The performance of low volatility strategy during 2010—2019

Summary

Longly have people commonly believed that higher risks yield higher returns; however, Robert Haugen and Nardin Baker observed the exact opposite relationship in Russel 3000 constituents' performances during 1979 and 1993. Following their approach, we have been able to examine this claim on a random list of 35 stocks' performances between 2010 and 2019, by:

1) Selecting **control variables** based on commonality in the determinants of expected stock returns, as introduced by Haugen and Baker;
2) Constructing volatility factors, which include **standard deviation, market beta and average true range (ATR);**
3) Conduct **four groups of cross-section regressions** on historical data of the sample stocks, in which the first three groups use different volatility factor, and the last group regresses on different periods;
4) Calculate expected returns based on 3), and compare **expected returns with corresponding realized returns** using their descriptive statistics.

In the end, we reached the conclusions that:

1) Coefficients of volatility factors generally display negative average values over the decade, with positive skewness and negative mean;
2) Though slightly, low volatility strategy does perform better than the market index over the last decade. Among different tiers of stocks sorted based on the expected returns, volatility tends to be negatively correlated with expected returns;
3) Comparatively, the standard deviation of the stock returns is better than market beta and ATR as a proxy for volatility in terms of the prediction performances;
4) The drawback of this factor model lies majorly in its strict requirement for sample building, without which various biases can exist.

Factor Model

- **Model**

In every trading **week** between the end of 2009 and the end of 2019, we conducted cross-section regressions using ordinary least squares (OLS) method. The model is as follows:

$$r_{j,t} = \sum_i \hat{P}_{i,t} * F_{j,i,t-1} + u_{j,t}$$

Where:

$r_{j,t}$ = rate of return to stock j in month t,
$\hat{P}_{i,t}$ = regression coefficient to factor i in month t,
$F_{j,t,t-1}$ = exposure to factor i (i.e., volatility, liquidity, technical, growth potential and price level factors) for stock j at the end of month t-1,
$u_{j,t}$ = unexplained component of return for stock j in month t.

Four different groups of regressions are conducted. The first three groups use standard deviation, market beta and ATR as the proxy for volatility factor, respectively. The last group starts regressing on 12/16/2011, after when all the stocks in the list are listed.

After obtaining the coefficient history data, we take their moving averages on a window of 50 weeks (i.e., one year) to make out-of-sample expected return projections. The model is as follows:

$$E(r_{j,t}) = \sum_i E(P_{i,t}) * F_{j,i,t-1}$$

Where:

$E(r_{j,t})$ = expected rate of return to stock j in month t,
$E(P_{i,t})$ = expected regression coefficient to factor i in month t (trailing 50-week mean),
$F_{j,t,t-1}$ = exposure to factor i (i.e., volatility, liquidity, technical, growth potential and price level factors) for stock j at the end of month t-1.

By dividing the stocks at each cross-section into 3 or 11 tiers, we then compared the differences between expected returns and realized returns. We also observed the relationship between volatility and returns using different descriptive statistics.

- ## Variable Context and Definition

Given the fact that the market is not efficient, it is fair to follow the same framework of choosing independent variables, which include factors from the following five categories: volatility factor, liquidity factor, price level factor, growth potential factor, and technical factor. Among them, one week and one-month excess returns should be negatively correlated with return rates, because of the short-term reversal pattern introduced by Haugen and Baker. However, one-year excess returns should be positively correlated with return rates because of the intermediate-term inertia patterns. The ratio of trading volume to market capitalization should be negatively correlated with return rates because investors expect lower returns for liquid stocks. Both the ratio of earnings to price and ROA should be positively associated with return rates, with investors expecting higher returns when these two factors are higher. Finally, with lower volatility, there should be higher returns, which is one of the main conclusions discussed by Haugen and Baker.

Volatility Factors:

1) **Standard deviation**: volatility of the (natural base) log return in the past 50 weeks. 50-week window is chosen to smoothen the short-term noise (e.g., pricing bias) and to take into consideration the data of a complete fiscal year. In the meantime, the low volatility strategy entails long-term investments.
2) **Market beta**: coefficients of the excess return of S&P 500 over risk-free rates, in regressions that have individual stocks' excess returns as response variables. Data from t-49 to t are used to get the market beta at time t.
3) **ATR**: 14-day moving average of max(|high - low|, |high – adj.close|, |adj.close – low|)[1], in which "high" is today's highest price, "low" is today's lowest price, and "adj.close" is yesterday's adjusted close price. ATR is believed to better represent the volatility one may encounter in real trading.

Liquidity Factor:

Trading Volume / Market Capitalization: (trailing 50-week average weekly trading volume to most recently available market capitalization)

Price Level Factor:

Earnings (per share) / Price: most recently available 4-quarters earnings to most recently available number of shares to current market price

Growth Potential Factor:

Return on Assets: most recently available net income to total assets

Technical Factors:

Excess Return (relative to the S&P 500) in Previous **1 week**;
Excess Return (relative to the S&P 500) in Previous **4 weeks (i.e., one month)**;
Excess Return (relative to the S&P 500) in Previous **50 weeks (i.e., one year)**.

[1] J. Welles Wilder Jr, *New Concepts in Technical Trading Systems* (Trend Research, 1978), 22.

- **Data Source and pre-regression processing**

Weekly Historical **Price Data**: obtained through Python package YFinance, which scrape data from Yahoo Finance.
Weekly **Risk-free Rates**: obtained from the Fama-French research database[2].
Historical **Financial Data**: obtained from S&P 500 Capital IQ database; used to calculate liquidity factor, price level factor and growth potential factor.

