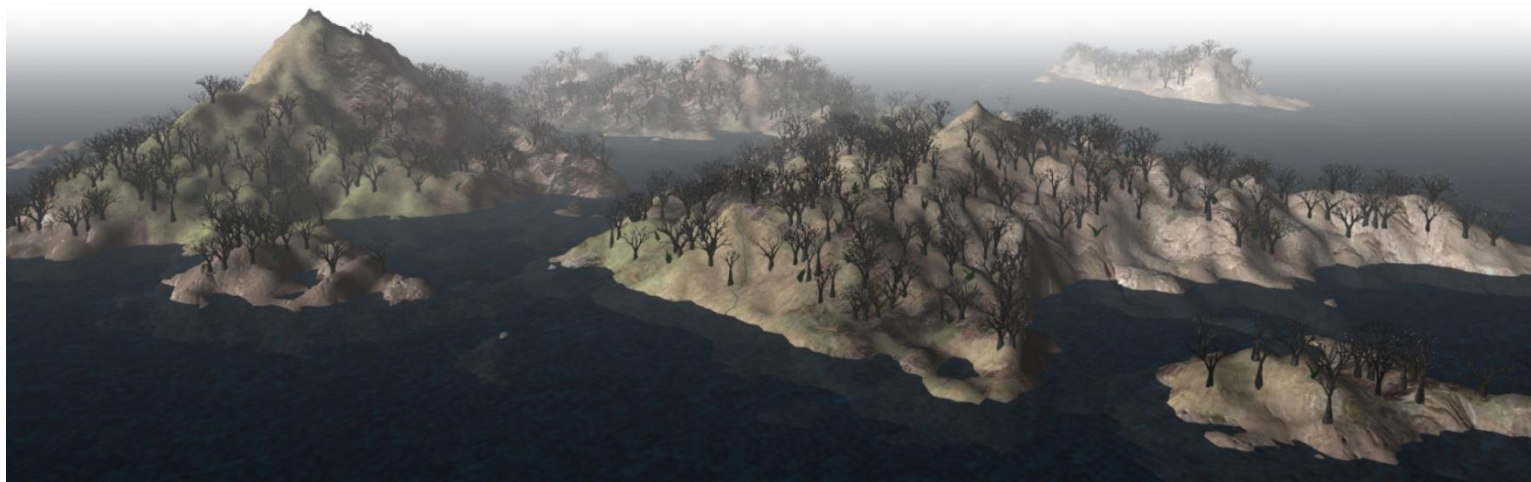


# Fraktalna geometrija prirode

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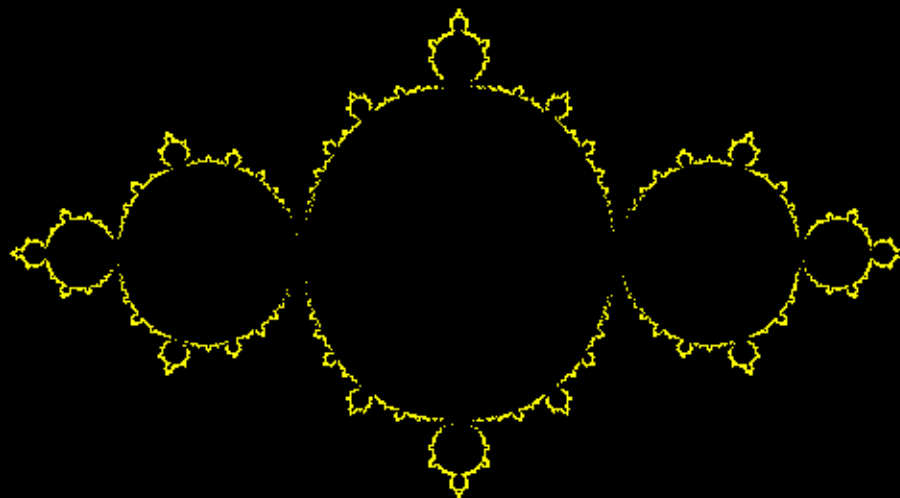
Clouds are not spheres, mountains are not cones, coastlines are not circles ...  
... Responding to this challenge, I conceived and developed a new geometry of nature.

***Benoit Mandelbrot u uvodu knjige  
“The Fractal Geometry of Nature”***

---

# Nelinearne iteracije

---



# Iteracije na brojevnom pravcu

---

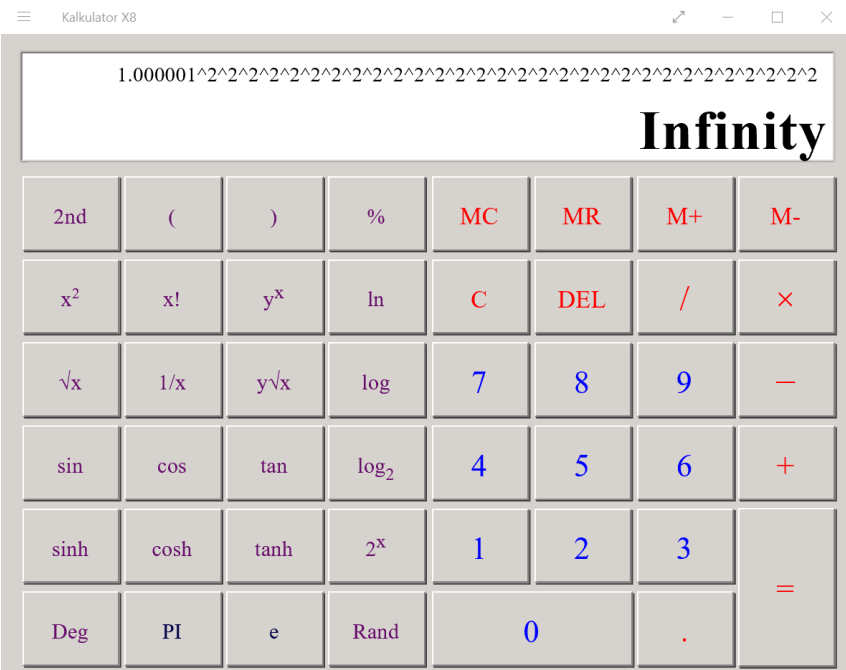
- iteracija: svaki sljedeći element niza dobiva se iz prethodnog – opetovano se primjenjuje ista funkcija
- primjer jednostavne nelinearne iteracije:

$$f(x) = x^2 \Rightarrow x_{k+1} = x_k^2$$

- na brojevnom pravcu, tj. za  $x_0 \in \mathbb{R}$  područje konvergencije je  $[-1, 1]$
-

