

Concept	1-Categorical construction	$\infty$ -Categorical construction	Intuition
$F$ -Cartesian edge	<p><math>F : X \rightarrow S</math> a functor, <math>f : x \rightarrow y</math> a morphism in <math>X</math> is <math>F</math>-cartesian if the induced map</p> $X_{/f} \rightarrow X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$ <p>is an isomorphism of categories. ([nLa25], Prop 2.4)</p>	<p><math>F : X \rightarrow S</math> an inner fibration of simplicial sets, <math>f : x \rightarrow y</math> an edge in <math>X</math> is <math>F</math>-cartesian if the induced map</p> $X_{/f} \rightarrow X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$ <p>is a trivial Kan fibration. ([Lur09], Def 2.4.1.1)</p>	<p>In the model structure on <b>sSet</b>, the fibrations are Kan fibrations and the weak equivalences are weak homotopy equivalences ([Lur09], A.2.7). A trivial fibration in a model category is a map which is both a fibration and a weak equivalence, which in <b>sSet</b> is equivalent to the definition given in this table. Thus, being related by a Kan fibration is a higher categorical notion of ‘sameness’. <b>Why not a categorical equivalence?</b> [Lur09] Rem 1.2.5.5 implies this is stronger, which would match more with the fact that the 1-categorical version is defined in terms of an isomorphism (not equivalence) of categories.</p>
Category	Collection of objects $\mathcal{C}$ , set $\text{Hom}(X, Y)$ for every $X, Y \in \mathcal{C}$ , associative composition and identity morphisms	Simplicial set $\mathcal{C} : \Delta^{\text{op}} \rightarrow \mathbf{Set}$ satisfying the inner horn filling condition. ([Lur09], Def 1.1.2.4)	Category with objects $\mathcal{C}_0$ , morphisms $\mathcal{C}_1$ , morphisms between morphisms $\mathcal{C}_2$ , etc. Inner horn filling defines a non-unique composition.
Colimit	A colimit for $F : J \rightarrow \mathcal{C}$ is an initial cone on $F$ .	A colimit for $F : X \rightarrow \mathcal{C}$ ( $X$ a simplicial set, $\mathcal{C}$ an $\infty$ -category) is an initial object of $\mathcal{C}_{F/}$ . ([Lur09], Def 1.2.13.4)	??
Essentially surjective functor	$F : \mathcal{C} \rightarrow \mathcal{D}$ is essentially surjective if for every $D \in \mathcal{D}$ , there exists some $C \in \mathcal{C}$ with $FC \cong D$ .	$F : \mathcal{C} \rightarrow \mathcal{D}$ is essentially surjective if $\text{h}F : \text{h}\mathcal{C} \rightarrow \text{h}\mathcal{D}$ is essentially surjective. ([Lur09], Def 1.2.10.1)	Essentially surjective up to homotopy.
Faithful functor	$F : \mathcal{C} \rightarrow \mathcal{D}$ is faithful if $\text{Hom}(X, Y) \rightarrow \text{Hom}(FX, FY)$ is injective for all $X, Y \in \mathcal{C}$ .	$F : \mathcal{C} \rightarrow \mathcal{D}$ is faithful if $\text{h}F : \text{h}\mathcal{C} \rightarrow \text{h}\mathcal{D}$ is faithful. ([Lur09], Def 1.2.10.1)	Faithful up to homotopy.
Final object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$ , there exists a unique morphism $C' \rightarrow C$ .	Object $C \in \mathcal{C}$ such that $C$ is final in $\text{h}\mathcal{C}$ , regarded as an enriched category over $\mathcal{H}$ . ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$ , there exists a unique (up to homotopy) morphism $C' \rightarrow C$ .
Full functor	$F : \mathcal{C} \rightarrow \mathcal{D}$ is full if $\text{Hom}(X, Y) \rightarrow \text{Hom}(FX, FY)$ is surjective for all $X, Y \in \mathcal{C}$ .	$F : \mathcal{C} \rightarrow \mathcal{D}$ is full if $\text{h}F : \text{h}\mathcal{C} \rightarrow \text{h}\mathcal{D}$ is full. ([Lur09], Def 1.2.10.1)	Full up to homotopy.
Functor	Functor.	Natural transformation of simplicial sets. ([Lur09], 1.2.7)	
Groupoid	Category whose morphisms are all invertible.	Kan complex.	<p>Not only can you find (non-unique) ‘composites’, but you can also fill in diagrams like</p>
Initial object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$ , there exists a unique morphism $C \rightarrow C'$ .	Object $C \in \mathcal{C}$ such that $C$ is initial in $\text{h}\mathcal{C}$ , regarded as an enriched category over $\mathcal{H}$ . ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$ , there exists a unique (up to homotopy) morphism $C \rightarrow C'$ .

