}}$ determine an equivalence of categories $\mathcal{C}^{\otimes}_{[n]} \to \mathcal{C}^{\otimes}_{\{0,1\}} \times ... \times \mathcal{C}^{\otimes}_{\{n-1,n\}} \simeq (\mathcal{C}^{\otimes}_{[1]})^n$.</td>
<td>Cocartesian fibration of simplicial sets $p : \mathcal{C}^{\otimes} \to N(\Delta)^{\mathrm{op}}$ such that, for each $n \geq 0$, the associated functors $\mathcal{C}^{\otimes}_{[n]} \to \mathcal{C}^{\otimes}_{\{i,i+1\}}$ determine an equivalence[6] of ∞-categories
$$\mathcal{C}^{\otimes}_{[n]} \to \mathcal{C}^{\otimes}_{\{0,1\}} \times ... \times \mathcal{C}^{\otimes}_{\{n-1,n\}} \simeq (\mathcal{C}^{\otimes}_{[1]})^n$$
([Lur07], Def 1.1.2)[7]</td>
<td>In the 1-categorical case, you recover the original category by setting $\mathcal{C} := \mathcal{C}^{\otimes}_{[1]}$. The unit is $\mathcal{C}^{\otimes}_{[0]}$, the tensor product $\otimes$ is induced by the outer inclusion $\{0 < 2\} = [1] \subseteq [2]$, the unitors and associators come from the commutative diagrams
$$\begin{tikzcd} \{0,1\} & \{0,1,2\} \arrow[l] \arrow[r] & \{1,2\} \\ & \{0,2\} \arrow[ul] \arrow[u] \arrow[ur] & \end{tikzcd}$$
$$\begin{tikzcd} \{0,3\} \arrow[r] \arrow[d] & \{0,1,3\} \arrow[d] \\ \{0,2,3\} \arrow[r] & \{0,1,2,3\} \end{tikzcd}$$
in Δ. Conversely, given a monoidal category $\mathcal{C}$, define $\mathcal{C}^{\otimes}$ to have objects finite sequences $[C_1, ..., C_n]$ of objects of $\mathcal{C}$, and a morphism $[C_1, ..., C_n] \to [C'_1, ..., C'_m]$ to be a map $[m] \to [n]$ and a collection of morphisms $C_{f(i-1)+1} \otimes \cdots \otimes C_{f(i)} \to C'_i$ for $1 \leq i \leq m$.[8] Then the forgetful functor to Δ^{op} is the required cocartesian fibration. ([Lur07], p5-6)</td>
</tr>
</table>

[6] Weak or categorical?

[7] I consider this a temporary definition, because I can't yet reconcile it with [Lur17], Def 4.1.1.10.

[8] You "tensor along the gap", if that makes any sense.

