An Application to Optimize Lap-times for Race Cars

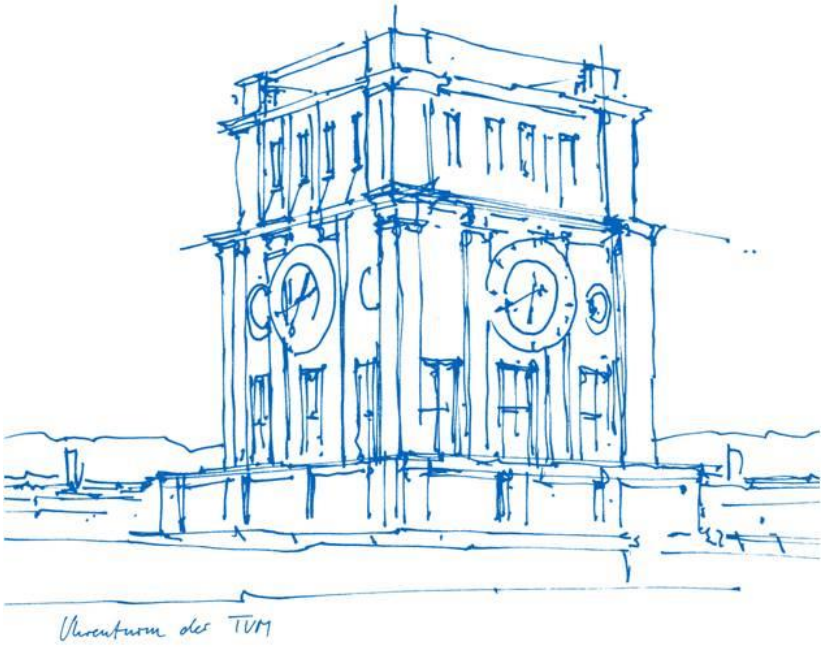
Group Members:

Alaa Bashir ge73cec

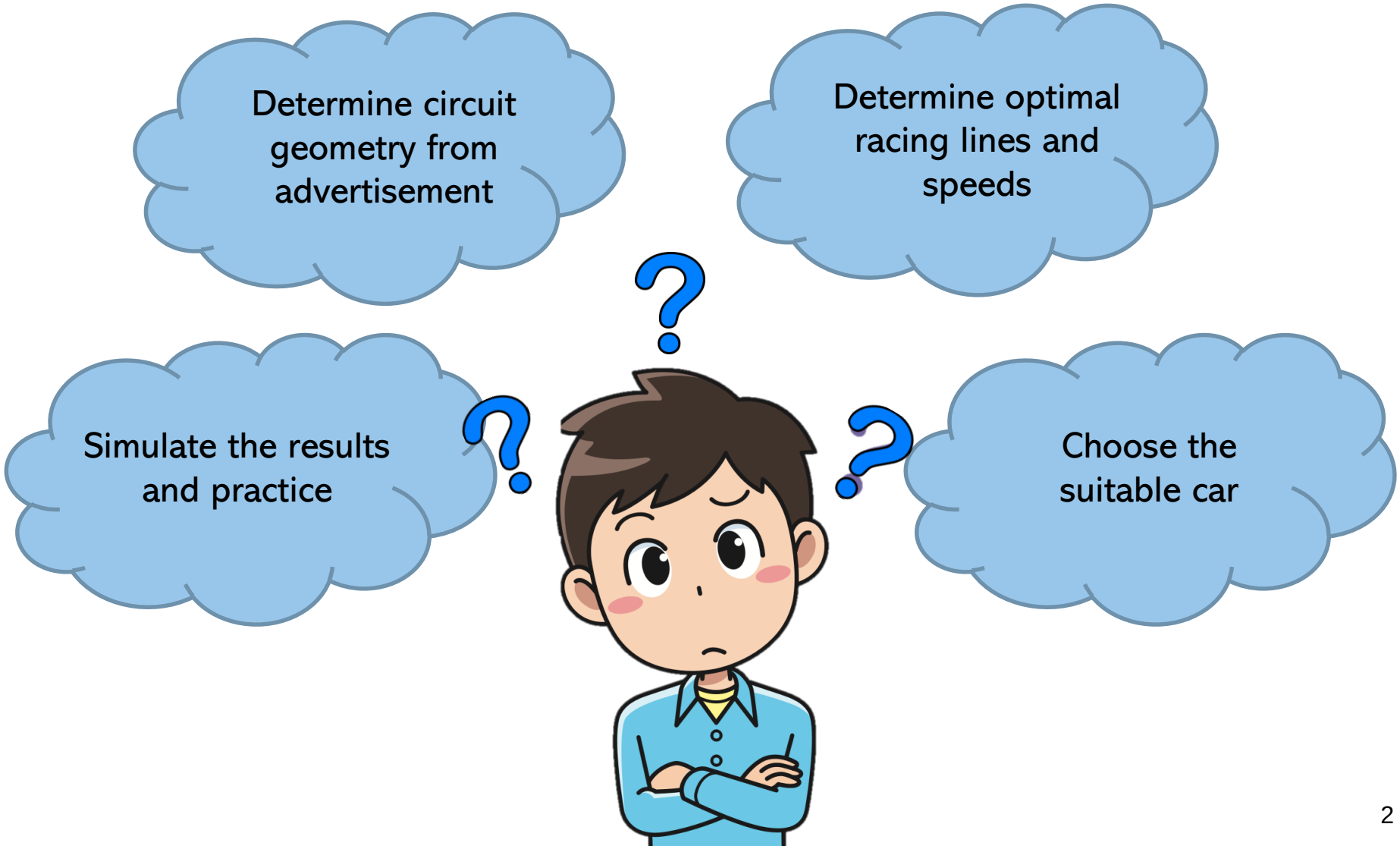
Ankit Khot ge73xuq

Máté Kelemen ge49pil

Supervised by: Jan Janse van Rensburg and Steve Schaefer



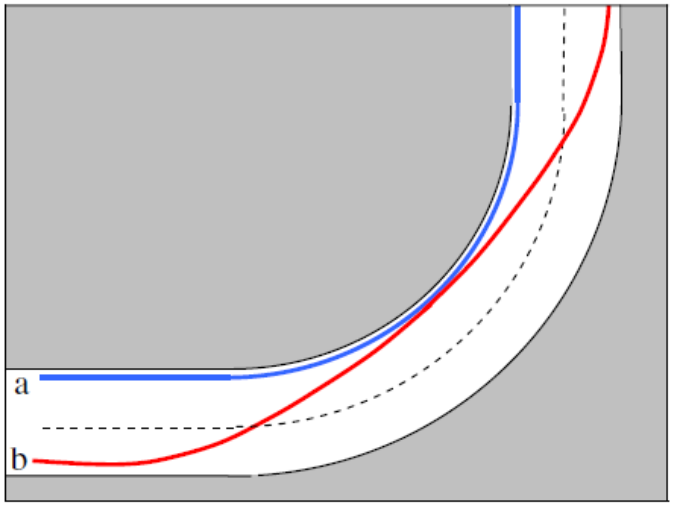
Outline:


Determine circuit geometry from advertisement
Determine optimal racing lines and speeds
Simulate the results and practice
Choose the suitable car

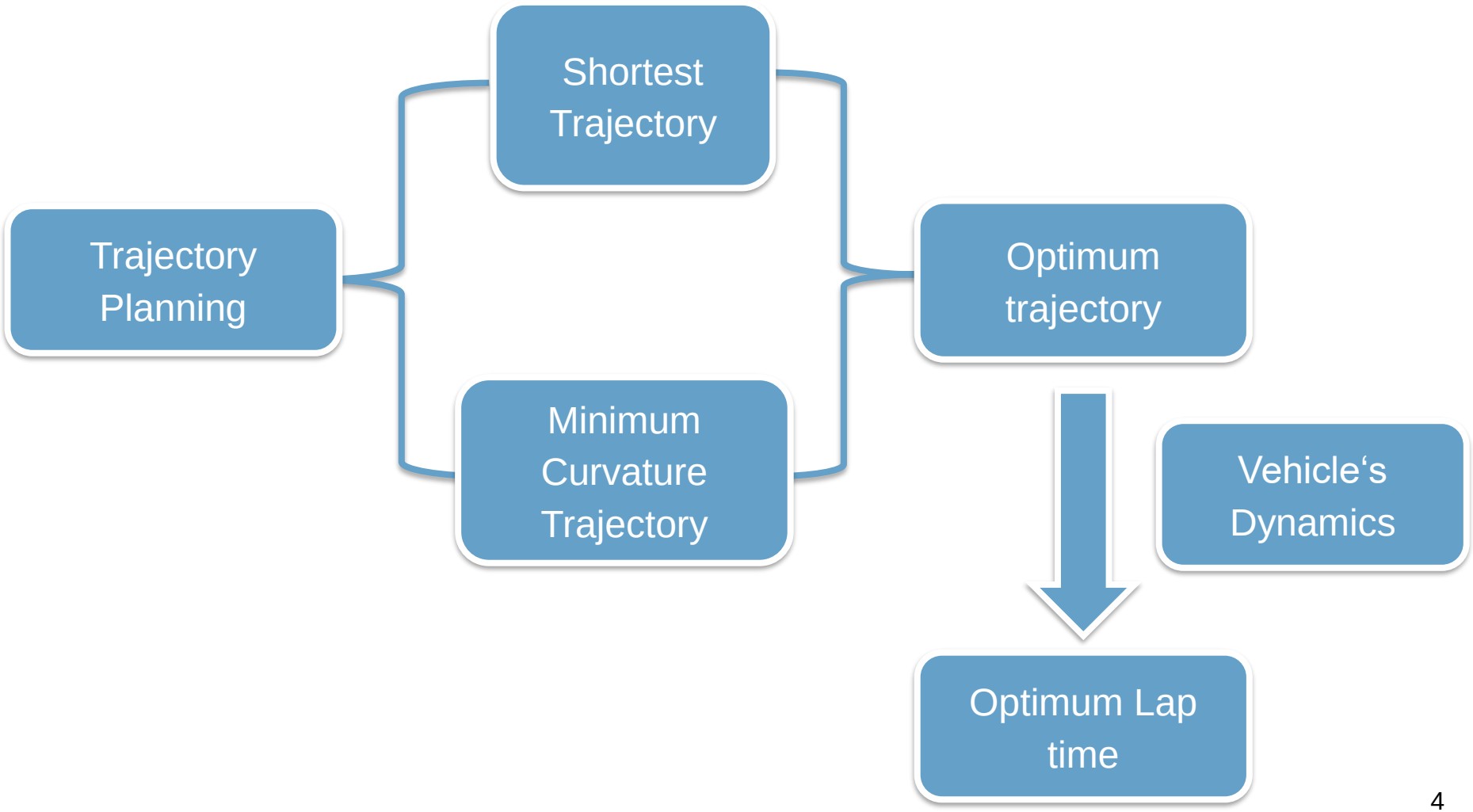
Goals and Problem Statement

- **Goals :**
  - Determine optimal racing lines
  - Minimize total lap time
  - Simulate the results for different tracks

