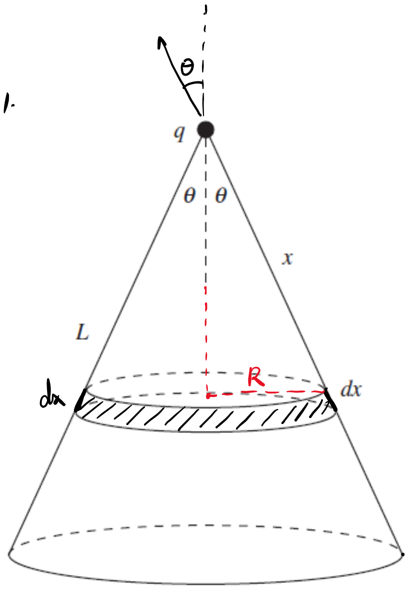
Q1.



$$dF = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dQ\,q}{x^2}\cos\theta$$

$$dF_{ring} = \int \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cos\theta}{x^2}\,dQ = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cos\theta}{x^2}Q_{ring}$$

$$Q_{ring} = dA\cdot\sigma = 2\pi R\,dx\,\sigma = \sigma\,2\pi x\sin\theta\,dx$$

$$dF_{ring} = \frac{1}{2\cancel{4\pi}\varepsilon_0}\cdot\frac{q\cos\theta}{x^{\cancel{2}}}\cdot\sigma\cdot\cancel{2\pi}\cancel{x}\sin\theta\,dx$$

$$= \frac{q\sigma\sin\theta\cos\theta}{2\varepsilon_0}\cdot\frac{dx}{x}$$

$$F = \int\frac{q\sigma\sin\theta\cos\theta}{2\varepsilon_0}\cdot\frac{dx}{x}$$

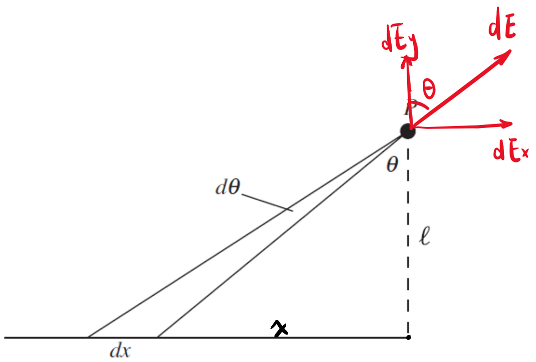
$$= \frac{q\sigma\sin\theta\cos\theta}{2\varepsilon_0}\int\frac{dx}{x}$$

1° $F = \frac{q\sigma\sin\theta\cos\theta}{2\varepsilon_0}\cdot\int_0^L\frac{dx}{x}$ $\qquad\int\frac{1}{x}dx = \ln|x| + C$

2° $F = \frac{q\sigma\cdot\sin\theta\cos\theta}{2\varepsilon_0}\int_{\frac{1}{2}L}^{L}\frac{dx}{x} = \frac{q\sigma\sin\theta\cos\theta}{2\varepsilon_0}\ln 2 = \frac{q\sigma}{4\varepsilon_0}\ln 2\cdot\underline{\sin 2\theta}$

$(\sin 2\theta = 2\sin\theta\cos\theta)$ $\qquad 2\theta = \frac{\pi}{2}$ $\qquad \theta = \frac{\pi}{4}$

Q2.



$r = \frac{l}{\cos\theta}$ $\qquad \frac{x}{l} = \tan\theta$ $\qquad x = l\tan\theta$

$$dx = d(l\tan\theta) = \frac{l}{\cos^2\theta}d\theta$$

$$dq = \lambda\,dx = \frac{\lambda l}{\cos^2\theta}d\theta$$

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{\lambda\cancel{l}}{\cancel{\cos^2\theta}}d\theta}{\frac{l^{\cancel{2}}}{\cancel{\cos^2\theta}}}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\,d\theta}{l} = \frac{\lambda}{4\pi\varepsilon_0 l}\cdot d\theta$$

$y = f(x)$ $\qquad dy = \left(\frac{dy}{dx}\right)dx = f'(x)dx$

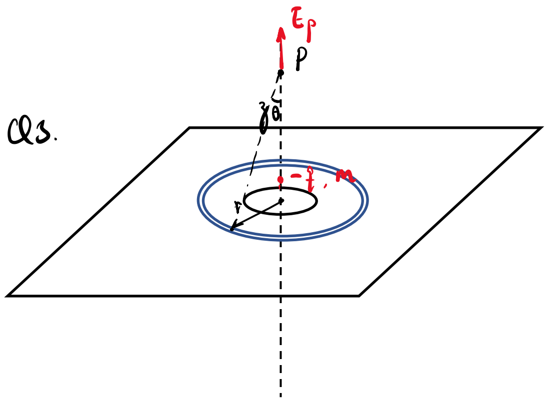
$$dE_x = dE\cdot\sin\theta = \frac{\lambda}{4\pi\varepsilon_0 l}\cdot\sin\theta\,d\theta$$