(Coloured) operad	• A collection of objects $\mathcal{O}$. • For every finite set I, every I-indexed collection of objects $\{X_i\}_{i\in I}$ of $\mathcal{O}$, and every $Y \in \mathcal{O}$, a set $\mathrm{Hom}(\{X_i\}_{i\in I}, Y)$. • For every map of finite sets $I \to J$ having fibres $\{I_j\}_{j\in J}$, every finite collection of objects $\{X_i\}_{i\in I}$, every finite collection of objects $\{Y_j\}_{j\in J}$, and every object $Z \in \mathcal{O}$, a composition map $\prod_j \mathrm{Hom}(\{X_i\}_{i\in I_j}, Y_j) \times \mathrm{Hom}(\{Y_j\}_j, Z) \to \mathrm{Hom}(\{X_i\}_i, Z)$, which is associative. • Units $\mathrm{id}_X \in \mathrm{Hom}(\{X\}, X)$. ([Lur17], Def 2.1.1.1)	Functor $p : \mathcal{O}^{\otimes} \to N(\mathbf{Fin}_*)$ between ∞-categories which satisfies the following conditions: • For every inert morphism $f : \langle m\rangle \to \langle n\rangle$ in $N(\mathbf{Fin}_*)$ and every object $C \in \mathcal{O}^{\otimes}_{\langle m\rangle}$, there exists a p-cocartesian morphism $\overline{f} : C \to C'$ in $\mathcal{O}^{\otimes}$ lifting f. • Let $C \in \mathcal{O}^{\otimes}_{\langle m\rangle}$ and $C' \in \mathcal{O}^{\otimes}_{\langle n\rangle}$ be objects, let $f : \langle m\rangle \to \langle n\rangle$ be a morphism in $\mathbf{Fin}_*$, and let $\mathrm{Hom}^f_{\mathcal{O}^{\otimes}}(C, C')$ be the union of those connected components of $\mathrm{Hom}_{\mathcal{O}^{\otimes}}(C, C')$ which lie over $f \in \mathrm{Hom}_{\mathbf{Fin}_*}(\langle m\rangle, \langle n\rangle)$. Choose p-cocartesian morphisms $C' \to C'_i$ lying over the inert morphisms $\rho^i : \langle n\rangle \to \langle 1\rangle$ for $1 \le i \le n$. Then the induced map $$\mathrm{Hom}^f_{\mathcal{O}^{\otimes}}(C, C') \to \prod_{1\le i\le n} \mathrm{Hom}^{\rho^i f}_{\mathcal{O}^{\otimes}}(C, C'_i)$$ is a homotopy equivalence. • For every finite collection of objects $C_1, ..., C_n \in \mathcal{O}^{\otimes}_{\langle 1\rangle}$, there exists an object $C \in \mathcal{O}^{\otimes}_{\langle n\rangle}$ and a collection of p-cocartesian morphisms $C \to C_i$ covering $\rho^i : \langle n\rangle \to \langle 1\rangle$.	[todo]
(Coloured) operad map	[todo]	For ∞-operads $p : \mathcal{O}^{\otimes} \to N(\mathbf{Fin}_*)$, $q : \mathcal{O}'^{\otimes} \to N(\mathbf{Fin}_*)$, an ∞-operad map $\mathcal{O}^{\otimes} \to \mathcal{O}'^{\otimes}$ is a map of simplicial sets $f : \mathcal{O}^{\otimes} \to \mathcal{O}'^{\otimes}$ such that the diagram below commutes, $$\begin{tikzcd} \mathcal{O}^{\otimes} \arrow[rr] \arrow[dr] & & \mathcal{O}'^{\otimes} \arrow[dl] \\ & N(\mathbf{Fin}_*) & \end{tikzcd}$$ and the functor f carries inert morphisms[9] in $\mathcal{O}^{\otimes}$ to inert morphisms in $\mathcal{O}'^{\otimes}$.	[todo]
Opposite category	$\mathcal{C}^{\mathrm{op}}$ has the same objects as $\mathcal{C}$, and $\mathrm{Hom}_{\mathcal{C}^{\mathrm{op}}}(X, Y) = \mathrm{Hom}_{\mathcal{C}}(Y, X)$.	$\mathcal{C}^{\mathrm{op}}_n = \mathcal{C}([n]^{\mathrm{op}})$, where $\{0 < 1 < ... < n\}^{\mathrm{op}} = \{0 > 1 > ... > n\}$. ([Lur09], 1.2.1)	A map $x \to y$ is an edge $\Delta^1 \to \mathcal{C}$ where $0 \mapsto x$ and $1 \mapsto y$. In $\mathcal{C}^{\mathrm{op}}$ 0 and 1 swap roles, so we instead get a map $y \to x$.

[9] I.e. p-cocartesian morphisms g with $p(g) : \langle m\rangle \to \langle n\rangle$ such that for each $i \in \langle n\rangle^{\circ}$, the inverse image $g^{-1}\{i\}$ has exactly one element ([Lur17] Def 2.1.1.8 and 2.1.2.3).

