Magnetic Moment and Modeling

USIYPT presentation based on thesis

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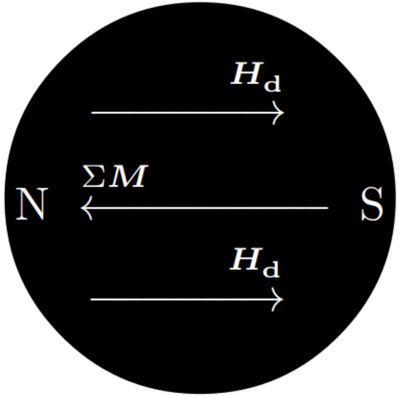
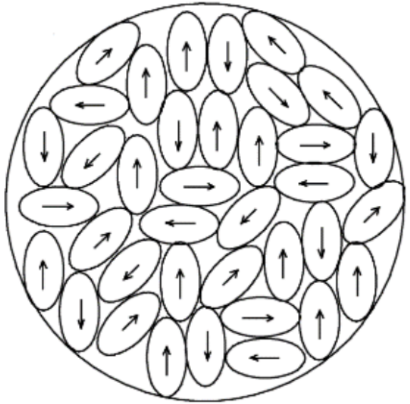
PARAMETERS

Terms	Parameters	Terms	Parameters
Diameter	N35	**Remanence**	1.17 T
Galvanization	Nickel + Argentum	**Volume**	16.2 cm^3
Density	7.4 g cm^{-3}	**Relative permeability**	1.09

PARAMETERS





- ▲ Figure (a) shown left: Magnetization vector of the #N35 sphere itself;
- ▲ Figure (b) shown right: The demagnetization curves of typical *NdFeB* and *Samarium Cobalt* permanent magnets that we have used.

REMANENCE *Br*

"Br"$\rightarrow$ "m"

Using thermodynamic methods, only the changes from the initial equilibrium state to the final state equilibrium state are studied. The work W_{env} that the outer-environment does is mostly translated into the inherent energy

$$A(\mathcal{B}\boldsymbol{r}) = \int \delta A = \int_0^B H_{\mathrm{total}} \mathrm{d}B - \frac{1}{2}\mu_0\mu_r H_0^2 = \mu_{\mathrm{sphere}} \int_0^M H \mathrm{d}M$$

REMANENCE *Br*

By the First Law of Thermodynamics, we have

$$\mathrm{d}U = T\mathrm{d}S + \delta A = T\mathrm{d}S - P_0\mathrm{d}V + \mu_r\mu_0 H\mathrm{d}M$$

Since the magnetization process was completed during an isochoric process,

$$P_0\mathrm{d}V = 0$$

And, the internal energy is related to the magnetic induction force received, so this internal energy should be counted when dealing out the potential. Then, the actual potential energy

$$U_{\mathrm{total}} = V_{\mathrm{mag}}(\mathbf{B}, \mathbf{m}, \mathbf{r}) + \boxed{A_{\mathrm{char}}(\mathrm{Br})}$$

Magnetic Moment “**m**”

$$
\begin{aligned}
\mathbf{m} &:= \textstyle\sum_i \mathbf{M}_i \,\mathrm{d}V \approx \chi_\mathrm{m}\,\mathbf{B}/\mu_0 \\
&= \int_0^\pi \Big\{ \int_0^{2\pi} \Big[\int_0^R \mathbf{M}(\boldsymbol{h}, \boldsymbol{\mu_B}, \boldsymbol{\mu_S})\ \mathrm{d}r \Big] \mathrm{d}\varphi \Big\}\, \mathrm{d}\theta
\end{aligned}
$$

($\boldsymbol{\mu_B}$ is *Bohr magneton*, and $\boldsymbol{\mu_S}$ is *Magnetic moment of electron spin*)

