Alexandra Chang

Homework 7

1. The Paper Towel Experiment

The paper towel absorbency experiment is described on page 303. Three brands of paper towels were compared (factor A at 3 levels). For each brand, both white and printed towels were evaluated (factor B, 1 = white, 2 = printed). For each observation, water was dripped from above a towel, which was horizontally suspended between two pairs of books on a flat surface, until the water began leaking through to the surface below. The time to collect each observation was measured in seconds. Absorbency was measured as the number of water drops absorbed per square inch of towel. The rate at which the water droplets fell to the towel was measured (in drops per second) as a covariate. The data are reproduced in Table 9.9.

(a) [3 pts] Plot absorbency versus rate, using the treatment level as the plotting symbol. Based on the plot, discuss the appropriateness of the analysis of covariance model, and discuss whether there appear to be treatment effects.

(b) [3 pts] Fit the one-way analysis of covariance model to the data in which you shall include the treatment effects and the covariate. Show your SAS code and the ANOVA table.

(c) [2 pts] Test for equality of treatment effects, using a significance level $\alpha = 0.05$.

(d) [2 pts] What is the estimate of the slope for the covariate? What does the negative value of this estimate say about the effects of rate on the absorbency? [Hint: consider the solution option in the model statement.]

(e) [2 pts] Test the hypothesis $H_0 : \beta = 0$ versus $H_1 : \beta \neq 0$ using a significance level $\alpha = 0.05$, where β denotes the slope for the covariate. Find the p-value and state your conclusion.

2. The Video Game Experiment

The video game experiment is described on page 426.

(a) [3 pts] Write down an appropriate model for this experiment. Explain each term in the model.

(b) [3 pts] Provide 95% simultaneous confidence intervals for all pairwise comparisons of the treatment using the best method that yields the shortest intervals.

(c) [3 pts] Based on the intervals in (b), which sound mode(s) do you recommend or not recommend? Specify your reasons.

(d) [2 pts] Find a 95% confidence interval for the "music versus no music" contrast $\frac{1}{3}(\tau_1+\tau_2+\tau_3)-\frac{1}{2}(\tau_4+\tau_5)$.

1. The Paper Towel Experiment

The paper towel absorbency experiment is described on page 303. Three brands of paper towels were compared (factor A at 3 levels). For each brand, both white and printed towels were evaluated (factor B, 1 = white, 2 = printed). For each observation, water was dripped from above a towel, which was horizontally suspended between two pairs of books on a flat surface, until the water began leaking through to the surface below. The time to collect each observation was measured in seconds. Absorbency was measured as the number of water drops absorbed per square inch of towel. The rate at which the water droplets fell to the towel was measured (in drops per second) as a covariate. The data are reproduced in Table 9.9.

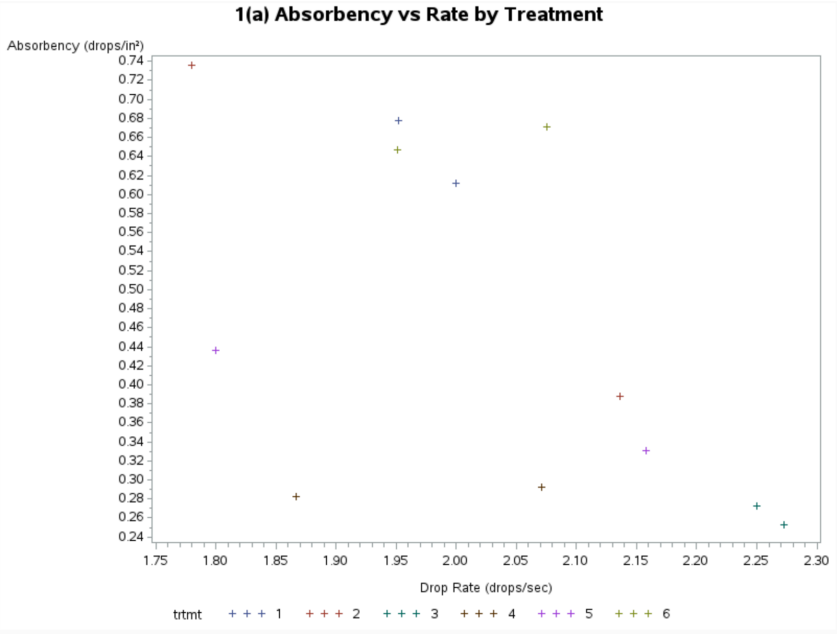
Table 9.9 Data for the paper tower absorbancy experiment