Join	$\mathcal{C} \star \mathcal{D}$ has objects $\text{ob } \mathcal{C} \sqcup \text{ob } \mathcal{D}$ , and $\text{Hom}_{\mathcal{C} \star \mathcal{D}}(X, Y)$ is given by: $\begin{cases} \text{Hom}_{\mathcal{C}}(X, Y) & X, Y \in \mathcal{C}, \\ \text{Hom}_{\mathcal{D}}(X, Y) & X, Y \in \mathcal{D}, \\ \emptyset & X \in \mathcal{D}, Y \in \mathcal{C}, \\ * & X \in \mathcal{C}, Y \in \mathcal{D}. \end{cases}$ ([Lur09], 1.2.8)	$\mathcal{C} \star \mathcal{D}$ has $n$ -simplices $(\mathcal{C} \star \mathcal{D}) = \mathcal{C}_n \cup \mathcal{D}_n \cup \bigcup_{i+j=n-1} \mathcal{C}_i \times \mathcal{D}_j$ . ([Lur09], Def 1.2.8.1)	??
Left cone	$\mathcal{C}^{\triangleleft} := [0] \star \mathcal{C}$ .	$\mathcal{C}^{\triangleleft} := \Delta^0 \star \mathcal{C}$ . ([Lur09], Not 1.2.8.4)	??
Left Kan extension (along the inclusion of a full subcategory)	<p>Given a commutative diagram</p> $\begin{array}{ccc} \mathcal{C}^0 & \xrightarrow{F_0} & \mathcal{D} \\ \downarrow \iota & \nearrow F & \\ \mathcal{C} & & \end{array}$ <p>, <math>F</math> is a left Kan extension of <math>F_0</math> along <math>\iota</math> if there is a natural transformation <math>\eta : F_0 \rightarrow F\iota</math> such that for any other pair <math>(G : \mathcal{C} \rightarrow \mathcal{D}, \gamma : F_0 \rightarrow G\iota)</math>, there exists a unique natural transformation <math>\alpha : F \rightarrow G</math> such that <math>\gamma = (\alpha * \iota) \circ \eta</math>. ([Rie16], Def 6.1.1)</p>	<p>Given a commutative diagram</p> $\begin{array}{ccc} \mathcal{C}^0 & \xrightarrow{F_0} & \mathcal{D} \\ \downarrow \iota & \nearrow F & \\ \mathcal{C} & & \end{array}$ <p>tension of <math>F_0</math> along <math>\iota</math> if for all <math>C \in \mathcal{C}</math>, the induced diagram</p> $\begin{array}{ccc} \mathcal{C}_{/C}^0 & \xrightarrow{F_C} & \mathcal{D} \\ \downarrow & \nearrow & \\ (\mathcal{C}_{/C}^0)^{\triangleright} & & \end{array}$ <p>exhibits <math>FC</math> as a colimit of <math>F_C</math>. ([Lur09], Def 4.3.2.2)</p>	??
Limit	A limit for $F : J \rightarrow \mathcal{C}$ is a terminal cone on $F$ .	A limit for $F : X \rightarrow \mathcal{C}$ ( $X$ a simplicial set, $\mathcal{C}$ an $\infty$ -category) is a final object of $\mathcal{C}_{/F}$ . ([Lur09], Def 1.2.13.4)	??
Opposite category	$\mathcal{C}^{\text{op}}$ has the same objects as $\mathcal{C}$ , and $\text{Hom}_{\mathcal{C}^{\text{op}}}(X, Y) = \text{Hom}_{\mathcal{C}}(Y, X)$ .	$\mathcal{C}_n^{\text{op}} = \mathcal{C}([n]^{\text{op}})$ , where $\{0 < 1 < \dots < n\}^{\text{op}} = \{0 > 1 > \dots > n\}$ . ([Lur09], 1.2.1)	A map $x \rightarrow y$ is an edge $\Delta^1 \rightarrow \mathcal{C}$ where $0 \mapsto x$ and $1 \mapsto y$ . In $\mathcal{C}^{\text{op}}$ 0 and 1 swap roles, so we instead get a map $y \rightarrow x$ .
Overcategory	<p>For <math>C \in \mathcal{C}</math>, the category <math>\mathcal{C}_{/C}</math> satisfies the following universal property: for any category <math>\mathcal{D}</math>, there is a bijection</p> $\text{Hom}(\mathcal{D}, \mathcal{C}_{/C}) \simeq \text{Hom}_{\mathcal{C}}(\mathcal{D} \star [0], \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors <math>\mathcal{D} \star [0] \rightarrow \mathcal{C}</math> whose restriction to <math>[0]</math> coincides with <math>C</math>. ([Lur09], 1.2.9)</p>	<p>For <math>f : S \rightarrow \mathcal{C}</math>, <math>S</math> a simplicial set and <math>\mathcal{C}</math> an <math>\infty</math>-category, the <math>\infty</math>-category <math>\mathcal{C}_{/f}</math> satisfies the following universal property: for any simplicial set <math>X</math>, there is a bijection</p> $\text{Hom}(X, \mathcal{C}_{/f}) \simeq \text{Hom}_f(X \star S, \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors <math>X \star S \rightarrow \mathcal{C}</math> whose restriction to <math>S</math> coincides with <math>f</math>. ([Lur09], Prop 1.2.9.2)</p>	??
Right cone	$\mathcal{C}^{\triangleright} := \mathcal{C} \star [0]$ .	$\mathcal{C}^{\triangleright} := \mathcal{C} \star \Delta^0$ . ([Lur09], Not 1.2.8.4)	??
Subcategory	Subcategory $\mathcal{C}' \subseteq \mathcal{C}$ .	<p>Subsimplicial set <math>\mathcal{C}' \subseteq \mathcal{C}</math> arising as a pullback</p> $\begin{array}{ccc} \mathcal{C}' & \longrightarrow & \mathcal{C} \\ \downarrow \lrcorner & & \downarrow \\ N(\text{h}\mathcal{C})' & \longrightarrow & N(\text{h}\mathcal{C}) \end{array}$ <p>where <math>(\text{h}\mathcal{C})' \subseteq \text{h}\mathcal{C}</math> is a subcategory. ([Lur09], 1.2.11)</p>	??

