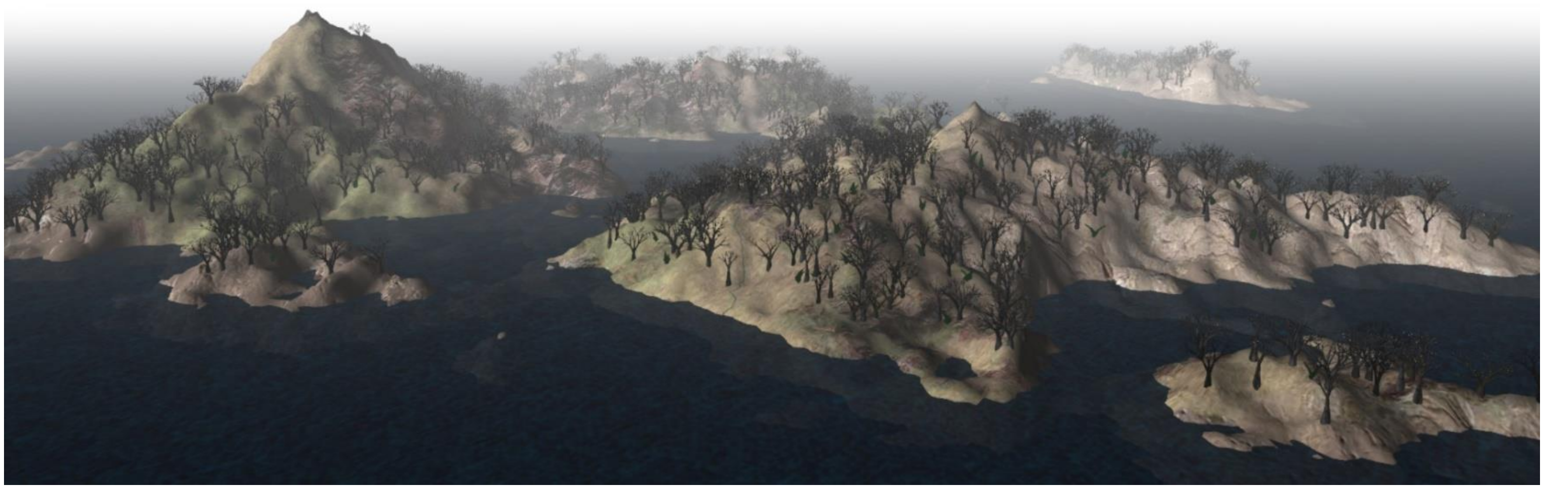
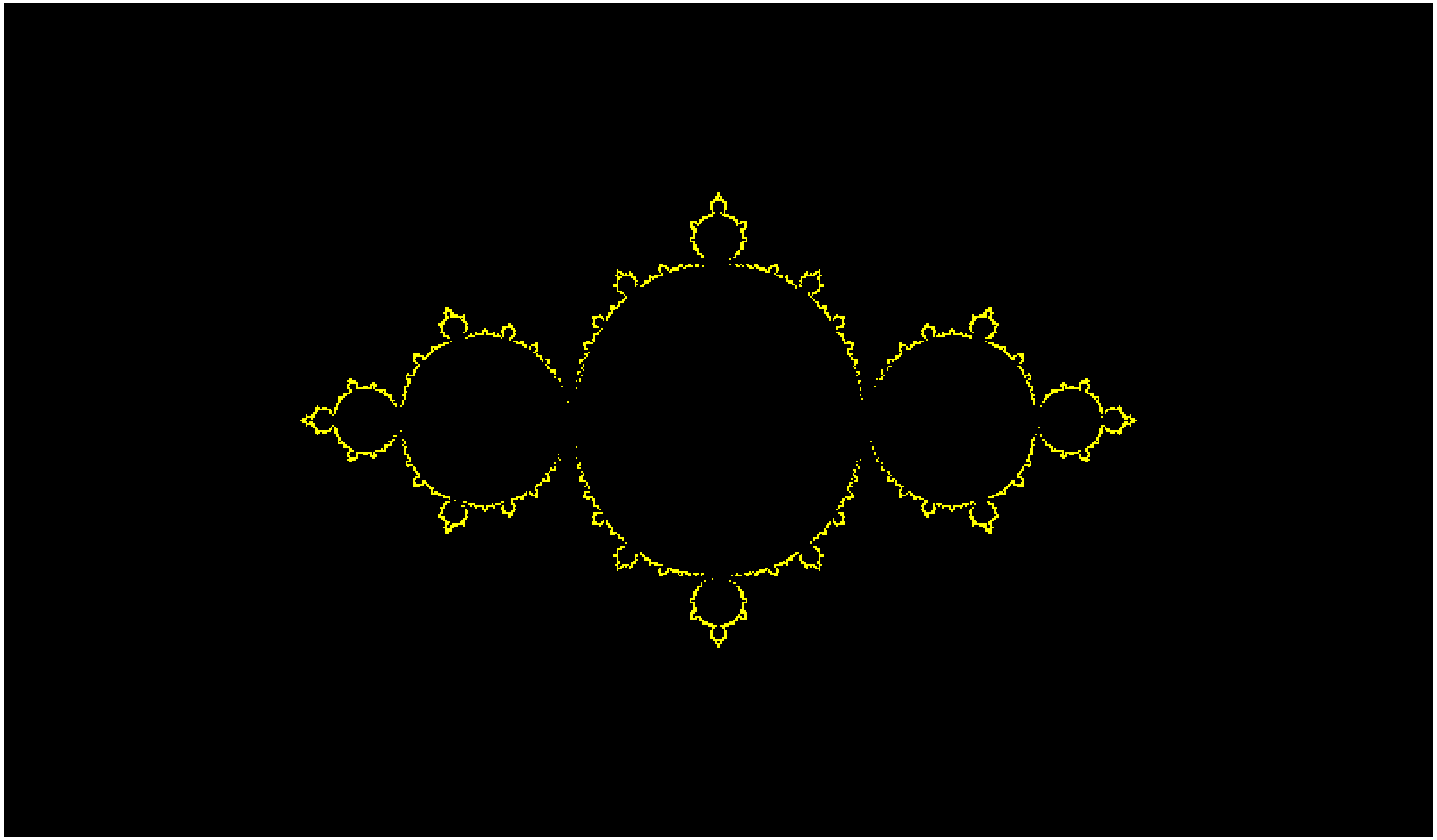
Fraktalna geometrija prirode



Clouds are not spheres, mountains are not cones, coastlines are not circles … … Responding to this challenge, I conceived and developed a new geometry of nature.

