Concept	1-Categorical construction	∞-Categorical construction	Intuition
F-Cartesian edge	$F : X \to S$ a functor, $f : x \to y$ a morphism in X is F-cartesian if the induced map $$X_{/f} \to X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$$ is an isomorphism of categories. ([nLa25a], Prop 2.4)	$F : X \to S$ an inner fibration of simplicial sets, $f : x \to y$ an edge in X is F-cartesian if the induced map $$X_{/f} \to X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1)	In the model structure on **sSet**, the fibrations are Kan fibrations and the weak equivalences are weak homotopy equivalences ([Lur09], A.2.7). A trivial fibration in a model category is a map which is both a fibration and a weak equivalence, which in **sSet** is equivalent to the definition given in this table. Thus, being related by a Kan fibration is a higher categorical notion of 'sameness'. Why not a categorical equivalence? [Lur09] Rem 1.2.5.5 implies this is stronger, which would match more with the fact that the 1-categorical version is defined in terms of an isomorphism (not equivalence) of categories.
Category	Collection of objects $\mathcal{C}$, set $\mathrm{Hom}(X, Y)$ for every $X, Y \in \mathcal{C}$, associative composition and identity morphisms	Simplicial set $\mathcal{C} : \Delta^{\mathrm{op}} \to \mathbf{Set}$ satisfying the inner horn filling condition. ([Lur09], Def 1.1.2.4)	Category with objects $\mathcal{C}_0$, morphisms $\mathcal{C}_1$, morphisms between morphisms $\mathcal{C}_2$, etc. Inner horn filling defines a non-unique composition.
Colimit	A colimit for $F : J \to \mathcal{C}$ is an initial cone on F.	A colimit for $F : X \to \mathcal{C}$ (X a simplicial set, $\mathcal{C}$ an ∞-category) is an initial object of $\mathcal{C}_{F/}$. ([Lur09], Def 1.2.13.4)	??
Essentially surjective functor	$F : \mathcal{C} \to \mathcal{D}$ is essentially surjective if for every $D \in \mathcal{D}$, there exists some $C \in \mathcal{C}$ with $FC \cong D$.	$F : \mathcal{C} \to \mathcal{D}$ is essentially surjective if $\mathrm{h}F : \mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is essentially surjective. ([Lur09], Def 1.2.10.1)	Essentially surjective up to homotopy.
Faithful functor	$F : \mathcal{C} \to \mathcal{D}$ is faithful if $\mathrm{Hom}(X, Y) \to \mathrm{Hom}(FX, FY)$ is injective for all $X, Y \in \mathcal{C}$.	$F : \mathcal{C} \to \mathcal{D}$ is faithful if $\mathrm{h}F : \mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is faithful. ([Lur09], Def 1.2.10.1)	Faithful up to homotopy.
Final object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C' \to C$.	Object $C \in \mathcal{C}$ such that C is final in $\mathrm{h}\mathcal{C}$, regarded as an enriched category over $\mathcal{H}$. ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C' \to C$.
Full functor	$F : \mathcal{C} \to \mathcal{D}$ is full if $\mathrm{Hom}(X, Y) \to \mathrm{Hom}(FX, FY)$ is surjective for all $X, Y \in \mathcal{C}$.	$F : \mathcal{C} \to \mathcal{D}$ is full if $\mathrm{h}F : \mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is full. ([Lur09], Def 1.2.10.1)	Full up to homotopy.
Functor	Functor.	Natural transformation of simplicial sets. ([Lur09], 1.2.7)	-
Groupoid	Category whose morphisms are all invertible.	Kan complex.	Not only can you find (non-unique) 'composites', but you can also fill in diagrams like $\begin{tikzcd} C \arrow[r, "\mathrm{id}"] \arrow[d, "f"'] & C \\ D \arrow[ur, dashed] & \end{tikzcd}$ $\begin{tikzcd} C \arrow[r, "\mathrm{id}"] \arrow[d, dashed] & D \\ C \arrow[ur, "f"'] & \end{tikzcd}$
Initial object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C \to C'$.	Object $C \in \mathcal{C}$ such that C is initial in $\mathrm{h}\mathcal{C}$, regarded as an enriched category over $\mathcal{H}$. ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C \to C'$.

