Concept	1-Categorical construction	∞ -Categorical construction	Intuition
F-Cartesian edge	$F: X \to S$ a functor, $f: x \to y$ a morphism in X is F -cartesian if the induced map $X_{/f} \to X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$ is an isomorphism of categories. ([nLa25a], Prop 2.4)	$F: X \to S$ an inner fibration of simplicial sets, $f: x \to y$ an edge in X is F -cartesian if the induced map $ X_{/f} \to X_{/y} \times_{S_{/F(y)}} S_{/F(f)} $ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1)	In the model structure on sSet, the fibrations are Kan fibrations and the weak equivalences are weak homotopy equivalences ([Lur09], A.2.7). A trivial fibration in a model category is a map which is both a fibration and a weak equivalence, which in sSet is equivalent to the definition given in this table. Thus, being related by a Kan fibration is a higher categorical notion of 'sameness'. Why not a categorical equivalence? [Lur09] Rem 1.2.5.5 implies this is stronger, which would match more with the fact that the 1-categorical version is defined in terms of an isomorphism (not equivalence) of
Category	Collection of objects C , set $\operatorname{Hom}(X,Y)$ for every $X,Y\in C$, associative composition and identity morphisms	Simplicial set $C: \Delta^{op} \to \mathbf{Set}$ satisfying the inner horn filling condition. ([Lur09], Def 1.1.2.4)	categories. Category with objects C_0 , morphisms C_1 , morphisms between morphisms C_2 , etc. Inner horn filling defines a non-unique composition.
F-Cocartesian edge	$F: X \to S$ a functor, $f: x \to y$ a morphism in X is F -cocartesian if the induced map $X_{f/} \to X_{x/} \times_{S_{F(y)/}} S_{F(f)/}$ is an isomorphism of categories. ([nLa25a], Prop 2.4)	$F: X \to S$ an inner fibration of simplicial sets, $f: x \to y$ an edge in X is F -cartesian if the induced map $X_{f/} \to X_{x/} \times_{S_{F(y)/}} S_{F(f)/}$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1 / Prop 2.4.1.8)	Note that the definitions of an inner fibration and a Kan fibration are invariant under taking opposites. For other intuition, see: F-cartesian edge.
Colimit	A colimit for $F: J \to \mathcal{C}$ is an initial cone on F .	A colimit for $F: X \to \mathcal{C}$ (X a simplicial set, \mathcal{C} an ∞ -category) is an initial object of $\mathcal{C}_{F/}$. ([Lur09], Def 1.2.13.4)	[todo]
Dual object	[todo]	[todo]	[todo]
Essentially surjective functor	$F: \mathcal{C} \to \mathcal{D}$ is essentially surjective if for every $D \in \mathcal{D}$, there exists some $C \in \mathcal{C}$ with $FC \cong D$.	$F: \mathcal{C} \to \mathcal{D}$ is essentially surjective if $hF: hC \to h\mathcal{D}$ is essentially surjective. ([Lur09], Def 1.2.10.1)	Essentially surjective up to homotopy.
Faithful functor	$F: \mathcal{C} \to \mathcal{D}$ is faithful if $\operatorname{Hom}(X,Y) \to \operatorname{Hom}(FX,FY)$ is injective for all $X,Y \in \mathcal{C}$.	$F: \mathcal{C} \to \mathcal{D}$ is faithful if $hF: h\mathcal{C} \to h\mathcal{D}$ is faithful. ([Lur09], Def 1.2.10.1)	Faithful up to homotopy.
Final object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C' \to C$.	Object $C \in \mathcal{C}$ such that C is final in $h\mathcal{C}$, regarded as an enriched category over \mathcal{H} . ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C' \to C$.
Full functor	$F: \mathcal{C} \to \mathcal{D}$ is full if $\operatorname{Hom}(X,Y) \to \operatorname{Hom}(FX,FY)$ is surjective for all $X,Y \in \mathcal{C}$.	$F: \mathcal{C} \to \mathcal{D}$ is full if $hF: h\mathcal{C} \to h\mathcal{D}$ is full. ([Lur09], Def 1.2.10.1)	Full up to homotopy.
