Concept	1-Categorical construction	∞ -Categorical construction	Intuition
F-Cartesian edge	$F: X \to S$ a functor, $f: x \to y$ a morphism in X is F -cartesian if the induced map $X_{/f} \to X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$ is an isomorphism of categories. ([nLa25a], Prop 2.4)	$F: X \to S$ an inner fibration of simplicial sets, $f: x \to y$ an edge in X is F -cartesian if the induced map $ X_{/f} \to X_{/y} \times_{S_{/F(y)}} S_{/F(f)} $ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1)	In the model structure on sSet, the fibrations are Kan fibrations and the weak equivalences are weak homotopy equivalences ([Lur09], A.2.7). A trivial fibration in a model category is a map which is both a fibration and a weak equivalence, which in sSet is equivalent to the definition given in this table. Thus, being related by a Kan fibration is a higher categorical notion of 'sameness'. Why not a categorical equivalence? [Lur09] Rem 1.2.5.5 implies this is stronger, which would match more with the fact that the 1-categorical version is defined in terms of an isomorphism (not equivalence) of
Category	Collection of objects C , set $\operatorname{Hom}(X,Y)$ for every $X,Y\in C$, associative composition and identity morphisms	Simplicial set $C: \Delta^{\text{op}} \to \mathbf{Set}$ satisfying the inner horn filling condition. ([Lur09], Def 1.1.2.4)	categories. Category with objects C_0 , morphisms C_1 , morphisms between morphisms C_2 , etc. Inner horn filling defines a non-unique composition.
F-Cocartesian edge	$F: X \to S$ a functor, $f: x \to y$ a morphism in X is F -cocartesian if the induced map $X_{f/} \to X_{x/} \times_{S_{F(y)/}} S_{F(f)/}$ is an isomorphism of categories. ([nLa25a], Prop 2.4)	$F: X \to S$ an inner fibration of simplicial sets, $f: x \to y$ an edge in X is F -cartesian if the induced map $X_{f/} \to X_{x/} \times_{S_{F(y)/}} S_{F(f)/}$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1 / Prop 2.4.1.8)	Note that the definitions of an inner fibration and a Kan fibration are invariant under taking opposites. For other intuition, see: F-cartesian edge.
Colimit	A colimit for $F: J \to \mathcal{C}$ is an initial cone on F .	A colimit for $F: X \to \mathcal{C}$ (X a simplicial set, \mathcal{C} an ∞ -category) is an initial object of $\mathcal{C}_{F/}$. ([Lur09], Def 1.2.13.4)	??
Essentially surjective functor	$F: \mathcal{C} \to \mathcal{D}$ is essentially surjective if for every $D \in \mathcal{D}$, there exists some $C \in \mathcal{C}$ with $FC \cong D$.	$F: \mathcal{C} \to \mathcal{D}$ is essentially surjective if $hF: hC \to h\mathcal{D}$ is essentially surjective. ([Lur09], Def 1.2.10.1)	Essentially surjective up to homotopy.
Faithful functor	$F: \mathcal{C} \to \mathcal{D}$ is faithful if $\operatorname{Hom}(X,Y) \to \operatorname{Hom}(FX,FY)$ is injective for all $X,Y \in \mathcal{C}$.	$F: \mathcal{C} \to \mathcal{D}$ is faithful if $hF: h\mathcal{C} \to h\mathcal{D}$ is faithful. ([Lur09], Def 1.2.10.1)	Faithful up to homotopy.
Final object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C' \to C$.	Object $C \in \mathcal{C}$ such that C is final in $h\mathcal{C}$, regarded as an enriched category over \mathcal{H} . ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C' \to C$.
Full functor	$F: \mathcal{C} \to \mathcal{D}$ is full if $\operatorname{Hom}(X,Y) \to \operatorname{Hom}(FX,FY)$ is surjective for all $X,Y \in \mathcal{C}$.	$F: \mathcal{C} \to \mathcal{D}$ is full if $hF: h\mathcal{C} \to h\mathcal{D}$ is full. ([Lur09], Def 1.2.10.1)	Full up to homotopy.
