# Lecture Notes: Singular Value Decomposition, Principal Component Analysis, and Applications

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# 1 Singular Value Decomposition

## 1.1 SVD Overview[3]

Singular value decomposition (SVD) is a method of matrix decomposition that separates a rank-r matrix $A \in \mathbb{R}^{m\times n}$ into a sum of $r$ rank-1 matrices, each written as a column times row. Specifically, we can find:

1) orthonormal vectors $\vec{u}_1, \ldots, \vec{u}_r \in \mathbb{R}^m$

2) orthonormal vectors $\vec{v}_1, \ldots, \vec{v}_r \in \mathbb{R}^n$

3) real, positive numbers $\sigma_1, \ldots, \sigma_r$ such that

$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T + \cdots + \sigma_r \vec{u}_r \vec{v}_r^T$$

The numbers $\sigma_1, \ldots, \sigma_r$ are called singular values and, by convention, we order them from the largest to smallest:

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$$

In its original form $A$ has $mn$ entries to be stored. In the SVD form each of the $r$ terms is the product of a column of $m$ entries with a row of $n$ entries; therefore we need $r(m+n)$ numbers to store. This is an advantage when $r$ is small relative to $m$ and $n$, that is $r(m+n) \ll mn$. In a typical application the exact rank $r$ may not be particularly small, but we may find that the first few singular values, say $\sigma_1, \ldots, \sigma_{\hat{r}}$ are much bigger than the rest, $\sigma_{\hat{r}+1}, \ldots, \sigma_r$. Then it is reasonable to discard the small singular values and approximate $A$ as

$$A \approx \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T + \cdots + \sigma_{\hat{p}} \vec{u}_{\hat{p}} \vec{v}_{\hat{f}}^T$$

which has rank $= \hat{r}$, thus $\hat{r}(m+n) \ll mn$ numbers to store.

Besides enabling data compression, SVD allows us to extract important features of a data set as illustrated in the next example.

Example (Netflix): Suppose we have a $m \times n$ matrix that contains the ratings of $m$ viewers for $n$ movies. A truncated SVD as suggested above not only saves memory; it also gives insight into the preferences of each viewer. For example we can interpret each rank-1 $\text{matrix}\, \sigma_i \vec{u}_i \vec{v}_i^T$ to be due to a particular attribute, $e.g.$, comedy, action, sci-fi, or romance content. Then $\sigma_i$ determines how strongly the ratings depend on the $i$ th attribute, the entries of $\vec{v}_i^T$ score each movie with respect to this attribute, and the entries of $\vec{u}_i$ evaluate how much each viewer cares about this particular attribute. Then truncating the SVD as in (8) amounts to identifying a few key attributes that underlie the ratings.

This is useful, for example, in making movie recommendations.

Numerical Example: Finding the SVD

To find a SVD for $A$ we use either the $n \times n$ matrix $A^T A$ or the $m\times$ $m$ matrix $AA^T$. We will see later that these matrices have only real eigenvalues, $r$ of which are positive and the remaining zero, and a complete set of orthonormal eigenvectors. For now we take this as a fact and outline the following procedure to find a SVD using $A^T A$ :

1. Find the eigenvalues $\lambda_i$ of $A^T A$ and order them from the largest to smallest, so that $\lambda_1 \geq \cdots \geq \lambda_r > 0$ and $\lambda_{r+1} = \cdots = \lambda_n = 0$

2. Find orthonormal eigenvectors $\vec{v}_i$, so that

$$A^T A \vec{v}_i = \lambda_i \vec{v}_i \quad i = 1, \ldots, r$$

3. Let $\sigma_i = \sqrt{\lambda_i}$ and obtain $\vec{u}_i$ from

$$A\vec{v}_i = \sigma_i \vec{u}_i \quad i = 1, \ldots, r$$

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Example: Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$$

Next:

$$A^T A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 12 \end{bmatrix}$$

Finding the eigenvalues of $A^T A$, we get:

$$\det(\lambda I - A) = \det \begin{bmatrix} \lambda - 3 & -6 \\ -6 & \lambda - 12 \end{bmatrix} = \lambda^2 - 15\lambda = \lambda(\lambda - 15) = 0$$

