

Concept	1-Categorical construction	$\infty$ -Categorical construction	Intuition
Accessible category	$\mathcal{C}$ is locally small, admits $\kappa$ -filtered colimits, and there is a set of $\kappa$ -compact objects that generate the category under $\kappa$ -filtered colimits. ([nLa25a], Def 2.1)	$\mathcal{C}$ is locally small, admits $\kappa$ -filtered colimits, the full subcategory $\mathcal{C}^\kappa \subseteq \mathcal{C}$ of $\kappa$ -compact objects is essentially small, and $\mathcal{C}^\kappa$ generates $\mathcal{C}$ under small, $\kappa$ -filtered colimits. ([Lur09], Prop 5.4.2.2)	[todo]
$F$ -Cartesian edge	$F : X \rightarrow S$ a functor, $f : x \rightarrow y$ a morphism in $X$ is $F$ -cartesian if the induced map $X_{/f} \rightarrow X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$ is an isomorphism of categories. ([nLa25b], Prop 2.4)	$F : X \rightarrow S$ an inner fibration of simplicial sets, $f : x \rightarrow y$ an edge in $X$ is $F$ -cartesian if the induced map $X_{/f} \rightarrow X_{/y} \times_{S_{/F(y)}} S_{/F(f)}$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1)	In the model structure on <b>sSet</b> , the fibrations are Kan fibrations and the weak equivalences are weak homotopy equivalences ([Lur09], A.2.7). A trivial fibration in a model category is a map which is both a fibration and a weak equivalence, which in <b>sSet</b> is equivalent to the definition given in this table. Thus, being related by a Kan fibration is a higher categorical notion of ‘sameness’. Why not a categorical equivalence? [Lur09] Rem 1.2.5.5 implies this is stronger, which would match more with the fact that the 1-categorical version is defined in terms of an isomorphism (not equivalence) of categories.
Category	Collection of objects $\mathcal{C}$ , set $\text{Hom}(X, Y)$ for every $X, Y \in \mathcal{C}$ , associative composition and identity morphisms	Simplicial set $\mathcal{C} : \Delta^{\text{op}} \rightarrow \mathbf{Set}$ satisfying the inner horn filling condition. ([Lur09], Def 1.1.2.4)	Category with objects $\mathcal{C}_0$ , morphisms $\mathcal{C}_1$ , morphisms between morphisms $\mathcal{C}_2$ , etc. Inner horn filling defines a non-unique composition.
$F$ -Cocartesian edge	$F : X \rightarrow S$ a functor, $f : x \rightarrow y$ a morphism in $X$ is $F$ -cocartesian if the induced map $X_{f/} \rightarrow X_{x/} \times_{S_{/F(y)/}} S_{/F(f)/}$ is an isomorphism of categories. ([nLa25b], Prop 2.4)	$F : X \rightarrow S$ an inner fibration of simplicial sets, $f : x \rightarrow y$ an edge in $X$ is $F$ -cocartesian if the induced map $X_{f/} \rightarrow X_{x/} \times_{S_{/F(y)/}} S_{/F(f)/}$ is a trivial Kan fibration. ([Lur09], Def 2.4.1.1 / Prop 2.4.1.8)	Note that the definitions of an inner fibration and a Kan fibration are invariant under taking opposites. For other intuition, see: $F$ -cartesian edge.
Colimit	A colimit for $F : J \rightarrow \mathcal{C}$ is an initial cocone on $F$ .	A colimit for $F : X \rightarrow \mathcal{C}$ ( $X$ a simplicial set, $\mathcal{C}$ an $\infty$ -category) is an initial object of $\mathcal{C}_{F/}$ . ([Lur09], Def 1.2.13.4)	The obvious extension of the definition of the undercategory $\mathcal{C}_{C/}$ for $C : \{*\} \rightarrow \mathcal{C}$ to $\mathcal{C}_{F/}$ for an arbitrary functor $F : J \rightarrow \mathcal{C}$ ends up being exactly <b>Cocone</b> ( $F$ ).
$\kappa$ -Compact object	Let $C \in \mathcal{C}$ , and let $j_C : \mathcal{C} \rightarrow \mathbf{Set}$ denote the functor represented by $C$ . If $\mathcal{C}$ admits $\kappa$ -filtered colimits, then $C$ is $\kappa$ -compact if $j_C$ commutes with filtered colimits. ([Lur09], 5.3.4)	Let $C \in \mathcal{C}$ , and let $j_C : \mathcal{C} \rightarrow \hat{S}$ denote the functor represented by $C$ . If $\mathcal{C}$ admits $\kappa$ -filtered colimits, then $C$ is $\kappa$ -compact if $j_C$ preserves $\kappa$ -filtered colimits. <sup>1</sup> ([Lur09], Def 5.3.4.5)	[todo]
Dual object	[todo]	[todo]	[todo]
Essentially small category	$\mathcal{C}$ equivalent to a small category.	$\mathcal{C}$ equivalent <sup>2</sup> to a small $\infty$ -category.	[todo]