Functor	Functor.	Natural transformation of simplicial sets. ([Lur09], 1.2.7)	-

Groupoid	Category whose morphisms are all invertible.	Kan complex.	Not only can you find (non-unique) 'composites', but you can also fill in diagrams like $C \xrightarrow{\mathrm{id}} C C \xrightarrow{\mathrm{id}} D$ $\downarrow \qquad \qquad \downarrow \qquad \qquad \uparrow$ $D \qquad \qquad C$
Initial object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique morphism $C \to C'$.	Object $C \in \mathcal{C}$ such that C is initial in $h\mathcal{C}$, regarded as an enriched category over \mathcal{H} . ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$, there exists a unique (up to homotopy) morphism $C \to C'$.
Join	$ \begin{array}{c c} \mathcal{C}\star\mathcal{D} \text{ has objects ob } \mathcal{C}\sqcup\text{ob }\mathcal{D},\\ \text{and } \operatorname{Hom}_{\mathcal{C}\star\mathcal{D}}(X,Y) \text{ is given by:}\\ \left\{ \begin{aligned} &\operatorname{Hom}_{\mathcal{C}}(X,Y) & X,Y\in\mathcal{C},\\ &\operatorname{Hom}_{\mathcal{D}}(X,Y) & X,Y\in\mathcal{D},\\ &\emptyset & X\in\mathcal{D},Y\in\mathcal{C},\\ &\ast & X\in\mathcal{C},Y\in\mathcal{D}. \end{aligned} \right. \\ \left([\operatorname{Lur}09], 1.2.8 \right) $	$ \begin{array}{c} \mathcal{C}\star\mathcal{D} \text{ has } n\text{-simplicies } (\mathcal{C}\star\mathcal{D}) = \\ \mathcal{C}_n\cup\mathcal{D}_n\cup\bigcup_{i+j=n-1}\mathcal{C}_i\times\mathcal{D}_j. \text{ The } \\ i\text{th boundary map } d_i:(\mathcal{C}\star\mathcal{D})_n\to \\ (\mathcal{C}\star\mathcal{D})_{n-1} \text{ is defined on } \mathcal{C}_n \text{ and } \\ \mathcal{D}_n \text{ using the } i\text{th boundary map } \\ \text{on } \mathcal{C} \text{ and } \mathcal{D}. \text{ Given } \sigma\in S_j, \tau\in T_k, \\ d_i(\sigma,\tau) \text{ is given by } \\ \begin{cases} (d_i\sigma,\tau) & i\leq j,\ j\neq 0, \\ (\sigma,d_{i-j-1}\tau) & i>j,\ k\neq 0. \end{cases} \\ \text{If } j=0, \text{ then } d_0(\sigma,\tau)=\tau, \text{ and } \\ \text{if } k=0, \text{ then } d_n(\sigma,\tau)=\sigma. \\ ([\text{Lur09}], \text{ Def } 1.2.8.1\ / \text{ [nLa25b]}) \\ \end{array}$	Objects are in both cases disjoint unions of objects from the two categories being joined. Morphisms are also exactly the same in both cases (you get all the morphisms from \mathcal{C} and \mathcal{D} , plus a morphism from $c \to d$ for every pair $(c,d) \in \mathcal{C}_0 \times \mathcal{D}_0$). Whenever you have an n -simplex in \mathcal{C} and an m -simplex in \mathcal{D} , you get an $(m+n+1)$ -simplex in $\mathcal{C} \star \mathcal{D}$, so in particular $\Delta^n \star \Delta^m \cong \Delta^{m+n+1}$.
Left cone	$\mathcal{C}^{\triangleleft} := [0] \star \mathcal{C}.$	$\begin{array}{cccc} \mathcal{C}^{\lhd} &:= & \Delta^0 \star \mathcal{C}. & ([Lur09], & Not \\ 1.2.8.4) & & & \end{array}$	\mathcal{C} with extra vertex (cone point) added, as well as a map from that cone point to every other vertex in \mathcal{C} (plus obligatory degenerate simplicies).
