CSE536 – Game Theory and Mechanism Design
Assignment 1, Spring Semester 2019

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1.

		COLUMN	
		Left	Right
ROW	Up	1	4
	Down	2	3

Table 1: Strategies of question 2(a)

2. (a) As the above strategies is of a zero-sum game, so we can also shown the above strategies as follows:

		COLUMN	
		Left	Right
ROW	Up	1,-1	4,-4
	Down	2,-2	3,-3

Table 2: Complete Strategies

Table 2 shows that COLUMN player gets more payoff in 'Left' move than 'Right' moves, no matter what is the ROW player's move. So, 'Right' is a dominated strategy. So, we can eliminated this strategy.

		COLUMN
		Left
ROW	Up	1,-1
	Down	2,-2

Table 3: After elimination of column 'Right'

Then, ROW player always select 'Down' for better payoff than 'Left' ($2 > 1$). So, here 'Up' is dominated. We can eliminated row 'Up'.

		COLUMN
		Left
ROW	Down	2,-2

Table 4: After elimination of row 'Up'

So, the nash equilibrium is ('Down','Left').

		COLUMN			
		Left	Right		Min
ROW	Up	1	4	→	1
	Down	2	3	→	2
		↓	↓		Max↓
	Max	2	4	Min→	2

Table 5: Minimax 2(a)

So, the nash equilibrium is ('Down','Left').

		COLUMN	
		Left	Right
ROW	Up	1	2
	Down	4	3

Table 6: Strategies question 2(b)

		COLUMN	
		Left	Right
ROW	Up	1,-1	2,-2
	Down	4,-4	3,-3

Table 7: Completed Strategies

(b) Table 9 shows that ROW player gets more payoff in 'Down' move than 'Up' move, no matter what is the COLUMN player's move. So, 'Up' is a dominated strategy. So, we can eliminated this strategy.

		COLUMN	
		Left	Right
ROW	Down	4,-4	3,-3

Table 8: After elimination of row 'Up'

Then, 'Left' is also dominated by 'Right'.

		COLUMN
		Right
ROW	Down	3,-3

Table 9: After elimination of column 'Left'

So, (Down, Right) is nash equilibrium.

		COLUMN			
		Left	Right		Min
ROW	Up	1	2	→	1
	Down	4	3	→	3
		↓	↓		Max↓
	Max	4	3	Min→	3

Table 10: Minimax 2(b)

So, (Down, Right) is nash equilibrium.

		COLUMN		
		Left	Middle	Right
ROW	Up	5	3	2
	Straight	6	4	3
	Down	1	6	2

Table 11: Strategies of question 2(c)

(c) Equivalent strategeis

		COLUMN		
		Left	Middle	Right
ROW	Up	5,-5	3,-3	2,-2
	Straight	6,-6	4,-4	3,-3
	Down	1,-1	6,-6	2,-2

Table 12: Strategies of question 2(c)

Middle column is dominated by Right column, so we can eliminated 'Middle' column.

		COLUMN	
		Left	Right
	Up	5,-5	2,-2
ROW	Straight	6,-6	3,-3
	Down	1,-1	2,-2

Table 13: After elimination of column Middle 2(c)

Up and Down row are dominated by Straight. So, Up and Down also are eliminated.

		COLUMN	
		Left	Right
ROW	Straight	6,-6	3,-3

Table 14: After elimination of row Up and Down 2(c)

		COLUMN
		Right
ROW	Straight	3,-3

Table 15: After elimination of column Left 2(c)

So, (Straight, Right) is nash equilibrium.

			COLUMN			
		Left	Middle	Right		Min
	Up	5	3	2	→	2
	Straight	6	4	3	→	3
ROW	Down	1	6	2	→	1
		↓	↓	↓		Max↓
	Max	6	6	3	Min→	3

Table 16: Minimax 2(c)

			COLUMN	
		Left	Middle	Right
	Up	5	3	1
ROW	Straight	6	1	2
	Down	1	0	0

Table 17: Strategies of question 2(d)

		COLUMN		
		Left	Middle	Right
	Up	5,-5	3,-3	1,-1
ROW	Straight	6,-6	1,-1	2,-2
	Down	1,-1	0,0	0,0

Table 18: Strategies of question 2(d)

(d)

Left column is dominated by other two.

