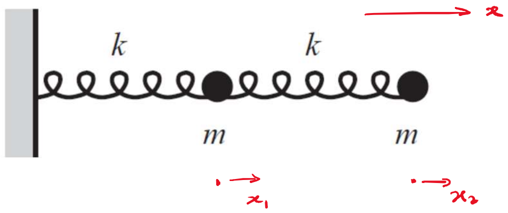
Q1:



$$F = -kx$$

$$F_1 = -kx_1 - k(x_1 - x_2) = m\ddot{x}_1$$

$$F_2 = -k(x_2 - x_1) = m\ddot{x}_2$$

$$\begin{cases} m\ddot{x}_1 + 2kx_1 - kx_2 = 0 \\ m\ddot{x}_2 - kx_1 + kx_2 = 0 \end{cases}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{i\alpha t} \qquad x_1 = Ae^{i\alpha t} \qquad x_2 = Be^{i\alpha t}$$

$$\begin{cases} m\cdot(-\alpha^2)\cdot A\cdot \cancel{e^{i\alpha t}} + 2k\cdot A\cancel{e^{i\alpha t}} - k\cdot B\cancel{e^{i\alpha t}} = 0 \\ m\cdot(-\alpha^2)B\cancel{e^{i\alpha t}} - k\cdot A\cancel{e^{i\alpha t}} + kB\cancel{e^{i\alpha t}} = 0 \end{cases}$$

$$\begin{cases} (2k - m\alpha^2)A - kB = 0 \\ -kA + (k - m\alpha^2)B = 0 \end{cases}$$

$$\begin{pmatrix} 2k - m\alpha^2 & -k \\ -k & k - m\alpha^2 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0 \qquad \det\begin{vmatrix} 2k - m\alpha^2 & -k \\ -k & k - m\alpha^2 \end{vmatrix} = 0$$

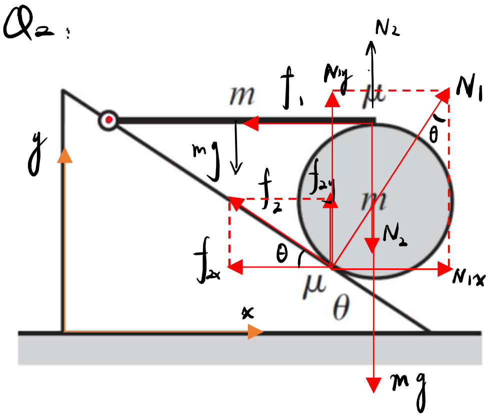
$$m^2\alpha^4 - 3km\alpha^2 + 2k^2 - k^2 = 0 \qquad m^2\alpha^4 - 3km\alpha^2 + k^2 = 0$$

let $u = \alpha^2$ $\quad m^2u^2 - 3km\cdot u + k^2 = 0 \quad u = \dfrac{3km \pm \sqrt{9k^2m^2 - 4k^2m^2}}{2m^2} = \dfrac{3k \pm \sqrt{5k^2}}{2m}$

$$\alpha^2 = \frac{3 \pm \sqrt{5}}{2}\cdot\frac{k}{m} \qquad \alpha = \pm\sqrt{\frac{k}{m}}\cdot\sqrt{\frac{3 \pm \sqrt{5}}{2}} \qquad = \frac{3 \pm \sqrt{5}}{2}\cdot\frac{k}{m}$$

$$\alpha = \pm\sqrt{\frac{k}{m}}\cdot\frac{\sqrt{5} \pm 1}{2}$$

Q2:



① centre of Cylinder $\sum M = 0$

$$f_1\cdot\cancel{R} = f_2\cdot\cancel{R} \qquad f_1 = f_2 = f$$

② Stick $\sum M = 0$

$$\frac{1}{2}l\cdot mg = N_2\cdot l \qquad N_2 = \frac{1}{2}mg$$

③ Cylinder: $\sum \vec{F} = 0$

$$\begin{cases} f_1 + f_2\cos\theta = N_1\sin\theta \\ f_2\sin\theta + N_1\cos\theta = N_2 + mg \end{cases}$$

$$f(1+\cos\theta) = N_1\sin\theta \qquad f = \frac{\sin\theta}{1+\cos\theta}N_1$$

$$f\sin\theta + N_1\cos\theta = \frac{3}{2}mg \qquad \frac{\sin^2\theta}{1+\cos\theta}\cdot N_1 + N_1\cos\theta = \frac{3}{2}mg$$

