

| Concept | 1-Categorical construction | ∞ -Categorical construction | Intuition |
|----------------------------|--|---|--|
| Accessible category | \mathcal{C} is locally small, admits κ -filtered colimits, and there is a set of κ -compact objects that generate the category under κ -filtered colimits. ([nLa25a], Def 2.1) | \mathcal{C} is locally small, admits κ -filtered colimits, the full subcategory $\mathcal{C}^\kappa \subseteq \mathcal{C}$ of κ -compact objects is essentially small, and \mathcal{C}^κ generates \mathcal{C} under small, κ -filtered colimits. ([Lur09], Prop 5.4.2.2) | [todo] |
| F -Cartesian edge | $F : X \rightarrow S$ a functor, $f : x \rightarrow y$ a morphism in X is F -cartesian if the induced map $X_{/f} \rightarrow X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$ is an isomorphism of categories. ([nLa25b], Prop 2.4) | $F : X \rightarrow S$ an inner fibration of simplicial sets, $f : x \rightarrow y$ an edge in X is F -cartesian if the induced map $X_{/f} \rightarrow X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1) | In the model structure on sSet , the fibrations are Kan fibrations and the weak equivalences are weak homotopy equivalences ([Lur09], A.2.7). A trivial fibration in a model category is a map which is both a fibration and a weak equivalence, which in sSet is equivalent to the definition given in this table. Thus, being related by a Kan fibration is a higher categorical notion of ‘sameness’. Why not a categorical equivalence? [Lur09] Rem 1.2.5.5 implies this is stronger, which would match more with the fact that the 1-categorical version is defined in terms of an isomorphism (not equivalence) of categories. |
| Category | Collection of objects \mathcal{C} , set $\text{Hom}(X, Y)$ for every $X, Y \in \mathcal{C}$, associative composition and identity morphisms | Simplicial set $\mathcal{C} : \Delta^{\text{op}} \rightarrow \mathbf{Set}$ satisfying the inner horn filling condition. ([Lur09], Def 1.1.2.4) | Category with objects \mathcal{C}_0 , morphisms \mathcal{C}_1 , morphisms between morphisms \mathcal{C}_2 , etc. Inner horn filling defines a non-unique composition. |
| F -Cocartesian edge | $F : X \rightarrow S$ a functor, $f : x \rightarrow y$ a morphism in X is F -cocartesian if the induced map $X_{f/} \rightarrow X_{x/} \times_{S_{F(y)/}} S_{F(f)/}$ is an isomorphism of categories. ([nLa25b], Prop 2.4) | $F : X \rightarrow S$ an inner fibration of simplicial sets, $f : x \rightarrow y$ an edge in X is F -cartesian if the induced map $X_{f/} \rightarrow X_{x/} \times_{S_{F(y)/}} S_{F(f)/}$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1 / Prop 2.4.1.8) | Note that the definitions of an inner fibration and a Kan fibration are invariant under taking opposites. For other intuition, see: F -cartesian edge. |
| Colimit | A colimit for $F : J \rightarrow \mathcal{C}$ is an initial cocone on F . | A colimit for $F : X \rightarrow \mathcal{C}$ (X a simplicial set, \mathcal{C} an ∞ -category) is an initial object of $\mathcal{C}_{F/}$. ([Lur09], Def 1.2.13.4) | The obvious extension of the definition of the undercategory $\mathcal{C}_{C/}$ for $C : \{*\} \rightarrow \mathcal{C}$ to $\mathcal{C}_{F/}$ for an arbitrary functor $F : J \rightarrow \mathcal{C}$ ends up being exactly Cocone (F). |
| κ -Compact object | Let $C \in \mathcal{C}$, and let $j_C : \mathcal{C} \rightarrow \mathbf{Set}$ denote the functor represented by C . If \mathcal{C} admits κ -filtered colimits, then C is κ -compact if j_C commutes with filtered colimits. ([Lur09], 5.3.4) | Let $C \in \mathcal{C}$, and let $j_C : \mathcal{C} \rightarrow \hat{S}$ denote the functor represented by C . If \mathcal{C} admits κ -filtered colimits, then C is κ -compact if j_C preserves κ -filtered colimits. ¹ ([Lur09], Def 5.3.4.5) | [todo] |
| Dual object | [todo] | [todo] | [todo] |
| Essentially small category | \mathcal{C} equivalent to a small category. | \mathcal{C} equivalent ² to a small ∞ -category. | [todo] |

