STAT 525

Chapter 24
Multi-Factor Studies

Dr. Qifan Song

Data for Three-Factor ANOVA

- Y is the response variable
- Factor A has levels $i = 1, 2, ..., a$
- Factor B has levels $j = 1, 2, ..., b$
- Factor C has levels $k = 1, 2, ..., c$
- Y_{ijkl} is the l^{th} observation from cell (i, j, k)
- Now $l = 1, 2, ..., n_{ijk}$

Example (Page 1005)

- Interested in the influence of several factors on exercise tolerance
- Age range 25-35 years old
- Three factors were
  - Gender ($a = 2$)
  - Percent body fat ($b = 2$)
  - Smoking history ($c = 2$)
- Y is exercise tolerance (minutes until fatigue on a bicycle test)
- Set up as balanced design ($n = 3$)

General Plan (As Always)

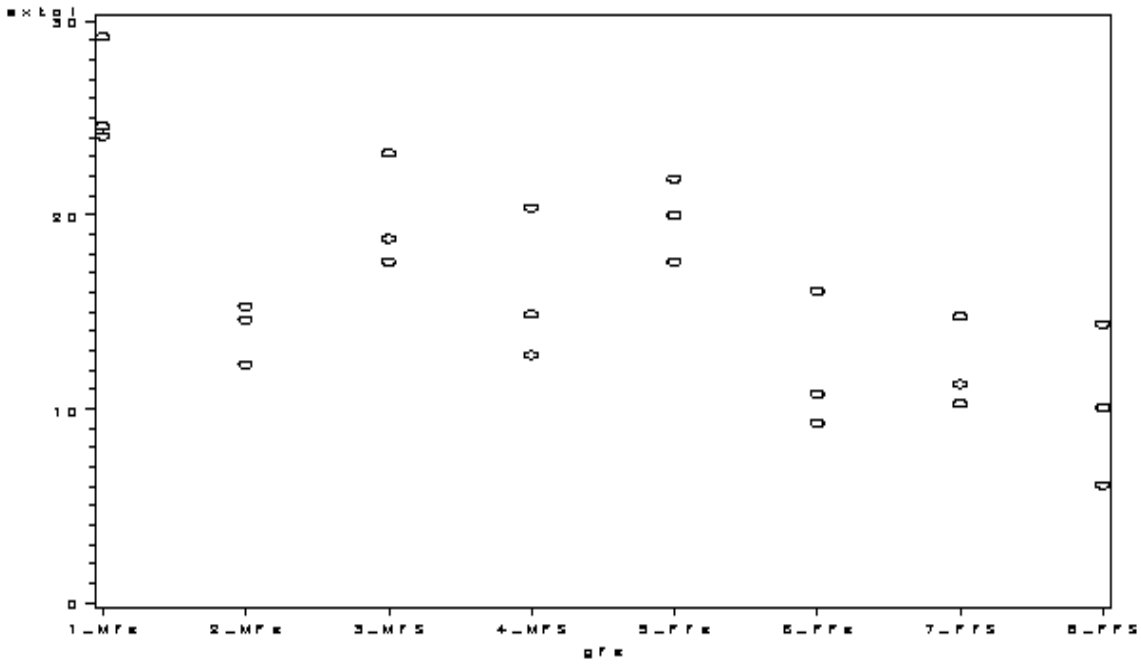
- Construct scatterplot / interaction plots
- Run full model
- Check assumptions
  - Residual plots
  - Histogram / QQplot
  - Ordered residuals plot
- Check significance of interaction

```
data a1;
    infile 'u:\.www\datasets525\CH24TA04.txt';
    input extol gender fat smoke;
proc print; run; quit;
```

