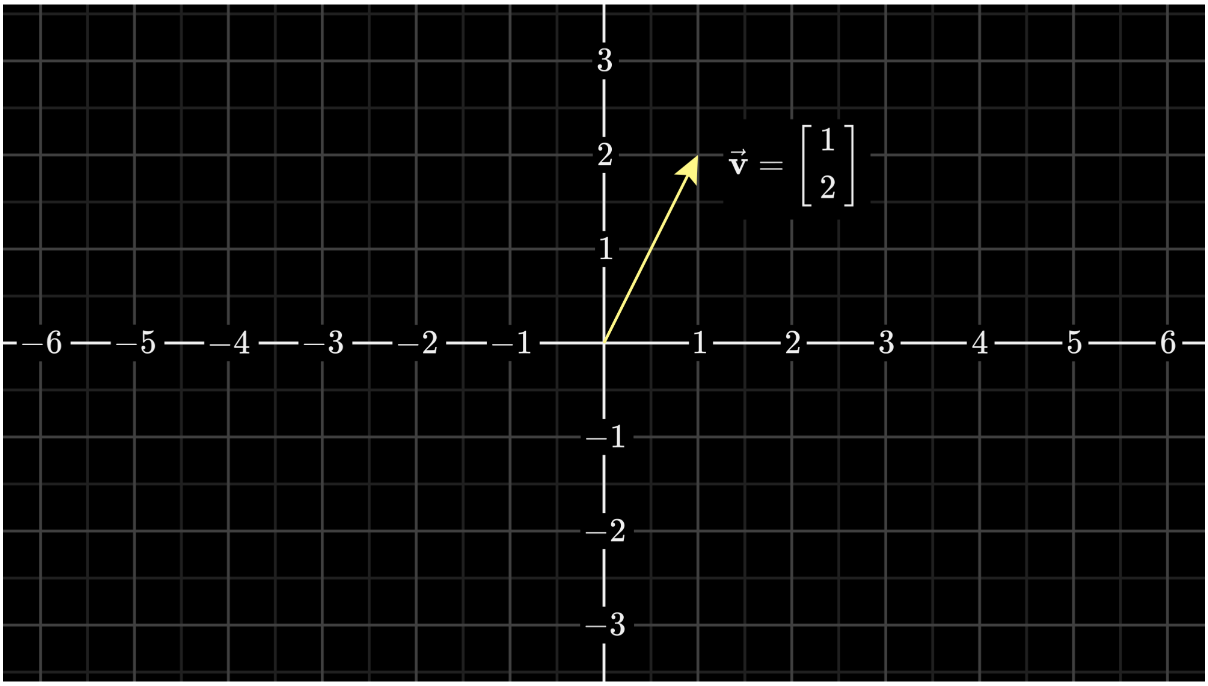
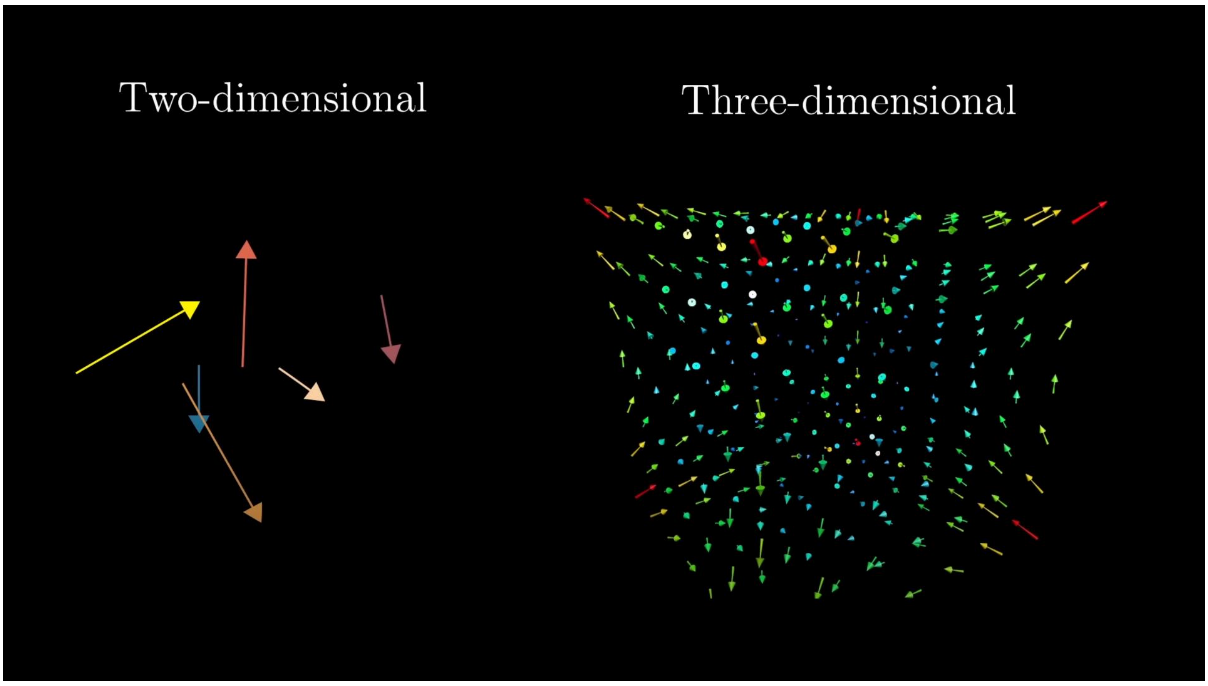
Chapter 1: Vectors, what even are they?

