To stop, or not to Stop: Time non-separable preferences

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Abstract

We investigate history dependent stopping behavior and identify factors that motivate people in non-monetary environments. In the context of chess games, we find that the history of wins/losses is an indicator of further participation. In particular, we find that there are two types of people: those who get discouraged by a loss and quit; and others, who get encouraged by a loss and keep playing until a win. We find that an individual's type is time-invariant over years. We propose a behavioral dynamic choice model where enjoyment from the next game is directly affected by the outcome of the previous game. We structurally estimate this time non-separable preference model and conduct counterfactuals to meet several objectives. In an environment where practice improves overall outcomes, type individualized incentives can improve welfare. We find that, in the context of chess games, adjusted incentives can increase the length of play by at least 60%.

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1 Introduction

What determines our decision of when to stop a given endeavor? Do our past successes motivate the stopping decision, or is failure the primary determining factor? In this paper, we study history dependence of stopping behavior in the context of chess games, in an environment that is free of monetary incentives. We find a behavioral friction, a person's past experience with successes and failures is a key determinant of the stopping choice. We argue that such a history dependence is not explained using existing economic theories and we propose an alternative behavioral theory, the time non-separable preference model, which rationalizes data extracted from a popular online chess platform.

We collect and analyze data from the most prominent online chess platform, chess.com, which has over 30 million users, and hosts 3 million chess games every day. We find two main empirical patterns. First, we detect strong history dependence in stopping decisions. The result of the previous game significantly changes the probability of playing another game. Second, we find heterogeneity amongst players. The same history can have opposite motivational effects on different people. We can classify the entire population into three broad groups: loss-stopper, win-stopper, and neutral types. Loss-stoppers are discouraged by a loss and decide not to play another game, while win-stoppers feel challenged by a loss and play until they win. We refer to these two types as behavioral types. The rest of the population are non-behavioral, meaning that the last game score does not have significant effect on their stopping decision.

We introduce a theory where *future* game utility depends on the history of play and a player's type, which we show can explain patterns observed in our descriptive analysis of the data. For some people the loss in a given game taints the next game and makes it less enjoyable. For the other behavioral type, the loss encourages further play and increases overall joy. Hence, the enjoyment from the next game is determined by the outcome of the current game and a player's type. We examine existing behavioral theories, such as reference dependence, gambler's fallacy, and fatigue, and we find that they do not capture the two types of behavioral patterns we identify in the data. Reference dependence would predict one type of the behavior we observe—stopping the session on a win. However, loss-stoppers are not accounted for by reference dependence. Similarly, all other theories could only account for one type of pattern but are silent on the other type.

To illustrate a practical use of the theory we propose, we estimate a structural model which classifies players according to the model types, and then conduct counterfactuals in which we show how a policy maker (or a platform designer) could leverage the type classification in order to meet certain welfare objectives. More precisely, a natural control for a policy maker or the platform is a player-to-player matching mechanism. The current matching mechanism on the chess platform is as follows. A player is randomly matched with another player with a rating within a defined range from their own rating. Suppose the platform wants to encourage *longer* play; can it achieve this objective by making its matching mechanism contingent on behavioral types?

The answer turns out to be positive. The intuition for why this is the case, is that incentive schemes for the two behavioral types should be different. To encourage longer play time, loss-stopper types can be matched with relatively lower rated individuals in their rating range, so they have a higher likelihood of winning and hence higher likelihood of continuing to play. Structurally estimating the model allows us to compute the welfare improvement by identifying types and providing individualized incentive structures. In our environment, type-identification and individualized incentives increase the session length by at least 60%.

One application of our theory is to the environments where practice could improve overall outcomes. For example, being able to identify a student's type could allow a teacher or a tutor to individualize the curriculum in such a way that would improve the student's learning outcomes. Loss-stopper types may need a more gradual introduction to new concepts, while win-stopper types may benefit from facing bigger challenges to keep them interested for longer. By observing children's behavior and identifying their type, parents could frame an environment or a problem in ways to accommodate their child's personality type and to better ensure their success.[1]

Another application of our theory is to the mobile app industry, which is one of the fastest growing industries that generated $88 billion in 2017 and it is projected to reach $189 billion by 2020. About three-quarters of total revenue are generated by mobile games, gained through advertising, in-app purchases, sponsorships, and freemium features.[2] The

[1] Our approach takes the types as given and creates an environment that could benefit all types of individuals. Alan et al. (2019) show that grit, a skill that has been identified as an important component for success and future achievement, is malleable in childhood and it can be developed in classroom environment (see also Heckman et al. (2006), Heckman et al. (2013), Sutter et al. (2013),Levitt et al. (2016), Alan and Ertac (2018)). However, the long term consequences of changing one's type and the stability of such modification are still unclear.

[2] The word freemium is a combination of the words "free" and "premium" used to describe a business model that offers basic services for free for the user to try and more advanced or additional features at a premium. This is a common practice with many software companies, especially game companies. Everybody is welcome to play the game for free, but additional features are available for the player at a cost. Sources

longer a player stays on a platform, the more value that can be generated by an app. However, little is known of what determines length of play; there could be many reasons that explain play time. Nevertheless, in this paper, we find that there are strong behavioral forces that drive players' stopping decisions.

This paper complements the literature spurred by Camerer et al. (1997) which, based on an analysis of New York City taxi drivers' labor supply, proposes that drivers stop working once a target level of earnings is reached. In contrast to that literature, we study stopping behavior in an environment that is free of monetary incentives, which avoids issues related to liquidity constraints and focuses on pure behavioral forces. From a broad perspective, the above literature posits that people have targets (reference points) and work persistently until they achieve those targets. Formally, loss aversion and reference dependent utility are sufficient to rationalize such behavioral patterns. In this paper, we show that there exists a behavioral pattern which is the mirror image of working persistently until a target is achieved. Such a pattern is exhibited by loss-stoppers—people who get discouraged by a failure. Being able to capture the heterogeneity in stopping behavior is crucial in designing incentives to encourage longer participation or for meeting other welfare objectives.

