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Measuring the Forces Between Magnetic Dipoles

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We describe a simple undergraduate lab in which students determine how the force between two magnetic dipoles depends on their separation. We consider the case where both dipoles are permanent and the case where one of the dipoles is induced by the field of the other (permanent) dipole. Agreement with theoretically expected results is quite good.

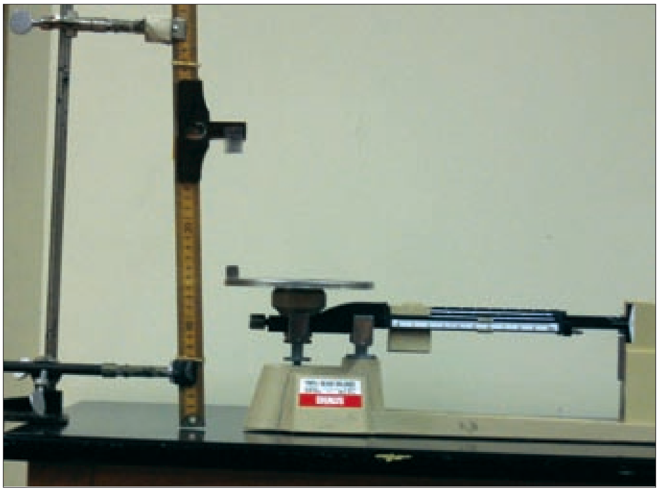

Fig. 1. The materials required are only very basic equipment.

Students have experience with the interaction between two magnetic dipoles, experience that dates back to early elementary school. However, even within a calculus-based introductory physics class it is uncommon to specifically detail the way that interaction depends on the separation of the dipoles. Due to the prevalence of well-explained inverse-square forces in introductory physics, it is little surprise that students often form incorrect assumptions about the force between dipole magnets. After all, inverse-square forces get weaker with increasing distance, and that's what happens with magnets, so since students are often not told differently, they may suppose that this force too is inverse square.

Several experimental arrangements to enable undergraduate students to measure a magnetic force have been proposed over the years,[1-8] with some of the earliest interpreting their results in terms of an inverse-square force.[1-3] We wish to suggest an approach that requires only very basic equipment, allows one to also measure the force between a permanent magnet and a small piece of a ferromagnetic material, and gives extremely good agreement with theoretical expectations for the distance dependence of the magnetic force.

Equipment

Our instrument consists of a triple-beam balance, a meter stick (or half-meter stick), and two powerful magnets, as seen in Fig. 1. The magnets we've used in our experiments are cube-shaped neodymium-iron-boron Magcraft permanent magnets, measuring 10.27 mm on a side. Based on the manufacturer's specifications, we expect each magnet to have a dipole moment of approximately 1.1 A·m^2. Such magnets are quite powerful and also materially fragile, so care must be taken to prevent injury to students and damage to the magnets.

One magnet is secured with tape near the edge of

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the balance pan. If the magnets are kept near the edge of the balance pan, we find only a negligible interaction between the magnets and the pan itself. Near the center of the pan, however, the magnets will interact significantly with the magnetic damping system of the balance. The other magnet is taped onto a support directly overhead (see Fig. 1). The support is bound to the meter stick with rubber bands but is capable of being moved up and down the length of the meter stick. Though we've used wooden supports, they could be made of any nonferromagnetic material. The edge of the pan is not perfectly horizontal even when in balance, so the meter stick should not be oriented vertically but should instead be inclined at a slight angle so that the faces of the magnets are parallel to each other.

Measuring the Magnetic Force

We vary the distance r between the centers of the magnets by sliding the support on the meter stick. For each separation distance, we can measure the force from the reading of the balance. The scale reports a mass reading based on the assumption that only the gravitational force and the normal force of the balance pan act on the object being measured. Since there is an additional magnetic force, the force applied by the balance pan to the object on the pan is given by

$$F_{\text{balance}} = mg \pm F_{\text{mag}}, \tag{1}$$

where F_{mag} is the magnetic force between the two magnets, m is the mass of the magnet on the pan, and g is the acceleration due to gravity. The sign in Eq. (1) is positive when the magnets are attracting each other and negative when they are repulsive. (We find it much easier to take data with the magnets in a repulsive orientation because the equilibrium of the balance pan is very unstable when the attractive force is large.) The "apparent mass" m_{app} read from the balance can be related to the actual mass of the object on the balance pan since

$$m_{\text{app}} = \frac{F_{\text{balance}}}{g} = m \pm \frac{F_{\text{mag}}}{g}. \tag{2}$$