<table>
<tr><td>Join</td><td>$\mathcal{C} \star \mathcal{D}$ has objects $\mathrm{ob}\,\mathcal{C} \sqcup \mathrm{ob}\,\mathcal{D}$, and $\mathrm{Hom}_{\mathcal{C}\star\mathcal{D}}(X,Y)$ is given by: $$\begin{cases} \mathrm{Hom}_{\mathcal{C}}(X,Y) & X,Y \in \mathcal{C}, \\ \mathrm{Hom}_{\mathcal{D}}(X,Y) & X,Y \in \mathcal{D}, \\ \emptyset & X \in \mathcal{D}, Y \in \mathcal{C}, \\ * & X \in \mathcal{C}, Y \in \mathcal{D}. \end{cases}$$ ([Lur09], 1.2.8)</td><td>$\mathcal{C} \star \mathcal{D}$ has n-simplicies $(\mathcal{C} \star \mathcal{D}) = \mathcal{C}_n \cup \mathcal{D}_n \cup \bigcup_{i+j=n-1} \mathcal{C}_i \times \mathcal{D}_j$. The ith boundary map $d_i : (\mathcal{C}\star\mathcal{D})_n \to (\mathcal{C}\star\mathcal{D})_{n-1}$ is defined on $\mathcal{C}_n$ and $\mathcal{D}_n$ using the ith boundary map on $\mathcal{C}$ and $\mathcal{D}$. Given $\sigma \in S_j, \tau \in T_k$, $d_i(\sigma,\tau)$ is given by $$\begin{cases} (d_i\sigma, \tau) & i \le j,\ j \ne 0, \\ (\sigma, d_{i-j-1}\tau) & i > j,\ k \ne 0. \end{cases}$$ If $j = 0$, then $d_0(\sigma,\tau) = \tau$, and if $k = 0$, then $d_n(\sigma,\tau) = \sigma$. ([Lur09], Def 1.2.8.1 / [nLa25b])</td><td>Objects are in both cases disjoint unions of objects from the two categories being joined. Morphisms are also exactly the same in both cases (you get all the morphisms from $\mathcal{C}$ and $\mathcal{D}$, plus a morphism from $c \to d$ for every pair $(c,d) \in \mathcal{C}_0 \times \mathcal{D}_0$). Whenever you have an n-simplex in $\mathcal{C}$ and an m-simplex in $\mathcal{D}$, you get an $(m+n+1)$-simplex in $\mathcal{C}\star\mathcal{D}$, so in particular $\Delta^n \star \Delta^m \cong \Delta^{m+n+1}$.</td></tr>
<tr><td>Left cone</td><td>$\mathcal{C}^{\triangleleft} := [0] \star \mathcal{C}$.</td><td>$\mathcal{C}^{\triangleleft} := \Delta^0 \star \mathcal{C}$. ([Lur09], Not 1.2.8.4)</td><td>Adds an extra vertex (cone point) to $\mathcal{C}$, with a map from that cone point to every other vertex in $\mathcal{C}$, but adds nothing else (except degenerate simplicies).</td></tr>
<tr><td>Left Kan extension (along the inclusion of a full subcategory)</td><td>Given a commutative diagram $\begin{tikzcd} \mathcal{C}^0 \arrow[r, "F_0"] \arrow[d, hook, "\iota"'] & \mathcal{D} \\ \mathcal{C} \arrow[ur, dashed, "F"'] \end{tikzcd}$, F is a left Kan extension of F_0 along ι if there is a natural transformation $\eta : F_0 \to F\iota$ such that for any other pair $(G : \mathcal{C} \to \mathcal{D}, \gamma : F_0 \to G\iota)$, there exists a unique natural transformation $\alpha : F \to G$ such that $\gamma = (\alpha * \iota) \circ \eta$. ([Rie16], Def 6.1.1)</td><td>Given a commutative diagram $\begin{tikzcd} \mathcal{C}^0 \arrow[r, "F_0"] \arrow[d, hook, "\iota"'] & \mathcal{D} \\ \mathcal{C} \arrow[ur, dashed, "F"'] \end{tikzcd}$, F is a left Kan extension of F_0 along ι if for all $C \in \mathcal{C}$, the induced diagram $\begin{tikzcd} \mathcal{C}^0_{/C} \arrow[r, "F_C"] \arrow[d, hook] & \mathcal{D} \\ (\mathcal{C}^0_{/C})^{\triangleright} \arrow[ur, dashed] \end{tikzcd}$ exhibits FC as a colimit of F_C. ([Lur09], Def 4.3.2.2)</td><td>??</td></tr>
<tr><td>Limit</td><td>A limit for $F : J \to \mathcal{C}$ is a terminal cone on F.</td><td>A limit for $F : X \to \mathcal{C}$ (X a simplicial set, $\mathcal{C}$ an ∞-category) is a final object of $\mathcal{C}_{/F}$. ([Lur09], Def 1.2.13.4)</td><td>??</td></tr>
<tr><td>Opposite category</td><td>$\mathcal{C}^{\mathrm{op}}$ has the same objects as $\mathcal{C}$, and $\mathrm{Hom}_{\mathcal{C}^{\mathrm{op}}}(X,Y) = \mathrm{Hom}_{\mathcal{C}}(Y,X)$.</td><td>$\mathcal{C}^{\mathrm{op}}_n = \mathcal{C}([n]^{\mathrm{op}})$, where $\{0 < 1 < \ldots < n\}^{\mathrm{op}} = \{0 > 1 > \ldots > n\}$. ([Lur09], 1.2.1)</td><td>A map $x \to y$ is an edge $\Delta^1 \to \mathcal{C}$ where $0 \mapsto x$ and $1 \mapsto y$. In $\mathcal{C}^{\mathrm{op}}$ 0 and 1 swap roles, so we instead get a map $y \to x$.</td></tr>
<tr><td>Overcategory</td><td>For $C \in \mathcal{C}$, the category $\mathcal{C}_{/C}$ satisfies the following universal property: for any category $\mathcal{D}$, there is a bijection $$\mathrm{Hom}(\mathcal{D}, \mathcal{C}_{/C}) \simeq \mathrm{Hom}_C(\mathcal{D} \star [0], \mathcal{C}),$$ where the subscript on the right indicates that we consider only those functors $\mathcal{D} \star [0] \to \mathcal{C}$ whose restriction to $[0]$ consides with C. ([Lur09], 1.2.9)</td><td>For $f : S \to \mathcal{C}$, S a simplicial set and $\mathcal{C}$ an ∞-category, the ∞-category $\mathcal{C}_{/f}$ satisfies the following universal property: for any simplicial set X, there is a bijection $$\mathrm{Hom}(X, \mathcal{C}_{/f}) \simeq \mathrm{Hom}_f(X \star S, \mathcal{C}),$$ where the subscript on the right indicates that we consider only those functors $X \star S \to \mathcal{C}$ whose restriction to S consides with f. ([Lur09], Prop 1.2.9.2)</td><td>??</td></tr>
<tr><td>Right cone</td><td>$\mathcal{C}^{\triangleright} := \mathcal{C} \star [0]$.</td><td>$\mathcal{C}^{\triangleright} := \mathcal{C} \star \Delta^0$. ([Lur09], Not 1.2.8.4)</td><td>Adds an extra vertex (cone point) to $\mathcal{C}$, with a map from every other vertex in $\mathcal{C}$ to that cone point, but adds nothing else (except degenerate simplicies).</td></tr>
</table>