Obs	extol	gender	fat	smoke
1	24.1	1	1	1
2	29.2	1	1	1
3	24.6	1	1	1
4	20.0	2	1	1
5	21.9	2	1	1
6	17.6	2	1	1
7	14.6	1	2	1
8	15.3	1	2	1
9	12.3	1	2	1
10	16.1	2	2	1
11	9.3	2	2	1
12	10.8	2	2	1
13	17.6	1	1	2
14	18.8	1	1	2
15	23.2	1	1	2
16	14.8	2	1	2
17	10.3	2	1	2
18	11.3	2	1	2
19	14.9	1	2	2
20	20.4	1	2	2
21	12.8	1	2	2
22	10.1	2	2	2
23	14.4	2	2	2
24	6.1	2	2	2

```
data a1; set a1;
    if (gender eq 1)*(fat eq 1)*(smoke eq 1) then gfs='1_Mfs';
    if (gender eq 1)*(fat eq 2)*(smoke eq 1) then gfs='2_MFs';
    if (gender eq 1)*(fat eq 1)*(smoke eq 2) then gfs='3_MfS';
    if (gender eq 1)*(fat eq 2)*(smoke eq 2) then gfs='4_MFS';
    if (gender eq 2)*(fat eq 1)*(smoke eq 1) then gfs='5_Ffs';
    if (gender eq 2)*(fat eq 2)*(smoke eq 1) then gfs='6_FFs';
    if (gender eq 2)*(fat eq 1)*(smoke eq 2) then gfs='7_FfS';
    if (gender eq 2)*(fat eq 2)*(smoke eq 2) then gfs='8_FFS';

/*----------- Scatterplot -----------*/
symbol1 v=circle i=none c=black;
proc gplot data=a1;
    plot extol*gfs/frame;
run; quit;
```



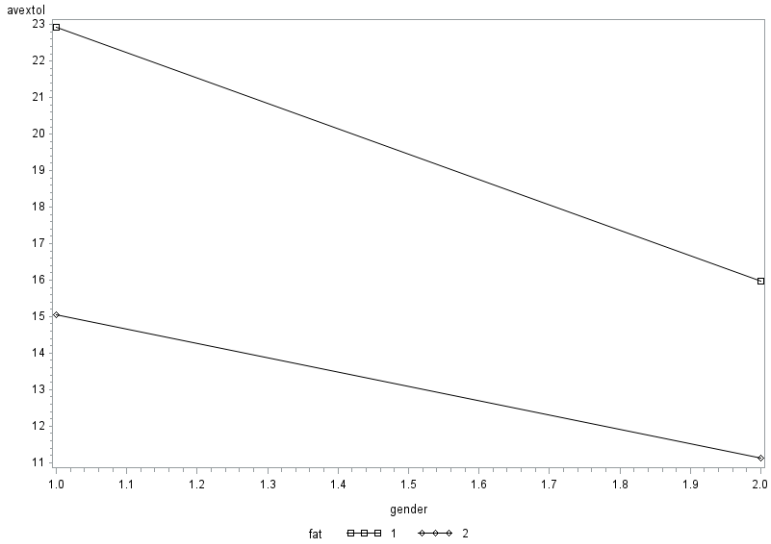
Interaction Plot

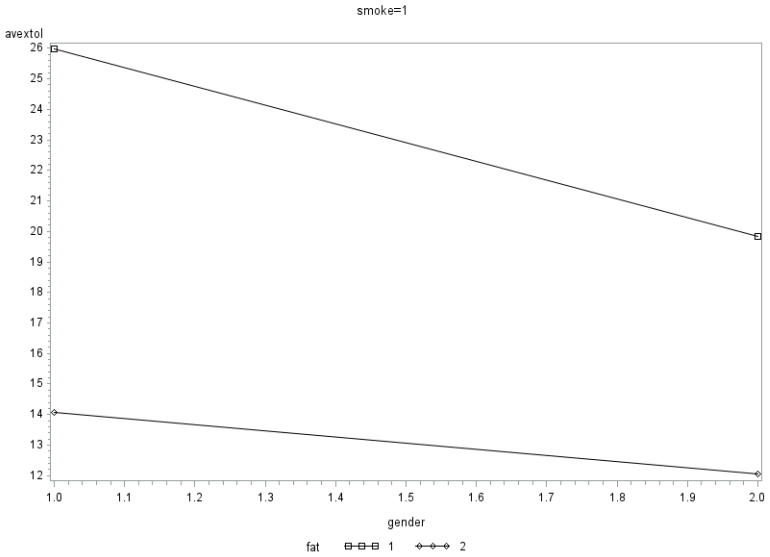
- Interaction plot between factors A and B, i.e., completely ignoring factor C

