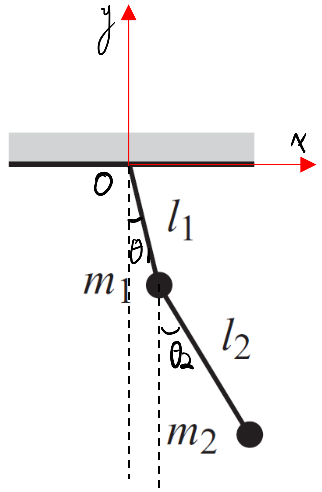

$$(x_1, y_1) = (l_1 \sin\theta_1, -l_1 \cos\theta_1)$$

$$(x_2, y_2) = (l_1 \sin\theta_1 + l_2 \sin\theta_2, -l_1\cos\theta_1 - l_2\cos\theta_2)$$

$$\vec{v}_1 = v_{1x}\hat{x} + v_{1y}\hat{y} = \frac{d\vec{r}_1}{dt} = \frac{dx_1}{dt}\cdot\hat{x} + \frac{dy_1}{dt}\cdot\hat{y}$$

$$\frac{dx_1}{dt} = l_1\cos\theta_1\cdot\dot{\theta}_1 \qquad \frac{dy_1}{dt} = l_1\sin\theta_1\cdot\dot{\theta}_1$$

$$K_1 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1(l_1^2\underline{\cos^2\theta_1}\cdot\dot{\theta}_1^2 + l_1^2\underline{\sin^2\theta_1}\cdot\dot{\theta}_1^2)$$

$$= \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2$$

$$\vec{v}_2 = v_{2x}\hat{x} + v_{2y}\hat{y} = \frac{dx_2}{dt}\hat{x} + \frac{dy_2}{dt}\hat{y}$$

$$\frac{dx_2}{dt} = l_1\cos\theta_1\cdot\dot{\theta}_1 + l_2\cos\theta_2\cdot\dot{\theta}_2$$

$$\boxed{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 = \cos(\theta_1-\theta_2)}$$

$$\frac{dy_2}{dt} = l_1\sin\theta_1\,\dot{\theta}_1 + l_2\sin\theta_2\,\dot{\theta}_2$$

$$K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_2\left(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)\right)$$

$$V = (-m_1 g\, l_1\cos\theta_1) + (-m_2 g\,(l_1\cos\theta_1 + l_2\cos\theta_2))$$

$$L = K - V = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2 + \underline{\frac{1}{2}m_2}(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + \underline{2l_1l_2}\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2))$$
$$+ m_1 g l_1\cos\theta_1 + m_2 g(l_1\cos\theta_1 + l_2\cos\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right) = \frac{\partial L}{\partial\theta_1}:$$

$$\frac{d}{dt}\left(m_1 l_1^2\dot{\theta}_1 + m_2 l_1^2\dot{\theta}_1 + m_2 l_1 l_2\,\underline{\dot{\theta}_2\cos(\theta_1-\theta_2)}\right) = \underline{-m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)} - m_1 g l_1\sin\theta_1 - m_2 g l_1\sin\theta_1$$

$$\uparrow$$

$$(m_1+m_2)l_1^2\ddot{\theta}_1 + m_2 l_1 l_2\ddot{\theta}_2\cos(\theta_1-\theta_2) - m_2 l_1 l_2\dot{\theta}_2\cdot\sin(\theta_1-\theta_2)(\cancel{\dot{\theta}_1}-\dot{\theta}_2) = \underline{\qquad}$$

$$\underline{(m_1+m_2)l_1^2\ddot{\theta}_1 + m_2 l_1 l_2\ddot{\theta}_2\cos(\theta_1-\theta_2) + m_2 l_1 l_2\dot{\theta}_2^2 + (m_1+m_2)g l_1\sin\theta_1 = 0}$$

$$m_2 l_1 l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_2}\right) = \frac{\partial L}{\partial\theta_2}:$$

$$\frac{d}{dt}\left(m_2 l_2^2\dot{\theta}_2 + m_2 l_1 l_2\dot{\theta}_1\cos(\theta_1-\theta_2)\right) = -m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)\cdot(-1) - m_2 g l_2\sin\theta_2$$

$$m_2 l_2^2\ddot{\theta}_2 + m_2 l_1 l_2\ddot{\theta}_1\cos(\theta_1-\theta_2) - m_2 l_1 l_2\dot{\theta}_1\ \sin(\theta_1-\theta_2)(\dot{\theta}_1-\cancel{\dot{\theta}_2}) = \cancel{m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2)} - m_2 g l_2\sin\theta_2$$

$$\underline{m_2 l_2^2\ddot{\theta}_2 + m_2 l_1 l_2\ddot{\theta}_1\cos(\theta_1-\theta_2) - m_2 l_1 l_2\dot{\theta}_1^2\sin(\theta_1-\theta_2) + m_2 g l_2\sin\theta_2 = 0}$$

Disscussion: $\theta_1 \to 0$, $\theta_2 \to 0$ $\quad \sin\theta_1 \approx \theta_1$, $\sin\theta_2 \approx \theta_2$

$\sin(\theta_1-\theta_2) \to 0 \quad \cos(\theta_1-\theta_2) \to 1$

$$\begin{cases} (m_1+m_2)l_1^{\cancel{2}}\ddot{\theta}_1 + m_2 \cancel{l_1} l_2 \ddot{\theta}_2 + (m_1+m_2) g \cancel{l_1} \theta_1 = 0 \\ \cancel{m_2} l_2^{\cancel{2}} \ddot{\theta}_2 + \cancel{m_2} l_1 \cancel{l_2} \ddot{\theta}_1 + \cancel{m_2} g \cancel{l_2} \theta_2 = 0 \end{cases}$$

$$\begin{cases} (m_1+m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 + (m_1+m_2) g\theta_1 = 0 \\ l_2\ddot{\theta}_2 + l_1\ddot{\theta}_1 + g\theta_2 = 0 \end{cases}$$

if $l_1 = l_2 \equiv l$
$$\begin{cases} (m_1+m_2) l \ddot{\theta}_1 + m_2 l \ddot{\theta}_2 + (m_1+m_2) g\theta_1 = 0 & ① \\ l\ddot{\theta}_2 + l\ddot{\theta}_1 + g\theta_2 = 0. & ② \end{cases}$$

$m_2 l \ddot{\theta}_2 + m_2 l \ddot{\theta}_1 + m_2 g \theta_2 = 0$ ③

①-③ $\underline{m_1 l \ddot{\theta}_1 + (m_1+m_2) g \theta_1 - m_2 g \theta_2 = 0}$ (a)

$(m_1+m_2) l \ddot{\theta}_2 + (m_1+m_2) l \ddot{\theta}_1 + (m_1+m_2) g\theta_2 = 0$ ④

$\underline{m_1 l \ddot{\theta}_2 + (m_1+m_2) g\theta_2 - (m_1+m_2) g \theta_1 = 0}$ (b)

$$\theta_1 = \frac{m_1 l \ddot{\theta}_2 + (m_1+m_2) g\theta_2}{(m_1+m_2) g} = \frac{m_1}{(m_1+m_2)g} l \ddot{\theta}_2 + \theta_2$$

$$\ddot{\theta}_1 = \frac{m_1}{(m_1+m_2)g} l \theta_2^{(4)} + \theta_2^{(2)} \to \underline{\theta_2^{(4)} \quad \theta_2^{(2)} \quad \theta_2} \Rightarrow a x_2^4 + b x_2^3 + c x_2^2 + d x_2 + e = 0$$

$$\omega_\pm = \sqrt{\frac{g}{l}} \cdot \sqrt{\frac{m_1+m_2 \pm \sqrt{m_1 m_2 + m_2^2}}{m_1}}$$

$$\underline{\begin{pmatrix} \theta_1(t) \\ \theta_2(t) \end{pmatrix} = A \cdot \begin{pmatrix} \frac{\mp\sqrt{m_2}}{\sqrt{m_1+m_2}} \end{pmatrix} \cos(\omega_\pm t + \varphi_\pm)}$$