Electromagnetism

…the part concerning our study

A brief summary of the related topics

- Electricity
  - Electric charge and electric field
  - Gauss's Law
  - Electric potential
- Magnetism
  - Sources of magnetic field
  - Electromagnetic induction
- Maxwell's equations

The fundamentals: Maxwell's equations

- "The basic equations for all electromagnetism. All of electromagnetism is contained in this set of four equations"

- $\oint \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q}{\epsilon_0}$
- $\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$
- $\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi_B}{dt}$
- $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Equations Explained

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Electricity

A brief intro of Electrostatics,
Classical Electrodynamics

Basics

- Conceptual Analogy
  - Mech
    - mass **m**, acceleration **a**, height **h**, altitude △**h**, gravitational potential energy **G=mgh**
  - EM
    - quantity of electricity **q**, field strength **E**, electric potential **φ**, difference of potential △**φ=U**, electric potential energy **qφ**
- Point charge
  - Symbol: q, unit: Coulomb (C)
  - Essence: charge carrying particles, i.e. electrons, protons, etc.
  - Magnitude of charge in an electron: q_e = -1.602x10^-19 C
  - Charge in a proton: q_p = 1.602x10/^-19 C
- Electric field strength E:=F/q, attractive force $\mathbf{F} = q\mathbf{E}(\mathbf{r}) = -\boldsymbol{\nabla} E_p$

Coulomb's Law

- Condition
  - Between point charges
  - Static field
  - Vacuum

$$\mathbf{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^3}(\mathbf{r}_2 - \mathbf{r}_1) \qquad (1)$$

$$\varepsilon_0 = 8.8541878128(13) \times 10^{-12}$$

- Applying the Nabla operator $\rightarrow$ 2 of Maxwell equations

$$\begin{aligned}\nabla \cdot \mathbf{E}(\mathbf{r}) &= \nabla \cdot \int \frac{1}{4\pi\epsilon_0}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\rho(\mathbf{r}')dV' \\ &= \frac{1}{4\pi\epsilon_0}\int \nabla \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\rho(\mathbf{r}')dV' \\ &= \frac{1}{4\pi\epsilon_0}\int 4\pi\delta(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')dV' \\ &= \frac{\rho(\mathbf{r})}{\epsilon_0}\end{aligned}$$

$$\begin{aligned}\nabla \times \mathbf{E}(\mathbf{r}) &= \nabla \times \int \frac{1}{4\pi\epsilon_0}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\rho(\mathbf{r}')dV' \\ &= \frac{1}{4\pi\epsilon_0}\int \nabla \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\rho(\mathbf{r}')dV' \\ &= \frac{1}{4\pi\epsilon_0}\int \mathbf{0}\rho(\mathbf{r}')dV' \\ &= \mathbf{0}\end{aligned}$$

Gauss' Law

(from@小时)

积分形式

在空间中任意选取一个闭合曲面 $\mathcal{S}$，电场在这个曲面上从内向外的通量等于被曲面包围的总电荷量除以真空中的介电常数.

$$\oint_{\mathcal{S}} \mathbf{E}(\mathbf{r}) \cdot \mathrm{d}\mathbf{s} = \frac{1}{\epsilon_0} \int_{\mathcal{S}} \rho(\mathbf{r}) \,\mathrm{d}V \tag{1}$$

作为另一种形象的理解，我们可以想象正电荷会以固定速率向周围释放或吸收一种不可压缩的流体，负电荷则吸收这种流体．释放和吸收的速率和电荷绝对值成正比，电场可以看作该流体的速度场．所以无论取什么形状的闭合曲面，单位时间流经曲面的总流量都只取决于曲面内部的电荷总量.

Current

- Definition $$\boldsymbol{I} = \frac{\mathrm{d}q}{\mathrm{d}t} = n\boldsymbol{v}qS = \oiint_S \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S}$$

Capacitor

- Definition $$C = \frac{Q}{U}$$
- Multiply resistance and capacitance, we get time, we call it the time constant of the circuit $$RC = \frac{U}{I} \times \frac{Q}{U} = \frac{U}{I} \times \frac{It}{U} = t$$

$$i_C = C\frac{dU_C}{dt} \qquad U_R = i_C R = RC\frac{dU_C}{dt}$$

- Capacitive reactance $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \ (unit\colon \Omega)$$

$$U_C = E\left(1 - e^{\frac{-t}{RC}}\right) (unit\colon V)$$

Magnetism

Sources of magnetic field
Electromagnetic induction

Physical quantities definition

- Magnetic field **B** (tesla)
  - a vector field in the neighbourhood of a magnet, electric current, or changing electric field, in which magnetic forces are observable.
  - An equivalent way: $\boldsymbol{dF} = I\ \boldsymbol{dl}\ \times \boldsymbol{B}$
  - $\boldsymbol{dF}$is the infinitesimal force acting on a differential length $\boldsymbol{dl}$ of the wire

Their relation between each other

Key rules

- $d\boldsymbol{F} = I\, d\boldsymbol{l}\ \times \boldsymbol{B}$
- (or when B is uniform): $\boldsymbol{F} = I\boldsymbol{l}\ \times \boldsymbol{B}$
  - l is the length of wire
  - The magnitude could be found by $\boldsymbol{F} = IlB\ sin\theta$
- $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$
  - (a moving charge q with a velocity of v)
  - The magnitude of which is $F = qvBsin\theta$

Key rules (2)

- Ampere's law: the line integral of the magnetic field B around any closed loop is equal to μ_0 times the to the total net current I_{encl} enclosed by the loop
- $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I_{encl}$
- The Biot-Savart law
- $d\boldsymbol{B} = \frac{\mu_0 I}{4\pi} \frac{d\boldsymbol{l} \times \hat{\boldsymbol{r}}}{r^2}$
- dB is the contribution to the total field at some point P due to a current I along an infinitesimal length dl of its path, and r is the unit vector along the direction of the displacement vector r from dl to P. The total field B will be the integral over all dB