Q1

(e)

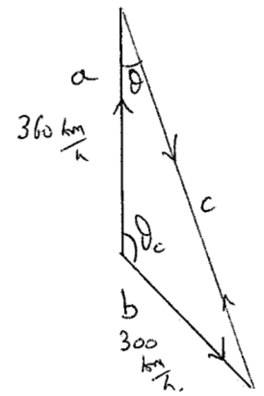


Diagram ✓

length, $a = \frac{2}{3} \times 360 = 240$ km

$b = \frac{2}{3} \times 300 = 200$ km

$\theta_c = 90 + 45 = 135°$ ✓

Cosine rule.

$c^2 = 240^2 + 200^2 - 2 \cdot 240 \cdot 200 \cos 135$

$c = 406.80$ km

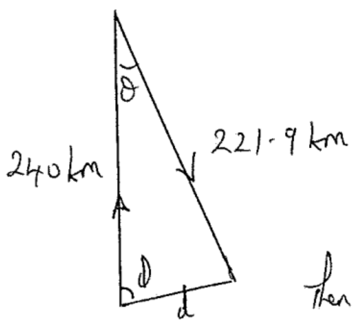
Combined speed of approach along c is 660 km/h. ✓

Time taken to meet is $\frac{406}{660} = 0.616$ h ✓

$= \underline{36.98 \text{ minutes}}$

In this time, plane 'a' travels $360 \times 0.616 = \underline{221.9 \text{ km}}$

meeting point found. ✓



From the top diagram

using sine rule

$\frac{c}{\sin 135} = \frac{b}{\sin \theta}$ $\quad \underline{\theta = 20.34°}$

Then cosine rule,

$d^2 = 240^2 + 221.9^2 - 2 \cdot 240 \cdot 221.9 \cos 20.34°$

$d = \underline{83.5 \text{ km}}$ ✓

Then $\frac{d}{\sin \theta} = \frac{221.9}{\sin \phi}$ $\quad \underline{\phi = 67.5°}$ ✓

[Need find two answers for (7) marks.
Either one or other gives (6) marks].

(7)

Q2

page 2

(c) (i) $\frac{ds}{dt} = 6t^2 - 18t + 12$

$\dot{s} = 0 \Rightarrow \quad t_0^2 - 3t_0 + 2 = 0$

$t_0 = 1\,s,\ 2\,s$ ✓

(ii) $\frac{d^2s}{dt^2} = 12t - 18$

$\ddot{s} = 0 \Rightarrow \quad t_{a=0} = \frac{3}{2} = 1.5\ s$ ✓

(iii) $\dot{s}$ at $t = \frac{3}{2}\ s$

$\dot{s}_{a=0} = V = 6\left(\frac{3}{2}\right)^2 - 18\cdot\frac{3}{2} + 12$

$= -\frac{3}{2} = -1.5\ ms^{-1}$ ✓

(iv) $\ddot{s}$ at $t = 1s, 2s$.

$a_1 = 12t - 18 = -6\ ms^{-2}$ ✓

$a_2 = 12t - 18 = +6\ ms^{-2}$

4

Q3

(f) ⑥ A

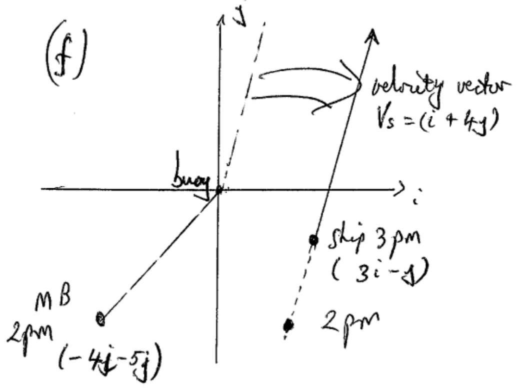


Diagram ✓

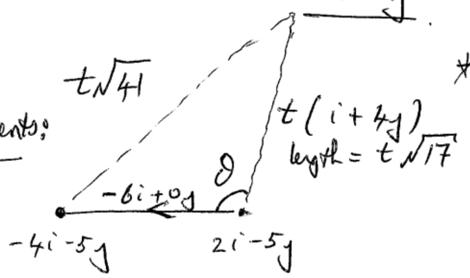
(i) Position of ship at 2pm is vector position at 3pm − 1 hour × $\underline{V_s}$ ✓

$= 3i - j - 1(i + 4j)$

$= \underline{2i - 5j}$ ✓

(ii)

Displacements:



* (✓✓) - 2 marks for clear working to obtain t and V_{MB}. If not clear, give 1 or even 0 marks for the working. But give the marks for each answer.

So, no working ⑦ → ⑤.

Using the cosine rule, $t^2 41 = 36 + t^2 17 - 2t\sqrt{17} \cdot \sqrt{36} \cdot \cos\theta$

and $\cos\theta = \dfrac{\underline{a} \cdot \underline{b}}{|a||b|} = \dfrac{(6i + 0j)}{\sqrt{36}} \cdot \dfrac{t(i + 4j)}{t \cdot \sqrt{17}}$

$= \dfrac{-6t}{\sqrt{36} \cdot t \cdot \sqrt{17}}$

$= -\dfrac{1}{\sqrt{17}}$

$\therefore \quad 41t^2 = 36 + 17t^2 + 2t\sqrt{17} \cdot \sqrt{36} \cdot \dfrac{1}{\sqrt{17}}$