<sup>1</sup>Lurie introduces the term  $\kappa$ -continuous for such functors, but in ordinary category theory this generally means a functor which preserves  $\kappa$ -small limits; a functor preserving  $\kappa$ -filtered colimits is called  $\kappa$ -finitary. I have thus steered clear of this term.

<sup>2</sup>Categorically, or weakly?

Essentially surjective functor	$F : \mathcal{C} \rightarrow \mathcal{D}$ is essentially surjective if for every $D \in \mathcal{D}$ , there exists some $C \in \mathcal{C}$ with $FC \cong D$ .	$F : \mathcal{C} \rightarrow \mathcal{D}$ is essentially surjective if $hF : h\mathcal{C} \rightarrow h\mathcal{D}$ is essentially surjective. ([Lur09], Def 1.2.10.1)	Essentially surjective up to homotopy.
Faithful functor	$F : \mathcal{C} \rightarrow \mathcal{D}$ is faithful if $\text{Hom}(X, Y) \rightarrow \text{Hom}(FX, FY)$ is injective for all $X, Y \in \mathcal{C}$ .	$F : \mathcal{C} \rightarrow \mathcal{D}$ is faithful if $hF : h\mathcal{C} \rightarrow h\mathcal{D}$ is faithful. ([Lur09], Def 1.2.10.1)	Faithful up to homotopy.
$\kappa$ -Filtered category	For a regular cardinal $\kappa$ , $\mathcal{C}$ is $\kappa$ -filtered if, for every $\kappa$ -small category $J$ and every functor $F : J \rightarrow \mathcal{C}$ , there exists a cocone on $F$ .	For a regular cardinal $\kappa$ , $\mathcal{C}$ is $\kappa$ -filtered if, for every $\kappa$ -small simplicial set $X$ and every map $f : X \rightarrow \mathcal{C}$ , there exists a map $\bar{f} : K^\triangleright \rightarrow \mathcal{C}$ extending $f$ . ([Lur09], Def 5.3.1.7)	A cocone on $F$ is a collection of compatible maps $(\lambda_j : F(j) \rightarrow C)$ . Define $\bar{F} : J \star [0] \rightarrow \mathcal{C}$ to be $F$ on $J$ , send the cone point to $C$ , and send the unique morphisms $*_j$ from $j \in J$ to the cone point to the $\lambda_j$ . Conversely, if you have some $\bar{F}$ extending $F$ , define $\lambda_j := F(*_j)$ .
Final object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$ , there exists a unique morphism $C' \rightarrow C$ .	Object $C \in \mathcal{C}$ such that $C$ is final in $h\mathcal{C}$ , regarded as an enriched category over $\mathcal{H}$ . ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$ , there exists a unique (up to homotopy) morphism $C' \rightarrow C$ .
Full functor	$F : \mathcal{C} \rightarrow \mathcal{D}$ is full if $\text{Hom}(X, Y) \rightarrow \text{Hom}(FX, FY)$ is surjective for all $X, Y \in \mathcal{C}$ .	$F : \mathcal{C} \rightarrow \mathcal{D}$ is full if $hF : h\mathcal{C} \rightarrow h\mathcal{D}$ is full. ([Lur09], Def 1.2.10.1)	Full up to homotopy.
Functor	Functor.	Natural transformation of simplicial sets. ([Lur09], 1.2.7)	-
Groupoid	Category whose morphisms are all invertible.	Kan complex.	Not only can you find (non-unique) ‘composites’, but you can also fill in diagrams like $\begin{array}{ccc} C & \xrightarrow{\text{id}} & C \\ f \downarrow & \nearrow & \\ D & & \end{array} \quad \begin{array}{ccc} C & \xrightarrow{\text{id}} & D \\ \downarrow & \nearrow f & \\ C & & \end{array}$
Initial object	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$ , there exists a unique morphism $C \rightarrow C'$ .	Object $C \in \mathcal{C}$ such that $C$ is initial in $h\mathcal{C}$ , regarded as an enriched category over $\mathcal{H}$ . ([Lur09], Def 1.2.12.1)	Object $C \in \mathcal{C}$ such that for any other object $C' \in \mathcal{C}$ , there exists a unique (up to homotopy) morphism $C \rightarrow C'$ .
Join	$\mathcal{C} \star \mathcal{D}$ has objects $\text{ob } \mathcal{C} \sqcup \text{ob } \mathcal{D}$ , and $\text{Hom}_{\mathcal{C} \star \mathcal{D}}(X, Y)$ is given by: $\begin{cases} \text{Hom}_{\mathcal{C}}(X, Y) & X, Y \in \mathcal{C}, \\ \text{Hom}_{\mathcal{D}}(X, Y) & X, Y \in \mathcal{D}, \\ \emptyset & X \in \mathcal{D}, Y \in \mathcal{C}, \\ * & X \in \mathcal{C}, Y \in \mathcal{D}. \end{cases}$ ([Lur09], 1.2.8)	$\mathcal{C} \star \mathcal{D}$ has $n$ -simplices $(\mathcal{C} \star \mathcal{D}) = \mathcal{C}_n \cup \mathcal{D}_n \cup \bigcup_{i+j=n-1} \mathcal{C}_i \times \mathcal{D}_j$ . The $i$ th boundary map $d_i : (\mathcal{C} \star \mathcal{D})_n \rightarrow (\mathcal{C} \star \mathcal{D})_{n-1}$ is defined on $\mathcal{C}_n$ and $\mathcal{D}_n$ using the $i$ th boundary map on $\mathcal{C}$ and $\mathcal{D}$ . Given $\sigma \in S_j, \tau \in T_k$ , $d_i(\sigma, \tau)$ is given by $\begin{cases} (d_i\sigma, \tau) & i \leq j, j \neq 0, \\ (\sigma, d_{i-j-1}\tau) & i > j, k \neq 0. \end{cases}$ If $j = 0$ , then $d_0(\sigma, \tau) = \tau$ , and if $k = 0$ , then $d_n(\sigma, \tau) = \sigma$ . ([Lur09], Def 1.2.8.1 / [nLa25c])	Objects are in both cases disjoint unions of objects from the two categories being joined. Morphisms are also exactly the same in both cases (you get all the morphisms from $\mathcal{C}$ and $\mathcal{D}$ , plus a morphism from $c \rightarrow d$ for every pair $(c, d) \in \mathcal{C}_0 \times \mathcal{D}_0$ ). Whenever you have an $n$ -simplex in $\mathcal{C}$ and an $m$ -simplex in $\mathcal{D}$ , you get an $(m+n+1)$ -simplex in $\mathcal{C} \star \mathcal{D}$ , so in particular $\Delta^n \star \Delta^m \cong \Delta^{m+n+1}$ .
Left cone	$\mathcal{C}^\triangleleft := [0] \star \mathcal{C}$ .	$\mathcal{C}^\triangleleft := \Delta^0 \star \mathcal{C}$ . ([Lur09], Not 1.2.8.4)	$\mathcal{C}$ with extra vertex (cone point) added, as well as a map from that cone point to every other vertex in $\mathcal{C}$ (plus obligatory degenerate simplices).