Run	Treatment	AB	Drops	Time	Area	Rate	Absorbancy
1	2	12	89	50	121.00	1.780	0.7355
2	4	22	28	15	99.00	1.867	0.2828
3	2	12	47	22	121.00	2.136	0.3884
4	1	11	82	42	121.00	1.952	0.6777
5	5	31	54	30	123.75	1.800	0.4364
6	1	11	74	37	121.00	2.000	0.6116
7	4	22	29	14	99.00	2.071	0.2929
8	6	32	80	41	123.75	1.951	0.6465
9	3	21	25	11	99.00	2.272	0.2525
10	3	21	27	12	99.00	2.250	0.2727
11	6	32	83	40	123.75	2.075	0.6707
12	5	31	41	19	123.75	2.158	0.3313

(a) [3 pts] Plot absorbency versus rate, using the treatment level as the plotting symbol. Based on the plot, discuss the appropriateness of the analysis of covariance model, and discuss whether there appear to be treatment effects.

```
data papertowel;
  input run trtmt ab drops time area rate absorb;
  lines;
1 2 12 89 50 121.00 1.780 0.7355
2 4 22 28 15  99.00 1.867 0.2828
3 2 12 47 22 121.00 2.136 0.3884
4 1 11 82 42 121.00 1.952 0.6777
5 5 31 54 30 123.75 1.800 0.4364
6 1 11 74 37 121.00 2.000 0.6116
7 4 22 29 14  99.00 2.071 0.2929
8 6 32 80 41 123.75 1.951 0.6465
9 3 21 25 11  99.00 2.273 0.2525
10 3 21 27 12  99.00 2.250 0.2727
11 6 32 83 40 123.75 2.075 0.6707
12 5 31 41 19 123.75 2.158 0.3313
;
run;

proc gplot data=papertowel;
  plot absorb*rate = trtmt;
  label rate   = "Drop Rate (drops/sec)"
        absorb = "Absorbency (drops/in²)";
  title "1(a) Absorbency vs Rate by Treatment";
run;
```



- Parallelism of the treatment-specific trends
  - ↳ All 6 treatments symbols lie roughly along the same downward-sloping cloud of points.
  - ↳ No obvious "fan-shaped" or crossing pattern that would suggest different slopes for different treatments.
    - ⇒ The equal-slopes assumption of ANCOVA (single common β) appears reasonable.

- Vertical separation at a given Rate
  - ↳ For any fixed drop-rate, some treatments' symbols tend to cluster above others.
  - ↳ This vertical offset indicates differences in adjusted mean absorbancy among treatments.
    - ⇒ There are treatment effects worth modeling.

(b) [3 pts] Fit the one-way analysis of covariance model to the data in which you shall include the treatment effects and the covariate. Show your SAS code and the ANOVA table.

```
1  data papertowel;
2    input run trtmt ab drops time area rate absorb;
3    lines;
4  1 2 12 89 50 121.00 1.780 0.7355
5  2 4 22 28 15  99.00 1.867 0.2828
6  3 2 12 47 22 121.00 2.136 0.3884
7  4 1 11 82 42 121.00 1.952 0.6777
8  5 5 31 54 30 123.75 1.800 0.4364
9  6 1 11 74 37 121.00 2.000 0.6116
10 7 4 22 29 14  99.00 2.071 0.2929
11 8 6 32 80 41 123.75 1.951 0.6465
12 9 3 21 25 11  99.00 2.273 0.2525
13 10 3 21 27 12  99.00 2.250 0.2727
14 11 6 32 83 40 123.75 2.075 0.6707
15 12 5 31 41 19 123.75 2.158 0.3313
16 ;
17 run;
18
19 proc glm data = papertowel;
20 class trtmt;
21 model absorb=trtmt rate/solution;
22 run;
```