$$dE_y = dE\cdot\cos\theta = \frac{\lambda}{4\pi\varepsilon_0 l}\cdot\cos\theta\,d\theta$$

$$E_x = \int_0^{\frac{\pi}{2}}\frac{\lambda}{4\pi\varepsilon_0 l}\cdot\sin\theta\,d\theta = \frac{\lambda}{4\pi\varepsilon_0 l}\cdot(-\cos\theta)\Big|_0^{\frac{\pi}{2}} = \frac{\lambda}{4\pi\varepsilon_0 l}$$

$$E_y = \int_0^{\frac{\pi}{2}}\frac{\lambda}{4\pi\varepsilon_0 l}\cos\theta\,d\theta = \frac{\lambda}{4\pi\varepsilon_0 l}\cdot(\sin\theta)\Big|_0^{\frac{\pi}{2}} = \frac{\lambda}{4\pi\varepsilon_0 l}$$

Q3.



$$dQ = 2\pi r\,dr \cdot \sigma$$

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dQ}{r^2+z^2}\cdot\frac{z}{\sqrt{r^2+z^2}}$$

$$= \frac{1}{2\cancel{4\pi}\varepsilon_0}\cdot\frac{\cancel{2\pi} r\,dr\,\sigma\cdot z}{(r^2+z^2)^{\frac{3}{2}}}$$

$$= \frac{\sigma z}{2\varepsilon_0}\cdot\frac{r\,dr}{(r^2+z^2)^{\frac{3}{2}}}$$

$$E = \int_R^\infty \frac{\sigma z}{2\varepsilon_0}\cdot\frac{r\,dr}{(r^2+z^2)^{\frac{3}{2}}} = \frac{\sigma z}{2\varepsilon_0}\cdot\left(-(r^2+z^2)^{-\frac{1}{2}}\right)\Big|_R^\infty = \frac{\sigma z}{2\varepsilon_0}\cdot\left(0-\left(-(R^2+z^2)^{-\frac{1}{2}}\right)\right)$$

$$= \frac{\sigma z}{2\varepsilon_0\sqrt{R^2+z^2}}$$

$$\underline{(-q)E = m\ddot{z}} \qquad z\to 0 \quad \boxed{z \ll R} \Rightarrow E \approx \frac{\sigma z}{2\varepsilon_0 R}$$

$$m\ddot{z} + q\frac{\sigma z}{2\varepsilon_0 R} = 0 \qquad \ddot{z} + \frac{q\sigma}{2\varepsilon_0 m R} z = 0 \qquad \omega^2 = \frac{q\sigma}{2\varepsilon_0 m R}$$

$$\omega = \sqrt{\frac{q\sigma}{2\varepsilon_0 m R}} = \sqrt{\frac{10^{-8}\cdot 10^{-6}}{2\cdot 8.85\times 10^{-12}\cdot 10^{-3}\cdot 0.1}} = 2.4 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = 0.4\,(Hz) \qquad W_F = \Delta E_k$$

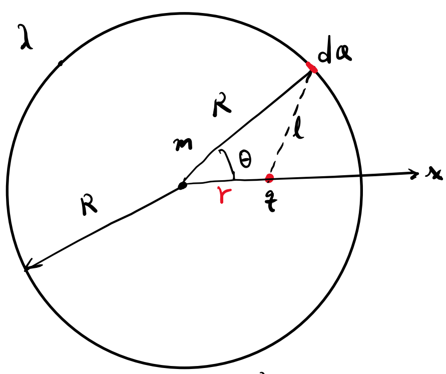
$$W_F = \int_z^0 (-q)E\,dz = \int_0^z qE\,dz = \int_0^z \frac{q\sigma\cdot z}{2\varepsilon_0\sqrt{R^2+z^2}}\,dz = \frac{q\sigma}{2\varepsilon_0}\cdot(R^2+z^2)^{\frac{1}{2}}\Big|_0^z$$

$$= \frac{q\sigma}{2\varepsilon_0}\cdot\left(\sqrt{R^2+z^2}-R\right) = \frac{1}{2}mv^2 \qquad v = \sqrt{\frac{q\sigma}{m\varepsilon_0}\left(\sqrt{R^2+z^2}-R\right)}$$

$$v = \sqrt{\frac{q\sigma}{m\varepsilon_0}\left(z\sqrt{\cancel{\frac{R^2}{z^2}}+1}-R\right)} = \sqrt{\frac{q\sigma z}{m\varepsilon_0}}$$

Assignment 6.