- **Problem Statement:**
  - Optimal race lines is a dual problem
  - Compromises are needed
  - Car dynamics must be considered



Literature Review: Roadmap



Literature Review

- Several researches covered the geometric consideration
- "**Race driver model**" by F. Braghin, F. Cheli, S. Melzi, E. Sabbioni covered the geometric consideration and vehicle dynamics

Introduction : Point Mass Model

Point mass model of the vehicle :

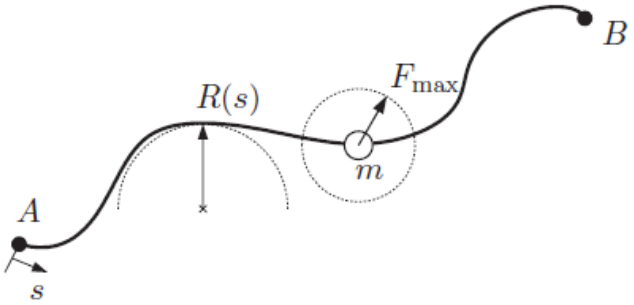
- The point mass model is the simplest model, considers vehicle as a point mass
- The dynamics of the point mass model are :

  Acceleration : a (assumed constant)

  Weight : mg

  Height of CG from ground level : h

  Overall friction coefficient : **μ**



Frictional force $F_{frictional} = \mu mg$

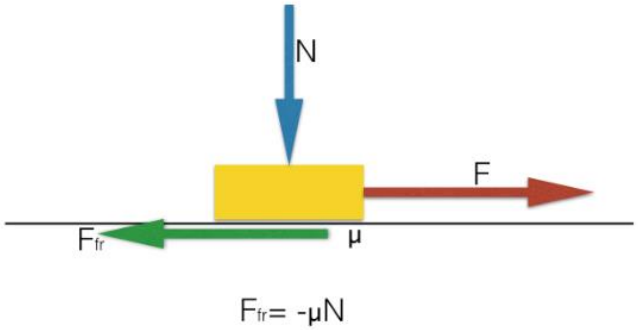
Centripetal force $F_{centripetal} = \frac{mv^2}{R}$

$V_{critical} = \sqrt{\mu Rg}$ (Sliding)OR

$V_{critical} = \sqrt{(g/h)}$ R (Overturning)

Illustrative example : L-bend

- **Forces and constraints on a body going around the corner**

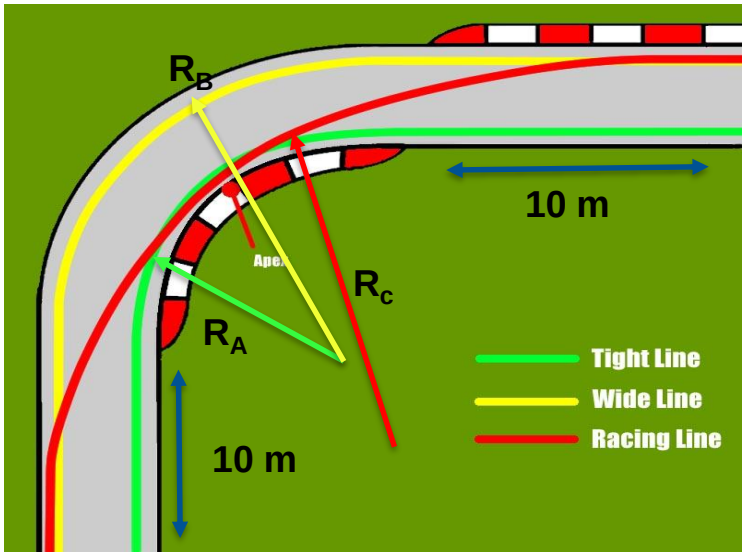


$F_{fr} = -\mu N$

Force Balance :

$F_{centripetal} = F_{frictional}$

$V_{critical} = \sqrt{\mu R g}$



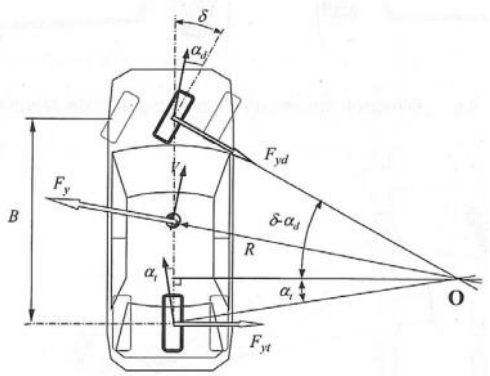
Results:	
$V_{critical\_A}$	12.13 m/s
$V_{critical\_B}$	14.35 m/s
$V_{critical\_C}$	18.79 m/s
t_A	2.62 s
t_B	2.70 s
t_C	2.01 s

Source : https://lifeatlean.com/what-is-the-racing-line/

Vehicle Dynamics : Literature Review

- Vehicle Models :

- Single Track Model :
  3 DOFs : Longitudinal, Lateral and Yaw
  Does not consider the rolling effect



- Double Track Model :
  6DOFs : Longitudinal, Lateral, Vertical
  Yaw, Roll,Pitch

- These models involve modelling of the Tire, Suspension system, Chassis, Transmission system etc

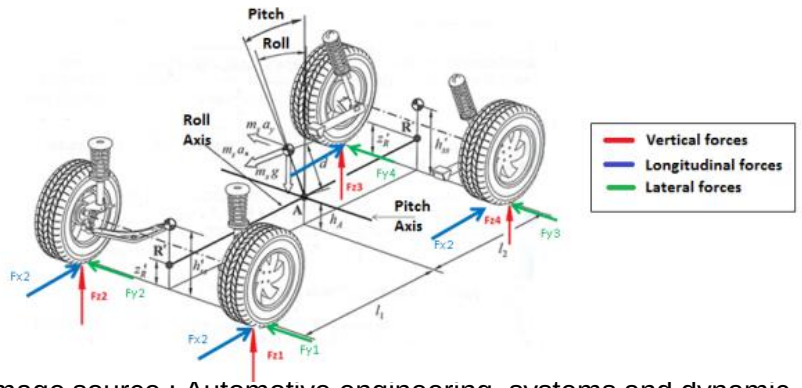


Image source : Automotive engineering, systems and dynamic behavior by Pablo Luque.

- Simulink

Trajectory Planning

➢ Track Discretization

$$\vec{\mathbf{P}}_i = x_i\vec{\mathbf{i}} + y_i\vec{\mathbf{j}}$$
$$= [x_{r,i} + \alpha_i(x_{l,i} - x_{r,i})]\vec{\mathbf{i}} + [y_{r,i} + \alpha_i(y_{l,i} - y_{r,i})]\vec{\mathbf{j}}$$

➢ Shortest Trajectory

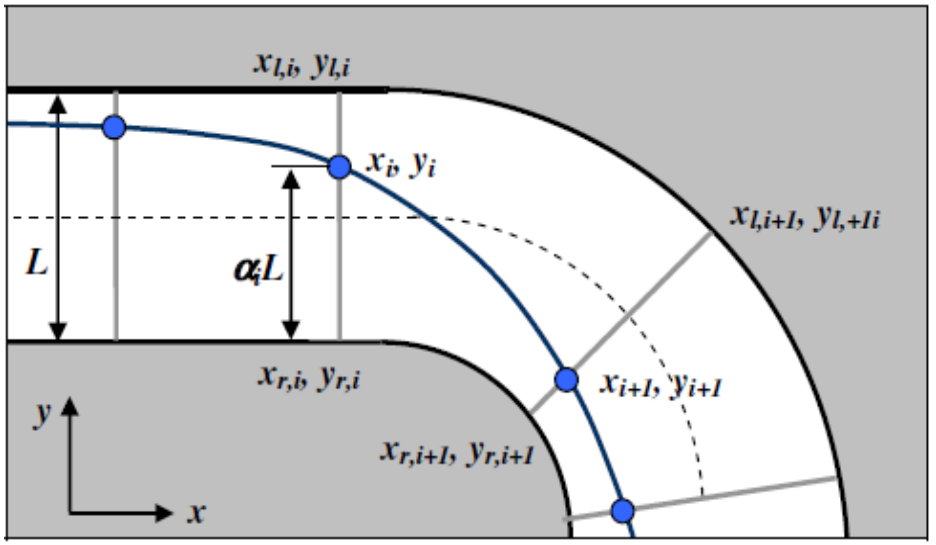
- Estimate line integral of the track:

$$S^2 = \sum_{i=1}^{n} \Delta P_{x,i}^{\mathrm{T}} \Delta P_{x,i} + \Delta P_{y,i}^{\mathrm{T}} \Delta P_{y,i}$$

➢ Minimum curvature trajectory

- Find the track curvature $\hat{\Gamma}$ using:

$$\hat{\Gamma}^2 = \left(\frac{\mathrm{d}^2 x(s)}{\mathrm{d}s^2}\right)^2 + \left(\frac{\mathrm{d}^2 y(s)}{\mathrm{d}s^2}\right)^2$$