The GLM Procedure

Class Level Information		
Class	Levels	Values
trtmt	6	1 2 3 4 5 6

Number of Observations Read	12
Number of Observations Used	12

The GLM Procedure

Dependent Variable: absorb

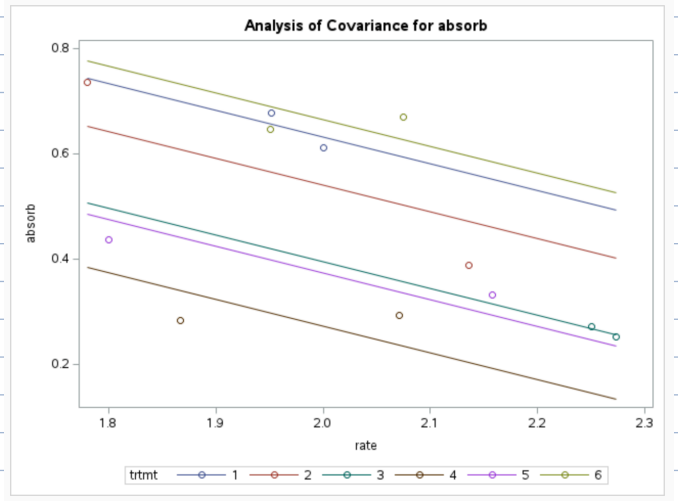
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	0.35669848	0.05944975	10.64	0.0101
Error	5	0.02794128	0.00558826		
Corrected Total	11	0.38463976			

R-Square	Coeff Var	Root MSE	absorb Mean
0.927357	16.02171	0.074755	0.466583

Source	DF	Type I SS	Mean Square	F Value	Pr > F
trtmt	5	0.31614510	0.06322902	11.31	0.0093
rate	1	0.04055338	0.04055338	7.26	0.0431

Source	DF	Type III SS	Mean Square	F Value	Pr > F
trtmt	5	0.24198648	0.04839730	8.66	0.0167
rate	1	0.04055338	0.04055338	7.26	0.0431

Parameter	Estimate		Standard Error	t Value	Pr > |t|
Intercept	1.680495956	B	0.38300747	4.39	0.0071
trtmt 1	-0.032732986	B	0.07507910	-0.44	0.6810
trtmt 2	-0.124570655	B	0.07546973	-1.65	0.1597
trtmt 3	-0.269849406	B	0.08821112	-3.06	0.0281
trtmt 4	-0.393086524	B	0.07521308	-5.23	0.0034
trtmt 5	-0.292010041	B	0.07502871	-3.89	0.0115
trtmt 6	0.000000000	B	.	.	.
rate	-0.507648264		0.18844626	-2.69	0.0431



(c) [2 pts] Test for equality of treatment effects, using a significance level $\alpha = 0.05$.

trtmt p-value: 0.0167 > 0.05

Using result from (b), we reject H_0, meaning significant differences among treatment means.

(d) [2 pts] What is the estimate of the slope for the covariate? What does the negative value of this estimate say about the effects of rate on the absorbency? [Hint: consider the solution option in the model statement.]

rate = -0.5076.

Parameter	Estimate		Standard Error	t Value	Pr > |t|
Intercept	1.680495956	B	0.38300747	4.39	0.0071
trtmt 1	-0.032732986	B	0.07507910	-0.44	0.6810
trtmt 2	-0.124570655	B	0.07546973	-1.65	0.1597
trtmt 3	-0.269849406	B	0.08821112	-3.06	0.0281
trtmt 4	-0.393086524	B	0.07521308	-5.23	0.0034
trtmt 5	-0.292010041	B	0.07502871	-3.89	0.0115
trtmt 6	0.000000000	B	.	.	.
rate	-0.507648264		0.18844626	-2.69	0.0431

- Negative value: As rate ↑, absorbancy ↓.
  - ↳ Higher rates of water droplets falling on the towel result in ↓ absorbancy.