<p> <b>1.1</b> <b>Introduction</b>  <b>1.2</b> <b>Background</b>  <b>1.3</b> <b>Objectives</b>  <b>1.4</b> <b>Scope</b>  <b>1.5</b> <b>Methodology</b>  <b>1.6</b> <b>Results</b>  <b>1.7</b> <b>Conclusion</b>  <b>1.8</b> <b>References</b>  <b>1.9</b> <b>Appendix</b>  <b>1.10</b> <b>Index</b>  <b>1.11</b> <b>Glossary</b>  <b>1.12</b> <b>Abbreviations</b>  <b>1.13</b> <b>Acronyms</b>  <b>1.14</b> <b>Footnotes</b>  <b>1.15</b> <b>Endnotes</b>  <b>1.16</b> <b>References</b>  <b>1.17</b> <b>Appendix</b>  <b>1.18</b> <b>Index</b>  <b>1.19</b> <b>Glossary</b>  <b>1.20</b> <b>Abbreviations</b>  <b>1.21</b> <b>Acronyms</b>  <b>1.22</b> <b>Footnotes</b>  <b>1.23</b> <b>Endnotes</b>  <b>1.24</b> <b>References</b>  <b>1.25</b> <b>Appendix</b>  <b>1.26</b> <b>Index</b>  <b>1.27</b> <b>Glossary</b>  <b>1.28</b> <b>Abbreviations</b>  <b>1.29</b> <b>Acronyms</b>  <b>1.30</b> <b>Footnotes</b>  <b>1.31</b> <b>Endnotes</b>  <b>1.32</b> <b>References</b>  <b>1.33</b> <b>Appendix</b>  <b>1.34</b> <b>Index</b>  <b>1.35</b> <b>Glossary</b>  <b>1.36</b> <b>Abbreviations</b>  <b>1.37</b> <b>Acronyms</b>  <b>1.38</b> <b>Footnotes</b>  <b>1.39</b> <b>Endnotes</b>  <b>1.40</b> <b>References</b>  <b>1.41</b> <b>Appendix</b>  <b>1.42</b> <b>Index</b>  <b>1.43</b> <b>Glossary</b>  <b>1.44</b> <b>Abbreviations</b>  <b>1.45</b> <b>Acronyms</b>  <b>1.46</b> <b>Footnotes</b>  <b>1.47</b> <b>Endnotes</b>  <b>1.48</b> <b>References</b>  <b>1.49</b> <b>Appendix</b>  <b>1.50</b> <b>Index</b>  <b>1.51</b> <b>Glossary</b>  <b>1.52</b> <b>Abbreviations</b>  <b>1.53</b> <b>Acronyms</b>  <b>1.54</b> <b>Footnotes</b>  <b>1.55</b> <b>Endnotes</b>  <b>1.56</b> <b>References</b>  <b>1.57</b> <b>Appendix</b>  <b>1.58</b> <b>Index</b>  <b>1.59</b> <b>Glossary</b>  <b>1.60</b> <b>Abbreviations</b>  <b>1.61</b> <b>Acronyms</b>  <b>1.62</b> <b>Footnotes</b>  <b>1.63</b> <b>Endnotes</b>  <b>1.64</b> <b>References</b>  <b>1.65</b> <b>Appendix</b>  <b>1.66</b> <b>Index</b>  <b>1.67</b> <b>Glossary</b>  <b>1.68</b> <b>Abbreviations</b>  <b>1.69</b> <b>Acronyms</b>  <b>1.70</b> <b>Footnotes</b>  <b>1.71</b> <b>Endnotes</b>  <b>1.72</b> <b>References</b>  <b>1.73</b> <b>Appendix</b>  <b>1.74</b> <b>Index</b>  <b>1.75</b> <b>Glossary</b>  <b>1.76</b> <b>Abbreviations</b>  <b>1.77</b> <b>Acronyms</b>  <b>1.78</b> <b>Footnotes</b>  <b>1.79</b> <b>Endnotes</b>  <b>1.80</b> <b>References</b>  <b>1.81</b> <b>Appendix</b>  <b>1.82</b> <b>Index</b>  <b>1.83</b> <b>Glossary</b>  <b>1.84</b> <b>Abbreviations</b>  <b>1.85</b> <b>Acronyms</b>  <b>1.86</b> <b>Footnotes</b>  <b>1.87</b> <b>Endnotes</b>  <b>1.88</b> <b>References</b>  <b>1.89</b> <b>Appendix</b>  <b>1.90</b> <b>Index</b>  <b>1.91</b> <b>Glossary</b>  <b>1.92</b> <b>Abbreviations</b>  <b>1.93</b> <b>Acronyms</b>  <b>1.94</b> <b>Footnotes</b>  <b>1.95</b> <b>Endnotes</b>  <b>1.96</b> <b>References</b>  <b>1.97</b> <b>Appendix</b>  <b>1.98</b> <b>Index</b>  <b>1.99</b> <b>Glossary</b>  <b>1.100</b> <b>Abbreviations</b>  <b>1.101</b> <b>Acronyms</b>  <b>1.102</b> <b>Footnotes</b>  <b>1.103</b> <b>Endnotes</b>  <b>1.104</b> <b>References</b>  <b>1.105</b> <b>Appendix</b>  <b>1.106</b> <b>Index</b>  <b>1.107</b> <b>Glossary</b>  <b>1.108</b> <b>Abbreviations</b>  <b>1.109</b> <b>Acronyms</b>  <b>1.110</b> <b>Footnotes</b>  <b>1.111</b> <b>Endnotes</b>  <b>1.112</b> <b>References</b>  <b>1.113</b> <b>Appendix</b>  <b>1.114</b> <b>Index</b>  <b>1.115</b> <b>Glossary</b>  <b>1.116</b> <b>Abbreviations</b>  <b>1.117</b> <b>Acronyms</b>  <b>1.118</b> <b>Footnotes</b>  <b>1.119</b> <b>Endnotes</b>  <b>1.120</b> <b>References</b>  <b>1.121</b> <b>Appendix</b>  <b>1.122</b> <b>Index</b>  <b>1.123</b> <b>Glossary</b>  <b>1.124</b> <b>Abbreviations</b>  <b>1.125</b> <b>Acronyms</b>  <b>1.126</b> <b>Footnotes</b>  <b>1.127</b> <b>Endnotes</b>  <b>1.128</b> <b>References</b>  <b>1.129</b> <b>Appendix</b>  <b>1.130</b> <b>Index</b>  <b>1.131</b> <b>Glossary</b>  <b>1.132</b> <b>Abbreviations</b>  <b>1.133</b> <b>Acronyms</b>  <b>1.134</b> <b>Footnotes</b>  <b>1.135</b> <b>Endnotes</b>  <b>1.136</b> <b>References</b>  <b>1.137</b> <b>Appendix</b>  <b>1.138</b> <b>Index</b>  <b>1.139</b> <b>Glossary</b>  <b>1.140</b> <b>Abbreviations</b>  <b>1.141</b> <b>Acronyms</b>  <b>1.142</b> <b>Footnotes</b>  <b>1.143</b> <b>Endnotes</b>  <b>1.144</b> <b>References</b>  <b>1.145</b> <b>Appendix</b>  <b>1.146</b> <b>Index</b>  <b>1.147</b> <b>Glossary</b>  <b>1.148</b> <b>Abbreviations</b>  <b>1.149</b> <b>Acronyms</b>  <b>1.150</b> <b>Footnotes</b>  <b>1.151</b> <b>Endnotes</b>  <b>1.152</b> <b>References</b>  <b>1.153</b> <b>Appendix</b>  <b>1.154</b> <b>Index</b>  <b>1.155</b> <b>Glossary</b>  <b>1.156</b> <b>Abbreviations</b>  <b>1.157</b> <b>Acronyms</b>  <b>1.158</b> <b>Footnotes</b>  <b>1.159</b> <b>Endnotes</b>  <b>1.160</b> <b>References</b>  <b>1.161</b> <b>Appendix</b>  <b>1.162</b> <b>Index</b>  <b>1.163</b> <b>Glossary</b>  <b>1.164</b> <b>Abbreviations</b>  <b>1.165</b> <b>Acronyms</b>  <b>1.166</b> <b>Footnotes</b>  <b>1.167</b> <b>Endnotes</b>  <b>1.168</b> <b>References</b>  <b>1.169</b> <b>Appendix</b>  <b>1.170</b> <b>Index</b>  <b>1.171</b> <b>Glossary</b>  <b>1.172</b> <b>Abbreviations</b>  <b>1.173</b> <b>Acronyms</b>  <b>1.174</b> <b>Footnotes</b>  <b>1.175</b> <b>Endnotes</b>  <b>1.176</b> <b>References</b>  <b>1.177</b> <b>Appendix</b>  <b>1.178</b> <b>Index</b>  <b>1.179</b> <b>Glossary</b>  <b>1.180</b> <b>Abbreviations</b>  <b>1.181</b> <b>Acronyms</b>  <b>1.182</b> <b>Footnotes</b>  <b>1.183</b> <b>Endnotes</b>  <b>1.184</b> <b>References</b>  <b>1.185</b> <b>Appendix</b>  <b>1.186</b> <b>Index</b>  <b>1.187</b> <b>Glossary</b>  <b>1.188</b> <b>Abbreviations</b>  <b>1.189</b> <b>Acronyms</b>  <b>1.190</b> <b>Footnotes</b>  <b>1.191</b> <b>Endnotes</b>  <b>1.192</b> <b>References</b>  <b>1.193</b> <b>Appendix</b>  <b>1.194</b> <b>Index</b>  <b>1.195</b> <b>Glossary</b>  <b>1.196</b> <b>Abbreviations</b>  <b>1.197</b> <b>Acronyms</b>  <b>1.198</b> <b>Footnotes</b>  <b>1.199</b> <b>Endnotes</b>  <b>1.200</b> <b>References&lt;/</b></p>
--

