7 The supply of a competitive firm

In economics, when we say that an agent is **competitive** (in a market) we mean that it takes the price (in that market) as given. That is, it cannot affect (significantly) the terms of trade in that market. Another way of saying that an agent is competitive is to say that the agent is a **price-taker**. The opposite of being competitive in a market is to have **market power**. Thus, competitive agents do not have market power.

Note that we have assumed (implicitly) that consumers were competitive in the markets of all the goods. Also, we have assumed that firms were competitive in the inputs markets: both treated the prices (of goods, in the case of consumers, of inputs, in the case of firms) as parameters they could not affect. In this section, we are going to assume that the firm is also competitive in the market where it sells its output.

A few words of why this may be. It is common to justify price-taking (if you think of it, a behavior assumption) as a result of the agent being small in size. Indeed, if there are many (similarly small) firms in a market, then there is little that a firm can do unilaterally to affect the terms of trade with its clients. By withholding output it cannot create scarcity, since buyers can always find alternative sources to buy the good. Likewise, by ramping up production it cannot clog the market, since it would still represent a tiny fraction of the total amount of the good available to buyers. Put in other words, if it pretends to charge a higher price (e.g., by withholding output), buyers will turn to alternative providers, and the price cut necessary to ramp up sales would be very tiny, to all effect negligible.

We now consider one such firm with access to some technology, described by the production function, f, which allows it to produce some good whose price in the market is (given, from its point of view and) equal to p. As we have said, that firm needs to decide how much to produce, q, and how to produce (i.e., using what combination of inputs to produce) that output. We have already analyzed this last decision, and we have summarized it with the conditional demand functions of inputs and the resulting cost function $C(q)$. Thus, the only thing left to study is the choice of q. Remember that the firm's objective when making these decisions is to maximize profits, i.e., the difference between revenues and spending. The latter is simply $C(q)$, once inputs are chosen optimally. The revenues, for a **competitive firm**, are simply $p \times q$: the price at which it can selle its ouput times the number of units it (produces and) sells.

Thus, a competitive firm's optimal choice of output is the solution to the problem:

$$\max_q \{p \times q - C(q)\}.$$

This is a simple problem indeed, and if it has an interior solution (that is, if it is optimal to produce at all), it needs to satisfy the first order condition:

$$p - MC(q) = 0.$$

Read this last expression. What it says is: the optimal q, **if positive**, is the level of output at which the cost of producing one more unit (or more precisely, the per-unit cost of producing a tiny little more) equals the extra revenue which that unit generates: the price. It makes sense, doesn't it? If the extra cost was smaller that the extra revenue, the firm should not stop at that q, but should produce that extra unit (and perhaps more). If the extra cost was smaller than the extra revenue (equivalently, the extra cost of producing the last unit was higher than the extra revenue), the firm should not have produced the last unit (and perhaps other units either).

Another way to write the same condition is:

$$q = MC^{-1}(p),$$

and so the right hand side is the mapping from prices to quantity: given price p, the optimal output is $MC^{-1}(p)$. (The notation MC^{-1} simply means inverse of the function MC. That is, literally, as we said before, the value of q such that MC attains the value p, which, remember, is given to the firm.

We can call this mapping, $MC^{-1}(p)$, the **supply function** of the firm, which is often written as $S(p)$. That is, $S(p) \equiv MC^{-1}(p)$ is the output that maximizes the profits of a competitive firm when the price in the market is p.

Remark 2 *Remember that the cost function typically depends also on the prices of inputs. (For compactness, we are not writting these prices.) Thus, the supply function, which is nothing but the marginal cost function inverted, also depends on these prices. If the price of an input increases, the marginal cost changes (typically it increases), and so the supply "curve" (q plotted against p) shifts (typically to the left: for any price, less output).*

7.1 Willingness to accept

When we obtained the demand function for a consumer, $D(p)$, we discuss how the inverse of that function, $P^d(q)$, represented the consumer's willingness to pay for an additional unit when she was consuming q units. (Or, more rigorously, the per-unit willingness to pay for a little tiny increase in consumption of that good.) We are using the superscript d now to indicate that we are talking about the **inverse demand**. What does the **inverse of the supply function**, which we can also represent by $P^s(q)$, measure?

