**BPHO & PUPC Class No. 20210612**

**Assignment 2 Kinematic Relations 20/06/2021** **Due on 11:00 pm 26/06/2021**

**Please try your best to finish the assignment. You may not be able to complete every question, however, please write as much as you can. It is required that all the answers are written independently by yourself.**

**Please print the documents, write your solutions to each question and scan it so that you can post yours to our study group directly. It is better for you to combine all your documents in a single .pdf profile. Other format of documents is acceptable as well, please compress them in a single file with your name.**

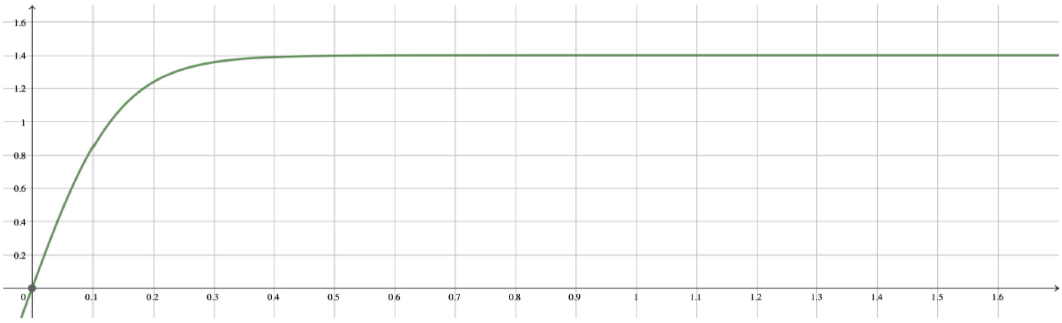
**This assignment is totally worth 30 points.**

**Good luck!**

**Name: ___________ Score: ____________**

**Q1(10 points)**

If a light body falls through air, it encounters some degree of air resistance such that its velocity increases and asymptotically approaches a limit value $v_L$.
The graph below represents the mathematical model according to which the friction force due to air resistance increases when the velocity of the body increases. Assume that a baking cup of mass 0.7 g and diameter 10 cm falls through air. Its velocity as a function of time varies as in the graph.
Notice from the graph that the velocity of the baking cup, which is initially at rest, increases and asymptotically approaches the limit value $v_L = 1.4 \text{ ms}^{-1}$.




This graph describes a function of the form

$$v(t) = \alpha \cdot \frac{e^{\beta t} - 1}{e^{\beta t} + 1}$$

where $\alpha$ and $\beta$ are positive real values; $t$ is time in seconds and $v$ is velocity in metres per second.

(i) Describe the variation of the forces acting on the baking cup during the fall and explain why the function $v(t)$ must satisfy the following conditions:

- the value of the function $v(t)$ must asymptotically approach the limit value $v_L = 1.4 \text{ ms}^{-1}$;
- the acceleration $a = \mathrm{d}v/\mathrm{d}t$ of the baking cup at time $t = 0$ coincides with the acceleration of gravity $g = 9.8 \text{ ms}^{-2}$.

Determine the values of parameters $\alpha$ and $\beta$, along with their respective units of measurement, such that the function $v(t)$ satisfies the above mentioned conditions.
Find the coordinates of some points of the function $v(t)$ for $t \geq 0$ and verify that they coincide with those of the function in the graph above. Particularly, compare them for $t = 0.1$ s; $t = 0.2$ s; $t = 0.3$ s; $t = 0.4$ s.

(ii) The mathematical model used is based on the assumption that the intensity of the friction force acting on the falling baking cup increases with velocity $v$ according to the relation $F_f = kv^2$, where $k$ is the friction coefficient.
From the graph of $v(t)$ one deduces that, for $t \geq 0.5$ s, the velocity of the baking cup can be considered to be constant.
By applying the principle of inertia (Newton's First Law) to this stage of the fall and using again the fact that acceleration at time $t = 0$ must equal $g$, express $\alpha$ and $\beta$ as a function of the mass $m$ of the baking cup, friction coefficient $k$ and acceleration of gravity $g$.

(iii) From point (ii) one deduces that the velocity depends on both time $t$ and friction coefficient $k$. Precisely, given

$$z = \sqrt{k} \quad \text{and} \quad b = 2t\sqrt{\frac{g}{m}}$$

the function of the fall velocity can be written in the form:

$$v = \frac{\sqrt{mg}}{z} \cdot \frac{e^{bz} - 1}{e^{bz} + 1}.$$

Determine how the function $v$ is modified if the parameter $k$ (and, consequently, $z$) decreases until it vanishes. Interpret the physical meaning of the result obtained.