Missing values are substituted either with the cross-section mean (e.g., log-returns of stocks not listed at a previous time), or with the nearest time-series mean (e.g., one missing data point of ROA with available earlier and later data). This method would bias the result to a certain extent, which is trivial compared with dropping all other available data at periods when missing values exist. However, to further justify the rationality, we conducted another regression which starts from 12/16/2011, after when the first kind of missing values does not exist.

Ticker **APC** and **ARNC** have been **removed** from the sample from the beginning because APC has been delisted, and ARNC experienced a merger, which makes its historical data inconsistent among different databases. The whole research is conducted on the remaining 33 stocks.

Because calculating moving average values would cause a percentage of earliest data to be removed, out of 523 cross-sections, 51^{st} to 523^{rd} (473 in total) cross-sections are retained after the regressions, 100^{th} to 522^{nd} (423 in whole) cross-sections are finally reserved in predictions. In the projections, 523^{rd} cross-section is also removed, because its (natural base) log-returns are used as the realized returns for 522^{nd} cross-section. For another regression that starts from 12/16/2011, 100^{th} to 419^{th} (320 in total) cross-sections are finally reserved.

Regression Results and Prediction Performances

- **Standard Deviation as the proxy for volatility**

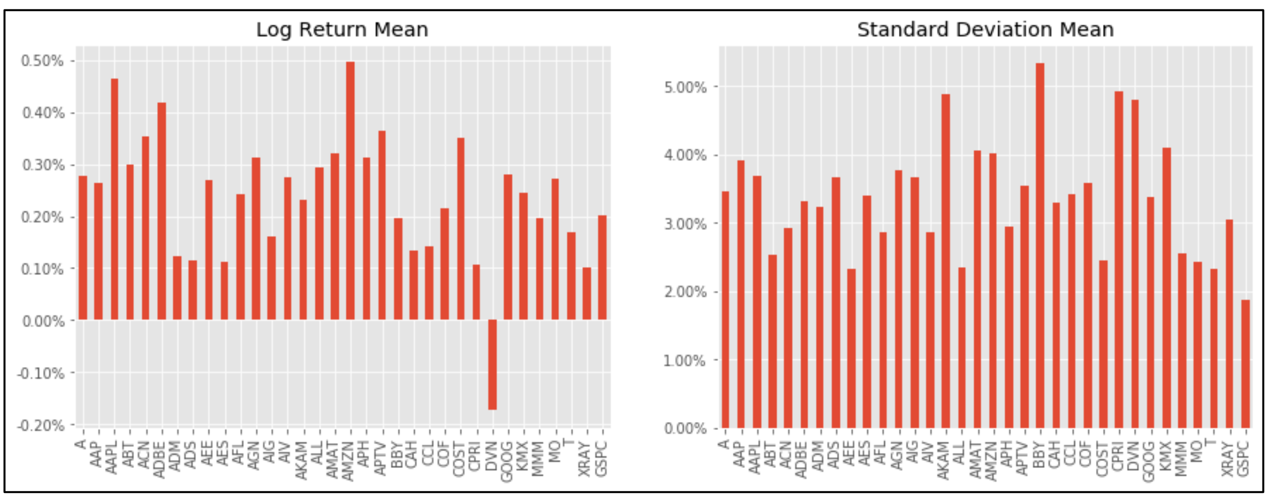

Figure 1 Average values for log returns and standard deviation

Over the past decade, the 33 stocks, along with the S&P 500, majorly recorded positive average weekly return rates from 0.1% to 0.5%. Among them, only ticker DVN recorded a negative value of -0.17%. The average weekly standard deviations of these stocks range from 2.32% to 5.34%, while the S&P 500 marked the lowest value at 1.87%. Variations across the sample stocks are not drastic in terms of these two descriptive statistics.