Trajectory Planning

➢ Minimum curvature trajectory

- Interpolate track points with piecewise cubic splines
- (Numerically) integrate curvature on the entire path
- Minimize integrated curvature with respect to the track point coefficients

Spline segment and its derivatives

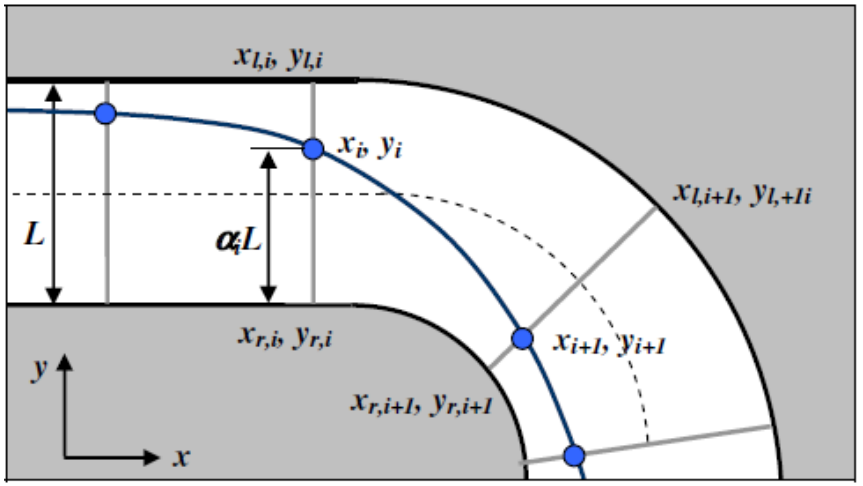
$$\boldsymbol{x}(t) = \boldsymbol{a}_0 + \boldsymbol{a}_1 t + \boldsymbol{a}_2 t^2 + \boldsymbol{a}_3 t^3$$

$$\boldsymbol{x}'(t) = \boldsymbol{a}_1 + 2\boldsymbol{a}_2 t + \mathbf{3}\boldsymbol{a}_3 t^2$$

