Assignment 1

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1.

		Column	
		left	right
Row	Up	1	4
	Down	2	3

Table 1: Original Strategies

2. (a) As the above strategies is of a zero-sum game, so we can also shown the above strategies as follows:

		Column	
		Left	Right
Row	Up	1,-1	4,-4
	Down	2,-2	3,-3

Table 2: Complete Strategies

Table 2 shows that COLUMN player gets more payoff in 'Left' move than 'Right' moves, no matter what is the ROW player's move. So, 'Right' is a dominated strategy. So, we can eliminated this strategy.

		Column
		Left
Row	Up	1,-1
	Down	2,-2

Table 3: After elimination of column 'Right'

Then, ROW player always select 'Down' for better payoff than 'Left' ($2 > 1$). So, here 'Up' is dominated. So, the nash equilibrium is ('Down','Left').

		Column
		Left
Row	Down	2,-2

Table 4: After elimination of row 'Up'

		Column	
		left	right
Row	Up	1	2
	Down	4	3

Table 5: Original Strategies

		Column	
		left	right
Row	Up	1,-1	2,-2
	Down	4,-4	3,-3

Table 6: Completed Strategies

(b) Table 6 shows that ROW player gets more payoff in 'Down' move than 'Up' move, no matter what is the COLUMN player's move. So, 'Up' is a dominated strategy. So, we can eliminated this strategy.

		Column		
		left	Middle	right
Row	Up	5	3	2
	Straight	6	4	3
	Down	1	6	2

		Column		
		left	Middle	right
Row	Up	5	3	1
	Straight	6	1	2
	Down	1	0	0

		Column	
		left	right
Row	Up	3,1	4,2
	Down	5,2	2,3

Table 7: Strategies of question 3(a)

3. (a) If ROW player selects 'Up' then COLUMN player select 'Right' and if ROW player selects 'Down' then COLUMN player selects 'Right' also. So, we can eliminate 'left' column. Then the strategies table looks as follows:

		COLUMN
		Right
ROW	Up	4,2
	Down	2,3

Table 8: After eliminaiton of column 'Left'

Now, the ROW player always selects 'Up' to maximize its payoff (4). So, we can eliminate the row 'Down'. Then the strategies table looks as follows:

		COLUMN
		Right
ROW	Up	4,2

Table 9: After elimination of row 'Down'

So, this state ('Up', 'Right') is the Nash equilibrium state.

		COLUMN	
		left	Right
ROW	Up	0,0	0,0
	Down	0,0	1,1

Table 10: Strategies of question 3(b)

(b) **('Up','left')** Nither player change their move because thus they cannot increase their payoff.

('Up','Right') ROW player wants to take a move as 'Down' instead of 'Up' because thus it can get more payoff (1) than 0.

('Down','left') COLUMN player wants to take a move as 'Right' instead of 'left' because thus it can get more payoff (1) than 0.

('Down','Right') Nither player change their move because thus they cannot increase their payoff.

So, the equilibrium states are ('Up','left') and ('Down','Right').

		COLUMN		
		Left	Middle	Right
ROW	Up	2,9	5,5	6,2
	Straight	6,4	9,2	5,3
	Down	4,3	2,7	7,1

Table 11: Strategies of question 3(c)

(c)
- If COLUMN player selects ‘Left’ then ROW player selects ‘Straight’ and If ROW player selects ‘Straight’ then COLUMN player selects ‘Left’. So, (Straight,Left) is a nash equilibrium.
- If COLUMN player selects ‘Middle’ then ROW player selects ‘Straight’ and If ROW player selects Straight’ then COLUMN player selects ‘Left’. So, this is not a nash equilibrium.
- If COLUMN player selects ‘Right’ then ROW player selects ‘Up’ and If ROW player selects ‘Up’ then COLUMN player selects ‘Left’. So, this is not a nash equilibrium.

		COLUMN		
		Left	Middle	Right
ROW	Up	5,3	7,2	2,1
	Straight	1,2	6,3	1,4
	Down	4,2	6,4	3,5

Table 12: Strategies of question 3(d)

(d)
- If COLUMN player selects ‘Left’ then ROW player selects ‘Up’ and If ROW player selects ‘Up’ then COLUMN player selects ‘Left’. So, (Up, Left) is a nash equilibrium.
- If COLUMN player selects ‘Middle’ then ROW player selects ‘Up’ and If ROW player selects ‘Up’ then COLUMN player selects ‘Left’. So, this is not a nash equilibrium.
- If COLUMN player selects ‘Right’ then ROW player selects ‘Down’ and If ROW player selects ‘Down’ then COLUMN player selects ‘Right’. So, (Down, Right) is a nash equilibrium.