[2] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

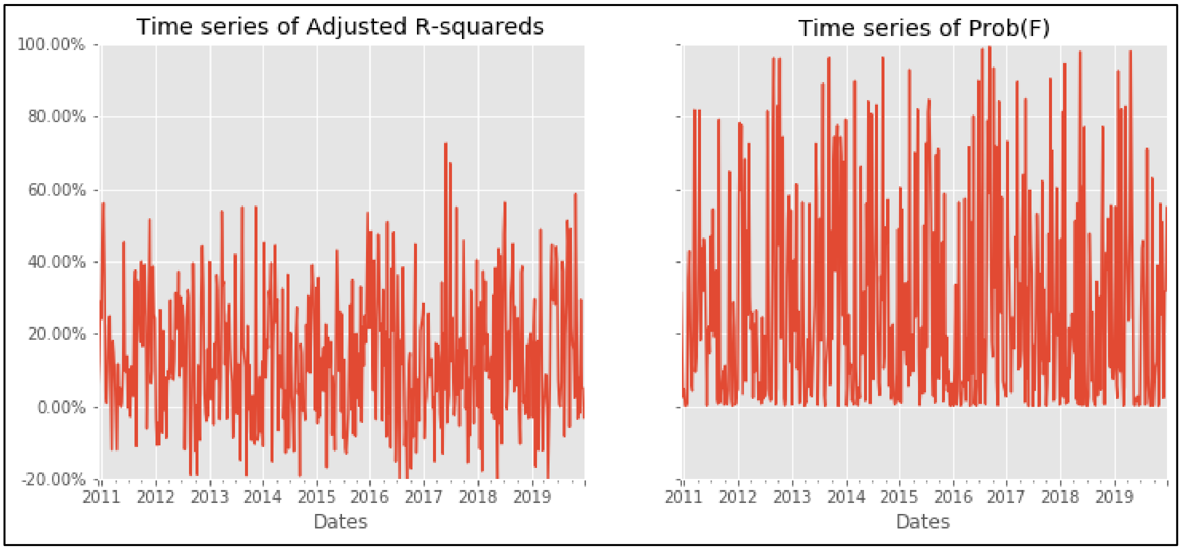

Figure 2 Regression quality

Figure 2 shows the time series of Adjusted R-squareds and the time series of Probability of F-statistics being significant. It is easy to tell that the regressions do not indicate high overall significances along the regression period. Most adjusted R-squareds distribute around 20%, and some cross-section regressions even have negative values for the coefficient of determinations. In the meantime, only a small portion of the cross-sections have Probabilities of F-statistics below 10%, which means the overall goodness of linear fit is not satisfying for most cross-sections. However, such is a universal nature of cross-section OLS regression method. At this level, we are still able to proceed with the following analysis.

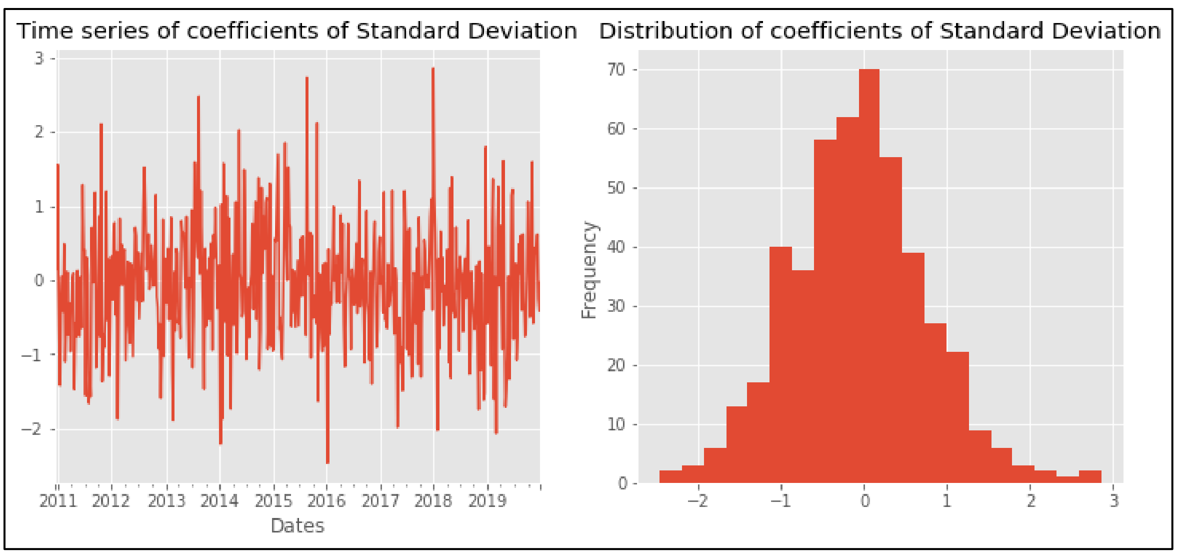

Figure 3 Time series and distribution of coefficients of standard deviation

Figure 3 shows the time series and distribution of coefficients of standard deviation. The coefficients have a skewness at 0.18 and a mean at -7.00%, indicating more of them are negative both in distribution and overall absolute values. Therefore, for more than half of the time, volatilities are negatively correlated with stock returns.