Left Kan extension (along the inclusion of a full subcategory)	tension of F_0 along ι if there is a natural transformation $\eta: F_0 \to F\iota$ such that for any other pair $(G: \mathcal{C} \to \mathcal{D}, \gamma: F_0 \to G\iota)$, there exists a unique natural transformation $\alpha: F \to G$ such that $\gamma = (\alpha * \iota) \circ \eta$. ([Rie16], Def 6.1.1)	$C \in \mathcal{C}$, the induced diagram $C^0_{/C} \xrightarrow{F_C} \mathcal{D}$ exhibits FC as $(C^0_{/C})^{\triangleright}$ a colimit of F_C . ([Lur09], Def 4.3.2.2)	[todo]
Limit	A limit for $F: J \to \mathcal{C}$ is a terminal cone on F .	A limit for $F: X \to \mathcal{C}$ (X a simplicial set, \mathcal{C} an ∞ -category) is a final object of $\mathcal{C}_{/F}$. ([Lur09], Def 1.2.13.4)	[todo]
Monoidal category	[todo]	[todo]	[todo]
Operad	[todo]	[todo]	[todo]
Opposite category	\mathcal{C}^{op} has the same objects as \mathcal{C} , and $\text{Hom}_{\mathcal{C}^{\text{op}}}(X,Y) = \text{Hom}_{\mathcal{C}}(Y,X)$.	$C_n^{\text{op}} = C([n]^{\text{op}}), \text{ where } \{0 < 1 < < n\}^{\text{op}} = \{0 > 1 > > n\}.$ ([Lur09], 1.2.1)	A map $x \to y$ is an edge $\Delta^1 \to \mathcal{C}$ where $0 \mapsto x$ and $1 \mapsto y$. In \mathcal{C}^{op} 0 and 1 swap roles, so we instead get a map $y \to x$.

Overcategory	For $C \in \mathcal{C}$, the category $\mathcal{C}_{/C}$ satisfies the following universal property: for any category \mathcal{D} , there is a bijection $\operatorname{Hom}(\mathcal{D},\mathcal{C}_{/C}) \simeq \operatorname{Hom}_{C}(\mathcal{D}\star[0],\mathcal{C}),$	For $f: S \to \mathcal{C}$, S a simplicial set and \mathcal{C} an ∞ -category, the ∞ -category $\mathcal{C}_{/f}$ satisfies the following universal property: for any simplicial set X , there is a bijection $\operatorname{Hom}(X, \mathcal{C}_{/f}) \simeq \operatorname{Hom}_f(X \star S, \mathcal{C}),$	[todo]
	where the subscript on the right indicates that we consider only those functors $\mathcal{D} \star [0] \to \mathcal{C}$ whose restriction to [0] consides with C . ([Lur09], 1.2.9)	where the subscript on the right indicates that we consider only those functors $X \star S \to \mathcal{C}$ whose restriction to S consides with f .	
		([Lur09], Prop 1.2.9.2)	
Presentable category	[todo]	[todo]	[todo]
Right cone	$\mathcal{C}^{\triangleright} := \mathcal{C} \star [0].$	$\mathcal{C}^{\triangleright} := \mathcal{C} \star \Delta^{0}.$ ([Lur09], Not 1.2.8.4)	\mathcal{C} with extra vertex (cone point) added, as well as a map from every other vertex in \mathcal{C} to that cone point (plus obligatory degenerate simplicies).