First, note that we are talking about the inverse of $S(p) \equiv MC^{-1}(p)$: *the inverse of the inverse* of the marginal cost curve! The inverse of the inverse is... yes, the original function. In this case, the marginal cost curve: That is, $P^s(q) = MC(q)$: the supply function, when read from quantity to price, is just the marginal cost function. So it measures how much the firm has to spend to produce an additional unit. This is also what we can call the firm´s (marginal) *willingness to accept*. That is, if we were to ask the firm what would be the minimum that it would accept in exchange for one more unit of output when it is already selling (and producing) q units, wouldn't the answer be $MC(q)$?

Therefore, when we look at the inverse supply curve for a firm we can see the willingness to accept, just as when we looked at a consumer's inverse demand function we saw that consumer's willingness to pay. The latter is the value of the extra unit for the consumer, and the former is the cost of the extra unit for the firm.

7.2 Corner solution: shutting down

So, **in case the firm decides to produce a positive amount of output** (an interior solution), that quantity is $q = MC^{-1}(p)$. That is, it satisfies that $p = MC(q)$. Here "corner" could only mean $q = 0$: not producing at all, or "shutting down." Indeed, there is no upper bound on how much a (competitive) firm (that is, one that sees no effect of its decisions on input or output markets) can produce.

Thus, if the best output is not $q = MC^{-1}(p)$, it must be $q = 0$. When $q = 0$, the revenues are certainly 0. Thus, $q = 0$ means profits equal to

$$\pi(0) = -C(0).$$

Therefore, the firm should **shut down** if

$$p \times q^* - C(q^*) < -C(0),$$

where $q^* = MC^{-1}(p)$, and otherwise it should produce q^*.

Let us play around with this condition.Taking $C(q^*)$ to the right of the inequality and dividing both sides by q^*, we can write this same condition as

$$p < \frac{C(q^*) - C(0)}{q^*}.$$

But remember, at $q = q^*$, $MC(q^*) = p$. That is how we defined q^*: the quantity at which the price equalled the marginal cost. Therefore, we can also write the above condition as

$$MC(q^*) < \frac{C(q^*) - C(0)}{q^*}. \tag{16}$$

7.3 Short- and long-run, again

The discussion so far works just as well whether we are considering the firm's long-run or short-run decisions. That is, whether the firm is considering to leave the market for good or not enter it (long run) or to shut down temporarily (in the short run). Obviously, in one case the firm will consider the long-run cost functions and in the other the short-run cost functions.

Now look at condition (16). When the decision is how much to produce before starting operations or for a time when no input commitment is binding (long-run horizon), obviously the cost of no producing, $C(0)$, is simply 0. That is, in the right-hand side of (16) we simply have $\frac{C(q^*)}{q^*} = AC(q^*)$. Therefore, the firm would rather not produce at all (shut down) if, at q^* where $MC(q^*) = p$ the MC is below the AC.

Look at Figure 30. If the price in the market is p', then the condition (16) is satisfied: at the q at which $MC(q) = p'$ (q' in the figure), the MC is below the AC. Think of what this means: the per unit revenue, p' (which at that q equals the MC) is below the per unit cost, $AC(q)$. That is, the profits are negative at that output level. But remember: that is the **positive** output level that results in highest profit! Thus, there is nothing better than shutting down: for such prices, the firm's supply is 0.

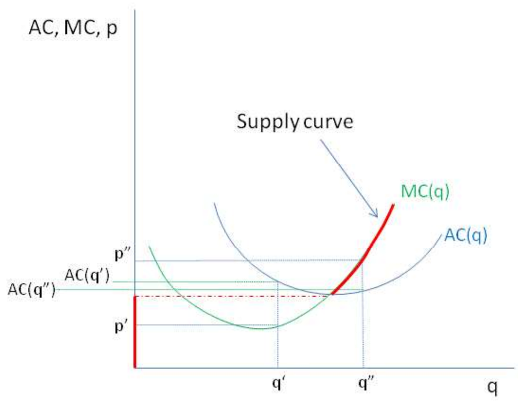

Figure 30

If the price is p”, then condition (16) is not satisfied, so at that price, and for q at which $MC(q) = p$” (q” in the figure), the per-unit revenue is higher than the per-unit cost, and so profits are positive. Thus, at that price the firm should indeed produce output q such that $MC(q) = p$”.

Note the difference between the prices p' and p”: the former is below the **minimum of the** AC **curve**, and the latter is above! Indeed, for any price below that minimum of the AC, the best option for the firm is to shut down, and for any price above, the best option is q at which the marginal cost equals the price!