Q1



$$l = \sqrt{R^2+r^2-2Rr\cos\theta}$$

$$dU = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,dQ}{l} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot R\,d\theta\cdot\lambda}{\sqrt{R^2+r^2-2Rr\cos\theta}}$$

$$U(r) = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{q\lambda R\,d\theta}{\sqrt{R^2+r^2-2Rr\cos\theta}}$$

$$= \frac{q\lambda}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{d\theta}{\sqrt{1+\underbrace{\left(\frac{r}{R}\right)^2-2\left(\frac{r}{R}\cos\theta\right)}_{\varepsilon}}}$$

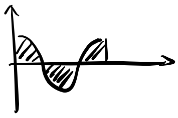
$$\frac{1}{\sqrt{1+\varepsilon}} = 1-\frac{\varepsilon}{2}+\frac{3\varepsilon^2}{8}$$

$$\varepsilon = \frac{r^2}{R^2} - 2\frac{r}{R}\cos\theta$$

$$U(r) = \frac{q\lambda}{4\pi\varepsilon_0}\int_0^{2\pi}\left(1-\frac{1}{2}\cdot\left(\frac{r^2}{R^2}-2\frac{r}{R}\cos\theta\right)+\frac{3}{8}\cdot\left(\frac{r^2}{R^2}-2\frac{r}{R}\cos\theta\right)^2\right)d\theta$$

$$U(r) = \frac{q\lambda}{4\pi\varepsilon_0}\int_0^{2\pi}\left(1-\frac{r^2}{2R^2}+\frac{r}{R}\cos\theta+\frac{3}{8}\left(\cancel{\frac{r^4}{R^4}}-\cancel{4\frac{r^3}{R^3}\cos\theta}+4\frac{r^2}{R^2}\cos^2\theta\right)\right)d\theta$$

$$= \frac{q\lambda}{4\pi\varepsilon_0}\int_0^{2\pi}\left(1+\frac{r^2}{2R^2}(3\cos^2\theta-1)+\frac{r}{R}\cos\theta\right)d\theta$$



$$= \frac{q\lambda}{4\pi\varepsilon_0}\left(2\pi+\int_0^{2\pi}\frac{r^2}{2R^2}(3\cos^2\theta-1)\,d\theta+0\right)$$

$$\left(\cos^2\theta=\frac{1+\cos 2\theta}{2}\right) \quad = \frac{q\lambda}{4\pi\varepsilon_0}\left(2\pi+\int_0^{2\pi}\frac{r^2}{2R^2}\cdot\left(3\cdot\frac{1+\cos 2\theta}{2}-1\right)d\theta\right)$$

$$= \frac{q\lambda}{4\pi\varepsilon_0}\left(2\pi+\frac{\pi r^2}{2R^2}\right) = \frac{q\lambda}{2\varepsilon_0}+\frac{q\lambda r^2}{8\varepsilon_0 R^2}$$

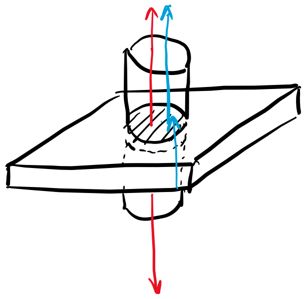
$$F(r) = -\frac{dU}{dr} = -\frac{q\lambda\cdot 2r}{8\varepsilon_0 R^2} = -\frac{q\lambda r}{4\varepsilon_0 R^2} = m\ddot{r}$$

$$\ddot{r}+\frac{q\lambda}{4m\varepsilon_0 R^2}r=0 \qquad \omega^2=\frac{q\lambda}{4m\varepsilon_0 R^2} \qquad \omega=\sqrt{\frac{q\lambda}{4m\varepsilon_0 R^2}} \qquad \lambda=\frac{Q}{2\pi R}$$

$$\omega \approx 21 \text{ rad/s} \qquad f=\frac{\omega}{2\pi} \doteq 3\text{Hz}$$

Gauss's Law

Q2. * $E=\dfrac{\sigma}{2\varepsilon_0}$ $\quad \sigma=\rho x$



$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_0}{(R+x)^2}+\frac{\rho x}{2\varepsilon_0}$$

$$E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{4}{3}\pi R^3\rho_0}{R^2}-\frac{\rho x}{2\varepsilon_0} \qquad E_B > E_A$$

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{4}{3}\pi R^3\rho_0}{(R+x)^2}+\frac{\rho x}{2\varepsilon_0} > \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{4}{3}\pi R^3\rho_0}{R^2}-\frac{\rho x}{2\varepsilon_0}$$

$$\frac{\rho x}{\cancel{\varepsilon_0}} > \frac{\frac{\cancel{4}}{3}\cancel{\pi}R^3\rho_0}{\cancel{4}\cancel{\pi}\cancel{\varepsilon_0}}\cdot\left(\frac{1}{R^2}-\frac{1}{(R+x)^2}\right)$$

$$\rho\cancel{x} > \frac{\rho_0 R^{\cancel{3}}}{3}\left(\frac{1}{\cancel{R^2}}-\frac{1}{\cancel{R^2}(1+\frac{x}{R})^2}\right) = \frac{\rho_0 R}{3}\left(1-\frac{1}{(1+\frac{x}{R})^2}\right)$$

$$= \frac{\rho_0 R}{3}\cdot\frac{\cancel{1}+\frac{2x}{R}+\cancel{\frac{x^2}{R^2}}-\cancel{1}}{(1+\frac{x}{R})^2} \approx \frac{\cancel{R}\rho_0}{3}\cdot\frac{2x}{\cancel{R}} = \frac{2}{3}\rho_0\cancel{x}$$

$$\rho > \frac{2}{3}\rho_0$$