Subcategory	Subcategory $C' \subseteq C$.	Subsimplicial set $\mathcal{C}' \subseteq \mathcal{C}$ arising as $\begin{array}{ccc} \mathcal{C}' & \longrightarrow \mathcal{C} \\ \text{a pullback} & \downarrow^{-1} & \downarrow \\ & N(\text{h}\mathcal{C})' & \longrightarrow N(\text{h}\mathcal{C}) \\ \text{where } (\text{h}\mathcal{C})' \subseteq \text{h}\mathcal{C} \text{ is a subcategory.} \\ ([\text{Lur09}], 1.2.11) \end{array}$	[todo]
Symmetric monoidal category	[todo]	[todo]	[todo]
Symmetric monoidal functor	[todo]	[todo]	[todo]
Topos	[todo]	[todo]	[todo]
Undercategory	For $C \in \mathcal{C}$, the category $\mathcal{C}_{C/}$ satisfies the following universal property: for any category \mathcal{D} , there is a bijection $\operatorname{Hom}(\mathcal{D},\mathcal{C}_{C/}) \simeq \operatorname{Hom}_{C}([0]\star\mathcal{D},\mathcal{C}),$ where the subscript on the right indicates that we consider only those functors $[0]\star\mathcal{D}\to\mathcal{C}$ whose restriction to $[0]$ consides with C . $([\operatorname{Lur09}], 1.2.9)$	For $f: S \to \mathcal{C}$, S a simplicial set and \mathcal{C} an ∞ -category, the ∞ -category $\mathcal{C}_{f/}$ satisfies the following universal property: for any simplicial set X , there is a bijection $\operatorname{Hom}(X,\mathcal{C}_{f/}) \simeq \operatorname{Hom}_f(S \star X,\mathcal{C}),$ where the subscript on the right indicates that we consider only those functors $S \star X \to \mathcal{C}$ whose restriction to S consides with f . ([Lur09], Prop 1.2.9.2)	[todo]

Equivalences		
Name	Between	Definition
Strong equivalence	Topological categories \mathcal{C}, \mathcal{D}	$\mathcal{C} \to \mathcal{D}$ is an equivalnce in the sense of enriched
		category theory. ([Lur09], Def 1.1.3.1)
(Weak) equivalence	Topological categories \mathcal{C}, \mathcal{D}	The induced functor $h\mathcal{C} \to h\mathcal{D}$ is an equiva-
		lence of \mathcal{H} -enriched categories. ([Lur09], Def
		1.1.3.6)
Categorical equivalence	Simplicial sets X, S	The induced functor $hX \to hS$ is an equiva-
		lence of \mathcal{H} -enriched categories. ([Lur09], Def
		1.1.5.14)
Weak (homotopy) equivalence	Simplicial sets X, S	The induced map $ X \rightarrow S $ is a weak
		homotopy equivalence of topological spaces.
		([Lur09], 1.1.4)
Equivalence	Simplicial categories \mathcal{C}, \mathcal{D}	The induced functor $h\mathcal{C} \to h\mathcal{D}$ is an equiva-
		lence of \mathcal{H} -enriched categories. ([Lur09], Def
		1.1.4.4)

Fibrations and anodyne morphisms		
Name	Describes	Definition
Acyclic Kan fibration	$f: X \to S$ map of simplicial sets	see: trivial Kan fibration. ([nLa23])
Anodyne	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with
		$p: Y \to T$ a Kan fibration,
		$X \longrightarrow Y$
		$ \begin{array}{ccc} f \downarrow & \downarrow p \\ S & \longrightarrow T \end{array} $
		$S \xrightarrow{\sim} T$
Cartesian fibration	$f: X \to S$ map of simplicial sets	there exists a dotted lift. ([Lur09], Ex 2.0.0.1) f is an inner fibration such that for every edge
Cartesian infration	$J: X \to S$ map of simplicial sets	$g: x \to y$ of S and every vertex \tilde{y} of X with
		$f(\tilde{y}) = y$, there exists an f-cartesian edge \tilde{g} :
		$\tilde{x} \rightarrow \tilde{y}$ with $f(\tilde{g}) = g$. ([Lur09], Def 2.4.2.1)
Categorical fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with
		$p: Y \to T$ both a cofibration and a categorical
		equivalence,
		$V \longrightarrow V$
		$Y \longrightarrow X$ $\downarrow f$ $T \longrightarrow S$
		$T \longrightarrow S$
		there exists a dotted lift. ([Lur09], p90)
Cocartesian fibration	$f: X \to S$ map of simplicial sets	f is an inner fibration such that for every edge
		$g: x \to y$ of S and every vertex \tilde{x} of X with
		$f(\tilde{x}) = x$, there exists an f-cocartesian edge
Cofibration	$f: X \to S$ map of simplicial sets	$\tilde{g}: \tilde{x} \to \tilde{y} \text{ with } f(\tilde{g}) = g. \text{ ([Lur09], Def 2.4.2.1)}$ $f \text{ is a monomorphism. ([Lur09], A.2.7)}$