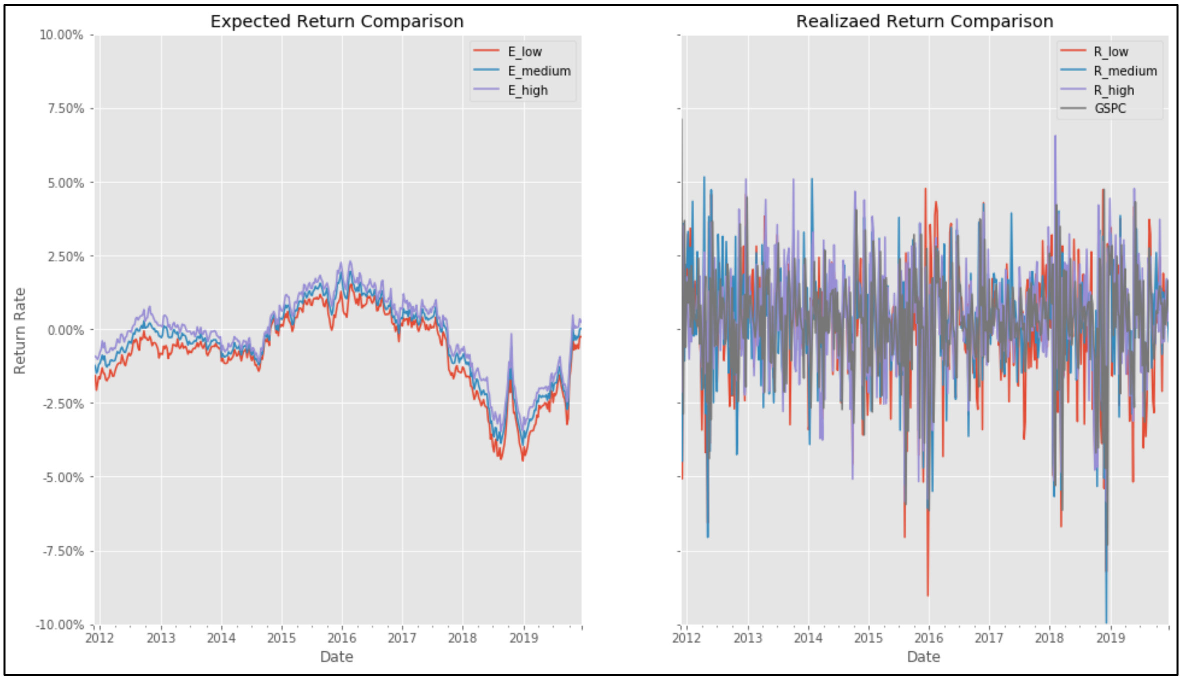

Figure 4 Return comparisons

After cross-section regressions, we got the coefficients of control variables, the moving average at a window of 50 weeks of which are then used to calculate the expected returns of time t+1 at time t. At each cross-section, we divide the stocks into three tiers based on their expected returns. Figure 4 shows the comparisons of expected returns and realized returns. It is hard to discern among tiers in the subplot of realized return comparison. The low volatility strategy does not show supremacy over the market, as the high expected return tier underperformed other tiers and the market in many cross-sections (purple lines being below other lines). Despite this, we may still say that comparatively high expected return tier has blower bound and higher bound over other tiers and the market. Also, for most of the time, this tier performs at least around the average level.

Table1 Descriptive Statistics of Realized Returns

	Skewness	Mean
Standard deviation coefficients	0.18	-7.00%
High expected return tier	-0.24	0.40%
Medium expected return tier	-0.64	0.34%
Low expected return tier	-0.65	0.10%
S&P 500	-0.62	0.23%

Skewness and mean can help compare realized returns. As exhibited in table 1, the high expected return tier has the highest weekly realized return at 0.40%, while the S&P 500 performed only better than the low expected return tier. This sounds more reasonable. Moreover, the fact that the high expected return tier has a negative skewness with the lowest absolute value demonstrates that this tier is the least positive skewed. Yet the same tier has the highest average weekly return, so it should be that in periods when the market booms, this tier performs the best. In contrast, in normal days, it generally does not perform worse than other tiers or the market, which coincides with the observation from figure 4.

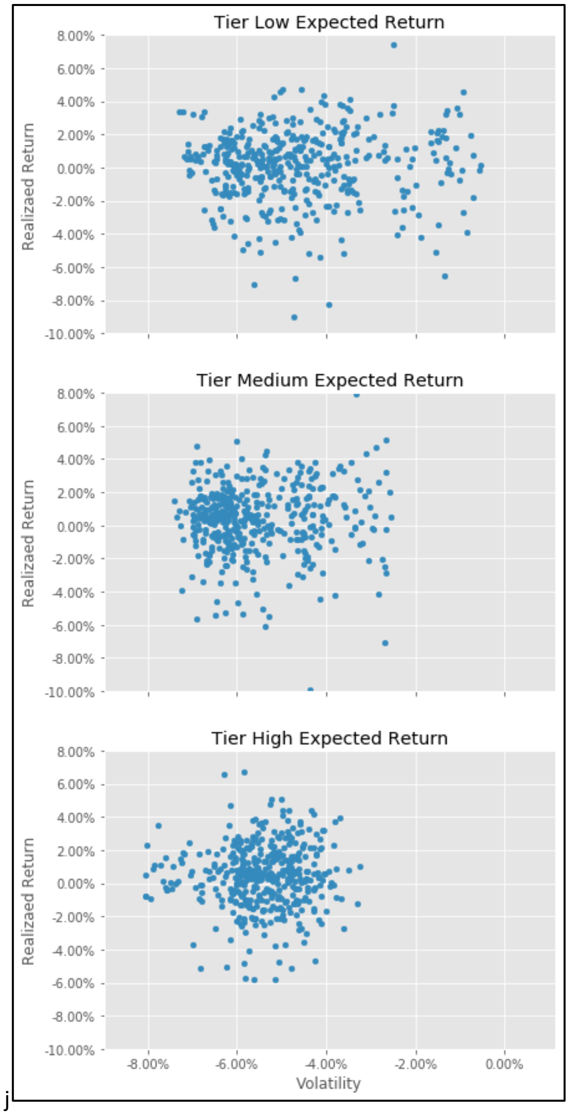

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Figure 5 Distribution of realized returns against standard deviation

It is clearer to conclude that generally lower volatility delivers higher returns by looking at figure 5, which shows the scatter point plots of realized returns against volatility levels. At the same level of volatility, the high expected return tier typically has higher realized returns. In contrast, at the same level of realized returns, the same tier typically has lower volatility level. Nevertheless, we should also note that such rank is not significantly common, as the supremacy among the tiers mostly exists in terms of outliers rather than the majority of data points in each tier.