In the context of chess games, Anderson and Green (2018) show that personal best rating acts as reference point for a player: a player ends a session whenever she sets her new personal best rating, and she plays longer otherwise. Similar to our paper, they also study history dependent stopping behavior. In contrast to our paper, they do not consider heterogeneity in stopping behavior across players. We find such heterogeneity in the data, and that it has crucial implications for understanding players' stopping behavior. Moreover, the instances where personal best rating can be improved are much more rare compared to all the other times a person plays a game.

In this paper we establish a behavioral friction that is evidence of time non-separable preferences[3] and we find that the observed patterns are robust and consistent over time. The results discussed in the main part of the paper are based on data gathered from games played in 2017. We use matching data from 2018 to test for time consistency of behavioral types. We find that there is a 75% match between these two consecutive years, highlighting that an individual's behavioral pattern does not change over time. In addition, we find that behavioral type classification is not affected by players' rating, that is, the fraction of people

www.statista.com and www.newzoo.com.

[3] See Braun et al. (1993) and Dragone and Ziebarth (2017) for evidence of time non-separable preferences in aggregate consumption and novelty consumption, respectively. See Turnovsky and Monteiro (2007) for the effects of consumption externalities under time non-separable preferences.

that get discouraged by a loss is the same with amateur players as it is with professional players.

The rest of the paper is organized as follows. Section 2 provides details on the data collection and descriptive statistics. In Section 3, we introduce the model and the identification strategy. Section 4 presents the results of the structural estimation. Section 5 concludes.

2 Data

In this section we first provide details on the data collection and information about the platform. We then provide descriptive statistics that illustrate consistent behavioral patterns. We end the section by considering various theories that could account for the behavioral patterns observed in the data.

2.1 Collection

We have collected the data from an online chess platform, chess.com, which was started in 2005 and is the most frequently visited chess website.[4] The website has over 30 million users, and it hosts 3 million chess games every day. The platform users' span from the amateur players to the world's best chess players, such as Magnus Carlsen, the World Chess Champion.[5] This platform is free to use, and anyone can register to play against other people or against a computer. The website also provides some lessons and chess puzzles; however, we only use games played between humans.

We use the public Application Programming Interface (API) to collect data. We divided the data into different groups according to the rating of players on this website. Each observation includes information about the players and the game. We observe their usernames, their self-identified country of association, and platform ratings. In addition, we see information about the game itself: time the game was played, which player had white pieces, the length of the game, and the final results.

2.2 Preliminaries

Throughout the paper we use terms to describe the data and understand our results. For clarity of exposition let us first define all the terms used before proceeding to the descriptive results.

[4] Based on Alexa internet rating, www.alexa.com.

[5] Magnus Carlsen, a Norwegian chess grandmaster and chess prodigy, is the highest rated player in the world, and the highest rated player in the history of chess.

A *game*, g, is a single game with a human opponent. A collection of games ordered by time stamp, $(g_1, g_2, ..., g_n)$, is called a *session* if there is no game played T minutes before g_1, there is no game played after g_n for at least T minutes and for any $i \in \{1, ..., n-1\}$, the time between g_i and g_{i+1} is less than T.[6] If a session contains only one game ($n = 1$), we refer to such session as *only game* (O-game). For sessions with $n \geq 2$, g_1 is the first game, g_n is the last game and any game in between the first and the last is referred to as *middle games* ($k \in \{2, 3, .., n-1\}$).

Based on the terms above, we can only have the following types of sessions: the only game $S = (O)$, two games in a session $S = (F, L)$ or $S = (F, M_1, ..., M_m, L)$ when there are 3 or more games in a session ($m \in \{1, ..., n-1\}$). Let $f_W(\cdot)$ be a function that calculates the fraction of wins in a particular type of game, for example, $f_W(L)$ is a player's average winning score in the last games. In some cases, when the context is clear, instead of writing $f_W(F)$, $f_W(M)$, $f_W(L)$, $f_W(O)$, we simply write F, M, L, and O, and we mean the average score in all first, middle, last, and only games, respectively.

2.3 Descriptive Results

In this section, we first establish that session-stopping behavior is history dependent. Then we evaluate the history dependence and ask whether players tend to get discouraged by a loss and stop the game or whether players get encouraged by a loss and keep playing. We find that large fraction of players fall into two behavioral categories: some people consistently leave the session after they win the game and others stop the session after a loss; we refer to these types *win-* and *loss-stoppers*, respectively. That is, win-stoppers are more likely to continue playing after a loss than a win. Conversely, loss-stoppers get discouraged after a loss, and they are more likely to stop playing for a while. At the end of this section, we discuss theoretical work that could potentially explain such patterns of behavior identified in the data. We conclude that none of the established theories can account for all the behavioral types and stopping patterns. The latter observation leads us to the next section where we present our theory model that provides an explanation for observed behavior and accounts for the behavioral patterns in the data.

[6] For the main section of the results we set $T = 30$ minutes, we vary T to check the robustness of our results and we find no substantial differences.

2.3.1 History Dependence

We first take all sessions in our data that lasted at least 3 games and calculate the average winning frequency in first, middle and last games for each player as defined in Section 2.2. If a decision to stop the game is made randomly and the stopping behavior is history independent, then the score in the last game should be similar to the average score in any other game, such as the score in the middle game. Hence, our first null hypothesis is that there is no difference in wining frequency in the last game vs the middle game.[7]

Figure 1a presents the relationship between average scores in the last and middle games. We can notice that last and middle game scores are not positively correlated. To the contrary, if there is any correlation it seems negative. Secondly, we calculate the correlation between winning fractions in the last and middle games, which is $-.32$ and statistically different from 1 with $p < 0.001$. This result is the first piece of evidence that the decision to stop is not random and depends on other factors. Next, we investigate what factors could explain the stopping behavior.

