1. A trainee examined a set of experimental data to find comparisons that "look promising" and calculated a family of Bonferroni confidence intervals for these comparisons with a 90 percent family confidence coefficient. Upon being informed that the Bonferroni procedure is not applicable in this case because the comparisons had been suggested by the data, the trainee stated: "This makes no difference. I would use the same formulas for the point estimates and the estimated standard errors even if the comparisons were not suggested by the data." Respond.

The trainee's statement that the Bonferroni adjustment does not affect the point estimates and estimated standard errors is indeed correct. However, the value of g in the adjustment ought to be the number of total comparisons, not just the number of comparisons that the data suggests looks promising.

2. Consider the following linear combinations of interest in a single-factor study involving four factor levels:

$$\text{(i)} \quad \mu_1 + 3\mu_2 - 4\mu_3$$
$$\text{(ii)} \quad 0.3\mu_1 + 0.5\mu_2 + 0.1\mu_3 + 0.1\mu_4$$
$$\text{(iii)} \quad \frac{\mu_1+\mu_2+\mu_3}{3} - \mu_4$$

a. *Which of the linear combinations are contrasts? State the coefficients for each of the contrasts.*

For a linear combination to be a contrast, their coefficients must sum of zero. Then (i) is a contrast with coefficients $1, 3, -4$, and 0, and (iii) is a contrast with coefficients $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$, and 1.

b. *Give an unbiased estimator for each of the linear combinations. Also give the estimated variance of each estimator assuming that $n_i = n$.*

By **(17.20)** and **(17.22)**,

(i) $\hat{L} = \hat{Y}_1 + 3\hat{Y}_2 - 4\hat{Y}_3$; $s^2\{\hat{L}\} = \frac{MSE(1+9+16)}{n} = 26 * \frac{MSE}{n}$.

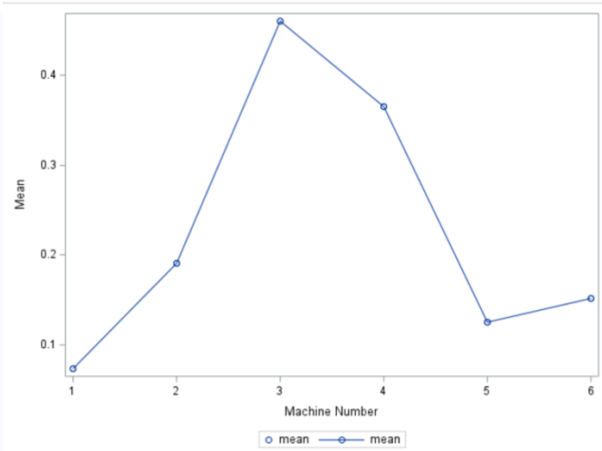
(ii) $\hat{L} = 0.3\hat{Y}_1 + 0.5\hat{Y}_2 + 0.1\hat{Y}_3 + 0.1\hat{Y}_4$; $s^2\{\hat{L}\} = \frac{MSE(0.09+0.25+0.01+0.01)}{n} = 0.36 * \frac{MSE}{n}$.

(iii) $\hat{L} = \frac{1}{3}\hat{Y}_1 + \frac{1}{3}\hat{Y}_2 + \frac{1}{3}\hat{Y}_3 + 1\hat{Y}_4$; $s^2\{\hat{L}\} = \frac{MSE\left(\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+1\right)}{n} = 1.333 * \frac{MSE}{n}$.

3. Refer to Filing machines Problem 16.11.

a. *Prepare a main effects plot of the estimated factor level means $\bar{Y}_i$. What does this plot suggest regarding the variation in the mean fills for the six machines?*

Level of machine_number	N	deviation	
		Mean	Std Dev
1	20	0.07350000	0.19252546
2	20	0.19050000	0.18540070
3	20	0.46000000	0.16463676
4	20	0.36550000	0.16535369
5	20	0.12500000	0.17273344
6	20	0.15150000	0.17348669



The SAS System

The GLM Procedure

Dependent Variable: deviation

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	2.28934667	0.45786933	14.78	<.0001
Error	114	3.53060000	0.03097018		
Corrected Total	119	5.81994667			

R-Square	Coeff Var	Root MSE	deviation Mean
0.393362	77.29873	0.175983	0.227667

There is high variance in the mean fill amounts between the six machines.

b. *Construct a 95 percent confidence interval for the mean fill for machine 1.*

machine_number	deviation LSMEAN	95% Confidence Limits	
1	0.073500	-0.004454	0.151454

c. *Obtain a 95 percent confidence interval for $D = \mu_2 - \mu_1$. Interpret your interval estimate.*

$\hat{D} = \bar{Y}_2 - \bar{Y}_1 = 0.1905 - 0.0735 = 0.117$; $s\{\hat{D}\} = \sqrt{s^2\{\bar{Y}_1\} + s^2\{\bar{Y}_2\}} = \sqrt{(0.192525)^2 + (0.185401)^2} = 0.26728$; $t(0.9725, 114) = 1.980992$. Then the interval is $0.117 \pm (0.26728)(1.980992) = (-0.41248, 0.64648)$. Because this interval contains zero, at the 95% confidence level, there is significant statistical evidence to suggest that the mean fill levels of machines 1 and 2 are identical.

d. Prepare a paired comparison plot and interpret it.