Left Kan extension (along the inclusion of a full subcategory)	<p>Given a commutative diagram</p> $\begin{array}{ccc} \mathcal{C}^0 & \xrightarrow{F_0} & \mathcal{D} \\ \iota \downarrow & \nearrow F & \\ \mathcal{C} & & \end{array}$ <p>, <math>F</math> is a left Kan extension of <math>F_0</math> along <math>\iota</math> if there is a natural transformation <math>\eta : F_0 \rightarrow F\iota</math> such that for any other pair <math>(G : \mathcal{C} \rightarrow \mathcal{D}, \gamma : F_0 \rightarrow G\iota)</math>, there exists a unique natural transformation <math>\alpha : F \rightarrow G</math> such that <math>\gamma = (\alpha * \iota) \circ \eta</math>. ([Rie16], Def 6.1.1)</p>	<p>Given a commutative diagram</p> $\begin{array}{ccc} \mathcal{C}^0 & \xrightarrow{F_0} & \mathcal{D} \\ \iota \downarrow & \nearrow F & \\ \mathcal{C} & & \end{array}$ <p>tension of <math>F_0</math> along <math>\iota</math> if for all <math>C \in \mathcal{C}</math>, the induced diagram</p> $\begin{array}{ccc} \mathcal{C}_{/C}^0 & \xrightarrow{F_C} & \mathcal{D} \\ \downarrow & \nearrow & \\ (\mathcal{C}_{/C}^0)^{\triangleright} & & \end{array}$ <p>exhibits <math>FC</math> as a colimit of <math>F_C</math>. ([Lur09], Def 4.3.2.2)</p>	<p>In the 1-categorical case, the colimit of <math>\mathcal{C}_{/C}^0 \rightarrow \mathcal{C}^0 \xrightarrow{F_0} \mathcal{D}</math> for each <math>C \in \mathcal{C}</math> defines the left Kan extension of <math>F_0</math> along <math>\iota</math> ([Rie16], Thm 6.2.1). <b>I think Lurie is saying exactly the same thing in the <math>\infty</math>-categorical case, just in a slightly confusing way.</b></p>
Limit	A limit for $F : J \rightarrow \mathcal{C}$ is a terminal cone on $F$ .	A limit for $F : X \rightarrow \mathcal{C}$ ( $X$ a simplicial set, $\mathcal{C}$ an $\infty$ -category) is a final object of $\mathcal{C}_{/F}$ . ([Lur09], Def 1.2.13.4)	The obvious extension of the definition of the overcategory $\mathcal{C}_{/C}$ for $C : \{*\} \rightarrow \mathcal{C}$ to $\mathcal{C}_{/F}$ for an arbitrary functor $F : J \rightarrow \mathcal{C}$ ends up being exactly <b>Cone</b> ( $F$ ).
Locally small category	For every $X, Y \in \mathcal{C}$ , $\text{Hom}(X, Y)$ is a set.	For every $X, Y \in \mathcal{C}$ , the space $\text{Hom}(X, Y)$ is essentially small. ([Lur09], Prop 5.4.1.7)	[todo]
Monoidal category	[todo]	<b>Cocartesian fibration of <math>\infty</math>-operads <math>\mathcal{C}^{\otimes} \rightarrow \text{Assoc}^{\otimes}</math>. ([Lur17], Def 4.1.1.10)</b>	[todo]