Overcategory	For $C \in \mathcal{C}$, the category $\mathcal{C}_{/C}$ satisfies the following universal property: for any category $\mathcal{D}$, there is a bijection $$\mathrm{Hom}(\mathcal{D}, \mathcal{C}_{/C}) \simeq \mathrm{Hom}_C(\mathcal{D} \star [0], \mathcal{C}),$$ where the subscript on the right indicates that we consider only those functors $\mathcal{D} \star [0] \to \mathcal{C}$ whose restriction to $[0]$ consides with C. ([Lur09], 1.2.9)	For $f : S \to \mathcal{C}$, S a simplicial set and $\mathcal{C}$ an ∞-category, the ∞-category $\mathcal{C}_{/f}$ satisfies the following universal property: for any simplicial set X, there is a bijection $$\mathrm{Hom}(X, \mathcal{C}_{/f}) \simeq \mathrm{Hom}_f(X \star S, \mathcal{C}),$$ where the subscript on the right indicates that we consider only those functors $X \star S \to \mathcal{C}$ whose restriction to S consides with f. Explicitly, $$(\mathcal{C}_{/f})_n := \mathrm{Hom}_f(\Delta^n \star S, \mathcal{C}).$$ ([Lur09], Prop 1.2.9.2)	If $S = \Delta^0$, writing $C \in \mathcal{C}$ for the object picked out by f, we have $(\mathcal{C}_{/C})_n = \mathrm{Hom}_C(\Delta^n \star \Delta^0, \mathcal{C}) \cong \mathrm{Hom}_C(\Delta^{n+1}, \mathcal{C})$ (where the subscript indicates that we only consider morphisms sending the $(n+1)$st vertex to C). In other words, the objects are maps to C, the morphisms are commuting triangles over C, and so on; these are exactly the objects and morphisms in the 1-categorical case.
Presentable category	[todo]	[todo]	[todo]
Presheaf	[todo]	[todo]	[todo]
Representable functor	[todo]	[todo]	[todo]
Right cone	$\mathcal{C}^{\triangleright} := \mathcal{C} \star [0]$.	$\mathcal{C}^{\triangleright} := \mathcal{C} \star \Delta^0$. ([Lur09], Not 1.2.8.4)	$\mathcal{C}$ with extra vertex (cone point) added, as well as a map from every other vertex in $\mathcal{C}$ to that cone point (plus obligatory degenerate simplicies).
Right dualisable object	Object $C \in \mathcal{C}$ such that there exists some ${}^*C \in \mathcal{C}$ and maps $e : {}^*C \otimes C \to \mathbb{1}$, $c : \mathbb{1} \to C \otimes {}^*C$ such that the composites $C \to C \otimes {}^*C \otimes C \to C$ and ${}^*C \to {}^*C \otimes C \otimes {}^*C \to {}^*C$ are equal to the identity.	Object $C \in \mathcal{C}$ such that there exists some ${}^*C \in \mathcal{C}$ and maps $e : {}^*C \otimes C \to \mathbb{1}$, $c : \mathbb{1} \to C \otimes {}^*C$ such that the composites $C \to C \otimes {}^*C \otimes C \to C$ and ${}^*C \to {}^*C \otimes C \otimes {}^*C \to {}^*C$ are homotopic to the identity.	C has a right dual up to homotopy.
Small category	$\mathcal{C}$ has a set's worth of objects, and between any two objects, a set's worth of morphisms.	$\mathcal{C}$ has a set's worth of nondegenerate simplicies.[10] [Lur25, Section 03PP]	-

[10] In other words, $\mathcal{C}$ is a simplicial set. So, by our definition, "∞-category" means the same as "small ∞-category".