Solving the polynomial, we find that $\lambda_1 = 15$ and $\lambda_2 = 0$. Next, we find an eigenvector $\vec{v}_1$ from

$$\begin{bmatrix} \lambda_1 - 3 & -6 \\ -6 & \lambda_1 - 12 \end{bmatrix} \vec{v}_1 = \begin{bmatrix} 12 & -6 \\ -6 & 3 \end{bmatrix} \vec{v}_1 = 0$$

We then normalize the eigenvector.

$$\vec{v}_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

We compute the singular value from $\sigma_1 = \sqrt{\lambda_1} = \sqrt{15}$, and $\vec{u}_1$:

$$\vec{u}_1 = \frac{1}{\sigma_1} A \vec{v}_1 = \frac{1}{\sqrt{15}} \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{\sqrt{15}} \frac{1}{\sqrt{5}} \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Thus we have obtained the SVD:

$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T = \sqrt{15} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

### 1.2 Applications of SVD

For mathematical applications, We can use SVD in computing the pseudoinverse, matrix approximation, and determining the rank, range and null space of a matrix. Some other applications of SVD include the study of inverse and regularization problems like Tikhonov regularization, often taught in machine learning courses as ridge regression. Ridge regression is used to compensate for multicollinearity, the state of a system having predictor variables that are not linearly independent from each other. This compromises model-building methods like least-squares regression, and thus reduces our analysis of the data. For an estimate of how multicollinearity can affect a system, we can perform SVD to show that the estimated coefficients can exhibit drastic changes after only slight changes in the training data.

We can also use SVD in recommender systems, which use data from sources such as internet searches and video history to personalize advertising and customize search results. Another application you are likely familiar with is Netflix's reccomendation algorithm, which predicts what rating a user might give a film provided with their existing review history (as discussed earlier in the notes). We can also use low-rank SVD to detect disease hotspots during outbreaks, which is particularly relevant given the severity of the ongoing COVID-19 pandemic; finding disease hotspots could allow travelers and first responders to assess the risk of infection in a particular area, if given access to disease statistics for areas close by.

## 2 Principal Component Analysis

### 2.1 Overview of PCA

PCA is an application of SVD in statistics whose goal is to find the directions (principal components) that explain a maximal amount of the set's variance.

Suppose the $m \times n$ matrix $A$ contains $n$ measurements from $m$ samples. Each measurement for a single row in the matrix would correspond to a different variable; for example, $n$ test scores for $m$ students, assuming that all the scores for any one student correspond to a different test. If we subtract from the sample mean from each measurement the average over all samples, then each column of $A$ is an $m$ -vector with zero mean, and the $n \times n$ matrix

$$\frac{1}{m-1} A^T A$$

constitutes the covariance matrix. Recall that the eigenvalues of this matrix are the singular values of $A$ except for the scaling factor $m-1$, and its orthonormal eigenvectors correspond to $\vec{v}_{1,\ldots,\vec{v}_n}$ in the SVD of $A$.

The vectors $\vec{v}_1, \vec{v}_2, \ldots$ corresponding to large singular values are called principal components and identify dominant directions in the data set along which the samples are clustered. The most significant direction is $\vec{v}_1$ corresponding to $\sigma_1$.[4]

Some points to remember about the covariance matrix are that it is necessarily an $n \times n$ matrix, where $n$ is the number of dimensions (or variables, in the case of a real-world classification problem). This matrix represents the covariances of all possible pairs of initial variables; thus, we can examine the elements along the main diagonal to find the variances (squared standard deviation) of each variable, since the diagonal consists of the covariance of each variable with itself.

Note that a positive covariance indicates positive correlation between the two variables in question, while a negative correlation indicates inverse correlation.

When performing PCA, be sure to examine the angles the principal components make with the axes; the principal components should be skewed towards the axes that represent variables with larger variances, so this could be a way to check your work. Additionally, we can consider the eigenvalues for insight about the percentage of variance each eigenvalue accounts for; larger eigenvalues account for a larger amount of the variance, while small eigenvalues represent variables that account for less of the variance.

