Counting

Order:	Matters	Doesn't Matter
w/ Replacement	n^k	$\binom{n+k-1}{k}$
wo/ Replacement	${}_nP_k = \frac{n!}{(n-k)!}$	$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Binomial: $(x+y)^n = \sum \binom{n}{k} x^k y^{n-k}$
Multinomial:

$$(x_1 + x_2 + ... + x_r)^n = \sum_{\substack{(n_1, n_2, ... n_r): \\ n_1 + ... + n_r = n}} \frac{n!}{n_1! n_2! ... n_r!} x_1^{n_1} x_2^{n_2} ... x_r^{n_r}$$

There are $\binom{n+k-1}{k-1}$ distinct nonnegative integer-valued vectors $< x_1, ..., x_r >$ satisfying $x_1 + ... + x_k = n$

Probability

$\left(\bigcup_{i=1}^n E_i\right)^c = \bigcap_{i=1}^n E_i$ (DeMorgan's Law)
$\left(\bigcap_{i=1}^n E_i\right)^c = \bigcup_{i=1}^n E_i$ (DeMorgan's Law)
$P(E^c) = 1 - P(E)$ (complement)
$P(E \cup F) = P(E) + P(F) - P(EF)$ (double counting)
$P(E|F) = \frac{P(EF)}{P(F)}$ (conditional)
$P(E) = P(EF) + P(EF^c) = P(F)P(E|F) + P(F^c)P(E|F^c) = P(F)P(E|F) + (1 - P(F))P(E|F^c)$
$P(E) = \sum_{i=1}^n P(E|F_i)P(F_i)$ (Law of Total Probability)
Two events are independent if $P(EF) = P(E)P(F)$
Bayes' Theorem: Suppose that $F_1, F_2, ...F_n$ are mutually exclusive events such that $\bigcup_{i=1}^n F_i = S$ (sample space), then $P(F_j|E) = \frac{P(EF_j)}{P(E)} = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^n P(E|F_i)P(F_i)}$

Random Variables

PMF: $p_X(a) = P\{X = a\}$
CDF: $F_X(a) = P\{X \le a\} = \sum_{x \le a} p_X(a)$
EV : $E[X] = \sum_x x p_X(x)$
EV of $g(X)$:
$E[g(X)] = \sum_x g(x) p_X(x)$
$Var(X) = E[X^2] - (E[X])^2$
$Var(aX + b) = a^2 Var(X)$
$SD(X) = \sqrt{Var(X)}$

Name	PMF	μ	Var
$Bern(p)$	$P\{X = 1\} = p$, $P\{X = 0\} = 1 - p$	p	$p(1-p)$
$Bin(n, p)$	$P\{X = k\} = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, ..., n$	np	$np(1-p)$
$Geom(p)$	$P\{X = n\} = p(1-p)^{n-1}, n = 0, 1, 2, ...$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
$Pois(\lambda)$	$P\{X = n\} = e^{-\lambda} \frac{\lambda^n}{n!}, n = 0, 1, ...$	λ	λ
$NBin(r, p)$	$P\{X = r\} = \binom{n-1}{r-1} p^r (1-p)^{n-r}, n = r, r+1, ...$	$\frac{r}{p}$	$r\frac{(1-p)}{p^2}$
h.g.	$P\{X = i\} = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$ $i = 0, 1, 2, ..., min(n, m)$	$\frac{nm}{N}$	$\frac{N-n}{N-1} np(1-p)$ $p = m/N$

PDF: $f_X(a) = \frac{1}{\epsilon} P\{a - \frac{\epsilon}{2} <= X \le a + \frac{\epsilon}{2}\}$
CDF: $F_X(a) = P\{X \le a\} = \int_{-\infty}^a p_X(a) dx$
EV: $E[X] = \int_{-\infty}^{\infty} x f_X(x)$
EV of $g(X)$:
$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)$
$P\{X \in B\} = \int B f_X(x) dx$
$f_X(x) = \frac{d}{dx} F_X(x)$