(a) Last and Middle Game Scores (b) Score Density

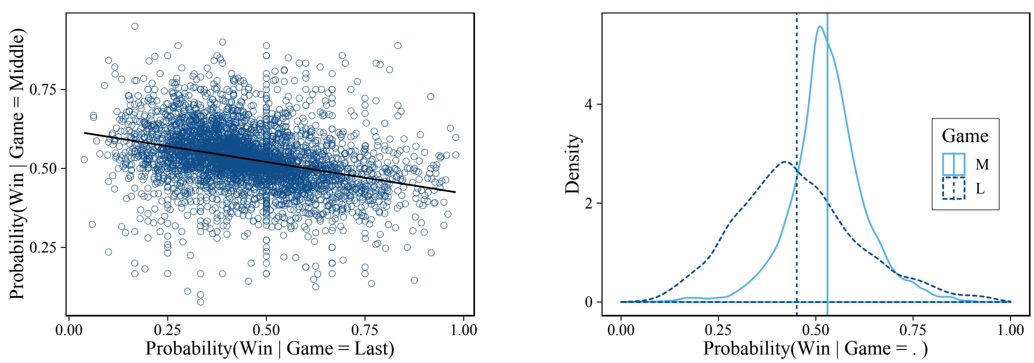

Figure 1: Correlation Matrix and Score Density

2.3.2 Behavioral Types

We rejected the hypothesis of history independence; however, the alternative hypothesis simply states that there is a relationship but does not identify its nature. The question is, are people motivated by success and failure? Put differently, in our environment, are players more likely to stop playing for a while after a win or after a loss? We state the

[7] We do not include first games at this stage, because the data on first games is used later to check robustness of the findings.

following two alternative hypotheses:

H_A^1: *players are more likely to stop playing after a win.*

H_A^2: *players are more likely to stop playing after a loss.*

If only one of the two alternative hypothesis is correct, we should see a skew or a shift of the distribution of last game scores compared to the middle game score distribution (Figure 1 presents both distributions). The test of medians for the two distributions shows that the median score in the last game is statistically lower than the score in the middle game. If we only look at this aggregate result, it supports the hypothesis that players are more likely to stop playing after a loss. However, if we take a broader look at the distribution, we see an interesting pattern. The standard deviation of the last score is twice that of the middle score. Figure 1 informs us that there are some people with much higher scores in the last game compared to the middle game, and that there are others with much lower scores in the last game compared to the middle game. Based on this evidence, we introduce Definition 1 and look for two behavioral types: players that are more likely to leave the session on a loss and others who are more likely to leave the session on a win.

Let the probability of winning conditional on this being the final game be $P_{W|Last} := \Pr(Win|\ g = L)$ and let $P_{W|Middle} := \Pr(Win|g = M)$.

Definition 1 *A player is behavioral type at the tolerance level of τ and she is*

- *a win-stopper if $P_{W|Last} > P_{W|Middle} + \tau$,*
- *a loss-stopper if $P_{W|Last} < P_{W|Middle} - \tau$,*

Otherwise, we call a player a non-behavioral type, $P_{W|Last} \in [P_{W|Middle} - \tau, P_{W|Middle} + \tau]$.

We vary the tolerance $\tau \in [0, .2]$ and we classify players in the data set according to the Definition 1 (see Figure 5 in the appendix). Intuitively, as we increase the tolerance level, less players are classified as behavioral types. Interestingly, while we change τ the ratio of win-stoppers to loss stoppers stays the same at around 40%. At tolerance level of 5%, 84% of the players are classified as behavioral types, with about 30% of them being win-stoppers and a larger fraction, 70%, loss-stoppers.[8]

2.3.3 The only and last games

To further examine that the Definition 1 captures the behavioral patterns in the data, let us examine only-games—sessions that contain only one game. Note that sessions with only

[8] Unless specified otherwise, we use 5% tolerance level.

one game have not been used in the classification, only sessions of length 3 and above are used. For ease of exposition, let us take Definition 1 to the extreme, where we assume that a win-stopper type **always** stops after a win and a loss-stopper type **always** stops after a loss. This extreme definition of types implies the following two observations: (1) the winning probability for win-stopper types in the only-games must be 1 (if a player wins the game she would stop playing, making a session 1 game long, and if a player loses the first game she will start another game, making this session at least 2 games long); (2) the winning probability for loss-stopper types in the only games must be 0 (if a player losses the first game she would stop playing, making a session 1 game long, and if a player wins the first game she will start another game, making this session at least 2 games long). Combining these two observations leads to the following prediction.

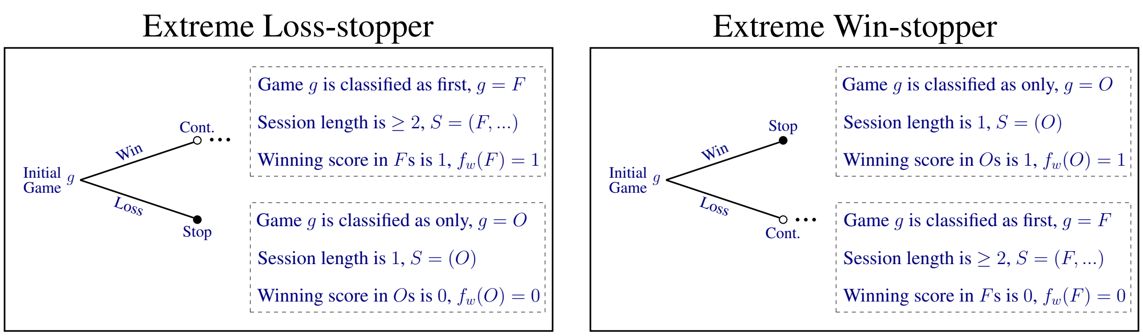

Figure 2: Extreme Behavioral Types' Actions

Prediction 1 *Correlation between the last and only game scores is positive.*

Note that we classify our players using data that contained at least 3 games per session. Now we are looking at sessions that contain only one game, which is not used for the type classification. We calculate the average scores of win-stopper and loss-stopper types in the only games as defined in Definition 1 and win-stoppers' score in the only-game is two times higher than that of loss-stoppers'. Figure 3a presents a scatter plot with the last-game scores on x-axis and the only-game scores on the y-axis. A strong and significant positive relationship between last- and only-game scores implies that the types who are more likely to stop playing on a win similarly have higher scores in the only game and vice versa, as predicted by the discussion in the previous paragraph.