In this vein, variables that vary significantly in magnitude (for example, human wrist circumference and human height) must be standardized before conducting PCA to ensure that equally significant dimensions are given equal weight in analysis regardless of differing magnitudes. Ideally, the data should all be centered around zero, as explained above, and have a standard deviation of one. However, this is not necessary for datasets where dimensions are of comparable magnitude, such as our test score example. For data such as examination score results, the data are largely of the same order of magnitude and fall within a certain limited range (e.g., 0-100) so we can go ahead with PCA analysis after centering.

In certain multidimensional datasets (such as the iris dataset used in the first coding assignment) smaller eigenvalues can provide a valuable opportunity for dimensionality reduction, because PCA analysis can demonstrate which variables are significant to the variance and which are less so. For example, if we wanted to classify a tetrapod dataset based on species, a variable that accounted for the number of limbs a certain animal had would not explain any of the variance in classification, since all tetrapods have four limbs. Even if we did not know that was the case, the eigenvalue corresponding to limb count would likely be one of the smallest eigenvalues in the set; hence, we could confidently eliminate the variable, and continue the classification problem with fewer dimensions.

As a technique, PCA is widely applicable in several fields. For example, principal components are highly valuable in the prediction of stock prices and financial risk analysis, using variables like earnings yield and book to market ratio. Financial applications of PCA often use Hankel matrices, which are square matrices where all ascending skew-diagonals are constant. And as covered in the

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EE18B course, PCA is especially useful in image analysis. Often, images of the same object will be taken multiple times under different lighting (i.e, green light, infrared, ultraviolet) which may lead to different features about the object being clearly captured in each image. PCA relies on the redundancy between these images as a reference to help obtain a final clear image. The first few principal components will yield the most accurate pictures, and subsequent principal components yield pictures that are progressively less defined.

# 3 Minimum Norm

Consider solving a linear system of equations in the form $Xw = y$.
$X \in \mathbb{R}^{nxd}$ is your data vector with n data points and d dimensions. $w$ is your feature weights or coefficients. $y$ are your observations or labels that you wish to predict or estimate.

With training data $X_{train}$ and $y_{train}$, or any data $X$ and $y$ which you wish yo model, you want to be able to come up with an estimate for $w$ in the form $\hat{y} = x^T w$ that minimizes the error or residual $\|Xw - y\|_2^2$.

## 3.1 4 Scenarios of Linear Equations

There are 4 scenarios in which we can determine a solution for $w$:

1. X is square ($n = d$) and full rank.
2. X is tall or over determined ($n > d$) and full rank.
3. X is wide or under determined ($n < d$) and full rank.
4. X is rectangular ($n \neq d$) and not full rank.

For (1.), to find $w$ we can simply invert $X$ to get our solution:

$$w = X^{-1}y$$

Note that $X^{-1}X = XX^{-1} = I_{nxn}$.

For (2.), we want to solve a linear regression problem where our estimate or prediction is of the form $\hat{y} = Xw$, where for each point we have $\hat{y}_i = x_i^T w$. We can try to find an approximate solution for $w$ that will minimize the error or residual between $\hat{y}$ and $y$: $\min_w \|Xw - y\|_2^2$. This will be our least squares solution.

$$w = (X^T X)^{-1} X^T y$$

Note that $(X^T X)^{-1} X^T X = I_{nxn}$, making $(X^T X)^{-1} X^T$ a left inverse for $X$.

For (3.), our problem $Xw = y$ has infinite solutions. In such a scenario, we are interested in the minimum-norm solution, so we can reformulate our problem:

$$\min_w \|w\| \text{ s.t. } Xw = y$$

Generally, these type of problems are useful for constrained optimization and control problems, which you may have encountered in other engineering courses.

All of our solutions to this problem will have the form $\{w|Xw = y\} = \{w_r + w_n | w_r \in R(X), w_n \in N(X)\}$, where $R(X)$ is the range of the columns of $X$ and $N(X)$ is the null space of the columns of $X$.

The least norm solution will be:

$$w = X^T (XX^T)^{-1} y$$

Note that $XX^T (XX^T)^{-1} = I_{nxn}$, making $X^T (XX^T)^{-1}$ a right inverse for $X$.

