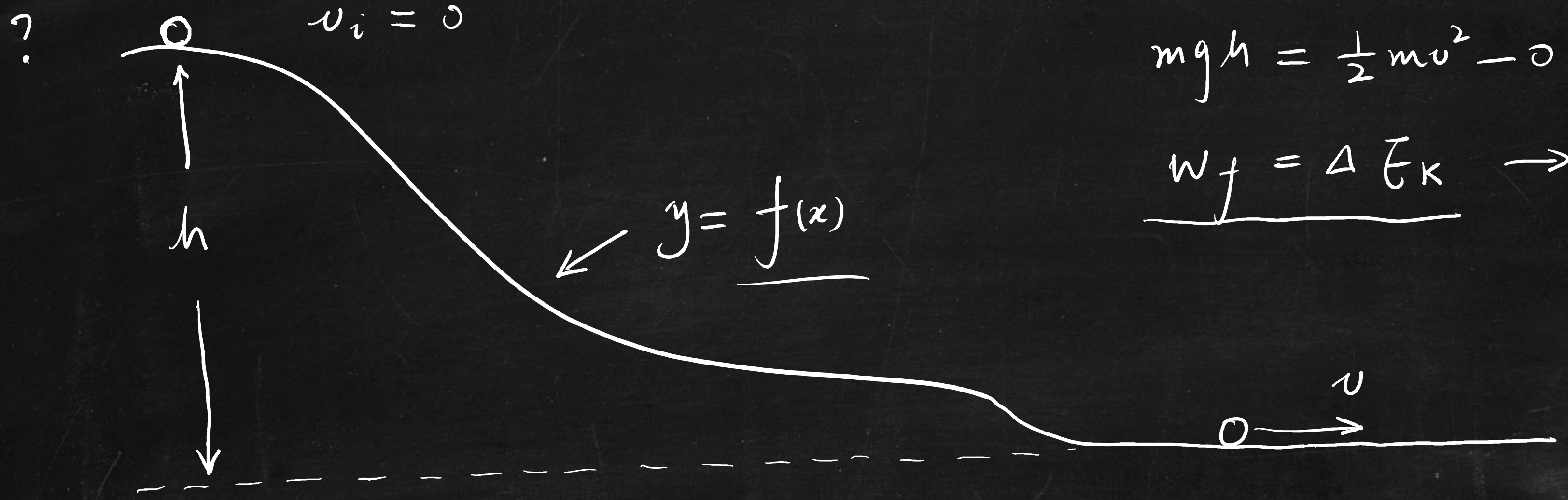



$$mgh = \frac{1}{2}mv^2 - 0 \qquad v = \sqrt{2gh}$$

$$\underline{W_f = \Delta E_K} \rightarrow \vec{F} = m\vec{a}$$

Mechanical Enegy

① G.P.E.

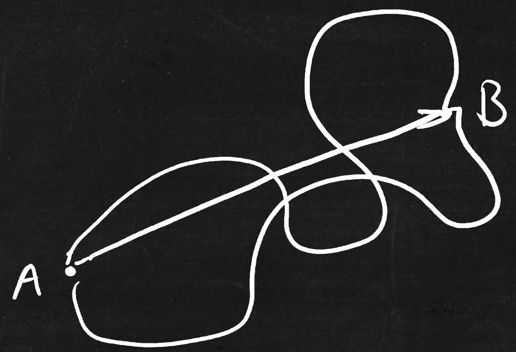
② K.E.

③ E.P.E

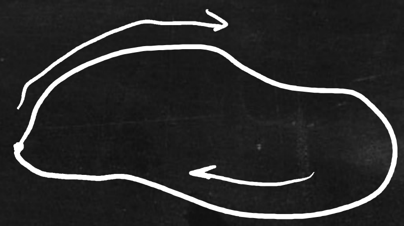
$$W_f = \Delta E_K$$

# Conservative Force

$$W = \int_{\vec r_A}^{\vec r_B} \vec F \cdot d\vec r$$




1° $\displaystyle \oint_{(l)} \vec F \cdot d\vec r = 0$



2° $\vec F = -\vec\nabla V$

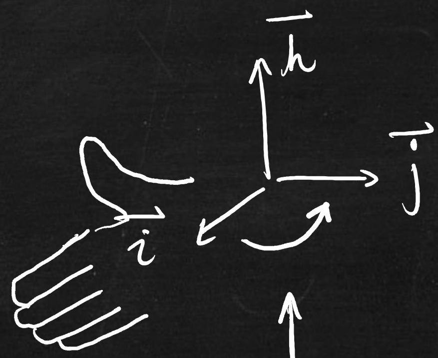
$$\vec\nabla = \frac{\partial}{\partial x}\vec i + \frac{\partial}{\partial y}\vec j + \frac{\partial}{\partial z}\vec k$$ gradient operator

