

Exercises – Disks

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1. **Exercise:** computation of access time for **RAID 0** disk architectures.

A storage server consists of 10 disks with RAID 0 organization. The disks have the following characteristics: an average seek time of 5ms, 7200 RPM rotation speed, average transfer rate of 8 MB/sec, 1KB of block size (stripe unit) and a stripe width of 4 disks. Compute the minimum time to access all the data of a file of 400 KB, assuming that all the disks have constant rotation speed and are synchronized. Controller time is set to zero.

Solution:

Mean access time for 1 block = **seek + latency (1/2 round) + transfer (1block)** =

$$5 + (60 \times 1000 / 7200) / 2 + 1000 / (8 \times 1024) = 5 + 4.17 + 0.12 = 9.29 \text{ ms}$$

The access time to read all the file with a single disk:

$$400 \times 9.29 / 1000 = 3.716 \text{ sec}$$

Finally we can compute the *minimum read time* for the file, taking into account that there are 4 blocks each read, one per each disk in the stripe width:

$$3.716 / 4 = 0.929 \text{ sec}$$

3. **Exercise:** Mean Time to Data Loss of a **RAID 0**.

A RAID 0 array consists of 8 disks, the MTTF for each disk is 1000 days. The MTTR is 10 days. Which is the array's MTDDL?

Solution:

MTTDL in this case is not affected by MTTR!

$$\text{MTTDL} = \text{MTTF (of a disk)} / n \text{ disks composing the array} = 1000 / 8 = 125 \text{ days}$$

4. **Exercise:** Mean Time to Data Loss of a **RAID 1**.

A RAID 1 array consists of two disks. The MTTF for each one of the 2 disks is 1000 days. The MTTR is 10 days. Which is the probability of a second failure while the first disk is being repaired?

Solution:

$$p(\text{array RAID1 failure}) = 1/\text{MTTDL};$$

$$p(\text{array RAID1 failure}) = p(1^\circ \text{ failure}) \times p(2^\circ \text{ failure} < \text{MTTR});$$

$$p(1^\circ \text{ failure}) = n / \text{MTTF}(\text{disk}) = 2 / 1000;$$

$$p(2^\circ \text{ failure: of the mirror disk}) = \text{MTTR} / \text{MTTF} = 10 / 1000;$$

$$p(\text{array RAID1 failure}) = (2 / 1000 \times 10 / 1000) = 0,00002;$$

$$\text{MTTDL} = 1 / 0,00002 = 50000 \text{ days.}$$

5. **Exercise:** Mean Time to Data Loss of a **RAID 1 + 0**.

A storage server consists of 16 disks organized in 8 groups of RAID 0 disks, and each group consists of 2 RAID 1 disks. Which is the MTTDL for the entire storage server knowing that the MTTF of a disk is 1000 days and the MTTR is 10 days?

Solution:

$$\text{Total number of disks} = 16 (8 \times 2);$$

$$p(1^\circ \text{ failure}) = \text{number of disks} / \text{MTTF}(\text{disk}) = 16 / 1000 = 0,016;$$

$$p(2^\circ \text{ failure: of the mirror disk of the } 1^\circ \text{ failed}) = \text{MTTR} / \text{MTTF} = 10 / 1000 = 0,01;$$

$$p(\text{storage failure}) = p(1^\circ \text{ failure}) \times p(2^\circ \text{ failure}) = 0.016 \times 0.01$$

$$\text{MTTDL} = 1 / (0,016 \times 0,01) = 6250 \text{ days}$$

6. **Exercise:** Mean Time to Data Loss of **RAID 0 + 1** organization.

Let's consider 2 group (RAID 1) each of 8 disks (RAID 0). Compute the MTTDL of the array given that the MTTF of a disk is 1000 days and the MTTR is 10 days.

Solution:

Total number of disks = 16 (8×2);

$p(1^\circ \text{ failure}) = \text{one among the 16 disks} = n / \text{MTTF}(\text{disk}) = 16 / 1000 = 0,016$;

$p(2^\circ \text{ failure: a disk fails in the other mirror group}) = \text{MTTR} / (\text{MTTF} \times 2/n)$;

$\text{MTTDL} = 1/[(16/1000) \times (10 \times 8/1000)] = 781 \text{ days}$

7. Exercise: Mean Time to Data Loss of an array of RAID 5.

Let's consider an array of RAID 5 disks, composed of $G=5$ disks. The MTTF of a disk is 1000 days and the MTTR is 10 days.

- Calculate the probability that one of the 5 disks fail.
- Calculate the probability that one of the remaining 4 disks fails during the repairing interval of the first failed disk.
- Calculate the probability that the array of RAID 5 disks fails.
- Calculate the MTTDL.