¹Lurie introduces the term κ -continuous for such functors, but in ordinary category theory this generally means a functor which preserves κ -small limits; a functor preserving κ -filtered colimits is called κ -finitary. I have thus steered clear of this term.

²Categorically, or weakly?

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| Essentially surjective functor | $F : \mathcal{C} \rightarrow \mathcal{D}$ is essentially surjective if for every $D \in \mathcal{D}$, there exists some $C \in \mathcal{C}$ with $FC \cong D$. | $F : \mathcal{C} \rightarrow \mathcal{D}$ is essentially surjective if $hF : h\mathcal{C} \rightarrow h\mathcal{D}$ is essentially surjective. ([Lur09], Def 1.2.10.1) | Essentially surjective up to homotopy. |
| Faithful functor | $F : \mathcal{C} \rightarrow \mathcal{D}$ is faithful if $\text{Hom}(X, Y) \rightarrow \text{Hom}(FX, FY)$ is injective for all $X, Y \in \mathcal{C}$. | $F : \mathcal{C} \rightarrow \mathcal{D}$ is faithful if $hF : h\mathcal{C} \rightarrow h\mathcal{D}$ is faithful. ([Lur09], Def 1.2.10.1) | Faithful up to homotopy. |
| κ -Filtered category | For a regular cardinal κ , \mathcal{C} is κ -filtered if, for every κ -small category J and every functor $F : J \rightarrow \mathcal{C}$, there exists a cocone on F . | For a regular cardinal κ , \mathcal{C} is κ -filtered if, for every κ -small simplicial set X and every map $f : X \rightarrow \mathcal{C}$, there exists a map $\bar{f} : K^\triangleright \rightarrow \mathcal{C}$ extending f . ([Lur09], Def 5.3.1.7) | A cocone on F is a collection of compatible maps $(\lambda_j : F(j) \rightarrow C)$. Define $\bar{F} : J \star [0] \rightarrow \mathcal{C}$ to be F on J , send the cone point to C , and send the unique morphisms $*_j$ from $j \in J$ to the cone point to the λ_j . Conversely, if you have some \bar{F} extending F , define $\lambda_j := F(*_j)$. |
| Final object | Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C' \rightarrow C$. | Object $C \in \mathcal{C}$ such that C is final in $h\mathcal{C}$, regarded as an enriched category over \mathcal{H} . ([Lur09], Def 1.2.12.1) | Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C' \rightarrow C$. |
| Full functor | $F : \mathcal{C} \rightarrow \mathcal{D}$ is full if $\text{Hom}(X, Y) \rightarrow \text{Hom}(FX, FY)$ is surjective for all $X, Y \in \mathcal{C}$. | $F : \mathcal{C} \rightarrow \mathcal{D}$ is full if $hF : h\mathcal{C} \rightarrow h\mathcal{D}$ is full. ([Lur09], Def 1.2.10.1) | Full up to homotopy. |
| Functor | Functor. | Natural transformation of simplicial sets. ([Lur09], 1.2.7) | - |