Subcategory	Subcategory $\mathcal{C}' \subseteq \mathcal{C}$.	Subsimplicial set $\mathcal{C}' \subseteq \mathcal{C}$ arising as a pullback $\begin{tikzcd} \mathcal{C}' \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & \mathcal{C} \arrow[d] \\ N(\mathrm{h}\mathcal{C})' \arrow[r] & N(\mathrm{h}\mathcal{C}) \end{tikzcd}$ where $(\mathrm{h}\mathcal{C})' \subseteq \mathrm{h}\mathcal{C}$ is a subcategory. ([Lur09], 1.2.11)	Expected definition: Subsimplicial set $\mathcal{C}' \subseteq \mathcal{C}$ satisfying inner horn filling. These are actually equivalent: suppose we have a such a subsimplicial set. Then $\begin{tikzcd} \mathcal{C}' \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & \mathcal{C} \arrow[d] \\ N(\mathrm{h}\mathcal{C}') \arrow[r] & N(\mathrm{h}\mathcal{C}) \end{tikzcd}$ is a pullback. Conversely, suppose we're given a pullback as in the box to the left. Then the diagram $\begin{tikzcd} \Delta^n \arrow[drr, dashed] \arrow[dr, dashed] \arrow[ddr, dashed] & \Lambda^n_i \arrow[l, hook'] \arrow[d] \arrow[dr] & \\ & \mathcal{C}' \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & \mathcal{C} \arrow[d] \\ & N(\mathrm{h}\mathcal{C})' \arrow[r] & N(\mathrm{h}\mathcal{C}) \end{tikzcd}$ shows that $\mathcal{C}'$ also satisfies inner horn filling, where the outer two dotted maps come from horn filling for $\mathcal{C}$ and $N(\mathrm{h}\mathcal{C})'$, and the inner dotted map comes from the fact that the square is a pullback.
Symmetric monoidal category	Opfibration of categories $p : \mathcal{C}^{\otimes} \to \mathbf{Fin}_*$ such that, for each $n \geq 0$, the associated functors $\mathcal{C}^{\otimes}_{\langle n\rangle} \to \mathcal{C}^{\otimes}_{\{*,i\}}$ determine an equivalence of categories $\mathcal{C}^{\otimes}_{\langle n\rangle} \to \mathcal{C}^{\otimes}_{\{*,1\}} \times ... \times \mathcal{C}^{\otimes}_{\{*,n\}} \simeq (\mathcal{C}^{\otimes}_{\langle 1\rangle})^n$.	Cocartesian fibration of simplicial sets $p : \mathcal{C}^{\otimes} \to N(\mathbf{Fin}_*)$ such that, for each $n \geq 0$, the associated functors $\mathcal{C}^{\otimes}_{\langle 1\rangle} \to \mathcal{C}^{\otimes}_{\{*,i\}}$ determine an equivalence[11] of ∞-categories $$\mathcal{C}^{\otimes}_{\langle n\rangle} \to \mathcal{C}^{\otimes}_{\{*,1\}} \times ... \times \mathcal{C}^{\otimes}_{\{*,n\}} \simeq (\mathcal{C}^{\otimes}_{\langle 1\rangle})^n$$ ([Lur17], Def 2.0.0.7)	In the 1-categorical case, you recover the original category by setting $\mathcal{C} := \mathcal{C}^{\otimes}_{\langle 1\rangle}$. The unit is $\mathcal{C}^{\otimes}_{\langle 0\rangle}$, the tensor product $\otimes$ is induced by $\langle 2\rangle \to \langle 1\rangle$, the symmetric isomorphism comes from the swapping map $\langle 2\rangle \to \langle 2\rangle$, the unitors and associators come from similar commutative diagrams to those in the intuition for a monoidal ∞-category. Conversely, given a symmetric monoidal category $\mathcal{C}$, define $\mathcal{C}^{\otimes}$ to have objects finite sequences $[C_1, ..., C_n]$ of objects of $\mathcal{C}$, and a morphism $[C_1, ..., C_n] \to [C'_1, ..., C'_m]$ to be a map $\langle n\rangle \to \langle m\rangle$ and a collection of morphisms $\bigotimes_{i\in\alpha^{-1}\{j\}} C_i \to C'_j$ for $1 \leq j \leq m$. Then the forgetful functor to $\mathbf{Fin}_*$ is the required cocartesian fibration. ([Lur17], p165-168)
Symmetric monoidal functor	[todo]	Let $p : \mathcal{C}^{\otimes} \to N(\mathbf{Fin}_*)$, $q : \mathcal{D} \to N(\mathbf{Fin}_*)$ be symmetric monoidal ∞-categories. Then an ∞-operad map $f \in \mathbf{Alg}_{\mathcal{C}}(\mathcal{D})$ is a symmetric monoidal functor if it carries p-cocartesian morphisms to q-cocartesian morphisms. ([Lur17], Def 2.1.3.7)	[todo]
Topos	[todo]	[todo]	[todo]

[11] Weak or categorical?