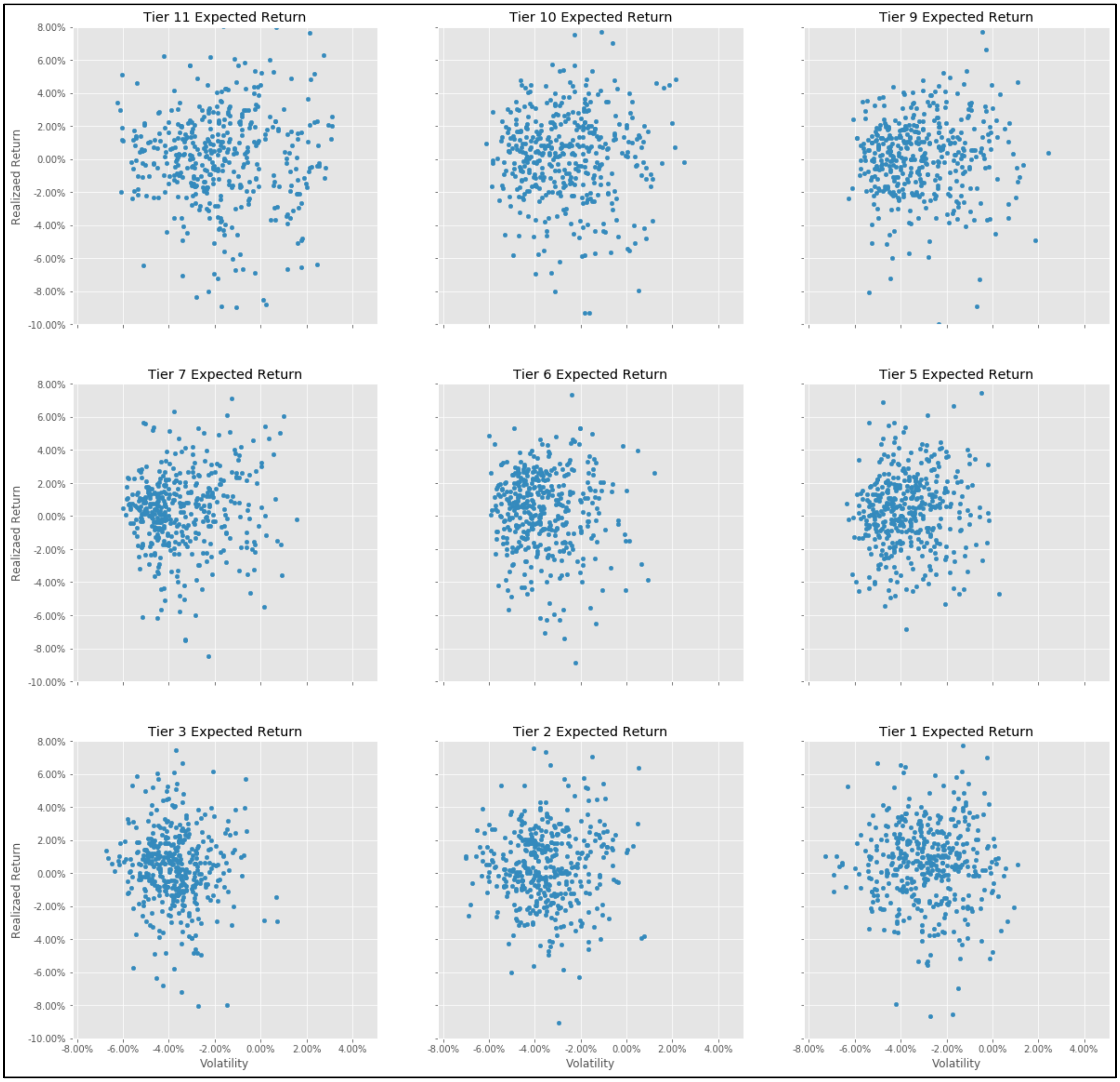

Figure 6 Distribution of realized returns against standard deviation – 11 tiers

To further examine the observation above, we divided the stocks into 11 tiers, with a smaller tier index indicating higher expected return, and produced the scatter point plots for 9 of the 11 tiers. Once again, though vaguely, we can observe the negative correlation relationship between volatilities and realized returns. In the meantime, with more tiers, we are able to find that the supremacy among close tiers (e.g., tier 1, tier 2 and tier 3) is even more indiscernible. Scatter points of tier 3 are denser than tier 1. For tiers farther from each other, the supremacy can be more discernible. Tier 11 has sparser scatter points than both tier 3 and tier 1 do. On the one hand, this fact demonstrates that the low volatility strategy is not predominantly better than simply buying the market index, or even random selection strategy; on the other hand, this may be the result of sample stocks being too few compared to the market composite.

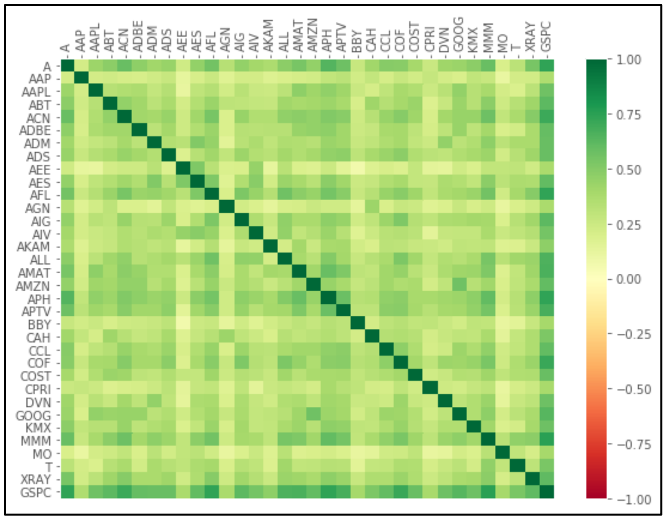

Figure 7 (Natural base) Log return correlation heatmap

The correlation heatmap can further illustrate the possible sample bias. As is shown in figure 7, none of the sample stocks are negatively correlated. The (natural base) log-returns of most of them are at least weakly positively correlated (correlations around 0.25) with each other. And almost all of them are highly positively correlated (correlations around 0.75) with the S&P 500. Therefore, even though volatilities vary among the stocks, in periods where higher expected return tiers perform well, the lower expected return tiers will not perform too bad. This kind of correlation among the sample stocks minimized the significances of the factor model built in this research, and maybe the reasons for many statistical insignificances observed above. Analyzing the market portfolio solve this problem, but the difficulty of collecting and processing data then becomes the endogenous drawback of this factor model, as with higher demand for data, more possible biases or inaccuracies can happen during the process, as mentioned earlier.