Inner anodyne	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with
inner anoughe	y 111 / S map of simplicial sees	$p: Y \to T$ an inner fibration,
		,
		$X \longrightarrow Y$
		$f \downarrow p$
		$S \xrightarrow{\checkmark} T$
		there exists a dotted lift. ([Lur09], Def 2.0.0.3)

$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Inner fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 < i < n$,
$F: C \to \mathcal{D} \text{ map of } \infty\text{-categories} \qquad F \text{ is an inner fibration such that for all } C \in \mathcal{C}$ and every isomorphism $u: D \to FC$ in \mathcal{D} (i.e. $[u]$ is an isomorphism $u: D \to FC$ in \mathcal{D} (i.e. $[u]$ is an isomorphism $u: D \to C$ in C such that $F(\overline{u}) = u$. $[Luv25, De DiEN]$ (Kan) fibration $f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with $0 \le i \le n$, $A_i^n \longrightarrow X$ $\downarrow f$ $\downarrow f$ there exists a dotted lift. ([Luv0], A.2.7) For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text$			
$F: C \to \mathcal{D} \text{ map of } \infty\text{-categories} \qquad F \text{ is an inner fibration such that for all } C \in \mathcal{C}$ and every isomorphism $u: D \to FC$ in \mathcal{D} (i.e. $[u]$ is an isomorphism $u: D \to FC$ in \mathcal{D} (i.e. $[u]$ is an isomorphism $u: D \to C$ in C such that $F(\overline{u}) = u$. $[Luv25, De DiEN]$ (Kan) fibration $f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with $0 \le i \le n$, $A_i^n \longrightarrow X$ $\downarrow f$ $\downarrow f$ there exists a dotted lift. ([Luv0], A.2.7) For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text{ map of simplicial sets}$ For every solid arrow diagram as below, with f $\downarrow f: X \to S \text$			there exists a dotted lift
$f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with } 0 \le i \le n, \\ A_{i}^{p} \longrightarrow X \\ A_{i}^{p} \longrightarrow S \\ \text{there exists a dotted lift. ([Lur09], A.2.7)}$ Left anodyne $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with } p: Y \to T \text{ a left fibration}$ $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with } p: Y \to T \text{ a left fibration}$ For every solid arrow diagram as below, with $0 \le i < n, \qquad A_{i}^{n} \longrightarrow X \\ A_{i}^{n} \longrightarrow X \\ A_{i}^{n} \longrightarrow S \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)}$ Right anodyne $f: X \to S \text{ map of simplicial sets} \qquad \text{for every solid arrow diagram as below, with } p: Y \to T \text{ a right fibration},$ $X \longrightarrow Y \\ f \downarrow \qquad A_{i}^{n} \longrightarrow S \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)}$ Right fibration $f: X \to S \text{ map of simplicial sets} \qquad \text{for every solid arrow diagram as below, with } 0 < i \le n, \\ A_{i}^{n} \longrightarrow X \\ A_{i}^{n} \longrightarrow S$	Isofibration	$F:\mathcal{C} o \mathcal{D}$ map of ∞ -categories	F is an inner fibration such that for all $C \in \mathcal{C}$ and every isomorphism $u:D \to FC$ in \mathcal{D} (i.e. $[u]$ is an isomorphism in $h\mathcal{D}$) there exists an isomorphism $\overline{u}:\overline{D}\to C$ in \mathcal{C} such that
Left anodyne $f: X \to S \text{ map of simplicial sets} \qquad for every solid arrow diagram as below, with } p: Y \to T \text{ a left fibration}, \\ X \longrightarrow Y \\ \downarrow \downarrow \qquad \downarrow p \\ S \longrightarrow T \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } 0 \le i < n, \\ A_i^n \longrightarrow X \\ \downarrow A^n \longrightarrow S \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Right anodyne} \qquad f: X \to S \text{ map of simplicial sets} \qquad \text{for every solid arrow diagram as below, with } 0 \le i < n, \\ A_i^n \longrightarrow X \\ \downarrow A^n \longrightarrow S \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } p: Y \to T \text{ a right fibration}, \\ X \longrightarrow Y \\ \downarrow \downarrow \qquad \downarrow p \\ S \longrightarrow T \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } 0 < i \le n, \\ A_i^n \longrightarrow X \\ \downarrow \qquad \downarrow f \\ A^n \longrightarrow S \\ \text{here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } 0 < i \le n, \\ A_i^n \longrightarrow X \\ \downarrow \qquad \downarrow f \\ A^n \longrightarrow S \\ \text{here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } 0 < i \le n, \\ A_i^n \longrightarrow X \\ \downarrow \qquad \downarrow f \\ A^n \longrightarrow S \\ \text{here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } 0 < i \le n, \\ A_i^n \longrightarrow X \\ \downarrow \qquad \downarrow f \\ A^n \longrightarrow S \\ \text{here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } 0 < i \le n, \\ A_i^n \longrightarrow X \\ \downarrow \qquad \downarrow f \\ A^n \longrightarrow S \\ \text{here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Here exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ Here exists a dotted lif$	(Kan) fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 \le i \le n$,
Left anodyne $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with } \\ p: Y \to T \text{ a left fibration}, \\ X \longrightarrow Y \\ \downarrow \downarrow \\ S \longrightarrow T \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } \\ 0 \le i < n, \\ A_i^n \longrightarrow X \\ \downarrow \\ \Delta^n \longrightarrow S \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } \\ p: Y \to T \text{ a right fibration}, \\ Y \to Y \text{ a right fibration}, \\ X \longrightarrow Y \\ \downarrow \\ S \longrightarrow T \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Right fibration}$ $f: X \to S \text{ map of simplicial sets}$ $f: X \to S map of simplicia$			$ \begin{array}{ccc} \Lambda_i^n & \longrightarrow X \\ \downarrow & & \downarrow f \\ \Delta^n & \longrightarrow S \end{array} $
Left anodyne $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with } \\ p: Y \to T \text{ a left fibration}, \\ X \longrightarrow Y \\ \downarrow \downarrow \\ S \longrightarrow T \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } \\ 0 \le i < n, \\ A_i^n \longrightarrow X \\ \downarrow \\ \Delta^n \longrightarrow S \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{For every solid arrow diagram as below, with } \\ p: Y \to T \text{ a right fibration}, \\ Y \to Y \text{ a right fibration}, \\ X \longrightarrow Y \\ \downarrow \\ S \longrightarrow T \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Right fibration}$ $f: X \to S \text{ map of simplicial sets}$ $f: X \to S map of simplicia$			there exists a dotted lift. ([Lur09], A.2.7)
Left fibration $f: X \to S \text{ map of simplicial sets} \qquad \begin{array}{c} \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ For every solid arrow diagram as below, with \\ 0 \leq i < n, \\ & & & & & & \\ & & & & & \\ & & & & & $	Left anodyne	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $p: Y \to T$ a left fibration,
Left fibration $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with } 0 \le i < n, \\ & & & & & & \\ & & & & & \\ & & & & & $			$ \begin{array}{ccc} X & \longrightarrow & Y \\ \downarrow f & & \downarrow p \\ S & \longrightarrow & T \end{array} $
Left fibration $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with } 0 \le i < n, \\ & & & & & & \\ & & & & & \\ & & & & & $			there exists a dotted lift. ([Lur09], Def 2.0.0.3)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Left fibration	$f: X \to S$ map of simplicial sets	$0 \le i < n,$
Right anodyne $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with} \\ p: Y \to T \text{ a right fibration,} \\ X \longrightarrow Y \\ f \downarrow \\ S \longrightarrow T \\ \text{there exists a dotted lift. ([Lur09], Def 2.0.0.3)} \\ \text{Right fibration} \qquad f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with} \\ 0 < i \le n, \\ \Lambda_i^n \longrightarrow X \\ \downarrow \\ \Delta^n \longrightarrow S \\ \end{cases}$			$\begin{array}{c} \downarrow & \downarrow^f \\ \Delta^n \longrightarrow S \end{array}$