The fundamental building block for linear algebra is the vector, so it's worth making sure we're all on the same page about *what* exactly a vector *is*.



Vectors that live in a flat plane are two-dimensional, and those sitting in the broader space that we live in are three-dimensional.



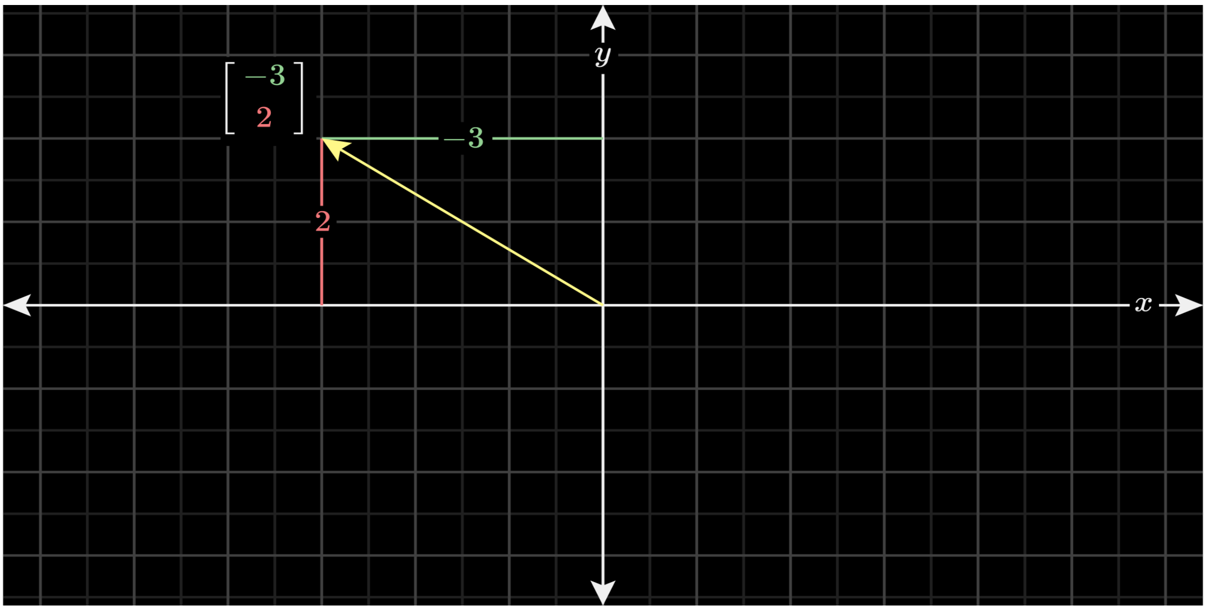
Coordinate systems are fundamental to understanding linear algebra, as they provide a framework for visualizing and working with vectors and transformations. In two dimensions, the most commonly used system consists of two perpendicular lines: the **x-axis**, which runs horizontally, and the **y-axis**, which runs vertically. These axes intersect at the **origin** (0,0), creating a grid where every point in the plane can be represented by a pair of numbers, often referred to as coordinates.

