BPHO & PUPC Class No. 20210612

Assignment 3 Oscillation and Rotation 28/06/2021 **Due on 11:00 pm 04/07/2021**

Please try your best to finish the assignment. You may not be able to complete every question, however, please write as much as you can. It is required that all the answers are written independently by yourself.

Please print the documents, write your solutions to each question and scan it so that you can post yours to our study group directly. It is better for you to combine all your documents in a single .pdf profile. Other format of documents is acceptable as well, please compress them in a single file with your name.

This assignment is totally worth 30 points.

Good luck!

Name: ___________ Score: ____________

Q1(10 points)

Consider figure (4.14). In this case, we will have two masses, m_1 and m_2, placed on a friction-less surface and attached to three ideal sprigs with identical spring constants, k.

(i) Initially the springs are held in their equilibrium positions, m_1 is kept at rest whilst m_2 is given a velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$. Ignoring gravity, write the equations of motion for the two masses in terms of x_1 ad x_2 their relative displacements from the equilibrium positions.

(ii) Label the normal modes as $y_1 = x_1 + x_2$ and $y_2 = x_1 - x_2$. Show that the modes obey the equation of motion which describes simple harmonic motion, and that the ratio of the oscillation frequencies of y_1 and y_2 is $\frac{1}{\sqrt{3}}$.

(iii) The two external springs are fixed to stationary walls, use these boundary conditions to show that the solution to y_1 is

$$y_1 = v\sqrt{\frac{m}{k}}\sin\left(t\sqrt{\frac{k}{m}}\right)$$

(iv) Now consider the addition of air friction as a damping factor. The friction force opposes the motion of the two masses and has a magnitude of βv_r, where v_r is defined as the relative velocity between the two masses. Write down the equation of motion of the masses in terms of x_1 ad x_2 with the addition of the damping force.

(v) Using the equations of motion obtained above in terms of y_1 and y_2, consider a trial solution for y_2 of Ae^{qt}. Show that $x_1 = x_2$ as $t \to \infty$, and hence derive the maximal amplitude of oscillation of either of the two masses as $t \to \infty$.

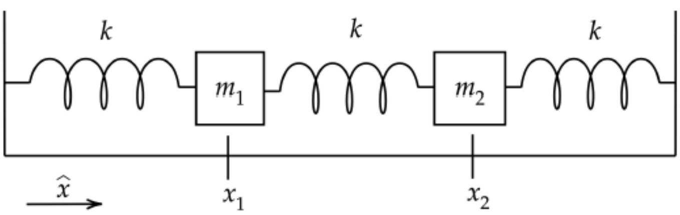


Figure 4.14.: Illustrating the setup of the coupled oscillator, with two masses and three springs.

Q2(4 points)

Let's consider an extended body pendulum, as seen in the figure below. The body has a total mass M, formed by a rod of length L and mass m_1 and a spherical ball of radius l and mass m_2. Clearly, the length of the whole extended pendulum will be $L + 2l$.

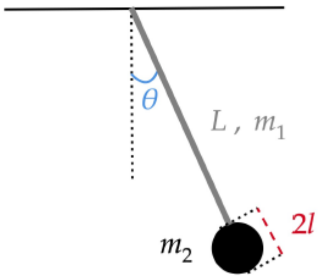


Figure 4.17.:

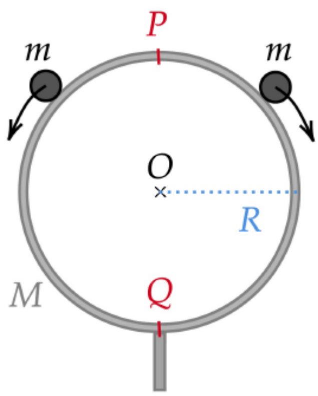
The equation of motion takes the form

$$\left(m_2 + \frac{m_1}{3}\right) L^2 \ddot{\theta} + \left(m_2 + \frac{m_1}{2}\right) gL\theta = 0$$

Find the frequency of the oscillatory motion of the extended pendulum. Then, using the solution to the *simple pendulum exercise* earlier in this chapter, show that the frequency determined is that of a simple pendulum as $m_1 \to 0$.

Q3(4 points)

Two beads of mass m are placed at the top of a friction-less hoop of mass M and radius R which is at rest in the vertical plane on top of a friction-less vertical support. The beads are given a tiny impulse and due to gravity they slide down from position (P), *seen in the image below*, in the clockwise and anticlockwise direction.



Determine the minimum value of $X = \frac{m}{M}$, X_{MIN} for which the loop will rise off the support before the beads will have reached the bottom position (Q).

Q4(12 points)

Fig. 4.15

Fig. 4.16

(a) Two identical masses m are constrained to move on a horizontal hoop. Two identical springs with spring constant k connect the masses and wrap around the hoop (see Fig. 4.15). Find the normal modes.

(b) Three identical masses are constrained to move on a hoop. Three identical springs connect the masses and wrap around the hoop (see Fig. 4.16). Find the normal modes.

(c) Now do the general case with N identical masses and N identical springs.