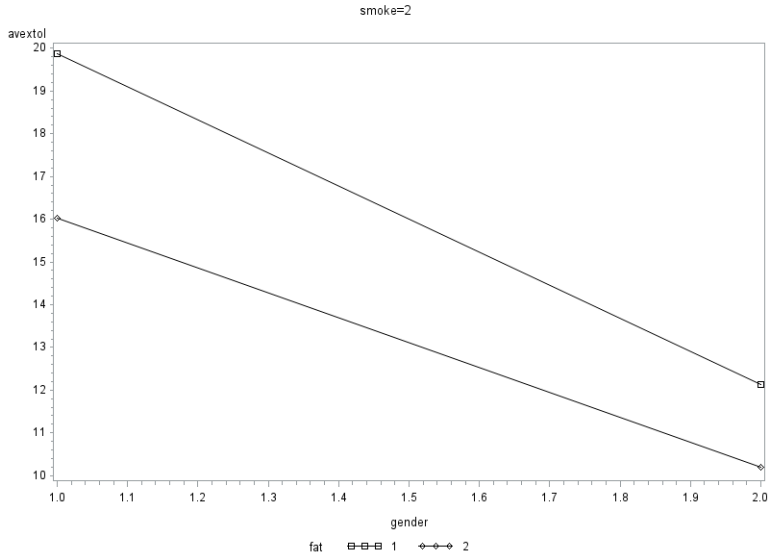
```
proc sort data=a1;
    by gender fat smoke;
proc means data=a1;
    var extol; by gender fat;
    output out=a2 mean=avextol;
symbol1 v=square i=join c=black; symbol2 v=diamond i=join c=black;
proc gplot data=a2;
    plot avextol*gender=fat/frame;
run;
```

- Interaction plot between factors A and B under a fixed level of C

```
proc sort data=a1;
    by gender fat smoke;
proc means data=a1;
    var extol; by gender fat smoke;
    output out=a2 mean=avextol;
proc sort data=a2;
    by smoke;
symbol1 v=square i=join c=black; symbol2 v=diamond i=join c=black;
proc gplot data=a2;
    plot avextol*gender=fat/frame;
    by smoke;
run;
```


avextol
23
22
21
20
19
18
17
16
15
14
13
12
11
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
gender
fat
1
2


smoke=1
avextol
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
gender
fat
1
2


smoke=2
avextol
20
19
18
17
16
15
14
13
12
11
10
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
gender
fat
1
2

High order interaction

- 2-way interaction AB
  $\Leftrightarrow$ Does the effect of factor A depend on the level of factor B?
  $\Leftrightarrow$ Almost Parallel curves in the 2-way interaction plot

- 3-way interaction ABC
  $\Leftrightarrow$ Does the interaction effect between factors A and B depend on the level of factor C?
  $\Leftrightarrow$ The 2-way interaction plots between A and B under different levels of factor C shows similar "interaction" pattern (not similar plot)

The Cell Means Model

- Expressed numerically

$$Y_{ijkl} = \mu_{ijk} + \varepsilon_{ijkl}$$

  where μ_{ijk} is the theoretical mean or expected value of all observations in cell (i, j, k)

- The ε_{ijkl} are iid $N(0, \sigma^2)$ which implies the Y_{ijkl} are independent $N(\mu_{ijk}, \sigma^2)$

- Parameters
  - $\{\mu_{ijk}\}$, $i = 1, 2, .., a$, $j = 1, 2, ..., b$, $k = 1, 2, ..., c$
  - σ^2

Estimates

- Estimate μ_{ijk} by the sample mean of the observations in cell (i, j, k)

$$\widehat{\mu}_{ijk} = \overline{Y}_{ijk.}$$

- For each cell (i, j, k), also estimate of the variance

$$s^2_{ijk} = \sum (Y_{ijkl} - \overline{Y}_{ijk.})^2/(n_{ijk} - 1)$$

- These s^2_{ij} are pooled to estimate σ^2

Factor Effects Model

- Statistical model is

$$\begin{aligned} Y_{ijkl} &= \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + \\ &\quad (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkl} \end{aligned}$$

- Over-parameterized model
  - Must include model constraints
  - Conceptual Approach
    - μ – grand mean
    - $\alpha_i, \beta_j, \gamma_k$ – main effects of A, B, and C
    - $(\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\beta\gamma)_{jk}$ – two-factor/first-order interactions
    - $(\alpha\beta\gamma)_{ijk}$ – three-factor/second-order interaction
  - SAS Approach