Name	PDF	CDF	μ	Var
$Uni(a, b)$	$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a < x < b \\ 0 & \text{othewrise} \end{cases}$	$F(x) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & \text{if } a < x < b \\ 1 & x \ge b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$Expo(\lambda)$	$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \ge 0 \\ 0 & \text{othewrise} \end{cases}$	$F(x) = 1 - e^{-\lambda x}$ if $x \ge 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$Gamm(a, \lambda)$	$f(x) = \frac{1}{\Gamma(a)} (\lambda x)^a e^{-\lambda x} \frac{1}{x}$	???	a/λ	a/λ^2
$Norm(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$	$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du$	μ	σ^2

Continuous Random Variables

For any normal random variable X with parameters (μ, σ^2), $Z = \frac{X - \mu}{\sigma}$ is the standardized normal random variable.

De Moivre-Laplace Limit Theorem: If S_n denotes the number of successes that occur when n independent trials, each resulting in a success with probability p, are performed then, for any $a < b$, $P\{a \le \frac{S_n - np}{\sqrt{np(1-p)}} \le b\} -> \phi(b) - \phi(a)$

Memoryless RV: We say that a nonnegative random variable X is *memoryless* if $P\{X > s + t | X > t\} = P\{X > s\}$

Distribution of a Function of a RV: $f_Y(y) = f_X[g^{-1}(y)] |\frac{d}{dy} g^{-1}(y)|$ if $y = g(x)$ for some x

Jointly Distributed Random Variables[1]

Joint PDF: $f_{XY}(a, b) = P\{X = a, Y = b\}$
Conditional PMF: $p_{X|Y}(x|y) = P\{X = x | Y = y\} = \frac{P\{X = x, Y = y\}}{P\{Y = y\}} = \frac{p_{XY}(x, y)}{p_Y(y)}$

Joint CDF: $F_{XY}(a, b) = P\{X \le a, Y \le b\}$
Conditional CDF: $F_{X|Y}(x|y) = P\{X \le x | Y \le y\} = \sum_{a \le x} p_{X|Y}(a|y)$

[1]Replace summation with integration for continuous.

Marginal PMF of X: $p_X(a) = \sum_b p_{XY}(a,b)$
Marginal PMF of Y: $p_Y(b) = \sum_a p_{XY}(a,b)$

Marginal CDF of X: $F_{XY}(a) = F_{XY}(a,\infty)$
Marginal CDF of Y: $F_{XY}(b) = F_{XY}(\infty,b)$

Conditional Jointly Distributed RV

$F_{X\|A}(x) = \frac{F_X(x)-F_X(a)}{F_X(b)-F_X(a)}$ if $a \le x < b$	$f_{X\|A}(x) = \frac{f_X(x)}{P(A)}$ if A is some event between $a \le X < b$	$f_{X\|Y}(x,y) = \frac{f_{XY}(x,y)}{f_Y(y)}$
$E[X\|A] = \int_{-\infty}^{\infty} x f_{X\|A}(x)dx$	$E[g(X)\|A] = \int_{-\infty}^{\infty} g(x) f_{X\|A}(x)dx$	

Independent Random Variables: Two random variables X and Y are said to be independent if for every a and b we have $f_{XY}(a,b) = f_X(a)f_Y(b)$.

Summation of Independent Random Variables

CMF	$F_{X+Y}(a) = P\{X+Y <= a\} = \int_{-\infty}^{\infty} F_X(a-y)f_Y(y)dy$
PDF	$f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(a-y)f_Y(y)dy$

Sum of Normal RV: Mean and Variance are the summation of all the normal RV's mean and variance.