$$x_0 = 1,000001$$



# Kompleksna ravnina

- imaginarna jedinica

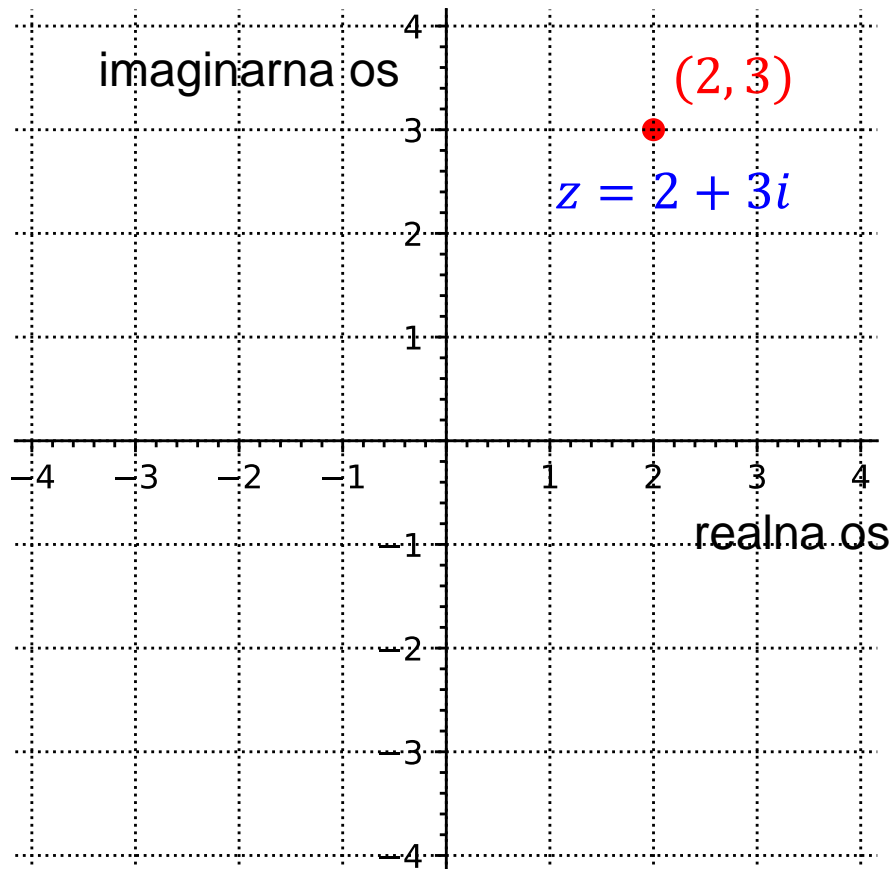
$$i \equiv \sqrt{-1}$$

- kompleksnom broju

$$z = x + iy$$

može se jednoznačno  
pridružiti točka  $(x, y)$  u  
kompleksnoj ravnini

- $x$  i  $y$  su realni brojevi,  
realni i kompleksni dio  
kompleksnog broja  $z$



# Iteracije u kompleksnoj ravnini

---

$$f(z) = z^2$$

$$z_{k+1} = z_k^2$$

$$z = x + iy$$

$$\begin{aligned} z^2 &= (x + iy)(x + iy) = \\ &= x^2 - y^2 + i 2xy \end{aligned}$$