$$\boldsymbol{x}''(t) = 2\boldsymbol{a}_2 + \mathbf{6}\boldsymbol{a}_3 t$$

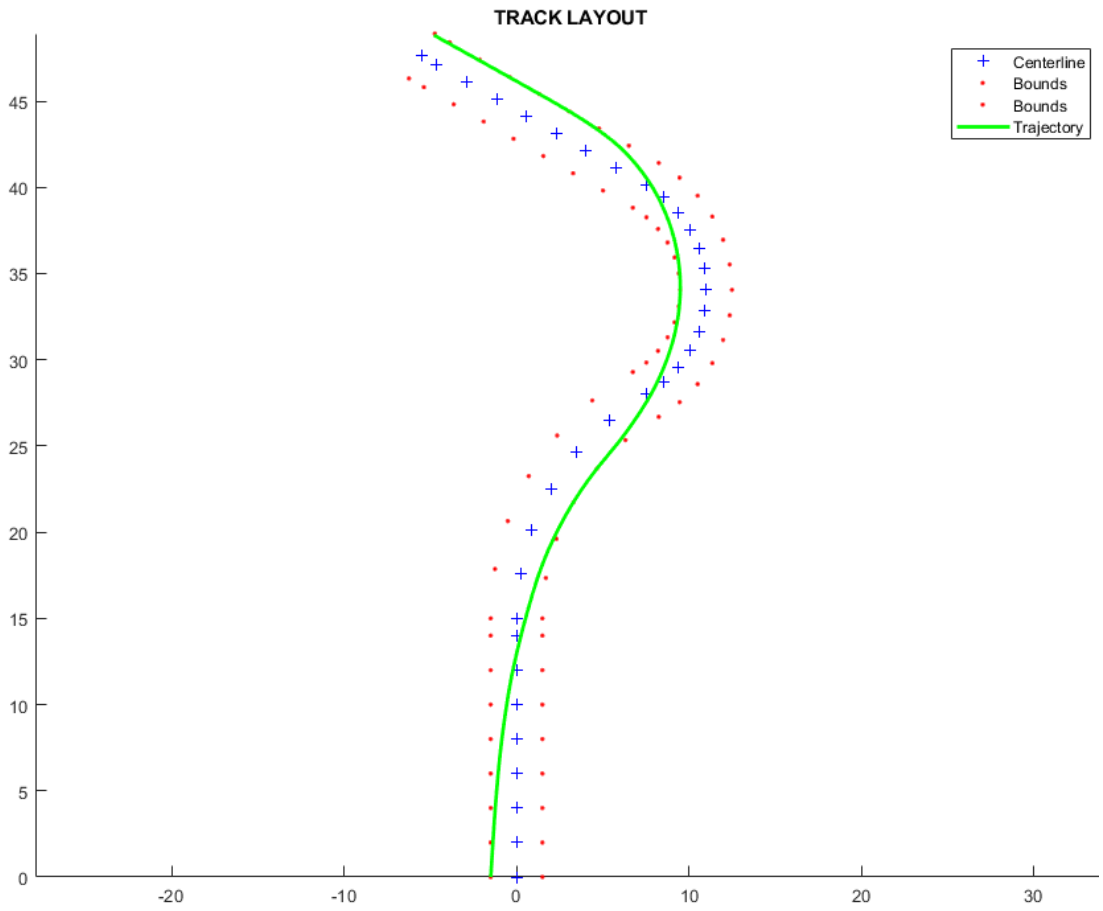
Curvature of a spline segment

$$\kappa = \frac{\boldsymbol{x}' \times \boldsymbol{x}''}{|\boldsymbol{x}'|^3}$$

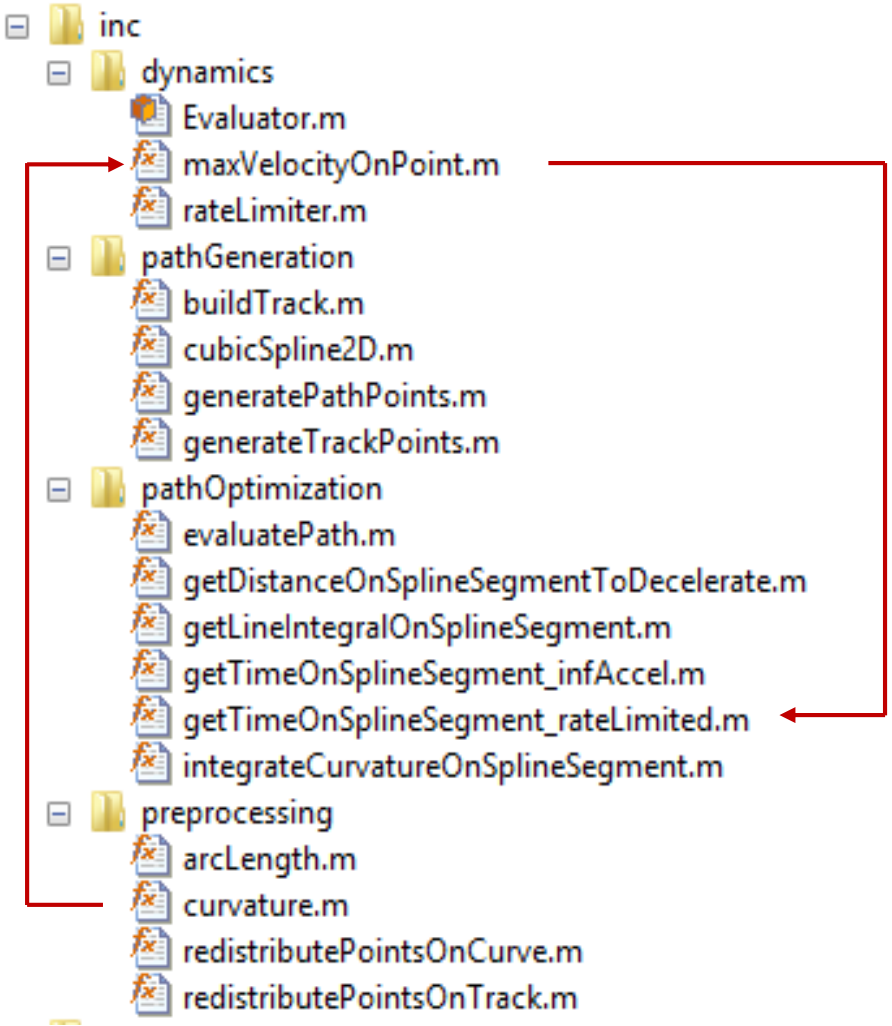


Trajectory Planning

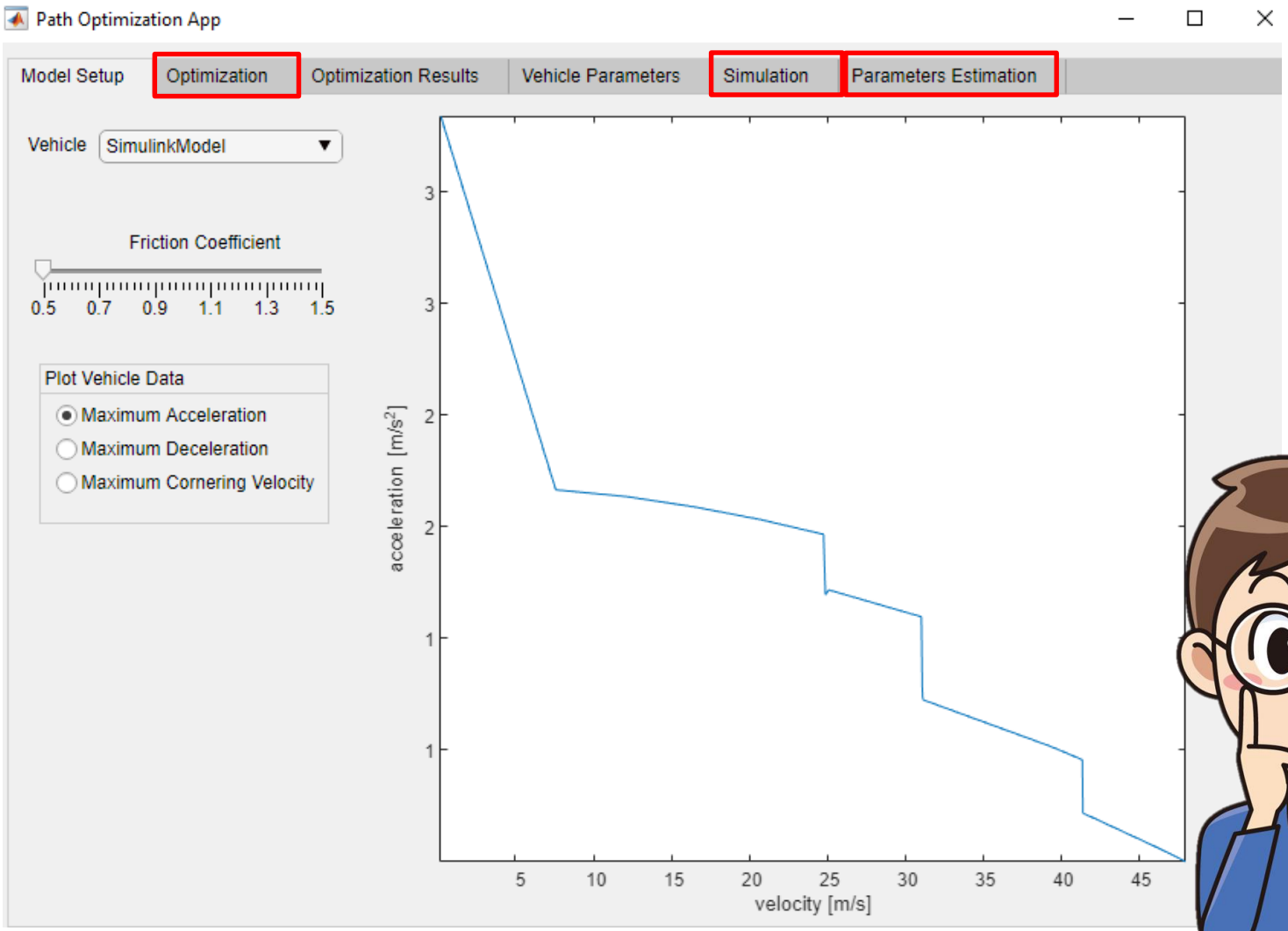
