BPHO & PUPC Class No. 20210612

Assignment 1 Kinematics 12/06/2021 **Due on 11:00 pm 19/06/2021**

Please try your best to finish the assignment. You may not be able to complete every question, however, please write as much as you can. It is required that all the answers are written independently by yourself.

Please print the documents, write your solutions to each question and scan it so that you can post yours to our study group directly. It is better for you to combine all your documents in a single .pdf profile. Other format of documents is acceptable as well, please compress them in a single file with your name.

This assignment is totally worth 30 points.

Good luck!

Name: ___________ Score: ____________

Q1(2 points)

Two planes set out at the same time from an aerodrome. The first flies north at $360\ km/h$, the second south-east at $300\ km/h$. After 40 minutes they both turn and fly towards each other. Calculate the distance of the meeting point from the aerodrome.

Q2(4 points)

The displacement of an object is determined by the following function:

$$s = 2t^3 - 9t^2 + 12t + 4$$

where s is the displacement in metres, and t the time elapsed in seconds. Determine
(a) the times when the object comes to rest,
(b) the time when the acceleration is zero,
(c) the object's velocity when its acceleration is zero,
(d) the object's accelerations when its velocity is zero.

Q3(4 points)

In this question, distances are measured in nautical miles and speeds in nautical miles per hour. A motorboat sets out at 2 p.m. from a point with position vector $-4\boldsymbol{i} - 5\boldsymbol{j}$ relative to a marker buoy (where $\boldsymbol{i}$ and $\boldsymbol{j}$ are two fixed perpendicular unit vectors) and travels at a steady speed of magnitude 6.4 in a straight line to intercept a ship S. The ship S maintains a steady velocity vector $\boldsymbol{i} + 4\boldsymbol{j}$ and at 3 p.m. is at a position $3\boldsymbol{i} - \boldsymbol{j}$ relative to the buoy. Find:
(a) the position vector of the ship at 2 p.m.
(b) the velocity vector of the motorboat
(c) the time of interception.

Q4(8 points)

A stone is projected with a speed u at a small angle α to the horizontal ground. It impacts on the ground at a distance d, having reached a maximum height h. At time t, its velocity makes an angle θ with the horizontal. Determine in terms of u and α:
(a) The time taken, T_g, to reach the ground and the distance d.
(b) The maximum height, h, reached.
(c)An expression for the radius of curvature of the trajectory, R, at height h.
(d) Express R in terms of d and h.
(e) Sketch a graph of $tan\theta$ against t, indicating the key points on the graph.
(f) Deduce, from (v), that there are no pair of points on the stone's trajectory, with velocities that are perpendicular if α less than 45°.

Q5(12 points)

An object with mass m is set free-falling at time $t = 0$, position $z = 0$ and $v_0 = 0$ in the Earth's gravitational field, with acceleration $-g$ along *z*-axis. The object in air feels a resistance opposite in direction to its speed: $\boldsymbol{F} = -\beta\boldsymbol{v}$**.**

(a) Give the expression for the terminal velocity v_T of the object in air at which the air resistance exactly compensates the gravitational force.

(b) Write down the forces acting on the object at a generic time *t*, and form a equation from Newton's 2nd Law of motion so that to get the expression for the velocity as a function of time *t*.

Hint: Some useful mathematical skills might be helpful

To solve the second order differential equation in the form:

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = f(t)$$

You have to find the general solution and particular solution so that the solution of the above second order differential equation could be written in the form:

$$x(t) = x_h(t) + x_p(t)$$

Where $x_h(t)$ *is the general solution and* $x_p(t)$ *is the particular solution*

The ways to find the general solution is to solve its homogeneous second order differential equation:

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0$$

The solution could always be written in the form:

$$x(t) = Ae^{r_1 t} + Be^{r_2 t}$$

Where A, B are constants and r_1, r_2 *are the real roots of the characteristic equation:*

$$ax^2 + bx + c = 0$$