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Excellent

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Exam Date : 10/5/2023
Mid Exam : One hour

Q.1]A) Solve the following system by Gauss elimination :

Subgrades

$$x_1 + 3x_3 = 1 \;...(1)$$
$$3x_1 + 3x_2 + x_3 = 0 \;...(2)$$

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Solution)

$[A|B] = \left[\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 3 & 3 & 1 & 0 \end{array}\right] \xrightarrow{-3R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 3 & -8 & -3 \end{array}\right]$

$\xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{8}{3} & -1 \end{array}\right]$

$x_1 + 3x_3 = 1$

$x_1 = 1 - 3x_3$

$x_2 - \frac{8}{3}x_3 = -1$

$x_2 = -1 + \frac{8}{3}x_3$

let $x_3 = s \in \mathbb{R}$

$x_1 = 1 - 3s$

$x_2 = -1 + \frac{8}{3}s$

$S = [\{(x_1, x_2, x_3) = (1-3s, -1+\frac{8}{3}s, s) \mid s \in \mathbb{R}\}$

Q.1]B) Solve the following system by invertibility method :

$$3x + y - 14 = 0 \;...(1)$$
$$-y + \frac{1}{4}x = -3 - y \;...(2)$$

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Solution)

$A = \begin{bmatrix} 3 & 1 \\ \frac{1}{4} & 0 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} 14 \\ -3 \end{bmatrix}$

$ad - bc = \frac{-1}{4}$ $A^{-1} = \frac{1}{\frac{-1}{4}} \begin{bmatrix} 0 & -1 \\ -\frac{1}{4} & 3 \end{bmatrix} = -4 \begin{bmatrix} 0 & -1 \\ -\frac{1}{4} & 3 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 1 & -12 \end{bmatrix}$

$14 + 36 = 50$

$X = A^{-1} B$

$$\underset{(2\times 2)}{\begin{bmatrix} 0 & 4 \\ 1 & -12 \end{bmatrix}} \underset{(2\times 1)}{\begin{bmatrix} 14 \\ -3 \end{bmatrix}} = \begin{bmatrix} -12 \\ 50 \end{bmatrix}$$

$x = -12 \quad y = 50$

$S = \{(-12, 50)\}$

Q.2]A) Let A be invertible matrix, with $adj(A) = \begin{bmatrix} 2 & -4 \\ 5 & 2 \end{bmatrix}$, $\det(3A) = 27$, *evaluate* A^{-1}.

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<u>*Solution)*</u>

$\det(A) = 9$

$$A^{-1} = \frac{1}{\det A} \times adj(A)$$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 2 & -4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{9} & \frac{-4}{9} \\ \frac{5}{9} & \frac{2}{9} \end{bmatrix}$$

Q.2]B)Evaluate $\begin{vmatrix} 9 & 1 & 2 \\ 3 & 4 & 0 \\ 6 & 1 & 1 \end{vmatrix}$, *using cofactor expansion along the second row.*

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<u>*Solution)*</u>

$M_{21} = -1$, $M_{22} = -3$, $M_{23} = 3$,
cof $= 1$, cof $= -3$, cof $= -3$,

det $= 3\times1 + -3\times4 + -3\times0 = 3 + -12 = -9$

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Q.3]A) Let $A = \begin{bmatrix} -2 & 1 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix}$, evaluate $(B - A^t)^2$. | 4 / 4

Solution)

$A^t = \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}$

$B - A^t = \begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 3 & -1 \end{bmatrix}$

$(B - A^t)^2 = \begin{bmatrix} 7 & 1 \\ 3 & -1 \end{bmatrix} \times \begin{bmatrix} 7 & 1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 52 & 6 \\ 18 & 4 \end{bmatrix}$

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Q.3]B) Write (3×3) matrix which is symmetric with nonzero entries. (ولا مدخلة فيها تساوي صفراً) | 2 / 2

Solution)

$A = \begin{bmatrix} 5 & 7 & 6 \\ 7 & 5 & 8 \\ 6 & 8 & 5 \end{bmatrix}$

$A^t = \begin{bmatrix}5 & 7 & 6\\ 7 & 5 & 8\\ 6 & 8 & 5\end{bmatrix}$

$A = A^t$

So A is Symmetric

Q.4]A) If $\det\left(\begin{bmatrix}a & b\\ c & d\end{bmatrix}\right) = 4$, evaluate $\det\left(\begin{bmatrix}2a & 2b\\ 3c & 3d\end{bmatrix}\right)$. (وضح خطوات الحل)

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det A det B

Solution)

$B = \frac{2}{3}\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$\det B = \frac{2}{3}\det(A)$

$\det B = \frac{2}{3} \times 4 = \frac{8}{3}$

Q.4]B) Let A and C be two invertible matrices of size (n), with $\det(A) = \frac{-1}{16}$, $\det(C^{-1}) = 0.5$, evaluate $\det(A^t \times C)$.

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Solution)

$\det C = \frac{1}{\det C^{-1}} = \frac{1}{\frac{1}{2}} = 2$

$\det A^t = \det A = \frac{-1}{16}$

$\det(A^t \times C) = \frac{-1}{16} \times 2 = \frac{-1}{8}$

THE END