Benoit Mandelbrot *u uvodu knjige*
“The Fractal Geometry of Nature”

Nelinearne iteracije



Iteracije na brojevnom pravcu

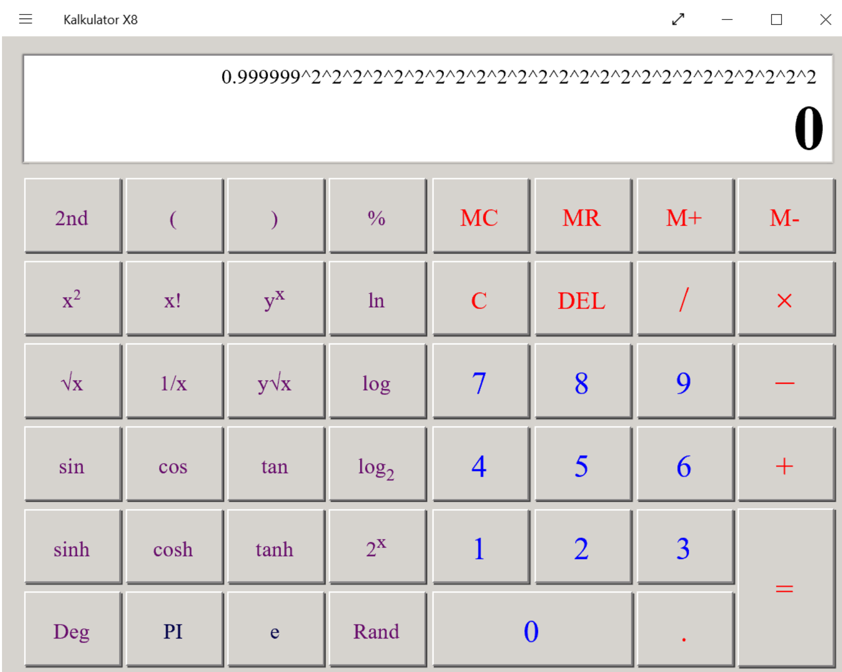
- iteracija: svaki sljedeći element niza dobiva se iz prethodnog – opetovano se primjenjuje ista funkcija
- primjer jednostavne nelinearne iteracije:

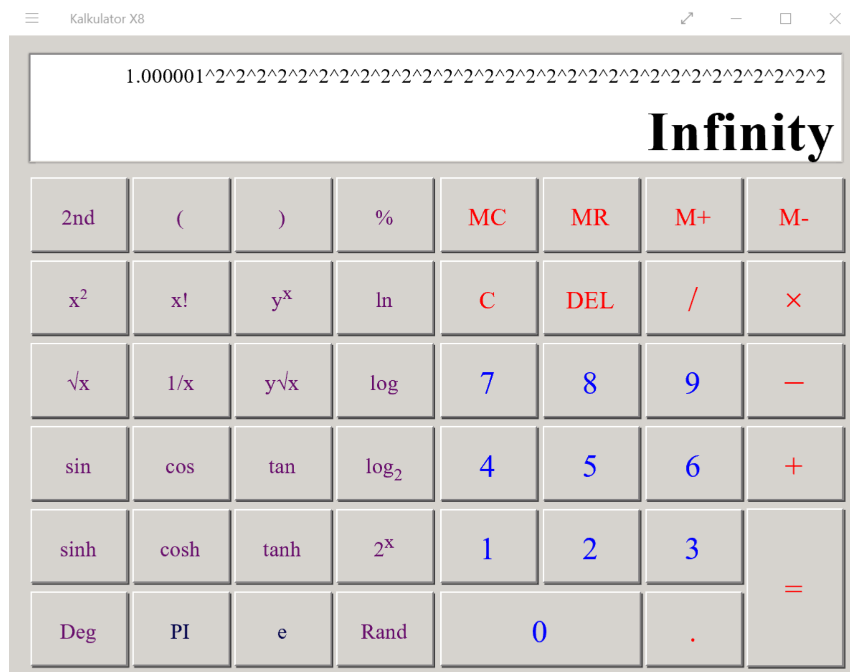
$$f(x) = x^2 \quad \Rightarrow \quad x_{k+1} = x_k^2$$

- na brojevnom pravcu, tj. za $x_0 \in \mathbb{R}$ područje konvergencije je $[-1, 1]$

Iteracije na brojevnom pravcu

$x_0 = 0{,}999999$ $\qquad\qquad$ $x_0 = 1{,}000001$





Kompleksna ravnina

- imaginarna jedinica

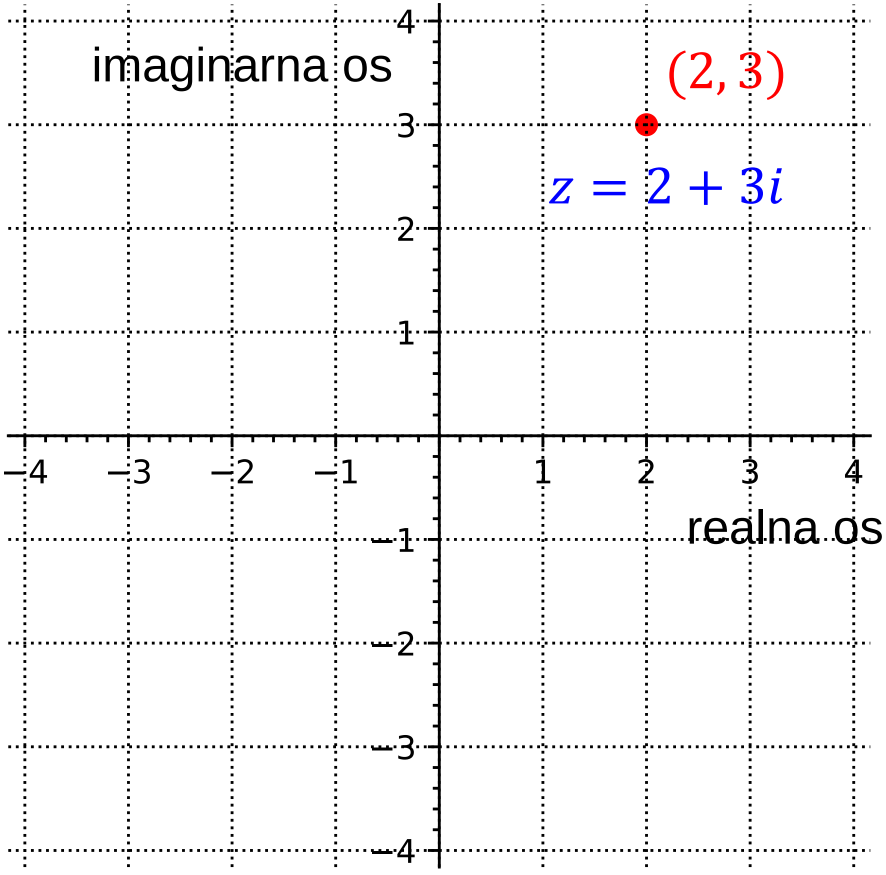
$$i \equiv \sqrt{-1}$$

- kompleksnom broju

$$z = x + iy$$

može se jednoznačno pridružiti točka (x, y) u kompleksnoj ravnini

- x i y su realni brojevi, realni i kompleksni dio kompleksnog broja z



Iteracije u kompleksnoj ravnini

$$f(z) = z^2$$

$$z_{k+1} = z_k^2$$

?