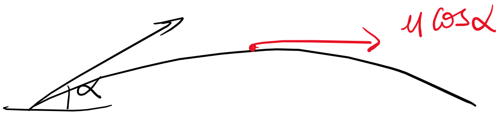
$24t^2 = 36 \quad + 2 \times 6t$

$2t^2 - t - 3 = 0$

$t = \dfrac{1 \pm \sqrt{1 + 24}}{4} = \dfrac{1}{4} \pm \dfrac{5}{4} \Rightarrow \underline{t = 1.5h}$ ✓

i.e. interception at 3:30pm

Q4



(a). $T_g = \frac{2u\sin\alpha}{g}$ $\quad d = u\cos\alpha \cdot T_g = \frac{2u\sin\alpha\cos\alpha}{g}$

(b) $h = \frac{(u\sin\alpha)^2}{2g} = \frac{u^2\sin^2\alpha}{2g}$

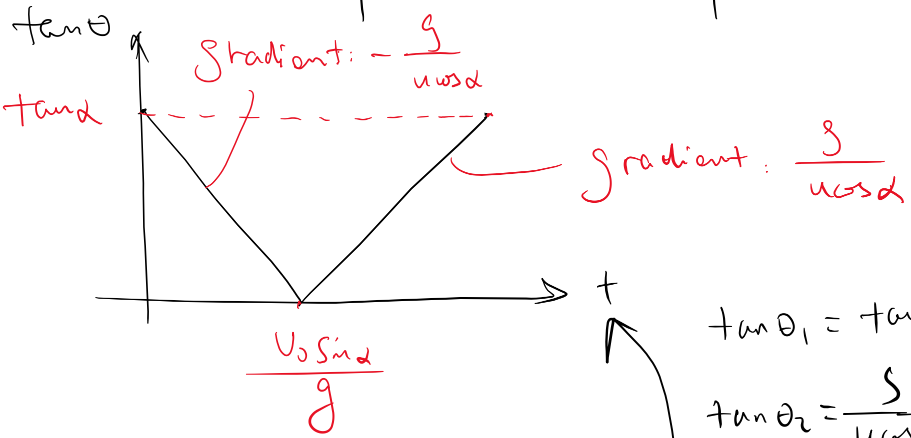
(c) $\cancel{m}g = \cancel{m}\frac{v^2}{R}$ $\quad R = \frac{v^2}{g} = \frac{u^2\cos^2\alpha}{g}$

(d) $R = \frac{u^2\cos^2\alpha}{g}$ $\quad d^2 = \frac{4u^2\sin^2\alpha\cos^2\alpha}{g}$ $\quad h = \frac{u^2\sin^2\alpha}{g}$

$$\frac{d^2}{h} = \frac{4u^{\cancel{4}\,2}\cancel{\sin^2}\alpha\cos^2\alpha}{g} \cdot \frac{\cancel{g}}{\cancel{u^2}\cancel{\sin^2\alpha}} = 4u^2\cos^2\alpha$$

$$R = \frac{1}{4}\cdot\frac{d^2}{h}\cdot\frac{1}{g} = \frac{d^2}{4gh}$$

(e) $\tan\theta = \left|\frac{u\sin\alpha - gt}{u\cos\alpha}\right| = \left|\tan\alpha - \frac{g}{u\cos\alpha}t\right|$



$\tan\theta_1 = \tan\alpha - \frac{g}{u\cos\alpha}t_1$

$\tan\theta_2 = \frac{g}{u\cos\alpha}\cdot t_2 - \tan\alpha$

f) perpendicular → $\theta_1 + \theta_2 = 90°$ $\quad \theta_1 + \theta_2 = 2\theta_1$ (Symmetric)

$\alpha < 45°$ $\quad \theta_1 < 45°$ $\quad \theta_2 < 45°$ $\quad = 2\tan\alpha - \frac{g}{u\cos\alpha}\cdot t_1 \leq 2\tan\alpha$

no perpendicular points of θ_1, θ_2

Q5

(i) The total force acting on the body in air, taking the downwards direction of the vertical axis to be positive, is

$$F = mg - \beta v.$$

Thus, the terminal velocity is

$$v = \frac{mg}{\beta}.$$

The ball falling in vacuum will have reached this velocity at time

$$t_f = \frac{m}{\beta}.$$

(ii) The equation of motion will be

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - \beta v.$$

Dividing both sides by RHS, we obtain

$$\frac{m}{mg - \beta v}\frac{\mathrm{d}v}{\mathrm{d}t} = 1,$$

which can be re-written as

$$\frac{\mathrm{d}v}{mg - \beta v} = \frac{\mathrm{d}t}{m}.$$

Multiplying both sides by $-\beta$, we have

$$\frac{\mathrm{d}v}{v - \frac{mg}{\beta}} = -\frac{\beta}{m}\mathrm{d}t.$$

Integrating both sides,

$$\ln\left(v - \frac{mg}{\beta}\right) = -\frac{\beta}{m}t + \widetilde{A} \implies v - \frac{mg}{\beta} = A\exp\left(-\frac{\beta}{m}t\right).$$

Isolating v and with the initial condition of speed and position zero at $t = 0$, $A = -mg/\beta$,

$$v = \frac{mg}{\beta}\left[1 - \exp\left(-\frac{\beta}{m}t\right)\right].$$

(iii) From the result above, at time t_f, its speed will be

$$v = \frac{mg}{\beta}(1 - e^{-1}) = \left(1 - \frac{1}{e}\right)\frac{mg}{\beta},$$

reduced by 30% w.r.t. the frictionless case.