Undercategory	For $C \in \mathcal{C}$, the category $\mathcal{C}_{C/}$ satisfies the following universal property: for any category $\mathcal{D}$, there is a bijection $$\mathrm{Hom}(\mathcal{D}, \mathcal{C}_{C/}) \simeq \mathrm{Hom}_C([0] \star \mathcal{D}, \mathcal{C}),$$ where the subscript on the right indicates that we consider only those functors $[0] \star \mathcal{D} \to \mathcal{C}$ whose restriction to $[0]$ consides with C. ([Lur09], 1.2.9)	For $f : S \to \mathcal{C}$, S a simplicial set and $\mathcal{C}$ an ∞-category, the ∞-category $\mathcal{C}_{f/}$ satisfies the following universal property: for any simplicial set X, there is a bijection $$\mathrm{Hom}(X, \mathcal{C}_{f/}) \simeq \mathrm{Hom}_f(S \star X, \mathcal{C}),$$ where the subscript on the right indicates that we consider only those functors $S \star X \to \mathcal{C}$ whose restriction to S consides with f. Explicitly, $$(\mathcal{C}_{f/})_n := \mathrm{Hom}_f(S \star \Delta^n, \mathcal{C}).$$ ([Lur09], Prop 1.2.9.2)	If $S = \Delta^0$, writing $C \in \mathcal{C}$ for the object picked out by f, we have $(\mathcal{C}_{C/})_n = \mathrm{Hom}_C(\Delta^0 \star \Delta^n, \mathcal{C}) \cong \mathrm{Hom}_C(\Delta^{n+1}, \mathcal{C})$ (where the subscript indicates that we only consider morphisms sending the 0th vertex to C). In other words, the objects are maps from C, the morphisms are commuting triangles under C, and so on; these are exactly the objects and morphisms in the 1-categorical case.

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Equivalences		
Name	Between	Definition
Strong equivalence	Topological categories $\mathcal{C}, \mathcal{D}$	$\mathcal{C} \to \mathcal{D}$ is an equivalnce in the sense of enriched category theory. ([Lur09], Def 1.1.3.1)
(Weak) equivalence	Topological categories $\mathcal{C}, \mathcal{D}$	The induced functor $\mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is an equivalence of $\mathcal{H}$-enriched categories. ([Lur09], Def 1.1.3.6)
Categorical equivalence	Simplicial sets X, S	The induced functor $\mathrm{h}X \to \mathrm{h}S$ is an equivalence of $\mathcal{H}$-enriched categories. ([Lur09], Def 1.1.5.14)
Weak (homotopy) equivalence	Simplicial sets X, S	The induced map $\lvert X\rvert \to \lvert S\rvert$ is a weak homotopy equivalence of topological spaces. ([Lur09], 1.1.4)
Equivalence	Simplicial categories $\mathcal{C}, \mathcal{D}$	The induced functor $\mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is an equivalence of $\mathcal{H}$-enriched categories. ([Lur09], Def 1.1.4.4)

Fibrations and anodyne morphisms		
Name	Describes	Definition
Acyclic Kan fibration	$f : X \to S$ map of simplicial sets	see: trivial Kan fibration. ([nLa23])
Anodyne	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p : Y \to T$ a Kan fibration, $\begin{tikzcd} X \arrow[r] \arrow[d, "f"'] & Y \arrow[d, "p"] \\ S \arrow[r] \arrow[ur, dashed] & T \end{tikzcd}$ there exists a dotted lift. ([Lur09], Ex 2.0.0.1)
Cartesian fibration	$f : X \to S$ map of simplicial sets	f is an inner fibration such that for every edge $g : x \to y$ of S and every vertex $\tilde{y}$ of X with $f(\tilde{y}) = y$, there exists an f-cartesian edge $\tilde{g} : \tilde{x} \to \tilde{y}$ with $f(\tilde{g}) = g$. ([Lur09], Def 2.4.2.1)
Categorical fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p : Y \to T$ both a cofibration and a categorical equivalence, $\begin{tikzcd} Y \arrow[r] \arrow[d, "p"'] & X \arrow[d, "f"] \\ T \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$ there exists a dotted lift. ([Lur09], p90)
Cocartesian fibration	$f : X \to S$ map of simplicial sets	f is an inner fibration such that for every edge $g : x \to y$ of S and every vertex $\tilde{x}$ of X with $f(\tilde{x}) = x$, there exists an f-cocartesian edge $\tilde{g} : \tilde{x} \to \tilde{y}$ with $f(\tilde{g}) = g$. ([Lur09], Def 2.4.2.1)
Cofibration	$f : X \to S$ map of simplicial sets	f is a monomorphism. ([Lur09], A.2.7)
Inner anodyne	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p : Y \to T$ an inner fibration, $\begin{tikzcd} X \arrow[r] \arrow[d, "f"'] & Y \arrow[d, "p"] \\ S \arrow[r] \arrow[ur, dashed] & T \end{tikzcd}$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)