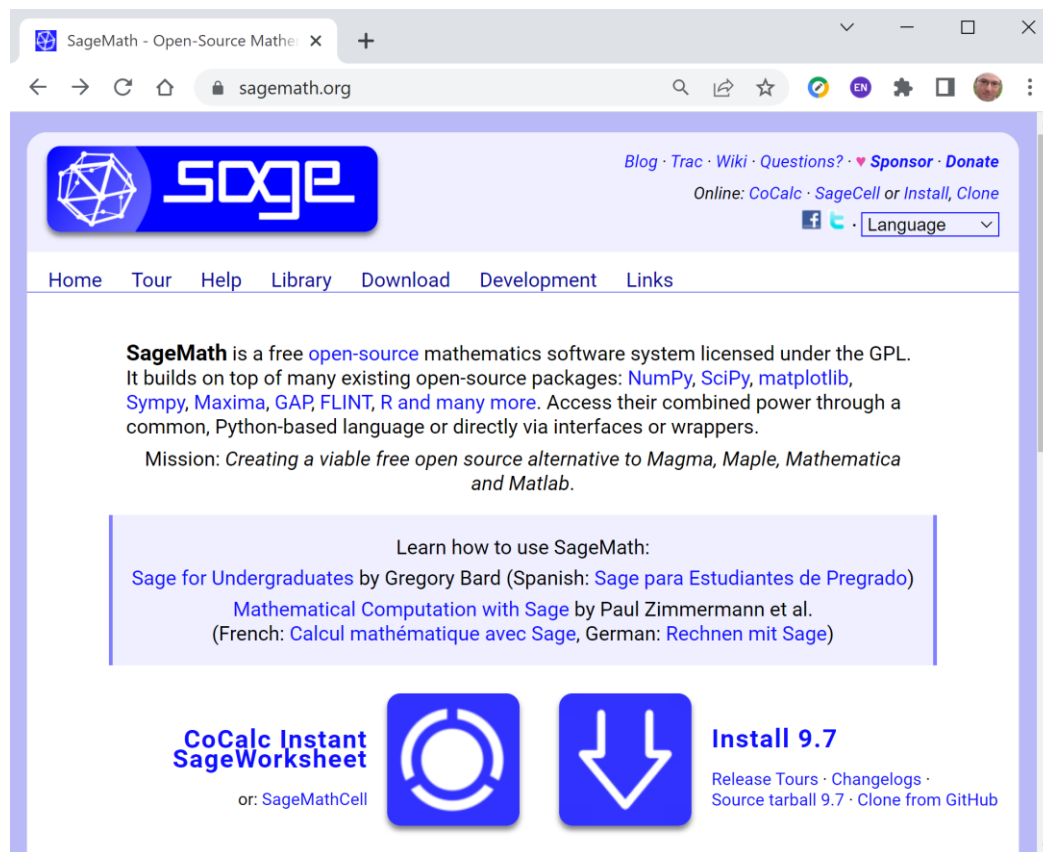


# SageMath

## Open-Source Mathematical Software System

### Prednosti:

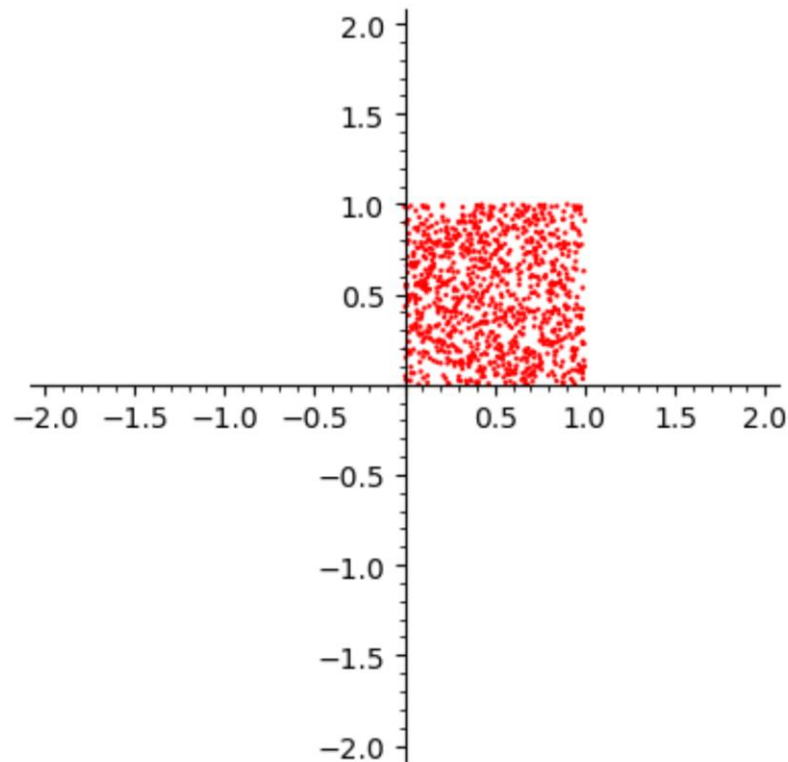
- temelji se na Pythonu
- korisničko sučelje je u pregledniku – dostupan na svim platformama
- besplatan i otvorenog koda
- ima jaku zajednicu korisnika





# Slučajni brojevi u kompleksnoj ravnini

---



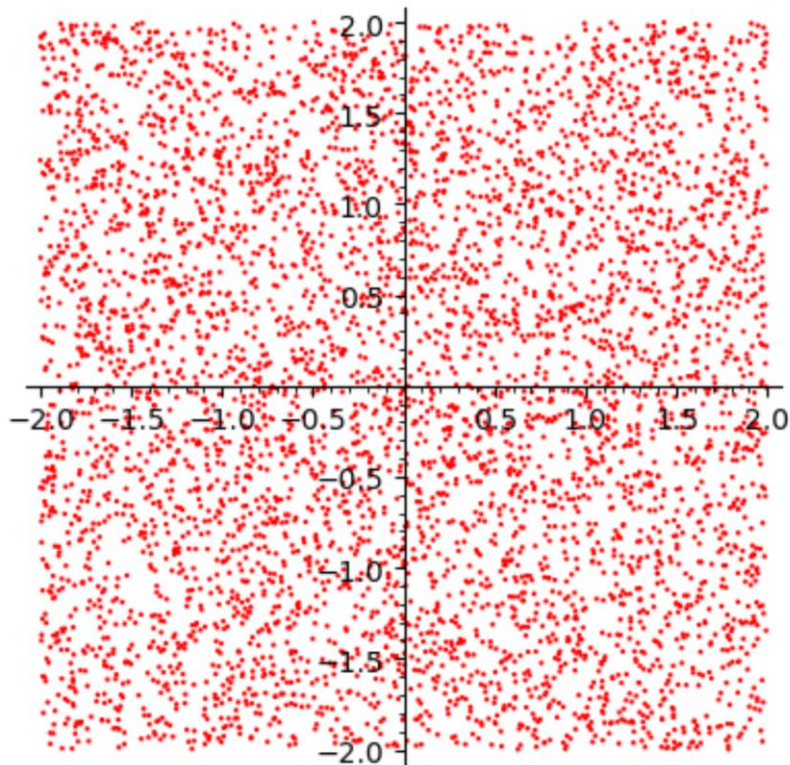
```
r = 2.0 # raspon prikaza
tocke = Graphics()

for i in range(1000):
    z = random() + random() * I
    tocke += point((z.real(), z.imag()), size = 3,
                  rgbcolor = (1, 0, 0))

tocke.show(aspect_ratio = 1,
           xmin = -r, xmax = r, ymin = -r, ymax = r)
```

# Slučajni brojevi u kompleksnoj ravnini

---



```
r = 2.0 # raspon prikaza
tocke = Graphics()

for i in range(5000):
    z = (random() - 0.5 + (random() - 0.5) * I) * 4
    tocke += point((z.real(), z.imag()), size = 3,
                  rgbcolor = (1, 0, 0))

tocke.show(aspect_ratio = 1,
           xmin = -r, xmax = r, ymin = -r, ymax = r)
```

# Iteracije u kompleksnoj ravnini

---

```
r = 2.0 # raspon prikaza
tocke = Graphics()
niter = 50

for i in range(20000):
    z0 = (random() - 0.5 + (random() - 0.5) * I) * 4
    z = z0
    i = 0
    while z.abs() < 2.0 and i < niter:
        z = z^2
        i += 1
    tocke += point((z0.real(), z0.imag()), size = 3,
        rgbcolor = (0, float(i) / float(niter), 0))

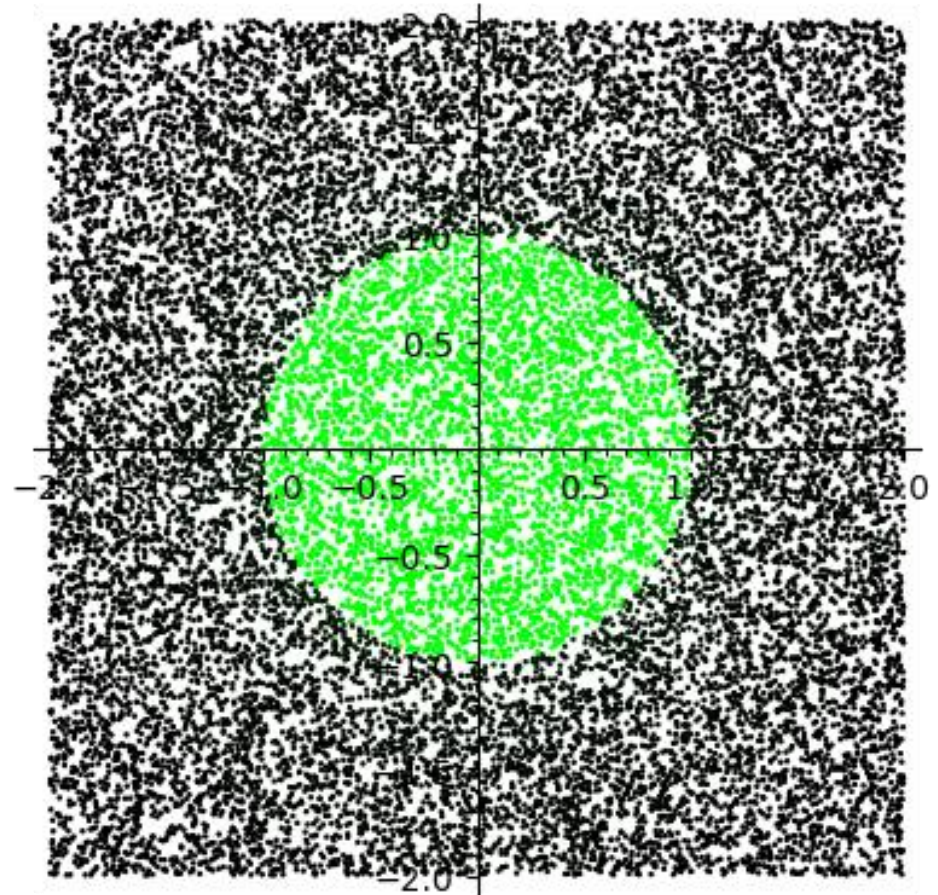
tocke.show(aspect_ratio = 1,
    xmin = -r, xmax = r, ymin = -r, ymax = r)
```