Jacobian Transformation (Joint Probability Distribution functions of RV)

$f_{Y_1,Y_2}(y_1,y_2) = \frac{f_{X_1,X_2}(x_1,x_2)}{\|J(x_1,x_2)\|}$	$J(x_1,x_2) = \begin{vmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} \end{vmatrix}$	$= \frac{\partial g_1}{\partial x_1}\frac{\partial g_2}{\partial x_2} - \frac{\partial g_1}{\partial x_2}\frac{\partial g_2}{\partial x_1} \neq 0$	$y_1 = g_1(x_1,x_2)$ and $y_2 = g_2(x_1,x_2)$

Properties of Expectation

Joint PMF: $E[g(X,Y)] = \sum_y \sum_x g(x,y)p(x,y)$
Joint PDF: $E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)p(x,y)dxdy$
$E[aX+b] = aE[X]+b$
$E[g(X)h(Y)] = E[g(X)]E[h(Y)]$ if X and Y are independent

Covariance:
$Cov(X,Y) = E[XY] - E[X]E[Y]$, $Cov(X,Y) = Cov(Y,X)$
$Cov(X,X) = Var(X)$, $Cov(aX,aY) = aCov(X,Y)$
$Cov(\sum_{i=1}^n X_i, \sum_{j=1}^m Y_j) = \sum_{i=1}^n \sum_{j=1}^m Cov(X_i,Y_j)$
$Var(\sum_{i=1}^n X_i) = \sum_{i=1}^n Var(X_i) + \sum_{i=1}^n \sum_{j\neq i} Cov(X_i,X_j)$
$Var(\sum_{i=1}^n X_i) = \sum_{i=1}^n Var(X_i)$ if $X_1,...,X_n$ are p.w. ind.

Correlation: $\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$, $-1 \le \rho(X,Y) \le 1$

Conditional Expectation:
$E[X|Y=y] = \sum_x x p_{X|Y}(x,y)$
$E[g(X)|Y=y] = \sum_x g(x) p_{X|Y}(x,y)$

Conditioning Property: $E[X] = E[E[X|Y]]$
Expectation of X: $\sum_y E[X|Y=y]P\{Y=y\}$

Conditional Variance: $Var(X|Y) = E[X^2|Y] - (E[X|Y])^2$
Uncond. var. of $X|Y$: $E[Var(X|Y)] = E[X^2] - E[(E[X|Y])^2]$

$Var(E[X|Y]) = E[(E[X|Y])^2] - (E[X])^2$
$Var(X) = E[Var(X|Y)] + Var(E[X|Y])$

Predictors:
General form: $E[(Y-g(X))^2] \ge E[(Y-E[Y|X])^2]$ (best case: $g(X) = E[Y|X]$)
Linear: $E[(Y-(a+bX))^2] \ge E[(Y-E[Y|X])^2]$, where $b = \rho\frac{\sigma_Y}{\sigma_X}$ and $a = E[Y] - bE[X]$

Moment Generating Functions:
n^{th} moment: $M^n(0) = E[X^n]$ $n \ge 0$
MGF of PDF: $M(t) = E[e^{tX}] = \sum_x e^{tx}p(x)$
MGF of PMF: $M(t) = E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx}f(x)dx$
More than 2 RV MGF: $M(t_1,...,t_n) = E[e^{t_1x_1+...+t_nX_n}]$
Individual MGF: $M_{X_i}(t) = M(0,...,0,t,0,...,0)$
Joint MGF: $M_{XY}(t,t') = E[e^{tX+T'Y}] = \sum_y \sum x e^{tx+t'y}p_{XY}(x,y)$
MGF of sum of two IRVs: $M_{X+Y}(t) = M_X(t)M_Y(t)$
Common MGFs:

Binomial	$M(t) = (pe^t+1-p)^n$
Poisson	$M(t) = exp\{\lambda(e^t-1)\}$
Exponential	$M(t) = \frac{\lambda}{\lambda-t}$ for $t<\lambda$
Standard Normal	$M(t) = e^{t^2/2}$
Normal	$M(t) = exp\{\frac{\sigma^2t^2}{2}+\mu t\}$

Limit Theorems

Markov's Inequality: If X is a random variable that takes only nonnegative values, then for any value $a > 0$, $P\{X \ge a|\} \le \frac{E[X]}{a}$

Chebyshev's Inequality: If X is a random variable with finite mean μ and variance σ^2, then for any value $k > 0$, $P\{|X-\mu| \ge k\} \le \frac{\sigma^2}{k^2}$