(Coloured) operad	<ul style="list-style-type: none"> <li>• A collection of objects <math>\mathcal{O}</math>.</li> <li>• For every finite set <math>I</math>, every <math>I</math>-indexed collection of objects <math>\{X_i\}_{i \in I}</math> of <math>\mathcal{O}</math>, and every <math>Y \in \mathcal{O}</math>, a set <math>\text{Hom}(\{X_i\}_{i \in I}, Y)</math>.</li> <li>• For every map of finite sets <math>I \rightarrow J</math> having fibres <math>\{I_j\}_{j \in J}</math>, every finite collection of objects <math>\{X_i\}_{i \in I}</math>, every finite collection of objects <math>\{Y_j\}_{j \in J}</math>, and every object <math>Z \in \mathcal{O}</math>, a composition map <math>\prod_j \text{Hom}(\{X_i\}_{i \in I_j}, Y_j) \times \text{Hom}(\{Y_j\}_{j \in J}, Z) \rightarrow \text{Hom}(\{X_i\}_{i \in I}, Z)</math>, which is associative.</li> <li>• Units <math>\text{id}_X \in \text{Hom}(\{X\}, X)</math>. ([Lur17], Def 2.1.1.1)</li> </ul>	<p>A functor <math>p : \mathcal{O}^{\otimes} \rightarrow N(\mathbf{Fin}_*)</math> between <math>\infty</math>-categories which satisfies the following conditions:</p> <ul style="list-style-type: none"> <li>• For every inert morphism <math>f : \langle m \rangle \rightarrow \langle n \rangle</math> in <math>N(\mathbf{Fin}_*)</math> and every object <math>C \in \mathcal{O}_{\langle m \rangle}^{\otimes}</math>, there exists a <math>p</math>-cocartesian morphism <math>\bar{f} : C \rightarrow C'</math> in <math>\mathcal{O}^{\otimes}</math> lifting <math>f</math>.</li> <li>• Let <math>C \in \mathcal{O}_{\langle m \rangle}^{\otimes}</math> and <math>C' \in \mathcal{O}_{\langle n \rangle}^{\otimes}</math> be objects, let <math>f : \langle m \rangle \rightarrow \langle n \rangle</math> be a morphism in <math>\mathbf{Fin}_*</math>, and let <math>\text{Hom}_{\mathcal{O}^{\otimes}}^f(C, C')</math> be the union of those connected components of <math>\text{Hom}_{\mathcal{O}^{\otimes}}(C, C')</math> which lie over <math>f \in \text{Hom}_{\mathbf{Fin}_*}(\langle m \rangle, \langle n \rangle)</math>. Choose <math>p</math>-cocartesian morphisms <math>C' \rightarrow C'_i</math> lying over the inert morphisms <math>\rho^i : \langle n \rangle \rightarrow \langle 1 \rangle</math> for <math>1 \leq i \leq n</math>. Then the induced map</li> </ul> $\text{Hom}_{\mathcal{O}^{\otimes}}^f(C, C') \rightarrow \prod_{1 \leq i \leq n} \text{Hom}_{\mathcal{O}^{\otimes}}^{\rho^i f}(C, C'_i)$ <p>is a homotopy equivalence.</p> <ul style="list-style-type: none"> <li>• For every finite collection of objects <math>C_1, \dots, C_n \in \mathcal{O}_{\langle 1 \rangle}^{\otimes}</math>, there exists an object <math>C \in \mathcal{O}_{\langle n \rangle}^{\otimes}</math> and a collection of <math>p</math>-cocartesian morphisms <math>C \rightarrow C_i</math> covering <math>\rho^i : \langle n \rangle \rightarrow \langle 1 \rangle</math>.</li> </ul>	[todo]
Opposite category	$\mathcal{C}^{\text{op}}$ has the same objects as $\mathcal{C}$ , and $\text{Hom}_{\mathcal{C}^{\text{op}}}(X, Y) = \text{Hom}_{\mathcal{C}}(Y, X)$ .	$\mathcal{C}_n^{\text{op}} = \mathcal{C}([n]^{\text{op}})$ , where $\{0 < 1 < \dots < n\}^{\text{op}} = \{0 > 1 > \dots > n\}$ . ([Lur09], 1.2.1)	A map $x \rightarrow y$ is an edge $\Delta^1 \rightarrow \mathcal{C}$ where $0 \mapsto x$ and $1 \mapsto y$ . In $\mathcal{C}^{\text{op}}$ 0 and 1 swap roles, so we instead get a map $y \rightarrow x$ .