Magnetic Moment "**m**"

$$\begin{aligned}\phi(\mathbf{r}) &= \left\{\sum_{\lambda=0}^{\infty}\left[A_\lambda r^\lambda + B_\lambda r^{-(\lambda+1)}\right]\right\}\left[\sum_{\epsilon_i}^{\epsilon_f} P_\epsilon(\cos(\theta))\right] \\ &= \frac{1}{4\pi}\int_V \frac{\rho_{\mathrm{m}}(\mathbf{r}\prime)}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}V(\mathbf{r}')\end{aligned}$$

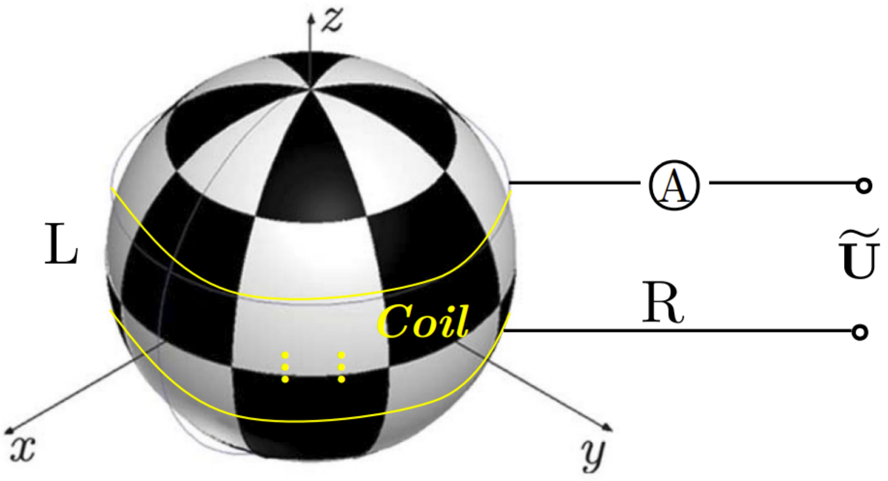
$$\begin{cases}\phi_{\mathrm{m}1} \to 0 \text{ for } r \to \infty \\ \phi_{\mathrm{m}1} = \phi_{\mathrm{m}2} \text{ for } r = R \\ \left(\frac{\partial \phi_{\mathrm{m}}}{\partial n}\right)_1 - \left(\frac{\partial \phi_{\mathrm{m}}}{\partial n}\right)_2 = -M_0\cos(\theta) \text{ for } r = R \\ \phi_{\mathrm{m}2} \neq \infty \text{ for } r = 0\end{cases}$$

$$\mathbf{m} = \iiint_{\mathrm{vol}} \mathbf{M}_i\,\mathrm{d}V = \frac{3}{4}\pi R^3 \mathbf{M}_{\mathrm{net}} = \sum_i I\Delta\mathbf{S}_{\mathrm{layer}(i)}$$

How To Obtain"**m**"?

Our instrument consists of a *student AC power supply*, a set of *copper coils*, a *voltmeter* with its peak voltage of $6\ \mathrm{V}$, a combination of *ammeters*, some *conductor wires*, a $5\ \Omega$ *protection resistance*, a *meter stick*, and two powerful *magnet spheres*. Specifically, the magnets we've used in our experiments are ball-shaped #N35 neodymium-iron-boron permanent magnets, measuring $1.57\ \mathrm{cm}$ in radius.

How To Obtain "**m**"?



▲ Figure (c): Schematic diagram of the experimental apparatus. (screenshotted from ***Modeling of Spherical Magnet Arrays Using the Magnetic Charge Model*** *B. vanNinhuijs, T. E. Motoasca, B. L. J. Gysen, and E. A. Lomonova Eindhoven University of Technology*; labeled accessorial apparatus)

How To Obtain "**m**"?

