Multi Compartment Model

Neuroprothetics SS 2017

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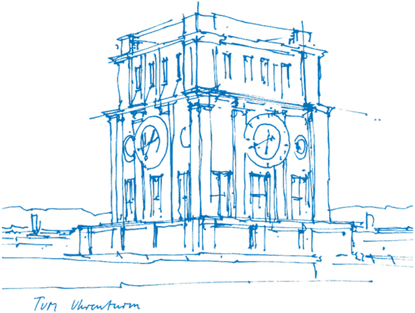

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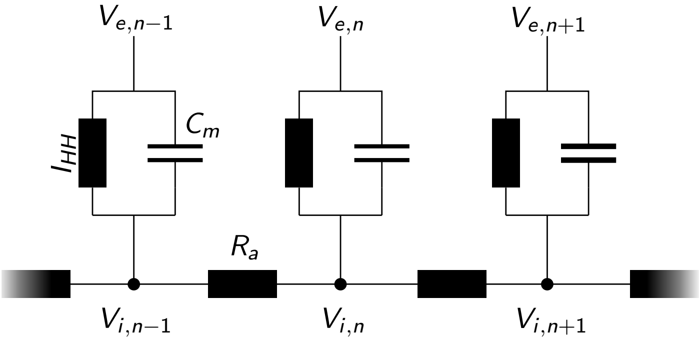
Equivalent Circuit



Using Kirchhoff's current law on the compartment n results in the following system of differential equations

$$0 = C_m \frac{dV_{m,n}}{dt} + I_{HH,n} + \frac{V_{i,n} + V_{i,n-1}}{R_a} + \frac{V_{i,n} - V_{i,n+1}}{R_a}$$

$$\frac{dV_{m,n}}{dt} = -\frac{1}{C_m} I_{HH,n} + \frac{1}{C_m} \frac{V_{i,n-1} - 2V_{i,n} + V_{i,n+1}}{R_a}$$

If we add a stimulation current:

$$\frac{dV_{m,n}}{dt} = \frac{1}{C_m}(-I_{HH,n} + I_{stim,n}) + \frac{1}{C_m} \frac{V_{i,n-1} - 2V_{i,n} + V_{i,n+1}}{R_a}$$

This system can also be written in matrix form:

$$\frac{d}{dt}\vec{V}_m = \frac{1}{C_m}(-\vec{I}_{HH} + \vec{I}_{stim}) + \frac{1}{C_m R_a}\mathbf{C}\vec{V}_i$$

Where $\vec{V}_m = \begin{pmatrix} V_{m,1} \\ V_{m,2} \\ \vdots \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -1 & 1 & & & 0 \\ 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \\ 0 & & & 1 & -1 \end{pmatrix}$

$\vec{V}_i, \vec{I}_{HH}, \vec{I}_{stim}$ are vectors similar to $\vec{V}_m$

We can substitute the internal potential using the following relation:
$\vec{V}_m = \vec{V}_i - \vec{V}_e$

$$\frac{d}{dt}\vec{V}_m = \frac{1}{C_m}(-\vec{I}_{HH} + \vec{I}_{stim}) + \frac{1}{C_m R_a}\mathbf{C}\vec{V}_m + \frac{1}{C_m R_a}\mathbf{C}\vec{V}_e$$

For this exercise, we set: $\mathbf{C}\vec{V}_e = 0$ as we do not apply any external potentials

Implicit Euler method

We use this system of differential equations using the implicit Euler method:

$$\begin{aligned} \frac{dV}{dt} =& f(t) \\ V(t+\Delta t) =& V(t) + \Delta t \cdot f(t+\Delta t) \end{aligned}$$

Linear system

$$\vec{V}_m(t+\Delta t) = \vec{V}_m(t) + \frac{\Delta t}{C_m}(-\vec{I}_{HH}(t+\Delta t) + \vec{I}_{stim}(t+\Delta t)) + \frac{\Delta t}{C_m R_a}\mathbf{C}\vec{V}_m(t+\Delta t)$$

$$\underbrace{\left(\mathbf{I} - \frac{\Delta t}{C_m R_a}\mathbf{C}\right)}_{\mathbf{A}} \cdot \underbrace{\vec{V}_m(t+\Delta t)}_{\vec{x}} = \underbrace{\vec{V}_m(t) + \frac{\Delta t}{C_m}(-\vec{I}_{HH}(t+\Delta t) + \vec{I}_{stim}(t+\Delta t))}_{\vec{b}}$$

The Result is a linear system of the Type:

$$\mathbf{A} \cdot \vec{x} = \vec{b}$$

A linear system of the type $\mathbf{A} \cdot \vec{x} = \vec{b}$ can easily be solved:

Matlab: $\mathrm{x} = \mathrm{A} \setminus \mathrm{b}$

Python: x = solve(A, b)
The solve function is provided by the Numpy package numpy.linalg

Questions ?