# Iteracije u kompleksnoj ravnini

---

$$f(z) = z^2$$

$$z_{k+1} = z_k^2$$

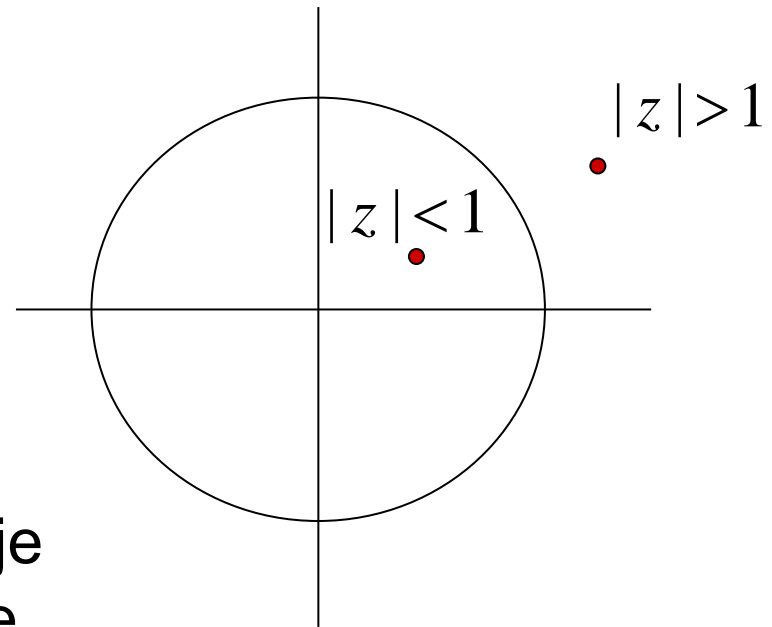


# Iteracije u kompleksnoj ravnini

---

$$f(z) = z^2$$

$$z_{k+1} = z_k^2$$



Područje konvergencije je unutar jedinične kružnice.

# Samo se dodaje konstanta...

---

$$f(z) = z^2 + c$$

$$z_{k+1} = z_k^2 + c$$

$c$  = konstanta

(kompleksni broj)

primjer:  $c = -0.4 + 0.6 i$





# Julijev skup

---

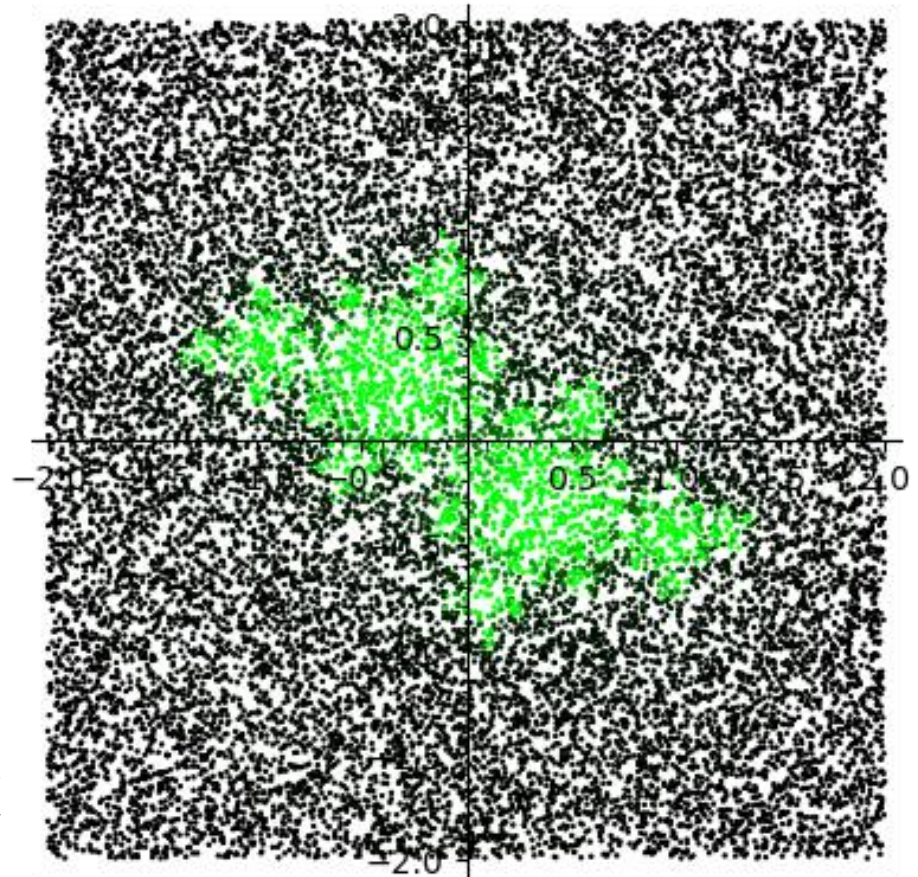
$$f(z) = z^2 + c$$

$$z_{k+1} = z_k^2 + c$$

$c$  = konstanta

(kompleksni broj)

primjer:  $c = -0.4 + 0.6 i$



Svaki puta se dodaje onaj  $z$  s kojim je započela iteracija...