- **Minimum curvature trajectory (MATLAB)**



MATLAB Implementation :



Path Optimization App



Controller Design Roadmap

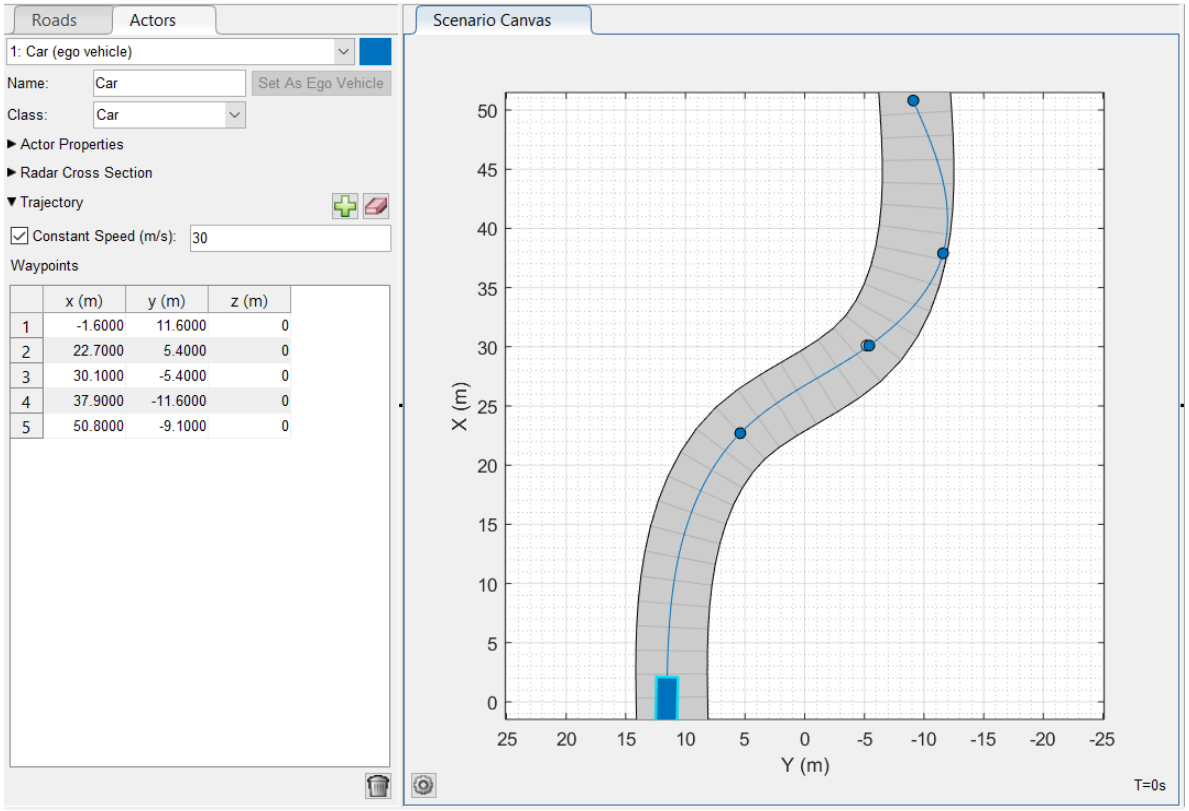
Test case creation → Build a control system → Testing and Visualization

$P \int D$

Test Case Creation

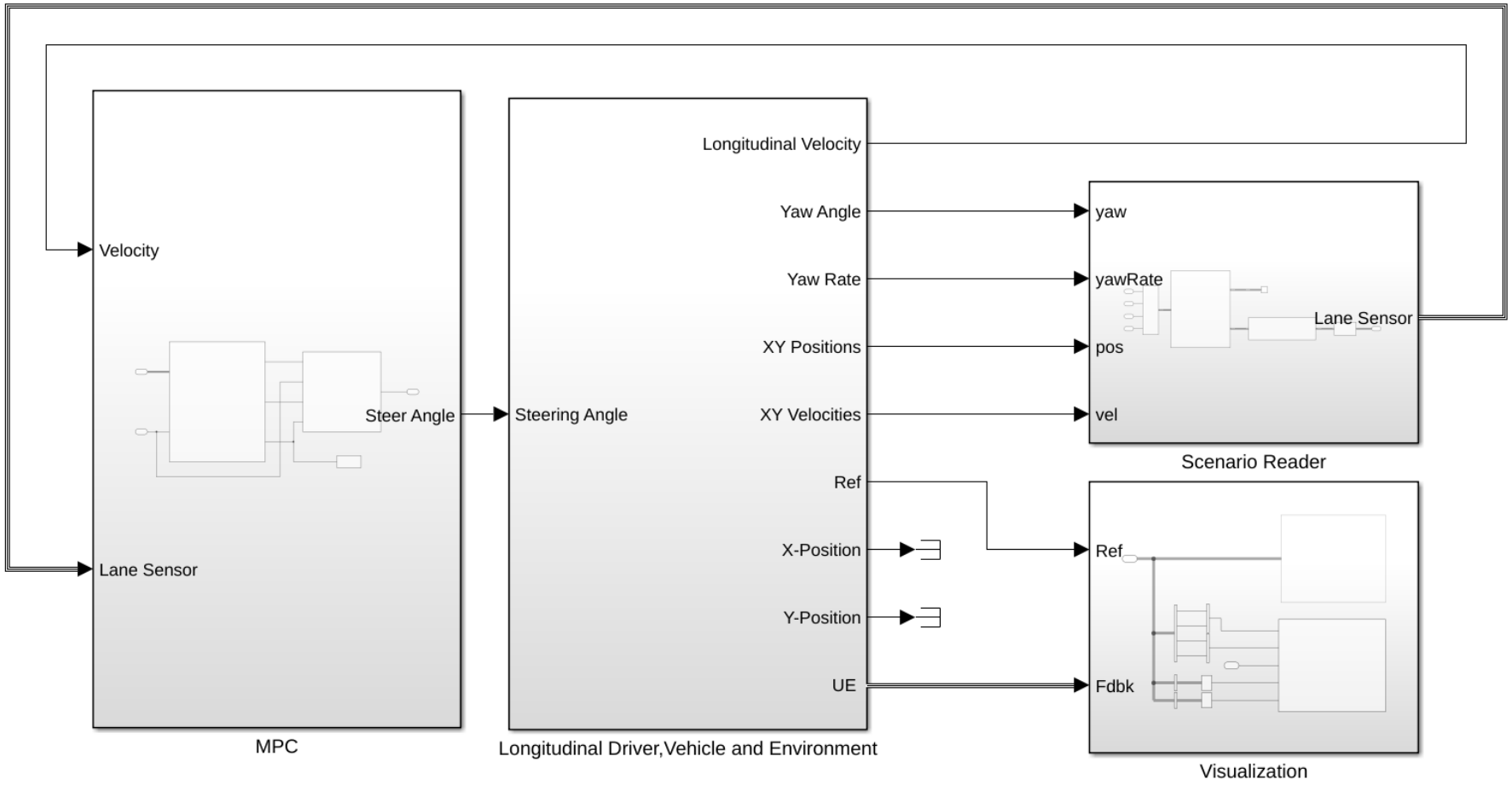
Using “Driving Scenario Designer” App to get:

- Reference position
- Road curvature
- Speed Profile
- Direction



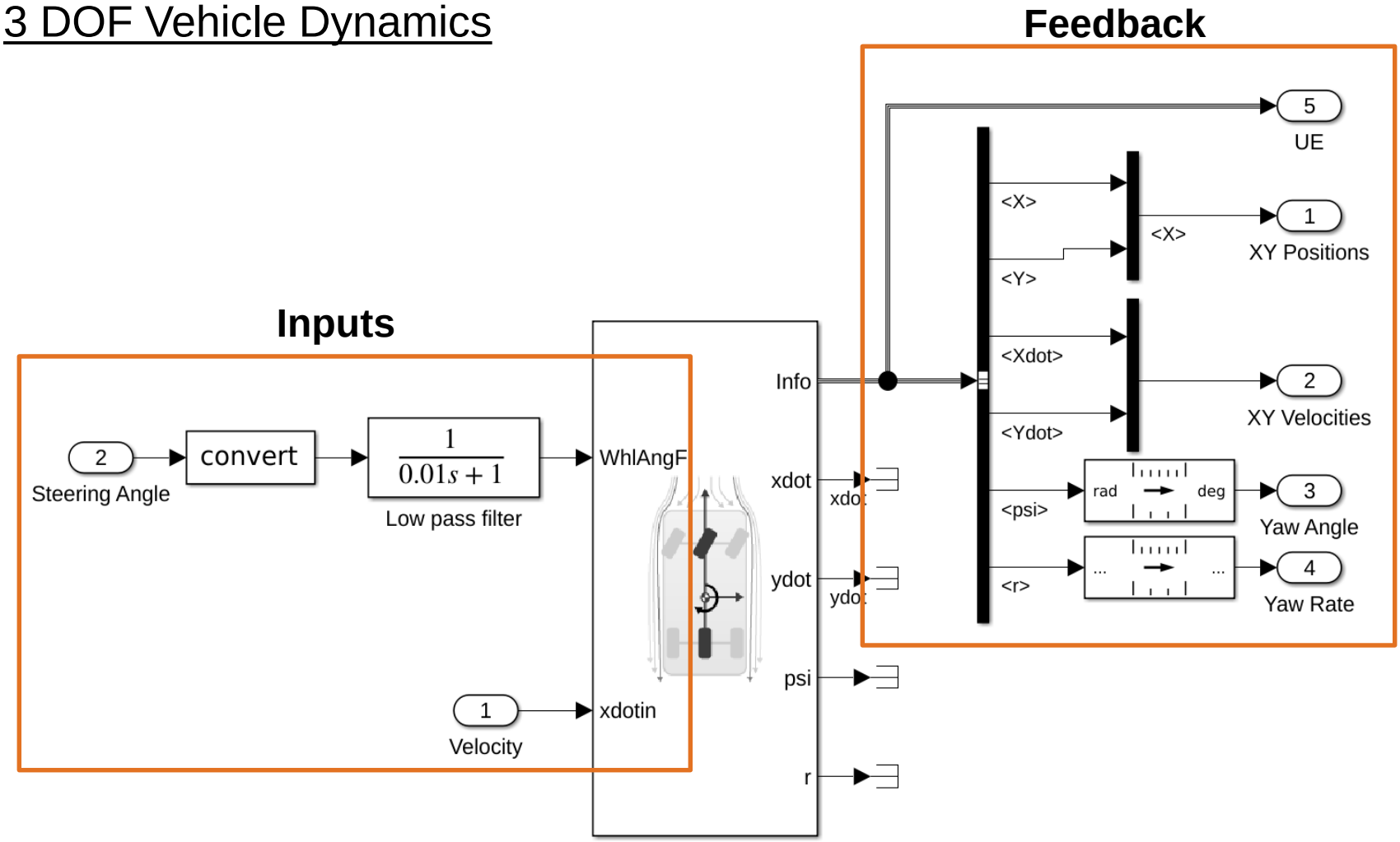
Simulation

Simulink Layout



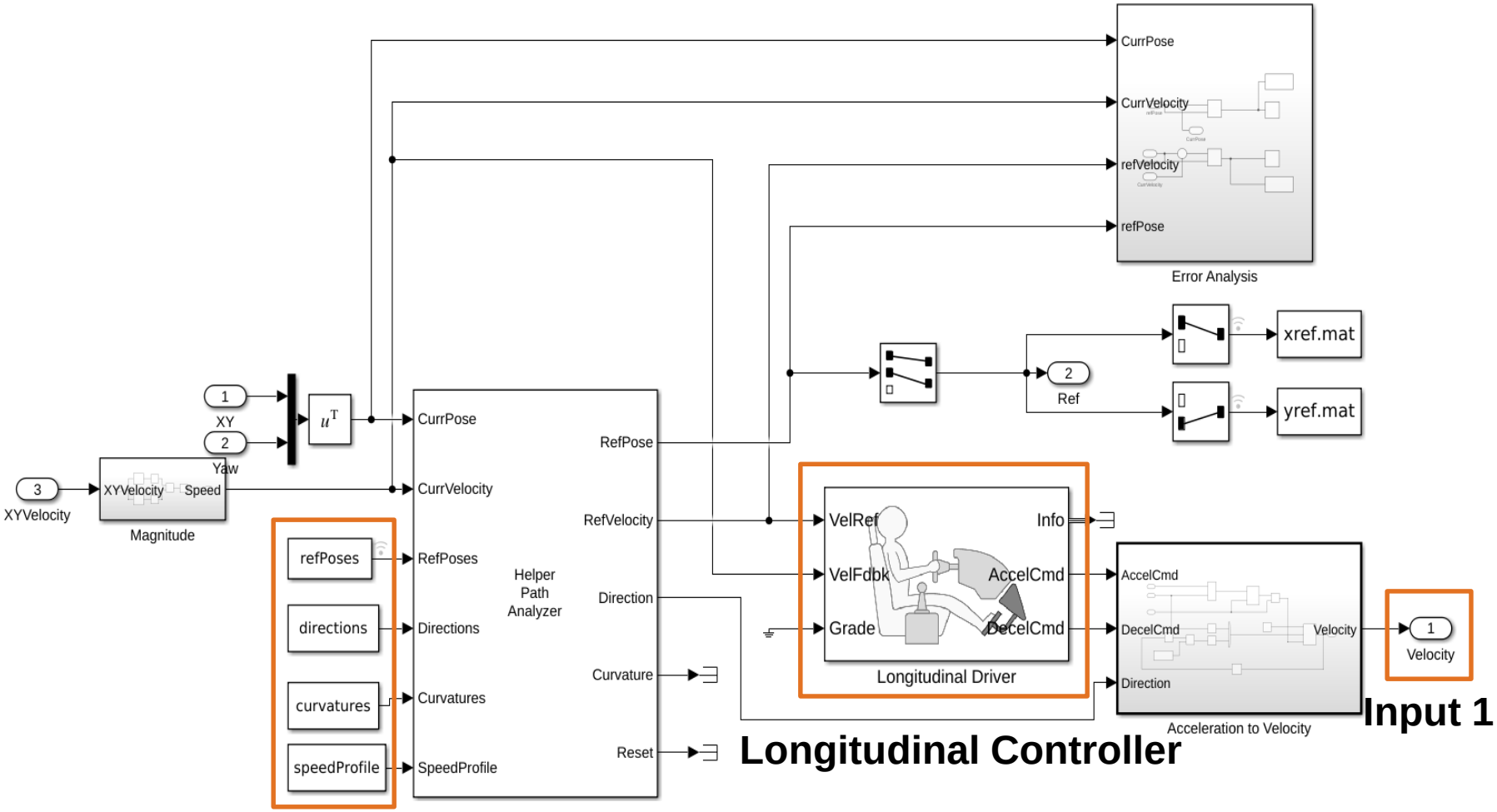
Simulation

3 DOF Vehicle Dynamics



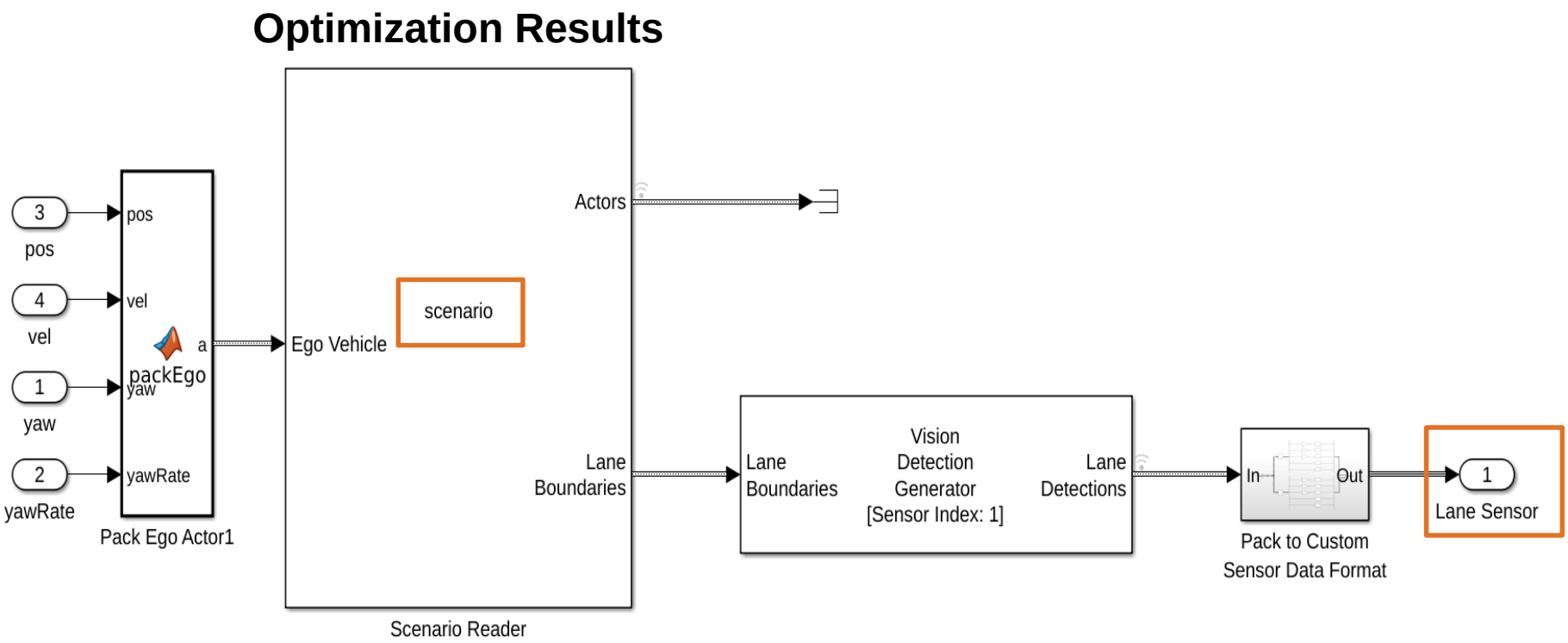
Simulation

Longitudinal Controller



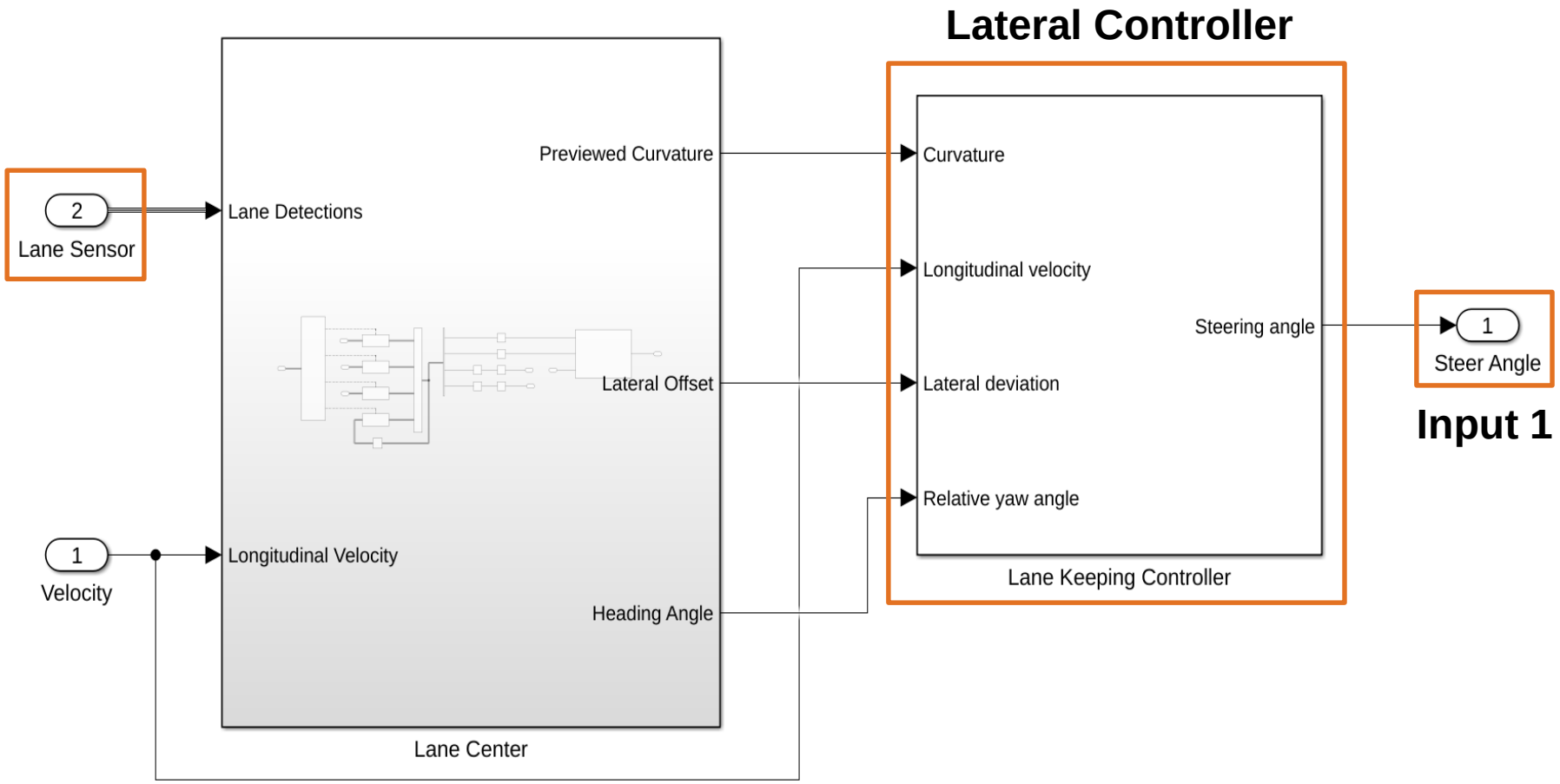
Simulation

Scenario Reader



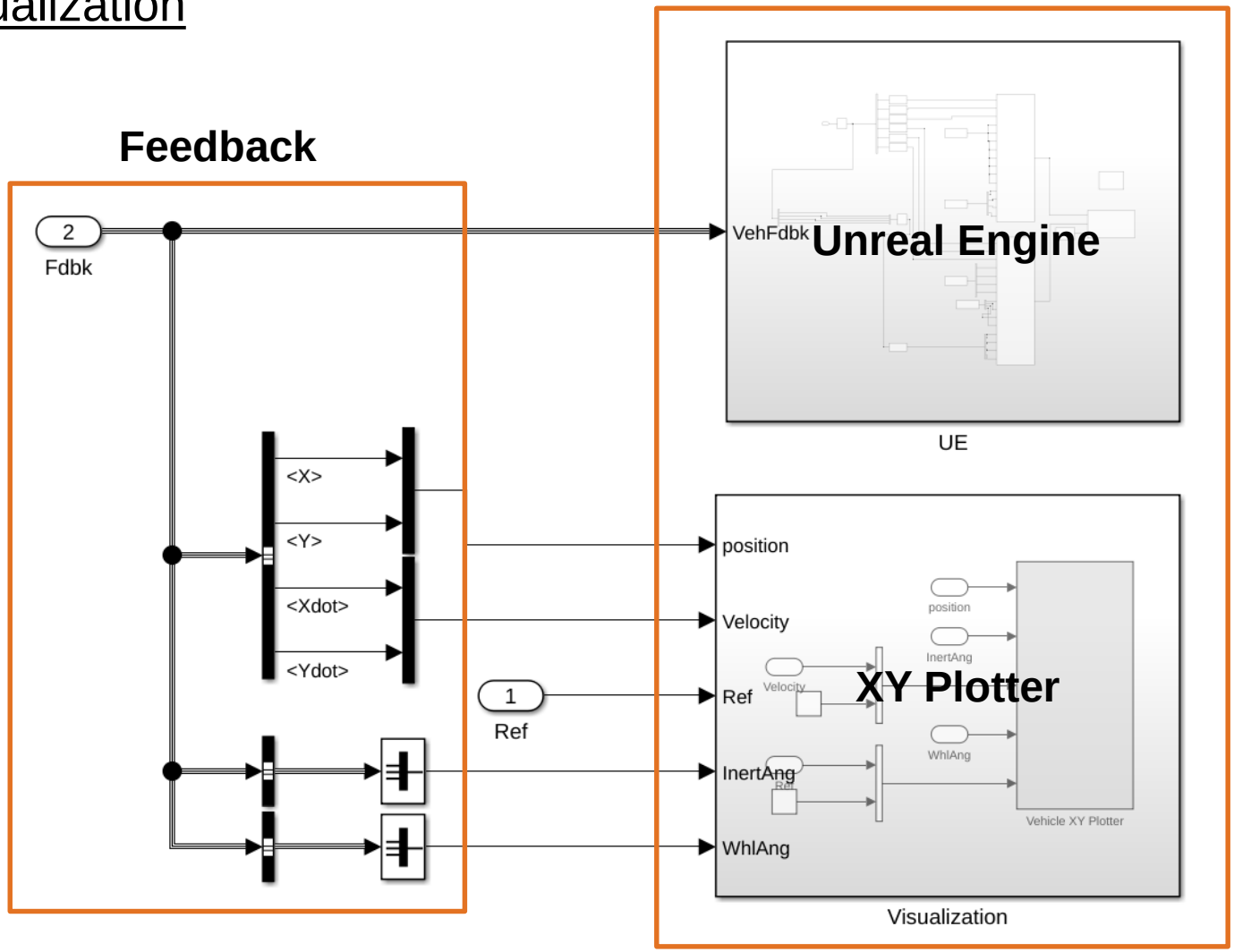
Simulation

Lateral Controller



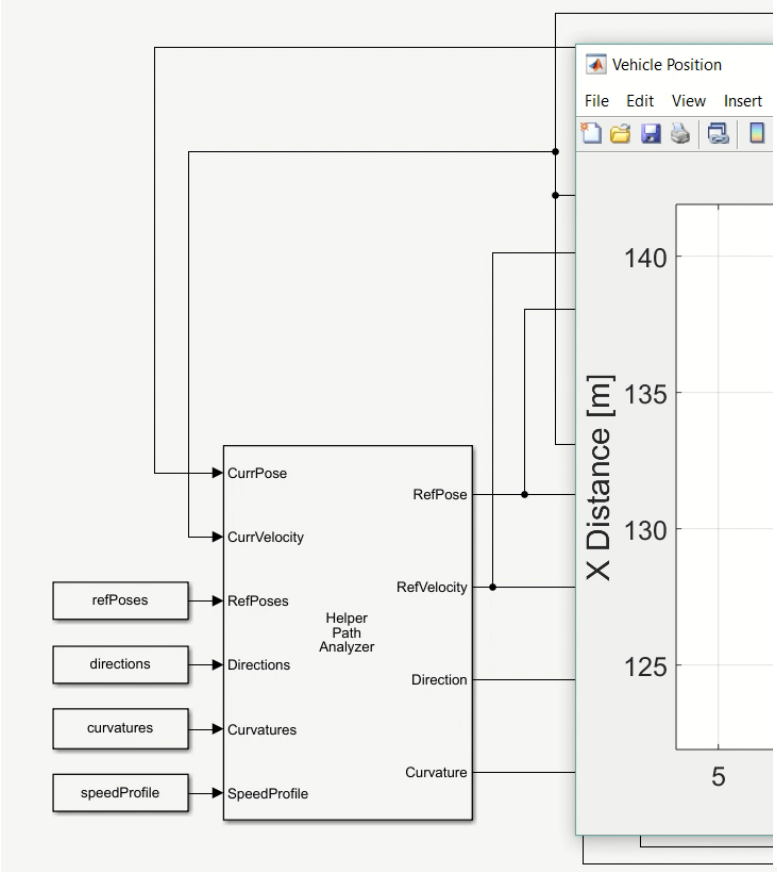
Simulation

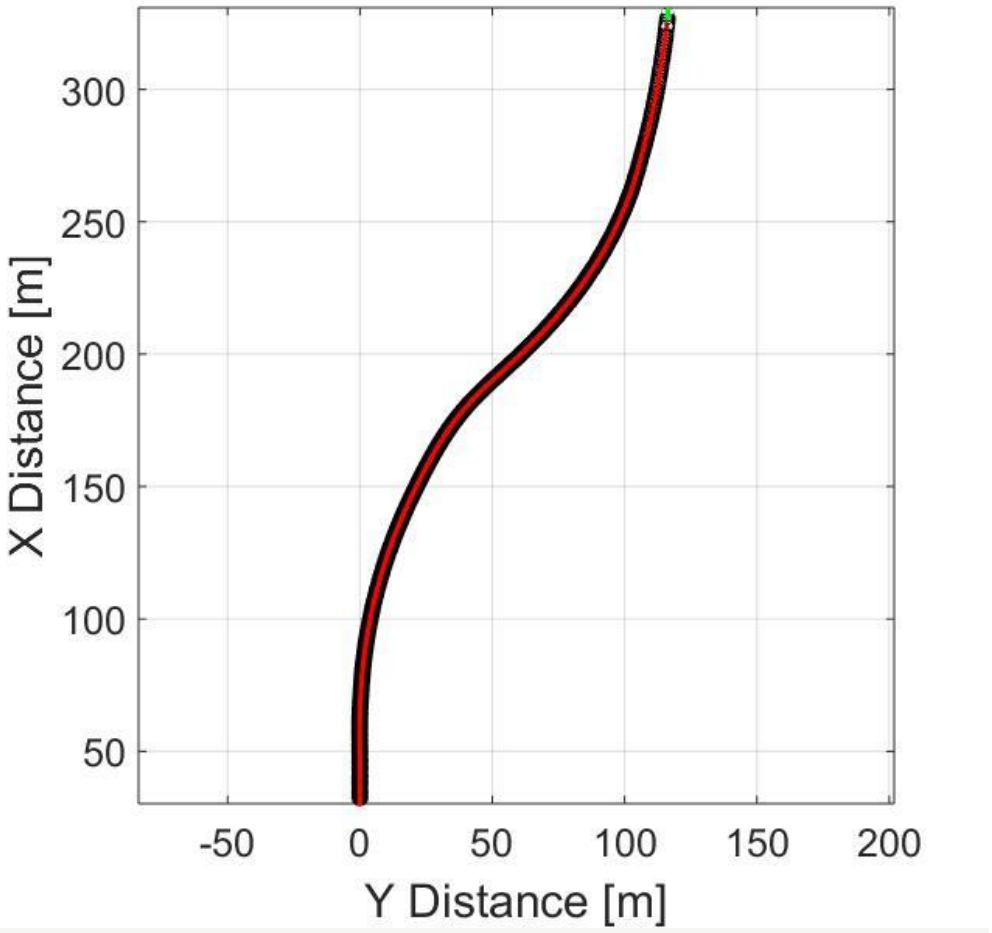