After reading the apparent mass of the magnet, we can calculate the strength of the magnetic force between the two magnets from

$$F_{\text{mag}} = |\, m_{\text{app}} - m \,|\, g. \tag{3}$$

Two Permanent Dipoles

If each of our permanent magnets were a uniformly magnetized sphere, the magnetic field produced by each magnet would be equivalent to that of a point dipole located at the magnet's center.[9] Since the magnets we used are cubes instead of spheres, such a simplification is only an approximation for our system, an approximation that becomes more inaccurate as one considers magnetic fields very close to the magnet. However, we find that within the range of separation distances probed in our experiments, the data are well explained by approximating each magnet as a point dipole located at its center.

Given that approximation, what kind of force measurements should we expect to observe? The strength of the magnetic field a distance r away from a dipole (along the dipole axis) is given by

$$B = \frac{\mu_0 m_1}{2\pi r^3}, \tag{4}$$

where m_1 is the magnetic dipole moment. The potential energy of a second dipole aligned either parallel or anti-parallel to the axis of the first dipole is

$$U = \pm m_2 B = \pm \frac{\mu_0 m_1 m_2}{2\pi r^3}, \tag{5}$$

where m_2 is the magnetic dipole moment of the second magnet. The force acting between the two magnets therefore has a magnitude of

$$F_{\text{mag}} = \left|\frac{dU}{dr}\right| = \frac{3\mu_0 m_1 m_2}{2\pi r^4}. \tag{6}$$

When both magnets are permanent dipoles, we should expect the force between them to vary as $1/r^4$—halving the separation distance should increase the strength of the force by a factor of 16.

Figure 2 shows a typical data set obtained using this equipment. The line is the best power-law fit: $F_{\text{mag}} = (1.21 \times 10^{-6})\, r^{-4.04}$. The agreement with the expected distance dependence is quite satisfactory. If we constrain the exponent to be exactly -4 in the fit, we obtain $F_{\text{mag}} = (1.37 \times 10^{-6})\, r^{-4}$. Assuming the two magnets to have essentially the same value of magnetic dipole moment, we can obtain an experimental measurement of that dipole moment from Eq. (6):

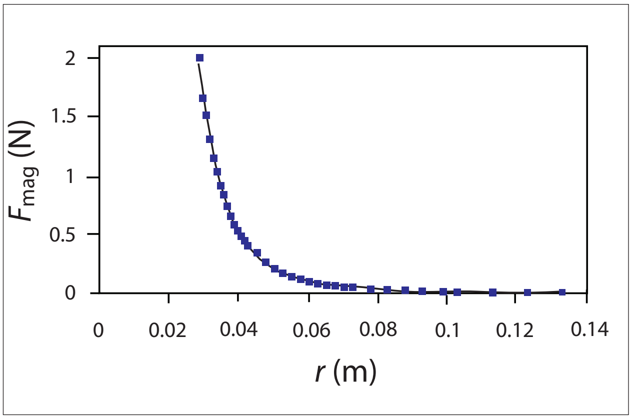

Fig. 2. Force between two permanent magnets as a function of their separation distance. The solid line is the best power-law fit.

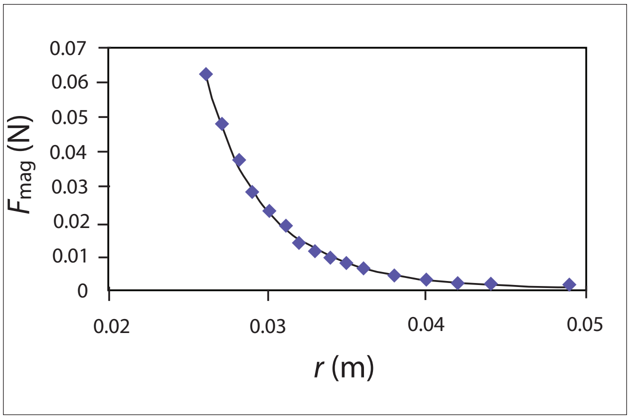

Fig. 3. Force between a permanent magnet and a pack of eight copper-clad steel BBs as a function of their separation distance. The solid line is the best power-law fit.

$$m_1 = m_2 = \sqrt{\frac{2\pi}{3\mu_0} \times 1.37 \times 10^{-6}} = 1.5 \ \ \mathrm{A \cdot m^2}, \quad (7)$$

which is reasonably consistent with the manufacturer's specifications for our magnets.