To see that $w$ above is the least norm solution, let us consider another solution $z$, so we have $y = Xz = Xw$. Then $X(z - w) = 0$. Then we see

$$(z - w)^T w = (z - w)^T X^T (XX^T)^{-1} y = (X(z - w))^T (XX^T)^{-1} y = 0$$

So

$$\|z\|^2 = \|z + w - w\|^2 = \|w\|^2 + \|z - w\|^2 \geq \|w\|^2$$

Note that cross terms cancel. From this, we see that our least norm solution has the smallest norm.

### 3.2 SVD Perspective

We will now see that we can solve the systems of linear equations using an SVD perspective to give us a way to create a form of an inverse for our solution. This inverse is called the Moore Penrose Pseudo-Inverse.

#### 3.2.1 Moore Penrose Pseudo-Inverse

When solving linear least squares problems in the form $Xw = y$ ($X$ data matrix, $w$ coefficients, $y$ observations to predict). Often, $X$ is not square or directly invertible.

The Moore-Penrose pseudoinverse $X^+$ gives us a generalization of the inverse matrix that can help us solve linear least squares solutions, $w = X^+ y$ .

A computationally simple and accurate of computing the pseudoinverse involves using the singular value decomposition. If $X = USV^T$, then the pseudo-inverse would be

$$X^+ = VS^+U^T$$

where $S^+$ of a diagonal matrix of singular values $S$ is obtained by taking the reciprocal of the nonzero diagonal entries, then transposing that matrix $(S^{-1})^T$.

#### 3.2.2 Solving linear equation with the Pseudoinverse

The pseudo-inverse works for the overdetermined and underdermined system of equations case. In Figure 1, we can see how we can use SVD to derive the Moore-Penrose Pseudoinverse to solve systems of linear equations

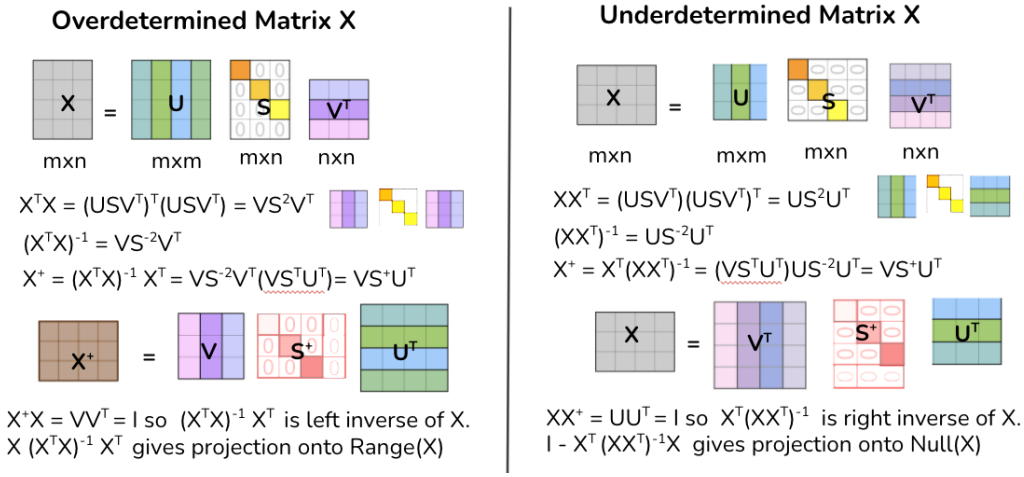



Figure 1: SVD form of the pseudoinverse for tall and wide matrices.

The following table summarizes the block form of the pseudo-inverse for all 4 scenarios listed above.