We also tested different proxies for volatility and different periods, results of which do not deviate from the above much.

- **ATR as the proxy for volatility**

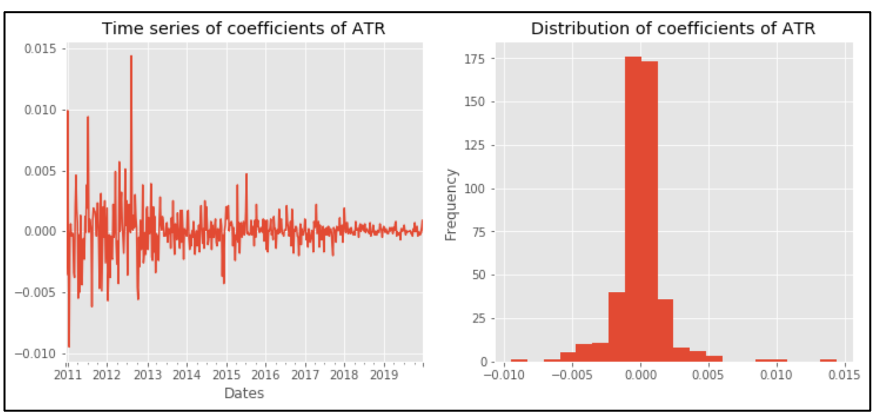

Figure 8 Time series and distribution of coefficients of ATR

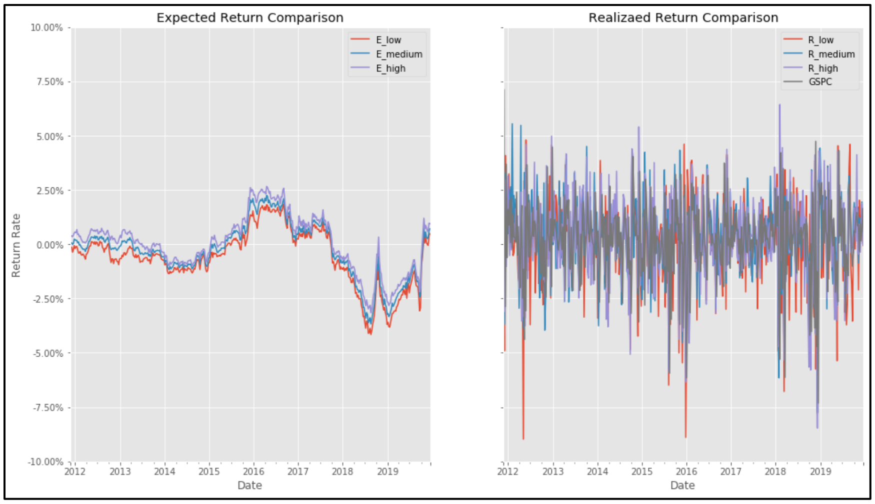

Figure 9 Return comparisons with ATR being the proxy for volatility

Figure 10 Distribution of realized returns against ATR – 11 tiers

Table2 Descriptive Statistics of Realized Returns with ATR Measuring Volatility

	Skewness	Mean
ATR Coefficients	1.12	0.0027%
High expected return tier	-0.38	0.38%
Medium expected return tier	-0.41	0.37%
Low expected return tier	-0.75	0.09%
S&P 500	-0.62	0.23%

The skewness and mean values of ATR coefficients are both positive, indicating a left-skewed distribution and an overall positive correlation with expected returns. Thus, when using ATR as the proxy for volatility, investors would expect risk premium instead of pay for lower volatility. The prediction performance is no better than using standard deviation as the proxy. Further, the 11-tier scatter plots are even more indiscernible. Tier 1 is even sparser than tier 11, which shows the low volatility strategy with ATR does not add value. Therefore, ATR is not a great proxy for volatility.