3° $\vec\nabla \times \vec F = 0$

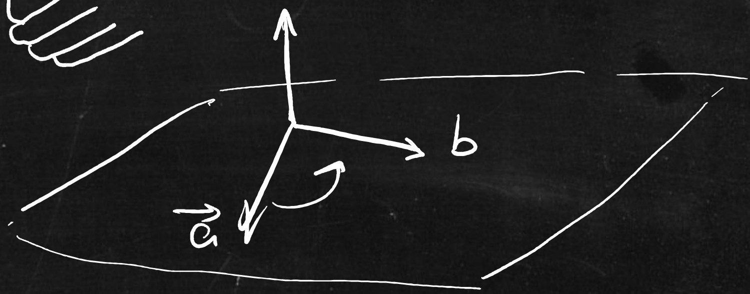
$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ scalar product

$\vec{a} \times \vec{b} = \vec{c}$ $|\vec{c}| = |\vec{a}||\vec{b}|\sin\theta$ right-hand rule

$\vec{i} \times \vec{j} = \vec{k}$




ex.


$\vec{r} \times \vec{F}$

$\vec{i} \times \vec{i} = 0$ $\vec{j} \times \vec{j} = 0$ $\vec{k} \times \vec{k} = 0$ $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

$i \times \vec{j} = \vec{k}$, $\vec{j} \times \vec{k} = \vec{i}$, $\vec{k} \times \vec{i} = \vec{j}$

$$y = f(x) = \sin x \qquad \frac{dy}{dx} = \cos x$$

$\partial$ d

$u = xy^2$ partial dirivative

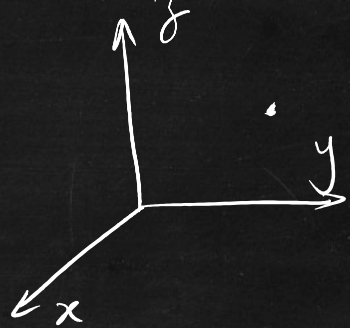
$$\frac{\partial u}{\partial x} = y^2 \qquad \frac{\partial u}{\partial y} = 2xy \qquad y = f(x) \quad u = xy^2 = x[f(x)]^2$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \qquad b = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \vec{i}\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \vec{j}\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \vec{k}\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= \vec{i}\,(a_yb_z - a_zb_y) - \vec{j}\,(a_xb_z - a_zb_x) + \vec{k}\,(a_xb_y - a_yb_x)$$

$$2^\circ \quad \vec{F} = -\vec{\nabla} V \qquad V = V(x, y, z)$$




$$-\vec{\nabla} V(x, y, z) = -\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right) V$$

$$= -\frac{\partial V}{\partial x}\vec{i} - \frac{\partial V}{\partial y}\vec{j} - \frac{\partial V}{\partial z}\vec{k}$$

$$\vec{F}_g = -mg\vec{k} \qquad V(x, y, z) = mgz$$

$$\cancel{-\frac{\partial V}{\partial x}\vec{i}} \cancel{-\frac{\partial V}{\partial y}\vec{j}} - \frac{\partial V}{\partial z}\vec{k} = -mg\vec{k} = \vec{F}_g$$

$$3^\circ \quad \vec{\nabla} \times \vec{F} = 0 \qquad \vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$\vec{F} = -2xy\vec{i} - x^2\vec{j} + 4\vec{k}$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2xy & -x^2 & 4 \end{vmatrix} = \vec{i} \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^2 & 4 \end{vmatrix} - \vec{j} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ -2xy & 4 \end{vmatrix} + \vec{k} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -2xy & -x^2 \end{vmatrix}$$

$$= 0 - 0 + (-2x - (-2x))\vec{k} = 0$$

Practice: A particle in 3-D has a potential energy given by

$$V = z^2 + y^3 + 2x^2y^2$$

Determin the equation of force $\vec{F}$ acting on the particle.

If the particle mous from origin $(0,0,0)$ to the position $(1,1,2)$, what is the change in kinetic energy assuming that only the force determined above is acting on the particle.

