Lab1

November 7, 2022

0.1 Project Documentation (Lab 1)

0.1.1 Programming language: Python 3.10.7

[1]:
```
!python --version
```

```
Python 3.10.7
```

0.1.2 Task 1.2 implementation (Norm$(\mathbf{mu}, \mathbf{sigma})$ empirical & theoretical CDF)

Data Source: randomly generated values for Normal distribution with parameters:

mu = 1.5

sigma = 4

Value Generator: scipy.stats.norm submodule of scipy.stats (Assuming scipy stats is a trusted data source and generates random values correctly)

[2]:
```
from random import random,randint
import os
import matplotlib.pyplot as plt
import numpy as np
from matplotlib.patches import Polygon
import matplotlib.pyplot as plt
import time
import pandas as pd
from statistics import mean
from fitter import Fitter, get_common_distributions, get_distributions
from scipy.stats import norminvgauss
from math import sqrt
```

Parameters for a Theoretical CDF are fitted below using the Fitter module Assuming the theoretical distribution function is either equivalent to Norm(mu,sigma) or is a distribution function contained in the get_common_distributions() method of the *Fitter* library

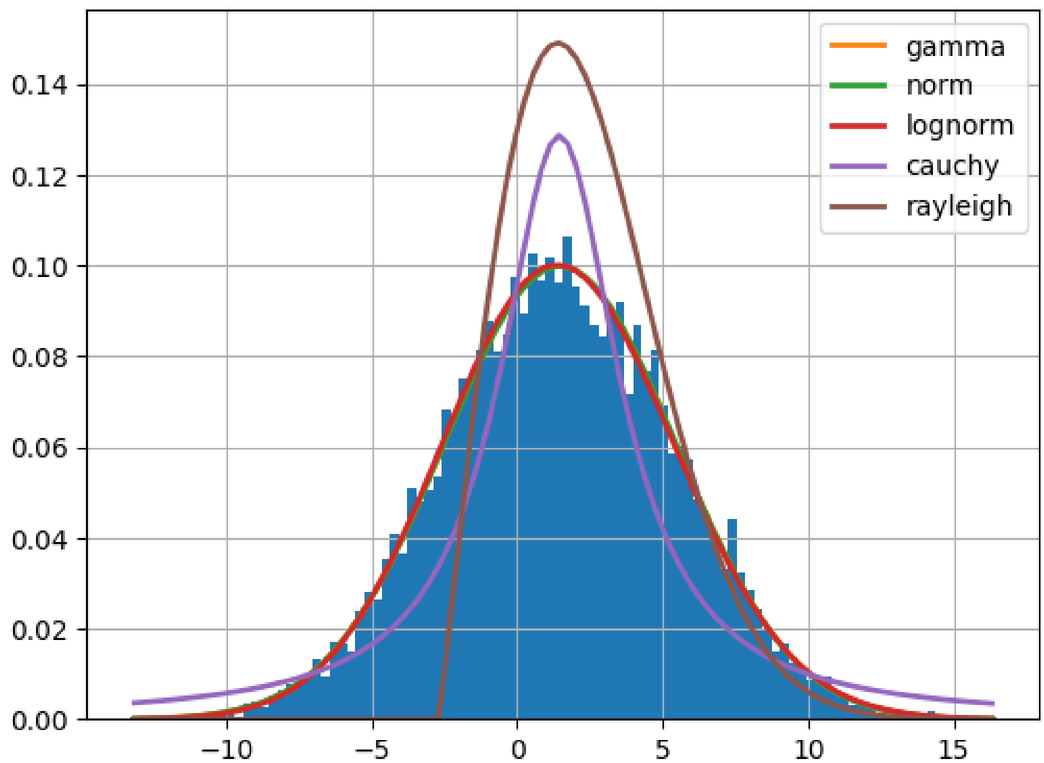
[3]:
```
print("common distributions:")
get_common_distributions()
```

```
common distributions:
```

[3]: ['cauchy',
 'chi2',
 'expon',
 'exponpow',
 'gamma',
 'lognorm',
 'norm',
 'powerlaw',
 'rayleigh',
 'uniform']