ANOVA Table

- Sources of variation are three main effects, three first-order interactions, one second-order interaction, and error
- If balanced
  - SS add up to model SS
  - Type I and Type III are the same
- Each effect is tested over MSE

Example (Page 1005)

```
proc glm data=a1;
    class gender fat smoke;
    model extol=gender fat smoke
                gender*fat gender*smoke fat*smoke
                gender*fat*smoke;
run; quit;
```

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	588.5829167	84.0832738	9.01	0.0002
Error	16	149.3666667	9.3354167		
Corrected Total	23	737.9495833			

R-Square	Coeff Var	Root MSE	extol Mean
0.797592	18.77833	3.055391	16.27083

Source	DF	Type I SS	Mean Square	F Value	Pr > F
gender	1	176.5837500	176.5837500	18.92	0.0005
fat	1	242.5704167	242.5704167	25.98	0.0001
smoke	1	70.3837500	70.3837500	7.54	0.0144
gender*fat	1	13.6504167	13.6504167	1.46	0.2441
gender*smoke	1	11.0704167	11.0704167	1.19	0.2923
fat*smoke	1	72.4537500	72.4537500	7.76	0.0132
gender*fat*smoke	1	1.8704167	1.8704167	0.20	0.6604

Source	DF	Type III SS	Mean Square	F Value	Pr > F
gender	1	176.5837500	176.5837500	18.92	0.0005
fat	1	242.5704167	242.5704167	25.98	0.0001
smoke	1	70.3837500	70.3837500	7.54	0.0144
gender*fat	1	13.6504167	13.6504167	1.46	0.2441
gender*smoke	1	11.0704167	11.0704167	1.19	0.2923
fat*smoke	1	72.4537500	72.4537500	7.76	0.0132
gender*fat*smoke	1	1.8704167	1.8704167	0.20	0.6604

Comments

- First examine interactions
- Some options when one or more interactions are significant
  - Interpret the plot of means
  - Run analyses for each level of one factor (slice)
  - Run as one-way with abc levels
  - Run as two-way with a and bc levels
  - Use contrasts
- If no interactions
  - Use contrasts
  - Multiple comparison procedure

There is no interactions b/w gender and (fat smoke).

