pe 63:

$\cancel{x = \log}$

$\boxed{n = \lfloor \log_{10} x + 1 \rfloor}$

$\boxed{x = a^n}, \ a \in \mathbb{R}$

$x = a^{\log_{10} x + 1}$

$\ln x = (\log_{10} x + 1) \ln a$

$\cancel{\ln x = \log_{10} x \ln a + \ln a}$

$\log_a x = \log_{10} x + 1$

$\log x + 1$

$x = a$

$x = a a^{\log x}$

$\ln x = \ln (a^{\log x + 1})$

$\ln x = (\log x + 1) \ln a$

$n = \log a^n + 1$

$n = n\log a + 1$

$n - 1 = n\log a$

$\frac{n-1}{n} = \log a$

$1 - \frac{1}{n} = \log a$ $\qquad \frac{1}{n}$

$\frac{1}{n} = -\log a + 1$

$\frac{1}{n} = 1 - \log a$

$n = \frac{1}{1 - \log a}$, $n \in W$
$a \in W$

$a \to \infty$

$\frac{1}{1 - \log a} > 0$

$a^n \; \overline{) \; 10^{n-1} < a^n < 10^n}$

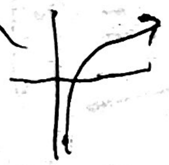
$10^{n-1} \leq a^n < 10^n$, $a \in \mathbb{Z}, a \neq 0$ / $n \in \mathbb{N}$

$\Rightarrow n-1 \leq n \log_{10} a < n$

$\Rightarrow n-1 \leq n \log a \wedge n \log a < n$

$\Rightarrow n \leq n \log a + 1$

$\Leftrightarrow$



$\therefore n \leq n \log a + 1$

$\forall n, a \in \mathbb{Z}, a \neq 0$

$n \log a < n$

$\Rightarrow \log_{10} a < 1$

$\Rightarrow a < 10$

but $a < 10$

$\therefore n \leq n \log a + 1 \quad \forall a < 10 \Rightarrow a \in (0, 10)$,

$\log 1 = 0 \Rightarrow$ ~~$n \leq 0+1$~~ $n \leq 1$, but $n \neq 0 \therefore n = 1$.

if $a = 1$: ↑

$a = 2$:

$\log 2 = 0.30103\ldots$

By deduction, to maximize n, we must maximize a $\therefore$  $\leftarrow \log x$

$\therefore$ let $a = 9$ and $n - n\log 9 \leq 1$

inverses are maximized when denominator is minimized!

$\Rightarrow n \leq n\log 9 + 1$

$\Rightarrow n - n\log 9 \leq 1$

$\Rightarrow n \leq 21.8$

$\therefore n \in [1, 21]$

Now to find how many n-digit positive integers exist w/ this property of also being an n^{th} power, we can cycle through $a \in (0, 10)$ manually and see if we find a trend w/ $n \in [1, 21]$.

$a = 1$: 9

$a = 2$: 6

$a = 3$:

numbers get too large to calculate manually. Lets find an algorithm / inequality: $\Rightarrow$

$a \in (0,10)$, $n \in (0,22)$

$\cancel{a^n}$ $10^{n-1} \leq a^n \cancel{\leq} < 10^n$

The answer are right in front of us w/ the same algorithm / inequality for $n \in (0,22)$ lets solve for a.

Can be sped up w/ code, but no need to write code here.

- Only consider left inequality, again
  $\therefore$ $n \log a < n$
  $\Rightarrow a < 10$, $a \in \mathbb{N}$

$\cancel{a \neq 1}$

$\cancel{n - \log n \leq 4}$

for $n \in (0,22)$, find how many $a \in (0,10)$ satisfy the requirements

$n=1$, $\Rightarrow n - n\log a \le 1$

$0 \le \log a$ $\Rightarrow n(1-\log a) \le 1$

$\Rightarrow 1 - \log a \le \frac{1}{n}$

$\log a > 0$ $\Rightarrow \log a \geq 1 - \frac{1}{n}$

$\therefore +9$

$n=2$: $\Rightarrow \cancel{\left(1-\frac{1}{n}\right) \log \le a}$

$\log a \geq 0.8$

$\therefore \boxed{\Rightarrow 10^{\left(1-\frac{1}{n}\right)} \le a}$

I extended the inequality to ease the calculations.

$10^{0.5} \le a$	$3.16 \le a$:	$+6$
$\vdots$		
$10^{\frac{2}{3}}$	4.64	$+5$
$10^{\frac{3}{4}}$	5.62	$+4$
$10^{4/5}$	6.3	$+3$
$10^{5/6}$	6.8	$+3$
$10^{6/7}$	7.20	$+2$
$10^{7/8}$	7.5	$+2$
$10^{8/9}$	7.74	$+2$
$10^{9/10}$	7.94	$+2$
$10^{10/11}$	8.11	1

Now that I have reached the point where only 1 n exists s.t. a^n has n digits, we can assume this is the case for this n_0 to n_{max} - 2

— proving this is logical intuitively and ~~[illegible]~~ doesn't require a rigorous proof for us.

∴ answer = 49 by summing

We assume that for $n \geq 11$ ($n \leq 21$) that only 1 a exists ~~since~~ (i.e. 9) since, we are just holding on at this point, soon exponentials without increasing the base that is $9^n, 9^{n+1}, 9^{21}$ for $a=9$ and $n \in [11, 21]$ cannot support an increase in digits per increase in exponent SINCE WE are multiplying by less than 10.

I got it right! 49!

I refused to deal w/ huge numbers in C++ (Java) or use Python and cheat! This was also more time efficient and clean.

I liked ~~how~~ I knew it would be able to be solved by hand (& calc.).

- Also solved this (and prior one ~~way~~ -kind of) after <u>outdoors</u> run and pushups. Man, the gym is powerfull!