$$z = x + iy$$

$$z^2 = (x + iy)(x + iy) =$$

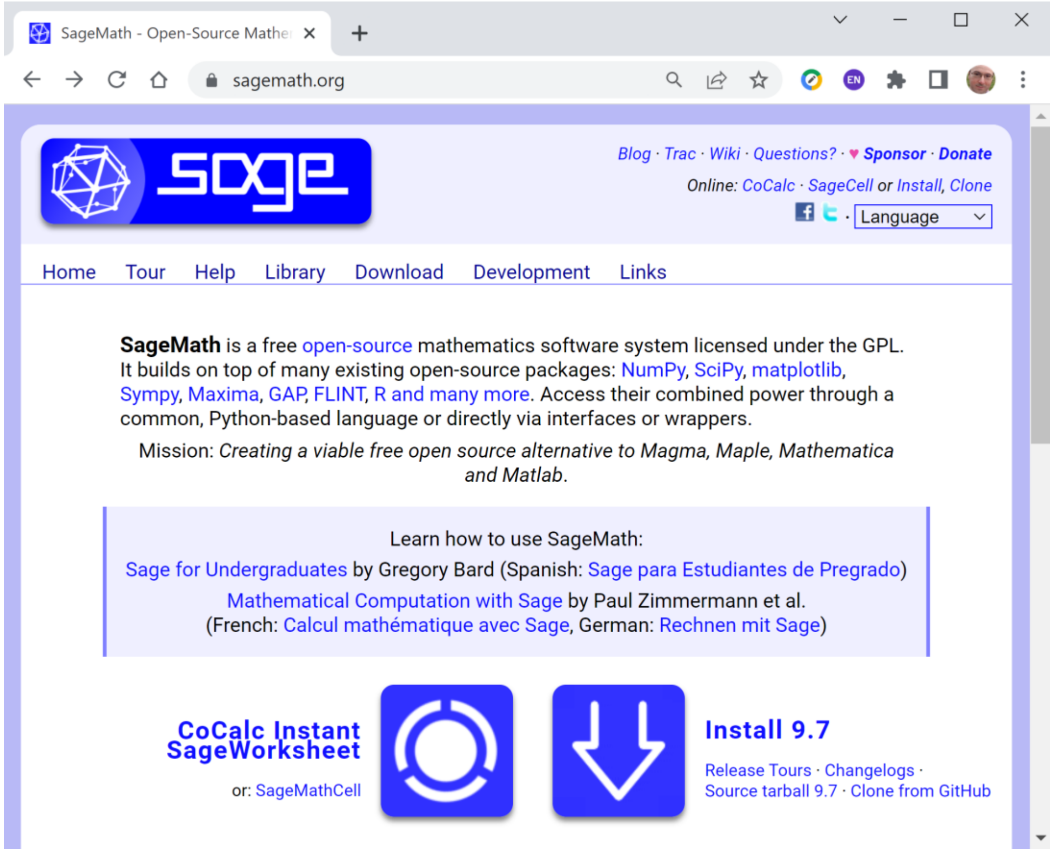
$$= \ x^2 - y^2 + i\,2xy$$

SageMath

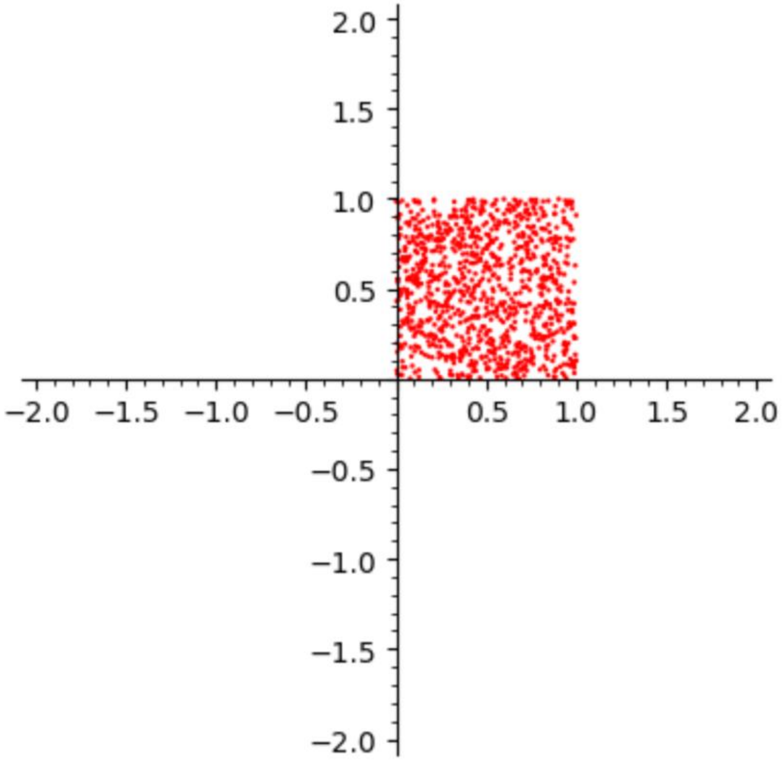
Open-Source Mathematical Software System

Prednosti:

- temelji se na Pythonu
- korisničko sučelje je u pregledniku – dostupan na svim platformama
- besplatan i otvorenog koda
- ima jaku zajednicu korisnika

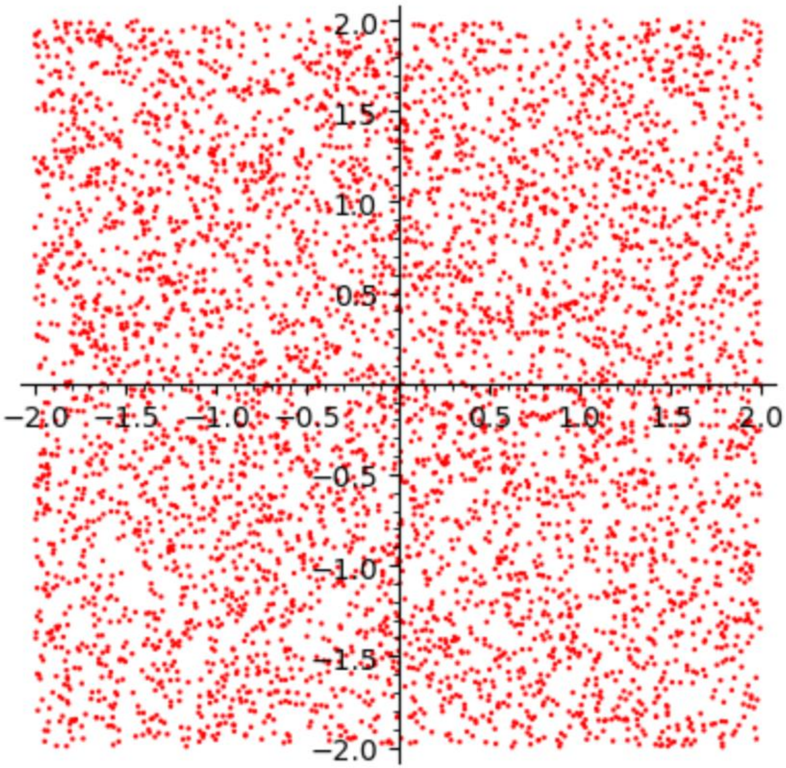


Slučajni brojevi u kompleksnoj ravnini




```
r = 2.0 # raspon prikaza
tocke = Graphics()

for i in range(1000):
    z = random() + random() * I
    tocke += point((z.real(), z.imag()), size = 3,
        rgbcolor = (1, 0, 0))

tocke.show(aspect_ratio = 1,
    xmin = -r, xmax = r, ymin = -r, ymax = r)
```

Slučajni brojevi u kompleksnoj ravnini




```
r = 2.0 # raspon prikaza
tocke = Graphics()

for i in range(5000):
    z = (random() - 0.5 + (random() - 0.5) * I) * 4
    tocke += point((z.real(), z.imag()), size = 3,
        rgbcolor = (1, 0, 0))

tocke.show(aspect_ratio = 1,
    xmin = -r, xmax = r, ymin = -r, ymax = r)
```