---

$$f(z) = z^2 + z_0$$

$$z_{k+1} = z_k^2 + z_0$$



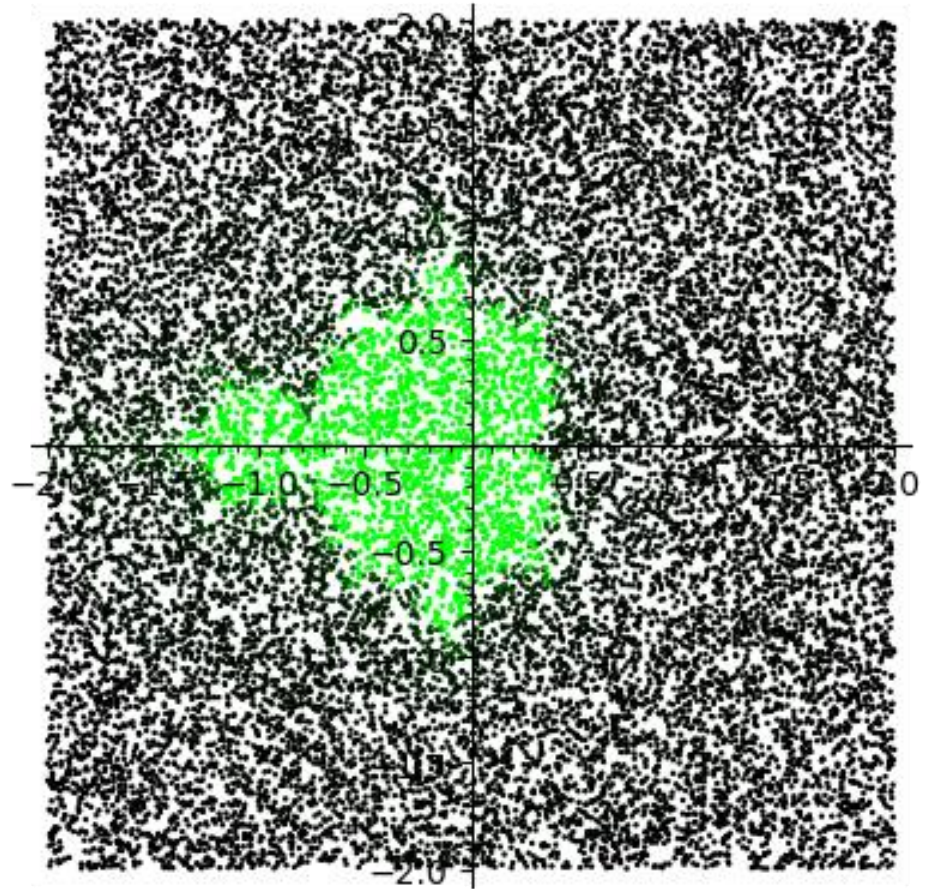


# Mandelbrotov skup

---

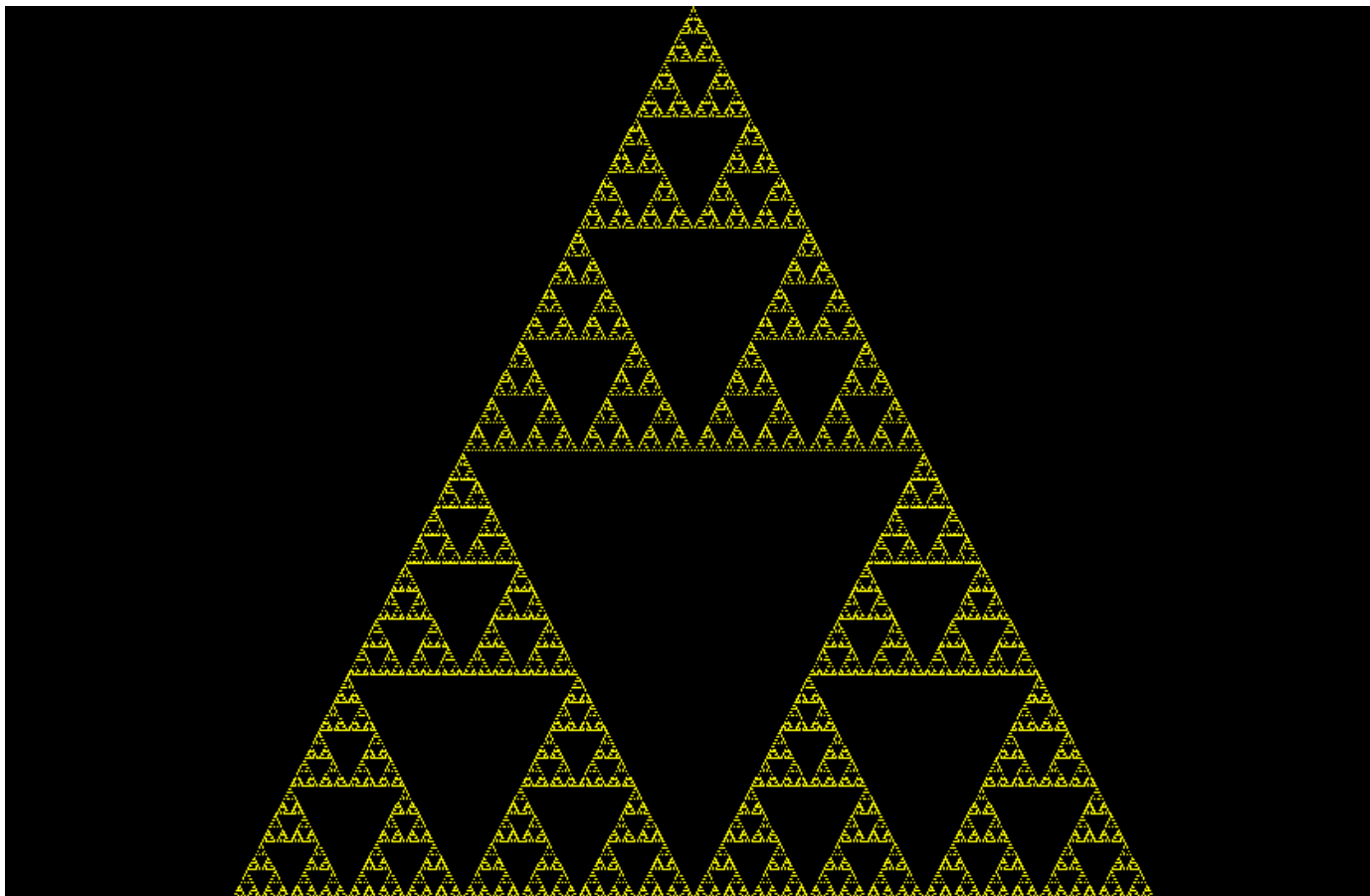
$$f(z) = z^2 + z_0$$

$$z_{k+1} = z_k^2 + z_0$$



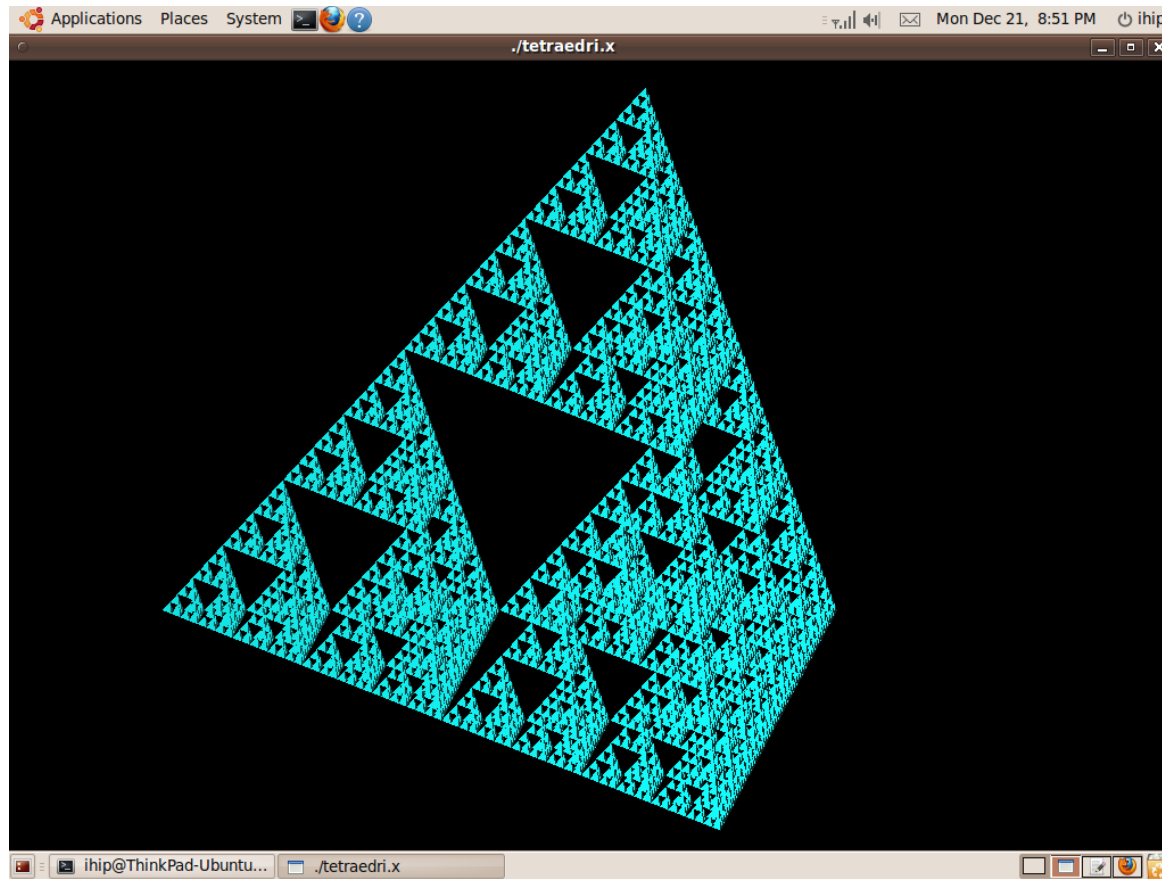
# Deterministički sebi-slični fraktali

---



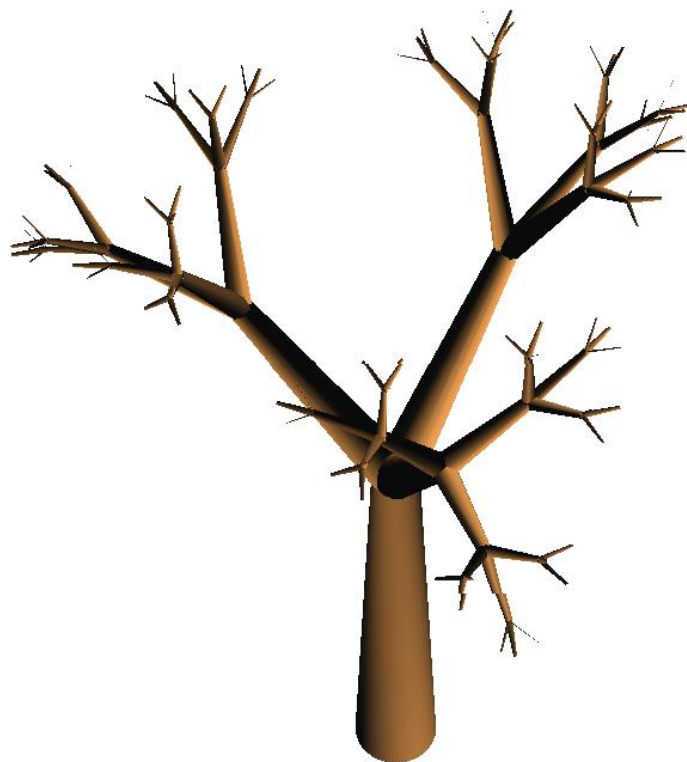
# Deterministički sebi-slični fraktali 2

