Alexandra Chang

Homework 1

Please upload only one pdf file as your solutions to Brightspace.

1. True or False Questions

Please write T (for true) or F (for false) in the parenthesis. A correct answer is worth 1 pt and a wrong answer is 0 pts.

For problems 1(a)-1(h), assume $Y_1, Y_2, \ldots, Y_n$ are a random sample from $N(\mu, \sigma^2)$ with the sample mean $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ and sample variance S^2.

(a) $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is $N(\mu, \sigma^2)$.

(b) $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is $N(\mu, \sigma^2/n)$.

(c) $\frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$.

(d) $\frac{\bar{Y}-\mu}{S/\sqrt{n}} \sim N(0,1)$.

(e) $\frac{\bar{Y}-\mu}{S/\sqrt{n}} \sim t_{n-1}$.

(f) $\bar{Y}$ and S are independent.

(g) S^2 has a χ^2 distribution.

(h) $\left(\frac{\bar{Y}-\mu}{S/\sqrt{n}}\right)^2$ has an F distribution.

2. Short Answer Questions

(a) [3 pts] Explain the difference or differences between a treatment factor and a treatment.

(b) [3 pts] Explain some differences between a completely randomized design and a randomized complete block design.

(c) [3 pts] What are some potential negative consequences if the treatments are assigned to experimental units without randomization?

3. Written Response Questions

Give an example design for each of the following: 1) Completely randomized design; 2) Block design; 3) Split-plot design. For each design, specify the treatment factors and their levels. You may come up with designs of your own or use examples in literature (books, papers or Internet). If you use an example in literature, please provide references. 5 pts for each design.

1. True or False Questions

False (a) $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is $N(\mu, \sigma^2)$. * Variance of sample mean is σ^2/n (Should not be reduced variance.*)

True (b) $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is $N(\mu, \sigma^2/n)$.

True (c) $\frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$.

False (d) $\frac{\bar{Y}-\mu}{S/\sqrt{n}} \sim N(0,1)$. When the pop. standard deviation (θ) is unknown and using sample standard deviation (S), result follows t-distribution $\sim t_{n-1}$

True (e) $\frac{\bar{Y}-\mu}{S/\sqrt{n}} \sim t_{n-1}$.

True (f) $\bar{Y}$ and S are independent.

False (g) S^2 has a χ^2 distribution. Cochran's Theorem: $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

True (h) $\left(\frac{\bar{Y}-\mu}{S/\sqrt{n}}\right)^2$ has an F distribution.

2. Short Answer Questions

(a) [3 pts] Explain the difference or differences between a treatment factor and a treatment.

- Treatment factor: variable whose effect we want to study. Ex) Fertilizer type, Temperature ...
- Treatment: specific level, or combination of levels of one or more treatment factors.

(b) [3 pts] Explain some differences between a completely randomized design and a randomized complete block design.

- Completely Randomized Design (CRD):
  - ↳ no blocking; all units treated homogeneous.
  - ↳ Treatments assigned randomly across all units.
  - ↳ Simpler to implement and analyze.
  - ↳ More sensitive to variation among units.

- Randomized Complete Block Design (RCBD):
  - ↳ Experimental units grouped into blocks (based on known nuisance variables)
  - ↳ Randomization happens within each block.
  - ↳ Each treatment appears once per block.
  - ↳ Controls for variability between blocks, increasing precision.

(c) [3 pts] What are some potential negative consequences if the treatments are assigned to experimental units without randomization?

- Introduce bias from confounding variables (eg. time, location, demographics).
- Violates assumptions of independence, reducing validity of statistical tests.
- Results may introduce systematic errors rather than treatment effects.
- Reduces the generalizability of findings.
- Weakens causal inference and compromises internal validity.

3. Written Response Questions

Give an example design for each of the following: 1) Completely randomized design; 2) Block design; 3) Split-plot design. For each design, specify the treatment factors and their levels. You may come up with designs of your own or use examples in literature (books, papers or Internet). If you use an example in literature, please provide references. 5 pts for each design.

1) Completely Randomized Design:

- Purpose: Studying the effects of 3 different stretching routine on athletic flexibility.
- Treatment factor: Types of Stretching
  - ↳ Level 1: Static Stretching
  - ↳ Level 2: Dynamic Stretching
  - ↳ Level 3: No stretching (control)
- Design: 30 athletes randomly assigned to one of the 3 groups (10 per group).
  Each athlete follows the assigned daily routine for 4 weeks, then takes a sit-and-reach test.

(Note: No blocks used b/c athletes are assumed to be similar in age and training background).

2) Block Design:

- Purpose: Testing the effectiveness of 3 online learning platforms on student performance.
- Treatment factor: Learning Platform
  - ↳ Level 1: Platform A
  - ↳ Level 2: Platform B
  - ↳ Level 3: Platform C
- Blocking factor: Prior GPA (High, Medium, Low)
- Design: Students are grouped into GPA blocks (eg. 6 students per block)
  Within each block, students are randomly assigned to one of the 3 platforms.
  After 2 months, students take the same standardized exam.

3) Split-plot Design:

- Purpose: Investigate how screen size and view quality affect view satisfaction.
- Whole-plot factor: Screen Size
  - ↳ Level 1: Small (13 inches)
  - ↳ Level 2: Large (27 inches)
- Sub-plot factor: Video Quality
  - ↳ Level 1: 480p
  - ↳ Level 2: 720p
  - ↳ Level 3: 1080p
- Design: 6 rooms are used: 3 with small screens, 3 with large screens (randomly assigned).
  In each room, 3 viewers watch a different quality level. (randomly assigned).
  Satisfaction rated on a 1-10 scale.

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(a) $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is $N(\mu, \sigma^2)$. False

(b) $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is $N(\mu, \sigma^2/n)$. True

(c) $\frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$. True

(d) $\frac{\bar{Y}-\mu}{S/\sqrt{n}} \sim N(0,1)$. False

(e) $\frac{\bar{Y}-\mu}{S/\sqrt{n}} \sim t_{n-1}$. True

(f) $\bar{Y}$ and S are independent. True

(g) S^2 has a χ^2 distribution. False

(h) $\left(\frac{\bar{Y}-\mu}{S/\sqrt{n}}\right)^2$ has an F distribution. True

2. Short Answer Questions

(a) [3 pts] Explain the difference or differences between a treatment factor and a treatment.

Give full credit as long as the answer is not completely wrong.

(b) [3 pts] Explain some differences between a completely randomized design and a randomized complete block design.

The former involves no blocks while the latter does; They also differ in how randomization is carried out. For the former, random assignments of units to treatments are done completely at random. For the latter, randomization is carried out within each block.

(c) [3 pts] What are some potential negative consequences if the treatments are assigned to experimental units without randomization?

It is likely to produce biased results that are not caused by the treatments.

3. Written Response Questions

Give an example design for each of the following: 1) Completely randomized design; 2) Block design; 3) Split-plot design. For each design, specify the treatment factors and their levels. You may come up with designs of your own or use examples in literature (books, papers or Internet). If you use an example in literature, please provide references. 5 pts for each design.

Give full credit if the answer seems sensible. Take off 2.5 points if they missed an important detail, but had the right idea. If the answer doesn't make any sense at all or there is no answer, then they lose all 5 points