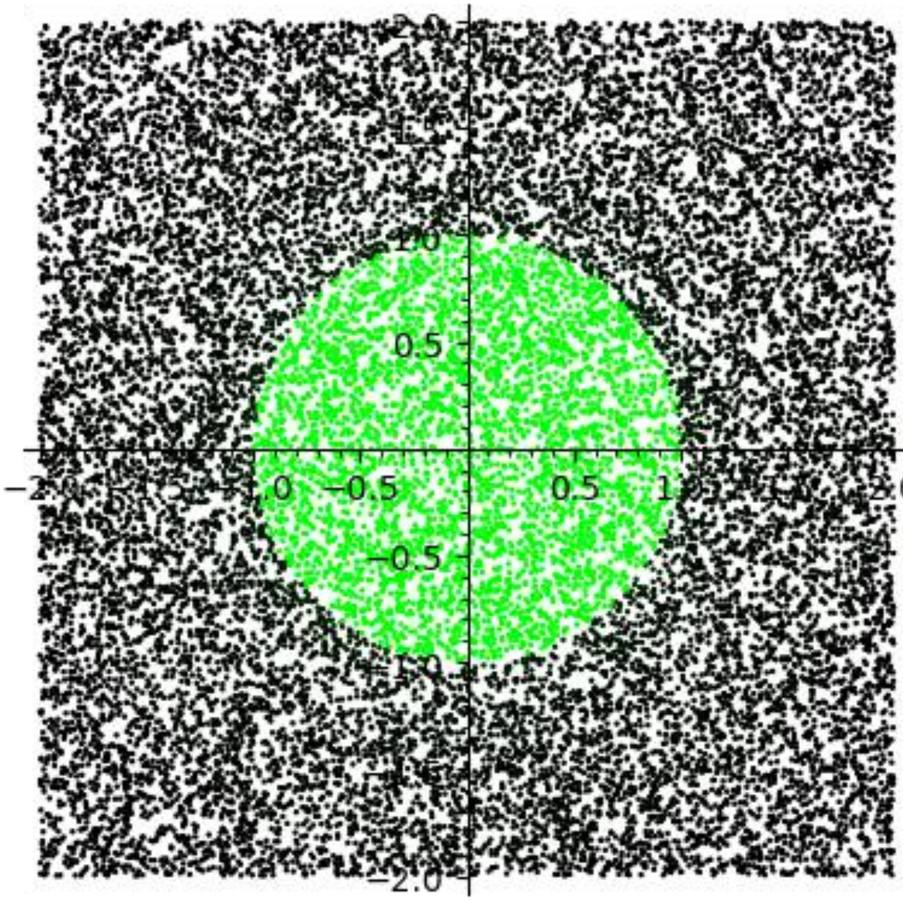
Iteracije u kompleksnoj ravnini

```
r = 2.0 # raspon prikaza
tocke = Graphics()
niter = 50

for i in range(20000):
    z0 = (random() - 0.5 + (random() - 0.5) * I) * 4
    z = z0
    i = 0
    while z.abs() < 2.0 and i < niter:
        z = z^2
        i += 1
    tocke += point((z0.real(), z0.imag()), size = 3,
        rgbcolor = (0, float(i) / float(niter), 0))

tocke.show(aspect_ratio = 1,
    xmin = -r, xmax = r, ymin = -r, ymax = r)
```

Iteracije u kompleksnoj ravnini

$$f(z) = z^2$$

$$z_{k+1} = z_k^2$$



Iteracije u kompleksnoj ravnini

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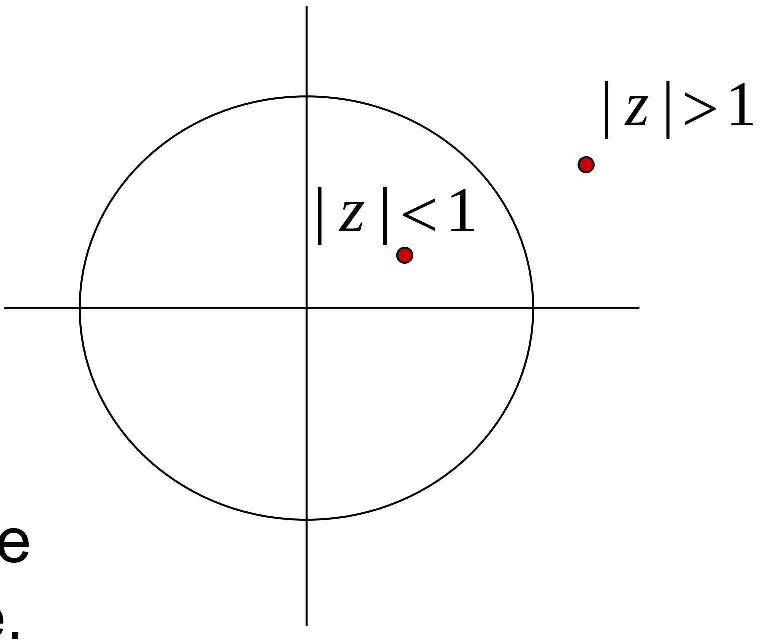


Područje konvergencije je unutar jedinične kružnice.

Samo se dodaje konstanta…

$$f(z) = z^2 + c$$

$$z_{k+1} = z_k^2 + c$$

?

c = konstanta

(kompleksni broj)

primjer: $c = -0.4 + 0.6\,i$

Julijev skup

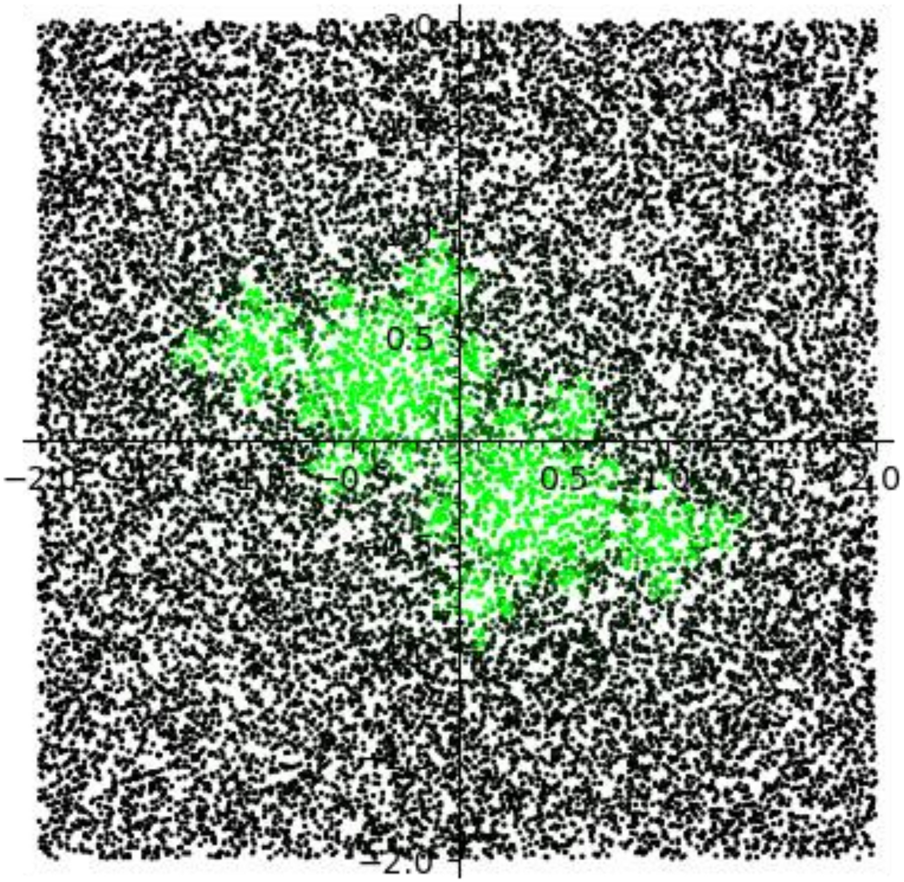
$$f(z) = z^2 + c$$

$$z_{k+1} = z_k^2 + c$$

c = konstanta

(kompleksni broj)

