charge

fundamental charge $e = 1.6 \times 10^{-19}$ (C)

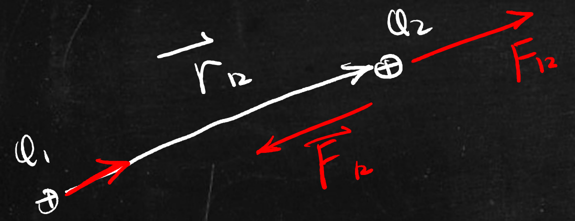
no relativity relation

Electric Field: Electroltatics

point charge

Coulomb's Law: $\vec{F}_{12} = k \cdot \dfrac{Q_1 Q_2}{r_{12}^2} \hat{r}_{12}$

$k = 8.99 \times 10^{9} \dfrac{N \cdot m^2}{C^2}$

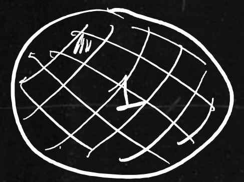


$= \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{Q_1 Q_2}{r_{12}^2} \hat{r}_{12}$

$k = \dfrac{1}{4\pi\varepsilon_0}$ ε_0 — permittivity

$= \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q_1 Q_2}{r_{12}^3} \cdot \vec{r}_{12}$

$\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/(N^2 m^2)$



$\vec{F} = \int k \dfrac{dq\, dq}{r_{12}^2} \hat{r}_{12}$

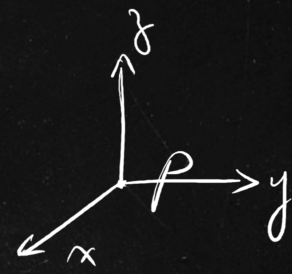
$\boxed{\text{Ethal}}$

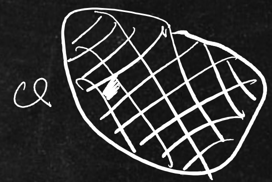
* charge $\rightleftharpoons$ field $\rightleftharpoons$ charge

Electric Field of a point charge Q_1

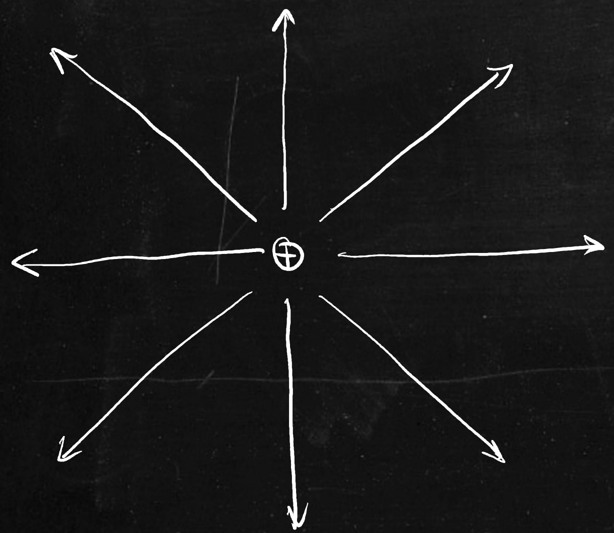
$$\vec{F}_{12} = k\frac{Q_1 Q_2}{r_{12}^2}\vec{r}_{12} = \boxed{\frac{kQ_1}{r_{12}^2}\vec{r}_{12}} \cdot Q_2$$

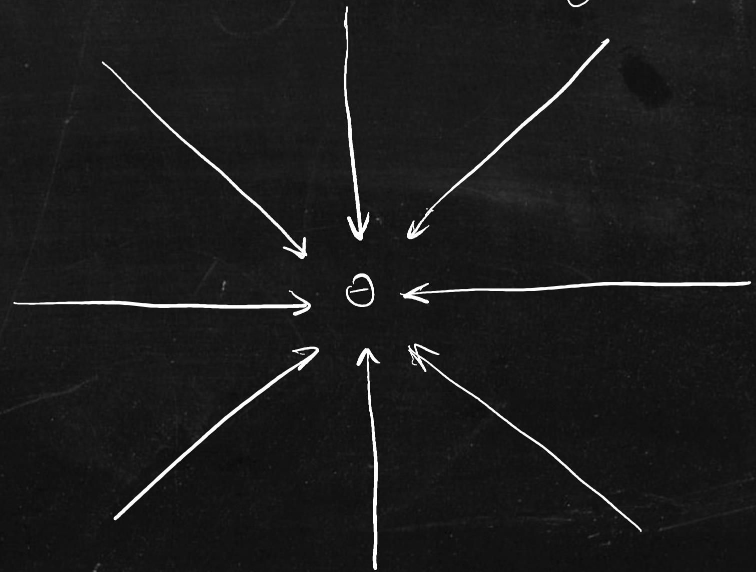
Electric Field Strength $\vec{E} = k\frac{Q}{r^2}\hat{r}$



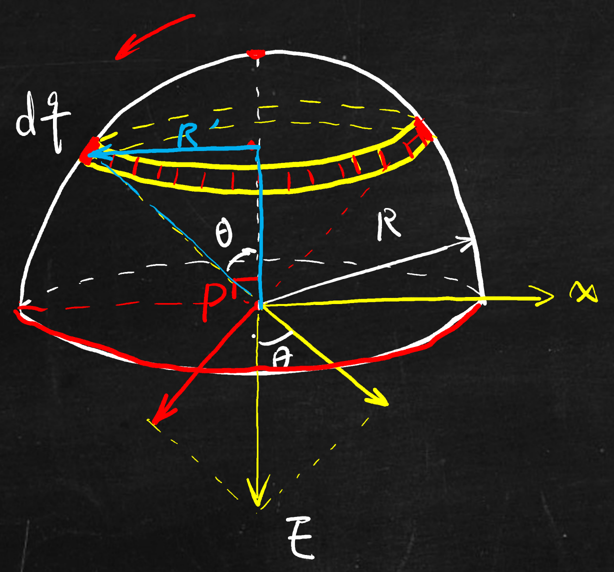


Electric Field Line





ex. charge surface density σ $\quad A = 4\pi R^2$ $\quad P$



Electric Field Strength at point P.

$$dE = k \cdot \frac{dq}{R^2} \cdot \cos\theta$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

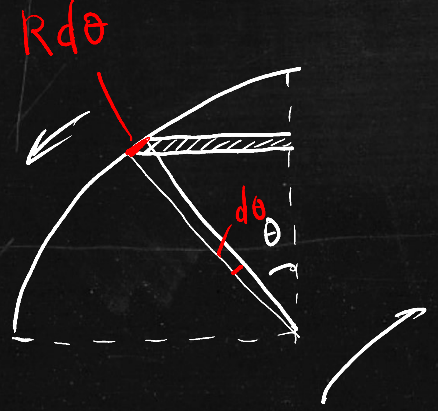
$$dq = dA \cdot \sigma = \sigma \cdot dA$$

$$dA = 2\pi R' \cdot dl \qquad R' = R\sin\theta$$

$$dA = 2\pi R\sin\theta \cdot R\,d\theta = 2\pi R^2 \sin\theta\, d\theta$$

$$-(-1)-(-1)$$

$$dE = k \cdot \frac{2\pi \cancel{R^2}\sigma \sin\theta\, d\theta \cos\theta}{\cancel{R^2}} = 2\pi k\sigma \sin\theta\, d\theta \cos\theta \qquad -\sigma-(-1)$$

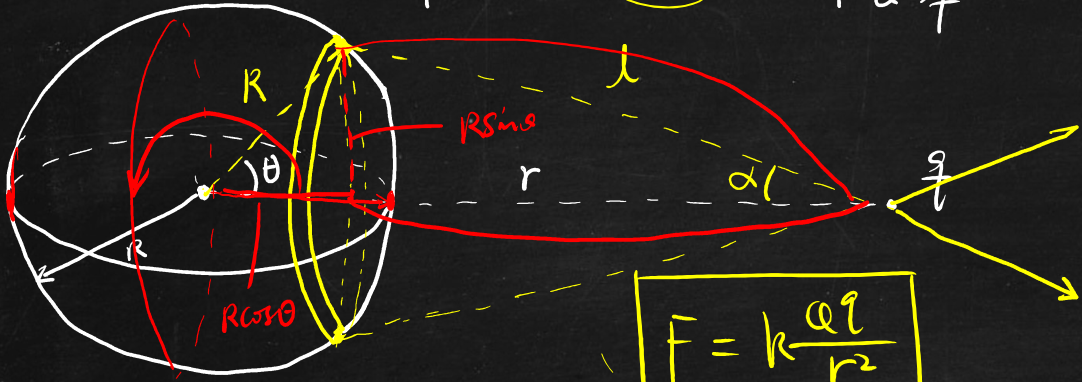


$$E = \int dE = \int_0^{\frac{\pi}{2}} 2\pi k\sigma \sin\theta \cos\theta\, d\theta = \frac{1}{\cancel{2}}\pi k\sigma \left(-\cos 2\theta\right)\Big|_0^{\frac{\pi}{2}}$$

$$= \cancel{2}k\pi\sigma = \cancel{2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\pi\cdot\sigma = \frac{\sigma}{4\varepsilon_0}.$$

ex

sphere : +Q $F_{Q\to q} = ?$ $\vec{F} = \vec{E}q$



$$\boxed{F = k\frac{Qq}{r^2}}$$

$$dE_x = k\frac{dQ}{l^2}\cdot\cos\alpha$$

$$dF = q\,dE_x = k\cdot\frac{dQ\,q}{l^2}\cos\alpha$$

$$dV = k\frac{dQ\,q}{l}$$

(cosine rule)

$$dQ = (2\cancel{\pi}\cancel{R}\sin\theta)(\cancel{R}\,d\theta)\cdot\frac{Q}{4\cancel{\pi}\cancel{R^2}} = \frac{1}{2}Q\sin\theta\,d\theta$$

$$l = \sqrt{R^2 + r^2 - 2Rr\cos\theta}$$

$$\cos\alpha = \frac{r - R\cos\theta}{l}$$

$$dF = \frac{1}{2}kqQ\frac{\sin\theta\cdot(r - R\cos\theta)}{(R^2+r^2-2Rr\cos\theta)^{\frac{3}{2}}}d\theta$$

$$F = \boxed{\int_0^{\pi}\frac{1}{2}kQq\frac{\sin\theta\,(r-R\cos\theta)}{(R^2+r^2-2Rr\cos\theta)^{\frac{3}{2}}}d\theta}$$

Conservative force

$$V = -\int\vec{F}\cdot d\vec{r} = -\int k\frac{Qq}{r^2}dr = k\frac{Qq}{r}$$

$$\boxed{F = -\frac{dV}{dr}}$$

$$V = -\int_0^{\pi}\frac{1}{2}kQq\frac{\sin\theta}{(R^2+r^2-2Rr\cos\theta)^{\frac{1}{2}}}d\theta = -\frac{1}{2}kQq\cdot\left((R^2+r^2-2Rr\cos\theta)^{\frac{1}{2}}\cdot 2\cdot\frac{1}{2Rr}\right)\Big|_0^{\pi} = k\frac{Qq}{r}$$

$r > R$