Undercategory	<p>For <math>C \in \mathcal{C}</math>, the category <math>\mathcal{C}_{C/}</math> satisfies the following universal property: for any category <math>\mathcal{D}</math>, there is a bijection</p> $\mathrm{Hom}(\mathcal{D}, \mathcal{C}_{C/}) \simeq \mathrm{Hom}_{\mathcal{C}}([0] \star \mathcal{D}, \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors <math>[0] \star \mathcal{D} \rightarrow \mathcal{C}</math> whose restriction to <math>[0]</math> coincides with <math>C</math>. ([Lur09], 1.2.9)</p>	<p>For <math>f : S \rightarrow \mathcal{C}</math>, <math>S</math> a simplicial set and <math>\mathcal{C}</math> an <math>\infty</math>-category, the <math>\infty</math>-category <math>\mathcal{C}_{f/}</math> satisfies the following universal property: for any simplicial set <math>X</math>, there is a bijection</p> $\mathrm{Hom}(X, \mathcal{C}_{f/}) \simeq \mathrm{Hom}_f(S \star X, \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors <math>S \star X \rightarrow \mathcal{C}</math> whose restriction to <math>S</math> coincides with <math>f</math>. ([Lur09], Prop 1.2.9.2)</p>	??
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Equivalences		
Name	Between	Definition
Strong equivalence	Topological categories $\mathcal{C}, \mathcal{D}$	$\mathcal{C} \rightarrow \mathcal{D}$ is an equivalence in the sense of enriched category theory. ([Lur09], Def 1.1.3.1)
(Weak) equivalence	Topological categories $\mathcal{C}, \mathcal{D}$	The induced functor $\mathrm{h}\mathcal{C} \rightarrow \mathrm{h}\mathcal{D}$ is an equivalence of $\mathcal{H}$ -enriched categories. ([Lur09], Def 1.1.3.6)
Categorical equivalence	Simplicial sets $X, S$	The induced functor $\mathrm{h}X \rightarrow \mathrm{h}S$ is an equivalence of $\mathcal{H}$ -enriched categories. ([Lur09], Def 1.1.5.14)
Weak (homotopy) equivalence	Simplicial sets $X, S$	The induced map $ X  \rightarrow  S $ is a weak homotopy equivalence of topological spaces. ([Lur09], 1.1.4)
Equivalence	Simplicial categories $\mathcal{C}, \mathcal{D}$	The induced functor $\mathrm{h}\mathcal{C} \rightarrow \mathrm{h}\mathcal{D}$ is an equivalence of $\mathcal{H}$ -enriched categories. ([Lur09], Def 1.1.4.4)