The x-axis typically represents the horizontal component of a point, while the y-axis represents the vertical component. Any point in this two-dimensional space can be described by a pair of numbers (x, y), where x corresponds to the point's position along the x-axis, and y corresponds to its position along the y-axis.

This coordinate system serves as the foundation for many concepts in linear algebra, such as vectors, transformations, and matrix operations, allowing us to express relationships and perform calculations visually and algebraically.

The place where they intersect is the *origin*, whichis the center of space and the root of all vectors.

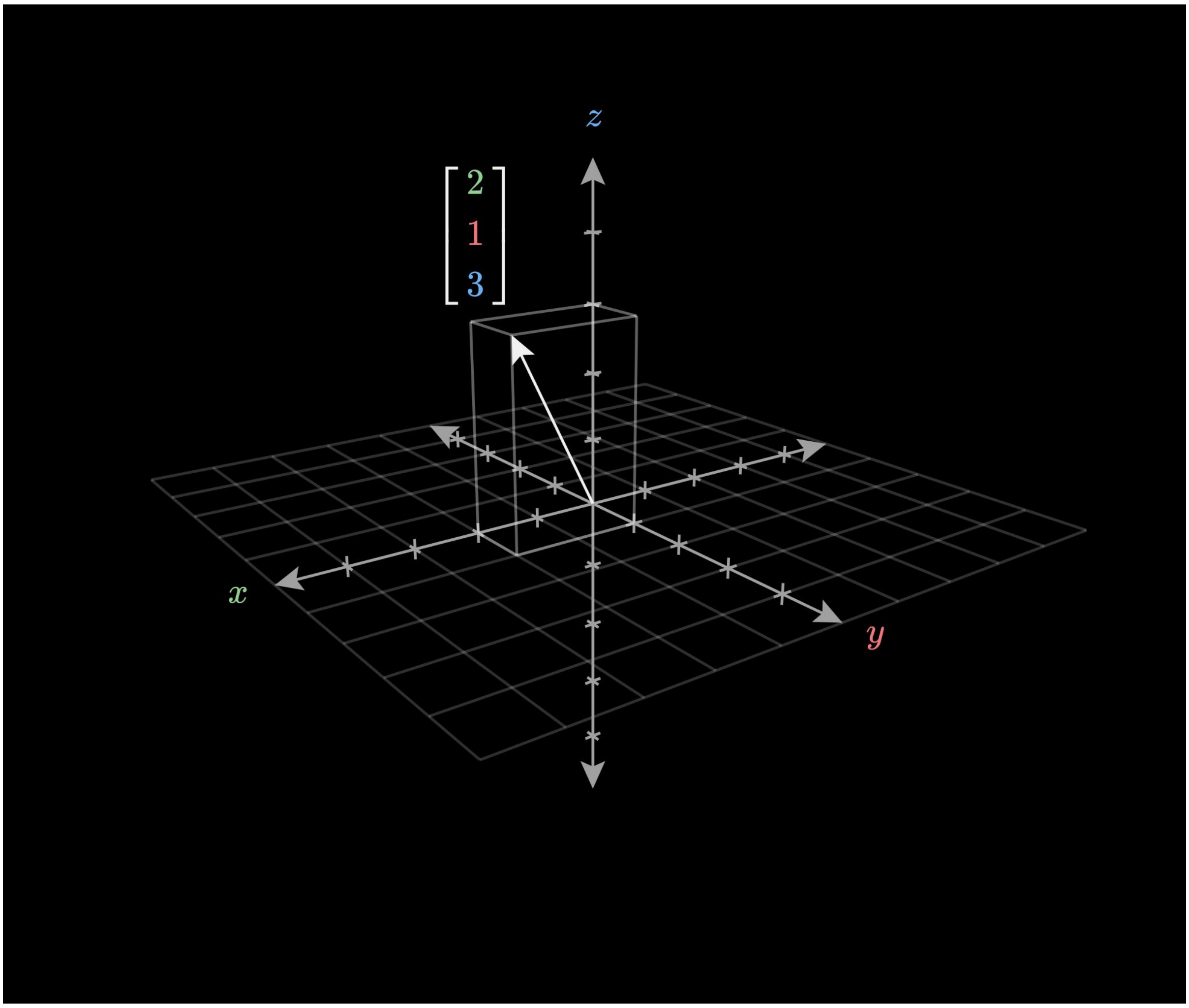
The coordinates of a vector are a pair of numbers that basically give instructions for how to get from the tail of that vector at the origin, to its tip. The first number tells you how far to walk along the xx-axis, with positive numbers indicating rightward motion and negative numbers indicating leftward motion, and the second number tells you how far to then walk parallel to the yy-axis, with positive numbers indicating upward motion, and negative numbers in