primjer: $c = -0.4 + 0.6\,i$



Svaki puta se dodaje onaj z s kojim je započela iteracija…

$$f(z) = z^2 + z_0$$

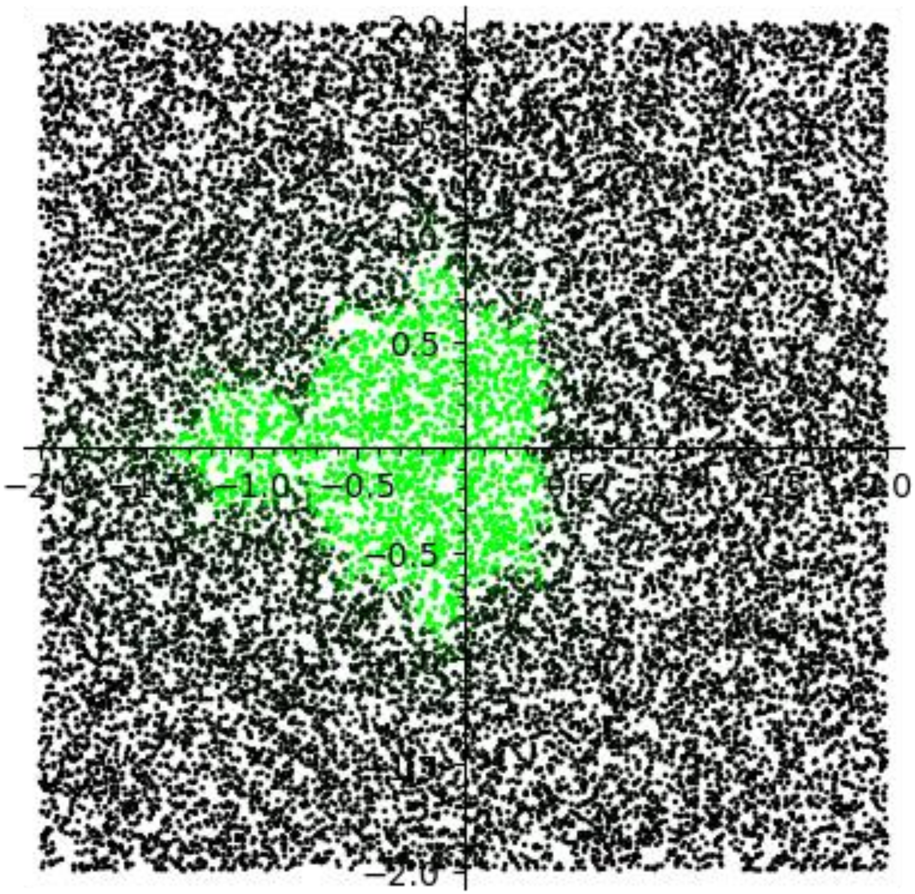
$$z_{k+1} = z_k^2 + z_0$$

?