Overcategory	<p>For <math>C \in \mathcal{C}</math>, the category <math>\mathcal{C}_{/C}</math> satisfies the following universal property: for any category <math>\mathcal{D}</math>, there is a bijection</p> $\mathrm{Hom}(\mathcal{D}, \mathcal{C}_{/C}) \simeq \mathrm{Hom}_C(\mathcal{D} \star [0], \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors <math>\mathcal{D} \star [0] \rightarrow \mathcal{C}</math> whose restriction to <math>[0]</math> coincides with <math>C</math>. ([Lur09], 1.2.9)</p>	<p>For <math>f : S \rightarrow \mathcal{C}</math>, <math>S</math> a simplicial set and <math>\mathcal{C}</math> an <math>\infty</math>-category, the <math>\infty</math>-category <math>\mathcal{C}_{/f}</math> satisfies the following universal property: for any simplicial set <math>X</math>, there is a bijection</p> $\mathrm{Hom}(X, \mathcal{C}_{/f}) \simeq \mathrm{Hom}_f(X \star S, \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors <math>X \star S \rightarrow \mathcal{C}</math> whose restriction to <math>S</math> coincides with <math>f</math>. Explicitly,</p> $(\mathcal{C}_{/f})_n := \mathrm{Hom}_f(\Delta^n \star S, \mathcal{C}).$ <p>([Lur09], Prop 1.2.9.2)</p>	<p>If <math>S = \Delta^0</math>, writing <math>C \in \mathcal{C}</math> for the object picked out by <math>f</math>, we have <math>(\mathcal{C}_{/C})_n = \mathrm{Hom}_C(\Delta^n \star \Delta^0, \mathcal{C}) \cong \mathrm{Hom}_C(\Delta^{n+1}, \mathcal{C})</math> (where the subscript indicates that we only consider morphisms sending the <math>(n+1)</math>st vertex to <math>C</math>). In other words, the objects are maps to <math>C</math>, the morphisms are commuting triangles over <math>C</math>, and so on; these are exactly the objects and morphisms in the 1-categorical case.</p>
Presentable category	[todo]	[todo]	[todo]
Presheaf	[todo]	[todo]	[todo]
Representable functor	[todo]	[todo]	[todo]
Right cone	$\mathcal{C}^{\triangleright} := \mathcal{C} \star [0]$ .	$\mathcal{C}^{\triangleright} := \mathcal{C} \star \Delta^0$ . ([Lur09], Not 1.2.8.4)	$\mathcal{C}$ with extra vertex (cone point) added, as well as a map from every other vertex in $\mathcal{C}$ to that cone point (plus obligatory degenerate simplices).
Subcategory	Subcategory $\mathcal{C}' \subseteq \mathcal{C}$ .	<p>Subsimplicial set <math>\mathcal{C}' \subseteq \mathcal{C}</math> arising as a pullback</p> $\begin{array}{ccc} \mathcal{C}' & \longrightarrow & \mathcal{C} \\ \downarrow \lrcorner & & \downarrow \\ N(\mathrm{h}\mathcal{C})' & \longrightarrow & N(\mathrm{h}\mathcal{C}) \end{array}$ <p>where <math>(\mathrm{h}\mathcal{C})' \subseteq \mathrm{h}\mathcal{C}</math> is a subcategory. ([Lur09], 1.2.11)</p>	<p>Expected definition: Subsimplicial set <math>\mathcal{C}' \subseteq \mathcal{C}</math> satisfying inner horn filling. These are actually equivalent: suppose we have a such a subsimplicial set. Then</p> $\begin{array}{ccc} \mathcal{C}' & \longrightarrow & \mathcal{C} \\ \downarrow \lrcorner & & \downarrow \\ N(\mathrm{h}\mathcal{C})' & \longrightarrow & N(\mathrm{h}\mathcal{C}) \end{array} \quad \text{is a pullback.}$ <p>Conversely, suppose we're given a pullback as in the box to the left. Then the diagram</p> $\begin{array}{ccccc} \Delta^n & \xleftarrow{\quad} & \Lambda_i^n & \xrightarrow{\quad} & \mathcal{C} \\ & \searrow \text{dotted} & \downarrow & \searrow \text{dotted} & \downarrow \\ & & \mathcal{C}' & \xrightarrow{\quad} & \mathcal{C} \\ & \searrow \text{dotted} & \downarrow \lrcorner & & \downarrow \\ & & N(\mathrm{h}\mathcal{C})' & \longrightarrow & N(\mathrm{h}\mathcal{C}) \end{array}$ <p>shows that <math>\mathcal{C}'</math> also satisfies inner horn filling, where the outer two dotted maps come from horn filling for <math>\mathcal{C}</math> and <math>N(\mathrm{h}\mathcal{C})'</math>, and the inner dotted map comes from the fact that the square is a pullback.</p>
Symmetric monoidal category	[todo]	[todo]	[todo]
Symmetric monoidal functor	[todo]	[todo]	[todo]
Topos	[todo]	[todo]	[todo]