[4]:
```
from scipy.stats import norm
mu=1.5
sigma=4
samples1=norm.rvs(mu,sigma,10000)


f1 = Fitter(samples1,distributions=get_common_distributions())
f1.fit()
f1.summary()
params1=f1.fitted_param["norm"]
print("(mu,sigma) ~=",params1)
```

```
Fitting 10 distributions:
100%|                              | 10/10
[00:00<00:00, 13.16it/s]

(mu,sigma) ~= (1.4705337269854863, 3.995563272831519)
```



Function drawEmpyricalCDF(axes, samples, label):

Arg: axes - matplotlib.pyplot axes object

Arg: samples - empyrical CDF values for every $x_i = x_{i-1} + delta$, where $delta = 1/n$Where n = length(samples), $i = \overline{\textit{0, n-2}}$

Arg: label - plot label

X - an array of segments $(samples[i], samples[i+1])$, $i = \overline{\textit{0, n-1}}$

Y - an array of CDF values (twice for every segment): $(\frac{(i_k + diff)}{n}, \frac{(i_k + diff)}{n})$ where $i_k = i_0 + k * delta, k = \overline{\textit{0, n-2}}$

Plot the CDF using the ax.plot() method

Function err_cdf(samples,dist,params):

Arg: *samples* - a random sample

Arg: *dist* - a theoretical distribution function

Arg: *params* theoretical distribution function parameters

for each *size* in [100,200,500,2000,3000,5000,7000,10000]:

Select a random subsample of size *size*

Draw the Theoretical CDF using the given parameters *params*

Draw an empyrical CDF based on the random subsample

Differentiate two CDFs and compute the standard error of the CDF difference. Store the resulting *standard error* in a numpy array