| Groupoid | Category whose morphisms are all invertible. | Kan complex. | Not only can you find (non-unique) ‘composites’, but you can also fill in diagrams like $\begin{array}{ccc} C & \xrightarrow{\text{id}} & C \\ f \downarrow & \nearrow \text{dashed} & \\ D & & \end{array} \quad \begin{array}{ccc} C & \xrightarrow{\text{id}} & D \\ \downarrow \text{dashed} & \nearrow f & \\ C & & \end{array}$ |
| Initial object | Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C \rightarrow C'$. | Object $C \in \mathcal{C}$ such that C is initial in $h\mathcal{C}$, regarded as an enriched category over \mathcal{H} . ([Lur09], Def 1.2.12.1) | Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C \rightarrow C'$. |
| Join | $\mathcal{C} \star \mathcal{D}$ has objects $\text{ob } \mathcal{C} \sqcup \text{ob } \mathcal{D}$, and $\text{Hom}_{\mathcal{C} \star \mathcal{D}}(X, Y)$ is given by: $\begin{cases} \text{Hom}_{\mathcal{C}}(X, Y) & X, Y \in \mathcal{C}, \\ \text{Hom}_{\mathcal{D}}(X, Y) & X, Y \in \mathcal{D}, \\ \emptyset & X \in \mathcal{D}, Y \in \mathcal{C}, \\ * & X \in \mathcal{C}, Y \in \mathcal{D}. \end{cases}$ ([Lur09], 1.2.8) | $\mathcal{C} \star \mathcal{D}$ has n -simplices $(\mathcal{C} \star \mathcal{D}) = \mathcal{C}_n \cup \mathcal{D}_n \cup \bigcup_{i+j=n-1} \mathcal{C}_i \times \mathcal{D}_j$. The i th boundary map $d_i : (\mathcal{C} \star \mathcal{D})_n \rightarrow (\mathcal{C} \star \mathcal{D})_{n-1}$ is defined on \mathcal{C}_n and \mathcal{D}_n using the i th boundary map on \mathcal{C} and \mathcal{D} . Given $\sigma \in S_j, \tau \in T_k$, $d_i(\sigma, \tau)$ is given by $\begin{cases} (d_i\sigma, \tau) & i \leq j, j \neq 0, \\ (\sigma, d_{i-j-1}\tau) & i > j, k \neq 0. \end{cases}$ If $j = 0$, then $d_0(\sigma, \tau) = \tau$, and if $k = 0$, then $d_n(\sigma, \tau) = \sigma$. ([Lur09], Def 1.2.8.1 / [nLa25c]) | Objects are in both cases disjoint unions of objects from the two categories being joined. Morphisms are also exactly the same in both cases (you get all the morphisms from \mathcal{C} and \mathcal{D} , plus a morphism from $c \rightarrow d$ for every pair $(c, d) \in \mathcal{C}_0 \times \mathcal{D}_0$). Whenever you have an n -simplex in \mathcal{C} and an m -simplex in \mathcal{D} , you get an $(m+n+1)$ -simplex in $\mathcal{C} \star \mathcal{D}$, so in particular $\Delta^n \star \Delta^m \cong \Delta^{m+n+1}$. |
| Left cone | $\mathcal{C}^\triangleleft := [0] \star \mathcal{C}$. | $\mathcal{C}^\triangleleft := \Delta^0 \star \mathcal{C}$. ([Lur09], Not 1.2.8.4) | \mathcal{C} with extra vertex (cone point) added, as well as a map from that cone point to every other vertex in \mathcal{C} (plus obligatory degenerate simplices). |

