1 Counting

Ordered Sampling with replacement	n^k
Ordered Sampling without replacement	${}^nP_k = \frac{n!}{(n-k)!}$
Unordered Sampling with replacement	$\binom{n}{k} = \frac{n!}{k!(n-k)!}$
Unordered Sampling with replacement	$\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$
The Binomial Theorem	$(x+y)^n = \sum \binom{n}{k} x^k y^{n-k}$
The Multinomial Theorem	$(x_1 + x_2 + ... + x_r)^n = \sum_{\substack{(n_1,n_2,...n_r): \\ n_1+...+n_r=n}} \frac{n!}{n_1!n_2!...n_r!} x_1^{n_1} x_2^{n_2} ... x_r^{n_r}$

There are $\binom{n+k-1}{k-1}$ distinct nonnegative integer-valued vectors $< x_1, ..., x_r >$ satisfying $x_1 + ... + x_k = n$

2 Probability

Probability Rules

$\left(\bigcup_{i=1}^n E_i\right)^c = \bigcap_{i=1}^n E_i$	$\left(\bigcap_{i=1}^n E_i\right)^c = \bigcup_{i=1}^n E_i$	$P(E^c) = 1 - P(E)$	$P(E \cup F) = P(E) + P(F) - P(EF)$	$P(E\|F) = \frac{P(EF)}{P(F)}$

$P(E) = P(EF) + P(EF^c) = P(F)P(E|F) + P(F^c)P(E|F^c) = P(F)P(E|F) + (1 - P(F))P(E|F^c)$

Bayes' Theorem	Suppose that $F_1, F_2, ...F_n$ are mutually exclusive events such that $\bigcup_{i=1}^n F_i = S$ (Sample Space), then $P(F_j\|E) = \frac{P(EF_j)}{P(E)} = \frac{P(E\|F_j)P(F_j)}{\sum_{i=1}^n P(E\|F_i)P(F_i)}$
Independent Events	Two events are independent if $P(EF) = P(E)P(F)$

3 Discrete Random Variable

Probability Mass Function (PMF)	$p_X(a) = P\{X = a\}$
Cumulative Distribution Function (CDF) of X	$F_X(a) = P\{X <= a\} = \sum_{x<=a} p_X(a)$
Expected Value	$E[X] = \sum_x x p_X(x)$
For any function of g	$E[g(X)] = \sum_x g(x) p_X(x)$
$Var(X) = E[X^2] - (E[X])^2$	$SD(X) = \sqrt{Var(X)}$ $Var(aX + b) = a^2 Var(X)$

Distributions

Name	PMF	Mean	Variance
$Bernoulli(p)$	$P\{X = 1\} = p$, $P\{X = 0\} = 1 - p$	p	$p((1-p)$
$Binomial(n, p)$	$P\{X = k\} = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, ..., n$	np	$np((1-p)$
$Geometric(p)$	$P\{X = n\} = p(1-p)^{n-1}, n = 0, 1, 2, ...$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
$Poisson(\lambda)$	$P\{X = n\} = e^{\lambda} \frac{\lambda^n}{n!}, n = 0, 1, ...$	λ	λ
$negbinomial(r, p)$	$P\{X = r\} = \binom{n-1}{r-1} p^r (1-p)^{n-r}, n = r, r+1, ...$	$\frac{r}{p}$	$r\frac{(1-p)}{p^2}$
$hypergeometric$	$P\{X = i\} = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}, i = 0, 1, 2, ..., min(n, m)$	$\frac{nm}{N}$	$\frac{N-n}{N-1} np(1-p) where p = m/N$

4 Continuous Random Variables

Probability Mass Function (PMF)	$f_X(a) = \frac{1}{\epsilon} P\{a - \frac{\epsilon}{2} <= X <= a + \frac{\epsilon}{2}\}$
Cumulative Distribution Function (CDF) of X	$F_X(a) = P\{X <= a\} = \int_{-\infty}^{a} p_X(a) dx$
$P\{X \in B\} = \int B f_X(x) dx$	$f_X(x) = \frac{d}{dx} F_X(x)$
Expected Value	$E[X] = \int_{-\infty}^{\infty} x f_X(x)$
For any function of g	$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)$
$Var(X) = E[X^2] - (E[X])^2$	$SD(X) = \sqrt{Var(X)}$ $Var(aX + b) = a^2 Var(X)$