Solution:

- Probability that a disk fails:

$$p(1^{\text{st}} \text{ fail}) = G \times p(\text{single disk fail}) = G / \text{MTTF}(\text{disk}) = 5 / 1000 = 0,005;$$

- Fail probability of one of the other 4 disks, during repair time of the first failed disk:

$$p(2^{\text{nd}} \text{ fail}) = \left[\frac{G-1}{\text{MTTF}(\text{Disk})} \text{MTTR} \right] = \frac{4}{1000} 10 = 0,04$$

- Probability that the array of RAID 5 disks fails.

$$P(\text{array fails}) = p(1^{\text{st}} \text{ fail}) p(2^{\text{nd}} \text{ fail}) = \left[\frac{G(G-1)}{\text{MTTF}(\text{Disk})^2} \text{MTTR} \right] = \frac{5}{1000} \frac{4}{1000} 10 = 0.0002$$

- $\text{MTTDL}_{\text{RAID 5}} = \frac{1}{P(\text{array fails})} = \frac{1}{0.0002} = 5000 \text{ days};$

8. Exercise: Mean Time to Data Loss of an array of RAID 5+0.

Let's consider an array of RAID 5+0 disks, composed of $N=25$ disks organized in 5 groups of $G=5$ disks each. Each group has a redundant disk. The MTTF of a disk is 1000 days and the MTTR is 10 days.

- Calculate the probability that one of the 5 disks in a group fail.
- Calculate the probability that one of the remaining 4 disks fails during the repairing interval of the first failed disk.
- Calculate the probability that a specific group fails.
- Calculate the probability that one of the 5 groups fails.
- Calculate the MTTDL.

Solution:

- a. Probability that a disk in a group fails:

$$p(1^{\text{st}} \text{ fail}) = G \times p(\text{single disk fail}) = G / \text{MTTF}(\text{disk}) = 5 / 1000 = 0,005;$$

- b. Fail probability of one of the other 4 disks, during repair time of the first failed disk:

$$p(2^{\text{nd}} \text{ fail}) = \left[\frac{G-1}{\text{MTTF}(\text{Disk})} \text{MTTR} \right] = \frac{4}{1000} 10 = 0,04$$

- c. Fail probability of one specific group:

$$p(\text{group fail}) = \left[\frac{G}{\text{MTTF}(\text{Disk})} \frac{G-1}{\text{MTTF}(\text{Disk})} \text{MTTR} \right] = \frac{5}{1000} \frac{4}{1000} 10 = 0.0002$$

- d. Fail probability of one of the 5 groups.

$$p(\text{array fails}) = \text{num. of groups} \times p(\text{group fail}) =$$

$$\frac{n.\text{groups} \times G \times (G-1) \times \text{MTTR}(\text{disk})}{\text{MTTF}(\text{Disk})^2} = \frac{N \times (G-1) \times \text{MTTR}(\text{disk})}{\text{MTTF}(\text{Disk})^2} = 5 \times 0,0002 = 0,001;$$

- e. $\text{MTTDL}_{\text{RAID 5+0}} = \frac{1}{p(\text{array fails})} = \frac{1}{0,001} = 1000 \text{ days}$

9. **Exercise:** Mean service time of an I/O operation.

Compute the mean service time of an I/O operation that read/write a sector of 512 Byte (0.5 KB). The device have a data transfer rate of 50 MB/sec and a rotation speed of 10000 RPM. The mean seek time is 6ms and the controller overhead is 0.2 ms.

Solution:

The mean I/O service time is given by seek time + latency + transfer time + controller overhead;

$$\text{Mean latency: } (60\text{s/min}) \times 1000 / (2 \times 10000 \text{ rpm}) = 3.0\text{ms (time for } \frac{1}{2} \text{ round)}$$

$$\text{Block transfer time: } (0.5\text{KB}) \times 1000 / (50 \times 1024\text{KB/s}) = 0.01\text{ms}$$

$$\text{Mean I/O service time} = 6\text{ms} + 3\text{ms} + 0.01\text{ms} + 0.2\text{ms} = 9.21\text{ms}$$

10. **Exercise:** Mean service time of an I/O operation considering **Data Locality**.

Calculate the mean service time of an I/O operation that read/write a sector of 512 Byte (0.5 KB). The device have a data transfer rate of 50 MB/sec and a rotation speed of 10000 RPM. We also know that the mean seek time is 6ms and the controller overhead is 0.2 ms. The locality of the data that must be read/write is 75%.

Solution: (same steps as previous exercise with the following exceptions)

(Seek time + Latency) affect only 25% of the operations.

$$\text{Mean I/O service time} = 0.25 \times (6\text{ms} + 3\text{ms}) + 0.01\text{ms} + 0.2\text{ms} = 2.46\text{ms}$$

11. Exercise: Storage system

Let's consider a storage system with 8 disks. Each disk has a MTTF equal to 108 days and a MTTR equal to 8 days.

Evaluate the MTDDL and the free storage capacity for each of the following configurations:

(a) All disks are configured as RAID4

(b) All disks are configured as RAID5

(c) Disks are divided in two groups composed by 4 disks each, each group is RAID5 and the two groups are in striping.

Solution:

- a. $n = 8$, $\text{MTTF} = 108 = 3^3 \cdot 2^2$, $\text{MTTR} = 8$, $\text{MTDDL} = \text{MTTF}^2 / (n \cdot (n-1) \cdot \text{MTTR}) = 3^6 \cdot 2^4 / (2^3 \cdot 2^3 \cdot 7) = 729/28$
- b. same as a.
- c. $n = 4$, $g = 2$, $\text{MTDDL}_{\text{group1}} = \text{MTTF}^2 / (n \cdot (n-1) \cdot \text{MTTR}) = 243/2$, $\text{MTDDL}_{\text{raid}} = \text{MTDDL}_{\text{group1}}/2 = 243/4$;