Functor	Functor.	Natural transformation of simplicial sets. ([Lur09], 1.2.7)	-

Groupoid	Category whose morphisms are all	Kan complex.	Not only can you find (non-
Groupoid	invertible.	Ivan complex.	unique) 'composites', but you
	miverensie.		can also fill in diagrams like
			C id C C id D
			$ \begin{array}{c cccc} C & \longrightarrow & C & \longrightarrow & D \\ \downarrow & & & \downarrow & & \downarrow \\ D & & & & C & & & & & \\ \end{array} $
			f
Initial object	Object $C \in \mathcal{C}$ such that for any	Object $C \in \mathcal{C}$ such that C is ini-	Object $C \in \mathcal{C}$ such that for any
	other object $C' \in \mathcal{C}$, there exists a	tial in hC , regarded as an enriched	other object $C' \in \mathcal{C}$, there exists
	unique morphism $C \to C'$.	category over \mathcal{H} . ([Lur09], Def	a unique (up to homotopy) mor-
т •	2 0 1 1: + 12++10	1.2.12.1)	$\begin{array}{c} \text{phism } C \to C'. \\ \end{array}$
Join	$\mathcal{C} \star \mathcal{D}$ has objects ob $\mathcal{C} \sqcup$ ob \mathcal{D} ,	$\mathcal{C} \star \mathcal{D}$ has <i>n</i> -simplicies $(\mathcal{C} \star \mathcal{D}) = \mathcal{C} \times \mathcal{D}$. The	Objects are in both cases disjoint
	and $\operatorname{Hom}_{\mathcal{C}\star\mathcal{D}}(X,Y)$ is given by:	$ \mathcal{C}_n \cup \mathcal{D}_n \cup \bigcup_{i+j=n-1} \mathcal{C}_i \times \mathcal{D}_j. \text{ The } $ $ ith boundary map \ d_i : (\mathcal{C} \star \mathcal{D})_n \to $	unions of objects from the two categories being joined. Mor-
	$ \begin{cases} \operatorname{Hom}_{\mathcal{C}}(X,Y) & X,Y \in \mathcal{C}, \\ \end{array} $	$(\mathcal{C} \star \mathcal{D})_{n-1}$ is defined on \mathcal{C}_n and	phisms are also exactly the same
	$ \begin{cases} \operatorname{Hom}_{\mathcal{D}}(X,Y) & X,Y \in \mathcal{D}, \end{cases} $	\mathcal{D}_n using the <i>i</i> th boundary map	in both cases (you get all the
		on \mathcal{C} and \mathcal{D} . Given $\sigma \in S_j, \tau \in T_k$,	morphisms from \mathcal{C} and \mathcal{D} , plus a
		$d_i(\sigma,\tau)$ is given by	morphism from $c \to d$ for every
	([Lur09], 1.2.8)		pair $(c,d) \in \mathcal{C}_0 \times \mathcal{D}_0$). Whenever
		$\int (d_i \sigma, \tau) \qquad i \le j, \ j \ne 0,$	you have an n -simplex in \mathcal{C} and
		$\begin{cases} (d_i \sigma, \tau) & i \leq j, \ j \neq 0, \\ (\sigma, d_{i-j-1} \tau) & i > j, \ k \neq 0. \end{cases}$	an m -simplex in \mathcal{D} , you get an
			$(m+n+1)$ -simplex in $\mathcal{C}\star\mathcal{D}$, so in
		If $j = 0$, then $d_0(\sigma, \tau) = \tau$, and	particular $\Delta^n \star \Delta^m \cong \Delta^{m+n+1}$.
		if $k = 0$, then $d_n(\sigma, \tau) = \sigma$.	
		([Lur09], Def 1.2.8.1 / [nLa25b])	
Left cone	$\mathcal{C}^{\triangleleft} := [0] \star \mathcal{C}.$	$\mathcal{C}^{\triangleleft} := \Delta^0 \star \mathcal{C}.$ ([Lur09], Not	\mathcal{C} with extra vertex (cone point)
		1.2.8.4)	added, as well as a map from that
			cone point to every other vertex
			in \mathcal{C} (plus obligatory degenerate simplicies).
Left Kan extension	Given a commutative diagram	Given a commutative diagram	??