(a) Last and Only Game Scores (b) Correlation Matrix

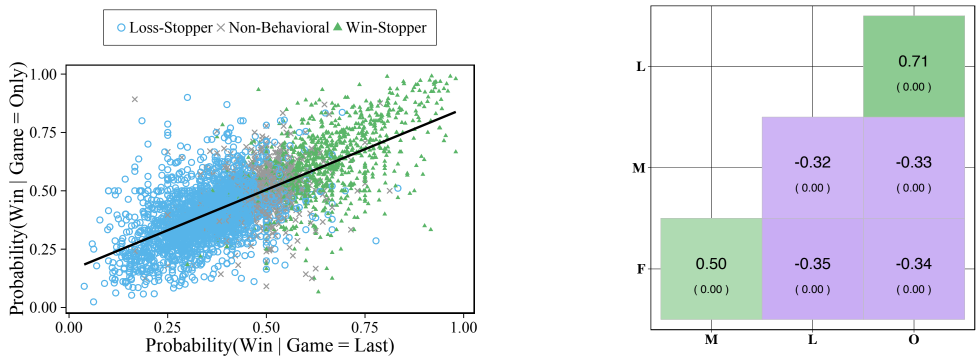

Figure 3: Correlation Matrix and Score Density

2.3.4 The first and only game

What is the relationship between the first game and the only game? For ease of exposition of the ideas let us take the Definition 1 to the extreme again, where we assume that a win-stopper type always stops after a win and a loss-stopper type always stops after a loss. The diagram in Figure 2 describes what would each type do if they won or lost the initial game. Note that while the prediction for last- and only-game relationship is positive, the prediction for first- and only-game relationship is negative.

Prediction 2 *Correlation between the first and only game scores is negative.*

Figure 3b presents the correlation matrix with p-values in parentheses. Negative relationship between first- and only-game scores implies that the types who are more likely to stop playing on a win have higher scores in the only game since they would have kept playing if they had lost the initial game. Similarly, the types who are more likely to stop playing on a loss have lower scores in the only game.

2.4 Theoretical Discussion

What would explain the behavioral patterns we find in the data? Can reference dependence, gambler's fallacy, or fatigue explain the observed patterns? Personal best rating could act as reference point for players: a player ends a session whenever she sets her new personal best rating, and she plays longer otherwise. Reference dependence would predict one type of the behavior we observe—stopping the session on a win. However, loss-stoppers are not accounted for by reference-dependence.

The gambler's fallacy would suggest that if a player wins a few games in a row then a player may think that it is less likely that she will win again, and therefore, she would stop on a win. While this explanation may fit win-stoppers, loss-stoppers are still to account for. Since with similar logic if a player has lost a few games in a row, she may think that she is more likely to win, therefore since she enjoys winning more than losing, she should stay and never leave on a loss. While gambler's fallacy could explain one of our types, it goes against the behavior of the other type.

Now, suppose as a player keeps playing games she gets fatigued over time; hence, fatigue would lead to worsened play over time. Such a conjecture would imply that the last game score should be lower than middle and first games. While lower last game scores are observed with loss-stoppers, it is completely opposite for win-stoppers, who have a much higher score in the final game. Some of these explanations account for one type or the other. However, to the best of our knowledge, no existing theories could explain both types of behavior that we have established in the data.

We introduce a theory where we propose that **future** game utility depends on the history of play and player's type. For some people the loss in a given game taints the next game and makes it less enjoyable. For the other type, the loss encourages further play and increases the overall joy. Hence, the enjoyment from the next game is determined by the outcome of the current game and player's type. We say there are two behavioral types of players and one non-behavioral type: *loss-encouraged*, *loss-discouraged*, and *loss-neutral*. We presume that expected utility from a next game is **reduced** by a loss in a current game for loss-discouraged types, the utility **increases** for a loss-encouraged types and stays the same for loss-neutral types.

To test our claim, we introduce a model that captures our intuition. We model the idea of history dependent utility by making the outside option more or less desirable based on the win/loss history in the previous game. We then structurally identify the model parameters and classify the players according to model's three types. By comparing the two classification types we find high correlation between the type identification based on the last game score and model types, which provides strong support for our claim.

3 The Model

We do not take a stand whether people get discouraged by a failure or whether people get motivated by it. Instead, we model both types of behavior and let the data inform us which hypothesis is more plausible (if not both). Our model allows both types of behavior by assuming that based on history of play, player's outside option increases or decreases based on whether the player had won or lost the previous game. We capture both types of people by allowing the outside option to be dependent on the previous game outcome and the type of the person.

We say that a person gets discouraged by a loss—referred to as loss-discouraged—if the outside option for this person becomes more attractive after a loss than what outside option would have been after a win. Similarly, we say that a person gets encouraged by a loss—referred to as loss-encouraged—if the outside option for this person becomes less attractive after a loss than what outside option would have been after a win. Loss-neutral types have the same outside option regardless of the previous game outcome.

The next subsections introduce our model and identification strategy.

3.1 Description

This section lays out a dynamic choice problem of a chess player. A chess player is characterized by her type that consists of an element observable to the player, player's opponents and econometrician, and an element privately known to the player.

Let x be a vector of characteristics of a player that are fixed over time, and are observable to the player, her opponents and econometrician (e.g. country). Let y be a vector of characteristics that may change over time, and are observable to the player, opponents and econometrician (e.g. player's rating). We assume that x and y live in finite spaces, $X = \{x^1, x^2, ..., x^{|X|}\}$ and $Y = \{y^1, y^2, ..., y^{|Y|}\}$, respectively.