Solution: $\underline{V = z^2 + y^2 + 2x^2y^2}$

$$F = -\vec{\nabla} V = -\left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)V$$

$$= -\frac{\partial V}{\partial x}\vec{i} - \frac{\partial V}{\partial y}\vec{j} - \frac{\partial V}{\partial z}\vec{k}$$

$$F_x = -\frac{\partial V}{\partial x} = -2y^2 \cdot 2x = -4xy^2$$

$$F_y = -\frac{\partial V}{\partial y} = -(2y + 4x^2y)$$

$$F_z = -\frac{\partial V}{\partial z} = -2z$$

$$\vec{F} = -4xy^2\vec{i} - (2y + 4x^2y)\vec{j} - 2z\vec{k}$$

$V\,\underline{(0,0,0)} = 0$

$\downarrow$

$\underline{V\,(1,1,2)} = 7\ (J)$

<u>potential energy</u> $\uparrow$

kinetic energy $\downarrow$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \qquad \begin{cases} \dfrac{\partial F_y}{\partial x} = \dfrac{\partial F_x}{\partial y} \\[2ex] \dfrac{\partial F_z}{\partial y} = \dfrac{\partial F_y}{\partial z} \\[2ex] \dfrac{\partial F_z}{\partial x} = \dfrac{\partial F_x}{\partial z} \end{cases}$$

$$V(\vec{r}) = -\int F(\vec{r}\,')\,d\vec{r}\,' \qquad \text{(Definition of P.E.)}$$

$$F_E = \underbrace{\frac{1}{4\pi\varepsilon_0}}_{k} \frac{q_1 q_2}{r^2} \cdot \hat{r} \qquad \text{(Coloumb's Law)}$$

$\varepsilon_0$ — permitivity in vacuum

$$q \bullet \xrightarrow{\vec{r}} \bullet\, q_2$$

$$V_E(\vec{r}) = -\int_0^{\vec{r}} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \underbrace{\hat{r}}_{1} \cdot d\vec{r} \qquad d\vec{r} = (dr) \cdot \hat{r}$$

$$= -\int_{r_1}^{\vec{r}_2} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \, dr = \left| \boxed{\frac{1}{4\pi\varepsilon_0} \, \frac{q_1 q_2}{r_2}} \right| - \frac{1}{4\pi\varepsilon_0} \, \frac{q_1 q_2}{r_1}$$

$$U_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

Oscillatory Motion 振动

$e^{i\alpha} = \cos\alpha + i\sin\alpha$

Simple Harmonic Motion. $F_{Net} = -kx$

Hooke's Law: $F_s = -kx$

$F_{Net} = ma$ SHM: $-kx = ma = m\frac{d^2x}{dt^2}$

Homogenous 2nd ODE $m\frac{d^2x}{dt^2} + kx = 0$ $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

$r^2 + \frac{k}{m} = 0$ $r = \pm\sqrt{\frac{k}{m}}\, i$ $\omega = \sqrt{\frac{k}{m}}$

$x(t) = \underline{Ae^{i\sqrt{\frac{k}{m}}t} + Be^{-i\sqrt{\frac{k}{m}}t}}$ $Ae^{\alpha t} + Be^{\beta t}$

欧拉公式 $= \underline{C\cos\sqrt{\frac{k}{m}}t + D\sin\sqrt{\frac{k}{m}}t}$ $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0$

$= \underline{C\cos\omega t + D\sin\omega t}$ $ar^2 + br + c = 0$

$$x(t) = C\cos\omega t + D\sin\omega t$$

$$= \sqrt{C^2+D^2}\left(\cos\omega t \cdot \underbrace{\frac{C}{\sqrt{C^2+D^2}}}_{\cos\varphi} + \sin\omega t \underbrace{\frac{D}{\sqrt{C^2+D^2}}}_{\sin\varphi}\right)$$

角频率(圆)　angular frequency

$$= \underset{\substack{\uparrow \\ \text{amplitude}}}{A} \cos(\overset{\downarrow}{\omega} t+\varphi) = A\sin(\omega t+\varphi)$$

$\omega t+\varphi$: phase 相位

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t+\varphi)$$

$$a = \frac{dv}{dt} = -A\omega^2\cos(\omega t+\varphi)$$

$$x(t) = Ae^{i\omega t} + Be^{-i\omega t}$$

$$\frac{dx}{dt}, \quad \frac{dv}{dt}$$

$$F = -\frac{mg}{l}x \qquad k = \frac{mg}{l} \qquad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{l}} \qquad T = \frac{2\pi}{\omega}$$

$$K = \frac{1}{2}m\dot{x}^2 \qquad V = \frac{1}{2}kx^2 = -\int F\,dx$$

$$E = K + V = \frac{1}{2}kA^2$$