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| Left Kan extension (along the inclusion of a full subcategory) | <p>Given a commutative diagram</p> $\begin{array}{ccc} \mathcal{C}^0 & \xrightarrow{F_0} & \mathcal{D} \\ \downarrow \iota & \nearrow F & \\ \mathcal{C} & & \end{array}$ <p>, F is a left Kan extension of F_0 along ι if there is a natural transformation $\eta : F_0 \rightarrow F\iota$ such that for any other pair $(G : \mathcal{C} \rightarrow \mathcal{D}, \gamma : F_0 \rightarrow G\iota)$, there exists a unique natural transformation $\alpha : F \rightarrow G$ such that $\gamma = (\alpha * \iota) \circ \eta$. ([Rie16], Def 6.1.1)</p> | <p>Given a commutative diagram</p> $\begin{array}{ccc} \mathcal{C}^0 & \xrightarrow{F_0} & \mathcal{D} \\ \downarrow \iota & \nearrow F & \\ \mathcal{C} & & \end{array}$ <p>tension of F_0 along ι if for all $C \in \mathcal{C}$, the induced diagram</p> $\begin{array}{ccc} \mathcal{C}_{/C}^0 & \xrightarrow{F_C} & \mathcal{D} \\ \downarrow & \nearrow & \\ (\mathcal{C}_{/C}^0)^{\triangleright} & & \end{array}$ <p>exhibits FC as a colimit of F_C. ([Lur09], Def 4.3.2.2)</p> | [todo] |
| Limit | A limit for $F : J \rightarrow \mathcal{C}$ is a terminal cone on F . | A limit for $F : X \rightarrow \mathcal{C}$ (X a simplicial set, \mathcal{C} an ∞ -category) is a final object of $\mathcal{C}_{/F}$. ([Lur09], Def 1.2.13.4) | The obvious extension of the definition of the overcategory $\mathcal{C}_{/C}$ for $C : \{*\} \rightarrow \mathcal{C}$ to $\mathcal{C}_{/F}$ for an arbitrary functor $F : J \rightarrow \mathcal{C}$ ends up being exactly $\mathbf{Cone}(F)$. |
| Locally small category | For every $X, Y \in \mathcal{C}$, $\text{Hom}(X, Y)$ is a set. | For every $X, Y \in \mathcal{C}$, the space $\text{Hom}(X, Y)$ is essentially small. ([Lur09], Prop 5.4.1.7) | [todo] |
| Monoidal category | [todo] | Cocartesian fibration of ∞ -operads $\mathcal{C}^{\otimes} \rightarrow \mathbf{Assoc}^{\otimes}$. ([Lur17], Def 4.1.1.10) | [todo] |
| (Coloured) operad | [todo] | [todo] | [todo] |
| Opposite category | \mathcal{C}^{op} has the same objects as \mathcal{C} , and $\text{Hom}_{\mathcal{C}^{\text{op}}}(X, Y) = \text{Hom}_{\mathcal{C}}(Y, X)$. | $\mathcal{C}_n^{\text{op}} = \mathcal{C}([n]^{\text{op}})$, where $\{0 < 1 < \dots < n\}^{\text{op}} = \{0 > 1 > \dots > n\}$. ([Lur09], 1.2.1) | A map $x \rightarrow y$ is an edge $\Delta^1 \rightarrow \mathcal{C}$ where $0 \mapsto x$ and $1 \mapsto y$. In \mathcal{C}^{op} 0 and 1 swap roles, so we instead get a map $y \rightarrow x$. |
| Overcategory | <p>For $C \in \mathcal{C}$, the category $\mathcal{C}_{/C}$ satisfies the following universal property: for any category \mathcal{D}, there is a bijection</p> $\text{Hom}(\mathcal{D}, \mathcal{C}_{/C}) \simeq \text{Hom}_{\mathcal{C}}(\mathcal{D} \star [0], \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors $\mathcal{D} \star [0] \rightarrow \mathcal{C}$ whose restriction to $[0]$ coincides with C. ([Lur09], 1.2.9)</p> | <p>For $f : S \rightarrow \mathcal{C}$, S a simplicial set and \mathcal{C} an ∞-category, the ∞-category $\mathcal{C}_{/f}$ satisfies the following universal property: for any simplicial set X, there is a bijection</p> $\text{Hom}(X, \mathcal{C}_{/f}) \simeq \text{Hom}_f(X \star S, \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors $X \star S \rightarrow \mathcal{C}$ whose restriction to S coincides with f. Explicitly,</p> $(\mathcal{C}_{/f})_n := \text{Hom}_f(\Delta^n \star S, \mathcal{C}).$ <p>([Lur09], Prop 1.2.9.2)</p> | If $S = \Delta^0$, writing $C \in \mathcal{C}$ for the object picked out by f , we have $(\mathcal{C}_{/C})_n = \text{Hom}_{\mathcal{C}}(\Delta^n \star \Delta^0, \mathcal{C}) \cong \text{Hom}_{\mathcal{C}}(\Delta^{n+1}, \mathcal{C})$ (where the subscript indicates that we only consider morphisms sending the $(n+1)$ st vertex to C). In other words, the objects are maps to C , the morphisms are commuting triangles over C , and so on; these are exactly the objects and morphisms in the 1-categorical case. |
| Presentable category | [todo] | [todo] | [todo] |
| Presheaf | [todo] | [todo] | [todo] |
| Representable functor | [todo] | [todo] | [todo] |
| Right cone | $\mathcal{C}^{\triangleright} := \mathcal{C} \star [0]$. | $\mathcal{C}^{\triangleright} := \mathcal{C} \star \Delta^0$. ([Lur09], Not 1.2.8.4) | \mathcal{C} with extra vertex (cone point) added, as well as a map from every other vertex in \mathcal{C} to that cone point (plus obligatory degenerate simplices). |