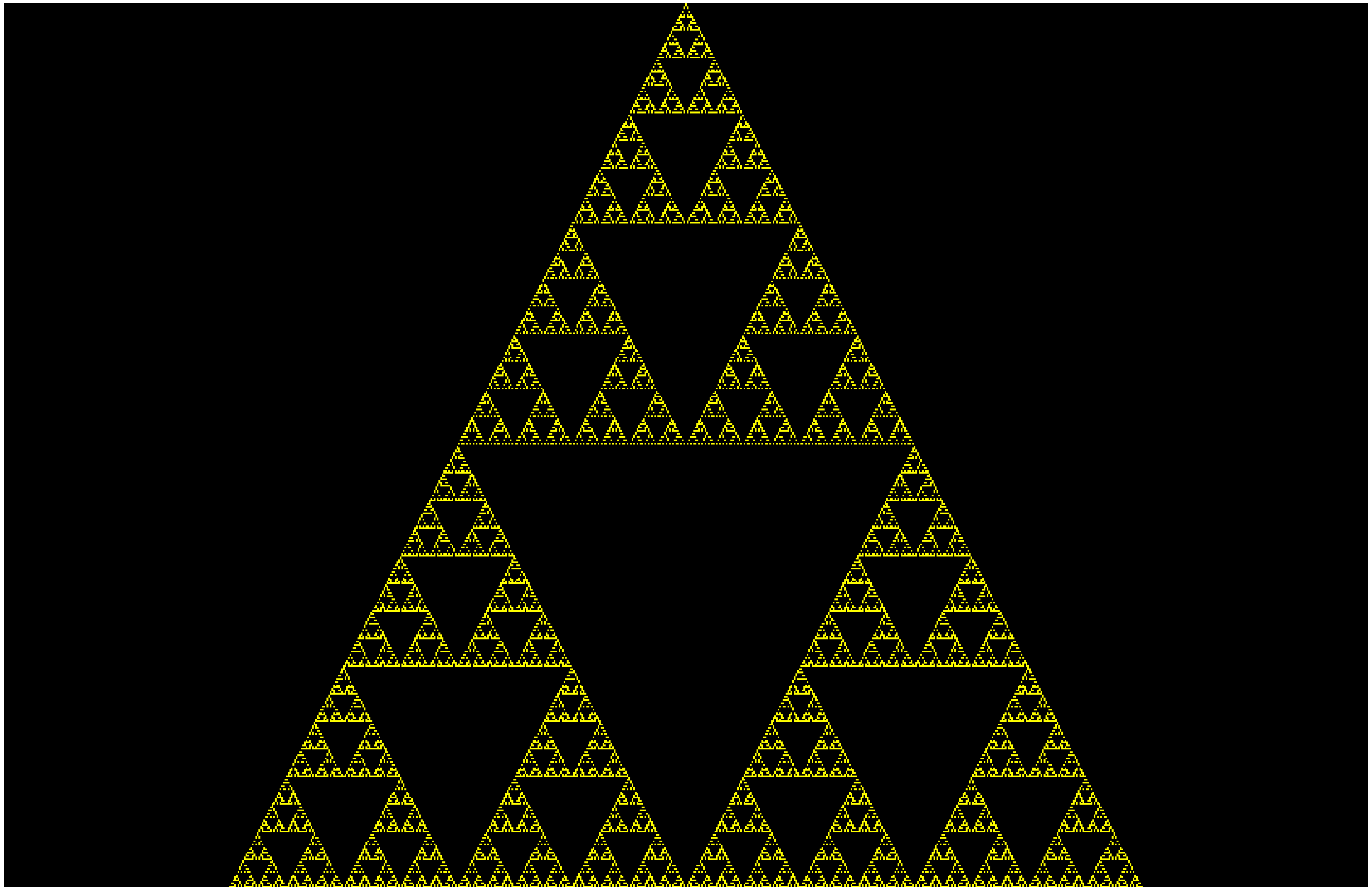
Mandelbrotov skup

$$f(z) = z^2 + z_0$$

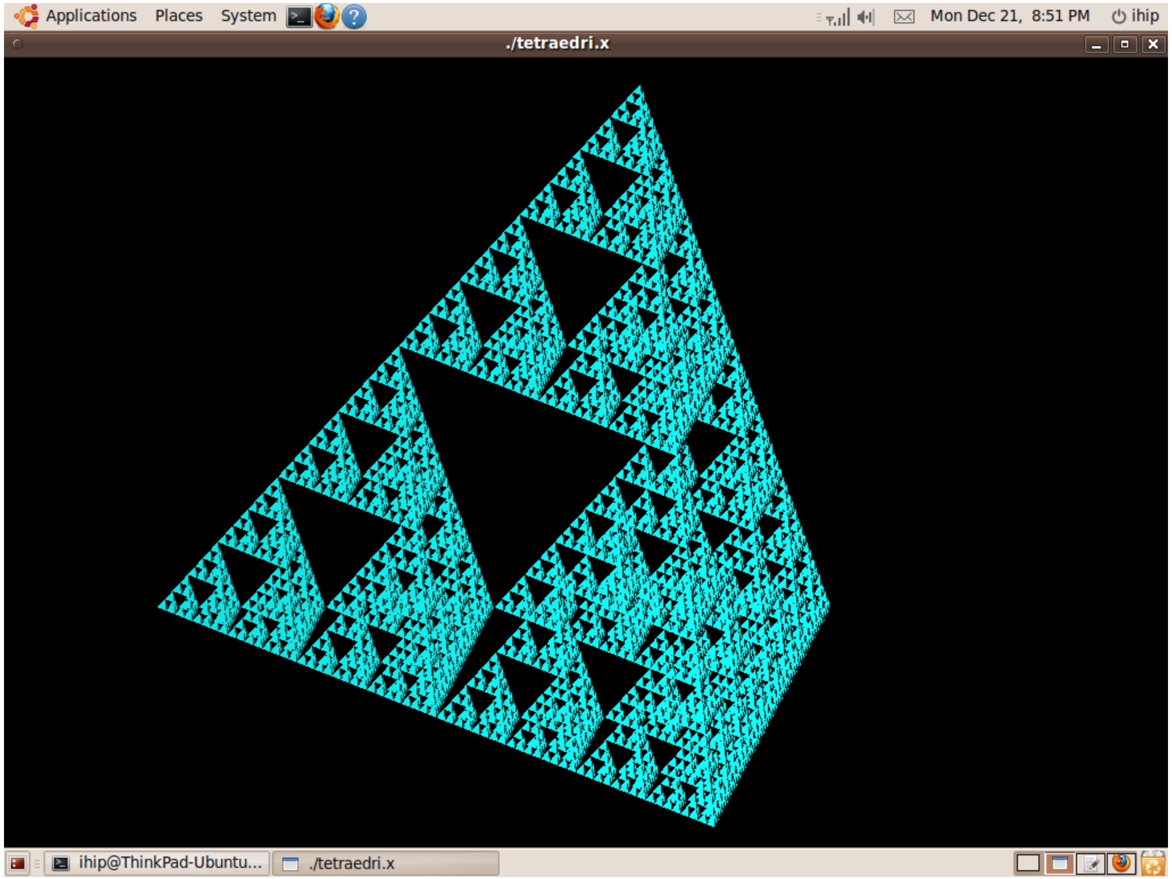
$$z_{k+1} = z_k^2 + z_0$$



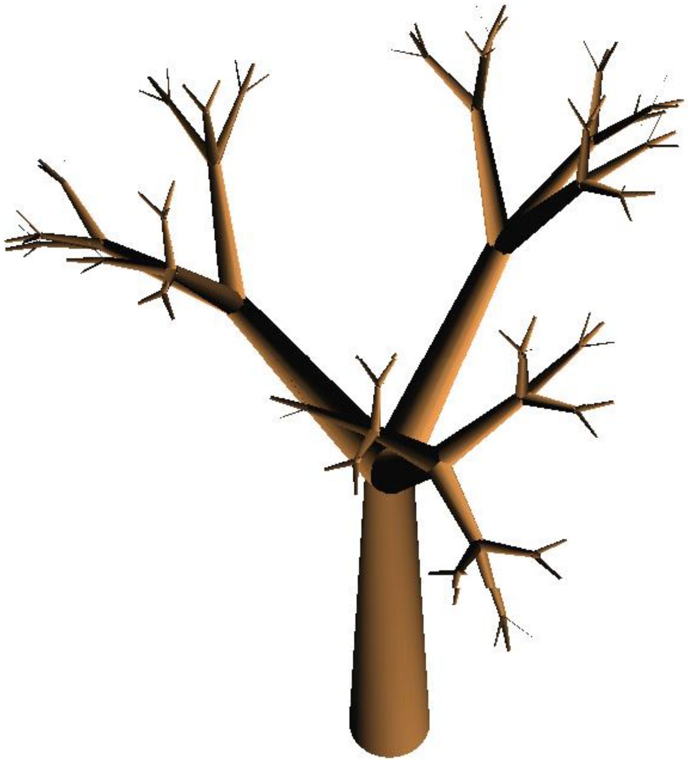
Deterministički sebi-slični fraktali



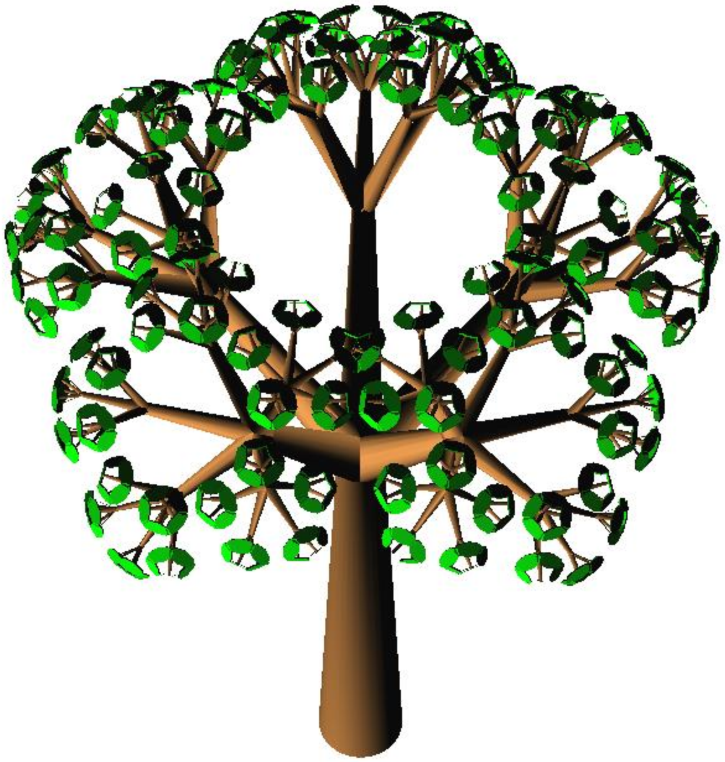
Deterministički sebi-slični fraktali 2



Fraktalno drvo