In three-dimensional space, a third axis, the **z-axis**, is added, perpendicular to both the **x-axis** and **y-axis**. Each point is represented by an ordered triplet $(x,y,z)(x, y, z)$(x,y,z), where:

- x indicates movement along the **x-axis**.
- y indicates movement along the **y-axis**.
- z indicates movement along the **z-axis**.

This coordinate system allows us to represent points and vectors in 3D space.

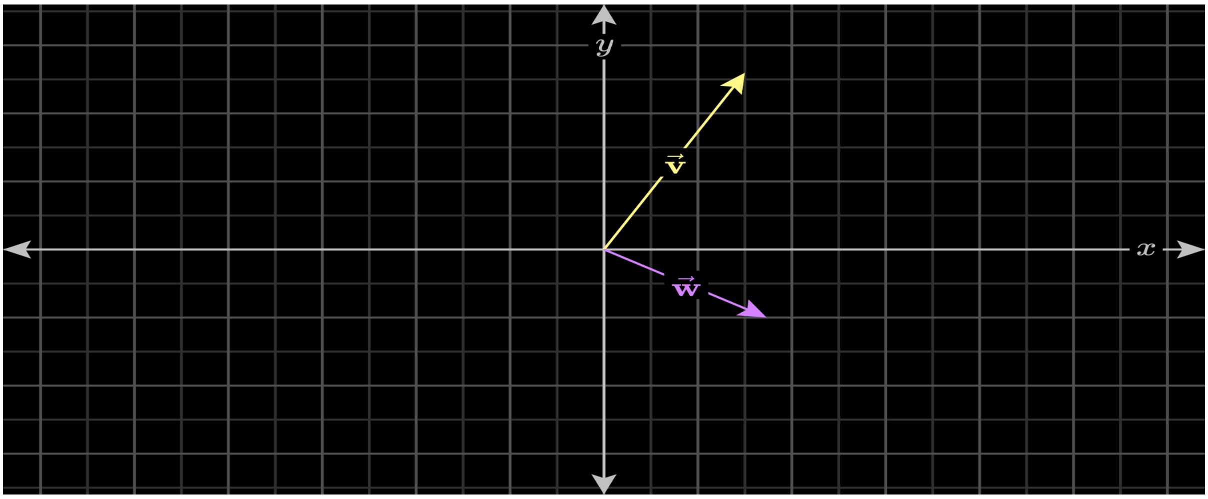


Vector Operations

So what about vector addition, and multiplying numbers by vectors? After all, every topic in linear algebra centers around these two operations. Luckily, these are both relatively straight-forward.

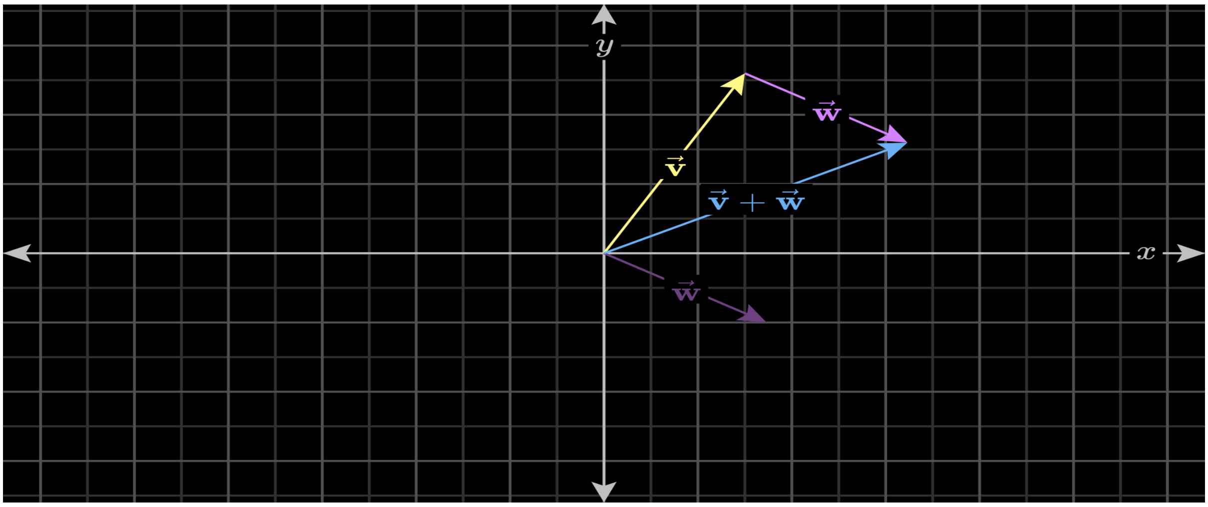
Addition

Let's say we have two vectors, one pointing up and a little to the right, and another pointing to the right and a little bit down.



To add two vectors, you can use the **tip-to-tail** method:

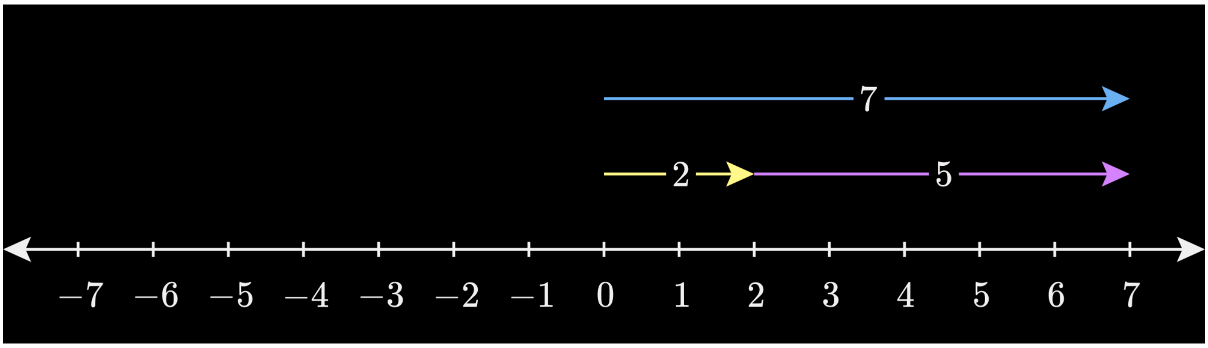
1 - Move the second vector, 2 - Draw the sum vector



This definition of vector addition is reasonable because it aligns with the idea that vectors represent **movements** or **displacements** in a certain direction and magnitude. When you add two vectors, you're essentially combining two movements, just like how you would combine steps along a number line.

To understand this, imagine you take a step along the first vector. Then, from that new position, you take a step along the second vector. The result is the same as if you had moved directly along the **sum** of the two vectors, as the overall displacement combines the distances and directions of both.

This mirrors the way we add numbers on a number line. For example, when adding *2+52 + 5*2+5, you can think of moving 2 steps to the right, followed by 5 more steps to the right. The total displacement is equivalent to just moving 7 steps to the right, reflecting the same idea of combining movements in a clear, straightforward way.



Reorganizing the steps of vector addition, you can think of it as first doing all the horizontal motion (rightward), and then doing all the vertical motion. Specifically, if you have two vectors with horizontal and vertical components, you begin by adding the horizontal components together. For example, if one vector moves 1 unit right and another moves 3 units right, you combine them to get a total of *1+3=41 + 3 = 4*1+3=4 units to the right.

Next, you move vertically. If one vector moves 2 units up and another moves 1 unit down, you subtract to get *2−1=12 - 1 = 1*2−1=1 unit up.

Thus, the new vector, which represents the combined movement, has the coordinates *(4,1)(4, 1)*(4,1), meaning 4 units to the right and 1 unit up. This approach simplifies vector addition into separate horizontal and vertical motions, making it easier to visualize and calculate.