Undercategory	<p>For <math>C \in \mathcal{C}</math>, the category <math>\mathcal{C}_{C/}</math> satisfies the following universal property: for any category <math>\mathcal{D}</math>, there is a bijection</p> $\mathrm{Hom}(\mathcal{D}, \mathcal{C}_{C/}) \simeq \mathrm{Hom}_{\mathcal{C}}([0] \star \mathcal{D}, \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors <math>[0] \star \mathcal{D} \rightarrow \mathcal{C}</math> whose restriction to <math>[0]</math> coincides with <math>C</math>. ([Lur09], 1.2.9)</p>	<p>For <math>f : S \rightarrow \mathcal{C}</math>, <math>S</math> a simplicial set and <math>\mathcal{C}</math> an <math>\infty</math>-category, the <math>\infty</math>-category <math>\mathcal{C}_{f/}</math> satisfies the following universal property: for any simplicial set <math>X</math>, there is a bijection</p> $\mathrm{Hom}(X, \mathcal{C}_{f/}) \simeq \mathrm{Hom}_f(S \star X, \mathcal{C}),$ <p>where the subscript on the right indicates that we consider only those functors <math>S \star X \rightarrow \mathcal{C}</math> whose restriction to <math>S</math> coincides with <math>f</math>. Explicitly,</p> $(\mathcal{C}_{f/})_n := \mathrm{Hom}_f(S \star \Delta^n, \mathcal{C}).$ <p>([Lur09], Prop 1.2.9.2)</p>	<p>If <math>S = \Delta^0</math>, writing <math>C \in \mathcal{C}</math> for the object picked out by <math>f</math>, we have <math>(\mathcal{C}_{C/})_n = \mathrm{Hom}_{\mathcal{C}}(\Delta^0 \star \Delta^n, \mathcal{C}) \cong \mathrm{Hom}_{\mathcal{C}}(\Delta^{n+1}, \mathcal{C})</math> (where the subscript indicates that we only consider morphisms sending the 0th vertex to <math>C</math>). In other words, the objects are maps from <math>C</math>, the morphisms are commuting triangles under <math>C</math>, and so on; these are exactly the objects and morphisms in the 1-categorical case.</p>
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Equivalences		
Name	Between	Definition
Strong equivalence	Topological categories $\mathcal{C}, \mathcal{D}$	$\mathcal{C} \rightarrow \mathcal{D}$ is an equivalence in the sense of enriched category theory. ([Lur09], Def 1.1.3.1)
(Weak) equivalence	Topological categories $\mathcal{C}, \mathcal{D}$	The induced functor $\mathrm{h}\mathcal{C} \rightarrow \mathrm{h}\mathcal{D}$ is an equivalence of $\mathcal{H}$ -enriched categories. ([Lur09], Def 1.1.3.6)
Categorical equivalence	Simplicial sets $X, S$	The induced functor $\mathrm{h}X \rightarrow \mathrm{h}S$ is an equivalence of $\mathcal{H}$ -enriched categories. ([Lur09], Def 1.1.5.14)
Weak (homotopy) equivalence	Simplicial sets $X, S$	The induced map $ X  \rightarrow  S $ is a weak homotopy equivalence of topological spaces. ([Lur09], 1.1.4)
Equivalence	Simplicial categories $\mathcal{C}, \mathcal{D}$	The induced functor $\mathrm{h}\mathcal{C} \rightarrow \mathrm{h}\mathcal{D}$ is an equivalence of $\mathcal{H}$ -enriched categories. ([Lur09], Def 1.1.4.4)