(iv) Given the following function

$$F(t) = A \ln\left(\frac{e^{14t} + 1}{2}\right) + Bt$$

where $A$ and $B$ are real values, verify that it can be a primitive (antiderivative) of the function

$$v(t) = 1.4 \cdot \frac{e^{\beta t} - 1}{e^{\beta t} + 1}.$$

Determine the units of measurement of the factor 1.4 in the expression for $v(t)$ and the value of coefficients $A, B$ and $\beta$.
Calculate the average function value for the function $v(t)$ in the time interval (expressed in seconds) $0 \leq t \leq 1$. Give a physical meaning to the value obtained. [*Hint: You may recall that the average value of a function $f(x)$ over the interval $a \leq x \leq b$ is given by:* $f_{\text{avg}} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,\mathrm{d}x$]

**Q2(6 points)**

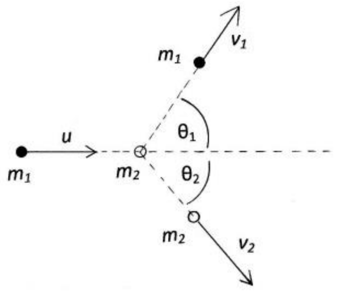



Figure 8.(1)

A particle of mass $m_1$ is incident with velocity $u$ on a stationary particle of mass $m_2 = 4\ m_1$. The incident particle is scattered through an angle $\theta_1$, with velocity $v_1$, and the other particle is scattered in a direction that makes an angle $\theta_2$ with the direction of $u$, with velocity $v_2$, **Figure 8.(1)**.

(a) Determine the velocity ratios $(v_1/v_2)$ and $(v_1/u)$ assuming momentum conservation only.

(b) Compare the initial kinetic energy, TI, with the final kinetic energy, TF, for the following situations:

(i) $\theta_1 = \theta_2 = 60.0°$

(ii) $\theta_1 = \theta_2 = 56.0°$

(iii) $v_1 = v_2$, $\theta_1 = 90°$

In each case determine if the energy is conserved.

(c) If $\theta_1$ and $\theta_2$ are small, verify that energy is conserved if $3\theta_1 = -8\theta_2$.

(For small $\theta$, $\sin\theta \approx \theta$)

**Q3(4 points)**

Three small identical steel balls A, B, and C are suspended by vertical threads of equal length from a horizontal support, with their centres in a horizontal line and separated by a small gap. A is raised by a height h, with the thread taut, and released. All subsequent collisions are elastic.

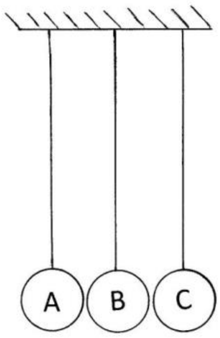
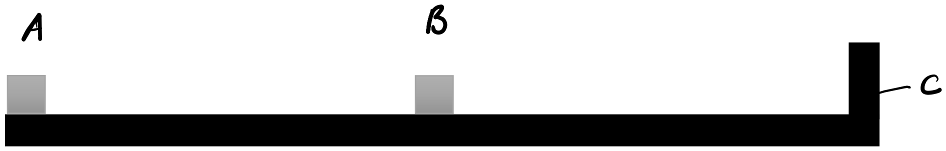



Figure 6.1

(i) When all the balls have mass M, determine the subsequent motion of the system and the height to which the centre of C is raised following its first collision.

(ii) When A has mass M, B has mass 2M and C has mass 3M, determine the height to which the centre of C is raised after its first collision.

**Q4(10 points)**




On a horizontal smooth surface, there is a long wooden flat C. A and B are small blocks placed on C. The initial distance between A and B is the same as that between B and C which is L. The dynamic coefficient between A and C, B and C is $\mu$. Initially, B and C are stationary. The mass of A, B and C are the same. Please discuss if the following conditions could happen. If so, what should the initial velocity of A $v_0$ should satisfy?

(i) Block A collide with Block B.

(ii) Block A collide with Block B and then Block B collide with C.

(iii) Block A collide with Block B, Block B collide with C and Block B collide with Block A again.

(iv) Block A collide with Block B, Block B collide with C and Block B collide with Block A again, after that, Block A falls on the ground.

(v) Block A collide with Block B, Block B collide with C and Block B collide with Block A again, after that, both Block A and Block B fall on the ground.