A player can be one of 3 types: loss-encouraged (denoted θ_E), loss-discouraged (denoted θ_D), or loss-neutral (denoted θ_N). Let $\Theta = \{\theta_E, \theta_D, \theta_N\}$ be the set of types and let θ denote an element from this set. The type of a player is fixed over time.

A player's type profile at time t, (x, y_t, θ), consists of player's fixed observable characteristics, x, time-variable characteristics, y_t, and fixed unobservable type, θ. We use variables without time subscripts to denote current states and 'prime' superscripts to denote next period's state. A player's opponents' variables have subscript $-i$.

In each period, a player is faced with the following decision: given the previous history of her play, she needs to decide whether to play an additional game, or to go offline and take an outside option. Before making this decision, she can calculate the expected utility of playing one more game,

$$EU(x,y) = \sum p(y_{-i}|x,y)\left[U^0(y,y_{-i}) + p(w|x,y,y_{-i})[U^1(y,y_{-i}) - U^0(y,y_{-i})]\right]$$

where $U^1(\cdot)$ and $U^0(\cdot)$ are utils from winning and losing. Those utils depend on certain observable about an opponent, $-i$, (e.g. defeating a higher rated player is more delightful) and the player. The conditional probability of being matched with an opponent of type (y_{-i}) is $p(y_{-i}|x, y)$. This probability is in practice determined by platform's matching technology. Finally, $p(w|x, y, y_{-i})$ is the conditional probability of winning.

An outside option, c, is drawn from a distribution with density $f(c)$, independently across players and over time. If a player goes offline (ends a session), she takes an outside option and accrues $c + (1 - \chi)l_\theta$, where l_θ is a parameter that may depend on player's behavioral type and χ is an indicator of whether a player lost or won the last game, taking value of 1 in case of winning.[9] Otherwise, she plays and expects a payoff $EU(x, y)$ from playing the game.

Definition 2 *A player is*

i) loss-discouraged, if $l_\theta > 0$*;*

ii) loss-encouraged, if $l_\theta < 0$*;*

iii) loss-neutral, if $l_\theta = 0$.

We have an optimal stopping decision problem,

$$\begin{aligned} V(\theta, x, y, c, \chi) = \max_{d \in \{0,1\}} \big\{ &(1-d)[c + (1-\chi)l_\theta] + \\ &d[EU(x,y) + \delta \sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y,\chi')p(\chi'|x,y)V(\theta, x, y', \chi')] \big\} \end{aligned} \tag{1}$$

In expression (1), $d = 1$ stands for finishing the session (stopping) and $V(\theta, x, y, \chi)$ is an expectation of $V(\theta, x, y, c, \chi)$ with respect to c. The next period's evolving type, y', depends on χ' and y. Note that the law of motion of y can be directly recovered from the data (e.g. player's rank updating rule).

3.2 Identification

In this section, we show the identification of players' behavioral types, outside option distribution, utilities from winning and losing, l_θ, δ, probabilities of winning, and matching

[9] Note that an observationally equivalent way of modeling behavioral types would be to instead make expected utility of playing the game dependent on behavioral type via previous game's result that is, U^0, U^1 could somehow depend on θ. One can choose to do so. However, note that if one wishes to model θ as affecting both, utility from playing a game and outside option, then identification of behavioral types would be problematic.

probabilities. The identification and estimation of the theory model is in the tradition of Hotz and Miller (1993). We show how we can forgo numerical dynamic programming to compute the value functions for every parameter vector, and propose an estimation procedure that is simple to implement and computationally efficient.

Claim 1 *Optimal stopping rule is a threshold rule in c.*

Let $c(\theta, x, y, \chi)$ denote a threshold such that a player of type (θ, x, y) having a result χ in the last game ends a session if and only if the realized c is at least as large as $c(\theta, x, y, \chi)$. At the interior $c(\theta, x, y, \chi)$ a player is indifferent between ending a session and continuing to play. If $c(\theta, x, y, \chi) = 0$ a player may strictly prefer to end a session at the threshold. However, we do not consider the case where $c(\theta, x, y, \chi) = 0$ as there is not even one player in our data whose behavior would be consistent with such a threshold. From now on, we assume that the model parameters are such that $c(\theta, x, y, \chi)$ is interior. From (1), we have,

$$c(\theta, x, y, \chi) + (1-\chi) l_\theta = EU(x, y) + \delta \sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi') p(\chi'|x, y) V(\theta, x, y', \chi') \quad (2)$$

3.2.1 Identification of Behavioral Types

The following proposition leads to the identification of behavioral types,

Proposition 1
i) $c(\theta_E, x, y, 0) > c(\theta_E, x, y, 1)$*;*
ii) $c(\theta_D, x, y, 0) < c(\theta_D, x, y, 1)$*;*
iii) $c(\theta_N, x, y, 0) = c(\theta_N, x, y, 1)$.

Proposition 1 implies that the loss-encouraged types' probability of playing one more game is higher if previous game was lost compared to when the previous game was won, and vice versa for the loss-discouraged types. For the loss-neutral types that probability is the same no matter the history of outcomes. By Proposition 1 we can identify a behavioral type of a player from the data by simply looking at her stopping probabilities after losses and wins.

3.3 Robustness of the Classification

In this paper we have two distinct definitions of behavioral types (Definitions 1 and 2). The two definitions are intuitively related, but these two methods of identifying behavioral types do not have to overlap at all. That is, given some data, a player could be identified as loss-encouraged according to the model but be classified as loss-stopper according to the last-game and middle-game definition. For example, suppose a player's complete playing history contains the following set of 3 sessions:

$$\{WWWW, WLW, WLL\},$$

Let us look at all the wins and calculate the stopping probability after a win. We have $\Pr(Stop|Win) = 2/7$. Now, calculate the stopping probability after a loss, $\Pr(Stop|Loss)$ $= 1/3$. Since probability to stop playing is higher after a loss than after a win, $\Pr(Stop|Loss)$ $> \Pr(Stop|Win)$, our model would qualify the player as loss-discouraged. However, according to our definition through the last game of the session, the player's behavior corresponds to win-stopper type, because the last game is won more often than middle games.