Fibrations and anodyne morphisms		
Name	Describes	Definition
Acyclic Kan fibration	$f : X \rightarrow S$ map of simplicial sets	see: trivial Kan fibration. ([nLa23])
Anodyne	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> a Kan fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Ex 2.0.0.1)</p>
Cartesian fibration	$f : X \rightarrow S$ map of simplicial sets	$F$ is an inner fibration such that for every edge $g : x \rightarrow y$ of $S$ and every vertex $\tilde{y}$ of $X$ with $f(\tilde{y}) = y$ , there exists an $f$ -cartesian edge $\tilde{g} : \tilde{x} \rightarrow \tilde{y}$ with $f(\tilde{g}) = g$ . ([Lur09], Def 2.4.2.1)
Cofibration	$f : X \rightarrow S$ map of simplicial sets	$f$ is a monomorphism. ([Lur09], A.2.7)

Inner anodyne	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> an inner fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>
Inner Fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>0 &lt; i &lt; n</math>,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift.</p>
Isofibration	$F : \mathcal{C} \rightarrow \mathcal{D}$ map of $\infty$ -categories	<p><math>F</math> is an inner fibration such that for all <math>C \in \mathcal{C}</math> and every isomorphism <math>u : D \rightarrow FC</math> in <math>\mathcal{D}</math> (i.e. <math>[u]</math> is an isomorphism in <math>\mathbf{h}\mathcal{D}</math>) there exists an isomorphism <math>\bar{u} : D \rightarrow C</math> in <math>\mathcal{C}</math> such that <math>F(\bar{u}) = u</math>. [Lur25, Def 01EN]</p>
(Kan) fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>0 \leq i \leq n</math>,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], A.2.7)</p>
Left anodyne	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> a left fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>
Left fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>0 \leq i &lt; n</math>,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>
Right anodyne	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> a right fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>

Right fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>0 &lt; i \leq n</math>,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>
Serre fibration	$f : Y \rightarrow Z$ map of topological spaces	<p>For every solid arrow diagram as below,</p> $\begin{array}{ccc} \{0\} \times  \Delta^n  & \longrightarrow & Y \\ \downarrow & \nearrow & \downarrow f \\ [0, 1] \times  \Delta^n  & \longrightarrow & Z \end{array}$ <p>there exists a dotted lift. [Lur25, Def 021R]</p>
Trivial (Kan) fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below,</p> $\begin{array}{ccc} \partial\Delta^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur25, Def 006W]/[Lur09], Def 2.0.0.2)</p>

Nerves		
Name	Domain object	Definition
Nerve	Category $\mathcal{C}$	$(N\mathcal{C})_n = \{n\text{-composable strings of morphisms in } \mathcal{C}\}$ .
Simplicial nerve	Simplicial category $\mathcal{C}$	$(N\mathcal{C})_n = \text{Hom}_{\mathbf{Cat}_\Delta}(\mathfrak{C}[\Delta^n], \mathcal{C})$ , where $\mathfrak{C}[\Delta^n]$ is the category whose objects are the same as $[n]$ , and $\text{Hom}_{\mathfrak{C}[\Delta^n]}(i, j) = \emptyset$ for $i < j$ and $N(P_{ij})$ for $i \geq j$ (where $P_{ij} = \{I \subseteq [n] : (i, j \in I) \wedge (\forall k \in I, i \leq k \leq j)\}$ ).
Topological nerve	Topological category $\mathcal{C}$	The simplicial nerve of $\text{Sing } \mathcal{C}$ .

Homotopy categories	
Domain object	Definition
$\infty$ -Category $\mathcal{C}$	The objects of $\text{h}\mathcal{C}$ are the vertices of $\mathcal{C}$ , and $\text{Hom}_{\text{h}\mathcal{C}}(X, Y)$ is the set of homotopy classes of edges $X \rightarrow Y$ in $\mathcal{C}$ . ([Lur09], Prop 1.2.3.9)
Simplicial category $\mathcal{C}$	$\text{h} \mathcal{C} $ . ([Lur09], 1.1.4)
Topological category $\mathcal{C}$	$\text{h}\mathcal{C}$ has the same objects as $\mathcal{C}$ , and $\text{Hom}_{\text{h}\mathcal{C}}(X, Y) = [\text{Hom}_{\mathcal{C}}(X, Y)]$ . ([Lur09], 1.1.3)

## References

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