---



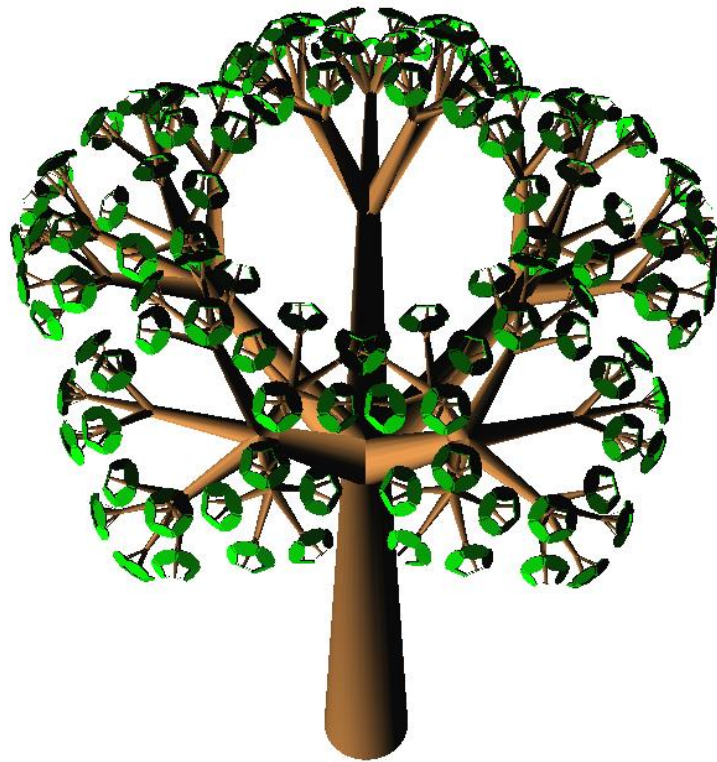
# Fraktalno drvo

---



# Fraktalno drvo s liščem

---



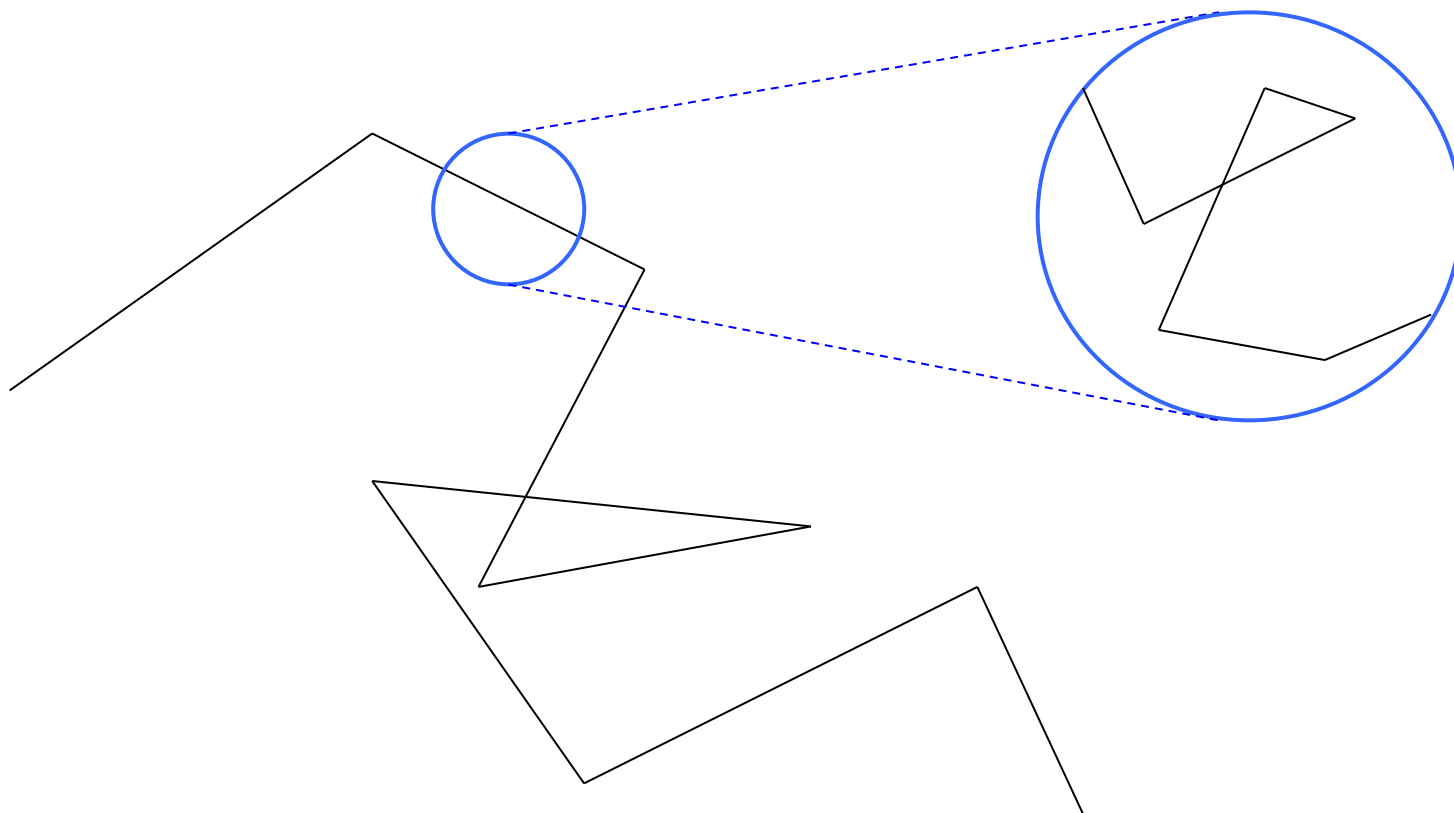
---

# **SLUČAJNI FRAKTALI**

---

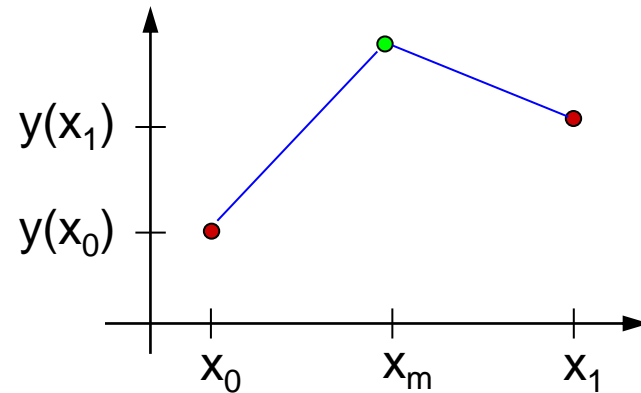
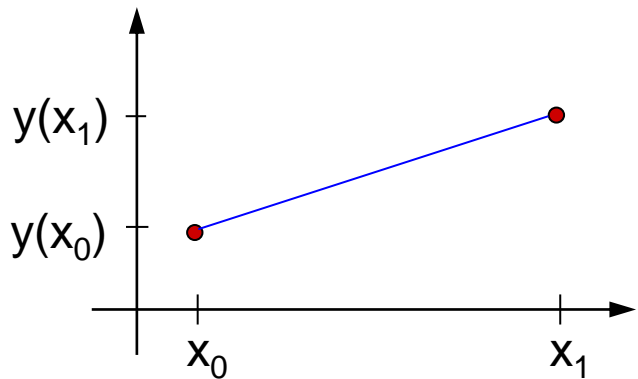
# Brownovo gibanje

---



# “Random midpoint-displacement methods”

---



$$y(x_m) = \frac{1}{2} [y(x_0) + y(x_1)] + r$$

$$r = s r_g (x_1 - x_0)$$

$r_g$  – slučajni broj iz  
Gaussove raspodjele  
(aritmetička sredina = 0  
i varijanca = 1)