The Weak Law of Large Numbers: Let $X_1, X_2, ...$ be a sequence of iid Random variables, each having finite mean. Then, for any $\epsilon > 0$, $P\{|\bar{X}-\mu| \ge \epsilon\} \to 0$ where $\bar{X} = \frac{X_1+X_2+...+X_n}{n}$ as $n \to \infty$

The Central Limit Theorem: Let $X_1, X_2, ...$ be a sequence of iid random variables, each having mean and variance. Then, the distribution of $Z = \frac{X_1+...+X_n-n\mu}{\sigma\sqrt{n}}$ tends to the standard normal as $n \to \infty$. That is, for $-\infty < a < \infty$, $P\{Z \le a\} = \phi(a) \to N(0,1)$ as $n \to \infty$

Random Processes

Random Processes: a collection of random variables usually indexed by time

Sample Function: the time function $x(t,s)$ associated with

the outcome s of an experiment
Ensemble: the set of all possible time function that can result from an experiment
Random Sequence: a random sequence X_n is an ordered sequence of random variables $X_0, X_1, ...$

Bernoulli Process: $X(t)$ is a sequence of Bernoulli trials; trials are independent of each other ($P\{X_n = 1\} = p = 1 - P\{X_n = 0\}$)

Counting Process: Given a stochastic process $N(t)$:
$N(0) = 0$, $N(t) \in \{0, 1, 2, ...\}$ for all $t \in [0, \infty)$
for $0 < s < t$, $N(t) - N(s)$ shows the # of events in $(s, t]$.

Poisson Process: Given $\lambda > 0$, a counting process $N(t), t \in [0, \infty)$ is called a Poisson process with rate λ if the following conditions hold:
$\rightarrow N(0) = 0$
$\rightarrow N(t)$ has independent and stationary increments
$\rightarrow$ The # of arrivals in any interval of length $\tau > 0$ has $Pois(\lambda\tau)$ distribution

PMF of $M = N(t_1) - N(t_0)$:

$$p_M(m) = \begin{cases} \frac{|\lambda|(t_1-t_0)|^m}{m!} e^{-\lambda(t_1-t_0} & m = 0, 1... \\ 0 & \text{otherwise} \end{cases}$$

Joint PMF of $N(t_1), ..., N(t_k)$, $t_1 < t_2 < ... < t_k$:

$$p_{N(t_1),...,N(t_k)}(n_1, .., n_k) = \begin{cases} \frac{\alpha_1^{n_1} e^{-\alpha_1}}{n_1!} \frac{\alpha_1^{n_2-n_1} e^{-\alpha_2}}{(n_2-n_1)!} ... \frac{\alpha_k^{n_k-n_{k-1}} e^{-\alpha_k}}{(n_k-n_{k-1})!} & m = 0, 1..., \alpha_i = \lambda(t_i - t_{i-1}) \\ 0 & \text{otherwise} \end{cases}$$

For a Poisson process of rate λ, the inter-arrival times $X_1, X_2, ...$ are an iid random sequence with the exponential PDF.

$P\{X_n - x' > x | X_n > x'\} = P\{X_n > x\} = e^{-\lambda x}$ (P.P. is memoryless)
A counting process with **independent exponential inter-arrivals** $X_1, X_2, ...$ with mean $E[X_i] = 1/\lambda$ is a Poisson process of rate λ

Brownian Motion Process: continuous-time, continuous-value process. $X(0) = 0$ and for $\tau > 0, X(t + \tau) - X(t)$ is a Gaussian random variable with mean 0 and variance $\alpha\tau$ independent of $X(t')$ for all $t' \leq t$:
$X(t + \delta) = X(t) + [X(t + \delta) - X(t)]$
PDF of Y_δ: $P_{Y_\delta}(y) = \frac{1}{\sqrt{2\pi\alpha\delta}} e^{-y^2/2\alpha\delta}$
Joint PDF: $f_{X(t_1),...,X(t_k)}(x_1, ..., x_k) = \prod_{i=1}^{k} \frac{1}{\sqrt{2\pi\alpha(t_n - t_{n-1})}} e^{-(x_n - x_{n-1})^2/2\alpha(t_n - t_{n-1})}$