Inner fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 < i < n$, $$\begin{tikzcd} \Lambda^n_i \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$$ there exists a dotted lift.
Isofibration	$F : \mathcal{C} \to \mathcal{D}$ map of ∞-categories	F is an inner fibration such that for all $C \in \mathcal{C}$ and every isomorphism $u : D \to FC$ in $\mathcal{D}$ (i.e. $[u]$ is an isomorphism in $\mathrm{h}\mathcal{D}$) there exists an isomorphism $\overline{u} : \overline{D} \to C$ in $\mathcal{C}$ such that $F(\overline{u}) = u$. [Lur25, Def 01EN]
(Kan) fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 \leq i \leq n$, $$\begin{tikzcd} \Lambda^n_i \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$$ there exists a dotted lift. ([Lur09], A.2.7)
Left anodyne	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p : Y \to T$ a left fibration, $$\begin{tikzcd} X \arrow[r] \arrow[d, "f"'] & Y \arrow[d, "p"] \\ S \arrow[r] \arrow[ur, dashed] & T \end{tikzcd}$$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Left fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 \leq i < n$, $$\begin{tikzcd} \Lambda^n_i \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Right anodyne	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p : Y \to T$ a right fibration, $$\begin{tikzcd} X \arrow[r] \arrow[d, "f"'] & Y \arrow[d, "p"] \\ S \arrow[r] \arrow[ur, dashed] & T \end{tikzcd}$$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Right fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 < i \leq n$, $$\begin{tikzcd} \Lambda^n_i \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)

Serre fibration	$f : Y \to Z$ map of topological spaces	For every solid arrow diagram as below, $$\begin{tikzcd} \{0\} \times \lvert\Delta^n\rvert \arrow[r] \arrow[d] & Y \arrow[d, "f"] \\ {[0,1]} \times \lvert\Delta^n\rvert \arrow[r] \arrow[ur, dashed] & Z \end{tikzcd}$$ there exists a dotted lift. [Lur25, Def 021R]
Trivial (Kan) fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, $$\begin{tikzcd} \partial\Delta^n \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$$ there exists a dotted lift. ([Lur25, Def 006W]/[Lur09], Def 2.0.0.2)

Nerves		
Name	Domain object	Definition
Nerve	Category $\mathcal{C}$	$(N\mathcal{C})_n = \{n\text{-composable strings of morphisms in } \mathcal{C}\}$. ([Lur09], p9)
Simplicial nerve	Simplicial category $\mathcal{C}$	$(N\mathcal{C})_n = \mathrm{Hom}_{\mathbf{Cat}_\Delta}(\mathfrak{C}[\Delta^n], \mathcal{C})$, where $\mathfrak{C}[\Delta^n]$ is the category whose objects are the same as $[n]$, and $\mathrm{Hom}_{\mathfrak{C}[\Delta^n]}(i,j) = \emptyset$ for $i < j$ and $N(P_{ij})$ for $i \geq j$ (where $P_{ij} = \{I \subseteq [n] : (i,j \in I) \wedge (\forall k \in I, i \leq k \leq j)\}$). ([Lur09], Def 1.1.5.5)
Topological nerve	Topological category $\mathcal{C}$	The simplicial nerve of $\mathrm{Sing}\,\mathcal{C}$. ([Lur09], Def 1.1.5.5)