| | row m, col n, rank r | $\boldsymbol{X}^+$ | = | $\boldsymbol{V}$ | $\boldsymbol{S}^+$ | $\boldsymbol{U}$ |
|---|---|---|---|---|---|---|
| 1. | $m = n = r$ | $\boldsymbol{X}^{-1}$ | = | $\begin{bmatrix} \boldsymbol{V}_R \end{bmatrix}$ | $\begin{bmatrix} \boldsymbol{S}^{-1} \end{bmatrix}$ | $\begin{bmatrix} \boldsymbol{U}_R^T \end{bmatrix}$ |
| 2. | $m > n, n = r$ | $\boldsymbol{X}^+$<br>$\boldsymbol{X}^{-L}$ | = | $\begin{bmatrix} \boldsymbol{V}_R \end{bmatrix}$ | $\begin{bmatrix} \boldsymbol{S}^{-1} & 0 \end{bmatrix}$ | $\begin{bmatrix} \boldsymbol{U}_R^T \\ \boldsymbol{U}_N^T \end{bmatrix}$ |
| 3. | $m = r, m < n$ | $\boldsymbol{X}^+$<br>$\boldsymbol{X}^{-R}$ | = | $\begin{bmatrix} \boldsymbol{V}_R & \boldsymbol{V}_N \end{bmatrix}$ | $\begin{bmatrix} \boldsymbol{S}^{-1} \\ 0 \end{bmatrix}$ | $\begin{bmatrix} \boldsymbol{U}_R^T \end{bmatrix}$ |
| 4. | $m \neq n \neq r$ | $\boldsymbol{X}^+$ | = | $\begin{bmatrix} \boldsymbol{V}_R & \boldsymbol{V}_N \end{bmatrix}$ | $\begin{bmatrix} \boldsymbol{S}^{-1} & 0 \\ 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} \boldsymbol{U}_R^T \\ \boldsymbol{U}_N^T \end{bmatrix}$ |

### 3.2.3 Optional Reading: Deriving Minimum Norm Solution Using Lagrangian Multipliers

For those who have taken multivariable calculus, we can see how the optimal minimum norm solution is derived using Lagrangian multipliers. Our problem is formulated as:

$$\min_w \|w\|_2$$

$$\text{s.t.} \quad Xw = y$$

Let us introduce Lagrange Multipliers:

$$L(w, \lambda) = w^T w + \lambda^T (Xw - y)$$

Taking derivatives:

$$\nabla_w L = 2w + X^T \lambda = 0 \Rightarrow w = -\frac{X^T \lambda}{2}$$

$$\nabla_\lambda L = Xw - y = 0$$

Then substituting $w$ from the first equation into the 2nd equation, we get:

$$Xw - y = -X\frac{X^T \lambda}{2} - y = 0 \Rightarrow \lambda = -2(XX^T)^{-1} y$$

And therefore we plug back into the first equation to get:

$$w = \frac{X^T \lambda}{2} = (X^T)(XX^T)^{-1} y$$

which is our minimum norm solution.

### 3.2.4 Optional Reading: Extension to General Norm Optimization

As you continue your engineering career, you may come across problems where constraints are involved. One example of these problems will be of the form

$$\min_w \|Xw - y\|$$

$$\text{subject to} \quad Cw = d$$

Here, least squares and least norm problems are a special case of the formulation above. The problem formulation is equivalent to saying:

$$\begin{aligned} \min_{w} \ & \frac{1}{2}\|Xw - y\|^2 \\ \text{subject to} \ & Cw = d \end{aligned}$$

Like the minimum norm solution, we can solve this problem using Lagrangian multipliers. The Lagrangian is:

$$\begin{aligned} L(x, \lambda) &= \frac{1}{2}\|Xw - y\|^2 + \lambda^T(Cw - d) \\ &= \frac{1}{2}w^T X^T X w - y^T X w + \frac{1}{2}y^T y + \lambda^T C w - \lambda^T d \end{aligned}$$

Taking partial derivative to $x$ and $\lambda$ and setting to 0:

$$\begin{aligned} \nabla_w L &= X^T X w - X^T y + C^T \lambda = 0 \\ \nabla_\lambda L &= Cw - d = 0 \end{aligned}$$

or

$$\begin{bmatrix} X^T X & X^T \\ X & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} X^T y \\ d \end{bmatrix}$$

If the left-most block matrix is invertible, then we will have:

$$\begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} X^T X & X^T \\ X & 0 \end{bmatrix}^{-1} \begin{bmatrix} X^T y \\ d \end{bmatrix}$$

# 4 Introduction to Brain Computer Interface

Brain Computer Interface (BCI) refers to the idea of connecting the human brain with an external device (e.g. computer, robot). Successful integration can be applied for various purposes such as research, augmenting human abilities, controlling devices through thought, or treating neurological disorders.