Comparisons significant at the 0.05 level are indicated by ***.

machine_number Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
3 - 4	0.09450	-0.07236	0.26136	
3 - 2	0.26950	0.10264	0.43636	***
3 - 6	0.30850	0.14164	0.47536	***
3 - 5	0.33500	0.16814	0.50186	***
3 - 1	0.38650	0.21964	0.55336	***
4 - 3	-0.09450	-0.26136	0.07236	
4 - 2	0.17500	0.00814	0.34186	***
4 - 6	0.21400	0.04714	0.38086	***
4 - 5	0.24050	0.07364	0.40736	***
4 - 1	0.29200	0.12514	0.45886	***
2 - 3	-0.26950	-0.43636	-0.10264	***
2 - 4	-0.17500	-0.34186	-0.00814	***
2 - 6	0.03900	-0.12786	0.20586	
2 - 5	0.06550	-0.10136	0.23236	
2 - 1	0.11700	-0.04986	0.28386	
6 - 3	-0.30850	-0.47536	-0.14164	***
6 - 4	-0.21400	-0.38086	-0.04714	***
6 - 2	-0.03900	-0.20586	0.12786	
6 - 5	0.02650	-0.14036	0.19336	
6 - 1	0.07800	-0.08886	0.24486	
5 - 3	-0.33500	-0.50186	-0.16814	***
5 - 4	-0.24050	-0.40736	-0.07364	***
5 - 2	-0.06550	-0.23236	0.10136	
5 - 6	-0.02650	-0.19336	0.14036	
5 - 1	0.05150	-0.11536	0.21836	
1 - 3	-0.38650	-0.55336	-0.21964	***
1 - 4	-0.29200	-0.45886	-0.12514	***
1 - 2	-0.11700	-0.28386	0.04986	
1 - 6	-0.07800	-0.24486	0.08886	
1 - 5	-0.05150	-0.21836	0.11536	

The confidence limits marked with *** indicate significant differences in mean fill levels between the two machines being compared.

e. The consultant is particularly interested in comparing the mean fills for machines 1, 4, and 5. Use the Bonferroni testing procedure for all pairwise comparisons among these three treatment means with family level of significance $\alpha = 0.10$. *Interpret your results and provide a graphic summary by preparing a line plot of the estimated factor level means with nonsignificant differences underlined. Do your conclusions agree with those in part (a)?*

We have three values of $\widehat{D}$:

i) $\widehat{D_1} = \bar{Y}_1 - \bar{Y}_4 = 0.0735 - 0.3655 = -0.2920$
ii) $\widehat{D_2} = \bar{Y}_1 - \bar{Y}_5 = 0.0735 - 0.1250 = -0.0515$
iii) $\widehat{D_3} = \bar{Y}_4 - \bar{Y}_5 = 0.3655 - 0.1250 = 0.2405$

For the $g = 3$ Bonferroni adjustment, we require $B = t\left(1 - \frac{0.1}{2(3)}, 114\right) = t(0.9833, 114) = 2.153323$. We find that $\left|\frac{\widehat{D}_1}{s\{\widehat{D}_1\}}\right|, \left|\frac{\widehat{D}_3}{s\{\widehat{D}_3\}}\right| > 2.153323$ but $\left|\frac{\widehat{D}_2}{s\{\widehat{D}_2\}}\right| < 2.153323$, so we conclude the mean fill differences are significant between machines 1 and 4, and machines 1 and 5, but not fail to find evidence that the mean fill differences are significant between machines 4 and 5.

f. *Would the Tukey testing procedure have been more efficient to use in part (e) than the Bonferroni testing procedure? Explain.*

First compute the Tukey multiplier: $q(0.9; 6; 114) = 3.709845$. Because the Tukey multiplier is greater than the Bonferroni multiplier computed in part (e), we conclude that Bonferroni is the better and more efficient testing procedure.

4. Refer to Filling machines Problem 16.11. Machines 1 and 2 were purchased new five years ago, machines 3 and 4 were purchased in a reconditioned state five years ago, and machines 5 and 6 were purchased new last year.

a. *Estimate the contrast,*

$$L = \frac{\mu_1 + \mu_2}{2} - \frac{\mu_3 + \mu_4}{2}$$

with a 95% confidence interval. Interpret your interval estimate.