Print errors and display the results

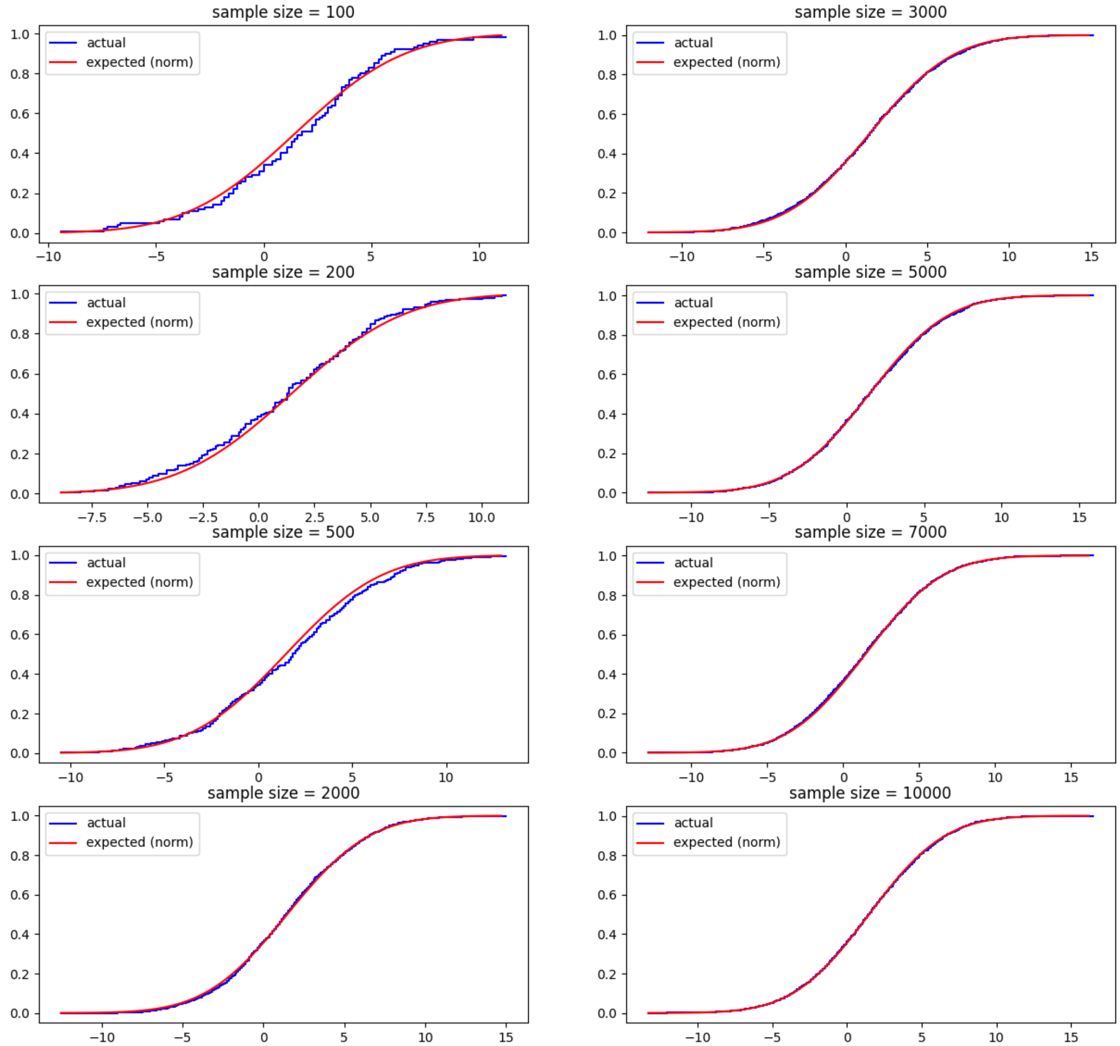
[5]:
```
def drawEmpyricalCDF(ax,_samples,label):
    mean=_samples.mean()
    var=np.array([x**2 for x in _samples]).mean()-mean**2
    samples=np.sort(_samples)

    n=len(samples)
    seg0=samples[0]
    seg1=None
    X=[]
    Y=[]
    i0=0
    diff=0
    for i in range(len(samples)-1):
        seg1=samples[i+1]
        diff+=1
        if abs(seg1-seg0)>0.1:
            X.append(seg0)
            X.append(seg1)
            Y.append((i0+diff)/n)
            Y.append((i0+diff)/n)
            seg0=samples[i+1]
            seg1=None
            i0=i
            diff=0

    ax.plot(X,Y,"b",label=label)

def err_cdf(samples,dist,params):
    print("params =",params)
    iarr=[100,200,500,2000,3000,5000,7000,10000]

    rep=10
    results=[0]*len(iarr)
    fig,axes=plt.subplots(4,2, figsize=(15,14))
    for r in range(rep):
        for _i in range(len(iarr)):
            ax=None
            if (_i>=4):
                ax=axes[_i-4,1]
            else:
                ax=axes[_i,0]
```

```
            i=iarr[_i]
            samp=np.random.choice(samples,i)
            samp.sort()

            bins=100


            actual,_samp = ax.
↪hist(samp,bins=bins,color="blue",density=True,label="actual")[:2]
            ax.cla()
            drawEmpyricalCDF(ax,samp,"actual")

            expected=[dist.pdf(a,*params) for a in _samp[:-1]]

            ax.plot(_samp[:-1],[dist.cdf(a,*params) for a in _samp[:
↪-1]],color="red",label="expected ("+str(dist.name)+")")

            diff = np.array([abs(actual[i]-expected[i]) for i in range(bins)])
            serr=sqrt(diff.std())
            results[_i]+=serr
            if (r==rep-1):
                ax.set_title("sample size = %d"%i)
                ax.legend()

    plt.show()
    results=[r/rep for r in results]

    plt.close()
    for i in range(len(iarr)):
        print("standard error for sample size = %d : %.6f"%(iarr[i],results[i]))
```

0.1.3 Empyrical & Theoretical CDF

[6]:
```
err_cdf(samples1,norm,params1)
```

```
params = (1.4705337269854863, 3.995563272831519)
```




```
standard error for sample size = 100 : 0.182192
standard error for sample size = 200 : 0.147523
standard error for sample size = 500 : 0.117449
standard error for sample size = 2000 : 0.083668
standard error for sample size = 3000 : 0.075714
standard error for sample size = 5000 : 0.069408
standard error for sample size = 7000 : 0.063295
standard error for sample size = 10000 : 0.060442
```