This example demonstrates that while the model and our behavioral type definitions are intuitively related, one does not imply the other. This fact strengthens any relationship we find between the two classifications, highlighting the consistency of our intuition with the proposed theory.

4 Structural Estimation

In this section we introduce our structural estimation results. First, we present model classification of types, and we compare it to the initial classification in Section 2.3. The rest of the section is devoted to the parameter estimates of the model and counterfactual estimation.

4.1 Consistency of Behavioral Types

We have identified each player using two classifications that are independent of each other; now, let us look at how they pair. We take all the players and in Figure 4a we put a plus sign if the model identifies a win-stopper player as loss-encouraged, loss-stopper as loss-discouraged and non-behavioral as loss-neutral. For 76% of the players in the data the two classifications match. This provides us with strong evidence that supports our model and the claim that the game outcome affects the utility of a next game.

Figure 4b presents a transition matrix from behavioral types to model types. We observe a large mass on the diagonal, meaning that the two classifications are consistent. For example, 71% of loss-encouraged types are win-stoppers. However, there are mismatches, for example, some non-behavioral types are classified as behavioral types by the model and vice versa. Notice that there are few cases in which a win-stopper (loss-stopper) is identified as loss-encouraged (loss-discouraged) by the model. Most of the errors come from the model and data pattern classificiation

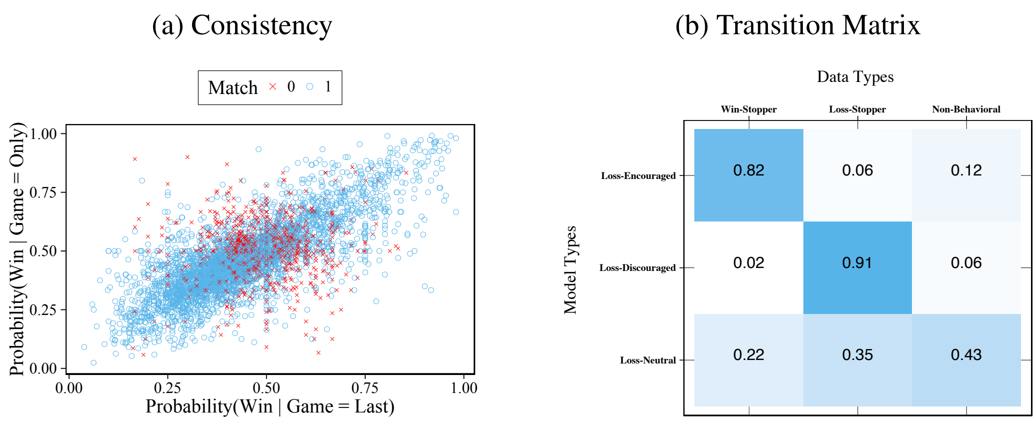

Figure 4: Model Identified Types and Consistency

4.2 Structural Estimates and Counterfactuals

Can the platform leverage information on behavioral types to increase expected time spent by players on the platform? If so, by how much can it increase this time? To answer these questions, in our counterfactual exercise we allow the platform to choose from matching mechanisms that can be contingent on players' behavioral types. Ideally, we would try to find an optimal matching mechanism in such class of mechanisms. However, due to the high dimensionality of the problem (we have $3n(n-1)$ variables to optimize over where n is the number of rating grids) we are not able to do so under our current computational capabilities. Instead, we try to answer the following question: at least, by how much can the platform increase expected time spent by players on the platform? To answer this question, we consider optimization over a smaller class of matching mechanism and find that the platform can increase this time by at least 60%.

5 Conclusion

In this paper we investigate stopping behavior in an environment free of monetary incentives and we identify factors that encourage or discourage people. We find consistent heterogeneity in stopping decisions. In particular, we identify two behavioral types of people: loss-encouraged and loss-discouraged types. Loss-encouraged types are much more likely to continue playing after a loss, than after a win. These types of players get challenged by a loss and their future utility from the next game increases. With conservative parameter values, we classify about 75% of players as behavioral types, one-third of which are loss-encouraged and the rest are loss-discouraged types. The latter player types receive lower utility from the next game if the current game was lost. That is, the future entertainment value of a game goes down by a loss in a current game: playing is no longer as enjoyable as it used to be, and players are more likely to stop playing. Being able to capture the heterogeneity in stopping behavior is crucial in designing incentives to encourage longer play or meeting other welfare objectives.

This paper highlights the importance of identifying a person's type in the environments where practice could improve overall outcomes. In the context of chess games, type-identification and individualized incentives increase the session length by 60%. Although mobile app games is huge industry, little is known about the determinants of length of play. In this paper, we document strong behavioral forces that drive players' stopping decisions. The results in the paper are not limited to a chess game, and the results can be applied to any environments where practice can influence outcomes. For example, identifying student types to improve educational outcomes from a personalized curriculum.

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Appendices

A Identification of the Model Parameters

Claim 1 Proof. Note that in equation 1, continuation values do not depend on the **current** realization of c. Hence, fixing the continuation values and current period utility from playing the game, the term multiplied by $(1-d)$ under the $\max$ operator is higher than the term multiplied by d, for sufficiently high c. So, we have a threshold, $c(\theta, x, y, \chi)$, such that for realizations of c above this threshold the player stops playing and takes her outside option. ■

Proposition 1 Proof. The proof follows by observing that the RHS of 2 does not depend on χ and the LHS is increasing in the threshold, $c(\theta, x, y, \chi)$. ■

The following parametric assumption is made on the distribution of outside option, $F(c)$,

Assumption 1 *$F(c)$ is exponential with parameter λ.*

We can identify the conditional probabilities of winning and matching probabilities directly from data.

