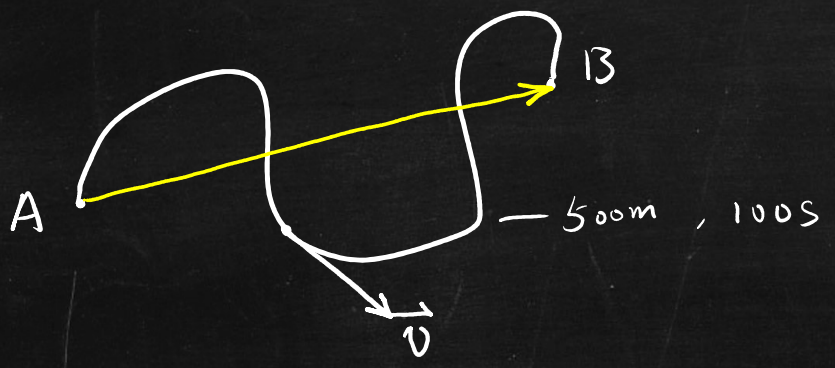
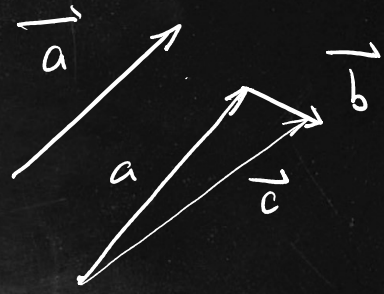
Mechanics

Motion 运动 → displacement 位移
velocity 速度
acceleration 加速度

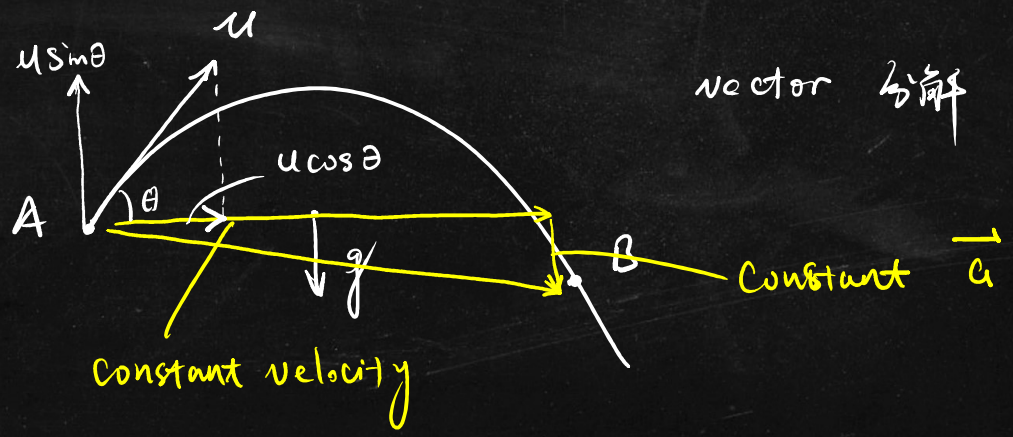
distance 路程
speed 速率

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$





constant acceleration 匀加速运动
projectile 抛物线

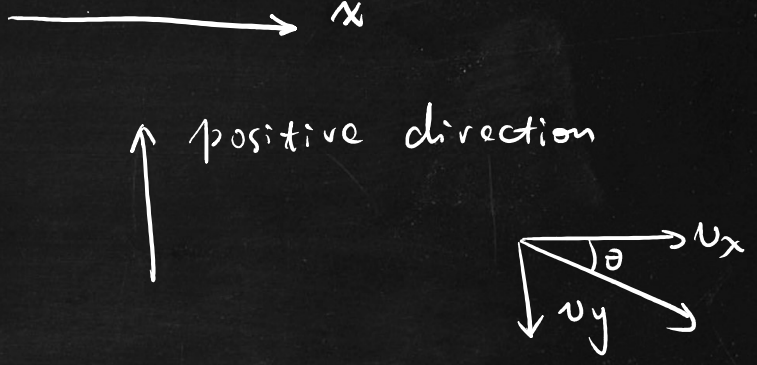


vector 分解

$a = \frac{\Delta v}{\Delta t}$ $\quad v \sim t \quad \Delta t = t - 0 = t$

$\Delta v = v - u$

$\star \; a = \frac{v-u}{t}$ $\quad v = u + at$



$v_y = u_y + at = u\sin\theta + (-g)t$

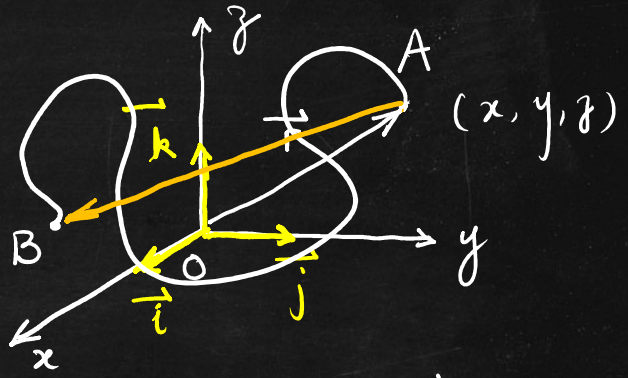
$v_y = u\sin\theta - gt$ $\quad v_x = u_x = u\cos\theta$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$ $\quad \tan\theta = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$