		COLUMN		
		Left	Middle	Right
ROW	Up	1,2	2,1	1,0
	Straight	0,5	1,2	7,4
	Down	-1,1	3,0	5,2

Table 13: Strategies of question 4

4.
- If COLUMN player selects ‘Left’ then ROW player selects ‘Up’ and If ROW player selects ‘Up’ then COLUMN player selects ‘Left’. So, (UP,Left) is a nash equilibrium.
- If COLUMN player selects ‘Middle’ then ROW player selects ‘Down’ and If ROW player selects ‘Down’ then COLUMN player selects ‘Right’. So, this is not a nash equilibrium.

- If COLUMN player selects 'Right' then ROW player selects 'Straight' and If ROW player selects 'Straight' then COLUMN player selects 'Left'. So, this is not a nash equilibrium.

We describe the equilibrium using the strategies of the players not merely by the payoff received by the players because of we don't know what is the actual move of the players in the runtime.

		COLUMN			
		w	x	y	z
ROW	A	7,5	-8,4	0,4	99,3
	B	5,0	4,1	15,9	100,8
	C	6,0	5,8	15,9	10,2
	D	2,6	7,-10	3,9	10,8
	E	1,6	2,10	1,7	8,6

Table 14: Original Strategies Table

		COLUMN		
		w	x	y
ROW	A	7,5	-8,4	0,4
	B	5,0	4,1	15,9
	C	6,0	5,8	15,9
	D	2,6	7,-10	3,9
	E	1,6	2,10	1,7

Table 15: After eliminatin of column 'z'

5. The nash equilibriums are (A,w),(B,y),(C,y).

6. (a)

$$\begin{aligned}
\Pi_1 &= (P_1 - 1) \times Q_1 \\
&= (P_1 - 1)(15 - P_1 - 0.5P_2) \\
&= 15P_1 - P_1^2 - 0.5P_1P2 - 15 + P_1 + 0.5P_2 \\
&= 16P_1 - P_1^2 - 0.5P_1P2 + 0.5P_2 - 15
\end{aligned}$$

$$\begin{aligned}
\tfrac{\partial \Pi_1}{\partial P_1} &= 16 - 2P_1 - 0.5P_2 = 0 \\
-2P_1 &= 0.5P_2 - 16 \\
P_1 &= 8 - 0.25P_2
\end{aligned}$$

So, best response rule for LaBoulangerie is $P_1 = 8 - 0.25P_2$.

$$\begin{aligned}
\Pi_2 &= (P_2 - 2) \times Q_2 \\
&= (P_2 - 2)(14 - P_2 - 0.5P_1) \\
&= 14P_2 - P_2^2 - 0.5P_1P2 - 28 + 2P_2 + P_1 \\
&= 16P_2 - P_2^2 - 0.5P_1P2 + P_1 - 28
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \Pi_2}{\partial P_2} &= 16 - 2P_2 - 0.5P_1 = 0 \\
-2P_2 &= 0.5P_1 - 16 \\
P_2 &= 8 - 0.25P_1
\end{aligned}
$$

So, best response rule for LaFromagerie's is $P_2 = 8 - 0.25P_1$.
Substituting, P_1's formula,

$$
\begin{aligned}
P_2 &= 8 - 0.25P_1 \\
&= 8 - 0.25(8 - 0.25P_2) \\
&= 8 - 2 + 0.625P_2 \\
0.9375P_2 &= 6 \\
P_2 &= 6.4\$
\end{aligned}
$$