- **Market beta as the proxy for volatility**

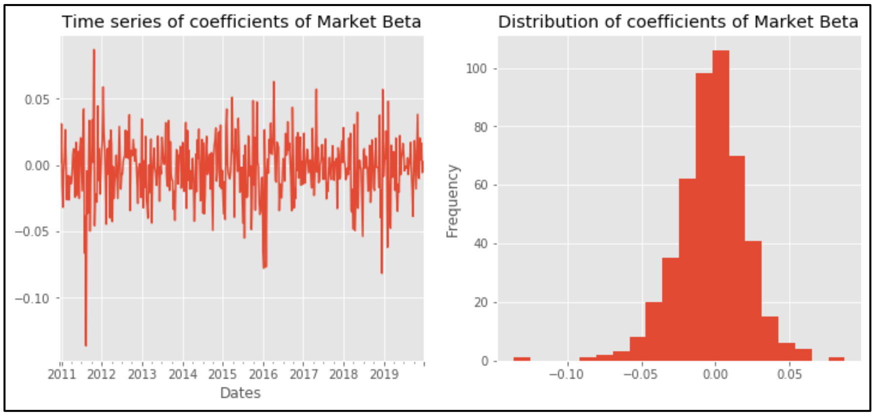

Figure 11 Time series and distribution of coefficients of market beta

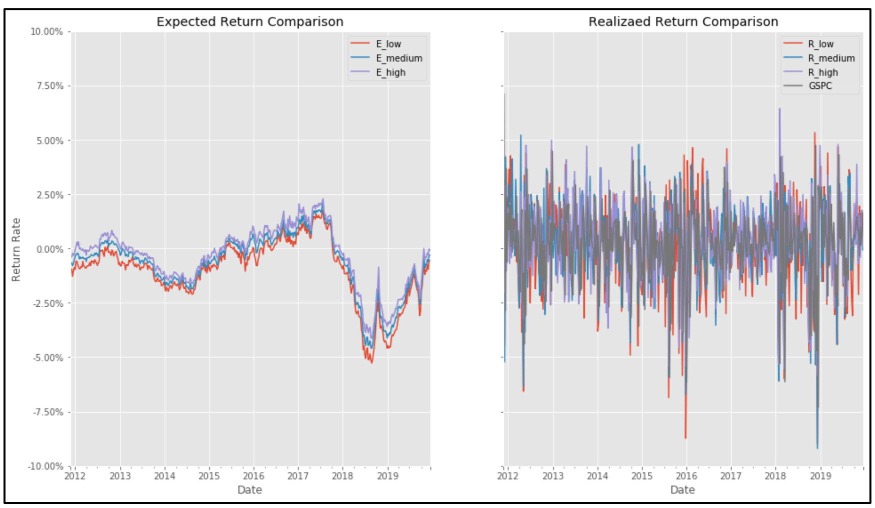

Figure 12 Return comparisons with market beta being the proxy for Volatility

Figure 13 Distribution of realized returns against market beta – 11 tiers

Table3 Descriptive Statistics of Realized Returns with Market Beta Measuring Volatility

	Skewness	Mean
Market Beta Coefficients	-0.48	-0.26
High expected return tier	-0.22	0.41%
Medium expected return tier	-0.70	0.31%
Low expected return tier	-0.53	0.12%
S&P 500	-0.62	0.23%

When using market beta as the proxy for volatility, the coefficients in regressions display a negative skewness, which means more of them tend to be positive. However, the average value for these coefficients are negative, so it should be that the collective influence of periods when the coefficients are negative is stronger. Overall, the relation between market beta and expected returns are negative. The only indicator the market beta can be better than standard deviation is that, high expected return tier and low expected return tier have higher average values than those with standard deviation as the proxy for volatility, which is not a strong evidence. In fact, it is even harder to see the negative correlation in figure 13. Thus, market beta does not outperform standard deviation. In consideration of the extra step of calculating the market betas, it is practically more advantageous to simply use standard deviation.

- **Regressions starting from 12/16/2011**

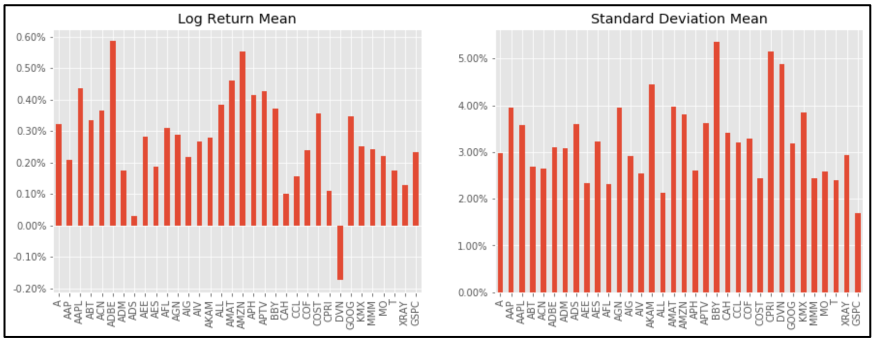

Figure 14 Average values for log returns and standard deviation after 12/16/2011

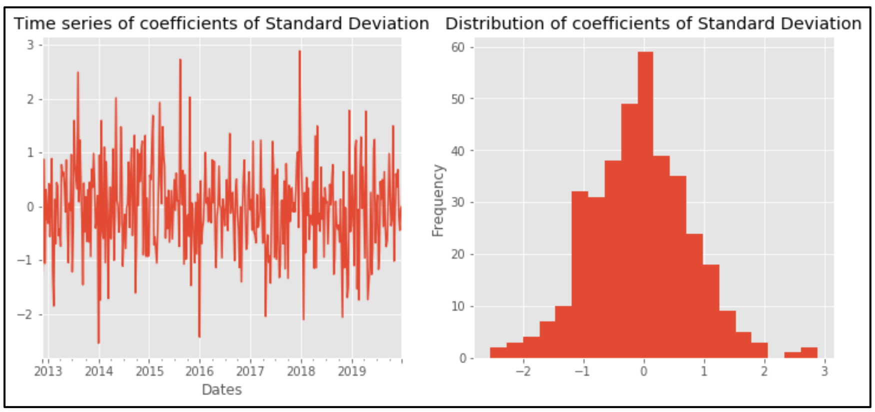

Figure 15 Time series and distribution of coefficients of standard deviation on regressions starting from 12/16/2011