Mechanics

Vector 向量 矢量　　Coordinates 坐标系

position vector $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x\vec{i} + y\vec{j} + z\vec{k}$



displacement 位移 $\vec{s} = \vec{r}_B - \vec{r}_A$

$= (x_B\vec{i} + y_B\vec{j} + z_B\vec{k}) - (x_A\vec{i} + y_A\vec{j} + z_A\vec{k})$

$\vec{s} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$

velocity 速度　　$y = x^2$　$\frac{dy}{dx} = 2x$　　$y = c$　$\frac{dy}{dx} = 0$

$$\vec{v} = \frac{d\vec{s}}{dt} = \frac{d(\vec{r} - \vec{r}_0)}{dt} = \frac{d\vec{r} - d\vec{r}_0}{dt} = \frac{d\vec{r}}{dt} - \frac{d\vec{r}_0}{dt}$$

$\frac{d\vec{r}_0}{dt} = 0$

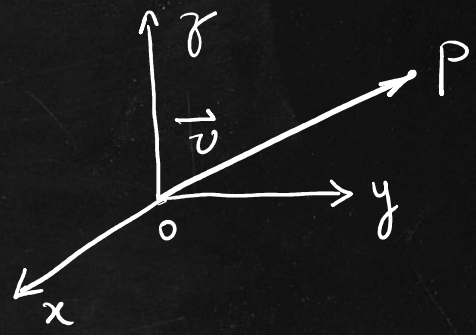
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr_x}{dt}\vec{i} + \frac{dr_y}{dt}\vec{j} + \frac{dr_z}{dt}\vec{k}$$

$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

constant velocity

Assume an object start from origin $\vec{r}_0 = (0,0,0)$

After time t, object is at P



$$\vec{r}_p = (x_p, y_p, z_p) = x_p\vec{i} + y_p\vec{j} + z_p\vec{k}$$

$$\vec{s} = \vec{v} \cdot t$$

$$\vec{s} = \vec{r}_p - \vec{r}_0 = \vec{r}_p = x_p\vec{i} + y_p\vec{j} + z_p\vec{k}$$

$$\vec{v}t = (v_x\vec{i} + v_y\vec{j} + v_z\vec{k})t$$

$$= v_xt\,\vec{i} + v_yt\,\vec{j} + v_zt\,\vec{k} = \vec{s}$$

$$\begin{cases} x_p = v_xt \\ y_p = v_yt \\ z_p = v_zt \end{cases} \longleftrightarrow x = vt \qquad \vec{s} = \vec{v}t$$

Acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$

constant acceleration. $\frac{d\vec{v}}{dt} = \vec{a}$ $\quad d\vec{v} = \vec{a}\,dt$

$$\int d\vec{v} = \int \vec{a}\,dt = \vec{a}\int dt$$

initial velocity

$$\vec{v} = \vec{a}t + \vec{c} \qquad t=0 \quad \vec{v} = \vec{v}_0 \quad \vec{u}$$

$$\vec{v}_0 = \vec{a}\cdot 0 + \vec{c} \qquad \vec{c} = \vec{v}_0 \text{ boundary condition}$$

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\begin{cases} v_x = v_{0x} + a_x t \\ v_y = v_{0y} + a_y t \\ v_z = v_{0z} + a_z t \end{cases} \qquad \begin{cases} x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \\ z = z_0 + v_{0z}t + \frac{1}{2}a_z t^2 \end{cases}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \qquad |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

integration

$y = x^2 \Leftarrow \frac{dy}{dx} = 2x$

$\frac{dy}{dx} = 2x \quad y = ?$

$y = x^2 + C$

$dy = 2x\,dx$ $\quad \int$

$\int 1\,dy = \int 2x\,dx$

$y + C_1 = x^2 + C_2$

$\boxed{y = x^2 + C}$

Free Falling object.

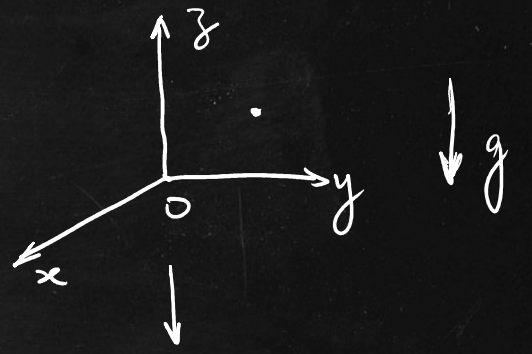
1° $\vec{v}_0 = 0$

2° Near the surface of the earth $\vec{a} = -g\vec{k}$

3° Air resistance, friction X

$\vec{G} = mg(-\vec{k})$

$= -mg\vec{k}$



$\vec{a} = -g\vec{k}$ $\quad \vec{v} = \vec{v}_0 + \vec{a}t = \vec{a}t$

$\vec{v} = -g\vec{k}\,t = -gt\vec{k}$ $\quad \vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$