(along the inclusion	$C_0 = F_0 \times T$	$C_0 = F_0$	
of a full subcate-	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccc} & \mathcal{C}^0 & \xrightarrow{F_0} & \mathcal{D} \\ & & & & \\ & \downarrow & & & \\ & & & & \\ & & & &$	
gory)	\downarrow	\downarrow	
	$\ \mathcal{C}'$	C	
	tension of F_0 along ι if there is a		
	natural transformation $\eta: F_0 \to$	$C \in \mathcal{C}$, the induced diagram	
	$F\iota$ such that for any other pair	$C^0_{/C} \xrightarrow{F_C} \mathcal{D}$ exhibits FC as	
	$(G: \mathcal{C} \to \mathcal{D}, \gamma: F_0 \to G\iota)$, there	exhibits FC as	
	exists a unique natural transformation $\alpha : F \rightarrow G$ such that		
	$\gamma = (\alpha * \iota) \circ \eta$. ([Rie16], Def 6.1.1)	$\left (\mathcal{C}_{/C}^0)^{ hd} \right $	
	(a * b) * η. ([thero], Der 0.1.1)	a colimit of F_C . ([Lur09], Def	
		4.3.2.2)	
Limit	A limit for $F: J \to \mathcal{C}$ is a terminal	A limit for $F: X \to \mathcal{C}$ (X a sim-	??
	cone on F .	plicial set, \mathcal{C} an ∞ -category) is a	
		final object of $\mathcal{C}_{/F}$. ([Lur09], Def	
		1.2.13.4)	
Opposite category	\mathcal{C}^{op} has the same objects as \mathcal{C} , and	$\mathcal{C}_n^{\text{op}} = \mathcal{C}([n]^{\text{op}}), \text{ where } \{0 < 1 < 1 < 1 < 1 < 1 < 1 < 1 < 1 < 1 < $	A map $x \to y$ is an edge $\Delta^1 \to \mathcal{C}$
	$\operatorname{Hom}_{\mathcal{C}^{\operatorname{op}}}(X,Y) = \operatorname{Hom}_{\mathcal{C}}(Y,X).$	$ < n\}^{\text{op}} = \{0 > 1 > > n\}.$	where $0 \mapsto x$ and $1 \mapsto y$. In \mathcal{C}^{op}
		([Lur09], 1.2.1)	0 and 1 swap roles, so we instead
			get a map $y \to x$.

Overcategory	For $C \in \mathcal{C}$, the category $\mathcal{C}_{/C}$ satisfies the following universal property: for any category \mathcal{D} , there is a bijection $\operatorname{Hom}(\mathcal{D},\mathcal{C}_{/C}) \simeq \operatorname{Hom}_C(\mathcal{D} \star [0],\mathcal{C}),$ where the subscript on the right indicates that we consider only those functors $\mathcal{D} \star [0] \to \mathcal{C}$ whose restriction to $[0]$ consides with C . ([Lur09], 1.2.9)	For $f: S \to \mathcal{C}$, S a simplicial set and \mathcal{C} an ∞ -category, the ∞ -category $\mathcal{C}_{/f}$ satisfies the following universal property: for any simplicial set X , there is a bijection $\operatorname{Hom}(X,\mathcal{C}_{/f}) \simeq \operatorname{Hom}_f(X \star S,\mathcal{C}),$ where the subscript on the right indicates that we consider only those functors $X \star S \to \mathcal{C}$ whose restriction to S consides with f . ([Lur09], Prop 1.2.9.2)	??
Right cone	$\mathcal{C}^{ hd}:=\mathcal{C}\star[0].$	$ \begin{array}{ccc} \mathcal{C}^{\triangleright} &:= \mathcal{C} \star \Delta^{0}. & ([\text{Lur09}], \text{ Not} \\ 1.2.8.4) \end{array} $	\mathcal{C} with extra vertex (cone point) added, as well as a map from every other vertex in \mathcal{C} to that cone point (plus obligatory degenerate simplicies).
Subcategory	Subcategory $C' \subseteq C$.	Subsimplicial set $\mathcal{C}' \subseteq \mathcal{C}$ arising as $\mathcal{C}' \longrightarrow \mathcal{C}$ a pullback $\downarrow \qquad \qquad \downarrow$ $N(h\mathcal{C})' \longrightarrow N(h\mathcal{C})$ where $(h\mathcal{C})' \subseteq h\mathcal{C}$ is a subcategory. ([Lur09], 1.2.11)	??