<table>
<tr><td>Subcategory</td><td>Subcategory $\mathcal{C}' \subseteq \mathcal{C}$.</td><td>Subsimplicial set $\mathcal{C}' \subseteq \mathcal{C}$ arising as a pullback $$\begin{tikzcd} \mathcal{C}' \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & \mathcal{C} \arrow[d] \\ N(\mathrm{h}\mathcal{C})' \arrow[r] & N(\mathrm{h}\mathcal{C}) \end{tikzcd}$$ where $(\mathrm{h}\mathcal{C})' \subseteq \mathrm{h}\mathcal{C}$ is a subcategory. ([Lur09], 1.2.11)</td><td>??</td></tr>
<tr><td>Undercategory</td><td>For $C \in \mathcal{C}$, the category $\mathcal{C}_{C/}$ satisfies the following universal property: for any category $\mathcal{D}$, there is a bijection $$\mathrm{Hom}(\mathcal{D}, \mathcal{C}_{C/}) \simeq \mathrm{Hom}_C([0] \star \mathcal{D}, \mathcal{C}),$$ where the subscript on the right indicates that we consider only those functors $[0] \star \mathcal{D} \to \mathcal{C}$ whose restriction to $[0]$ consides with C. ([Lur09], 1.2.9)</td><td>For $f : S \to \mathcal{C}$, S a simplicial set and $\mathcal{C}$ an ∞-category, the ∞-category $\mathcal{C}_{f/}$ satisfies the following universal property: for any simplicial set X, there is a bijection $$\mathrm{Hom}(X, \mathcal{C}_{f/}) \simeq \mathrm{Hom}_f(S \star X, \mathcal{C}),$$ where the subscript on the right indicates that we consider only those functors $S \star X \to \mathcal{C}$ whose restriction to S consides with f. ([Lur09], Prop 1.2.9.2)</td><td>??</td></tr>
</table>