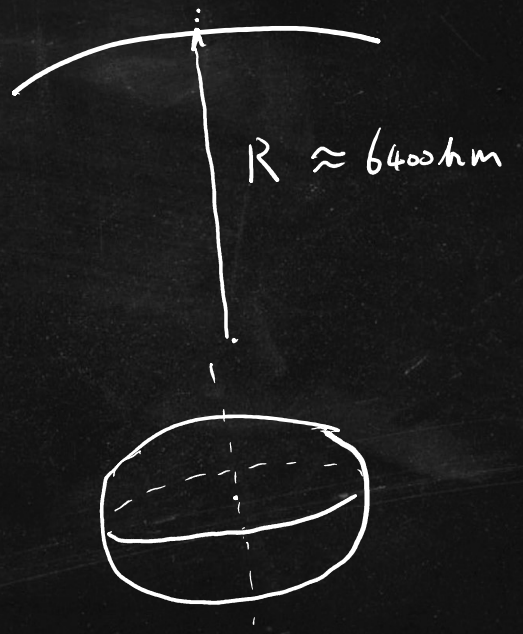
$= -gt\vec{k} + 0\vec{i} + 0\vec{j}$

$$\begin{cases} v_x = 0 \\ v_y = 0 \\ v_z = -gt \end{cases}$$

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$= r_{0x}\vec{i} + r_{0y}\vec{j} + r_{0z}\vec{k} + \frac{1}{2}\cdot(-g\vec{k})t^2$

$= r_{0x}\vec{i} + r_{0y}\vec{j} + (r_{0z} - \frac{1}{2}gt^2)\vec{k}$



ex. $\vec{v}_0 = 2\vec{i} + 3\vec{j} + \vec{k}$ (m/s) $\quad g = -9.8\vec{k}$ (m/s²)

$\vec{r}_0 = 2\vec{i} + 2\vec{j} + 2\vec{k}$ (m) $\quad t = 3$ (s) $\quad \vec{r} = ? \quad \vec{v} = ?$

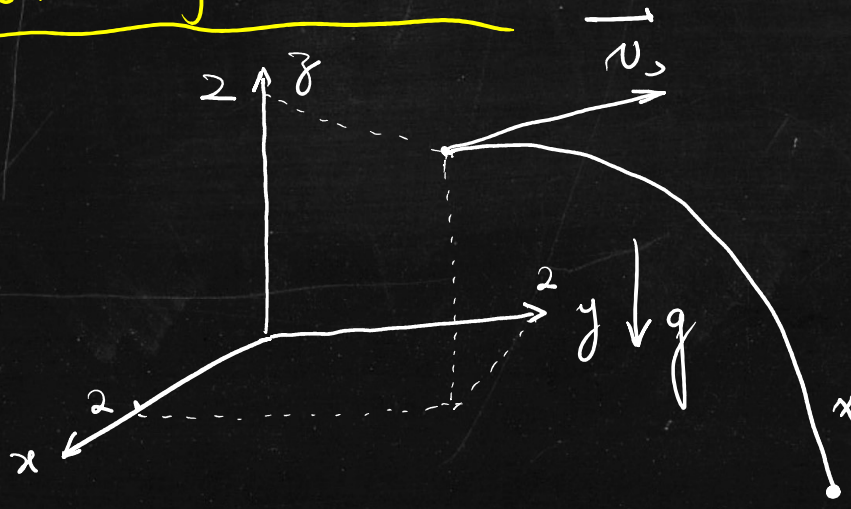
$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} g t^2 \rightarrow \frac{1}{2}\vec{a}t^2 \quad \vec{a} = -9.8\vec{k}$

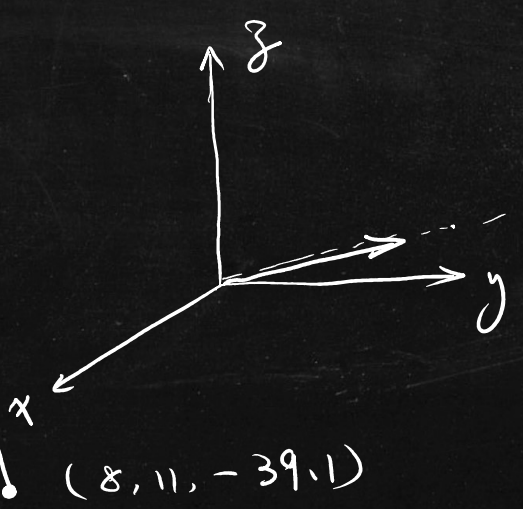
$\frac{1}{2}(-9.8)\cdot\vec{k}\cdot t^2 = -4.9t^2\vec{k}$

$\vec{r} = 2\vec{i} + 2\vec{j} + 2\vec{k} + (2\vec{i} + 3\vec{j} + \vec{k})\cdot 3 - 4.9\times 3^2\cdot\vec{k}$

$= 8\vec{i} + 11\vec{j} + (-39.1)\vec{k}$

4.9
9
44.1





projectile.