Fibrations and anodyne morphisms		
Name	Describes	Definition
Acyclic Kan fibration	$f : X \rightarrow S$ map of simplicial sets	see: trivial Kan fibration. ([nLa23])
Anodyne	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> a Kan fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Ex 2.0.0.1)</p>
Cartesian fibration	$f : X \rightarrow S$ map of simplicial sets	$f$ is an inner fibration such that for every edge $g : x \rightarrow y$ of $S$ and every vertex $\tilde{y}$ of $X$ with $f(\tilde{y}) = y$ , there exists an $f$ -cartesian edge $\tilde{g} : \tilde{x} \rightarrow \tilde{y}$ with $f(\tilde{g}) = g$ . ([Lur09], Def 2.4.2.1)

Categorical fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> both a cofibration and a categorical equivalence,</p> $\begin{array}{ccc} Y & \longrightarrow & X \\ p \downarrow & \nearrow & \downarrow f \\ T & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], p90)</p>
Cocartesian fibration	$f : X \rightarrow S$ map of simplicial sets	$f$ is an inner fibration such that for every edge $g : x \rightarrow y$ of $S$ and every vertex $\tilde{x}$ of $X$ with $f(\tilde{x}) = x$ , there exists an $f$ -cocartesian edge $\tilde{g} : \tilde{x} \rightarrow \tilde{y}$ with $f(\tilde{g}) = g$ . ([Lur09], Def 2.4.2.1)
Cofibration	$f : X \rightarrow S$ map of simplicial sets	$f$ is a monomorphism. ([Lur09], A.2.7)
Inner anodyne	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> an inner fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>
Inner fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>0 &lt; i &lt; n</math>,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift.</p>
Isofibration	$F : \mathcal{C} \rightarrow \mathcal{D}$ map of $\infty$ -categories	$F$ is an inner fibration such that for all $C \in \mathcal{C}$ and every isomorphism $u : D \rightarrow FC$ in $\mathcal{D}$ (i.e. $[u]$ is an isomorphism in $\mathbf{h}\mathcal{D}$ ) there exists an isomorphism $\bar{u} : D \rightarrow C$ in $\mathcal{C}$ such that $F(\bar{u}) = u$ . [Lur25, Def 01EN]
(Kan) fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>0 \leq i \leq n</math>,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], A.2.7)</p>
Left anodyne	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> a left fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>
Left fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>0 \leq i &lt; n</math>,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>