Equivalences		
Name	Between	Definition
Strong equivalence	Topological categories $\mathcal{C}, \mathcal{D}$	$\mathcal{C} \to \mathcal{D}$ is an equivalnce in the sense of enriched category theory. ([Lur09], Def 1.1.3.1)
(Weak) equivalence	Topological categories $\mathcal{C}, \mathcal{D}$	The induced functor $\mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is an equivalence of $\mathcal{H}$-enriched categories. ([Lur09], Def 1.1.3.6)
Categorical equivalence	Simplicial sets X, S	The induced functor $\mathrm{h}X \to \mathrm{h}S$ is an equivalence of $\mathcal{H}$-enriched categories. ([Lur09], Def 1.1.5.14)
Weak (homotopy) equivalence	Simplicial sets X, S	The induced map $\lvert X \rvert \to \lvert S \rvert$ is a weak homotopy equivalence of topological spaces. ([Lur09], 1.1.4)
Equivalence	Simplicial categories $\mathcal{C}, \mathcal{D}$	The induced functor $\mathrm{h}\mathcal{C} \to \mathrm{h}\mathcal{D}$ is an equivalence of $\mathcal{H}$-enriched categories. ([Lur09], Def 1.1.4.4)

<table>
<tr><th colspan="3">Fibrations and anodyne morphisms</th></tr>
<tr><th>Name</th><th>Describes</th><th>Definition</th></tr>
<tr><td>Acyclic Kan fibration</td><td>$f : X \to S$ map of simplicial sets</td><td>see: trivial Kan fibration. ([nLa23])</td></tr>
<tr><td>Anodyne</td><td>$f : X \to S$ map of simplicial sets</td><td>For every solid arrow diagram as below, with $p : Y \to T$ a Kan fibration, $$\begin{tikzcd} X \arrow[r] \arrow[d, "f"'] & Y \arrow[d, "p"] \\ S \arrow[r] \arrow[ur, dashed] & T \end{tikzcd}$$ there exists a dotted lift. ([Lur09], Ex 2.0.0.1)</td></tr>
<tr><td>Cartesian fibration</td><td>$f : X \to S$ map of simplicial sets</td><td>F is an inner fibration such that for every edge $g : x \to y$ of S and every vertex $\tilde{y}$ of X with $f(\tilde{y}) = y$, there exists an f-cartesian edge $\tilde{g} : \tilde{x} \to \tilde{y}$ with $f(\tilde{g}) = g$. ([Lur09], Def 2.4.2.1)</td></tr>
</table>