General Properties:

Random	Process	Sequence
EV	$\mu_X(t) = E[X(t)]$	n/a
Autocov.	$C_X(t, \tau) = Cov(X(t), X(t + \tau))$	$C_X[m, k] = Cov[X_m, X_{m+k}]$
Autocor.	$R_X(t, \tau) = E[X(t)X(t + \tau)]$	$R_X[m, k] = E[X_m, X_{m+k}]$

Stationary Properties:

Random	Process	Sequence
EV	$\mu_X(t) = \mu_X$	$E[X_m] = \mu_X$
Autocov.	$C_X(t, \tau) = R_X(\tau) - \mu_X^2 = C_X(\tau)$	$C_X[m, k] = R_X(\tau) - \mu_X^2$
Autocor.	$R_X(t, \tau) = R_X(0, \tau) = R_X(\tau)$	$R_X[m, k] = R_X[0, k] = R_X[k]$

WSS Properties:

Random	Process	Sequence
EV	$E[X(t)] = \mu_X$	$E[X_m] = \mu_X$
Autocor.	$R_X(t, \tau) = R_X(0, \tau) = R_X(\tau)]$	$R_X[m, k] = R_X[0, k] = R_X[k]$

Stationary Process: A random process X(t) is stationary if and only if for all sets of time constants $t_, ..., t_m$ and any time difference τ, $f_{X(t_1),...,X(t_k)}(x_1, ..., x_k) = f_{X(t_1+\tau),...,X(t_k+\tau)}(x_1, ..., x_k)$
Stationary Sequence: $f_{X_{n_1},...,X_{n_m}}(x_1, ..., x_m) = f_{X_{n_1+k},...,X_{n_m+k}}(x_1, ..., x_m)$
If $(X(t)/X_n$ is WSSP/WSRS:

Processes	$R_X(0) \geq 0$ $R_X(\tau) = R_X(-\tau)$ $\lvert R_X(\tau)\rvert \leq R_X(0)$

Sequences	$R_X(0) \geq 0$ $R_X(k) = R_X(-k)$ $\lvert R_X(k)\rvert \leq R_X(0)$

$C_X(t, \tau) = R_X(t, \tau) - \mu_X(t)\mu_X(t + \tau)$

Average Power of WSSP: $R_X(0) = E[X^2(t)]$

Random Signal Processing

LTI Filter Output: For $X(t)$ input to a LTI filter with impulse response $h(t)$, the output $Y(t)$ is the convolution of the input $X(t)$ with $h(t)$

If the input to LTI filter with impulse response $h(t)$ is WSSP $X(t)$, then WSSP output $Y(t)$ has the following:
$\mu_Y = \mu_X H(0)$ (mean)
$R_Y(\tau) = \int_{-\infty}^{\infty} h(u) \int_{-\infty}^{\infty} h(v) R_X(\tau + u - v) dv du$ (auto-correl.)

Power Spectral Density: For a WSSP $X(t)$, $R_X(\tau)$ and the power spectral density $S_X(f)$ are Fourier transform pairs:
$S_X(f) = \mathcal{F}\{R_X(\tau)\}$
$R_X(\tau) = \mathcal{F}^{-1}\{S_X(f)\}$
$S_X(f) \geq 0$ for all f

Properties of $S_X(f)$:
$E[X^2(t)] = \int_{-\infty}^{\infty} S_X(f) df$, $S_X(f) = S_X(-f)$
$S_Y(f) = S_X(f)|H(f)|^2$ where H is the input response