$\hat{L} = \frac{1}{2}(\bar{Y}_1 + \bar{Y}_2) - \frac{1}{2}(\bar{Y}_3 + \bar{Y}_4) = \frac{1}{2}(0.0735 + 0.1905) - \frac{1}{2}(0.4600 + 0.3655) = -0.28075$. And $s\{\hat{L}\} = MSE\left(\frac{0.25}{114}\right) = 0.03097\left(\frac{1}{114}\right) = 2.7167 * 10^{-4}$, $t(0.975{,}114) = 1.980992$. Then the confidence interval is $-0.28075 \pm (2.7167 * 10^{-4})(1.980992) = (-0.28128, -0.28021)$. Since this interval does not contain zero, we conclude that there is insufficient statistical evidence to suggest that machines 1 and 2 have different mean fill levels than machines 3 and 4.

b. *Estimate the following comparisons with a 90 percent family confidence coefficient. Use the most efficient multiple comparison procedure. Interpret your results. What can the consultant learn from these results about the differences between the six filling machines?*

Parameter	Estimate	Standard Error	t Value	Pr > |t|	99.334% Confidence Limits	
1 v 2	-0.11700000	0.05565085	-2.10	0.0377	-0.27081934	0.03681934
3 v 4	-0.11700000	0.05565085	-2.10	0.0377	-0.27081934	0.03681934
5 v 6	-0.11700000	0.05565085	-2.10	0.0377	-0.27081934	0.03681934
1&2 v 3&4	-0.28075000	0.03935110	-7.13	<.0001	-0.38951670	-0.17198330
1&2 v 5&6	-0.28075000	0.03935110	-7.13	<.0001	-0.38951670	-0.17198330
1&2&5&6 v 3&4	0.13412500	0.03407905	3.94	0.0001	0.03993028	0.22831972
1&2&3&4 v 5&6	0.13412500	0.03407905	3.94	0.0001	0.03993028	0.22831972

```
proc glm data = a1;
    class machine_number;
    model deviation = machine_number / clparm alpha=0.00666;
    estimate '1 v 2' machine_number 1 -1;
    estimate '3 v 4' machine_number 1 -1;
    estimate '5 v 6' machine_number 1 -1;
    estimate '1&2 v 3&4' machine_number 0.5 0.5 -0.5 -0.5;
    estimate '1&2 v 5&6' machine_number 0.5 0.5 -0.5 -0.5;
    estimate '1&2&5&6 v 3&4' machine_number 0.25 0.25 0.25 0.25 -0.5 -0.5;
    estimate '1&2&3&4 v 5&6' machine_number 0.25 0.25 0.25 0.25 -0.5 -0.5;
run; quit;
```

We will use the Bonferroni adjustment: $\alpha^* = \frac{0.1}{\frac{6(6-1)}{2}} = \frac{1}{150}$. By examining these confidence intervals, the consultant can learn which contrasts are significantly different from zero at the specified α level.

5. Refer to Filling machines Problem 16.11. Suppose the primary interest is in estimating the following comparisons:

$$\boldsymbol{L_1 = \mu_1 - \mu_2;\ L_2 = \mu_3 - \mu_4}$$
$$\boldsymbol{L_3 = \frac{\mu_1 + \mu_2}{2} - \frac{\mu_3 + \mu_4}{2}; L_4 = \frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4} - \frac{\mu_5 + \mu_6}{2}}$$

What would be the required sample sizes if the precision of each comparison is to not exceed ± 0.08 ounce, using the most efficient multiple comparison procedure with a 95 percent family confidence coefficient?

We use the adjusted Bonferroni multiplier: $B = t\left(1 - \frac{0.05}{2(4)}, 114\right) = t(0.99375,114) = 2.537956$. Also $MSE = 0.03097018$. After solving the equation $0.08 = 2.537956 * \sqrt{\frac{2(0.03097018)}{n}}$ for n, we conclude $n \geq 62$ samples per machine.

6. A student proposed in class that deviations of the observations $\boldsymbol{Y_{ij}}$ around the estimated overall mean $\boldsymbol{Y}$. be plotted to assist in evaluating the appropriateness of ANOVA model (16.2). Would these deviations be helpful in studying the independence of the error terms? The constancy of the variance of the error terms? The normality of the error terms? Discuss.

This plot would not be helpful in assessing the normality of the error terms; we would need a qq plot or something similar for that task. Neither would it be helpful for assessing the independence of error terms because residual versus factor level plots would be much more appropriate. But we would be able to assess the error terms' constancy of variance by examining the distribution of residuals in Y_{ij} vs $\bar{Y}$.