Generally in the process of implementing a BCI, a device will be used to record electrical activity from the brain, which can be placed superficially on the head or implanted in the brain through surgery. These signals will need to be processed and understood in order to extract meaningful information. We will give a basic overview of basic neural signal processing workflow, and how dimensionality reduction can be useful.

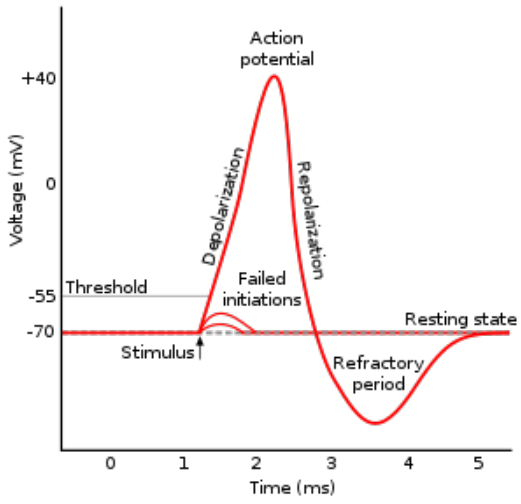
## 4.1 Electrophysiology Basics

Neurons conduct electrical signals in the brain via action potentials (AP). These occur when the electrochemical gradient across the cell membranes reaches a level that causes ion movement, generating a 'current' as the neuron propogates this signal across axons. Neurotransmitters, a type of chemical signal, will then be released to neighboring neurons. With enough neurotransmitter release, ion flow across neighboring neurons will activate another action potential and pass on the signal around the brain.

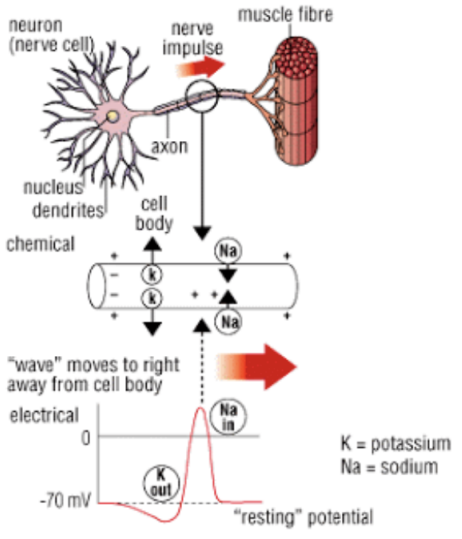
These electrical impulses can be picked up with sensors. Figure 2a shows us the form of an action potential, and Figure 2b shows how these action potentials propogate through neurons.

## 4.2 Neuronal Recordings and Applications

Electrical impulses from neurons can be picked up by electrodes, although the type and quality of the signal will depend upon many factors. At a small scale, it is possible to record from a single neuron. Usually in humans, recordings are done via extracellular recordings, where the electrodes are placed directly on the brain (called ECoG, or electrocorticography). These recordings usually pick up an ensemble of neuron signals, but some single neuron recordings can possibly be extracted. At an even higher and non-invasive level, electrical signals can be picked up by electrocorticography (EEG), which involved recording from the skin. These signals are usually noisy and give average electrical




(a) The form of the change in membrane voltage during an action potential.




(b) Propogation of an action potential down an axon.

Figure 2: Neuron Action Potentials.

activity in a large region of the brain. Usually, there can be high levels features that can be extracted, such as time, frequency, space, and phase.

For recordings at a level where single neuron spikes can be picked up, one technique widely used is spike detection and sorting. From ensemble waveforms, spikes can be extracted, and then classified based upon the waveform characteristics of the spike to be associated with individual neurons or independent spiking events. Knowing what class a spike comes from is useful for various applications by then looking at spike frequency. By classifying and bringing meaning to the neuronal signals picked up by the sensors, we can then use that to extract meaningful commands to help us control a computer or device! This is the appeal of brain computer interface!