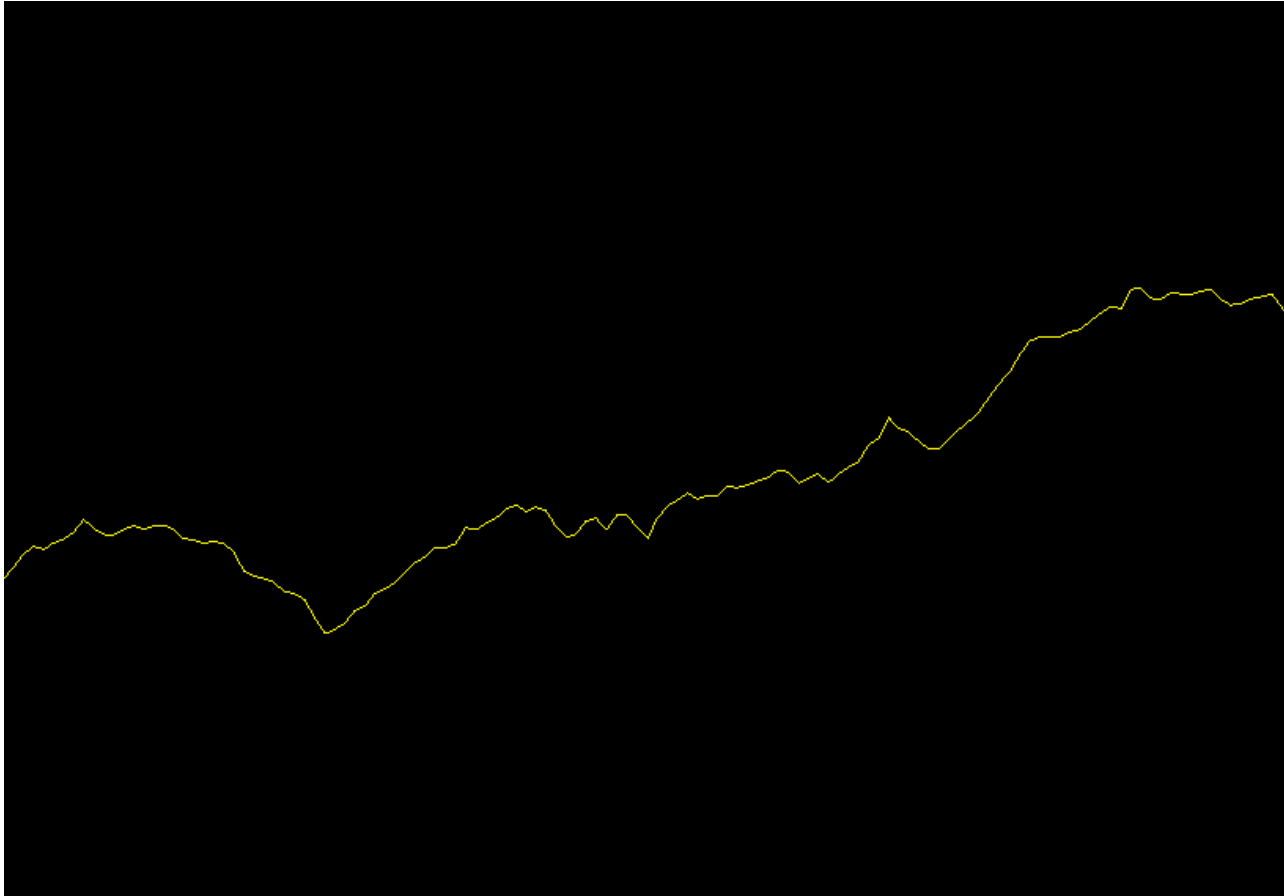
$s$  - služi za skaliranje

---



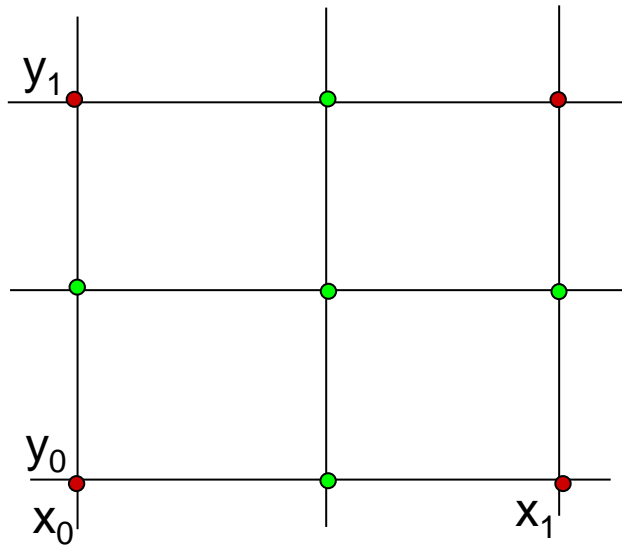
# Iskra

---



# Plazma-fraktali

---



$$z(x_m, y_k) = \frac{1}{2} [z(x_0, y_k) + z(x_1, y_k)] + r_x$$

$$z(x_k, y_m) = \frac{1}{2} [z(x_k, y_0) + z(x_k, y_1)] + r_y$$

$$r_x = s r_g (x_1 - x_0) \quad r_y = s \tilde{r}_g (y_1 - y_0)$$

$$z(x_m, y_m) = \frac{1}{4} [z(x_m, y_0) + z(x_1, y_m) + z(x_m, y_1) + z(x_0, y_m)] + r_m$$

$$r_m = s r_g (x_1 - x_0 + y_1 - y_0) / 2$$


---

# Plazma

---



# Model terena

---