$X \longrightarrow Y \\ f \downarrow \qquad \downarrow p \\ S \longrightarrow T$ there exists a dotted lift. ([Lur09], Def 2.0.0.3) Right fibration $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with } 0 < i \leq n,$ $A_i^n \longrightarrow X \\ \downarrow \qquad \downarrow f \\ \Delta^n \longrightarrow S$	Right anodyne	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with
Right fibration $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with} \\ 0 < i \le n, \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ &$			$X \longrightarrow Y$
Right fibration $f: X \to S \text{ map of simplicial sets} \qquad \text{For every solid arrow diagram as below, with} \\ 0 < i \le n, \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ &$			there exists a dotted lift. ([Lur09], Def 2.0.0.3)
	Right fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below, with $0 < i \le n$,
there exists a dotted lift. ([Lur09], Def 2.0.0.3)			$ \begin{array}{ccc} \Lambda_i^n & \longrightarrow X \\ \downarrow & & \downarrow f \\ \Delta^n & \longrightarrow S \end{array} $
			there exists a dotted lift. ([Lur09], Def 2.0.0.3)

Serre fibration	$f: Y \to Z$ map of topological	For every solid arrow diagram as below,
	spaces	
		$\{0\} \times \Delta^n \longrightarrow Y$
		$\{0\} \times \Delta^n Y$ $\downarrow \qquad \qquad \downarrow f$
		$[0,1] \times \Delta^n \longrightarrow Z$
		there exists a dotted lift. [Lur25, Def 021R]
Trivial (Kan) fibration	$f: X \to S$ map of simplicial sets	For every solid arrow diagram as below,
		$\partial \Delta^n \longrightarrow X$
		$ \begin{array}{ccc} 0\Delta^{n} & \longrightarrow & X \\ \downarrow & & \downarrow f \\ \Delta^{n} & \longrightarrow & S \end{array} $
		there exists a dotted lift. ([Lur25, Def 006W]/[Lur09], Def 2.0.0.2)

Nerves		
Name	Domain object	Definition
Nerve	Category \mathcal{C}	$(NC)_n = \{n\text{-composable strings of morphisms}\}$
		$\operatorname{in} \mathcal{C}$.
Simplicial nerve	Simplicial category \mathcal{C}	$(N\mathcal{C})_n = \operatorname{Hom}_{\mathbf{Cat}_{\Delta}}(\mathfrak{C}[\Delta^n], \mathcal{C}), \text{ where } \mathfrak{C}[\Delta^n] \text{ is }$
		the category whose objects are the same as $[n]$,
		and $\operatorname{Hom}_{\mathfrak{C}[\Delta^n]}(i,j) = \emptyset$ for $i < j$ and $N(P_{ij})$
		for $i \geq j$ (where $P_{ij} = \{I \subseteq [n] : (i, j \in I) \land$
		$(\forall k \in I, i \le k \le j)\}).$
Topological nerve	Topological category \mathcal{C}	The simplicial nerve of Sing \mathcal{C} .

Homotopy categories		
Domain object Definition		
∞ -Category \mathcal{C} The objects of \mathcal{C} are the vertices of \mathcal{C} , and		
	$\operatorname{Hom}_{\mathrm{h}\mathcal{C}}(X,Y)$ is the set of homotopy classes of edges	
	$X \to Y$ in \mathcal{C} . ([Lur09], Prop 1.2.3.9)	
Simplicial category \mathcal{C}	h C . ([Lur09], 1.1.4)	
Topological category \mathcal{C}	hC has the same objects as C , and $Hom_{hC}(X,Y) =$	
	$[\operatorname{Hom}_{\mathcal{C}}(X,Y)]. \ ([\operatorname{Lur}09], \ 1.1.3)$	

References

- [Lur09] Jacob Lurie. Higher Topos Theory. 2009.
- [Lur25] Jacob Lurie. Kerodon. https://kerodon.net. 2025.
- [nLa23] nLab (Urs Schreiber). acyclic Kan fibration. https://ncatlab.org/nlab/show/acyclic+Kan+fibration. Revision 5. 2023.
- $[nLa25a] \quad nLab \ authors. \ \textit{Cartesian morphism}. \ \texttt{https://ncatlab.org/nlab/show/Cartesian+morphism}. \\ \quad Revision \ 52. \ 2025.$
- [nLa25b] nLab authors. join of simplicial sets. https://ncatlab.org/nlab/show/join+of+simplicial+sets. Revision 62. 2025.
- [Rie16] Emily Riehl. Category Theory in Context. 2016. URL: https://emilyriehl.github.io/files/context.pdf.