BPHO&PUPC Class No.20210612

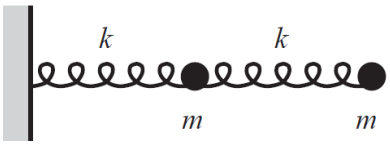
Mechanics Test 29/07/2021

1. You are allowed to use calculator.
2. The test lasts for ***3 hours*** from 19:15-22:15. You are allowed to read the questions from 19:00 to 19:15.
3. You are encouraged to try every question.
4. Total mark equals ***100 points***. Try your best to solve the physics part if mathematics is complicated.
5. Open your camera during the whole exam. You are allowed to go to the restroom or drink water if you would like.
6. This test is **closed book**. Dictionary is allowed.
7. After the exam, please take a photo of all your answers with a .zip file or .pdf file and send it to the study group. You will have 15mins to submit your answers. Make sure your solutions is submitted before 22:30

Good luck!

Problem 1(15 points)

Two identical springs and two identical masses are attached to a wall as a shown in the figure. The mass of each object if m and the stiffness spring constant is k. Find the equations of motions of the system the frequencies of the motion of the system in terms of m and k.

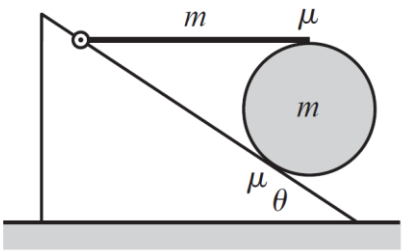


Problem 2(20 points)

A horizontal stick of mass m has its left end attached to a pivot on a plane inclined at an angle θ, while its right end rests on the top of a cylinder also of mass m which in turn rests on the plane, as shown in the figure. The coefficient of friction between the cylinder and both the stick and the plane is μ.

(a) Assuming that the system is at rest, what is the normal force from the plane on the cylinder?

(b) What is the smallest value of μ (in terms of θ) for which the system doesn't slip anywhere?

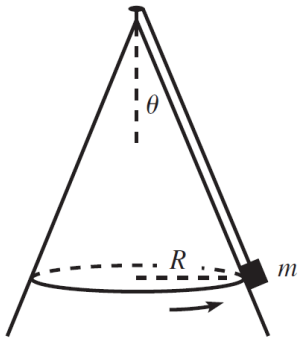


Problem 3(15 points)

When viewed from the side, the cone in the figure below subtends an angle 2θ at its tip. A block of mass m is connected to the tip by a massless string and moves in a horizontal circle of radius R around the surface. If the initial speed is v_0, and if the coefficient of kinetic friction between the block and the cone is μ, how much time does it take the block to stop?

Hint: You might need the mathematical formula of an integration:

$$\int \frac{1}{1-x^2}\,dx = \ln\left(\frac{1+x}{1-x}\right) + C$$

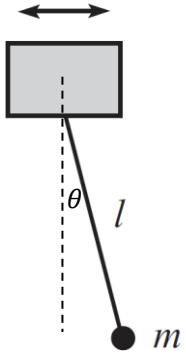


Problem 4(15 points)

Please drive the expressions of the general form of the velocity and acceleration in polar coordinates (r,θ). Assume the unit vector in the direction of r is $\hat{r}$, and the unit vector in the direction of θ is $\hat{\theta}$. Also, please use the expression you have derived to discuss what requirements needed to be satisfied if the net force applied on an object with mass *m* is a central force.

Problem 5(20 points)

A pendulum consists of a mass *m* and a massless stick of length *l*. The pendulum support oscillates horizontally with a position given by $x(t) = Acos(\omega t)$; see figure below. What is the general solution for the angle of the pendulum θ as function of time when the angle is quite small? Hint: You might use some approximation for small angle θ. *Hint: Lagrangian method is recommended.*



Problem 6(15 points)

A ladder of length l and uniform mass density stands on a frictionless floor and leans against a frictionless wall. It is initially held motionless, with its bottom end an infinitesimal distance from the wall. It is then released, whereupon the bottom end slides away from the wall, and the top end slides down the wall (see figure below). When it loses contact with the wall, what is the horizontal component of the velocity of the center of mass?