Cofibration	$f : X \to S$ map of simplicial sets	f is a monomorphism. ([Lur09], A.2.7)
Inner anodyne	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p : Y \to T$ an inner fibration, $\begin{tikzcd} X \arrow[r] \arrow[d, "f"'] & Y \arrow[d, "p"] \\ S \arrow[r] \arrow[ur, dashed] & T \end{tikzcd}$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Inner Fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 < i < n$, $\begin{tikzcd} \Lambda^n_i \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$ there exists a dotted lift.
Isofibration	$F : \mathcal{C} \to \mathcal{D}$ map of ∞-categories	F is an inner fibration such that for all $C \in \mathcal{C}$ and every isomorphism $u : D \to FC$ in $\mathcal{D}$ (i.e. $[u]$ is an isomorphism in $\mathrm{h}\mathcal{D}$) there exists an isomorphism $\overline{u} : \overline{D} \to C$ in $\mathcal{C}$ such that $F(\overline{u}) = u$. [Lur25, Def 01EN]
(Kan) fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 \leq i \leq n$, $\begin{tikzcd} \Lambda^n_i \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$ there exists a dotted lift. ([Lur09], A.2.7)
Left anodyne	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p : Y \to T$ a left fibration, $\begin{tikzcd} X \arrow[r] \arrow[d, "f"'] & Y \arrow[d, "p"] \\ S \arrow[r] \arrow[ur, dashed] & T \end{tikzcd}$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Left fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 \leq i < n$, $\begin{tikzcd} \Lambda^n_i \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Right anodyne	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p : Y \to T$ a right fibration, $\begin{tikzcd} X \arrow[r] \arrow[d, "f"'] & Y \arrow[d, "p"] \\ S \arrow[r] \arrow[ur, dashed] & T \end{tikzcd}$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)

Right fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 < i \leq n$, $\begin{tikzcd} \Lambda^n_i \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$ there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Serre fibration	$f : Y \to Z$ map of topological spaces	For every solid arrow diagram as below, $\begin{tikzcd} \{0\} \times \lvert\Delta^n\rvert \arrow[r] \arrow[d] & Y \arrow[d, "f"] \\ {[0,1]} \times \lvert\Delta^n\rvert \arrow[r] \arrow[ur, dashed] & Z \end{tikzcd}$ there exists a dotted lift. [Lur25, Def 021R]
Trivial (Kan) fibration	$f : X \to S$ map of simplicial sets	For every solid arrow diagram as below, $\begin{tikzcd} \partial\Delta^n \arrow[r] \arrow[d, hook] & X \arrow[d, "f"] \\ \Delta^n \arrow[r] \arrow[ur, dashed] & S \end{tikzcd}$ there exists a dotted lift. ([Lur25, Def 006W]/[Lur09], Def 2.0.0.2)

Nerves		
Name	Domain object	Definition
Nerve	Category $\mathcal{C}$	$(N\mathcal{C})_n = \{n\text{-composable strings of morphisms in } \mathcal{C}\}$.
Simplicial nerve	Simplicial category $\mathcal{C}$	$(N\mathcal{C})_n = \mathrm{Hom}_{\mathbf{Cat}_\Delta}(\mathfrak{C}[\Delta^n], \mathcal{C})$, where $\mathfrak{C}[\Delta^n]$ is the category whose objects are the same as $[n]$, and $\mathrm{Hom}_{\mathfrak{C}[\Delta^n]}(i,j) = \emptyset$ for $i < j$ and $N(P_{ij})$ for $i \geq j$ (where $P_{ij} = \{I \subseteq [n] : (i, j \in I) \wedge (\forall k \in I, i \leq k \leq j)\}$).
Topological nerve	Topological category $\mathcal{C}$	The simplicial nerve of $\mathrm{Sing}\, \mathcal{C}$.

Homotopy categories	
Domain object	Definition
∞-Category $\mathcal{C}$	The objects of $\mathrm{h}\mathcal{C}$ are the vertices of $\mathcal{C}$, and $\mathrm{Hom}_{\mathrm{h}\mathcal{C}}(X, Y)$ is the set of homotopy classes of edges $X \to Y$ in $\mathcal{C}$. ([Lur09], Prop 1.2.3.9)
Simplicial category $\mathcal{C}$	$\mathrm{h}\lvert\mathcal{C}\rvert$. ([Lur09], 1.1.4)
Topological category $\mathcal{C}$	$\mathrm{h}\mathcal{C}$ has the same objects as $\mathcal{C}$, and $\mathrm{Hom}_{\mathrm{h}\mathcal{C}}(X, Y) = [\mathrm{Hom}_{\mathcal{C}}(X, Y)]$. ([Lur09], 1.1.3)

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