Undercategory	For $C \in \mathcal{C}$, the category $\mathcal{C}_{C/}$ satisfies the following universal property: for any category \mathcal{D} , there is a bijection Hom $(\mathcal{D}, \mathcal{C}_{C/}) \simeq \operatorname{Hom}_{C}([0] \star \mathcal{D}, \mathcal{C})$, where the subscript on the right indicates that we consider only those functors $[0] \star \mathcal{D} \to \mathcal{C}$ whose restriction to $[0]$ consides with C . ([Lur09], 1.2.9)	For $f: S \to \mathcal{C}$, S a simplicial set and \mathcal{C} an ∞ -category, the ∞ -category $\mathcal{C}_{f/}$ satisfies the following universal property: for any simplicial set X , there is a bijection $\operatorname{Hom}(X,\mathcal{C}_{f/}) \simeq \operatorname{Hom}_f(S \star X,\mathcal{C}),$ where the subscript on the right indicates that we consider only those functors $S \star X \to \mathcal{C}$ whose restriction to S consides with f . ([Lur09], Prop 1.2.9.2)	??

Equivalences		
Name	Between	Definition
Strong equivalence	\parallel Topological categories \mathcal{C}, \mathcal{D}	$\mathcal{C} \to \mathcal{D}$ is an equivalnce in the sense of enriched
		category theory. ([Lur09], Def 1.1.3.1)
(Weak) equivalence	Topological categories \mathcal{C}, \mathcal{D}	The induced functor $h\mathcal{C} \to h\mathcal{D}$ is an equiva-
		lence of \mathcal{H} -enriched categories. ([Lur09], Def
		1.1.3.6)
Categorical equivalence	Simplicial sets X, S	The induced functor $hX \to hS$ is an equiva-
		lence of \mathcal{H} -enriched categories. ([Lur09], Def
		1.1.5.14)
Weak (homotopy) equivalence	Simplicial sets X, S	The induced map $ X \rightarrow S $ is a weak
		homotopy equivalence of topological spaces.
		([Lur09], 1.1.4)
Equivalence	Simplicial categories \mathcal{C}, \mathcal{D}	The induced functor $h\mathcal{C} \to h\mathcal{D}$ is an equiva-
		lence of \mathcal{H} -enriched categories. ([Lur09], Def
		1.1.4.4)

Fibrations and anodyne morphisms			
Name	Describes	Definition	
Acyclic Kan fibration	$f: X \to S$ map of simplicial sets	see: trivial Kan fibration. ([nLa23])	
Anodyne	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with	
		$p: Y \to T$ a Kan fibration,	
		$X \longrightarrow Y$	
		$ \begin{array}{ccc} f \downarrow & \downarrow p \\ S & \longrightarrow T \end{array} $	
		$S \longrightarrow T$	
		there exists a dotted lift. ([Lur09], Ex 2.0.0.1)	
Cartesian fibration	$f: X \to S$ map of simplicial sets	f is an inner fibration such that for every edge	
	J - 1	$g: x \to y$ of S and every vertex \tilde{y} of X with	
		$f(\tilde{y}) = y$, there exists an f-cartesian edge \tilde{g} :	
		$\tilde{x} \to \tilde{y}$ with $f(\tilde{g}) = g$. ([Lur09], Def 2.4.2.1)	
Categorical fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with	
		$p: Y \to T$ both a cofibration and a categorical	
		equivalence,	
		$Y \longrightarrow X$	
		$Y \longrightarrow X$ $\downarrow f$ $T \longrightarrow S$	
		1 1 1 1 1 1 1 1 1 1	
		$T \longrightarrow S$	
		there exists a dotted lift. ([Lur09], p90)	
Cocartesian fibration	$f: X \to S$ map of simplicial sets	f is an inner fibration such that for every edge	
		$g: x \to y$ of S and every vertex \tilde{x} of X with	
		$f(\tilde{x}) = x$, there exists an f-cocartesian edge	
9.01		$\tilde{g}: \tilde{x} \to \tilde{y} \text{ with } f(\tilde{g}) = g. \text{ ([Lur09], Def } 2.4.2.1)}$	
Cofibration	$f: X \to S \text{ map of simplicial sets}$ $f: X \to S \text{ map of simplicial sets}$	f is a monomorphism. ([Lur09], A.2.7)	
Inner anodyne	$J: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p: Y \to T$ an inner fibration,	
		p. 1 / 1 all limit instanton,	
		$X \longrightarrow Y$	
		$X \longrightarrow Y$ $f \downarrow \qquad \qquad \downarrow p$	
		$\downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \downarrow \qquad $	
		there exists a dotted lift. ([Lur09], Def 2.0.0.3)	
Inner fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with	
		0 < i < n,	
		$ \begin{array}{ccc} \Lambda_i^n & \longrightarrow X \\ \downarrow & & \downarrow f \end{array} $	
		$\Delta^n \xrightarrow{\qquad} S$	
		there exists a dotted lift.	
