USIYPT Team 2 Prep

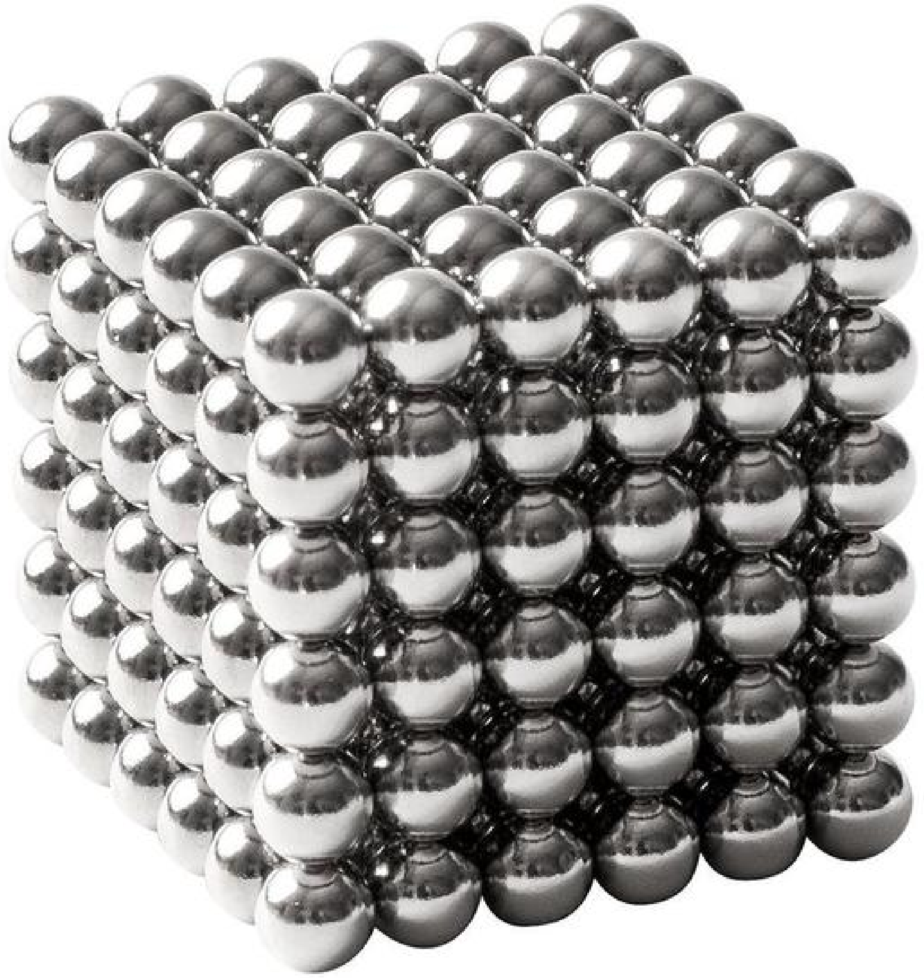
Understanding of the problem

A Magnetic Force Law

One of our first experiences with permanent magnets is that *like poles* repel and *opposite poles* attract. Charles-Augustin de Coulomb tried in vain to develop a simple magnetic force law, much like his electrostatic law, but these attempts ultimately failed since there is no magnetic corollary to charge. That said, for the simple case of spherical magnets one would expect a simple attractive force law.

For this problem you are to measure the attractive force between strong spherical magnets, which are free to rotate, as a function of the distance between their centers. Compare your measurements to the force predicted by classical electrodynamics as we now understand it.

Spherical magnets



Strong spherical magnets

- Not identical!
- More than 2!
- simple→general

Free to rotate

- 1. has angular acceleration
- 2. does not have angular acceleration
- 3. motionless

hypothesis

- The attractive force is inversely proportional to the distance between center of mass
- Analogy of law of universal gravitation and Coulomb Law

Reference

- https://totalelement.com/collections/sphere-magnets
- https://lcmagnet.en.made-in-china.com/product/vNoEOKfXcFhj/China-Strong-N52-Industry-Neodymium-Magnet-Ball-NdFeB-Sphere-Factory.html

Electromagnetism

…the part concerning our study

A brief summary of the related topics

- Electricity
  - Electric charge and electric field
  - Gauss’s Law
  - Electric potential
- Magnetism
  - Sources of magnetic field
  - Electromagnetic induction
- Maxwell’s equations

The fundamentals: Maxwell’s equations

- “The basic equations for all electromagnetism. All of electromagnetism is contained in this set of four equations”

- $\oint \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q}{\epsilon_0}$
- $\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$
- $\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi_B}{dt}$
- $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Equations Explained

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Electricity

A brief intro of Electrostatics,
Classical Electrodynamics

Basics

- Conceptual Analogy
  - Mech
    - mass **m**, acceleration **a**, height **h**, altitude △**h**, gravitational potential energy **G=mgh**
  - EM
    - quantity of electricity **q**, field strength **E**, electric potential **φ**, difference of potential △**φ=U**, electric potential energy **qφ**
- Point charge
  - Symbol: q, unit: Coulomb (C)
  - Essence: charge carrying particles, i.e. electrons, protons, etc.
  - Magnitude of charge in an electron: q_e = -1.602x10^-19 C
  - Charge in a proton: q_p = 1.602x10/^-19 C
- Electric field strength E:=F/q, attractive force $\mathbf{F} = q\mathbf{E}(\mathbf{r}) = -\boldsymbol{\nabla} E_p$

