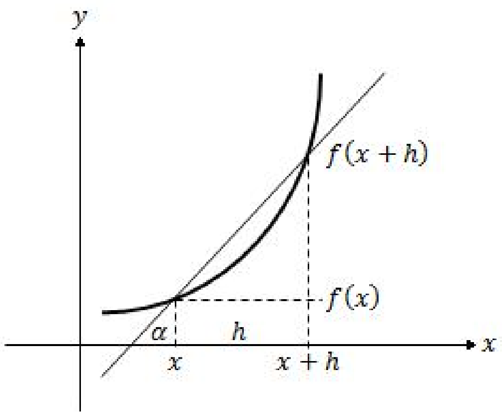
If f is a function of the independent variable x, the derivative of the function is defined by the equation:	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
	*$f(x+h) - f(x)$ is the height of the triangle.* *h is the base length of the triangle.* *The slope is:* $\tan\alpha = \frac{f(x+h) - f(x)}{(x+h) - x}$ *So when h tends to zero the expression become:* $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ *This is the slop of the tangent line to the function $f(x)$ at point x.*
***chain rule**: Suppose that: $y = y(u)$ and $u = u(x)$ then $\frac{dy}{dx}$ is defined by:*	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ $\quad du \neq 0$ *and* $dx \neq 0$
***multiplication rule**: If $f(x) = g(x) \cdot u(x)$ then f' is:*	$f'(x) = g'u + gu'$
***quotient rule**: If $f(x) = \frac{g(x)}{u(x)}$ then f' is:*	$f'(x) = \frac{g' \cdot u - g \cdot u'}{u^2}$ $\quad u \neq 0$
***Reciprocal rule**: If $f(x) = \frac{1}{u(x)}$ then f' is:*	$f'(x) = -\frac{u'}{u^2}$ $\quad u \neq 0$
***Addition rule**: If $f = f(x)$ and $g = g(x)$ and a and b are real numbers then $(af + bg)'$ is:*	$(af + bg)' = af' + bg'$
***Constant rule**: If $f(x)$ is a constant then f' is:*	$f' = 0$
If $f = f(x)$ then all the following notations for derivatives are valid:	*First derivative:* $\frac{df}{dx} \equiv f' \equiv \dot{f} \equiv f_x$ *Second derivative:* $\frac{d^2 f}{dx^2} \equiv \frac{d}{dx}\left(\frac{df}{dx}\right) \equiv f'' \equiv \ddot{f} \equiv f_{xx}$

$\frac{d}{dx}(c)=0$	$\frac{d}{dx}(cx)=c$	$\frac{d}{dx}(x^c)=cx^{c-1}$
$\frac{d}{dx}(c^x)=c^x\ln(c) \quad c>0$	$\frac{d}{dx}(x^x)=x^x(1+ln\,x)$	$\frac{d}{dx}(e^x)=e^x$
$\frac{d}{dx}\left(\frac{1}{x}\right)=\frac{-1}{x^2}$	$\frac{d}{dx}\left(\frac{1}{x^2}\right)=\frac{-2}{x^3}$	$\frac{d}{dx}\left(\frac{1}{x^n}\right)=\frac{-n}{x^{n+1}}$
$\frac{d}{dx}(\sqrt{x})=\frac{1}{2\sqrt{x}} \quad x>0$	$\frac{d}{dx}(\sqrt[3]{x})=\frac{1}{3\cdot\sqrt[3]{x^2}}$	$\frac{d}{dx}(\sqrt[n]{x})=\frac{1}{n\cdot\sqrt[n]{x^{n-1}}}$
$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)=\frac{-1}{2\sqrt{x^3}}$	$\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x}}\right)=\frac{-1}{3\cdot\sqrt[3]{x^4}}$	$\frac{d}{dx}\left(\frac{1}{\sqrt[n]{x}}\right)=\frac{-1}{n\cdot\sqrt[n]{x^{n+1}}}$
$\frac{d}{dx}(\ln x)=\frac{1}{x} \quad x>0$	$\frac{d}{dx}(x\cdot\ln x)=\ln x+1$	$\frac{d}{dx}(\log_c x)=\frac{1}{x\ln c} \quad c>0 \quad c\neq 1$
$\frac{d}{dx}\left(\frac{1}{lnx}\right)=\frac{-1}{x(\ln x)^2}$	$\frac{d}{dx}\left(\frac{1}{x\cdot\ln x}\right)=\frac{-(\ln x+1)}{(x\cdot\ln x)^2}$	$\frac{d}{dx}\left(\frac{1}{\log_c x}\right)=\frac{-1}{x\cdot\ln c\cdot(\log_c x)^2}$
$\frac{d}{dx}\left(\frac{1}{x+1}\right)=\frac{-1}{(x+1)^2}$	$\frac{d}{dx}\left(\frac{1}{(x+1)^2}\right)=\frac{-2}{(x+1)^3}$	$\frac{d}{dx}\left(\frac{1}{(x+1)^n}\right)=\frac{-n}{(x+1)^{n+1}}$
$\frac{d}{dx}\left(\frac{1}{\sqrt{x+1}}\right)=\frac{-1}{2\cdot\sqrt{(x+1)^3}}$	$\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x+1}}\right)=\frac{-1}{3\cdot\sqrt[3]{(x+1)^4}}$	$\frac{d}{dx}\left(\frac{1}{\sqrt[n]{x+1}}\right)=\frac{-1}{n\cdot\sqrt[n]{(x+1)^{n+1}}}$