Visualization



Testing and Visualization

Using Vehicle XY Plotter





Simulation

Vehicle Paremeters | Simulation | Parameters Estimation

Please choose how do you want to simulate the results:

Simulation Options: XY Plotter (2D) ▼

- XY Plotter (2D)
- Unreal Engine (3D)
- Both

Start

Parameters Estimation

Vehicle Paremeters | Simulation | Parameters Estimation

Which Parameter(s) do you want to optimize?

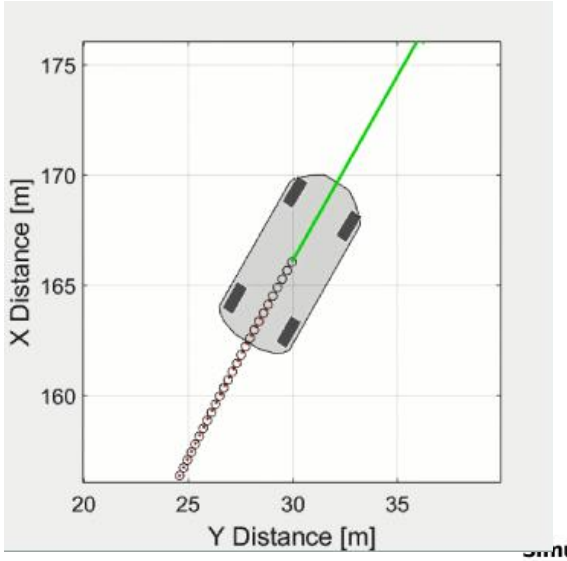
- [ ] Vehicle Mass
- [ ] Vehicle Wheelbase
- [ ] Location of center of gravity
- [ ] Axle Height