We now argue that under the assumption 1 and normalizing one parameter of our choice in the model, δ, λ, l_θ, $U^0(\cdot)$ and $U^1(\cdot)$ can be identified. In the estimation, we normalize $\lambda = 1$.

Let,

$$H(\theta, x, y) = EU(x, y) + \\ \delta \sum_{\chi', y' \in \{0,1\} \times Y} p(y'|y, \chi')p(\chi'|x, y)V(\theta, x, y', \chi')$$

Under the assumption 1 and from 2, we have that the probability of stopping and taking outside option, $h(\theta, x, y, \chi)$, is,

$$h(\theta, x, y, \chi) = e^{-\lambda(H(\theta,x,y)-(1-\chi)l_\theta)} \tag{3}$$

In 3, the left hand side can be calculated from the data. So, we can identify $\lambda H(\theta, x, y)$ and λl_θ. This is summarized in claim 2.

Claim 2 *$\lambda H(\theta, x, y)$ and λl_θ are identified for all (θ, x, y).*

Taking expectations of both hand sides of 1 with respect to c, multiplying both hand sides by λ and substituting $c(\theta, x, y, \chi)$ from 2, we get,

$$\lambda V(\theta, x, y, \chi) = \lambda H(\theta, x, y) + e^{-\lambda[H(\theta,x,y)-(1-\chi)l_\theta]} \tag{4}$$

Claim 2 and expression 4 imply the following,

Claim 3 *$\lambda V(\theta, x, y, \chi)$ are identified for all (θ, x, y, χ).*

By the definition of $H(\theta, x, y)$, claims 2 and 3 and the fact that matching and winning probabilities are identified, it follows that,

Claim 4 *δ and $\lambda EU(x, y)$ are identified.*

Claim 4 Proof.

We can consider the difference $\lambda(H(\theta, x, y) - H(\theta', x, y))$ for some $\theta \neq \theta'$. This gives us,

$$\delta = \frac{\lambda(H(\theta, x, y) - H(\theta', x, y))}{\lambda(\sum_{\chi', y' \in \{0,1\}\times Y} p(y'|y, \chi')p(\chi'|x, y)(V(\theta, x, y', \chi') - V(\theta', x, y', \chi')))}$$

By claims 2 and 3, numerator and denominator are identified in the above equation.

Finally, $\lambda EU(x, y)$ is identified from,

$$\lambda EU(x, y) = \lambda H(\theta, x, y) -$$
$$\lambda\delta \sum_{\chi', y' \in \{0,1\}\times Y} p(y'|y, \chi')p(\chi'|x, y)V(\theta, x, y', \chi')$$

■

Finally, we can normalize all the parameters and value functions by λ. This completes the identification of the parameters of the model.

B Robustness

B.1 Robustness of Tolerance Threshold

Figure 5 presents the players' population decomposition by types as we vary τ from 0 to .2. We see that behavioral types are robust to changing the allowed tolerance.

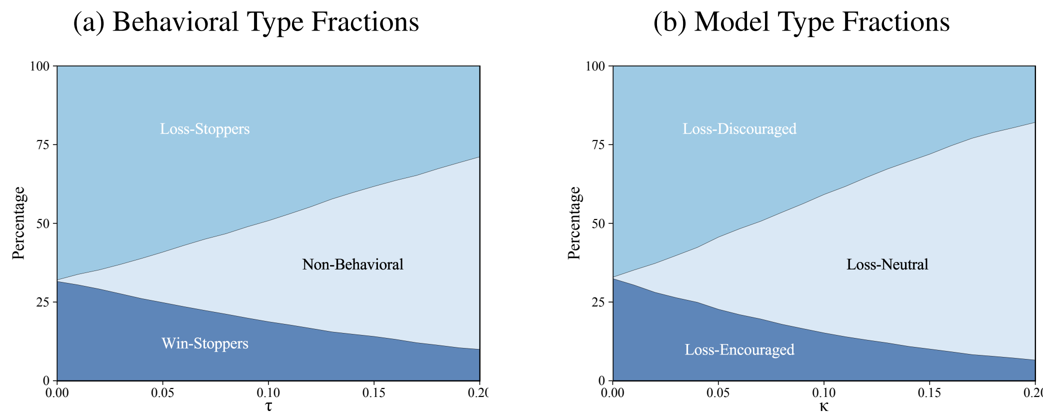

Figure 5: Type Decomposition

Similarly, changes in κ, also have consistent effect on model type decomposition, Figure 5b.

B.2 Rating and Time Invariance of Behavioral Types

In this section, we examine whether a person's type changes over time and whether the rating in the chess game is correlated with the type classification. To study these questions we take two approaches: (i) we use existing data, and (ii) we collect additional data.

Rating Figure 6a presents the break down of types over different rating categories. All three classification types are represented at every rating level. Moreover, the ratio of types for non-extreme rating levels are similar. Figure 6b presents the the distribution of the rating by type classification and we find that the distributions are practically the same. Kolmogorov-Smirnov test between win and loss-stoppers finds no difference with $p = 0.4$. Non-behavioral types have on average lower rating than behavioral types with $p < 0.01$.

To further examine and look for any possible differences in rating and type classification we collect data for a special subset of chess players that have rating in top 2000. We do not find any qualitative difference in the distribution of behavioral type or in the ratio of win and loss-stoppers. These result suggest that the rating in the chess game is not related to the player's type.

