Tutorial2 Mechanics2

Q1

Two identical masses m are constrained to move on a horizontal hoop. Two identical springs with spring constant k connect the masses and wrap around the hoop (see Fig. 4.14). One mass is subject to a driving force $F_d \cos \omega_d t$. Find the particular solution for the motion of the masses.

Fig. 4.14

Q2

(a) Two identical masses m are constrained to move on a horizontal hoop. Two identical springs with spring constant k connect the masses and wrap around the hoop (see Fig. 4.15). Find the normal modes.

(b) Three identical masses are constrained to move on a hoop. Three identical springs connect the masses and wrap around the hoop (see Fig. 4.16). Find the normal modes.

(c) Now do the general case with N identical masses and N identical springs.

Fig. 4.15

Fig. 4.16

Q3

A projectile of mass m is fired from the origin at speed v_0 and angle θ. It is attached to the origin by a spring with spring constant k and relaxed length zero.

(a) Find $x(t)$ and $y(t)$.

(b) Show that for small $\omega \equiv \sqrt{k/m}$, the trajectory reduces to normal projectile motion. And show that for large ω, the trajectory reduces to simple harmonic motion, that is, oscillatory motion along a line (at least before the projectile smashes back into the ground). What are the more meaningful statements that should replace "small ω" and "large ω"?

(c) What value should ω take so that the projectile hits the ground when it is moving straight downward?

Q4

A mass M collides with a stationary mass m. If $M < m$, then it is possible for M to bounce directly backward. However, if $M > m$, then there is a maximal angle of deflection of M. Show that this maximal angle equals $\sin^{-1}(m/M)$. *Hint*: It is possible to do this problem by working in the lab frame, but you can save yourself a lot of time by considering what happens in the CM frame, and then shifting back to the lab frame.

Q5

Assume that a cloud consists of tiny water droplets suspended (uniformly distributed, and at rest) in air, and consider a raindrop falling through them. What is the acceleration of the raindrop? Assume that the raindrop is initially of negligible size and that when it hits a water droplet, the droplet's water gets added to it. Also, assume that the raindrop is spherical at all times.

Q6

Consider a double pendulum made of two masses, m_1 and m_2, and two rods of lengths ℓ_1 and ℓ_2 (see Fig. 6.22). Find the equations of motion.

For small oscillations, find the normal modes and their frequencies for the special case $\ell_1 = \ell_2$ (and consider the cases $m_1 = m_2$, $m_1 \gg m_2$, and $m_1 \ll m_2$). Do the same for the special case $m_1 = m_2$ (and consider the cases $\ell_1 = \ell_2$, $\ell_1 \gg \ell_2$, and $\ell_1 \ll \ell_2$).

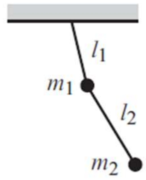

Fig. 6.22

Q7

A bead is released from rest at the origin and slides down a frictionless wire that connects the origin to a given point, as shown in Fig. 6.26. You wish to shape the wire so that the bead reaches the endpoint in the shortest possible time. Let the desired curve be described by the function $y(x)$, with downward taken to be positive. Show that $y(x)$ satisfies

$$1 + y'^2 = \frac{B}{y}, \tag{6.94}$$

where B is a constant. Then show that x and y may be written as

$$x = a(\theta - \sin\theta), \quad y = a(1 - \cos\theta). \tag{6.95}$$

This is the parametrization of a *cycloid*, which is the path taken by a point on the rim of a rolling wheel.

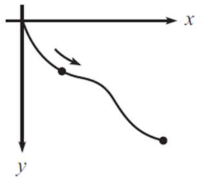

Fig. 6.26

Q8

A particle of mass m travels in a hyperbolic orbit past a mass M, whose position is assumed to be fixed. The speed at infinity is v_0, and the impact parameter is b (see Exercise 7.14).

(a) Show that the angle through which the particle is deflected is

$$\phi = \pi - 2\tan^{-1}(\gamma b) \quad \Longrightarrow \quad b = \frac{1}{\gamma}\cot\left(\frac{\phi}{2}\right), \tag{7.52}$$

where $\gamma \equiv v_0^2/GM$.

(b) Let $d\sigma$ be the cross-sectional area (measured when the particle is initially at infinity) that gets deflected into a solid angle of size $d\Omega$ at angle ϕ.[11] Show that

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\gamma^2\sin^4(\phi/2)}. \tag{7.53}$$

This quantity is called the *differential cross section*. *Note*: the label of this problem, *Rutherford scattering*, actually refers to the scattering of charged particles. But since the electrostatic and gravitational forces are both inverse-square laws, the scattering formulas look the same, except for a few constants.

Q9

A particle of mass m moves in a potential given by $V(r) = \beta r^k$. Let the angular momentum be L.

(a) Find the radius, r_0, of the circular orbit.

(b) If the particle is given a tiny kick so that the radius oscillates around r_0, find the frequency, ω_r, of these small oscillations in r.

(c) What is the ratio of the frequency ω_r to the frequency of the (nearly) circular motion, $\omega_\theta \equiv \dot{\theta}$? Give a few values of k for which the ratio is rational, that is, for which the path of the nearly circular motion closes back on itself.

Q10

A ladder of length ℓ and uniform mass density stands on a frictionless floor and leans against a frictionless wall. It is initially held motionless, with its bottom end an infinitesimal distance from the wall. It is then released, whereupon the bottom end slides away from the wall, and the top end slides down the wall (see Fig. 8.22). When it loses contact with the wall, what is the horizontal component of the velocity of the center of mass?

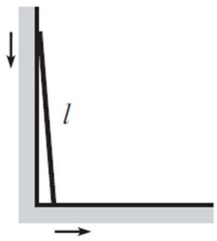

Fig. 8.22