[ ]:

```

```

0.1.4 Task 1.1 (additional), Analyzing the Triangulation algorithm execution time

0.1.5 Task 1.1 implementation:

Subtask1 (Triangulation algorithm execution time)

Load samples

Draw empyrical PDF

Fit parameters for a suggested distribution (norminvgauss) using Fitter python module

Draw empyrical & Theoretical PDF

Evaluate standard error between Empyrical & Theoretical PDF

Show that the standard error decreases as sample size grows (empirical PDF -> theoretical PDF, n->inf)

Subtask2 (Normal distribution empirical CDF)

Generate random Norm(mu,sigma) values

Draw empyrical CDF

Fit parameters for a suggested distribution (norm) using the Fitter python module

Evaluate standard error for the difference between Empyrical & Theoretical CDF

Show that the standard error limits to zero as sample size grows (empirical CDF -> theoretical CDF, n->inf)

Functions for saving/loading sample data

[7]:
```
def saveSamples(samples,location):
    with open(location,"w+") as f:
        for sample in samples:
            f.write(' '.join(map(str,sample))+'\n')

def readSamples(location):
    samples=[]
    with open(location,"r") as f:
        for line in f.readlines():
            samples.append(list(map(float,line.split())))
    return samples
```

[8]:
```
fname3="sample-data-3.txt"
```

File: "sample-data-3.txt" - Point set Triangulation algorithm execution time for 100 points (sample size: 10000)

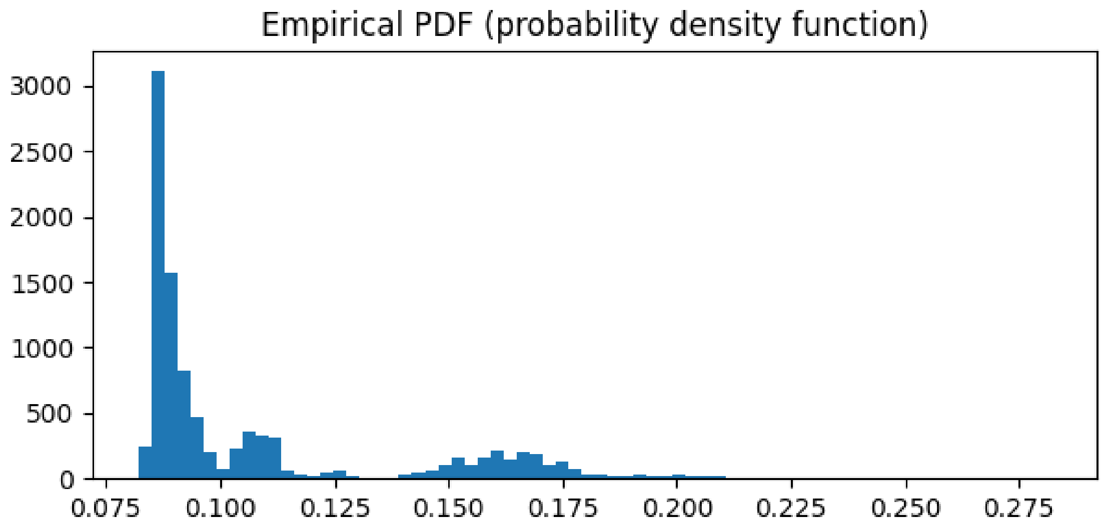
The data was obtained by executing the below algorithm: (Triangulation algorithm: https://github.com/al3xkras/ComputationalGeometryCourseProjectKNU)