If One of the Dipoles Is Induced

We have also used this experimental arrangement to investigate the force between a magnet and a nonmagnetized ferromagnetic object. For this case, the upper magnet is replaced with eight copper-coated steel BBs taped into a roughly cubic arrangement. We chose BBs primarily because they were readily available—we had a box of them in the lab. The main advantage of the BBs is that one can then arrange any number of them to choose the size and shape of the ferromagnetic object without needing to machine anything. However, any small piece of iron or steel should suffice.

The alignment of dipoles in the steel will produce a magnetization that in general has a notably nonlinear dependence on the applied magnetic field. However, as long as the applied field is not large enough to saturate the magnetization, the relationship can somewhat crudely be approximated as a linear one:

$$M \approx \chi_\mathrm{m} \frac{B}{\mu_0}, \quad (8)$$

where χ_m is an "effective" magnetic susceptibility of the steel. Think about it as a best-fit line to the true magnetization curve. Using Eq. (4), we can write the magnetization of the steel in terms of the dipole moment of the permanent magnet as

$$M \approx \frac{\chi_\mathrm{m} m_1}{2\pi r^3}. \quad (9)$$

The magnetic dipole moment of the steel is just the magnetization times the volume V of steel, which for our sample is $V \approx 3.33 \times 10^{-7} \mathrm{m}^3$. Using that to obtain m_2 in Eq. (6) leads to an expression for the force between the permanent magnet and the induced magnet:

$$F_\mathrm{mag} = \frac{3\chi_\mathrm{m}\mu_0 m_1^2 V}{4\pi^2 r^7}. \quad (10)$$

Since the magnetic dipole induced in the steel will be in the same orientation as that of the permanent magnet, the force between them will always be attractive.

Figure 3 shows a typical data set obtained for the force between a permanent magnet and the steel BBs. The line is the best power-law fit: $F_\mathrm{mag} = (4.68 \times 10^{-13})\ r^{-7.02}$. The agreement with the expected distance dependence is again quite satisfactory. If we constrain the exponent to be exactly -7 in the fit, we find $F_\mathrm{mag} = (5.01 \times 10^{-13})\ r^{-7}$. Using our experimentally obtained value for m_1, we can obtain an experimental measurement of the effective magnetic susceptibility of the steel:

$$\chi_\mathrm{m} = \frac{4\pi^2}{3\mu_0 m_1^2 V} \times 5.01 \times 10^{-13} \approx 7. \quad (11)$$

Though the magnetic susceptibility of a ferromagnetic material is not truly a constant independent of applied field [as we have supposed in Eq. (8)], we find that our data are adequately explained by treating the susceptibility as approximately constant within our range of applied magnetic fields.

Conclusion

We incorporated the experiment for two permanent dipoles into the second calculus-based introductory course last summer. The biggest problems we encountered were magnets that weren't secured in place well enough and the fact that many students took the vast majority of their data at relatively large separation distances. As a result, the power law fits for their data weren't very accurate. We conclude that one of the key things to stress in presenting this lab for a class is that they need to be told to take most of their data in a range where there is a significant magnetic force to be measured.

In summary, a direct measurement of the magnetic force between two dipoles allows us to observe a distance dependence that is quite distinct from "inverse-square" behavior. The specific dependence observed is consistent with theoretical expectations. This is true not only for a dipole-dipole interaction but also a dipole-induced dipole interaction. Moreover, we can obtain from the data both a value for a permanent magnet's dipole moment and an effective value for the magnetic susceptibility of a ferromagnetic material. All of this can be accomplished within the context of an undergraduate lab experience.

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