(e) [2 pts] Test the hypothesis $H_0 : \beta = 0$ versus $H_1 : \beta \neq 0$ using a significance level $\alpha = 0.05$, where β denotes the slope for the covariate. Find the p-value and state your conclusion.

rate p-value: 0.0431 > 0.05

Using result from (b), we reject H_0, meaning slope is significantly different from 0.

2. The Video Game Experiment

The video game experiment is described on page 426.

Video game experiment

Professor Robert Wardrop, of the University of Wisconsin, conducted an experiment in 1991 to evaluate in which of five sound modes he best played a certain video game. The first three sound modes corresponded to three different types of background music, as well as game sounds expected to enhance play. The fourth mode had game sounds but no background music. The fifth mode had no music or game sounds. Denote these sound modes by the treatment factor levels 1–5, respectively.
The experimenter observed that the game required no warm up, that boredom and fatigue would be a factor after 4–6 games, and that his performance varied considerably on a day-to-day basis. Hence, he used a Latin square design, with the two blocking factors being "day" and "time order of the game." The response measured was the game score, with higher scores being better. The design and resulting data are given in Table 12.16.

Table 12.16 Latin square design showing treatments and data for the video game experiment

		Day									
		1		2		3		4		5	
Time order	1	1	94	3	100	4	98	2	101	5	112
	2	3	103	2	111	1	51	5	110	4	90
	3	4	114	1	75	5	94	3	85	2	107
	4	5	100	4	74	2	70	1	93	3	106
	5	2	106	5	95	3	81	4	90	1	73

(a) [3 pts] Write down an appropriate model for this experiment. Explain each term in the model.

Standard Latin Square Design:

$$Y_{hqi} = \mu + \theta_h + \phi_q + T_i + \varepsilon_{hqi}$$

Y_{hqi} : Response
μ : Overall Mean
T_i : Treatment Effect
θ_h : Row Effect
ϕ_q : Column Effect
ε_{hqi} : Random Error

↓

2b ~ 2d

(b) [3 pts] Provide 95% simultaneous confidence intervals for all pairwise comparisons of the treatment using the best method that yields the shortest intervals.

```
DATA VIDEO;
  INPUT Order Day TRTMT Y;
  LINES;
1 1 1 94
1 2 3 100
1 3 4 98
1 4 2 101
1 5 5 112
2 1 3 103
2 2 2 111
2 3 1 51
2 4 5 110
2 5 4 90
3 1 4 114
3 2 1 75
3 3 5 94
3 4 3 85
3 5 2 107
4 1 5 100
4 2 4 74
4 3 2 70
4 4 1 93
4 5 3 106
5 1 2 106
5 2 5 95
5 3 3 81
5 4 4 90
5 5 1 73
;
RUN;
```

```
PROC GLM DATA=VIDEO;
  CLASS Order Day TRTMT;
  MODEL Y = Order Day TRTMT / SOLUTION;
  LSMEANS TRTMT / PDIFF=ALL CL ADJUST=TUKEY;
  TITLE '2(b) Tukey Simultaneous 95% CIs for Treatment Differences';
RUN;
```

2(b) Tukey Simultaneous 95% CIs for Treatment Differences

The GLM Procedure

Class Level Information		
Class	Levels	Values
Order	5	1 2 3 4 5
Day	5	1 2 3 4 5
TRTMT	5	1 2 3 4 5

Number of Observations Read	25
Number of Observations Used	25

2(b) Tukey Simultaneous 95% CIs for Treatment Differences

The GLM Procedure

Dependent Variable: Y

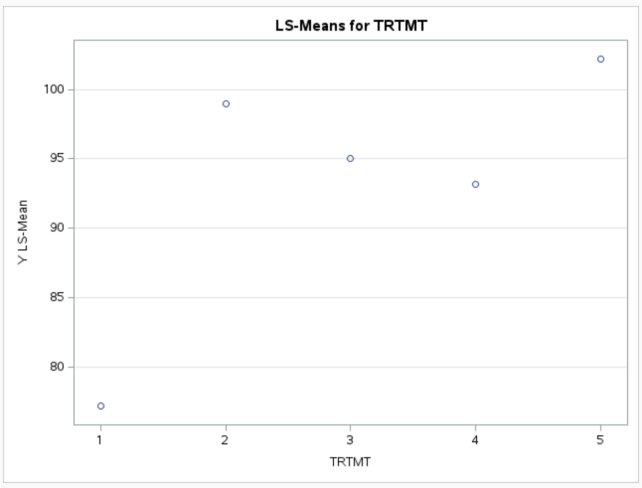
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	12	4094.720000	341.226667	2.34	0.0774
Error	12	1748.720000	145.726667		
Corrected Total	24	5843.440000			