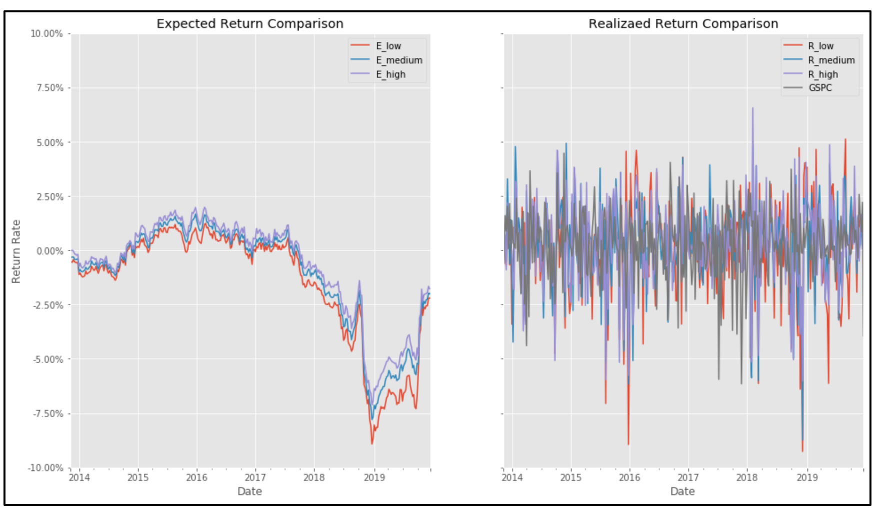

Figure 16 Return comparisons for regressions starting from 12/16/2011

Figure 17 Distribution of realized returns against standard deviation after 12/16/2011– 11 tiers

Table 4 Descriptive Statistics of Realized Returns after Regressions Starting from 12/16/2011

	Skewness	Mean
Standard Deviation Coefficients	0.15	-5.24%
High expected return tier	-0.42	0.31%
Medium expected return tier	-0.78	0.23%
Low expected return tier	-0.76	0.03%
S&P 500	-0.58	0.26%

Again, figures from 14 through 17 and table 4 are basically the same as the results discussed in the first part. Therefore, the robustness of all the conclusions derived above, at least in the given sample stocks over the last decade, is examined.

Conclusion

After four groups of regressions, we may fairly arrive at the following conclusions:

1) Compared to market beta and ATR, standard deviation is a better proxy for volatility, in terms of accuracy in predicting future returns and of the practical value with being easy to be calculated;
2) Low volatility strategy has performed slightly better than the market index over the last decade, as the negative relation between volatility and expected return indeed had been observed with multiple examinations. Further, stocks with higher expected returns tend to have lower volatilities;
3) Comparatively, the standard deviation of the stock returns is better than market beta and ATR as a proxy for volatility in terms of the prediction performances and of the practical value;
4) The drawback of this factor model lies majorly in its strict requirement for sample building. In order to get better accuracies, one has to employ as many control variables and sample stocks as possible. The sample stocks need to resemble the market portfolio for the best as well. Otherwise, the process of calculating expected returns can be inaccurate, causing the following prediction performs mundanely. However, these strict requirements entail the fact that multiple biases can occur during the process of collecting data and constructing ratios;

5) Another major drawback of the low volatility strategy is that the portfolio can become too concentrated. The very fact of focusing on a specific characteristic of stocks would add additional risks to the portfolio, especially when the stocks are positively correlated.

Appendix:

VBA code used to merge raw financial data downloaded from S&P Capital IQ database:

```
Sub Merge_Financial_DataFrames()

Dim i As Integer, j As Integer, k As Integer, m As Integer, n As Integer
Dim wb As Workbook, wb2 As Workbook, OC_R As Range
Dim C As Double, total As Double, filen As String, R As String

With ThisWorkbook.Worksheets("Sheet1").Range("A1")
Set wb = ThisWorkbook
i = 1
Do While .Cells(i, "AZ") <> ""
    filen = "C:\Users\Alaaa\Downloads\" & .Cells(i, "AZ")
    Set wb2 = Workbooks.Open(filen)

    Range("B10").Select
    Selection.Copy
    wb.Activate
    Worksheets("Sheet1").Activate
    Range("B14").Offset(0, i).Select
' Without a pause the paste does not work properly
    Application.Wait (Now + TimeValue("0:00:01"))
    ActiveSheet.Paste

    wb2.Activate
' Select different ranges for different tickers
    Range("C15:C48").Select
    Selection.Copy
    wb.Activate
    Worksheets("Sheet1").Activate
    Range("B15").Offset(0, i).Select
    Application.Wait (Now + TimeValue("0:00:01"))
    ActiveSheet.Paste
    ActiveWorkbook.Save
' Without close commands the computer may shut down with dozens of Worksheets open
    wb2.Close
i = i + 1
Loop
End With

End Sub
```

Description:
The goal of this part is to demonstrate your ability to develop a cross-sectional regression model and clearly present results. Imagine you have a potential investor who would like to find out whether buying securities with low volatility yields higher returns than buying securities with high volatility. To answer this question, please follow the methodology illustrated in Haugen's paper published in 1996 and construct an factor model using volatility as one or multiple factors (independent variable) in Python or R. You can define your own low volatility variables (price, return, residual return, beta, etc.) In addition to codes, you may also wish to include any other materials to help illustrate your analysis in more detail.

An arbitrary list of securities is attached. Historical price data for these securities can be found online from Yahoo Finance or Compustat. The date range is from 2009/12/31 to 2019/12/31, you can choose the frequency as you see fit.

Results:
Your analysis should aim to answer the following questions:
1, What is a better proxy to historical volatility?
2, How have the low volatility strategies performed historically over the last 10 years?
3, What is the special characteristic of volatility factor?
4, What are the limitations of this factor model?

Attachment:
Haugen's paper, a list of tickers