Coulomb's Law

- Condition
  - Between point charges
  - Static field
  - Vacuum

$$\mathbf{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\left|\mathbf{r}_2 - \mathbf{r}_1\right|^3}(\mathbf{r}_2 - \mathbf{r}_1) \qquad (1)$$

$$\varepsilon_0 = 8.8541878128(13) \times 10^{-12}$$

- Applying the Nabla operator $\rightarrow$ 2 of Maxwell equations

$$\begin{aligned}
\nabla \cdot \mathbf{E}(\mathbf{r}) &= \nabla \cdot \int \frac{1}{4\pi\epsilon_0}\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^3}\rho(\mathbf{r}')dV' \\
&= \frac{1}{4\pi\epsilon_0}\int \nabla \cdot \frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^3}\rho(\mathbf{r}')dV' \\
&= \frac{1}{4\pi\epsilon_0}\int 4\pi\delta(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')dV' \\
&= \frac{\rho(\mathbf{r})}{\epsilon_0}
\end{aligned}$$

$$\begin{aligned}
\nabla \times \mathbf{E}(\mathbf{r}) &= \nabla \times \int \frac{1}{4\pi\epsilon_0}\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^3}\rho(\mathbf{r}')dV' \\
&= \frac{1}{4\pi\epsilon_0}\int \nabla \times \frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^3}\rho(\mathbf{r}')dV' \\
&= \frac{1}{4\pi\epsilon_0}\int \mathbf{0}\rho(\mathbf{r}')dV' \\
&= \mathbf{0}
\end{aligned}$$

Gauss' Law

(from@小时)

积分形式

在空间中任意选取一个闭合曲面 $\mathcal{S}$，电场在这个曲面上从内向外的通量等于被曲面包围的总电荷量除以真空中的介电常数．

$$\oint_{\mathcal{S}} \mathbf{E}(\mathbf{r}) \cdot \mathrm{d}\mathbf{s} = \frac{1}{\epsilon_0} \int_{\mathcal{S}} \rho(\mathbf{r})\, \mathrm{d}V \tag{1}$$

作为另一种形象的理解，我们可以想象正电荷会以固定速率向周围释放或吸收一种不可压缩的流体，负电荷则吸收这种流体．释放和吸收的速率和电荷绝对值成正比，电场可以看作该流体的速度场．所以无论取什么形状的闭合曲面，单位时间流经曲面的总流量都只取决于曲面内部的电荷总量．

Current

- Definition

$$\boldsymbol{I} = \frac{\mathrm{d}q}{\mathrm{d}t} = n\boldsymbol{v}qS = \oiint_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S}$$

Capacitor

- Definition

$$C = \frac{Q}{U}$$

- Multiply resistance and capacitance, we get time, we call it the time constant of the circuit

$$RC = \frac{U}{I} \times \frac{Q}{U} = \frac{U}{I} \times \frac{It}{U} = t$$

$$i_C = C\frac{dU_C}{dt} \qquad U_R = i_C R = RC\frac{dU_C}{dt}$$

- Capacitive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \ (unit{:}\, \Omega)$$

$$U_C = E\left(1 - e^{\frac{-t}{RC}}\right)(unit{:}\, V)$$

Magnetism

Sources of magnetic field
Electromagnetic induction

Physical quantities definition

- Magnetic field **B** (tesla)
  - a vector field in the neighbourhood of a magnet, electric current, or changing electric field, in which magnetic forces are observable.
  - An equivalent way: $\boldsymbol{dF} = I\ \boldsymbol{dl}\ \times \boldsymbol{B}$
  - $\boldsymbol{dF}$ is the infinitesimal force acting on a differential length $\boldsymbol{dl}$ of the wire

Key rules

- $d\boldsymbol{F} = I\, d\boldsymbol{l}\ \times \boldsymbol{B}$
- (or when B is uniform): $\boldsymbol{F} = I\boldsymbol{l}\ \times \boldsymbol{B}$
  - l is the length of wire
  - The magnitude could be found by $\boldsymbol{F} = IlB\ sin\theta$
- $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$
  - (a moving charge q with a velocity of v)
  - The magnitude of which is $F = qvBsin\theta$

Key rules (2)

- Ampere's law: the line integral of the magnetic field B around any closed loop is equal to $\boldsymbol{\mu_0}$ times the to the total net current I_{encl} enclosed by the loop
- $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I_{encl}$
- The Biot-Savart law
- $d\boldsymbol{B} = \frac{\mu_0 I}{4\pi} \frac{d\boldsymbol{l} \times \hat{\boldsymbol{r}}}{r^2}$
- dB is the contribution to the total field at some point P due to a current I along an infinitesimal length dl of its path, and r is the unit vector along the direction of the displacement vector r from dl to P. The total field B will be the integral over all dB

future plan

• theoretical part

a.Physics：electromagnetism （9.7-10.13）

1. 《大学物理学》ch.19-20
2. 张梅老师电磁学教学视频
3. 老师、学长学姐资源
4. reading related papers

b. Math: calculus

1. MIT courses（Bilibili）

https://www.bilibili.com/video/BV1mx411S7M3?share_source=copy_web

c. others

1. Matlab
2. LaTex

Experiment apparatus

Apparatus

1. Spherical magnets of different sizes
2. Force sensors (redesigned)
3. Caliper (游标卡尺)
4. Meter ruler

Procedure

- 1. put magnets on redesigned force sensors
- 2. fixed the sensors positions, measures the magnets' radius and distance
- 3. measure the force between the magnets
- 4. adjust the magnets' radius and distance to obtain multiple data
- 5. Analyze the data

oral defense

- debaters
- mock debate