R-Square	Coeff Var	Root MSE	Y Mean
0.700738	12.93584	12.07173	93.32000

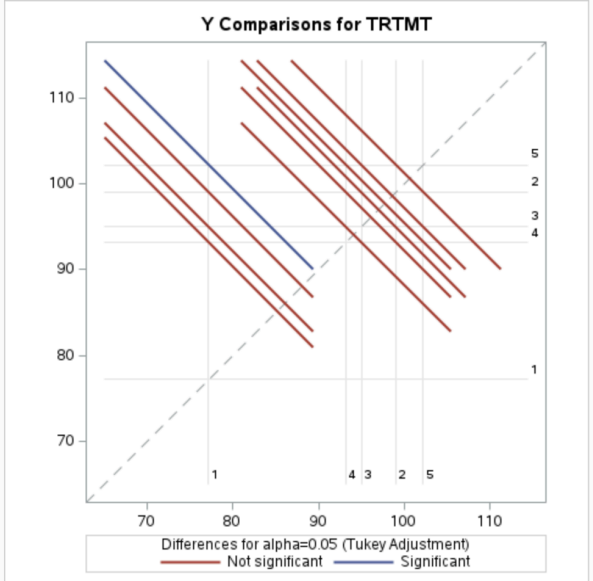
Source	DF	Type I SS	Mean Square	F Value	Pr > F
Order	4	514.240000	128.560000	0.88	0.5033
Day	4	1711.440000	427.860000	2.94	0.0661
TRTMT	4	1869.040000	467.260000	3.21	0.0523

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Order	4	514.240000	128.560000	0.88	0.5033
Day	4	1711.440000	427.860000	2.94	0.0661
TRTMT	4	1869.040000	467.260000	3.21	0.0523

Parameter	Estimate		Standard Error	t Value	Pr > |t|
Intercept	102.1600000	B	8.70504834	11.74	<.0001
Order 1	12.0000000	B	7.63483246	1.57	0.1420
Order 2	4.0000000	B	7.63483246	0.52	0.6099
Order 3	6.0000000	B	7.63483246	0.79	0.4472
Order 4	-0.4000000	B	7.63483246	-0.05	0.9591
Order 5	0.0000000	B	.	.	.
Day 1	5.8000000	B	7.63483246	0.76	0.4621
Day 2	-6.6000000	B	7.63483246	-0.86	0.4043
Day 3	-18.8000000	B	7.63483246	-2.46	0.0299
Day 4	-1.8000000	B	7.63483246	-0.24	0.8176
Day 5	0.0000000	B	.	.	.
TRTMT 1	-25.0000000	B	7.63483246	-3.27	0.0066
TRTMT 2	-3.2000000	B	7.63483246	-0.42	0.6825
TRTMT 3	-7.2000000	B	7.63483246	-0.94	0.3643
TRTMT 4	-9.0000000	B	7.63483246	-1.18	0.2613
TRTMT 5	0.0000000	B	.	.	.



Least Squares Means for Effect TRTMT				
i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	-21.800000	-46.135262	2.535262
1	3	-17.800000	-42.135262	6.535262
1	4	-16.000000	-40.335262	8.335262
1	5	-25.000000	-49.335262	-0.664738
2	3	4.000000	-20.335262	28.335262
2	4	5.800000	-18.535262	30.135262
2	5	-3.200000	-27.535262	21.135262
3	4	1.800000	-22.535262	26.135262
3	5	-7.200000	-31.535262	17.135262
4	5	-9.000000	-33.335262	15.335262



(c) [3 pts] Based on the intervals in (b), which sound mode(s) do you recommend or not recommend? Specify your reasons.