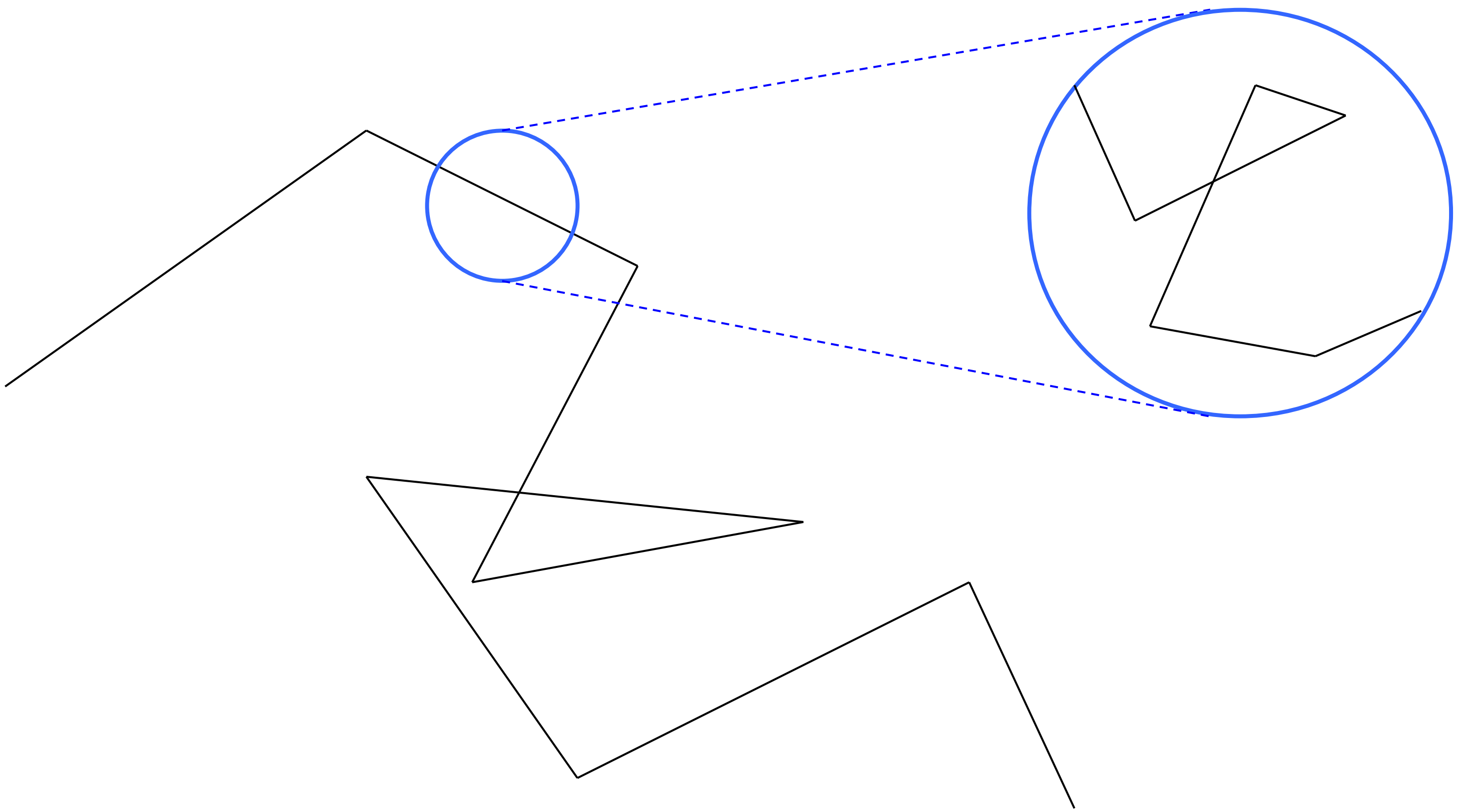


Fraktalno drvo s lišćem

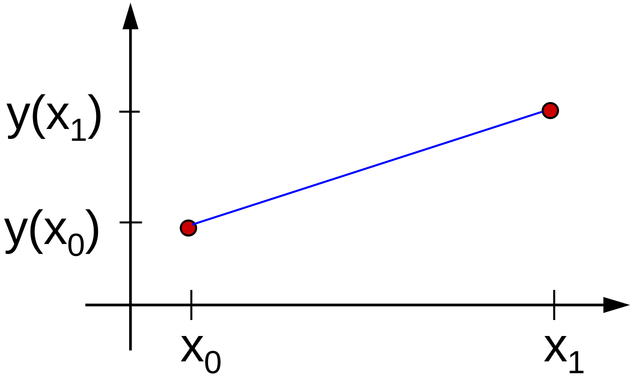


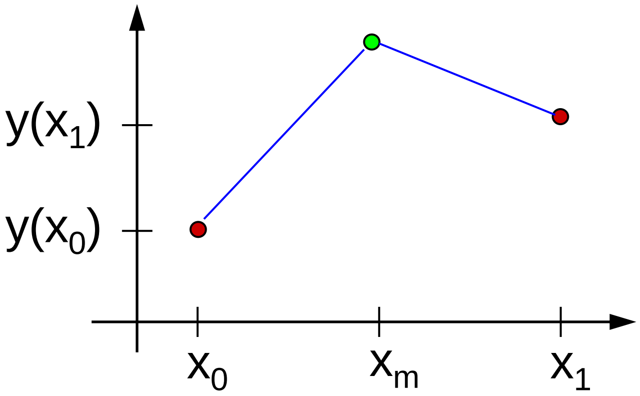
SLUČAJNI FRAKTALI

Brownovo gibanje



“Random midpoint-displacement methods”





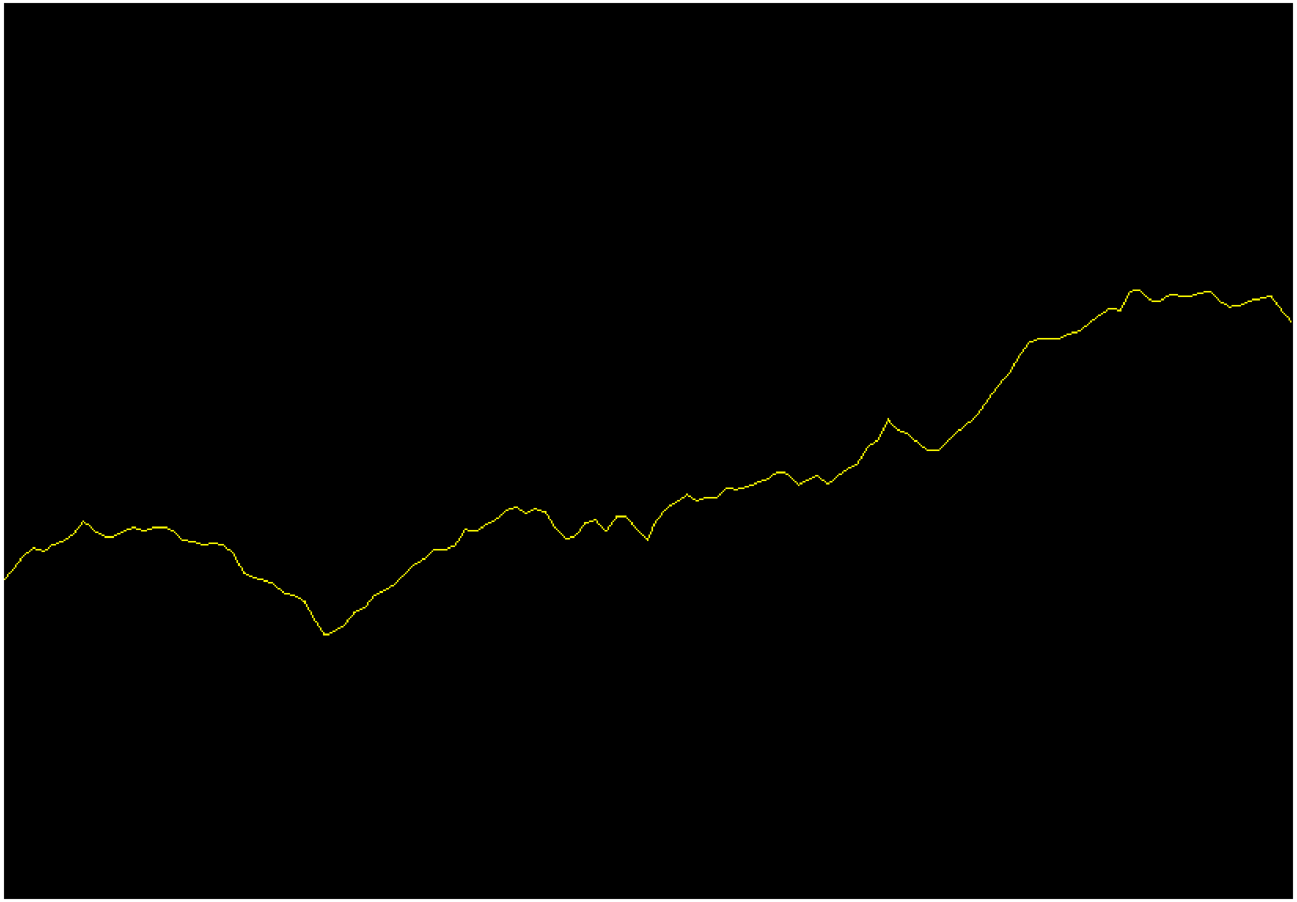
$$y(x_m) = \frac{1}{2}\left[y(x_0) + y(x_1)\right] + r$$