(1) Compute the mutual inductance $\rightarrow$ $L = \frac{\mu_r}{25}\frac{D-d}{D+d}N^2 = 23\ \mathrm{H} = 23\ \mathrm{VAs}^{-1}$

$$\xi_{\text{ind}} := LI$$

(2) Compute the induced current $\rightarrow$ $\Rightarrow \sum I = \frac{\xi_{\text{ind}}}{L} \approx \frac{1}{L}\left\langle\frac{\mathrm{d}\Phi}{\mathrm{d}\tau}\right\rangle_{\tau\in(0,2\pi)} = \frac{B\Delta S}{LT}$

(3) Compute the magnetic moment $\rightarrow$

$$\begin{aligned} m_{\text{total}} &= I_{\text{net}}\left[\textstyle\sum_i \Delta S_{\text{layer}(i)}\right]_0^{N_{\text{coil}}} \\ &= \frac{B_{(\text{in})}}{LT}\left(\int \mathrm{d}S\right)\left\{\lim_{\Delta r\to 0}\left[\pi N_{\text{coil}}(R-\Delta r)^2\right]\right\} \\ &= \frac{\mathcal{B}r}{LT}(4\pi R^2)(N\pi R^2) \approx 0.86\ \mathrm{Am}^2 \end{aligned}$$

MODELING PROCEDURE

Create parameters and functions

1

Establish static model

2

Make it work

3

Export the results of the operation

4

Examine the results

5

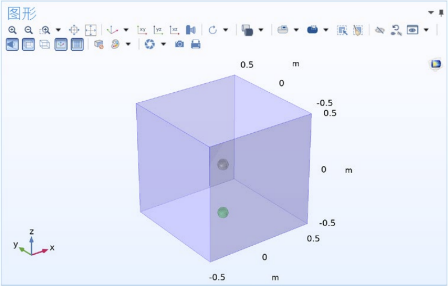
Create parameters and functions

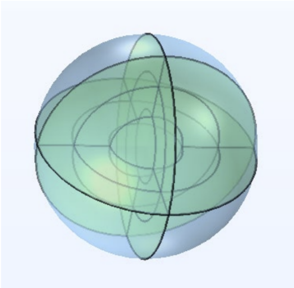
名称	表达式	值	描述
L	10[cm]	0.1 m	移动参数
mu_r	1.09	1.09	相对导磁率
B0	1.17[T]	1.17 T	剩磁（固有磁感应强度）
B_analytic	((3*mu_r)/(mu_r+2))*B0	1.2382 T	导磁球内部场的解析
dm	7.4[g/cm^3]	7400 kg/m³	磁球密度
r0	1.57[cm]	0.0157 m	半径，导磁球

属性	变量	表达式	单位
热膨胀系数	alpha_i...	alpha_p(pA,T)	1/K
平均摩尔质量	Mn	0.02897	kg/mol
本体黏度	muB	muB(T)	Pa·s
相对磁导率	mur_is...	1	1
相对介电常数	epsilon...	1	1
动力黏度	mu	eta(T)	Pa·s
比热率	gamma	1.4	1
电导率	sigma_i...	0[S/m]	S/m
恒压热容	Cp	Cp(T)	J/(kg·K)
密度	rho	rho(pA,T)	kg/m³
导热系数	k_iso ;...	k(T)	W/(m·K)
声速	c	cs(T)	m/s

▲ Table (b); (c): The COMSOL parameter list; the one of air column.

Establish static model

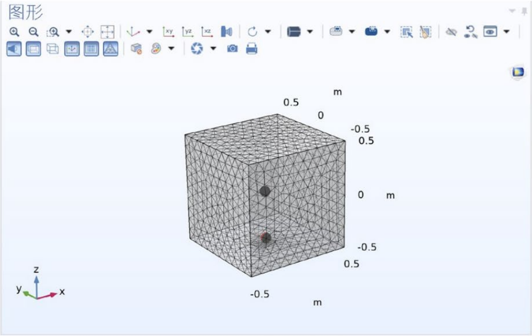


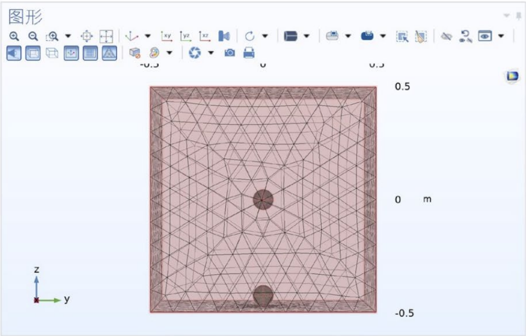


▲ Figure (d) shown left: A list of component models, comp1, including stable physical field, air gap, magnetic spheres, material definition, etc.

▲ Figure (e) shown right: The sphere is magnetized in divided layers.

▲ Figure (f) shown in the middle: Meshed rendering of experimental set-ups.

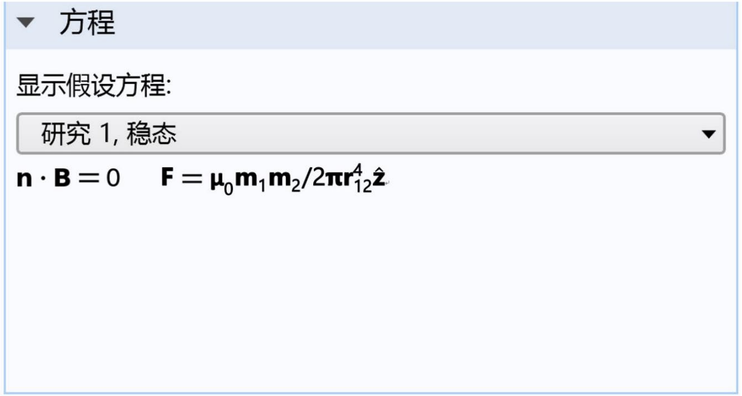




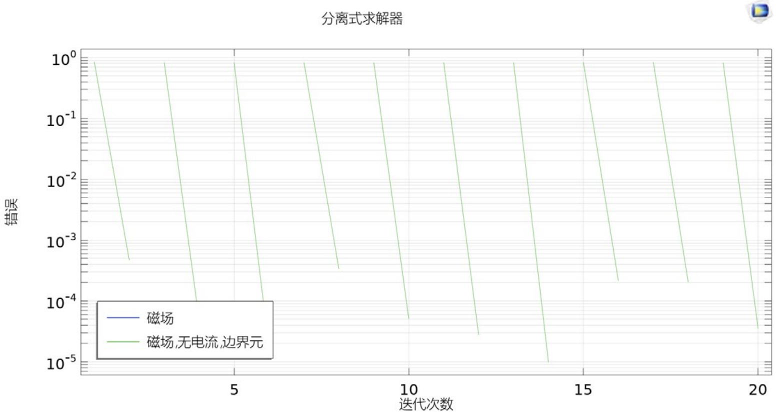
▲ Figure (g): The magnetic sphere and the air cuboid with quadrilateral grids.

Make it work

Rapid iteration ... $\rightarrow$ Approach ideal results



▲ Figure (i): Steady-state equations: boundary conditions and expression of F_{12}.



▲ Figure (i): The Convergence Trend Lines of magnetic flux density modulus.

Examine the results

(Roughly)

F_{12}[N]	x_{12}[cm]
0	15.131
0.01	10.621
0.02	6.403
0.03	5.411
0.05	4.252
0.08	4.722
0.11	3.576
0.21	3.553

F_2	x_2	in m^4
0	14.95	0.00049953
0.01	10.8	0.00013605
0.02	6.4	1.6777E-05
0.03	5.2	7.3116E-06
0.05	4.35	3.5806E-06
0.08	4.3	3.4188E-06
0.11	3.65	1.7749E-06
0.21	3.05	8.6537E-07

▲ Table (d): Some x-values taken from the experimental data and the corresponding F.

Citation

4 Citation

[1] PhET interactive simulations, University of Colorado at Boulder.

[2] Roald K. Wangsness, Electromagnetic Fields, 2nd edition (Wiley, New York, NY, 1986), p. 494.

[3] Horace Lamb, Hydrodynamics 16th edition (Dover, New York, 1993) Sec. 337, pp. 597-599.

[4] Magnetic Properties, Integrated Magnetics, http://www.intemag.com/magnetic properties.html.

[5] Shuohong Guo, Electrodynamics 2nd edition.

[6] David. J. Griffiths, Electrodynamics 4th edition.