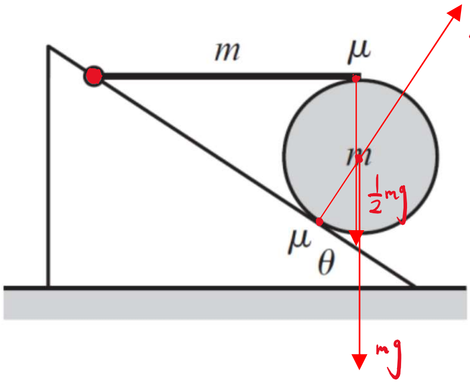
$$\left(\frac{1-\cos^2\theta}{1+\cos\theta} + \cos\theta\right)N_1 = \frac{3}{2}mg \qquad N_1 = \frac{3}{2}mg$$

$$f = \frac{3}{2}mg\frac{\sin\theta}{1+\cos\theta} \qquad \underline{f \le N_1\mu} \qquad \underline{f \le N_2 \mu}$$

$$\frac{3}{2}mg\frac{\sin\theta}{1+\cos\theta} \le \frac{3}{2}mg\mu \qquad \mu \ge \frac{\sin\theta}{1+\cos\theta}$$

$$\frac{3}{2}mg\frac{\sin\theta}{1+\cos\theta} \le \frac{1}{2}mg\mu \qquad \mu \ge \frac{3\sin\theta}{1+\cos\theta}$$

$$\Rightarrow \mu \ge \frac{3\sin\theta}{1+\cos\theta}$$

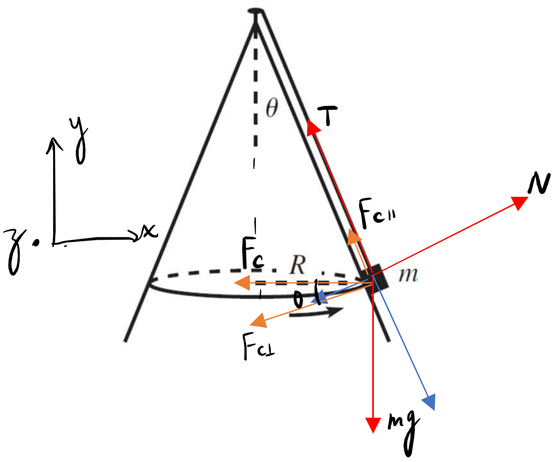


$N_1 = \frac{3}{2}mg$

Static Mechanics $\begin{cases} \sum \vec{F} = 0 \\ \sum \vec{M} = 0 \end{cases}$

Q3. $$\int \frac{1}{1-x^2}dx = \int \frac{1}{(1+x)(1-x)}dx = \int\left(\frac{1}{1+x}+\frac{1}{1-x}\right)\frac{1}{2}dx = \frac{1}{2}\int\left(\frac{1}{1+x}+\frac{1}{1-x}\right)dx$$

$$\int \frac{1}{x}dx = \ln|x| + C \qquad = \frac{1}{2}\left(\ln(1+x) - \ln(1-x)\right) + C = \frac{1}{2}\ln\frac{1+x}{1-x} + C$$



★ $$\underline{mg\sin\theta - N = m\frac{v^2}{R}\cos\theta}$$

$$N = mg\sin\theta - m\frac{v^2}{R}\cos\theta$$

$$f = \mu N = \mu\left(mg\sin\theta - m\frac{v^2}{R}\cos\theta\right)$$

$$\cancel{m}\cdot\frac{dv}{dt} = -f = -\mu\left(\cancel{m}g\sin\theta - \cancel{m}\frac{v^2}{R}\cos\theta\right)$$

$$\underline{\frac{dv}{g\sin\theta - (v^2/R)\cos\theta} = -\mu\, dt}$$

$$\frac{dv}{\left(1 - \frac{v^2\cos\theta}{gR\sin\theta}\right)} = -\mu g\sin\theta\, dt$$

$$\int_0^t -\mu g\sin\theta\, dt = \int_{v_0}^0 \frac{dv}{1-\frac{v^2}{gR\tan\theta}} \qquad \text{let } u \equiv v/\sqrt{gR\tan\theta} \qquad du = dv/\sqrt{gR\tan\theta}$$

$$\int_0^t -\mu g\sin\theta\, dt = \int_{v_0/\sqrt{gR\tan\theta}}^0 \frac{du}{1-u^2}\cdot\sqrt{gR\tan\theta} \qquad -\mu g\sin\theta\int_0^t dt = \sqrt{gR\tan\theta}\int_{\frac{v_0}{\sqrt{gR\tan\theta}}}^0 \frac{du}{1-u^2}$$

$$t = +\frac{1}{\mu g\sin\theta}\cdot\sqrt{gR\tan\theta}\cdot\frac{1}{2}\left[\cancel{\ln\left(\frac{1+0}{1-0}\right)} + \ln\left(\frac{1+\frac{v_0}{\sqrt{gR\tan\theta}}}{1-\frac{v_0}{\sqrt{gR\tan\theta}}}\right)\right] = \frac{1}{2\mu}\sqrt{\frac{R}{g\sin\theta\cos\theta}}\ln\left(\frac{\sqrt{gR\tan\theta}+v_0}{\sqrt{gR\tan\theta}-v_0}\right)$$

Q 4.