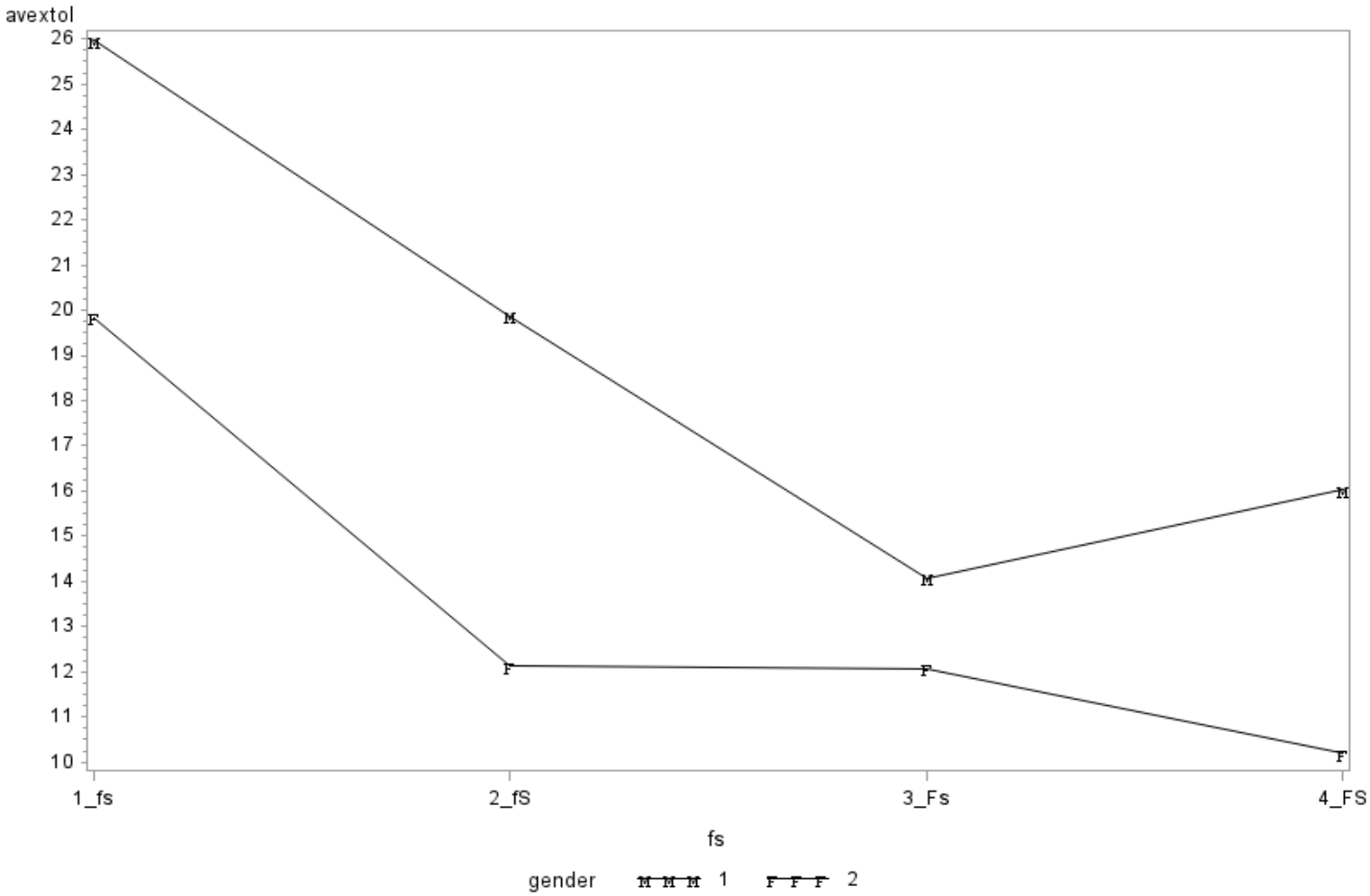
```
data a1; set a1;
    if (fat eq 1)*(smoke eq 1) then fs='1_fs';
    if (fat eq 1)*(smoke eq 2) then fs='2_fS';
    if (fat eq 2)*(smoke eq 1) then fs='3_Fs';
    if (fat eq 2)*(smoke eq 2) then fs='4_FS';

proc sort data=a1;
    by gender fs;

proc means data=a1;
    output out=a2 mean=avextol;
    by gender fs;

/*----------- Scatterplot -----------*/
symbol1 v='M' i=join c=black;
symbol2 v='F' i=join c=black;
proc gplot data=a2;
    plot avextol*fs=gender/frame;
run;

/*------------2Way-ANOVA------------------*/
proc glm data=a1;
    class gender fs;
    model extol=gender fs;
    means gender fs/tukey;
run; quit;
```


avextol
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
M
M
M
F
F
F
1_fs
2_fS
3_Fs
4_FS
fs
gender
M M M 1
F F F 2

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	561.9916667	140.4979167	15.17	<.0001
Error	19	175.9579167	9.2609430		
Corrected Total	23	737.9495833			

R-Square	Coeff Var	Root MSE	extol Mean
0.761558	18.70328	3.043180	16.27083

Source	DF	Type I SS	Mean Square	F Value	Pr > F
gender	1	176.5837500	176.5837500	19.07	0.0003
fs	3	385.4079167	128.4693056	13.87	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
gender	1	176.5837500	176.5837500	19.07	0.0003
fs	3	385.4079167	128.4693056	13.87	<.0001

Tukey's Studentized Range (HSD) Test for extol

Alpha	0.05
Error Degrees of Freedom	19
Error Mean Square	9.260943
Critical Value of Studentized Range	3.97655
Minimum Significant Difference	4.9404

	Mean	N	fs
A	22.900	6	1_fs
B	16.000	6	2_fS
B			
B	13.117	6	4_FS
B			
B	13.067	6	3_Fs

Unequal Sample Size

- Similar approach as two-way ANOVA
- Type I and Type III SS are different
- Type III is more commonly used
- `LSMEANS` is used for comparisons

Chapter Review

- 3-factor studies
  - Data
  - Model
  - Parameter Estimates
  - Inference
- Unequal sample size