$\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}} \quad \lvert x\rvert<1$	$\frac{d}{dx}\sinh^{-1}x=\frac{1}{\sqrt{1+x^2}}$
$\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}} \quad \lvert x\rvert<1$	$\frac{d}{dx}\cosh^{-1}x=\frac{1}{\sqrt{x^2-1}}$
$\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$	$\frac{d}{dx}\tanh^{-1}x=\frac{1}{1-x^2} \quad \lvert x\rvert<1$
$\frac{d}{dx}\cot^{-1}x=\frac{-1}{1+x^2}$	$\frac{d}{dx}\coth^{-1}x=\frac{1}{1-x^2} \quad \lvert x\rvert<1$
$\frac{d}{dx}\csc^{-1}x=\frac{-1}{x\sqrt{x^2-1}} \quad \lvert x\rvert>1$	$\frac{d}{dx}\mathrm{csch}^{-1}x=\frac{-1}{x\sqrt{x^2+1}}$
$\frac{d}{dx}\sec^{-1}x=\frac{1}{x\sqrt{x^2-1}} \quad \lvert x\rvert>1$	$\frac{d}{dx}\mathrm{sech}^{-1}x=\frac{-1}{x\sqrt{x^2-1}}$

Partial Derivatives definition: If f is a function of two variables or more $f = f(x, y)$, then the partial derivatives can be found according to both variables as: $\frac{\partial f}{\partial x}$ (y is kept constant) and $\frac{\partial f}{\partial y}$ (x kept constant)	$\frac{\partial f}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$ $\frac{\partial f}{\partial y} = \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$
Chain rule: for computing partial derivatives: If $f = f(x,y)$ is continuous and both derivatives exists and $x = x(r,s)$ and $y = y(r,s)$ then:	$\frac{\partial f}{\partial r} = \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial x}{\partial r}\right) + \left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial y}{\partial r}\right)$ $\frac{\partial f}{\partial s} = \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial x}{\partial s}\right) + \left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial y}{\partial s}\right)$
If $f = f(x,y)$ and their derivatives f_x, f_y are continuous in the range of the derivation then:	$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ $\quad (f_{xy} = f_{yx})$
If $f = f(x,y,z)$ and is differentiable in a range then the implicit differential is:	$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$
If $f = f(x,y,z)$ and the function is continuous in the domain, then the ∇ operator can be defined. The meaning of this operator is the gradient vector at the point (a).	$\nabla f(a) = \left(\frac{\partial f}{\partial x}(a) + \frac{\partial f}{\partial y}(a) + \frac{\partial f}{\partial z}(a)\right)$
Laplace operator in three dimentions: f should be twice differentiable in the domain.	$\Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$
Laplace operator in polar form of two variables $f = f(r,\theta)$:	$\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$
Laplace operator in spherical form $f = f(r,\theta,\varphi)$ θ – azimuth, φ – polar angle:	$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\varphi}\frac{\partial}{\partial \varphi}\left(\sin\varphi\frac{\partial f}{\partial \varphi}\right) + \frac{1}{r^2\sin^2\varphi}\frac{\partial^2 f}{\partial \theta^2}$
Second and higher derivatives notation: If we write $z = f(x,y)$, then the following symbols have the same meanings:	$\frac{\partial^2 z}{\partial x^2}$; $\frac{\partial^2 f}{\partial x^2}$; $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$; f_{xx}; z_{xx}

Integration by parts: *because* $d(uv) = udv + vdu$ *we can integrate both sides*:	$$\int \boldsymbol{u\,dv} = \boldsymbol{uv} - \int \boldsymbol{v\,du}$$
Example: *find the integral* $\int xe^x\,dx$	*Write this integral in the form* $\int udv$ $u = x$ *and* $dv = e^x dx$ *then* $du = dx$ *and* $v = e^x$ *The integration is*: $\int xe^x\,dx = xe^x - \int e^x\,dx$ $= xe^x - e^x + C = e^x(x-1) + C$
Integration by substitution: *If* f *is a continuous function, then*:	$$\int_a^b \boldsymbol{f(g(t))\,g'(t)dt} = \int_{g(a)}^{g(b)} \boldsymbol{f(x)\,dx}$$
Example: *find the integral* $\int_0^2 \frac{(2x+1)dx}{\sqrt{x^2+x+1}}$	*Substitute*: $u = x^2 + x + 1$ *Then* $du = (2x+1)dx$ $\int_1^7 \left(\frac{(2x+1)du}{\sqrt{u}(2x+1)}\right) = \int_1^7 \frac{du}{\sqrt{u}} = 2\sqrt{u}\,\Big\rvert_0^7 = 2(\sqrt{7}-1)$ *The new integration limits are*: $u(x=0) = 1$ *and* $u(x=2) = 7$ *Note*: *we could resubstitute the* $u = u(x)$ *value and leave the old integration limits.*
Integration contains the function and its derivative in the numerator:	$$\int \frac{\boldsymbol{f'(x)}}{\boldsymbol{f(x)}} = \boldsymbol{\ln\lvert f(x)\rvert}$$
Example: *find the integral* $\int \frac{\cos x}{\sin x + 2}dx$	*We see that* $\frac{d(\sin x + 2)}{dx} = \cos x$ *According to the above rule the integral is simple*: $\int \frac{\cos x}{\sin x + 2}dx = \ln\lvert\sin x + 2\rvert + C$
Example: *find the integral* $\int \frac{3x^2}{x^3+2}dx$	*Because*: $\frac{d(x^3+2)}{dx} = 3x^2$ *is the same as the numerator then the result of the integral is*: $\int \frac{3x^2}{x^3+2}dx = \ln\lvert x^3 + 2\rvert + C$