Independent Processes: If $X(t)$ $Y(t)$ are independent for any time sample of $t_1, .., t_n$, and $t'_1, ..., t'_m$, then:
$f_{X(t_1),...,X(t_n),Y(t'_1),...,Y(t'_m)}(x_1, ..., x_n, y_1, ..., y_m$
$= f_{X(t_1),...,X(t_n)}(x_1, ..., x_n,) f_{Y(t'_1),...,Y(t'_m)}(y_1, ..., y_m)$
Cross Correlation Func: $R_{XY}(t, \tau) = E[(X(t)Y(t + \tau)]$
Jointly WSSP: $R_{XY}(t, t + \tau) = R_{XY}(\tau)$ and $R_{XY}(\tau) = R_{XY}(-\tau)$
Cross Spectral Density: $S_{XY}(f) = \mathcal{F}\{R_{XY}(\tau)\}$
Input-Output Cross Correlation: When a WSSP $X(t)$ is the input to a LTI filter $h(t)$, the input-output cross correlation is $R_{XY}(t, t + \tau) = R_{XY}(\tau) = \int_{-\infty}^{\infty} h(u) R_X(\tau - u) du$. Both $X(t)$ and $Y(t)$ are jointly wide sense stationary.
Output Autocorrelation: When a wide stationary process $X(t)$ is the input to a LTI filter $h(t)$, the autocorrelation of the output $Y(t)$ is $R_Y(\tau) = \int_{-\infty}^{\infty} h(-w) R_{XY}(\tau - w) dw$.
Cross Spectral Properties: Let $X(t)$ be a WSS input to a LTI filter $H(f)$. The input $X(t)$ and output $Y(t)$ satisfy:
$S_{XY}(f) = H(f) S_X(f)$, and
$S_Y(f) = H(f) S_{XY}(f)$

Gaussian Process: $X(t)$ is a Gaussian random process if the joint PDF of $X(t_1,), ..., X(t_k)$ has the multivariate Gaussian density $f_{X(t_1),...X(t_k)} = \frac{1}{(2\pi)^{k/2}|C|^{1/2}} \exp\{-\frac{1}{2}(X - \mu x)^\tau C^{-1}(X - \mu x)\}$
If $X(t)$ is WSS Gaussian process, then $X(t)$ is a stationary Gaussian process.
$X(t)$ is a Gaussian RV if $Y = \int_0^T g(t) X(t) dt$ is a Gaussian random variable for every $g(t)$ such that $E[Y^2] < \infty$.
Properties of Gaussian Processes: Passing a stationary Gaussian process $X(t)$ through a linear filter $h(t)$ yields as the output Gaussian random process $Y(t)$ with the following properties.
Mean: $\mu_Y = \mu_X H(0)$
Autocorrelation: $R_Y(\tau) = \int_{-\infty}^{\infty} h(u) \int_{-\infty}^{\infty} h(v) R_X(\tau + u - v) dv du$

White Gaussian Noise Processes: Noise is modeled as stationary Gaussian random process $W(t)$, with no DC component.
$E[W(t_t 1)] = \mu_W = 0$
$R_W(\tau) = 0$ if $\tau \neq 0$
Power Spectral Density of $W(t)$: $S_W(f)$ is constant. The constant is 0 unless $R_W(\tau) = \frac{N_0}{2}\delta(\tau)$. N_0 is the the power per unit bandwidth of $W(t)$.
Average Noise Power: $E[W^2(t)] = R_W(0) = \infty$
Noise process output: $Y(t) = \int_0^t h(t - \tau) W(\tau) d\tau =$ Constant

Random Helpful Stuff

Don't panic.
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad$ for $|x| < 1$
$\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$
$\int u\, dv = uv - \int v\, du.$
$\Gamma(a) = \int\limits_0^\infty x^{a-1} e^{-x} dx$
$\Gamma(n) = (n-1)!$ if $n \in \mathbf{Z}^+$

Fourier Transforms

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \Leftrightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$e^{-a|t|}, \Re\{a\} > 0 \Leftrightarrow \frac{2a}{a^2 + \omega^2}$$

$$\cos(\omega_0 t + \theta) \Leftrightarrow \pi\left[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)\right]$$

$$e^{-\alpha t} \Leftrightarrow \frac{1}{\alpha + j\omega}$$

$$1 \Leftrightarrow 2\pi\delta(\omega)$$

$$-0.5 + u(t) \Leftrightarrow \frac{1}{j\omega}$$

$$\sin(\omega_0 t + \theta) \Leftrightarrow j\pi\left[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)\right]$$