### 4.3 Spike Sorting

With extracellular recordings, usually the resolution is at a level where we can extract signals from nearby individual neurons. Let us walk through the step of processing this signal:

1. Filtering: The first step after obtaining a recording is to perform filtering in order to extract the frequencies of interest, and remove noise. Removing noise is an important step to extract meaningful information from our signal. There are various techniques for filtering, but frequencies of interest should also be done into consideration of likely frequencies associated with the task of interest.

2. Spike Detection: With a cleaned signal, we want to extract parts of the signal may be associated with a single neuron. Usually this signals are louder than the surrounding noise. With a strong clean signal, simple peak detection methods work fine. For more complicated signals, algorithms such as nonlinear energy operator (NEO) attempts to identify peaks using the energy of the signal.

3. Waveform Extraction: With peaks identified, the waveforms contributing to those peaks can then be identified to represent the shape of the neuronal spike at that time. Usually a timeframe before and after the highest peak point are used as features to represent the waveform.

4. Waveform Classification Using Dimensionality Reduction: With the waveform extracted, it is not an unsupervised clustering problem to identify what waveforms belong to which neurons. Waveforms can be projected into a lower dimensional space using PCA as a dimensionality reduction algorithm. Then various methods can be used to identify clusters. With new signals and new waveforms, the waveform can be projected into the PCA space and classified based upon the most likely cluster that the waveform belongs to. This is the basis of spike sorting.

Figure 3 gives a pictoral overview of the steps outlines above.

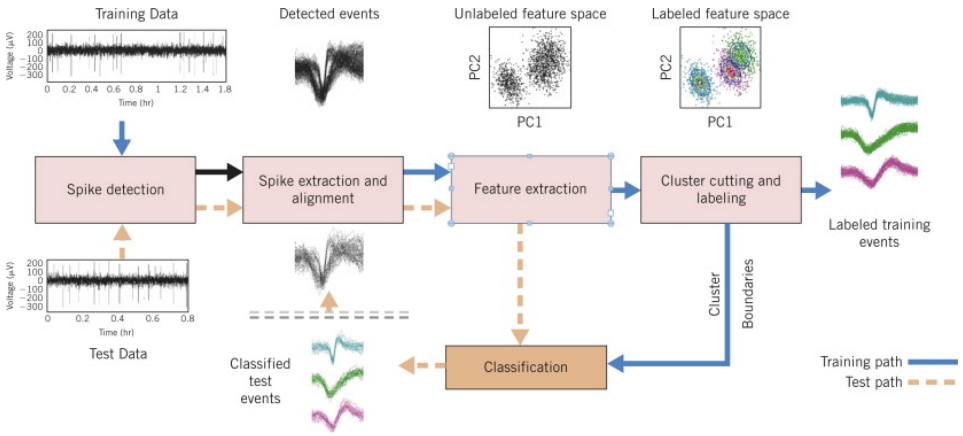



Figure 3: General Spike Sorting Pipeline.

### 4.4 Beyond Spike Sorting

Now that we understand basic neural signal processing, we can continue to apply machine learning concepts, such as dimensionality reduction and clustering, for applications of these neural signals.

#### 4.4.1 Application 1: Following individual neurons to conduct neuroscience experiments.

Using ECoG on humans or mice brain, you have just learned how to perform spike sorting to identify individual neuronal signals. By identifying individual neuronal signals, we can now perform experiments to observe neuron behavior in certain experimental settings, such as associated with a sensory signal or with a task.

One common signal that is followed is the spike train - which represents the spikes of an individual neuron or cluster over time. Using your spike sorting algorithm, you can now follow the spiking times and frequencies of your individual neuronal clusters. We can perform an experiment observing how the spiking of specific neurons change with a stimulus, such as observing the behavior of neurons in different regions of the brain when a student hears the word 'machine learning' (Figure 4). In this manner, we can ask questions to better understand the behavior of neurons and the brain.