Name	PMF	CDF	Mean	Variance
$uniform(a, b)$	$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a < x < b \\ 0 & \text{othewrise} \end{cases}$	$F(x) = \begin{cases} 0 & x <= a \\ \frac{x-a}{b-a} & \text{if } a < x < b \\ 1 & x >= a \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$Exponential(\lambda)$	$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x >= 0 \\ 0 & \text{othewrise} \end{cases}$	$F(x) = 1 - e^{-\lambda x}$ if $a >= 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$Normal(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$	$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du$ *Use normal table	μ	σ^2

For any normal random variable X with parameters (μ, σ^2), $Z = \frac{X-\mu}{\sigma}$ is the standardized normal random variable.
DeMoivere-Laplace Limit Theorem: If S_n denotes the number of successes that occur when n independent trials, each resulting in asuccess with probability p, are performed then, for any a ¡ b, $P\{a <= \frac{S_n - np}{\sqrt{np(1-p)}} <= b\} - > \phi(b) - \phi(a)$
Distribution of a Function of a RV: $f_Y(y) = f_X[g^{-1}(y)]|\frac{d}{dy}g^{-1}(y)|$ if $y = g(x)$ for some x

5 Jointly Distributed Random Variables

Note: Replace summation with integration for continuous

Joint CDF	$f_{XY}(a,b) = P\{X = a, Y = b\}$	Joint CDF	$F_{XY}(a,b) = P\{X <= a, Y <= b\}$
Marginal PMF of X	$p_X(a) = \sum_b p_{XY}(a,b)$	Marginal PMF of Y	$p_Y(b) = \sum_a p_{XY}(a,b)$
Marginal CDF of X	$F_{XY}(a) = F_{XY}(a, \infty)$	Marginal CDF of Y	$F_XY(b) = F_{XY}(\infty, b)$

Conditional PMF	$p_{X\|Y}(x\|y) = P\{X = x\|Y = y\} = \frac{P\{X=x,Y=y\}}{P\{Y=y\}} = \frac{p_{XY}(x,y)}{p_Y(y)}$	Conditional Jointly Distributed RV
Conditional CDF	$F_{X\|Y}(x\|y) = P\{X <= x\|Y <= y\} = \sum_{a<=x} p_{X\|Y}(a\|y)$	
Given Function $g(X,Y)$	$\sum_y \sum_x g(x,y) p_{XY}(x,y)$	
Joint MGF	$M_{XY}(t,t') = E[e^{tX+T'Y}] = \sum_y \sum x e^{tx+t'y} p_{XY}(x,y)$	
Conditional Expectation	$E[X\|Y = y] = \sum_x x p_{X\|Y}(x,y)$ $E[g(X)\|Y = y] = \sum_x g(x) p_{X\|Y}(x,y)$	
Expectation of X	$\sum_y E[X\|Y = y] P\{Y = y\}$	

$F_{X\|A}(x) = \frac{F_X(x) - F_X(a)}{F_X(b) - F_X(a)}$ if$a <= x < b$	$f_{X\|A}(x) = \frac{f_X(x)}{P(A)}$	$f_{X\|Y}(x,y) = \frac{f_{XY}(x,y)}{f_Y(y)}$
$E[X\|A] = \int_{-\infty}^{\infty} x f_{X\|A}(x) dx$	$E[g(X)\|A] = \int_{-\infty}^{\infty} g(x) f_{X\|A}(x) dx$	

Independent Random Variables: Two random variables X and Y are said to be independent if for every a and b we have $f_{XY}(a,b) = f_X(a) f_Y(b)$.

Summation of Independent Random Variables

CMF	$F_{X+Y}(a) = P\{X + Y <= a\} = \int_{-\infty}^{\infty} F_X(a-y) f_Y(y) dy$
PDF	$f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(a-y) f_Y(y) dy$

Jacobian Transformation (Joint Probability Distribution functions of RV)

$f_{Y_1,Y_2}(y_1,y_2) = \frac{f_{X_1,X_2}(x_1,x_2)}{\|J(x_1,x_2)\|}$	$J(x_1,x_2) = \begin{vmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} \end{vmatrix} = \frac{\partial g_1}{\partial x_1}\frac{\partial g_2}{\partial x_2} - \frac{\partial g_1}{\partial x_2}\frac{\partial g_2}{\partial x_1} \neq 0$	$y_1 = g_1(x_1,x_2)$ and $y_2 = g_2(x_1,x_2)$

6 Expectation