Isofibration	$F: \mathcal{C} \to \mathcal{D}$ map of ∞ -categories	there exists a dotted int. F is an inner fibration such that for all $C \in \mathcal{C}$	
	l	and every isomorphism $u: D \to FC$ in \mathcal{D}	
		(i.e. $[u]$ is an isomorphism in $h\mathcal{D}$) there exists	
		an isomorphism $\overline{u}:\overline{D}\to C$ in $\mathcal C$ such that	
		$F(\overline{u}) = u.$ [Lur25, Def 01EN]	
(Kan) fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with	
		$0 \le i \le n,$	
		$\Lambda_i^{r_i} \longrightarrow X$	
		$ \begin{array}{ccc} \Lambda_i^n & \longrightarrow X \\ \downarrow & & \downarrow f \end{array} $	
		$\Delta^n \xrightarrow{\dot{S}} \dot{S}$	
		there exists a dotted lift. ([Lur09], A.2.7)	
		uncre eaists a dotted int. ([Duivs], A.2.1)	

Left anodyne	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p: Y \to T$ a left fibration,
		$X \longrightarrow Y$ $f \downarrow \qquad \downarrow p$ $S \longrightarrow T$
I of flooring	f. V. Cf.ili.i.l	there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Left fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 \le i < n$,
		$ \begin{array}{ccc} \Lambda_i^n & \longrightarrow X \\ \downarrow & & \downarrow f \\ \Delta^n & \longrightarrow S \end{array} $
Di la		there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Right anodyne	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p: Y \to T$ a right fibration,
		$X \longrightarrow Y$ $f \downarrow \qquad \qquad \downarrow p$ $S \longrightarrow T$
		there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Right fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 < i \le n$,
		$\begin{array}{ccc} \Lambda_i^n & \longrightarrow X \\ & & \downarrow^f \\ \Delta^n & \longrightarrow S \end{array}$
		there exists a dotted lift. ([Lur09], Def 2.0.0.3)
Serre fibration	$f: Y \to Z$ map of topological spaces	For every solid arrow diagram as below,
		$\{0\} \times \Delta^n \longrightarrow Y$
		$ \begin{cases} \{0\} \times \Delta^n & \longrightarrow Y \\ \downarrow & \downarrow f \\ [0,1] \times \Delta^n & \longrightarrow Z \end{cases} $
		there exists a dotted lift. [Lur25, Def 021R]
Trivial (Kan) fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below,
		$ \begin{array}{ccc} \partial \Delta^n & \longrightarrow X \\ \downarrow & & \downarrow f \\ \Delta^n & \longrightarrow S \end{array} $
		there exists a dotted lift. ([Lur25, Def $006W$]/[Lur09], Def $2.0.0.2$)

Nerves		
Name	Domain object	Definition
Nerve	Category \mathcal{C}	$(NC)_n = \{n\text{-composable strings of morphisms}\}$
		$\inf C$.
Simplicial nerve	Simplicial category \mathcal{C}	$(N\mathcal{C})_n = \operatorname{Hom}_{\mathbf{Cat}_{\Delta}}(\mathfrak{C}[\Delta^n], \mathcal{C}), \text{ where } \mathfrak{C}[\Delta^n] \text{ is }$
		the category whose objects are the same as $[n]$,
		and $\operatorname{Hom}_{\mathfrak{C}[\Delta^n]}(i,j) = \emptyset$ for $i < j$ and $N(P_{ij})$
		for $i \geq j$ (where $P_{ij} = \{I \subseteq [n] : (i, j \in I) \land$
		$(\forall k \in I, i \le k \le j)\}).$
Topological nerve	Topological category \mathcal{C}	The simplicial nerve of $\operatorname{Sing} \mathcal{C}$.

Homotopy categories		
Domain object	Definition	
∞ -Category \mathcal{C}	The objects of hC are the vertices of C , and	
	$\operatorname{Hom}_{\mathrm{h}\mathcal{C}}(X,Y)$ is the set of homotopy classes of edges	
	$X \to Y \text{ in } \mathcal{C}. \ ([Lur09], Prop 1.2.3.9)$	
Simplicial category \mathcal{C}	h C . ([Lur09], 1.1.4)	
Topological category \mathcal{C}	hC has the same objects as C , and $Hom_{hC}(X,Y) =$	
	$[Hom_{\mathcal{C}}(X,Y)].$ ([Lur09], 1.1.3)	

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