TRTMT 1	-25.0000000
TRTMT 2	-3.2000000
TRTMT 3	-7.2000000
TRTMT 4	-9.0000000
TRTMT 5	0.0000000

TRTMT 1: -25.00 ∘ Not Recommended:
↳ Lowest mean

TRTMT 5: 0.00 ∘ Recommended:
↳ Highest mean

(d) [2 pts] Find a 95% confidence interval for the "music versus no music" contrast $\frac{1}{3}(\tau_1+\tau_2+\tau_3)-\frac{1}{2}(\tau_4+\tau_5)$.

Estimate $= \frac{1}{3}(-25.00 - 3.20 - 7.20) - \frac{1}{2}(-9.00 - 0.00) = -7.30$

$c_1 = c_2 = c_3 = 1/3$
$c_4 = c_5 = -1/2$

$$\sum_{i=1}^{5} \frac{c_i^2}{n_i} = \frac{3(1/3)^2 + 2(1/2)^2}{5} = \frac{1/3 + 1/2}{5} = \frac{5/6}{5} = 1/6$$

MSE = 145.73

⇓

$$\text{Standard Error} = \sqrt{MSE \times \sum_{i=1}^{5} \frac{c_i^2}{n_i}} = \sqrt{145.73 \cdot 1/6} = 4.928$$

$t_{0.025,12} = 2.179$

Hence, the CI for the contrast: $-7.3 \pm 2.179 \times 4.928 \approx -7.3 \pm 10.73$

$(-18.03, 3.43)$

Homework 7

1. The Paper Towel Experiment

The paper towel absorbency experiment is described on page 303. Three brands of paper towels were compared (factor A at 3 levels). For each brand, both white and printed towels were evaluated (factor B, 1 = white, 2 = printed). For each observation, water was dripped from above a towel, which was horizontally suspended between two pairs of books on a flat surface, until the water began leaking through, the surface below. The time, collect each observation was measured in seconds. Absorbency was measured as the number of water drops absorbed per square inch of towel. The rate at which the water droplets fell, the towel was measured (in drops per second) as a covariate. The data are reproduced in Table 9.9.

(a) [3 pts] Plot absorbency versus rate, using the treatment level as the plotting symbol. Based on the plot, discuss the appropriateness of the analysis of covariance model, and discuss whether there appear, be treatment effects.

Full points if figure appears, be correct.

(b) [3 pts] Fit the one-way analysis of covariance model, the data in which you shall include the treatment effects and the covariate. Show your SAS code and the ANOVA table.

SAS Code for Solution:

```
* paper.towel.absorbancy.sas, paper towel absorbancy experiment, Table 9.9, p303;
;
data papertowel;
input run trtmt AB drops time area rate absorb;
lines;
1 2 12 89 50 121.00 1.780 0.7355
2 4 22 28 15 99.00 1.867 0.2828
3 2 12 47 22 121.00 2.136 0.3884
4 1 11 82 42 121.00 1.952 0.6777
5 5 31 54 30 123.75 1.800 0.4364
6 1 11 74 37 121.00 2.000 0.6116
7 4 22 29 14 99.00 2.071 0.2929
8 6 32 80 41 123.75 1.951 0.6465
9 3 21 25 11 99.00 2.273 0.2525
10 3 21 27 12 99.00 2.250 0.2727
11 6 32 83 40 123.75 2.075 0.6707
12 5 31 41 19 123.75 2.158 0.3313

;
run;
proc print;
run;

proc glm data = papertowel;
class trtmt;
model absorb=trtmt rate/solution;
```

```
run;
```

The ANOVA table for the one-way analysis of covariance model is as follows:

Source	DF	Type III SS	Mean Square	F Value	Pr > F
trtmt	5	0.24198648	0.04839730	8.66	0.0167
rate	1	0.04055338	0.04055338	7.26	0.0431

(c) [2 pts] Test for equality of treatment effects, using a significance level $\alpha = 0.05$.