[9]:
```
samples=readSamples(fname3)[0]
len(samples)
```

[9]: 10000

[10]:
```
plt.gcf().set_size_inches((7,3))
plt.hist(samples, bins=70)
```

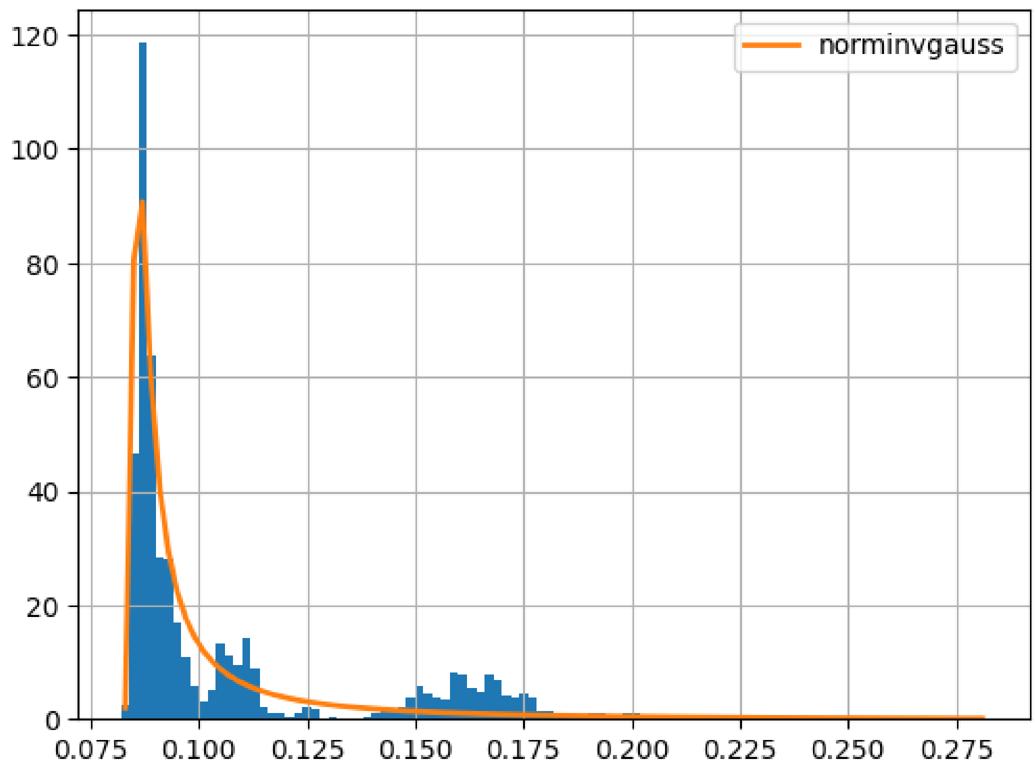
```
plt.title("Empirical PDF (probability density function)")
plt.show()
```



Theoretical distribution (suggested by Fitter): norminvgauss Normal inverse Gauss distribution parameters calculation process is shown below.

[11]:
```
f = Fitter(samples,distributions= ["norminvgauss"])
f.fit()
f.summary()
params=f.fitted_param["norminvgauss"]
print(params)
```

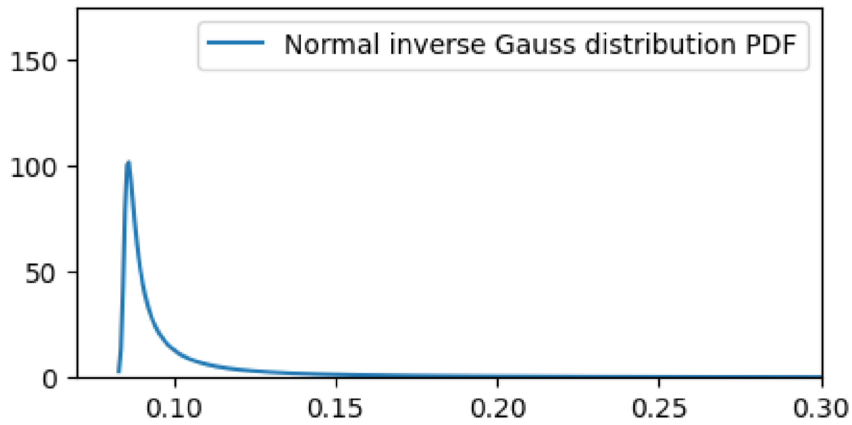
```
Fitting 1 distributions:
100%|                                | 1/1
[00:10<00:00, 10.58s/it]

(2.469494132931309, 2.46282262681328, 0.08435554713051517,
0.0017098074789276005)
```