Right anodyne	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>p : Y \rightarrow T</math> a right fibration,</p> $\begin{array}{ccc} X & \longrightarrow & Y \\ f \downarrow & \nearrow & \downarrow p \\ S & \longrightarrow & T \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>
Right fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below, with <math>0 &lt; i \leq n</math>,</p> $\begin{array}{ccc} \Lambda_i^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur09], Def 2.0.0.3)</p>
Serre fibration	$f : Y \rightarrow Z$ map of topological spaces	<p>For every solid arrow diagram as below,</p> $\begin{array}{ccc} \{0\} \times  \Delta^n  & \longrightarrow & Y \\ \downarrow & \nearrow & \downarrow f \\ [0, 1] \times  \Delta^n  & \longrightarrow & Z \end{array}$ <p>there exists a dotted lift. [Lur25, Def 021R]</p>
Trivial (Kan) fibration	$f : X \rightarrow S$ map of simplicial sets	<p>For every solid arrow diagram as below,</p> $\begin{array}{ccc} \partial\Delta^n & \longrightarrow & X \\ \downarrow & \nearrow & \downarrow f \\ \Delta^n & \longrightarrow & S \end{array}$ <p>there exists a dotted lift. ([Lur25, Def 006W]/[Lur09], Def 2.0.0.2)</p>

Nerves		
Name	Domain object	Definition
Nerve	Category $\mathcal{C}$	$(N\mathcal{C})_n = \{n\text{-composable strings of morphisms in } \mathcal{C}\}$ .
Simplicial nerve	Simplicial category $\mathcal{C}$	$(N\mathcal{C})_n = \text{Hom}_{\mathbf{Cat}_\Delta}(\mathfrak{C}[\Delta^n], \mathcal{C})$ , where $\mathfrak{C}[\Delta^n]$ is the category whose objects are the same as $[n]$ , and $\text{Hom}_{\mathfrak{C}[\Delta^n]}(i, j) = \emptyset$ for $i < j$ and $N(P_{ij})$ for $i \geq j$ (where $P_{ij} = \{I \subseteq [n] : (i, j \in I) \wedge (\forall k \in I, i \leq k \leq j)\}$ ).
Topological nerve	Topological category $\mathcal{C}$	The simplicial nerve of $\text{Sing } \mathcal{C}$ .

Homotopy categories	
Domain object	Definition
$\infty$ -Category $\mathcal{C}$	The objects of $\text{h}\mathcal{C}$ are the vertices of $\mathcal{C}$ , and $\text{Hom}_{\text{h}\mathcal{C}}(X, Y)$ is the set of homotopy classes of edges $X \rightarrow Y$ in $\mathcal{C}$ . ([Lur09], Prop 1.2.3.9)
Simplicial category $\mathcal{C}$	$\text{h} \mathcal{C} $ . ([Lur09], 1.1.4)
Topological category $\mathcal{C}$	$\text{h}\mathcal{C}$ has the same objects as $\mathcal{C}$ , and $\text{Hom}_{\text{h}\mathcal{C}}(X, Y) = [\text{Hom}_{\mathcal{C}}(X, Y)]$ . ([Lur09], 1.1.3)

Objects	
Name	Definition
<b>Assoc</b>	<a href="#">[Lur17]</a> , Def 4.1.1.1
$\text{Assoc}^\otimes$ (the associative $\infty$ -operad)	$N(\mathbf{Assoc}^\otimes)$ . ( <a href="#">[Lur17]</a> , Def 4.1.1.3)
$\mathbf{Assoc}^\otimes$	<a href="#">[Lur17]</a> , Def 4.1.1.3
<b>Kan</b>	The full subcategory of <b>sSet</b> spanned by the collection of small Kan complexes. ( <a href="#">[Lur09]</a> , Def 1.2.16.1)
<b>KAN</b>	The category of all Kan complexes. ( <a href="#">[Lur09]</a> , Rem 5.1.6.1)
$\mathcal{S}$ (the $\infty$ -category of spaces)	The simplicial <sup>3</sup> nerve $N(\mathbf{Kan})$ . ( <a href="#">[Lur09]</a> , Def 1.2.16.1)
$\widehat{\mathcal{S}}$	The simplicial nerve $N(\mathbf{KAN})$ . ( <a href="#">[Lur09]</a> , Rem 5.1.6.1)

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<sup>3</sup>**sSet** is a simplicial category, with  $\text{Hom}(X, S)_n = \text{Hom}_{\mathbf{sSet}}(\Delta^n \times X, S)$ . The subcategory **Kan** inherits this structure.



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