Homotopy categories	
Domain object	Definition
∞-Category $\mathcal{C}$	The objects of $\mathrm{h}\mathcal{C}$ are the vertices of $\mathcal{C}$, and $\mathrm{Hom}_{\mathrm{h}\mathcal{C}}(X,Y)$ is the set of homotopy classes of edges $X \to Y$ in $\mathcal{C}$. ([Lur09], Prop 1.2.3.9)
Simplicial category $\mathcal{C}$	$\mathrm{h}\lvert\mathcal{C}\rvert$. ([Lur09], 1.1.4)
Topological category $\mathcal{C}$	$\mathrm{h}\mathcal{C}$ has the same objects as $\mathcal{C}$, and $\mathrm{Hom}_{\mathrm{h}\mathcal{C}}(X,Y) = [\mathrm{Hom}_{\mathcal{C}}(X,Y)]$. ([Lur09], 1.1.3)

Objects	
Name	Definition
$\mathbf{Alg}_{\mathcal{C}}(\mathcal{D})$ ($p : \mathcal{C}^{\otimes} \to N(\mathbf{Fin}_*)$, $q : \mathcal{D}^{\otimes} \to N(\mathbf{Fin}_*)$ ∞-operads)	The full subcategory of $\mathbf{Fun}_{N(\mathbf{Fin}_*)}(\mathcal{C}^{\otimes}, \mathcal{D}^{\otimes})$[12] spanned by the ∞-operad maps. ([Lur17], Def 2.1.2.7)

[12] This notation is defined nowhere I can see, but must just mean maps $\mathcal{C}^{\otimes} \to \mathcal{D}^{\otimes}$ making the obvious triangle commute (but is this commuting on the nose, or only up to homotopy?).

Assoc (the associative operad)	The coloured operad with a single object $\mathfrak{a}$, and for every finite set I, $\mathrm{Hom}(\{\mathfrak{a}\}_i, \mathfrak{a})$ is the set of linear orderings on I. Given a map of finite sets $\alpha : I \to J$ together with operations $\phi_j \in \mathrm{Hom}(\{\mathfrak{a}\}_{\alpha(i)=j}, \mathfrak{a})$ and $\psi \in \mathrm{Hom}(\{\mathfrak{a}_j, \mathfrak{a}\})$, we identify each ϕ_j with a linear ordering $\leq_j$ on the set $\alpha^{-1}\{j\}$ and ψ with a linear ordering $\leq'$ on the set J. The composition of ψ with $\{\phi_j\}$ corresponds to the linear ordering $\leq$ on the set I which is defined by: $i \leq i'$ if either $\alpha(i) <_j \alpha(i')$ or $\alpha(i) = j = \alpha(i')$ and $i \leq_j i'$. ([Lur17], Def 4.1.1.1)
$\mathrm{Assoc}^{\otimes}$ (the associative ∞-operad)	$N(\mathbf{Assoc}^{\otimes})$. ([Lur17], Def 4.1.1.3)
$\mathbf{Assoc}^{\otimes}$	The category whose objects are the objects of $\mathbf{Fin}_*$, and a morphism $m \to n$ is given by a map $\alpha : \langle m \rangle \to \langle n \rangle$ in $\mathbf{Fin}_*$ together with a collection of linear orderings $\leq_j$ on $\alpha^{-1}\{j\}$, for $1 \leq j \leq n$. Composition of morphisms is determined by the composition laws on $\mathbf{Fin}_*$ and on **Assoc**. [Lur17], Def 4.1.1.3
$\mathbf{Fin}_*$	The category whose objects are the sets $\langle n \rangle = \{*, 1, 2, ..., n\}$, and a morphism $\langle m \rangle \to \langle n \rangle$ is a map $\alpha : \langle m \rangle \to \langle n \rangle$ such that $\alpha(*) = *$.
Kan	The full subcategory of **sSet** spanned by the collection of small Kan complexes. ([Lur09], Def 1.2.16.1)
KAN	The category of all Kan complexes. ([Lur09], Rem 5.1.6.1)
$\mathcal{S}$ (the ∞-category of spaces)	The simplicial[13]nerve $N(\mathbf{Kan})$. ([Lur09], Def 1.2.16.1)
$\widehat{\mathcal{S}}$	The simplicial nerve $N(\mathbf{KAN})$. ([Lur09], Rem 5.1.6.1)

[13]**sSet** is a simplicial category, with $\mathrm{Hom}(X, S)_n = \mathrm{Hom}_{\mathbf{sSet}}(\Delta^n \times X, S)$. The subcategory **Kan** inherits this structure.

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