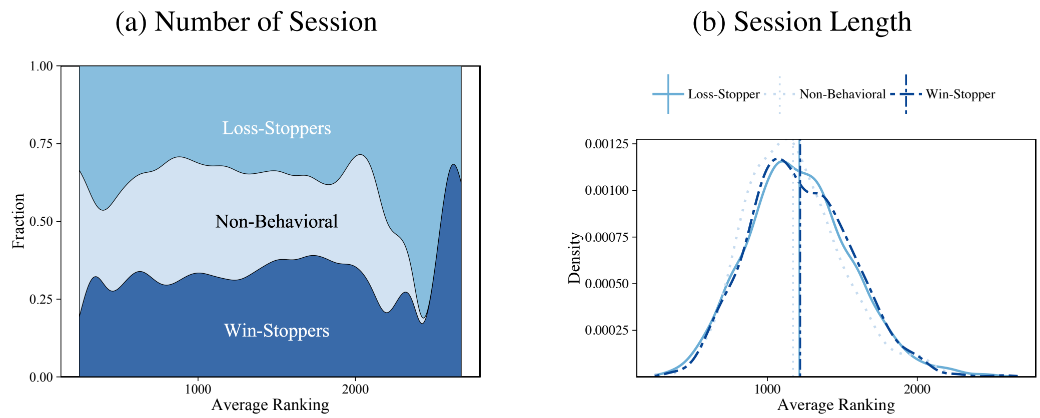

Figure 6: Ability and Behavioral Types

Time To study whether the behavioral friction that we find in this paper is an inherent trait that does not change over time we collect and analyze data from year 2018 and compare to the results that are based on the year 2017. We use the data for same players in 2018 and we find that there is 85% type match between this two consecutive years, highlighting that the individual patterns we observe do not change over time.

B.3 Number of sessions and session length

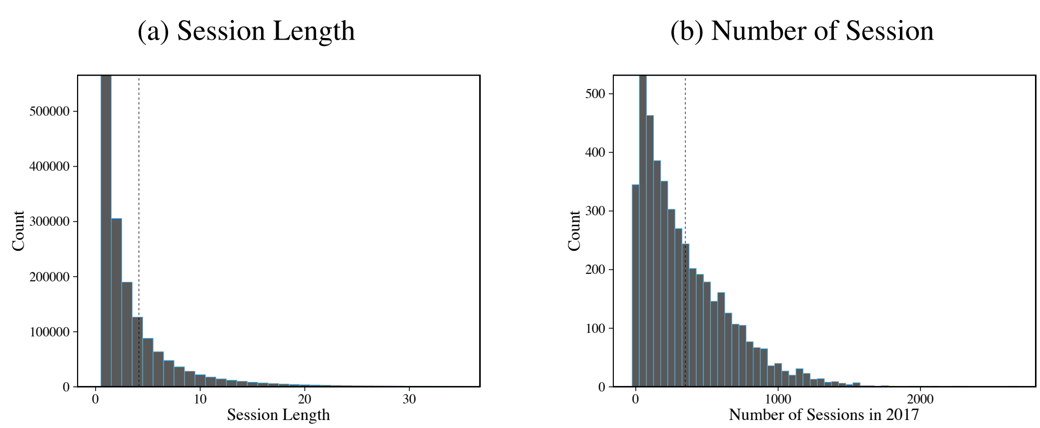

Figure 7: Session length and number of sessions per player

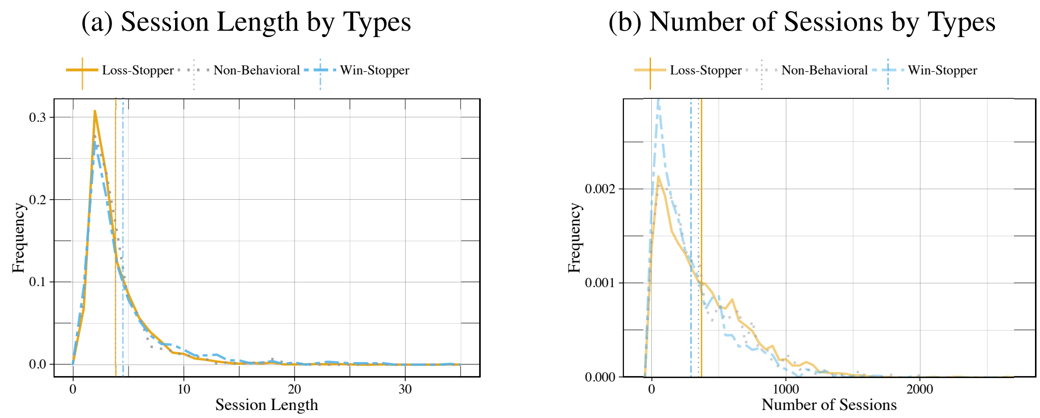

Figure 8: Session Statistics by Types

C Data Description

Observations	6614606
Players	4573
Average Session Length	4.14
Average Number of Session	348
Average Rating	1208
Rating Range	[100,2798]
Pr(Win | White)	52.4

Table 1: Data Description (Year 2017)

(a) Time Invariance of Types

(b) Transition From 2017 to 2018

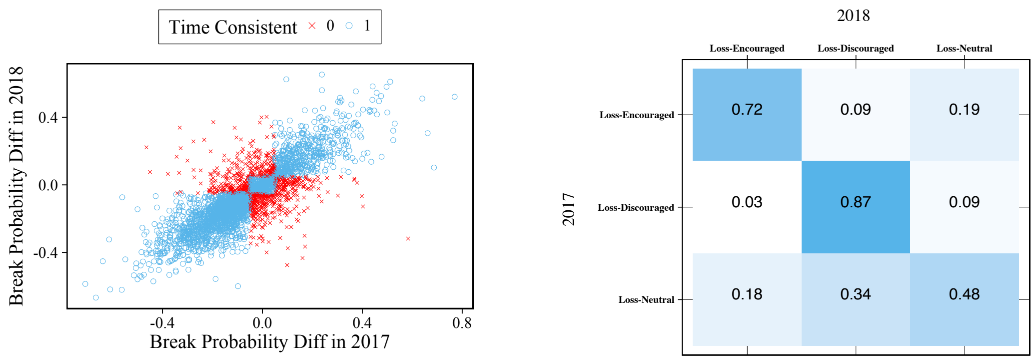

Figure 9: Time Invariance of Types (2017 to 2018)