$L = mr^2\dot{\theta}$ (angular momentum)

$F(r) = m(\ddot{r} - r\dot{\theta}^2)$ (Central force)

上面的第一个公式是角动量表达式，第二个式子是有心力的条件（今天notes上有的）

$$\ddot{r} = \frac{d}{dt}(\dot{r}) = \frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{d}{dt}\left(\frac{dr}{d\theta}\cdot\frac{d\theta}{dt}\right)$$

$$= \underline{\frac{d}{dt}\left(\frac{dr}{d\theta}\right)\cdot\frac{d\theta}{dt}} + \underline{\frac{dr}{d\theta}\cdot\frac{d^2\theta}{dt^2}}$$

$$\frac{d}{dt}\left(\frac{dr}{d\theta}\right)\frac{d\theta}{dt} = \frac{d}{d\theta}\cdot\left(\frac{dr}{d\theta}\right)\cdot\frac{d\theta}{dt}\cdot\frac{d\theta}{dt}$$

$$= \frac{d^2r}{d\theta^2}\cdot\left(\frac{d\theta}{dt}\right)^2$$

$$\frac{dr}{d\theta}\frac{d^2\theta}{dt^2} = \frac{dr}{d\theta}\frac{d}{dt}\left(\frac{d\theta}{dt}\right)$$

$$= \frac{dr}{d\theta}\cdot\frac{d}{d\theta}\left(\frac{d\theta}{dt}\right)\cdot\frac{d\theta}{dt}$$

$$\ddot{r} = \frac{d^2r}{d\theta^2}\left(\frac{d\theta}{dt}\right)^2 + \frac{dr}{d\theta}\frac{d}{d\theta}\left(\frac{d\theta}{dt}\right)\frac{d\theta}{dt}$$

$L = mr^2\frac{d\theta}{dt}$ $\quad \ddot{r} = \frac{d^2r}{d\theta^2}\left(\frac{L}{mr^2}\right)^2 + \frac{dr}{d\theta}\frac{d}{d\theta}\left(\frac{L}{mr^2}\right)\frac{L}{mr^2}$

$\frac{d\theta}{dt} = \frac{L}{mr^2}$ 再代入 $F(r) = m(\ddot{r} - r\dot{\theta}^2)$

$$m\left[\left(\frac{d^2r}{d\theta^2}\left(\frac{L}{mr^2}\right)^2+\frac{dr}{d\theta}\frac{d}{d\theta}\left(\frac{L}{mr^2}\right)\frac{L}{mr^2}\right)-r\left(\frac{L}{mr^2}\right)^2\right]=F(r)$$

两边同乘以 m：

$$\frac{d^2r}{d\theta^2}\cdot\frac{L^2}{r^4}+\frac{dr}{d\theta}\cdot\frac{d}{d\theta}\left(\frac{L}{r^2}\right)\frac{L}{r^2}-r\cdot\frac{L^2}{r^4}=mF(r)$$

$$\frac{d^2r}{d\theta^2}+r^2\frac{dr}{d\theta}\cdot\frac{d}{d\theta}\left(\frac{1}{r^2}\right)-r=\frac{mr^4}{L^2}F(r)$$

比耐方程