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| Subcategory | Subcategory $\mathcal{C}' \subseteq \mathcal{C}$. | Subsimplicial set $\mathcal{C}' \subseteq \mathcal{C}$ arising as $\begin{array}{ccc} \mathcal{C}' & \longrightarrow & \mathcal{C} \\ \downarrow \lrcorner & & \downarrow \\ N(\mathbf{h}\mathcal{C})' & \longrightarrow & N(\mathbf{h}\mathcal{C}) \end{array}$ a pullback where $(\mathbf{h}\mathcal{C})' \subseteq \mathbf{h}\mathcal{C}$ is a subcategory. ([Lur09], 1.2.11) | [todo] |
| Symmetric monoidal category | [todo] | [todo] | [todo] |
| Symmetric monoidal functor | [todo] | [todo] | [todo] |
| Topos | [todo] | [todo] | [todo] |
| Undercategory | For $C \in \mathcal{C}$, the category $\mathcal{C}_{C/}$ satisfies the following universal property: for any category \mathcal{D} , there is a bijection $\mathrm{Hom}(\mathcal{D}, \mathcal{C}_{C/}) \simeq \mathrm{Hom}_{\mathcal{C}}([0] \star \mathcal{D}, \mathcal{C}),$ where the subscript on the right indicates that we consider only those functors $[0] \star \mathcal{D} \rightarrow \mathcal{C}$ whose restriction to $[0]$ coincides with C . ([Lur09], 1.2.9) | For $f : S \rightarrow \mathcal{C}$, S a simplicial set and \mathcal{C} an ∞ -category, the ∞ -category $\mathcal{C}_{f/}$ satisfies the following universal property: for any simplicial set X , there is a bijection $\mathrm{Hom}(X, \mathcal{C}_{f/}) \simeq \mathrm{Hom}_f(S \star X, \mathcal{C}),$ where the subscript on the right indicates that we consider only those functors $S \star X \rightarrow \mathcal{C}$ whose restriction to S coincides with f . Explicitly, $(\mathcal{C}_{f/})_n := \mathrm{Hom}_f(S \star \Delta^n, \mathcal{C}).$ ([Lur09], Prop 1.2.9.2) | If $S = \Delta^0$, writing $C \in \mathcal{C}$ for the object picked out by f , we have $(\mathcal{C}_{C/})_n = \mathrm{Hom}_{\mathcal{C}}(\Delta^0 \star \Delta^n, \mathcal{C}) \cong \mathrm{Hom}_{\mathcal{C}}(\Delta^{n+1}, \mathcal{C})$ (where the subscript indicates that we only consider morphisms sending the 0th vertex to C). In other words, the objects are maps from C , the morphisms are commuting triangles under C , and so on; these are exactly the objects and morphisms in the 1-categorical case. |