$\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$ $\qquad \hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$ $\qquad \dot{\hat{r}} = (-\sin\theta\,\hat{x} + \cos\theta\,\hat{y})\dot\theta = \hat\theta\,\dot\theta$

$\dot{\hat\theta} = (-\cos\theta\,\hat{x} - \sin\theta\,\hat{y})\dot\theta = -\hat{r}\,\dot\theta$

$$\dot{\vec r} = \frac{d}{dt}(r\hat r) = \dot r\hat r + r\cdot\dot{\hat r} = \dot r\hat r + r\hat\theta\dot\theta$$

$$\ddot{\vec r} = \frac{d}{dt}(\dot r\hat r + r\hat\theta\dot\theta) = \ddot r\hat r + \dot r\hat\theta\dot\theta + \dot r\hat\theta\dot\theta + r\dot{\hat\theta}\dot\theta + r\hat\theta\ddot\theta$$

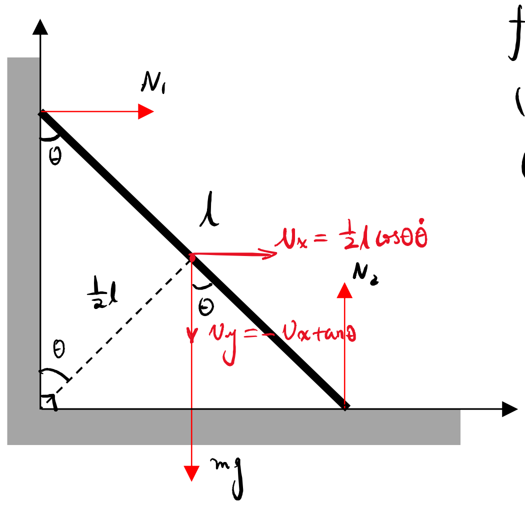
$$= \ddot r\hat r + 2\dot r\dot\theta\hat\theta - r\dot\theta\hat r + r\hat\theta\ddot\theta$$

$$= (\ddot r - r\dot\theta)\hat r + (2\dot r\dot\theta + r\ddot\theta)\hat\theta$$

$$\vec F(r) = \underline{F(r)}\cdot\hat r = m\ddot{\vec r} = m\underline{(\ddot r - r\dot\theta)}\hat r + \underbrace{m(2\dot r\dot\theta + r\ddot\theta)}_{\Rightarrow 0}\hat\theta$$

$$\begin{cases} F(r) = m(\ddot r - r\dot\theta) \\ 2\dot r\dot\theta + r\ddot\theta = 0 \end{cases}$$

Q 6.



$f = ma$

$(x, y) = \left(\frac{1}{2}l\sin\theta,\ \frac{1}{2}l\cos\theta\right)$

$(\dot x, \dot y) = \left(\underline{\frac{1}{2}l\cos\theta\cdot\dot\theta},\ -\frac{1}{2}l\sin\theta\dot\theta\right)$

$\frac{1}{2}mgl - \frac{1}{2}mgl\cos\theta = \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2$

$\cancel{\frac{1}{2}}mgl(1-\cos\theta) = \underbrace{\cancel{\frac{1}{2}}m\cdot\frac{1}{4}l^2\dot\theta^2}_{\text{translational}} + \underbrace{\cancel{\frac{1}{2}}I\omega^2}_{\text{rotational}}$

$I = \frac{1}{12}ml^2$ (moment of inertia)

$\cancel{m}g\cancel{l}(1-\cos\theta) = \frac{1}{4}ml^2\dot\theta^2 + \frac{1}{12}ml^2\dot\theta^2$

$= \frac{1}{3}\cancel{m}l^{\cancel{2}}\dot\theta^2$

$\dot\theta = \dfrac{\sqrt{v_x^2 + v_y^2}}{\frac{1}{2}l} = \dfrac{\sqrt{v_x^2 + v_x^2\tan^2\theta}}{\frac{1}{2}l} = \dfrac{2v_x}{l}\cdot\sqrt{1+\tan^2\theta}$

$= \dfrac{2v_x}{l}\cdot\dfrac{1}{\cos\theta}$

$g(1-\cos\theta) = \frac{1}{3}\cancel{l}\cdot\dfrac{4v_x^2}{l^{\cancel{2}}}\cdot\dfrac{1}{\cos^2\theta}$

$v_x^2 = \dfrac{3gl(1-\cos\theta)\cos^2\theta}{4}$

$1+\tan^2\theta = \sec^2\theta = \dfrac{1}{\cos^2\theta}$

$v_x = \sqrt{\dfrac{3gl(1-\cos\theta)\cos^2\theta}{4}}$

$\underline{f(\theta) = (1-\cos\theta)\cos^2\theta}$ $\qquad \cos\theta = \frac{2}{3}$

$\theta \approx 48.2^\circ$ $\qquad v_x = \sqrt{\dfrac{3gl\cdot\frac{1}{3}\cdot\frac{4}{9}}{4}}$

$= \dfrac{\sqrt{gl}}{3}$