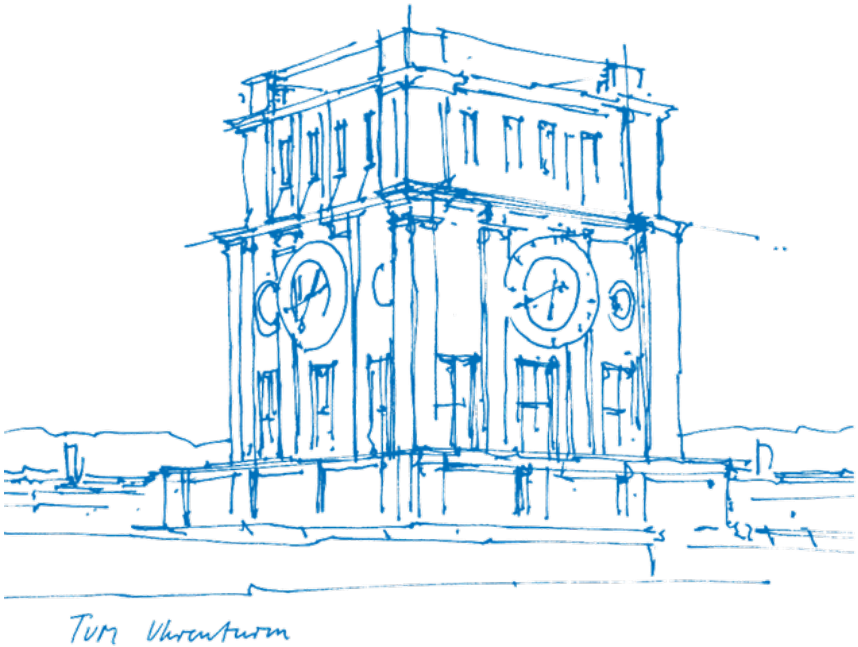
Mathematical Basics I

Neuroprosthetics SS 2018

Jörg Encke, Andrej Voss

Prof. Werner Hemmert
Technical University of Munich
Bio-inspired Information Processing



Technische Universität München

Fachgebiet für Bioanaloge Informationsverarbeitung

$$\frac{dV}{dt} = f(V, t)$$

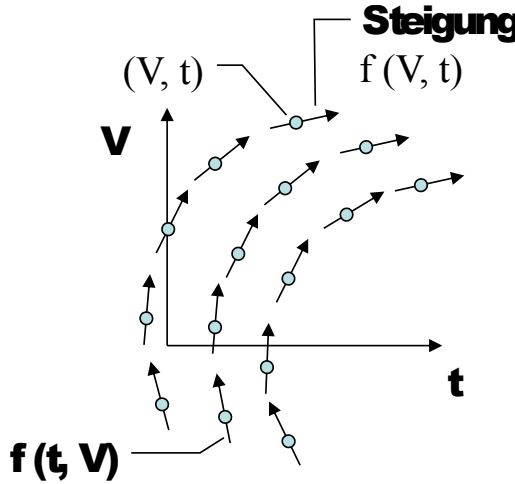
$$\frac{dV}{dt} = t^2 - V \rightarrow \text{analytically solvable}$$

$$\frac{dV}{dt} = t - V^2 \rightarrow \underline{\text{not}} \text{ (trivially) analytically solvable}$$

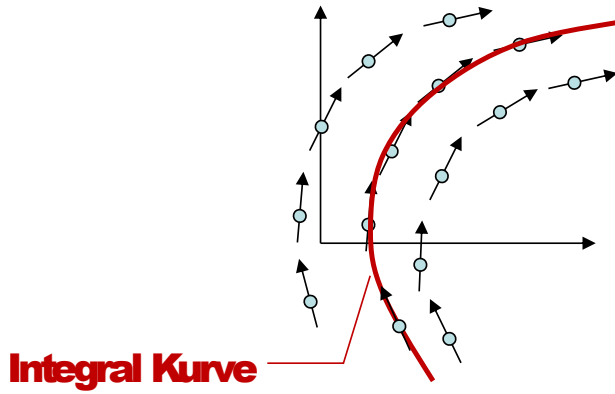
Geometric view on Differential Equations

Analytical $\Rightarrow$ **Geometrical**

$$\frac{dV}{dt} = f(V,t) \quad \Rightarrow \quad \text{slope field}$$



solution $V(t)$, with t_0, V_0 $\Rightarrow$ integral curve



Recipe for the slope field (Computer)

1. Select equally distributed points in space.
2. Calculate $f(V, t)$ for the points (V_n, t_n).
3. Draw the slope of the function at these points.

Recipe for the slope field (Human)

1. Select an interesting slope C.
2. Solve $C = f(V, t)$ for a selected number of interesting points t_n.
3. Draw the so-called *isoclines*.

Isoclines provide information about certain properties of the solution even if the general solution is not determinable, e.g.:

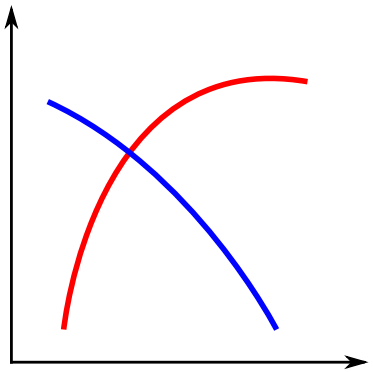
- Identification of areas where the solution has a steep slope.
- Identification of areas where the solutions stray.

Example: slope field

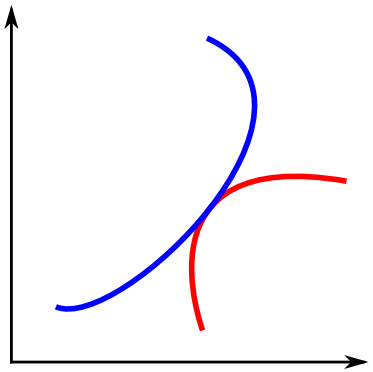
$$\frac{dV}{dt} = 1 + t - V$$

Table transcript

In general: Two integral curves don't intersect



Two integral curves don't touch.

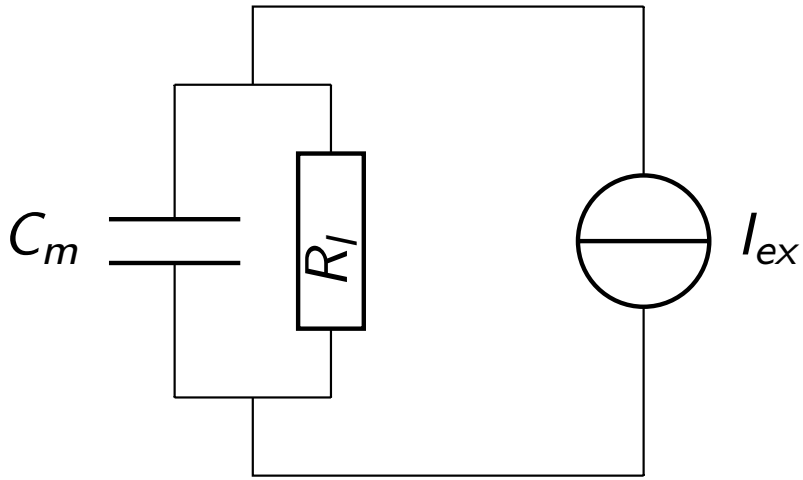


For each point (t_n, V_n) there is only one solution!

This approach especially helps to understand dynamic systems.

Fish population in a fishpond.
Table transcript

A simple equivalent circuit of a cell:



Questions?