Similarly,

$$P_1 = 6.4\$$$

(b)

$$
\begin{aligned}
\Pi &= \Pi_1 + \Pi_2 \\
&= 16P_1 - P_1^2 - 0.5P_1P2 + 0.5P_2 - 15 + 16P_2 - P_2^2 - 0.5P_1P2 + P_1 - 28 \\
&= 17P_1 + 16.5P_2 - P_1^2 - P_2^2 - P_1P_2 - 43
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \Pi}{\partial P_1} &= 17 - 2P_1 - P_2 = 0 \\
-2P_1 &= P_2 - 17 \\
P_1 &= 8.5 - 0.5P_2
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \Pi}{\partial P_2} &= 16.5 - 2P_2 - P_1 = 0 \\
-2P_2 &= P_1 - 16.5 \\
P_2 &= 8.25 - 0.5P_1 \\
P_2 &= 8.25 - 0.5(8.5 - 0.5P_2) \\
P_2 &= 0.25 - 4.25 + 0.25P_2 \\
0.75P_2 &= -4 \\
P_2 &= 5.3\$
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
P_1 &= 8.5 - 0.5(8.25 - 0.5P_1) \\
P_1 &= 8.5 - 4.125 + 0.25P_1 \\
75P_1 &= 4.375 \\
P_1 &= 5.83\$
\end{aligned}
$$

		Bluebert	
		Goes Straight	**Swerves**
Redbert,	**Goes Straight**	-6,-6	2,-2
	Swerves	-2,2	0,0

7. (a)

   (b) When both select 'Goes Straight' then both are crushed. So, then negative payoff (-6). And, if one selects 'Goes Straight', other selects 'Swerves' then who selects 'Goes Straight' get payoff 2 other one losses '2'. When both select 'Swerves' neither player loss, so same payoff gains (1).

   ('Goes Straight','Goes Straight') Redbert player wants to take a move as 'Swerves' instead of 'Goes Straight' because thus it can get more payoff (-2) than -6. Bluebert also try to select 'Swerves' when Redbert choices 'Goes Straight' for better payoff (-2).

   ('Goes Straight','Swerves') Nither player change their move because thus they cannot increase their payoff.

   ('Swerves','Goes Straight') Nither player change their move because thus they cannot increase their payoff.

   ('Swerves','Swerves') Both player change their move because thus they can increase their payoff.

   So, the pure strategy nash equilibrium are ('Goes Straight','Swerves') and ('Swerves','Goes Straight').

		COLUMN			
		Goes Straight	Swerves		Probability
ROW	Goes Straight	-6,-6	2,-2	$\rightarrow$	α
	Swerves	-2,2	1,1	$\rightarrow$	$1-\alpha$
		$\downarrow$	$\downarrow$		
	Probability	β	$1-\beta$		

Table 16: Mixed Strategies

Then,

$$\begin{aligned} -6\times\beta+2\times(1-\beta) &= -2\times\beta+1\times(1-\beta) \\ -6\beta+2-2\beta &= -2\beta+1-\beta \\ -8\beta+2 &= -3\beta+1 \\ -5\beta &= -1 \\ \beta &= \frac{1}{5} \end{aligned}$$

and,

$$
\begin{aligned}
-6 \times \alpha + 2 \times (1 - \alpha) &= -2 \times \alpha + 1 \times (1 - \alpha) \\
-6\alpha + 2 - 2\alpha &= -2\alpha + 1 - \alpha \\
-8\alpha + 2 &= -3\alpha + 1 \\
-5\alpha &= -1 \\
\alpha &= \frac{1}{5}
\end{aligned}
$$

So, mixed strategy nash equilibrium is $(\alpha, \beta) = (\frac{1}{5}, \frac{1}{5})$.

		COLUMN		
		x	**y**	**z**
	p	5,6	2,4	17,5
ROW	**q**	4,5	10,16	10,6
	r	0,6	3,4	15,-7

Table 17: Strategies

8. • COLUMN player has three strategies x, y, z and ROW player has also three strategies p, q, r.

				COLUMN				
		x		**y**		**z**		x
	p	6	$>$	4	$<$	5	$<$	6
ROW	**q**	5	$<$	16	$>$	6	$>$	5
	r	6	$>$	4	$>$	-7	$<$	6

Table 18: Column comparison

• Table 18 shows that no column is dominant over other any column.

		COLUMN		
		x	**y**	**z**
	p	5	2	17
		$\vee$	$\wedge$	$\vee$
	q	4	10	10
ROW		$\vee$	$\vee$	$\wedge$
	r	0	3	15
		$\wedge$	$\vee$	$\wedge$
	p	5	2	17

Table 19: Row comparison

Table 19 shows that no one row is dominant over other any row. So, there is no dominant strategy in this game.

- As, this game is not zero-sum or constant sum game so this game is not solvable using minmax method.
- From the Table 17 only (p,x) and (q,y) are the nash equilibrium. Because of for only these two state neither player change their strategy until other player don't change their strategy.