| Equivalences | | |
|-----------------------------|---|---|
| Name | Between | Definition |
| Strong equivalence | Topological categories \mathcal{C}, \mathcal{D} | $\mathcal{C} \rightarrow \mathcal{D}$ is an equivalence in the sense of enriched category theory. ([Lur09], Def 1.1.3.1) |
| (Weak) equivalence | Topological categories \mathcal{C}, \mathcal{D} | The induced functor $\mathbf{h}\mathcal{C} \rightarrow \mathbf{h}\mathcal{D}$ is an equivalence of \mathcal{H} -enriched categories. ([Lur09], Def 1.1.3.6) |
| Categorical equivalence | Simplicial sets X, S | The induced functor $\mathbf{h}X \rightarrow \mathbf{h}S$ is an equivalence of \mathcal{H} -enriched categories. ([Lur09], Def 1.1.5.14) |
| Weak (homotopy) equivalence | Simplicial sets X, S | The induced map $ X \rightarrow S $ is a weak homotopy equivalence of topological spaces. ([Lur09], 1.1.4) |
| Equivalence | Simplicial categories \mathcal{C}, \mathcal{D} | The induced functor $\mathbf{h}\mathcal{C} \rightarrow \mathbf{h}\mathcal{D}$ is an equivalence of \mathcal{H} -enriched categories. ([Lur09], Def 1.1.4.4) |

| Fibrations and anodyne morphisms | | |
|----------------------------------|--|---------------------------------------|
| Name | Describes | Definition |
| Acyclic Kan fibration | $f : X \rightarrow S$ map of simplicial sets | see: trivial Kan fibration. ([nLa23]) |

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| Anodyne | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $p : Y \rightarrow T$ a Kan fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Ex 2.0.0.1)</p> |
| Cartesian fibration | $f : X \rightarrow S$ map of simplicial sets | <p>f is an inner fibration such that for every edge $g : x \rightarrow y$ of S and every vertex \tilde{y} of X with $f(\tilde{y}) = y$, there exists an f-cartesian edge $\tilde{g} : \tilde{x} \rightarrow \tilde{y}$ with $f(\tilde{g}) = g$. ([Lur09], Def 2.4.2.1)</p> |
| Categorical fibration | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $p : Y \rightarrow T$ both a cofibration and a categorical equivalence,</p> $\begin{array}{ccc} Y & \longrightarrow & X \\ p \downarrow & \nearrow & \downarrow f \\ T & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], p90)</p> |
| Cocartesian fibration | $f : X \rightarrow S$ map of simplicial sets | <p>f is an inner fibration such that for every edge $g : x \rightarrow y$ of S and every vertex \tilde{x} of X with $f(\tilde{x}) = x$, there exists an f-cocartesian edge $\tilde{g} : \tilde{x} \rightarrow \tilde{y}$ with $f(\tilde{g}) = g$. ([Lur09], Def 2.4.2.1)</p> |
| Cofibration | $f : X \rightarrow S$ map of simplicial sets | <p>f is a monomorphism. ([Lur09], A.2.7)</p> |
| Inner anodyne | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $p : Y \rightarrow T$ an inner fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p> |
| Inner fibration | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $0 < i < n$,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift.</p> |
| Isofibration | $F : \mathcal{C} \rightarrow \mathcal{D}$ map of ∞ -categories | <p>F is an inner fibration such that for all $C \in \mathcal{C}$ and every isomorphism $u : D \rightarrow FC$ in \mathcal{D} (i.e. $[u]$ is an isomorphism in $\mathbf{h}\mathcal{D}$) there exists an isomorphism $\bar{u} : \bar{D} \rightarrow C$ in \mathcal{C} such that $F(\bar{u}) = u$. [Lur25, Def 01EN]</p> |
| (Kan) fibration | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $0 \leq i \leq n$,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], A.2.7)</p> |