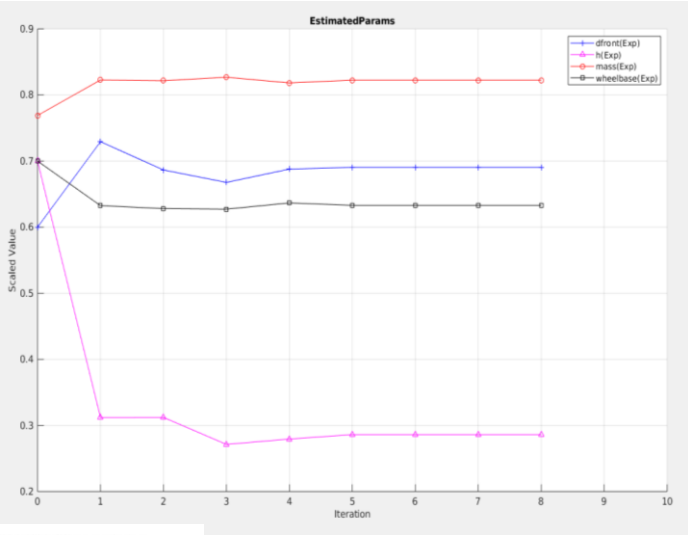
Results

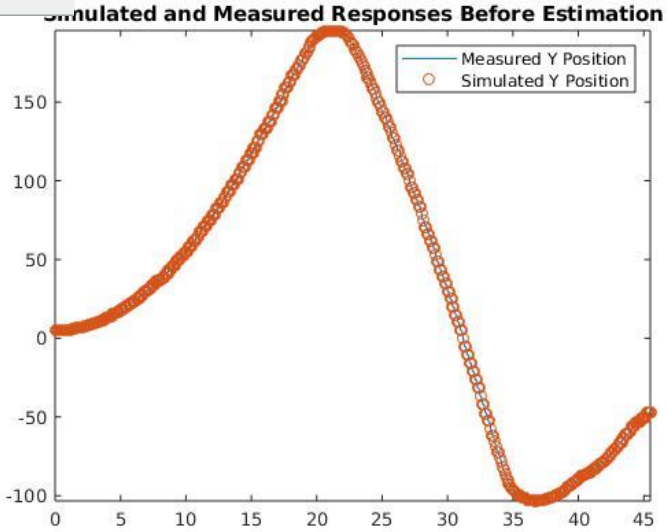
Mass	0
Wheelbase	0
COG	0
Axle Height	0

Start

Parameters Estimation

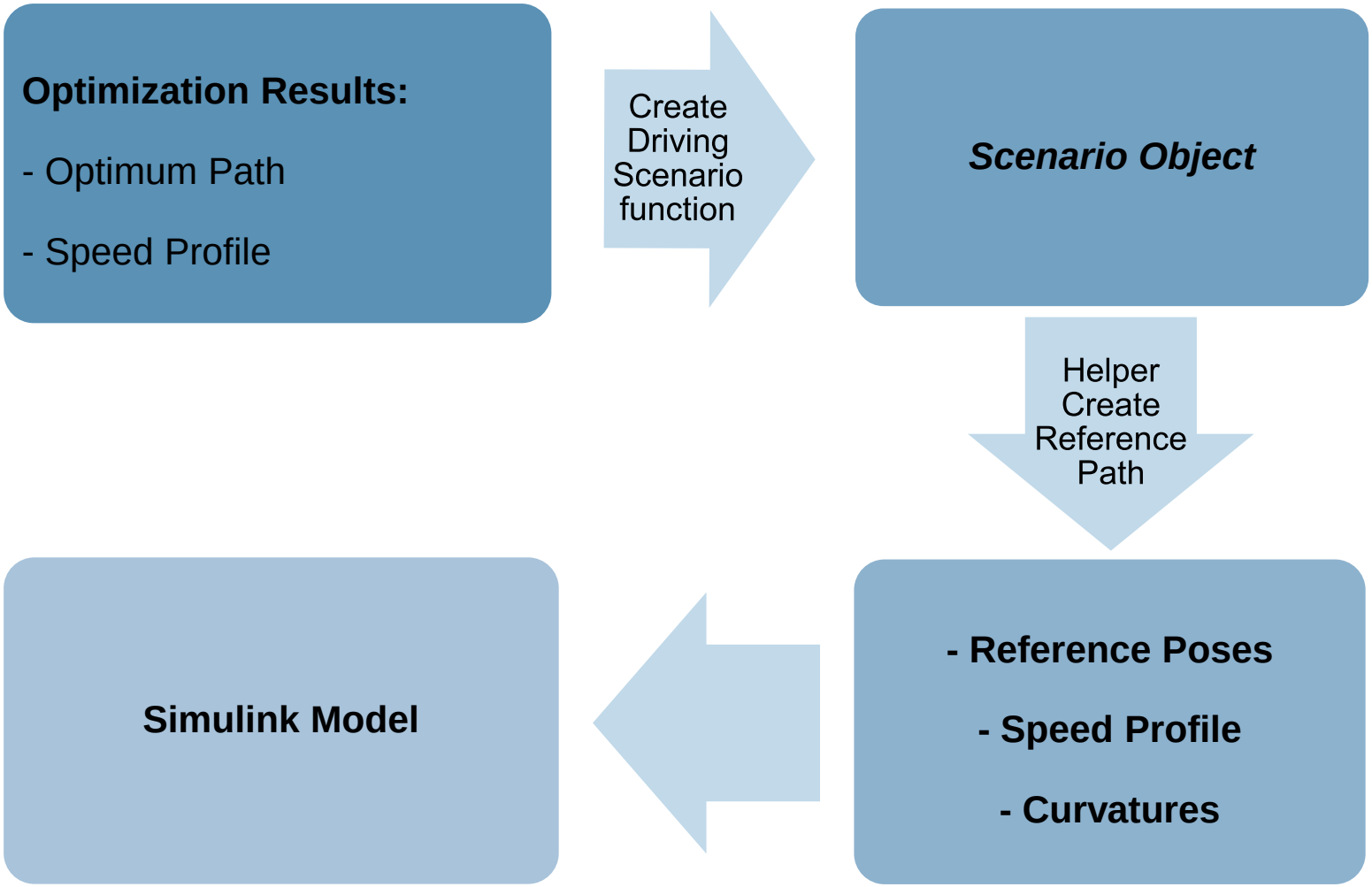




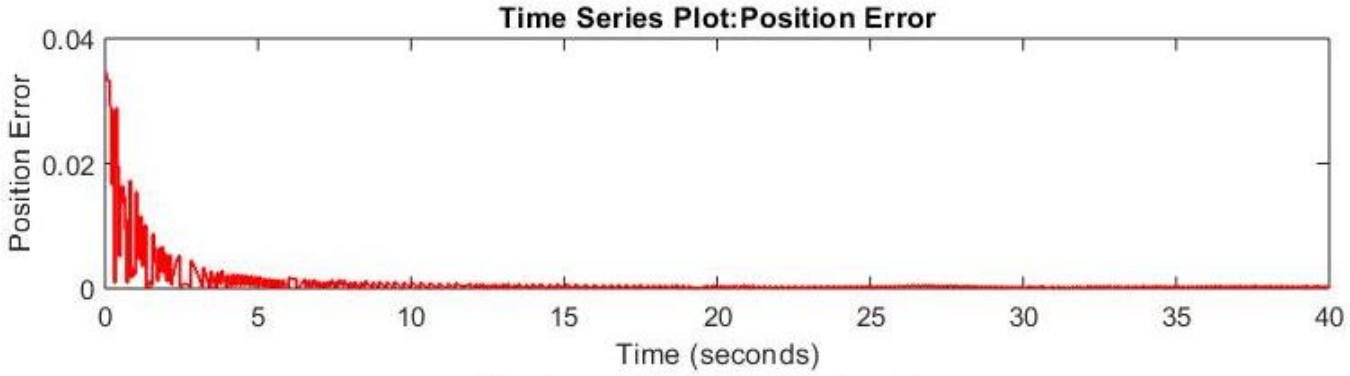


Simulation

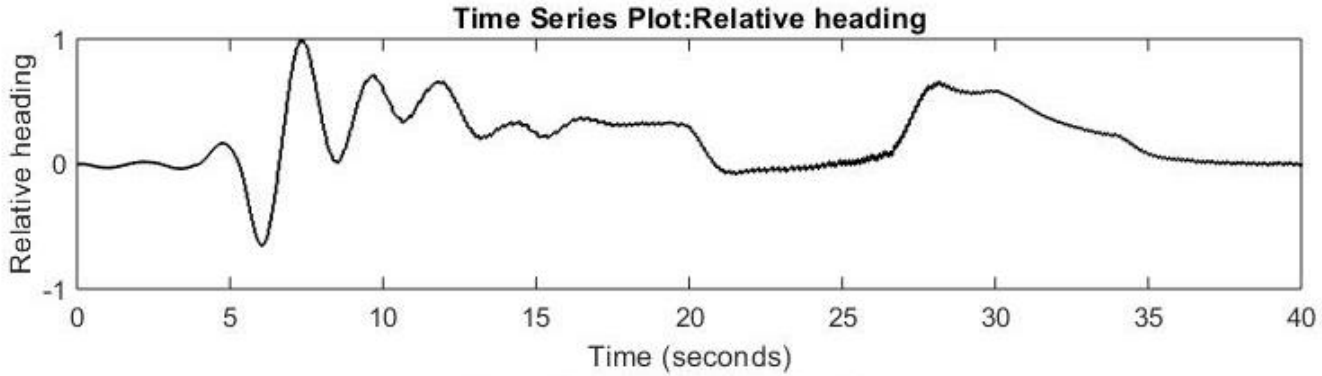
Results Preprocessing



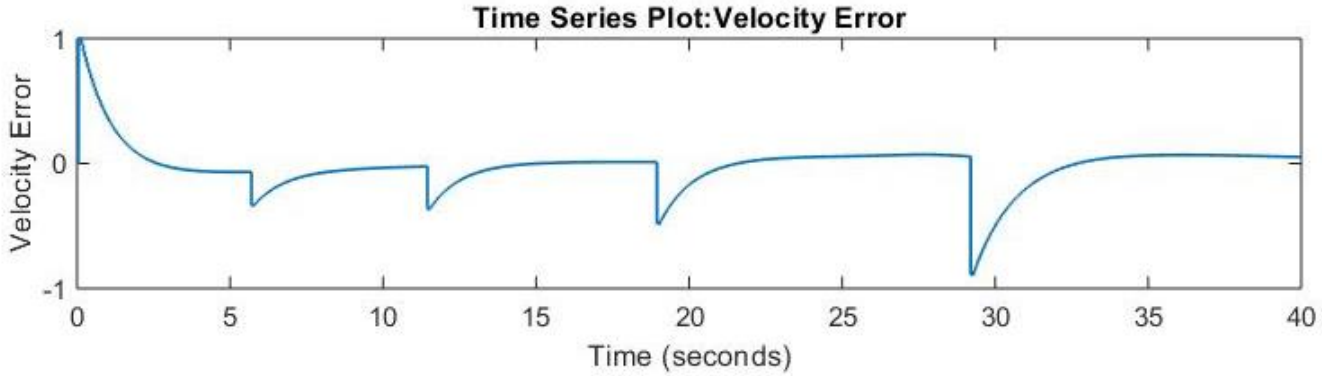
Controller Evaluation



Position Error



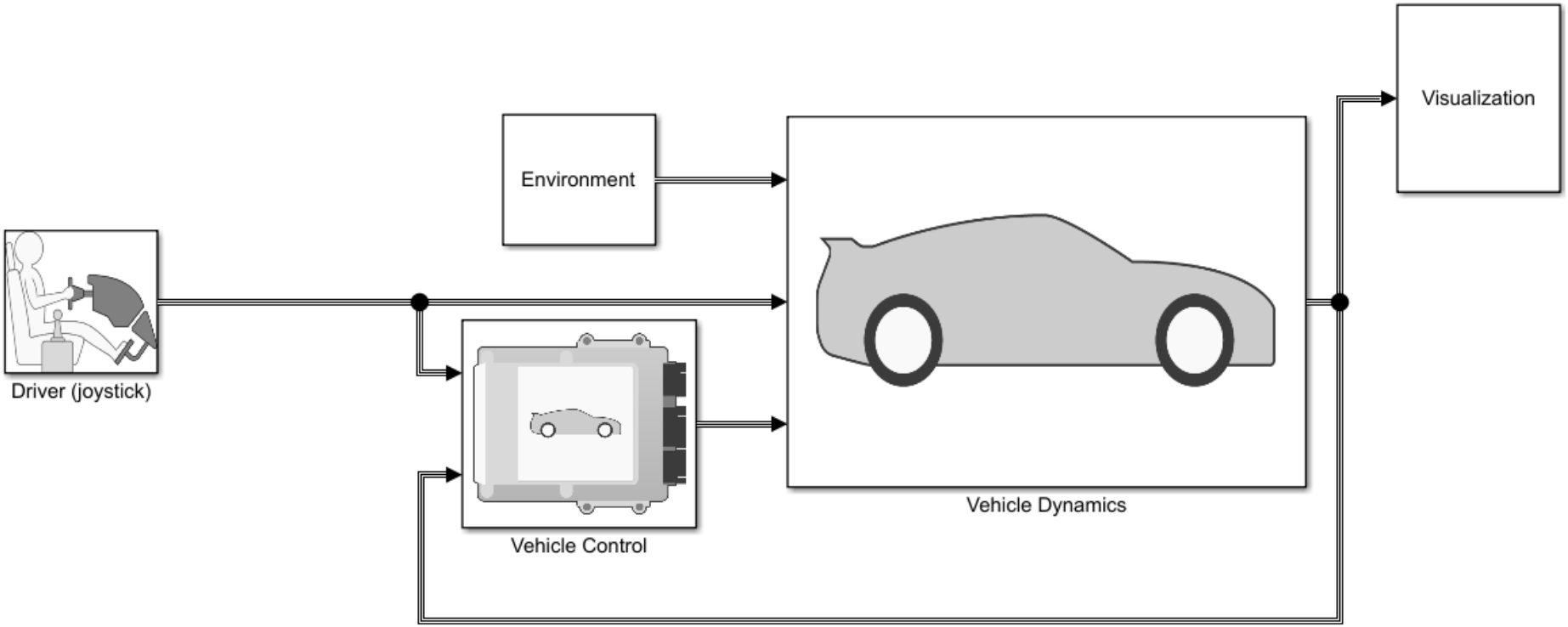
Relative Heading



Velocity Error

Interface to a 3D engine (Unreal+ Simulink)

- User-controlled simulation
- Provides a reference lap time



Interface to a 3D engine (Unreal+ Simuink)

Summary

- Project goals, importance and challenges
- Literature Review : two solutions possible
  - Shortest path
  - Least curvature path
- Illustrative example with a simple L-bend.
- MATLAB script for path optimization
- Simulink model with 3D visualization

THANK YOU
FOR LISTENING