PUPC+BPHO Class No.21210612

Tutorial 1 Mechanics1

Q1

A particle with mass m and initial speed V is subject to a velocity-dependent damping force of the form bv^n.

(a) For $n = 0, 1, 2, \ldots$, determine how the stopping time depends on m, V, and b.

(b) For $n = 0, 1, 2, \ldots$, determine how the stopping distance depends on m, V, and b.

Be careful! See if your answers make sense. Dimensional analysis gives the answer only up to a numerical factor. This is a tricky problem, so don't let it discourage you from using dimensional analysis. Most applications of dimensional analysis are quite straightforward.

Q2

Two equal masses are connected by a string that hangs over two pulleys (of negligible size), as shown in Fig. 1.6. The left mass moves in a vertical line, but the right mass is free to swing back and forth in the plane of the masses and pulleys. It can be shown (see Problem 6.4) that the equations of motion for r and θ (labeled in the figure) are

$$2\ddot{r} = r\dot{\theta}^2 - g(1 - \cos\theta),$$
$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} - \frac{g\sin\theta}{r}. \qquad (1.16)$$

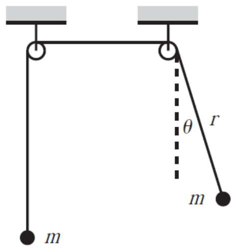


Fig. 1.6

Assume that both masses start out at rest, with the right mass making an initial angle of $10° = \pi/18$ with the vertical. If the initial value of r is 1 m, how much time does it take for it to reach a length of 2 m? Write a program to solve this numerically. Use $g = 9.8\,\mathrm{m/s^2}$.

Q3

Consider a projectile subject to a drag force $\mathbf{F} = -m\alpha\mathbf{v}$. If it is fired with speed v_0 at an angle θ, it can be shown that the height as a function of time is given by (just accept this here; it's one of the tasks of Exercise 3.53)

$$y(t) = \frac{1}{\alpha}\left(v_0\sin\theta + \frac{g}{\alpha}\right)\left(1 - e^{-\alpha t}\right) - \frac{gt}{\alpha}. \qquad (1.19)$$

Show that this reduces to the usual projectile expression, $y(t) = (v_0\sin\theta)t - gt^2/2$, in the limit of small α. What exactly is meant by "small α"?

Q4

(a) A chain with uniform mass density per unit length hangs between two given points on two walls. Find the general shape of the chain. Aside from an arbitrary additive constant, the function describing the shape should contain one unknown constant. (The shape of a hanging chain is known as a *catenary*.)

(b) The unknown constant in your answer depends on the horizontal distance d between the walls, the vertical distance λ between the support points, and the length ℓ of the chain (see Fig. 2.14). Find an equation involving these given quantities that determines the unknown constant.

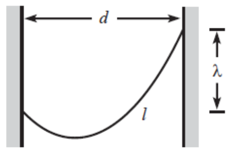


Fig. 2.14

Q5

A spool consists of an axle of radius r and an outside circle of radius R which rolls on the ground. A thread is wrapped around the axle and is pulled with tension T at an angle θ with the horizontal (see Fig. 2.24).

(a) Given R and r, what should θ be so that the spool doesn't move? Assume that the friction between the spool and the ground is large enough so that the spool doesn't slip.

(b) Given R, r, and the coefficient of friction μ between the spool and the ground, what is the largest value of T for which the spool remains at rest?

(c) Given R and μ, what should r be so that you can make the spool slip from the static position with as small a T as possible? That is, what should r be so that the upper bound on T in part (b) is as small as possible? What is the resulting value of T?

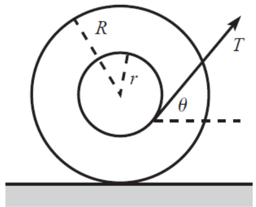


Fig. 2.24

Q6

A large number of sticks (with mass density per unit length ρ) and circles (with radius R) lean on each other, as shown in Fig. 2.26. Each stick makes an angle θ with the horizontal and is tangent to the next circle at its upper end. The sticks are hinged to the ground, and every other surface is *frictionless* (unlike in the previous problem). In the limit of a very large number of sticks and circles, what is the normal force between a stick and the circle it rests on, very far to the right? Assume that the last circle leans against a wall, to keep it from moving.

Fig. 2.26

Q7

(a) Consider the first bridge in Fig. 2.29, made of three equilateral triangles of beams. Assume that the seven beams are massless and that the connection between any two of them is a hinge. If a car of mass m is located at the middle of the bridge, find the forces (and specify tension or compression) in the beams. Assume that the supports provide no horizontal forces on the bridge.

(b) Same question, but now with the second bridge in Fig. 2.29, made of seven equilateral triangles.

(c) Same question, but now with the general case of $4n-1$ equilateral triangles.

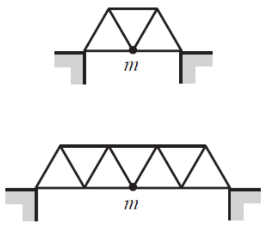
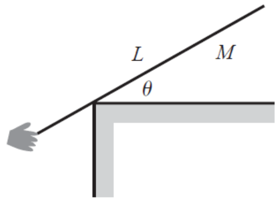


Fig. 2.29

Q8

You support one end of a stick of mass M and length L with the tip of your finger. A quarter of the way up the stick, it rests on a frictionless corner of a table, as shown in Fig. 2.35. The stick makes an angle θ with the horizontal. What is the magnitude of the force your finger must apply to keep the stick in this position? For what angle θ does your force point horizontally?



Q9

Two sticks, each of mass m and length ℓ, are connected by a hinge at their top ends. They each make an angle θ with the vertical. A massless string connects the bottom of the left stick to the right stick, perpendicularly as shown in Fig. 2.37. The whole setup stands on a frictionless table.

(a) What is the tension in the string?

(b) What force does the left stick exert on the right stick at the hinge? *Hint*: No messy calculations required!

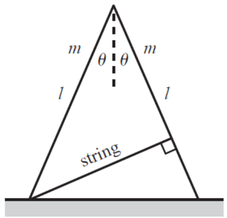


Fig. 2.37

Q10

N blocks of length ℓ are stacked on top of each other at the edge of a table, as shown in Fig. 2.40 for $N = 4$. What is the largest horizontal distance the rightmost point on the top block can hang out beyond the table? How does your answer behave for $N \to \infty$?[8]

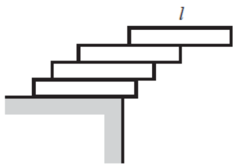


Fig. 2.40