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| Left anodyne | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $p : Y \rightarrow T$ a left fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p> |
| Left fibration | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $0 \leq i < n$,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p> |
| Right anodyne | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $p : Y \rightarrow T$ a right fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p> |
| Right fibration | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below, with $0 < i \leq n$,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p> |
| Serre fibration | $f : Y \rightarrow Z$ map of topological spaces | <p>For every solid arrow diagram as below,</p> $\begin{array}{ccc} \{0\} \times \Delta^n & \longrightarrow & Y \\ \downarrow & \nearrow & \downarrow f \\ [0, 1] \times \Delta^n & \longrightarrow & Z \end{array}$ <p>there exists a dotted lift. [Lur25, Def 021R]</p> |
| Trivial (Kan) fibration | $f : X \rightarrow S$ map of simplicial sets | <p>For every solid arrow diagram as below,</p> $\begin{array}{ccc} \partial\Delta^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur25, Def 006W]/[Lur09], Def 2.0.0.2)</p> |

| Nerves | | |
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| Name | Domain object | Definition |
| Nerve | Category \mathcal{C} | $(N\mathcal{C})_n = \{n\text{-composable strings of morphisms in } \mathcal{C}\}$. |
| Simplicial nerve | Simplicial category \mathcal{C} | $(N\mathcal{C})_n = \text{Hom}_{\mathbf{Cat}_\Delta}(\mathfrak{C}[\Delta^n], \mathcal{C})$, where $\mathfrak{C}[\Delta^n]$ is the category whose objects are the same as $[n]$, and $\text{Hom}_{\mathfrak{C}[\Delta^n]}(i, j) = \emptyset$ for $i < j$ and $N(P_{ij})$ for $i \geq j$ (where $P_{ij} = \{I \subseteq [n] : (i, j \in I) \wedge (\forall k \in I, i \leq k \leq j)\}$). |
| Topological nerve | Topological category \mathcal{C} | The simplicial nerve of $\text{Sing } \mathcal{C}$. |

| Homotopy categories | |
|------------------------------------|--|
| Domain object | Definition |
| ∞ -Category \mathcal{C} | The objects of $\text{h}\mathcal{C}$ are the vertices of \mathcal{C} , and $\text{Hom}_{\text{h}\mathcal{C}}(X, Y)$ is the set of homotopy classes of edges $X \rightarrow Y$ in \mathcal{C} . ([Lur09], Prop 1.2.3.9) |
| Simplicial category \mathcal{C} | $\text{h} \mathcal{C} $. ([Lur09], 1.1.4) |
| Topological category \mathcal{C} | $\text{h}\mathcal{C}$ has the same objects as \mathcal{C} , and $\text{Hom}_{\text{h}\mathcal{C}}(X, Y) = [\text{Hom}_{\mathcal{C}}(X, Y)]$. ([Lur09], 1.1.3) |

| Categories | |
|--|---|
| Name | Definition |
| Assoc [⊗] | [todo] |
| Kan | The full subcategory of sSet spanned by the collection of small Kan complexes. ([Lur09], Def 1.2.16.1) |
| KAN | The category of all Kan complexes. ([Lur09], Rem 5.1.6.1) |
| \mathcal{S} (the ∞ -category of spaces) | The simplicial ³ nerve $N(\mathbf{Kan})$. |
| $\widehat{\mathcal{S}}$ | The simplicial nerve $N(\mathbf{KAN})$. |

³**sSet** is a simplicial category, with $\text{Hom}(X, S)_n = \text{Hom}_{\mathbf{sSet}}(\Delta^n \times X, S)$. The subcategory **Kan** inherits this structure.

References

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