		COLUMN	
		Middle	Right
	Up	3,-3	1,-1
ROW	Straight	1,-1	2,-2
	Down	0,0	0,0

Table 19: After elimination of column Left 2(d)

		COLUMN	
		Middle	Right
ROW	Up	3,-3	1,-1
	Straight	1,-1	2,-2

Table 20: After elimination of row Down 2(d)

There is no pure nash equilibrium.

		COLUMN			
		Middle	Right		Min
	Up	3	1	$\rightarrow$	1
ROW	Straight	1	2	$\rightarrow$	1
		$\downarrow$	$\downarrow$		Max$\downarrow$
	Max	3	2	Min$\rightarrow$	1,2

Table 21: Minimax 2(d)

Not solvable by minimax.

		COLUMN	
		Left	Right
ROW	Up	3,1	4,2
	Down	5,2	2,3

Table 22: Strategies of question 3(a)

3. (a) If ROW player selects ‘Up’ then COLUMN player select ‘Right’ and if ROW player selects ‘Down’ then COLUMN player selects ‘Right’ also. So, we can eliminate ’Left’ column. Then the strategies table looks as follows:

		COLUMN
		Right
ROW	Up	4,2
	Down	2,3

Table 23: After eliminaiton of column ‘Left’

Now, the ROW player always selects ‘Up’ to maximize its payoff (4). So, we can eliminate the row ‘Down’. Then the strategies table looks as follows:

		COLUMN
		Right
ROW	Up	4,2

Table 24: After elimination of row ‘Down’

So, this state (‘Up’, ‘Right’) is the Nash equilibrium state.

		COLUMN	
		Left	Right
ROW	Up	0,0	0,0
	Down	0,0	1,1

Table 25: Strategies of question 3(b)

(b) **(‘Up’,‘’)** Nither player change their move because thus they cannot increase their payoff.

(‘Up’,‘Right’) ROW player wants to take a move as ‘Down’ instead of ‘Up’ because thus it can get more payoff (1) than 0.

(‘Down’,‘Left’) COLUMN player wants to take a move as ‘Right’ instead of ‘Left’ because thus it can get more payoff (1) than 0.

(‘Down’,‘Right’) Nither player change their move because thus they cannot increase their payoff.

So, the equilibrium states are (‘Up’,‘Left’) and (‘Down’,‘Right’).

		COLUMN		
		Left	Middle	Right
	Up	2,9	5,5	6,2
ROW	Straight	6,4	9,2	5,3
	Down	4,3	2,7	7,1

Table 26: Strategies of question 3(c)

(c)
- If COLUMN player selects 'Left' then ROW player selects 'Straight' and If ROW player selects 'Straight' then COLUMN player selects 'Left'. So, (Straight,Left) is a nash equilibrium.
- If COLUMN player selects 'Middle' then ROW player selects 'Straight' and If ROW player selects Straight' then COLUMN player selects 'Left'. So, this is not a nash equilibrium.
- If COLUMN player selects 'Right' then ROW player selects 'Up' and If ROW player selects 'Up' then COLUMN player selects 'Left'. So, this is not a nash equilibrium.

		COLUMN		
		Left	Middle	Right
	Up	5,3	7,2	2,1
ROW	Straight	1,2	6,3	1,4
	Down	4,2	6,4	3,5

Table 27: Strategies of question 3(d)

(d)
- If COLUMN player selects 'Left' then ROW player selects 'Up' and If ROW player selects 'Up' then COLUMN player selects 'Left'. So, (Up, Left) is a nash equilibrium.
- If COLUMN player selects 'Middle' then ROW player selects 'Up' and If ROW player selects 'Up' then COLUMN player selects 'Left'. So, this is not a nash equilibrium.
- If COLUMN player selects 'Right' then ROW player selects 'Down' and If ROW player selects 'Down' then COLUMN player selects 'Right'. So, (Down, Right) is a nash equilibrium.

		COLUMN		
		Left	Middle	Right
	Up	1,2	2,1	1,0
ROW	Straight	0,5	1,2	7,4
	Down	-1,1	3,0	5,2

Table 28: Strategies of question 4

4.
- If COLUMN player selects 'Left' then ROW player selects 'Up' and If ROW player selects 'Up' then COLUMN player selects 'Left'. So, (UP,Left) is a nash equilibrium.
- If COLUMN player selects 'Middle' then ROW player selects 'Down' and If ROW player selects 'Down' then COLUMN player selects 'Right'. So, this is not a nash equilibrium.

- If COLUMN player selects 'Right' then ROW player selects 'Straight' and If ROW player selects 'Straight' then COLUMN player selects 'Left'. So, this is not a nash equilibrium.

We describe the equilibrium using the strategies of the players not merely by the payoff received by the players because of we don't know what is the actual move of the players in the runtime.

		COLUMN			
		w	x	y	z
	A	7,5	-8,4	0,4	99,3
	B	5,0	4,1	15,9	100,8
ROW	C	6,0	5,8	15,9	10,2
	D	2,6	7,-10	3,9	10,8
	E	1,6	2,10	1,7	8,6

Table 29: Original Strategies Table

		COLUMN		
		w	x	y
	A	7,5	-8,4	0,4
	B	5,0	4,1	15,9
ROW	C	6,0	5,8	15,9
	D	2,6	7,-10	3,9
	E	1,6	2,10	1,7

Table 30: After eliminatin of column 'z'

5. The nash equilibriums are (A,w),(B,y),(C,y).

6. (a)

$$\begin{aligned}\Pi_1 &= (P_1 - 1) \times Q_1 \\ &= (P_1 - 1)(15 - P_1 - 0.5P_2) \\ &= 15P_1 - P_1^2 - 0.5P_1P2 - 15 + P_1 + 0.5P_2 \\ &= 16P_1 - P_1^2 - 0.5P_1P2 + 0.5P_2 - 15\end{aligned}$$

$$\begin{aligned}\tfrac{\partial \Pi_1}{\partial P_1} &= 16 - 2P_1 - 0.5P_2 = 0 \\ -2P_1 &= 0.5P_2 - 16 \\ P_1 &= 8 - 0.25P_2\end{aligned}$$

So, best response rule for LaBoulangerie is $P_1 = 8 - 0.25P_2$.

$$\begin{aligned}\Pi_2 &= (P_2 - 2) \times Q_2 \\ &= (P_2 - 2)(14 - P_2 - 0.5P_1) \\ &= 14P_2 - P_2^2 - 0.5P_1P2 - 28 + 2P_2 + P_1 \\ &= 16P_2 - P_2^2 - 0.5P_1P2 + P_1 - 28\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi_2}{\partial P_2} &= 16 - 2P_2 - 0.5P_1 = 0 \\
-2P_2 &= 0.5P_1 - 16 \\
P_2 &= 8 - 0.25P_1
\end{aligned}$$

So, best response rule for LaFromagerie's is $P_2 = 8 - 0.25P_1$.
Substituting, P_1's formula,

$$\begin{aligned}
P_2 &= 8 - 0.25P_1 \\
&= 8 - 0.25(8 - 0.25P_2) \\
&= 8 - 2 + 0.625P_2 \\
0.9375P_2 &= 6 \\
P_2 &= 6.4\$
\end{aligned}$$

Similarly,

$$P_1 = 6.4\$$$

(b)

$$\begin{aligned}
\Pi &= \Pi_1 + \Pi_2 \\
&= 16P_1 - P_1^2 - 0.5P_1P2 + 0.5P_2 - 15 + 16P_2 - P_2^2 - 0.5P_1P2 + P_1 - 28 \\
&= 17P_1 + 16.5P_2 - P_1^2 - P_2^2 - P_1P_2 - 43
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi}{\partial P_1} &= 17 - 2P_1 - P_2 = 0 \\
-2P_1 &= P_2 - 17 \\
P_1 &= 8.5 - 0.5P_2
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi}{\partial P_2} &= 16.5 - 2P_2 - P_1 = 0 \\
-2P_2 &= P_1 - 16.5 \\
P_2 &= 8.25 - 0.5P_1 \\
P_2 &= 8.25 - 0.5(8.5 - 0.5P_2) \\
P_2 &= 0.25 - 4.25 + 0.25P_2 \\
0.75P_2 &= -4 \\
P_2 &= 5.3\$
\end{aligned}$$

Similarly,

$$\begin{aligned}
P_1 &= 8.5 - 0.5(8.25 - 0.5P_1) \\
P_1 &= 8.5 - 4.125 + 0.25P_1 \\
75P_1 &= 4.375 \\
P_1 &= 5.83\$
\end{aligned}$$

		Bluebert	
		Goes Straight	**Swerves**
Redbert	**Goes Straight**	-6,-6	2,-2
	Swerves	-2,2	0,0

7. (a)

   (b) When both select 'Goes Straight' then both are crushed. So, then negative payoff (-6). And, if one selects 'Goes Straight', other selects 'Swerves' then who selects 'Goes Straight' get payoff 2 other one losses '2'. When both select 'Swerves' neither player loss, so same payoff gains (1).

   ('Goes Straight','Goes Straight') Redbert player wants to take a move as 'Swerves' instead of 'Goes Straight' because thus it can get more payoff (-2) than -6. Bluebert also try to select 'Swerves' when Redbert choices 'Goes Straight' for better payoff (-2).

   ('Goes Straight','Swerves') Nither player change their move because thus they cannot increase their payoff.

   ('Swerves','Goes Straight') Nither player change their move because thus they cannot increase their payoff.

   ('Swerves','Swerves') Both player change their move because thus they can increase their payoff.

   So, the pure strategy nash equilibrium are ('Goes Straight','Swerves') and ('Swerves','Goes Straight').

		COLUMN			
		Goes Straight	Swerves		Probability
	Goes Straight	-6,-6	2,-2	$\rightarrow$	α
ROW	Swerves	-2,2	1,1	$\rightarrow$	$1-\alpha$
		$\downarrow$	$\downarrow$		
	Probability	β	$1-\beta$		

Table 31: Mixed Strategies

Then,

$$
\begin{aligned}
-6 \times \beta + 2 \times (1-\beta) &= -2 \times \beta + 1 \times (1-\beta) \\
-6\beta + 2 - 2\beta &= -2\beta + 1 - \beta \\
-8\beta + 2 &= -3\beta + 1 \\
-5\beta &= -1 \\
\beta &= \frac{1}{5}
\end{aligned}
$$

and,

$$\begin{aligned} -6 \times \alpha + 2 \times (1-\alpha) &= -2 \times \alpha + 1 \times (1-\alpha) \\ -6\alpha + 2 - 2\alpha &= -2\alpha + 1 - \alpha \\ -8\alpha + 2 &= -3\alpha + 1 \\ -5\alpha &= -1 \\ \alpha &= \frac{1}{5} \end{aligned}$$

So, mixed strategy nash equilibrium is $(\alpha, \beta) = (\frac{1}{5}, \frac{1}{5})$.

		COLUMN		
		x	**y**	**z**
	p	5,6	2,4	17,5
ROW	**q**	4,5	10,16	10,6
	r	0,6	3,4	15,-7

Table 32: Strategies

8. • COLUMN player has three strategies x, y, z and ROW player has also three strategies p, q, r.

				COLUMN				
		x		**y**		**z**		x
	p	6	$>$	4	$<$	5	$<$	6
ROW	**q**	5	$<$	16	$>$	6	$>$	5
	r	6	$>$	4	$>$	-7	$<$	6

Table 33: Column comparison

• Table 33 shows that no column is dominant over other any column.

		COLUMN		
		x	**y**	**z**
	p	5	2	17
		$\vee$	$\wedge$	$\vee$
	q	4	10	10
ROW		$\vee$	$\vee$	$\wedge$
	r	0	3	15
		$\wedge$	$\vee$	$\wedge$
	p	5	2	17

Table 34: Row comparison

Table 34 shows that no one row is dominant over other any row. So, there is no dominant strategy in this game.

- As, this game is not zero-sum or constant sum game so this game is not solvable using minmax method.
- From the Table 32 only (p,x) and (q,y) are the nash equilibrium. Because of for only these two state neither player change their strategy until other player don't change their strategy.