For $\alpha = 0.05$, we reject the null hypothesis. Hence, there are significant differences among the treatment means.

(d) [2 pts] What is the estimate of the slope for the covariate? What does the negative value of this estimate say about the effects of rate on the absorbency? [Hint: consider the solution option in the model statement.]

The estimate of the slope for the covariate is -0.5076. The negative value of this estimate indicates that as the rate increases, the absorbency decreases. This suggests that higher rates of water droplets falling on the towel result in lower absorbency.

(e) [2 pts] Test the hypothesis $H_0 : \beta = 0$ versus $H_1 : \beta \neq 0$ using a significance level $\alpha = 0.05$, where β denotes the slope for the covariate. Find the p-value and state your conclusion.

The p-value for the test is 0.0431. We reject the null hypothesis for $\alpha = 0.05$. Hence, the slope is significantly different from 0.

2. The Video Game Experiment

The video game experiment is described on page 426.

(a) [3 pts] Write down an appropriate model for this experiment. Explain each term in the model.

The design is a standard Latin square design. The model can be written as:

$$Y_{hqi} = \mu + \theta_h + \phi_q + \tau_i + \varepsilon_{hqi}$$

where Y_{hqi} is the response, μ is the overall mean, τ_i is the treatment effect, θ_h is the row effect, ϕ_q is the column effect, and ε_{hqi} is the random error.

(b) [3 pts] Provide 95% simultaneous confidence intervals for all pairwise comparisons of the treatment using the best method that yields the shortest intervals.

SAS Code for Solution:

```
DATA VIDEO;
INPUT ORDER DAY TRTMT Y;
LINES;
1 1 1 94
1 2 3 100
1 3 4 98
1 4 2 101
1 5 5 112
2 1 3 103
2 2 2 111
2 3 1 51
```

```
2 4 5 110
2 5 4 90
3 1 4 114
3 2 1 75
3 3 5 94
3 4 3 85
3 5 2 107
4 1 5 100
4 2 4 74
4 3 2 70
4 4 1 93
4 5 3 106
5 1 2 106
5 2 5 95
5 3 3 81
5 4 4 90
5 5 1 73
;

PROC GLM DATA=VIDEO;
CLASS ORDER DAY TRTMT;
MODEL Y=ORDER DAY TRTMT / SOLUTION;
LSMEANS TRTMT /PDIFF=ALL CL ADJUST=TUKEY;

RUN;
```

The simultaneous confidence intervals using Tukey's method are as follows:

i	j	Difference Between Means	Simultaneous 95% Confidence Limits
1	2	-21.800000	(-46.135262, 2.535262)
1	3	-17.800000	(-42.135262, 6.535262)
1	4	-16.000000	(-40.335262, 8.335262)
1	5	-25.000000	(-49.335262, -0.664738)
2	3	4.000000	(-20.335262, 28.335262)
2	4	5.800000	(-18.535262, 30.135262)
2	5	-3.200000	(-27.535262, 21.135262)
3	4	1.800000	(-22.535262, 26.135262)
3	5	-7.200000	(-31.535262, 17.135262)
4	5	-9.000000	(-33.335262, 15.335262)

(c) [3 pts] Based on the intervals in (b), which sound mode(s) do you recommend or not recommend? Specify your reasons.

Treatment 1 has a significantly lower mean than any other treatment. Therefore, I do not recommend it. Treatment 5 has a significantly higher mean than other treatments, which I recommend.

(d) [2 pts] Find a 95% confidence interval for the "music versus no music" contrast $\frac{1}{3}(\tau_1+\tau_2+\tau_3)-\frac{1}{2}(\tau_4+\tau_5)$.

The estimate and standard error for the "music versus no music" contrast are given by:

$$\text{Estimate} = -7.30000000, \quad \text{Standard Error} = 4.92826316$$

With 12 degrees of freedom, $t_{0.025,12} = 2.1788128$. The confidence interval is given by:

$$-7.3 \pm 2.179 \times 4.928 \approx -7.3 \pm 10.73$$

Hence, the 95% confidence interval is approximately $(-18.03, 3.43)$.