		COLUMN	
		Left	**Right**
ROW	**Top**	4,2	0,4
	Bottom	2,4	6,0

Table 20: Pure Strategies

9. Best responses

   - If COLUMN selects 'left' then ROW selects 'Top' $(4 > 2)$.
   - If ROW selects 'Top' then COLUMN selects 'Right' $(4 > 2)$.
   - If COLUMN selects 'Right' then ROW selects 'Bottom' $(6 > 2)$.
   - If COLUMN selects 'Bottom' then ROW selects 'left' $(4 > 0)$.

   So, there is no nash equilibrium for pure strategy.

		COLUMN			
		left	Right		Probability
ROW	Top	4,2	0,4	$\rightarrow$	α
	Bottom	2,4	6,0	$\rightarrow$	$1-\alpha$
		$\downarrow$	$\downarrow$		
	Probability	β	$1-\beta$		

Table 21: Mixed Strategies

Then,

$$\begin{aligned}4 \times \beta + 0 \times (1-\beta) &= 2 \times \beta + 6 \times (1-\beta)\\ 4\beta &= 2 \times \beta + 6 - 6\beta\\ 8\beta &= 6\\ \beta &= \frac{3}{4}\end{aligned}$$

and,

$$\begin{aligned}2 \times \alpha + 4 \times (1-\alpha) &= 4 \times \alpha + 0 \times (1-\alpha)\\ 2\alpha + 4 - 4\alpha &= 4\alpha\\ -6\alpha &= -4\\ \alpha &= \frac{2}{3}\end{aligned}$$

So, the mixed strategy Nash Equilibrium is $(\alpha, \beta) = (\frac{2}{3}, \frac{3}{4})$.

10. Dominance solvable games are those where the equilibrium outcome is the result of elimination of dominated strategies. Here, the game has unique nash equilibrium with non-degenerated mixed strategy. So, neither strategy is dominated by other. So, this game is not dominance solvable.

		COLUMN			
		w	x	y	z
ROW	A	5,3	-2,3	2,4	1,5
	B	2,3	-1,16	16,3	3,15
	C	4,5	-10,16	19,5	0,7
	D	0,2	0,4	5,-7	-1,6

Table 22: Original Strategies Table

11. Elimination respect to COLUMN. COLUMN 'w' is eliminated.

		COLUMN		
		x	y	z
ROW	A	-2,3	2,4	1,5
	B	-1,16	16,3	3,15
	C	-10,16	19,5	0,7
	D	0,4	5,-7	-1,6

Table 23: After elimination of column 'w'

Elimination respect to ROW. ROW **'A'** is eliminated.

		COLUMN		
		x	y	z
ROW	B	-1,16	16,3	3,15
	C	-10,16	19,5	0,7
	D	0,4	5,-7	-1,6

Table 24: After elimination of row 'A'

Elimination respect to COLUMN. COLUMN **'y'** is eliminated.

		COLUMN	
		x	z
ROW	B	-1,16	3,15
	C	-10,16	0,7
	D	0,4	-1,6

Table 25: After elimination of column 'y'

Elimination respect to ROW. ROW **'C'** is eliminated.

		COLUMN	
		x	z
ROW	B	-1,16	3,15
	D	0,4	-1,6

Table 26: After elimination of row 'C'

Now, let α is probability of playing B by ROW player and β is probability of playing x by COLUMN player.

		COLUMN		
		x	**z**	**Probability**
ROW	B	-1,16	3,15	α
	D	0,4	-1,6	1-α
	Probability	β	1-β	

Then,

$$\begin{aligned} -1 \times \beta + 3 \times (1-\beta) &= 0 \times \beta + (-1) \times (1-\beta) \\ -\beta + 3 - 3\beta &= -1 + \beta \\ -5\beta &= -4 \\ \beta &= \frac{4}{5} \end{aligned}$$

and,

$$\begin{aligned} 16 \times \alpha + 4 \times (1-\alpha) &= 15 \times \alpha + 6 \times (1-\alpha) \\ 16\alpha + 4 - 4\alpha &= 15\alpha + 6 - 6\alpha \\ 12\alpha + 4 &= 9\alpha + 6 \\ 3\alpha &= 2 \\ \alpha &= \frac{2}{3} \end{aligned}$$

So, the mixed strategy Nash Equilibrium is $(\alpha, \beta) = (\frac{2}{3}, \frac{4}{5})$.