0.1.6 Plotting the Normal inverse Gauss distribution PDF (theoretical PDF)

[12]:
```
params=f.fitted_param['norminvgauss']

x = np.linspace(norminvgauss.ppf(0.001, *params),
                norminvgauss.ppf(0.999, *params),1000)
pdf = norminvgauss.pdf(x,*params)

plt.plot(x,pdf,label="Normal inverse Gauss distribution PDF")
plt.ylim(0,175)
plt.xlim(0.07,0.3)
plt.legend()
print("fitted parameters:",params)
plt.gcf().set_size_inches(5,2.5)
plt.show()
```

```
fitted parameters: (2.469494132931309, 2.46282262681328, 0.08435554713051517,
0.0017098074789276005)
```



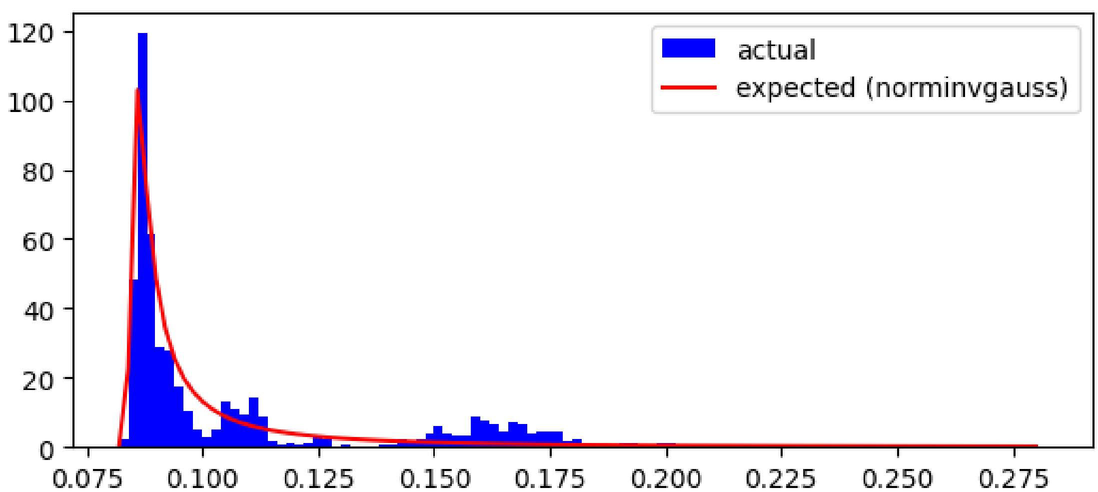
Calculating the standard error between theoretical & empirical PDF

(Depending on the sample size)

[13]:
```
def err(samples,dist,params):
    print("params =",params)
    iarr=[100,200,500,2000,5000,7000,10000]

    rep=10
    results=[0]*len(iarr)
    for r in range(rep):
        for _i in range(len(iarr)):
            i=iarr[_i]
            samp=np.random.choice(samples,i)
            samp.sort()

            bins=100
            plt.cla()
            actual,_samp = plt.
 ↪hist(samp,bins=bins,color="blue",density=True,label="actual")[:2]

            expected=[dist.pdf(a,*params) for a in _samp[:-1]]

            plt.plot(_samp[:-1],expected,color="red",label="expected␣
 ↪("+str(dist.name)+")")

            diff = np.array([abs(actual[i]-expected[i]) for i in range(bins)])
            serr=sqrt(diff.std())
            results[_i]+=serr

    results=[r/rep for r in results]
```

```
    for i in range(len(iarr)):
        print("standard error for sample size = %d : %.6f"%(iarr[i],results[i]))
    plt.gcf().set_size_inches(7,3)
    plt.legend()
    plt.show()

err(samples,norminvgauss,params)
```

```
params = (2.469494132931309, 2.46282262681328, 0.08435554713051517,
0.0017098074789276005)
standard error for sample size = 100 : 3.170142
standard error for sample size = 200 : 2.757837
standard error for sample size = 500 : 2.601013
standard error for sample size = 2000 : 2.354968
standard error for sample size = 5000 : 2.312885
standard error for sample size = 7000 : 2.281375
standard error for sample size = 10000 : 2.130213
```



[ ]: