Neuroprothetik Exercise 4
Hodgkin & Huxley Model

Jörg Encke, Korbinian Steger

WS 2016/2017

1. Time constants and steady state values

Derive the relationship between the rate equations for α_x, β_x and the time constant τ_x and steady state value x_∞ with $x \in \{m, n, h\}$. To gain these two variables bring the gating ODEs into the following form:

$$\frac{dx}{dt} = \frac{1}{\tau_x}(x_\infty - x)$$

Plot τ_x and x_∞ against the voltage V $\in$ [-100 mV, 100 mV] at 6.3 $°C$ and 28 $°C$. Explain what you can read from these plots.

2. Hodgkin & Huxley Neuron Model

Change the LIF model from the previous exercise to a HH type model.

To do this, remove the fixed spiking threshold and reset value as the HH model will spike on its own. Next, implement a function that calculates the ionic current density i_{ion}. For example:

$$[i\_ion, gate] = hh\_current(V, dt, last\_gate)$$

Use the exponential-euler solver which was implemented in the previous exercise to solve the gating ODEs.

2.1. Experiments

Run the model for 100 ms ($\Delta t = 0.01$ ms) with the following settings

1. At 6.3 °C induce a stair of five 5 ms long rectangular current pulses with a gap of 10 ms and the amplitudes 1 uA/cm^2, 2 uA/cm^2, 3 uA/cm^2, 4 uA/cm^2, 5 uA/cm^2
2. At 28 °C induce a stair of five 5 ms long rectangular current pulses with a gap of 10 ms and the amplitudes 2 uA/cm^2, 4 uA/cm^2, 8 uA/cm^2, 16 uA/cm^2, 32 uA/cm^2

Create the following Plots of the results from the two experiments:

1. Plot the membrane potential over time.
2. Plot the gating constants m, n, h over time
3. Plot the current densities i_{Na}, i_K over time.
4. Plot the current densities i_{Na}, i_K, i_L, over the membrane potential (phase plot).

2.2. Analysis of the Results

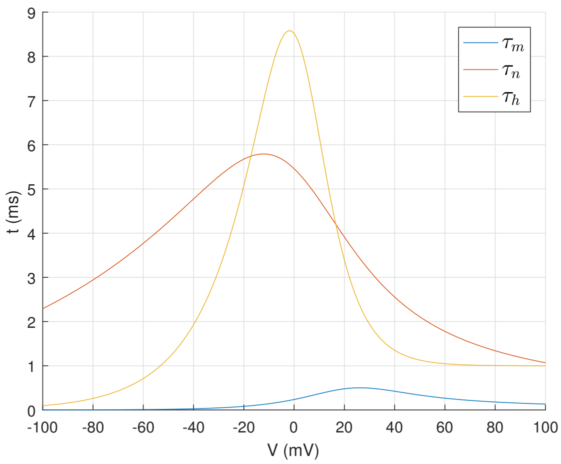
Answer the following questions by interpreting the results. For some answers, it might be helpful to replot the graphs in different scales.

- Describe the differences between the results at 6.3 °C and 28 °C
- How is an action potential generated and what is the role of the different currents and gating variables.
- Explain why consecutive action potentials decrease in amplitude (at at 28 °C)
- How can you interpret the phase plot.
- Explain the differences between the LIF and the HH model, why would you implement one or the other.

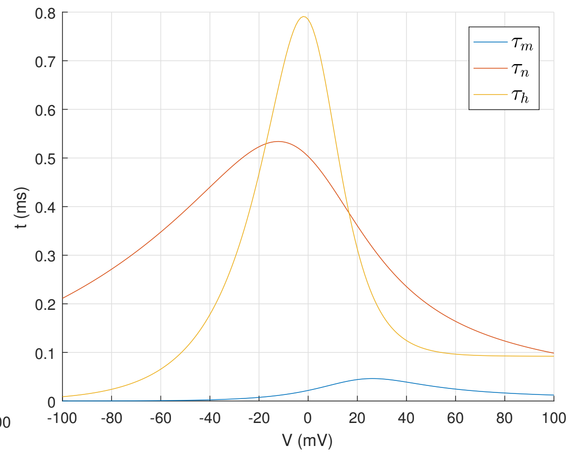
3. Solution

Here you can see how the resulting plots should look like. This is just to give you an idea if your results are valid.

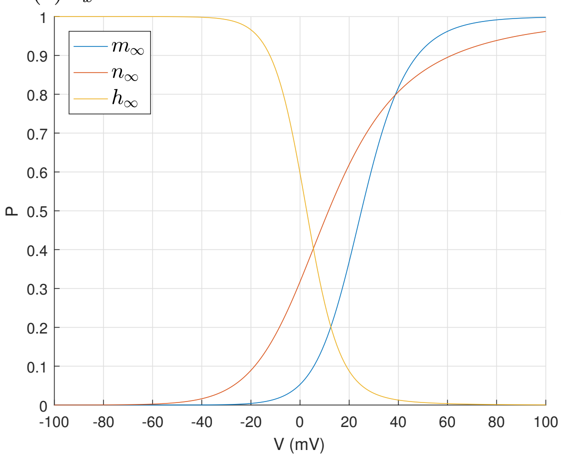
3.1. Solution to 1



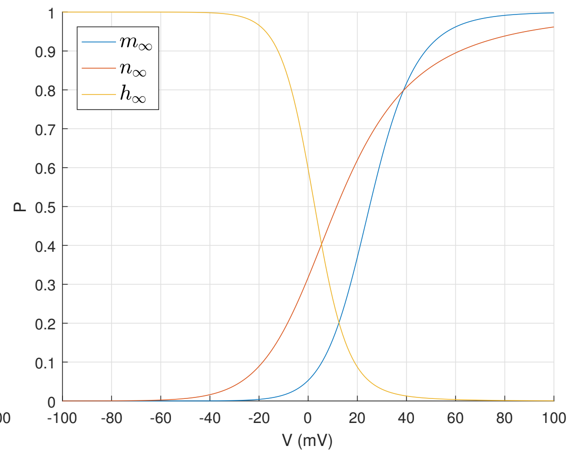
(a) τ_x for 6.3 °C



(b) τ_x for 28 °C



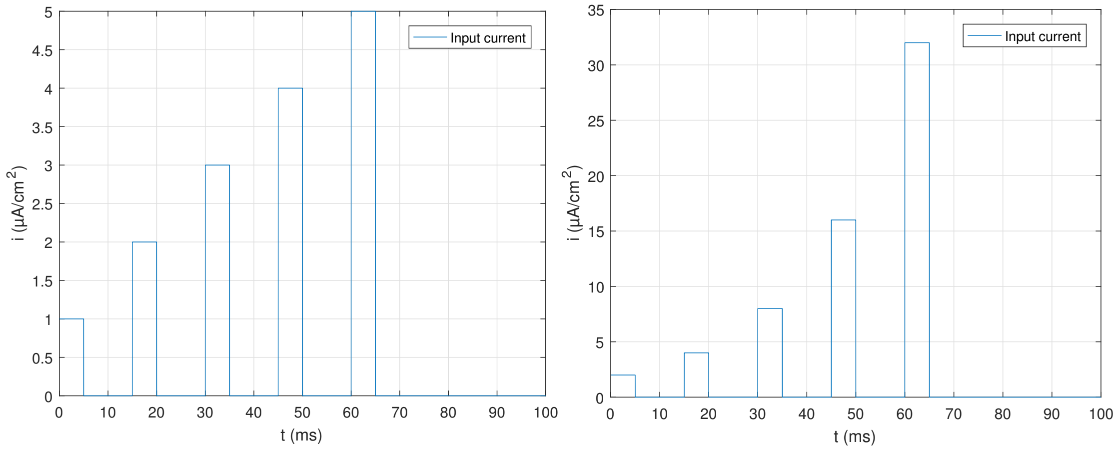
(c) x_∞ for 6.3 °C



(d) x_∞ for 28 °C

Abbildung 1: Parameters τ and x_∞ for the gating variables m, n and h for 6.3 °C and 28 °C

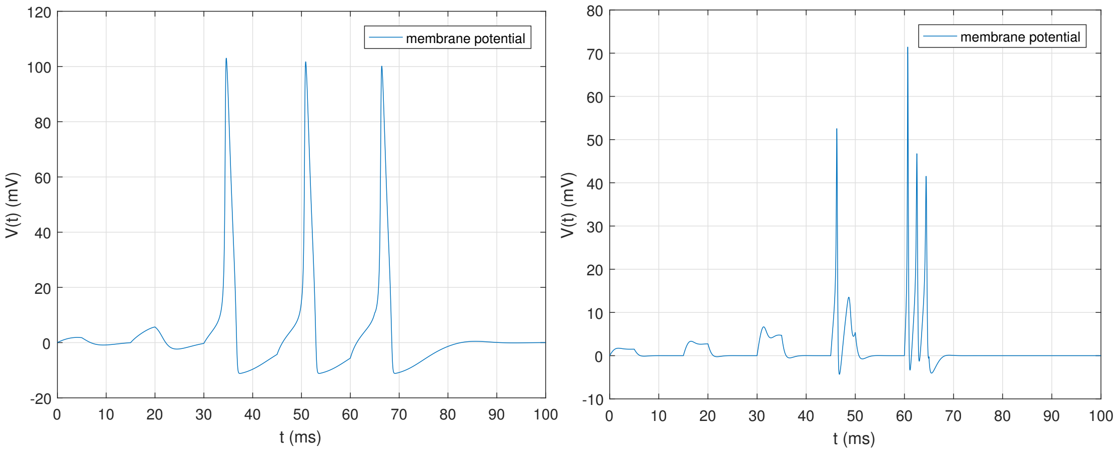
3.2. Solution to 2.1



(a) Input current densities for the simulation at 6.3 °C

(b) Input current densities for the simulation at 28 °C

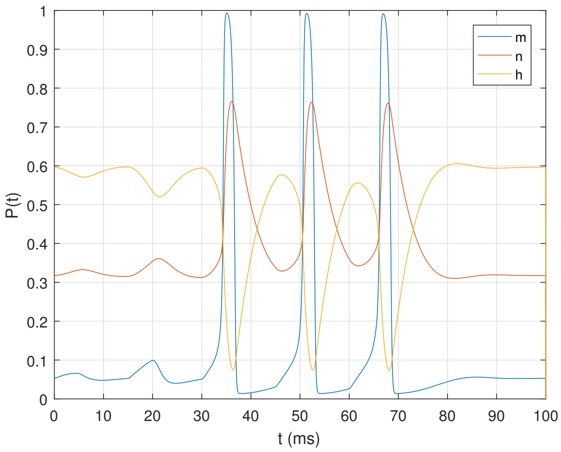
Abbildung 2: Input current densities for different temperatures



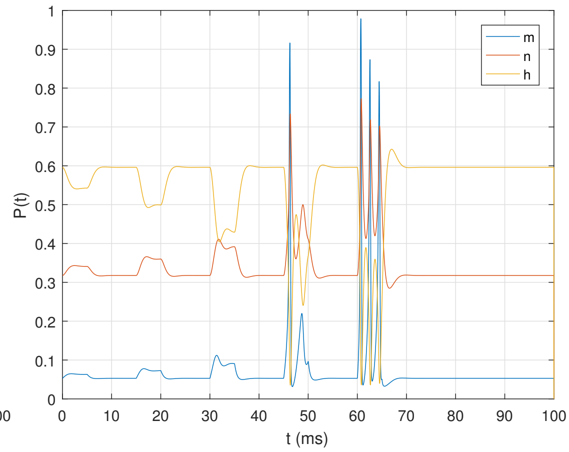
(a) Membrane potential for 6.3 °C and the input visible in figure 2a

(b) Membrane potential for 28 °C and the input visible in figure 2b

Abbildung 3: Membrane Potentials for the different cases

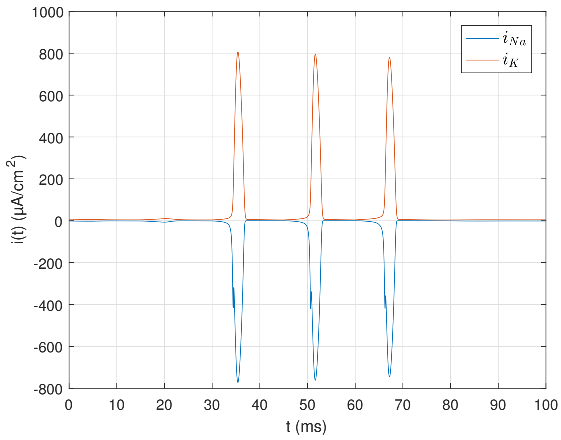


(a) Gating Variables m, n, h at 6.3 °C and the input visible in figure 2a

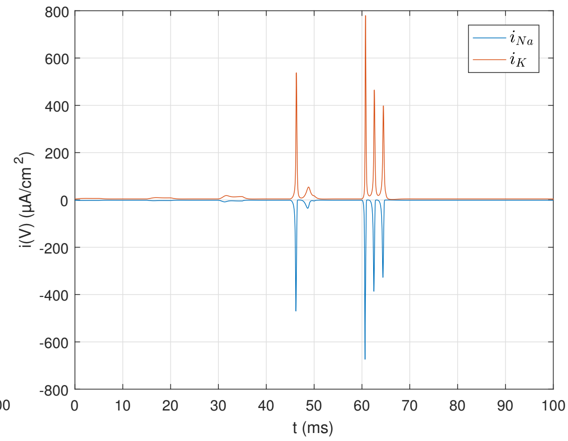


(b) Gating Variables m, n, h at 28 °C and the input visible in figure 2b

Abbildung 4: Gating Variables m, n, h for the different cases

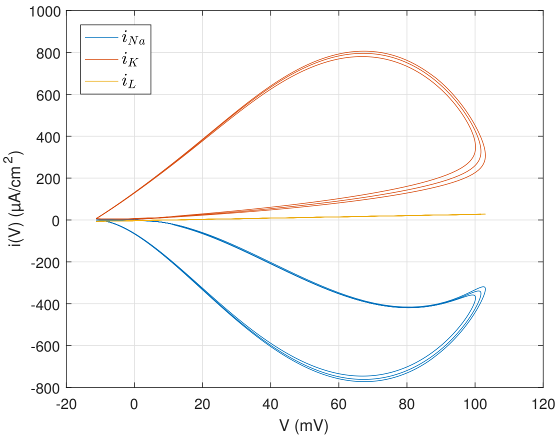


(a) Current densities i_{Na} and i_K for 6.3 °C and the input visible in figure 2a

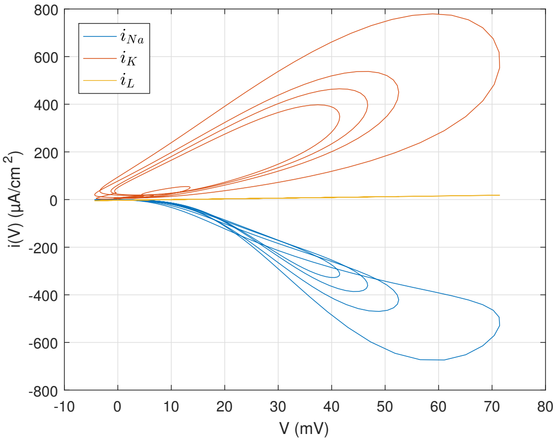


(b) Current densities i_{Na} and i_K for 28 °C and the input visible in figure 2b

Abbildung 5: Current densities i_{Na} and i_K for the different cases



(a) Phase plot for 6.3 °C and the input visible in figure 2a



(b) Phase plot for 28 °C and the input visible in figure 2b

Abbildung 6: Phase plot for different cases

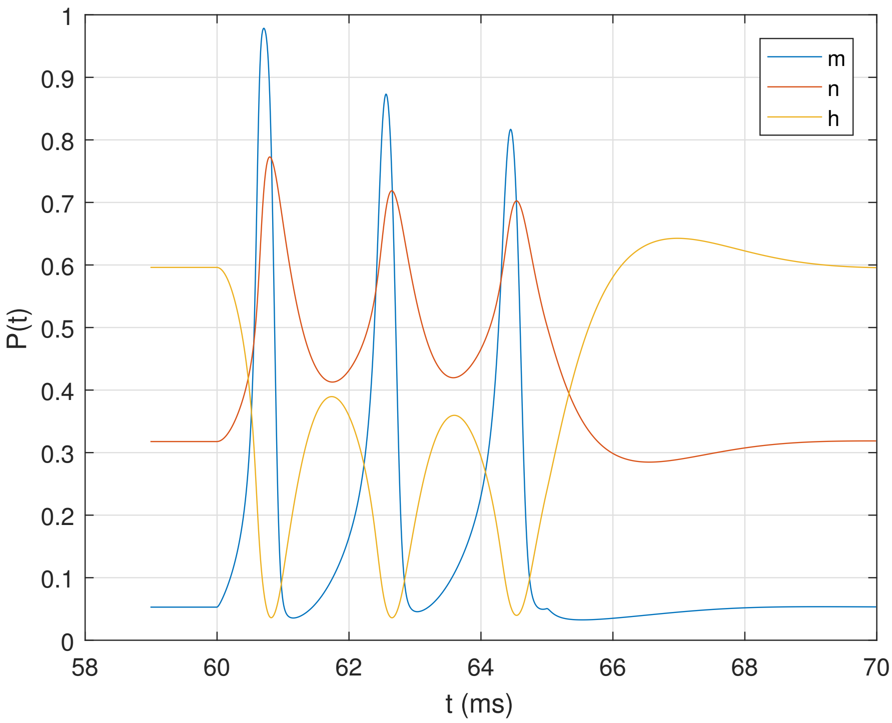


Abbildung 7: Closeup on gating variables m, n, h at 28 °C and for the input visible in figure 2b

A. Equations and Constants

All units are given as in the original publication. Hodgkin and Huxley obviously had no idea about SI units but if you use all units as given and ms for the time axis, all numbers can be used without conversions.

General equations:

$$\frac{dV}{dt} = \frac{1}{c}(-i_{ion} + i_{stimulus})$$
$$i_{ion} = i_{Na} + i_K + i_L$$
$$V = V_i - V_e - V_{rest}$$

Ionic currents:

$$i_{Na} = \bar{g}_{Na} m^3 h (V - V_{Na}) \qquad i_K = \bar{g}_K n^4 (V - V_K)$$
$$\frac{dm}{dt} = [\alpha_m(1-m) - \beta_m m]k \qquad \frac{dn}{dt} = [\alpha_n(1-n) - \beta_n n]k$$
$$\frac{dh}{dt} = [\alpha_h(1-h) - \beta_h h]k \qquad i_L = \bar{g}_L(V - V_L)$$

Temperature correction (T in °C):

$$k = 3^{0.1(T-6.3)}$$

Rate equations (V in mV):

$$\alpha_m = \frac{2.5 - 0.1V}{e^{(2.5-0.1V)} - 1} \qquad \beta_m = 4e^{-V/18}$$
$$\alpha_n = \frac{0.1 - 0.01V}{e^{(1-0.1V)} - 1} \qquad \beta_n = 0.125e^{-V/80}$$
$$\alpha_h = 0.07e^{-V/20} \qquad \beta_h = \frac{1}{e^{(3-0.1V)} + 1}$$

Constants

Conductances in $^{\mathrm{mS}}/_{cm^2}$			
$\bar{g}_{Na} = 120$	$\bar{g}_K = 36$	$\bar{g}_L = 0.3$	
Resting potentials in mV			
$V_{Na} = 115$	$V_K = -12$	$V_L = 10.6$	$V_{rest} = -70$
Other constants			
$c = 1\,^{\mu\mathrm{F}}/_{\mathrm{cm}^2}$			