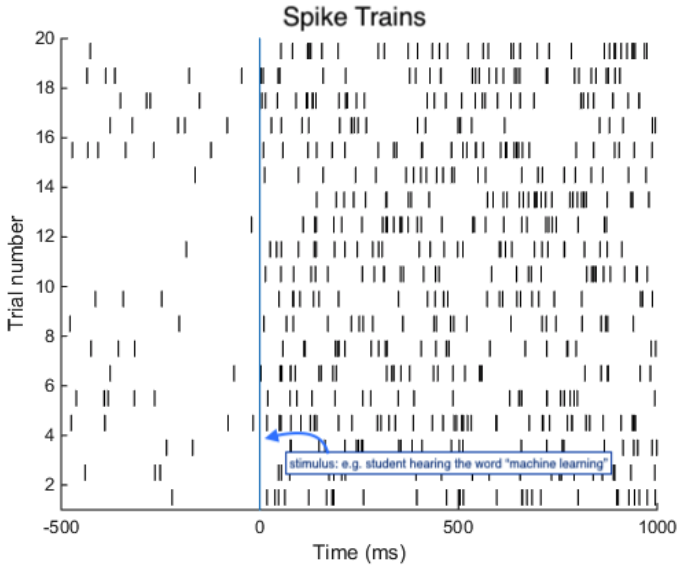



Figure 4: Example spike train, given a stimulus, over many trials.

#### 4.4.2 Application 2: Classifying Intention from Neuronal Activity

Often, one commercial application of noninvasive recording of neuronal activity through EEG, is for the purpose of classifying intention. By being able to distinguish intention in neuronal signals, this is how we can "control" the world with our minds!

For example, we may want to control a robotic arm, control the cursor on a screen, or control a steering wheel of a car. These applications may be formulated as a classification problem, such as classifying the intention to "move left hand" or "move right hand".

In this case, to create our algorithm, we can create our training data by recording many signals of left and right hand intention from human subjects. From the training data, we can extract features (such as spike waveform), and perform dimensionality reduction to classify these signals. That way, in the future, we can apply this model to new and unseen data to predict whether the person is intending to move their left or right hand (Figure 5).

#### 4.4.3 Application 3: Disease Detection

Another common application of neural signal processing is with applications to disease, such as epilepsy detection.

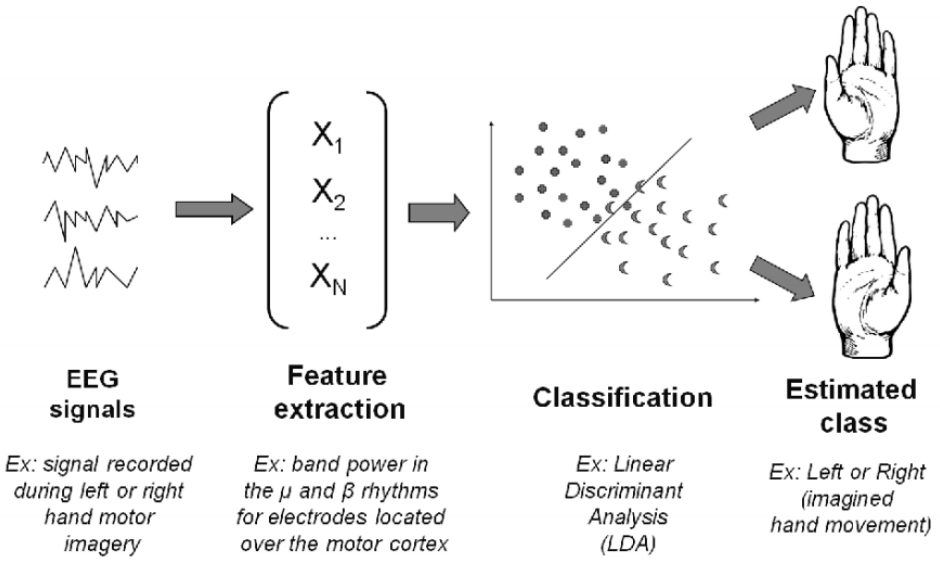



Figure 5: Visual example of the workflow to classify intention.

This can be formulated as a classification task on EEG data, into the labels of 'normal' or 'disease', for example. Again, this problem is just like the applications described above.

Given normal and disease EEG examples, we can extract features, perform dimensionality reduction, and classify the signals as normal or diseased. In this way, we can apply these models to detect seizures for a patient in the future!

As you can see, the applications for BCI and machine learning are endless. These examples should provide an overview of the applications of basic neural signal processing and classification.