		COLUMN	
		Left	**Right**
ROW	**Top**	4,2	0,4
	Bottom	2,4	6,0

Table 35: Pure Strategies

9. Best responses
   - If COLUMN selects 'Left' then ROW selects 'Top' ($4 > 2$).
   - If ROW selects 'Top' then COLUMN selects 'Right' ($4 > 2$).
   - If COLUMN selects 'Right' then ROW selects 'Bottom' ($6 > 2$).
   - If COLUMN selects 'Bottom' then ROW selects 'Left' ($4 > 0$).

   So, there is no nash equilibrium for pure strategy.

		COLUMN			
		Left	Right		Probability
ROW	Top	4,2	0,4	$\rightarrow$	α
	Bottom	2,4	6,0	$\rightarrow$	$1-\alpha$
		$\downarrow$	$\downarrow$		
	Probability	β	$1-\beta$		

Table 36: Mixed Strategies

Then,

$$\begin{aligned} 4 \times \beta + 0 \times (1-\beta) &= 2 \times \beta + 6 \times (1-\beta) \\ 4\beta &= 2 \times \beta + 6 - 6\beta \\ 8\beta &= 6 \\ \beta &= \frac{3}{4} \end{aligned}$$

and,

$$\begin{aligned} 2 \times \alpha + 4 \times (1-\alpha) &= 4 \times \alpha + 0 \times (1-\alpha) \\ 2\alpha + 4 - 4\alpha &= 4\alpha \\ -6\alpha &= -4 \\ \alpha &= \frac{2}{3} \end{aligned}$$

So, the mixed strategy Nash Equilibrium is $(\alpha, \beta) = (\frac{2}{3}, \frac{3}{4})$.

10. Dominance solvable games are those where the equilibrium outcome is the result of elimination of dominated strategies. Here, the game has unique nash equilibrium with non-degenerated mixed strategy. So, neither strategy is dominated by other. So, this game is not dominance solvable.

		COLUMN			
		w	x	y	z
ROW	A	5,3	-2,3	2,4	1,5
	B	2,3	-1,16	16,3	3,15
	C	4,5	-10,16	19,5	0,7
	D	0,2	0,4	5,-7	-1,6

Table 37: Strategies of question 11

11. Elimination respect to COLUMN. COLUMN 'w' is eliminated.

		COLUMN		
		x	y	z
ROW	A	-2,3	2,4	1,5
	B	-1,16	16,3	3,15
	C	-10,16	19,5	0,7
	D	0,4	5,-7	-1,6

Table 38: After elimination of column 'w'

Elimination respect to ROW. ROW **'A'** is eliminated.

		COLUMN		
		x	y	z
ROW	B	-1,16	16,3	3,15
	C	-10,16	19,5	0,7
	D	0,4	5,-7	-1,6

Table 39: After elimination of row 'A'

Elimination respect to COLUMN. COLUMN **'y'** is eliminated.

		COLUMN	
		x	z
ROW	B	-1,16	3,15
	C	-10,16	0,7
	D	0,4	-1,6

Table 40: After elimination of column 'y'

Elimination respect to ROW. ROW **'C'** is eliminated.

		COLUMN	
		x	z
ROW	B	-1,16	3,15
	D	0,4	-1,6

Table 41: After elimination of row 'C'

Now, let α is probability of playing B by ROW player and β is probability of playing x by COLUMN player.

		COLUMN		
		x	**z**	**Probability**
ROW	B	-1,16	3,15	α
	D	0,4	-1,6	1-α
	Probability	β	1-β	

Then,

$$\begin{aligned} -1 \times \beta + 3 \times (1-\beta) &= 0 \times \beta + (-1) \times (1-\beta) \\ -\beta + 3 - 3\beta &= -1 + \beta \\ -5\beta &= -4 \\ \beta &= \frac{4}{5} \end{aligned}$$

and,

$$\begin{aligned} 16 \times \alpha + 4 \times (1-\alpha) &= 15 \times \alpha + 6 \times (1-\alpha) \\ 16\alpha + 4 - 4\alpha &= 15\alpha + 6 - 6\alpha \\ 12\alpha + 4 &= 9\alpha + 6 \\ 3\alpha &= 2 \\ \alpha &= \frac{2}{3} \end{aligned}$$

So, the mixed strategy Nash Equilibrium is $(\alpha, \beta) = (\frac{2}{3}, \frac{4}{5})$.