$\int a\ dx = ax$	$\int a \cdot f(x)\ dx = a\int f(x)\ dx$
$\int \phi(y)\ dx = \int \frac{\phi(y)}{y'}dy \qquad when\ \ y' = \frac{dy}{dx}$	$\int (u+v)\ dx = \int udx + \int vdx \qquad u = u(x)\ \ v = v(x)$
$\int u\ dv = u\int dv - \int v\,du = uv - \int v\,du$	$\int u\frac{dv}{dx}\ dx = uv - \int v\frac{du}{dx}\ dx$
$\int x^n\ dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$	$\int \frac{f'(x)dx}{f(x)} = \log f(x) \qquad d\,f(x) = f'(x)dx$
$\int \frac{1}{x}dx = \log\|x\|$	$\int \frac{f'(x)dx}{2\sqrt{f(x)}} = \sqrt{f(x)} \qquad df(x) = f'(x)dx$
$\int e^x\ dx = e^x$	$\int e^{ax}\ dx = \frac{e^{ax}}{a}$
$\int a^x\ dx = \frac{a^x}{\ln a} \qquad a > 0,\ \ a \neq 1$	$\int b^{ax}\ dx = \frac{b^{ax}}{a\log b}$
$\int a^x \log a\ dx = a^x$	$\int \log x\ dx = x\log x - x$
$\int \frac{dx}{x^2} = -\frac{1}{x}$	$\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) = -\frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right)$	$\int \frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) = \frac{1}{2a}\log\frac{a+x}{a-x}$
$\int \frac{dx}{x^2-a^2} = -\frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) = \frac{1}{2a}\log\frac{x-a}{x+a}$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right)$
$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left(x + \sqrt{x^2 \pm a^2}\right)$	$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\cos^{-1}\left(\frac{a}{x}\right)$
$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a}\log\left(\frac{a+\sqrt{a^2 \pm x^2}}{x}\right)$	
$\int \frac{dx}{x\sqrt{a+bx}} = \frac{2}{\sqrt{-2}}\tan^{-1}\sqrt{\frac{a+bx}{-a}} = \frac{-2}{\sqrt{a}}\tanh^{-1}\sqrt{\frac{a+bx}{a}} = \frac{1}{\sqrt{a}}\log\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}$	
$\int \sqrt{ax^2+c}\ dx = \frac{x}{2}\sqrt{ax^2+c} + \frac{c}{2\sqrt{a}}\log\left(x\sqrt{a} + \sqrt{ax^2+c}\right) \qquad a > 0$ $= \frac{x}{2}\sqrt{ax^2+c} + \frac{c}{2\sqrt{-a}}\sin^{-1}\left(x\sqrt{\frac{-a}{c}}\right) \qquad a < 0$	
$\int \frac{dx}{\sqrt{a+bx}\sqrt{c+ex}} = \frac{2}{\sqrt{-be}}\tan^{-1}\sqrt{\frac{-e(a+bx)}{b(c+ex)}}$	$\int \sqrt{\frac{1+x}{1-x}}\ dx = \sin^{-1}x - \sqrt{1-x^2}$
$\int \frac{dx}{\sqrt{a \pm 2bx + cx^2}} = \frac{1}{\sqrt{c}}\log\left(\pm b + cx + \sqrt{c}\sqrt{a \pm 2bx + cx^2}\right)$	$\int \frac{ax}{\sqrt{a \pm 2bx - cx^2}} = \frac{1}{\sqrt{c}}\sin^{-1}\frac{cx \mp b}{\sqrt{b^2+ac}}$
$\int \frac{x\ dx}{\sqrt{a \pm 2bx + cx^2}} = \frac{1}{c}\sqrt{a \pm 2bx + cx^2} - \frac{b}{\sqrt{c^3}}\log\left(\pm b + cx + \sqrt{c}\sqrt{a \pm 2bx + cx^2}\right)$	
$\int \frac{x\ dx}{\sqrt{a \pm 2bx - cx^2}} = \frac{1}{c}\sqrt{a \pm 2bx - cx^2} \pm \frac{b}{\sqrt{c^3}}\sin^{-1}\frac{cx \mp b}{\sqrt{b^2+ac}}$	