$$r = s\, r_g\,(x_1 - x_0)$$

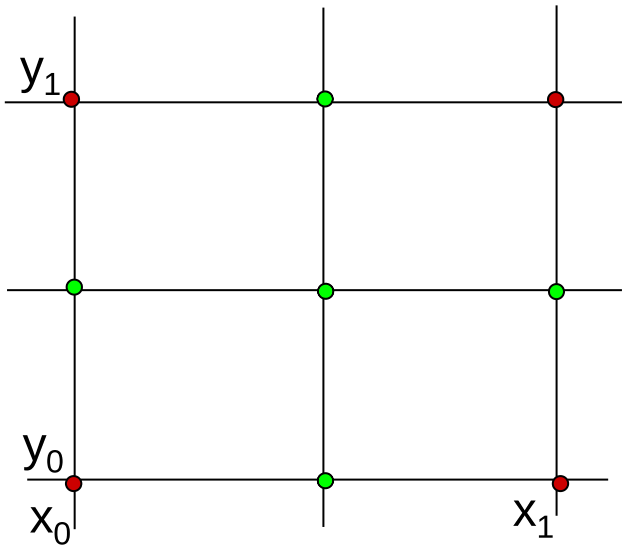
r_g – slučajni broj iz Gaussove raspodjele (aritmetička sredina = 0 i varijanca = 1)

s - služi za skaliranje

Iskra



Plazma-fraktali



$$z(x_m, y_k) = \frac{1}{2}\left[z(x_0, y_k) + z(x_1, y_k)\right] + r_x$$

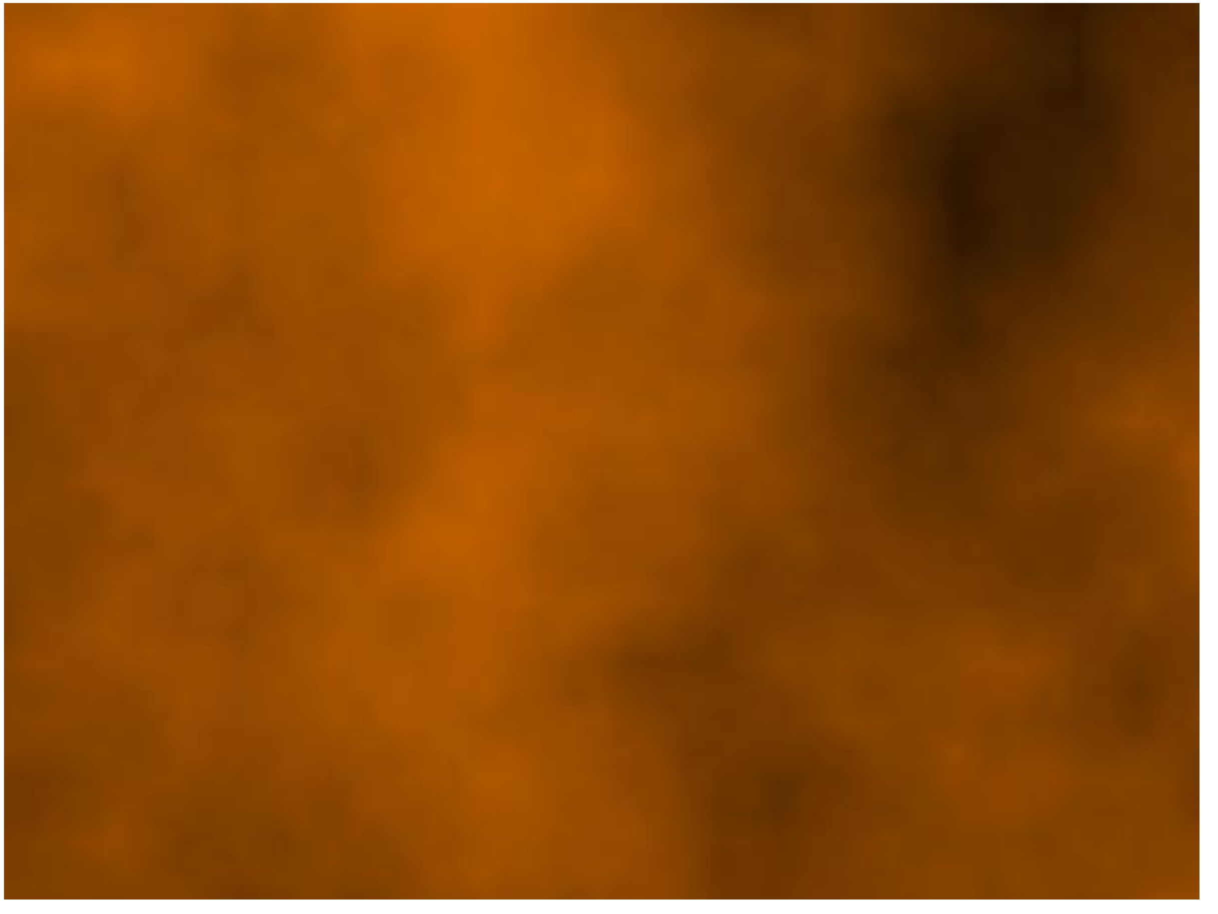
$$z(x_k, y_m) = \frac{1}{2}\left[z(x_k, y_o) + z(x_k, y_1)\right] + r_y$$

$$r_x = s\, r_g (x_1 - x_0) \qquad r_y = s\, \tilde{r}_g (y_1 - y_0)$$

$$z(x_m, y_m) = \frac{1}{4}\left[z(x_m, y_0) + z(x_1, y_m) + z(x_m, y_1) + z(x_0, y_m)\right] + r_m$$

$$r_m = s\, r_g (x_1 - x_0 + y_1 - y_0)/2$$

Plazma



Model terena