We said above that the supply function is $S(p) = MC^{-1}(p)$. Now you realize that we were sloppy. To be precise,

$$S(p) = \begin{cases} MC^{-1}(p) & \text{if } p \geq \min_q AC(q) \\ 0 & \text{if } p < \min_q AC(q). \end{cases}$$

(Identify the above function in Figure 30, in red!)

Notice that one obvious implication is that, if **competitive**, the firm **never produces with increasing returns** to scale. Indeed, as we have argued, if the MC (equals p and) is above AC, then the AC is increasing, which means that the returns to scale are decreasing.

Let us turn to the short run. As we know, if the firm is deciding for the short run it will have some fixed inputs, which means that it will have some fixed costs. That is, $FC \equiv C(0) > 0$. In that case the numerator of the right-hand side of (16) is not the total cost, but the **variable cost**. That is, the firm should shut down if (but only if) $\frac{VC(q^*)}{q^*} = AVC(q^*)$ is below (p and so below) $MC_{SR}(q^*)$. In other words, if p, the per-unit revenue (equal to $MC_{SR}(q^*)$), is lower than the per-unit **avoidable** costs (the average variable costs). Intuitive, isn't it? Sunk is sunk, and so those costs that are not avoidable should not matter for deciding whether to produce or not.

There is nothing mysterious in this: in the short run, a firm may still produce even if the price is below the minimum average cost (and so have negative profit) as long as it would have an even more negative profit by not producing, $\pi(0) = -C(0)$. This is the case in Figure 31 for price p'.

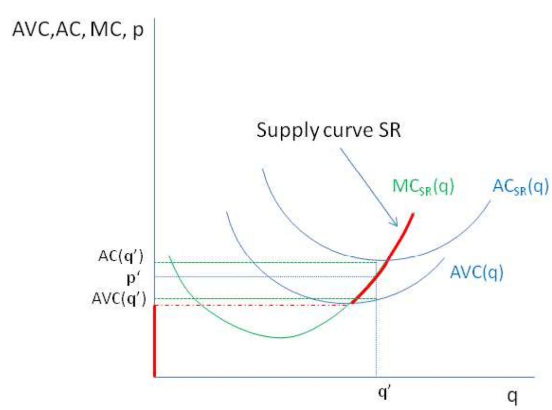

Figure 31

Thus, the supply function in the short run is

$$S_{SR}(p) = \begin{cases} MC_{SR}^{-1}(p) & \text{if } p \geq \min_q AVC(q) \\ 0 & \text{if } p < \min_q AVC(q). \end{cases}$$

Remark 3 *Note the subscript SR in MC^{-1}. The marginal cost is different in the long-run (where increasing output may be done changing the amount of all inputs) and in the short-run (where increasing output may not be done changing all inputs). However, as opposed to what happens with average costs, the marginal cost in the short run (for a given level of fixed input) needs not be higher than the long-run marginal cost for all output levels. Indeed, for levels of output for which the fixed input is excessive (from a long-run point of view), the marginal cost is lower in the short run than in the long run, and vice versa. You can check that with our example in expression (14) with $w = r = 1$ and $K = 100$ in the short run: For $q < 10,000$ (for which in the long run the conditional demand for K is below 100), $MC_{SR}(q) < MC(q)$. For $q > 10,000$ the opposite is true. Intuitively, when K is excessive for q, the firm has more K than it wants, and so it would be able to –and interested in– increase output using a lower increase of the other inputs.*

7.4 Market supply function

We have obtained the supply function of a competitive firm. In a competitive market there will be many such firms. So, what is the total supply of one good traded in a competitive market. The answer is simple: as in the case of the market demand, the market supply function will simply be the **horizontal sum** of all the firms' supply functions. That is, for each price p the market supply will be the sum of all the quantities supplied by all firms in the market.

For instance, if there are two firms (you know that in a competitive market we expect to encounter more than two firms, or put in other words, if we encounter only two firms we do not expect them to take the price as given), say firm 1 and firm 2, and their supply function is, respectively, $S_1(p)$ and $S_2(p)$, then the market supply is simply

$$S(p) = S_1(p) + S_2(p).$$

The only thing you should be careful with is not to add the supplies vertically. That is, as in the case of consumers, not to add the willingness to accept (the inverse supply function, i.e., the marginal cost functions) for each quantity, but to add the supplied quantity for each price!