(a) The energy levels, E_n, of the hydrogen atom are given by

$E_n = -13.6/n^2$ eV, where $n = 1, 2, 3, \ldots$

What is the wavelength of the photon that results from a transition from the second excited state to the ground state?

[3]

$n=1 \quad E_n = -13.6\,eV$

$n=2 \quad E_n = -\frac{13.6}{4} = -3.9\,eV$

$E = h\nu \quad h$

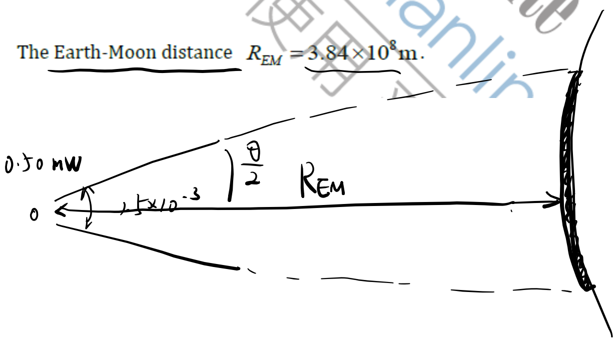
(b) A 0.50 mW laser, of wavelength 590 nm producing a beam with a divergence angle of 1.5×10^{-3} radians, is pointing at the Moon. What is the maximum number of photons arriving per second per square metre on the Moon?

[5]

photon

$E = hf = h\nu$

The Earth-Moon distance $R_{EM} = 3.84\times10^{8}$ m.



$\frac{\theta}{2} = \frac{1}{2}\times 1.5\times10^{-3}$

$r = R_{EM}\cdot\frac{\theta}{2}$

$A = \pi r^2$

$\frac{0.5\times10^{-3}}{h\cdot\frac{c}{\lambda}} = N$

(c) A photon of frequency f_1 and wavelength λ_1, momentum h/λ_1, is scattered by a stationary electron. A photon of frequency f_2 and wavelength λ_2 results. It travels in the opposite direction to the initial photon and the electron gains energy of 5.00 keV, with velocity v in the same direction as the incident photon. Determine numerically the value of λ_1.

[8]

$p = mc \quad E = hf = mc^2$

$p = mv$

$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$p = \sqrt{2mE_k}$

$f_1 \quad \lambda_1 \quad p = \frac{h}{\lambda_1}$ electron

$1\,eV = 1.6\times10^{-19}\,J$

$m_e = 9.11\times10^{-31}\,kg$

collision

$\begin{cases} M.C \\ E.C. \end{cases}$

$f_2 \quad \lambda_2 \quad \frac{h}{\lambda_2}$ electron

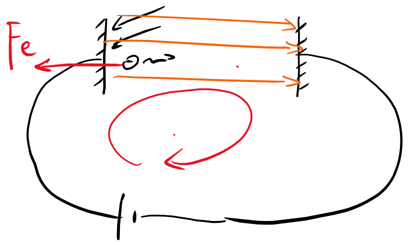
$\frac{h}{\lambda_1} = \sqrt{2m_eE_k} - \left(\frac{h}{\lambda_1} - \frac{E_k}{c}\right)$

$$\begin{cases} \frac{h}{\lambda_1} = \sqrt{2m_eE_k} - \frac{h}{\lambda_2} \\ hf_1 = hf_2 + E_k \Rightarrow h\frac{c}{\lambda_1} = h\frac{c}{\lambda_2} + E_k \end{cases}$$

$\frac{h}{\lambda_2} = \frac{h}{\lambda_1} - \frac{E_k}{c}$

$f = \frac{c}{\lambda}$

(d) Monochromatic light of wavelength 380 nm is incident on a metal surface. A potential of 1.32 V is required to cut off the flow of photoelectrons. What is the work function of the metal?



$E = hf$ $\qquad E_k = hf - \phi$ ← work function

$E = QV$ $\qquad E_k = 1.32\,eV$

$\phi = hf - E_k = h \cdot \frac{c}{\lambda} - 1.32 \times 1.6 \times 10^{-19}$

A photon of wavelength λ has momentum p and energy E_λ .

(i) Determine the relation between p and E_λ. An electric light bulb emits 20 W of radiation uniformly in all directions.

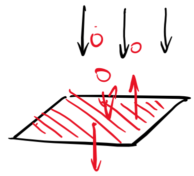
(ii) What is the maximum radiation pressure on a surface placed 2.0 m away from the bulb?

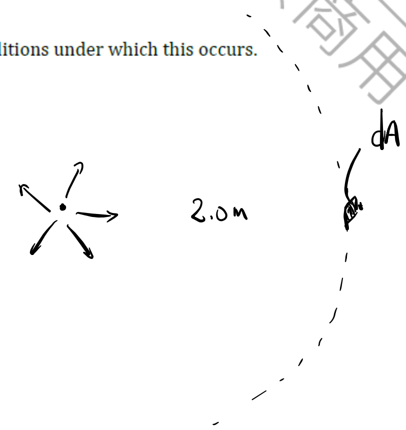
(iii)State the conditions under which this occurs.