7. Winding speeds. In a completely randomized design to study the effect of the speed of winding thread (1: slow; 2: normal; 3: fast; 4: maximum) onto 75-yard spools, 16 runs of 10,000 spools each were made at each of the four winding speeds. The response variable is the number of thread breaks during, the production run. Since the responses are counts, the researcher was concerned about the normality and equal variances assumptions of ANOVA model (16.2).

a. *Obtain the fitted values and residuals for ANOVA model.*

The SAS System

The GLM Procedure

Dependent Variable: length

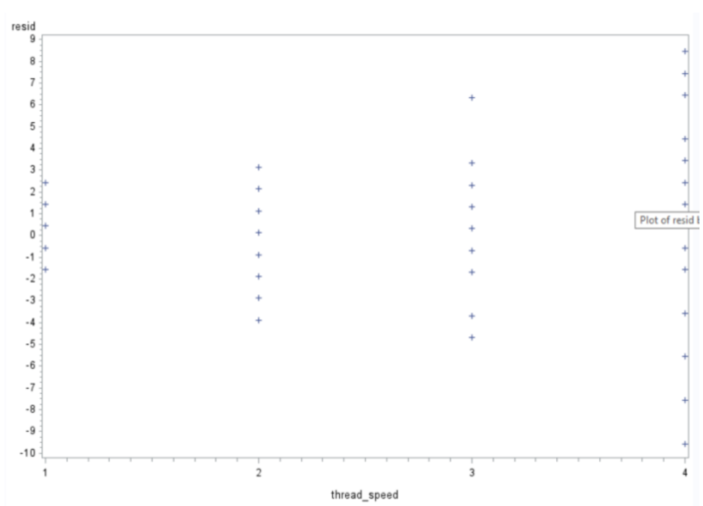
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	1588.046875	529.348958	47.47	<.0001
Error	60	669.062500	11.151042		
Corrected Total	63	2257.109375			

R-Square	Coeff Var	Root MSE	length Mean
0.703576	36.40823	3.339318	9.171875

Source	DF	Type I SS	Mean Square	F Value	Pr > F
thread_speed	3	1588.046875	529.348958	47.47	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
thread_speed	3	1588.046875	529.348958	47.47	<.0001

b. *Prepare suitable residual plots to study whether or not the error variances are equal for the four winding speeds. What are your findings?*



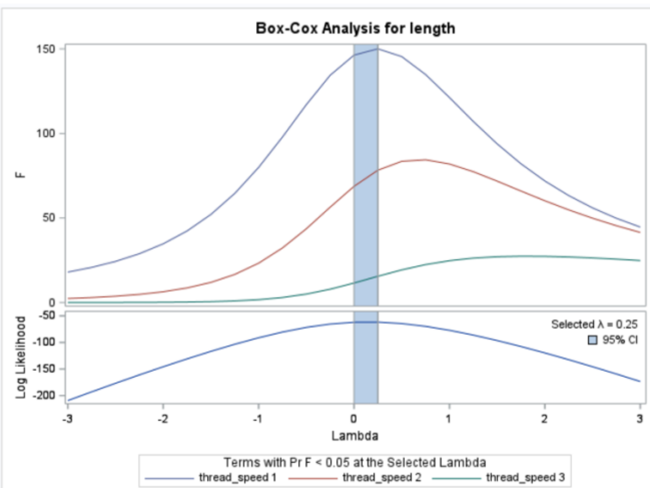
As evidenced by the residual plot, the variance of the error terms is not constant and increase with thread speed.

c. *Test by means of the Brown-Forsythe test whether or not the treatment error variances are equal; use* $\alpha = 0.05$. *What is the P-value of the test? Are your results consistent with the diagnosis in part (b)?*

Brown and Forsythe's Test for Homogeneity of length Variance ANOVA of Absolute Deviations from Group Medians					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
thread_speed	3	111.5	37.1823	9.54	<.0001
Error	60	233.8	3.8969		

The p-value of this test is less than 0.0001, so we conclude that at least one level of thread speed produces an error variance significantly different than the others. This is indeed consistent with the residual plot in part (b).

d. *For each winding speed, calculate* $\overline{Y}_i$ *and* s_i. *Examine the three relations found in the table on page 791 and determine the transformation that is most appropriate here. What do you conclude?*



Level of thread_speed	N	length	
		Mean	Std Dev
1	16	3.5625000	1.09354165
2	16	5.8750000	1.99582898
3	16	10.6875000	3.23972735
4	16	16.5625000	5.37858408

By the Box-Cox plot, we select $\lambda = 0.25$ so we consider a cube root transformation.