BPHO & PUPC Class No. 20210612

Assignment 7 Magnetic Field 19/08/2021   Due on 11:00 pm 25/08/2021

Please try your best to finish the assignment. You may not be able to complete every question, however, please write as much as you can. It is required that all the answers are written independently by yourself.

Please print the documents, write your solutions to each question and scan it so that you can post yours to our study group directly. It is better for you to combine all your documents in a single .pdf profile. Other format of documents is acceptable as well, please compress them in a single file with your name.

This assignment is totally worth 40 points.

Good luck!

Name: ___________   Score: ____________

Q1(15 points)

Consider a square loop with current I and side length a. The goal of this problem is to determine the magnetic field at a point a large distance r (with $r \gg a$) from the loop.

(a) At the distant point P in Fig. 6.36, the two vertical sides give essentially zero Biot–Savart contributions to the field, because they are essentially parallel to the radius vector to P. What are the Biot–Savart contributions from the two horizontal sides? These are easy to calculate because every little interval in these sides is essentially perpendicular to the radius vector to P. Show that the sum (or difference) of these contributions equals $\mu_0 I a^2/2\pi r^3$, to leading order in a.

(b) This result of $\mu_0 I a^2/2\pi r^3$ is not the correct field from the loop at point P. The correct field is half of this, or $\mu_0 I a^2/4\pi r^3$. We will eventually derive this in Chapter 11, where we will show that the general result is $\mu_0 I A/4\pi r^3$, where A is the area of a loop with arbitrary shape. But we *should* be able to calculate it via the Biot–Savart law. Where is the error in the reasoning in part (a), and how do you go about fixing it? This is a nice one – don't peek at the answer too soon!

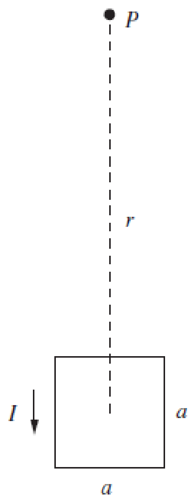

Figure 6.36.

Q2(15 points)

A ring with radius R carries a current I. Show that the magnetic field due to the ring, at a point in the plane of the ring, a distance a from the center (either inside or outside the ring), is given by

$$B = 2 \cdot \frac{\mu_0 I}{4\pi} \int_0^{\pi} \frac{(R - a\cos\theta)R\,d\theta}{(a^2 + R^2 - 2aR\cos\theta)^{3/2}}. \tag{6.94}$$

Q3(10 points)

Consider an infinite solenoid with circular cross section. The current is I, and there are n turns per unit length. Show that the magnetic field is zero outside and $B = \mu_0 n I$ (in the longitudinal direction) everywhere inside. Do this in three steps as follows.

(a) Show that the field has only a longitudinal component. *Hint:* Consider the contributions to the field from rings that are symmetrically located with respect to a given point.

(b) Use Ampère's law to show that the field has a uniform value outside and a uniform value inside, and that these two values differ by $\mu_0 n I$.

(c) Show that $B \to 0$ as $r \to \infty$. There are various ways to do this. One is to obtain an upper bound on the field contribution due to a given ring by unwrapping the ring into a straight wire segment, and then finding the field due to this straight segment.