[8]

$E_\lambda = h\nu = h \cdot \frac{c}{\lambda} = mc^2$

$\frac{h}{\lambda} = mc = p \qquad E_\lambda = p \cdot c$





Δt

$$P = \frac{\left(\frac{\frac{20 \cdot \cancel{\Delta t}}{h\nu}}{2 \cancel{4} \pi R^2} \cdot \cancel{dA}\right) \cdot 2p}{\frac{\cancel{\Delta t}}{\cancel{dA}}}$$

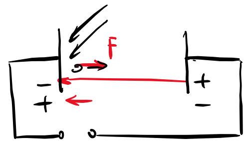
$$= \frac{20 \cdot \frac{E_\lambda}{c}}{2\pi h\nu R^2} = \frac{10 E_\lambda}{\pi h R^2 \cdot c\nu} = \frac{10 E_\lambda}{\pi h R^2 c \cdot \frac{c}{\lambda}}$$

$$= \frac{10 E_\lambda \cdot \lambda}{\pi h R^2 c^2}$$

(a)(i) Explain the photoelectric effect.

(ii) Derive a relation between the incident photon frequency, v, and the electron kinetic energy for a photocathode with work function φ.

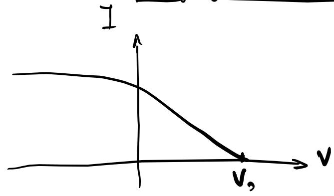
$E_K = h\nu - \varphi$



(iii) How does the classical explanation of this phenomenon differ from the quantum explanation?

wave

(iv) Sketch a graph of current, I, against voltage, V, from anode to cathode, for positive and negative V, in the presence of a constant beam of photons in a photoelectric experiment.

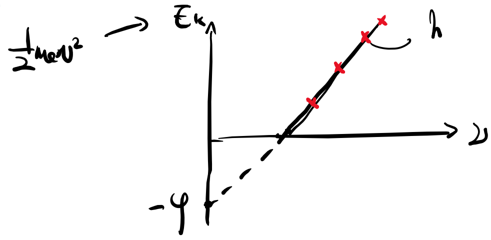


(v) How could one graphically determine φ from measurements of photon wavelength and electron velocity?

$\varphi = h\nu - E_K = h\frac{c}{\lambda} - E_K$

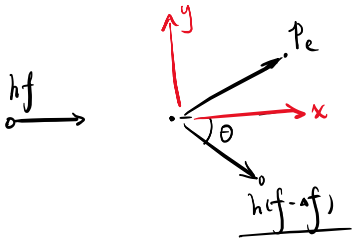
$$\boxed{E_K = h\frac{c}{\lambda} - \varphi}$$

[12]



The scattering of photons (Compton scattering) can be used to identify the composition of materials by the intensity of the scattered radiation. The scattered radiation is at a different frequency and in this problem you are asked to find out what happens when a photon is scattered off at an angle.

An incident photon, frequency f, momentum (hf/c), is scattered by a stationary electron producing a scattered photon of frequency ($f-\Delta f$) , where Δf is small compared with f. This photon travels in a direction that makes an angle θ with the direction of the incident photon. The electron, mass m_e, acquires a non-relativistic speed v.



(a) Draw a labelled vector triangle of the momenta of the particles.

[3]

(b) Write down the equation relating the magnitude of the momentum of the electron to that of the photons.

$$P_{ey} = \frac{h(f-\Delta f)}{c}\sin\theta \qquad P_{ex} = \frac{hf}{c} - \frac{h(f-\Delta f)}{c}\cos\theta$$

[4]

(c) Obtain the equation for energy conservation.

$$hf = h(f-\Delta f) + \frac{1}{2}m_e v^2$$

[2]

(d) Deduce an equation for Δf. When Δf is small much less than f and hf much less than $m_e c^2$, obtain the approximation,

$$\Delta f = \frac{hf^2(1-\cos\theta)}{m_e c^2}$$

[7]

$$\cancel{hf} = \cancel{hf} - h\Delta f + \frac{1}{2}m_e v^2$$

$$\Delta f = \frac{m_e v^2}{2h} \qquad P_e = \sqrt{P_{ex}^2 + P_{ey}^2} \qquad P_e^2 = P_{ex}^2 + P_{ey}^2$$

$$= \frac{P_e^2}{2m_e h} = \frac{P_{ex}^2 + P_{ey}^2}{2m_e h}$$

$$= \frac{1}{2m_e h}\left(\left(\frac{hf}{c} - \frac{h(f-\Delta f)}{c}\cos\theta\right)^2 + \left(\frac{h(f-\Delta f)}{c}\sin\theta\right)^2\right)$$

$$= \frac{1}{2m_e h}\left(\frac{h^2f^2}{c^2} - \frac{2h^2 f(f-\Delta f)}{c^2}\cos\theta + \frac{h^2(f-\Delta f)^2}{c^2}\cos^2\theta + \frac{h^2(f-\Delta f)^2}{c^2}\sin^2\theta\right)$$

$$= \frac{1}{2m_e \cancel{h}}\left(\frac{h^{\cancel{2}}f^2}{c^2} - \frac{2h^{\cancel{2}} f(f-\Delta f)}{c^2}\cos\theta + \frac{h^{\cancel{2}}(f-\Delta f)^2}{c^2}\right)$$

$$= \frac{h}{2m_e c^2}\left(f^2 - 2f(f-\Delta f)\cos\theta + (f-\Delta f)^2\right)$$

$$= \frac{hf^2}{2m_e c^2}\left(1 - \frac{2(f-\Delta f)\cos\theta}{f} + \left(1 - \cancel{\frac{\Delta f}{f}}\right)^2\right) \qquad \Delta f \ll f$$

$$= \frac{hf^2}{2m_e c^2}\left(2 - 2\left(1 - \cancel{\frac{\Delta f}{f}}\right)\cos\theta\right) = \frac{hf^2}{\cancel{2}m_e c^2}(\cancel{2} - \cancel{2}\cos\theta) = \frac{hf^2(1-\cos\theta)}{m_e c^2}$$