Dependability is the collective term used to describe the availability performance of a system, which is the readiness for correct service, and its influencing factors:

- Reliability, the continuity of correct service;
- Maintainability, the reparation to restore correct service;
- Maintenance support.

It can be described only with probabilistic laws.

The **probability of failure** f(t) is the probability that a component fails at a given time instant, it is the probability density function of the failure.

The cumulative distribution F(t) is the unreliability, which is the probability of having a failure before the time t, knowing that at t=0 the system was working. Its opposite is the **reliability** R(t), which is computed as R(t) = 1-F(t) and represents the probability that a component fails after the time t.
Some properties are:

- $R(0) = 1$, the system is working when it is bought;
- $lim_{t \to \infty} R(t) = 0$, the system will fail at a certain time,
- $f(x) = -\frac{dR(t)}{dt}$

Reliability is computed empirically: n independent and statistically identical elements are deployed at time t=0 in identical conditions, then the failure times $t_0, t_1, \ldots t_n$ are observed, knowing that they are not related to each other. The function n(t) is computed by interpolating its shape from the failure times, then the reliability R(t) is calculated as $R(t) = lim_{n \to \infty} \frac{n(t)}{n}$.

The mean of the reliability is the **mean time to failure** (MTTF), which is $\int_{\infty}^{0} t \cdot f(t)dt = \int_{\infty}^{0} R(t)dt$

The quantities can be rewritten considering the **failure rate** $\lambda(t)$, the probability of failure at instant t assuming that the component is working at time t, which differs from the probability of failure as it refers to a system which was working at time t=0.

The sources of failures can be many:

- Design failures: errors in designing;
- Infant mortality: errors that come out during the testing stage;
- Random failures: errors that show up randomly during the entire life of a system, they are those considered when calculating the previous mentioned quantities;
- Wear out: at the end of its life, some components cause the failure of the system, it can be reduced with maintenance.

These sources have different impact on different components.

- Electronic components: high infant mortality, high wear out;
- Mechanical components: low infant mortality, high wear out;
- Software: high infant mortality, no wear out, when it is upgraded its lifetime goes back to t=0 and this action must be considered to evaluate correctly the system

However, the only source that is considered are the random failures, which are supposed to have a constant rate, to be exponentially distributed and independent one from the other. This leads to a rewriting of the probability functions.

- $MTTF = E[X] = \int_{\infty}^{0} t \cdot \lambda e^{\lambda t}\, dt = \frac{1}{\lambda}$, so $\lambda = \frac{1}{MTTF}$
- $F(t) = 1 - e^{-\lambda}$, which becomes $F(t) = \frac{1}{MTTF}$ if $t \ll MTTF$
- $f(t) = \lambda e^{\lambda t}$

It was said that a system that comprises redundant components can tolerate a certain number of failed components, so the **life time** is the time t* after which the reliability becomes less than a fixed threshold, it can be increased by

- Using highly reliable element,
- Increasing the redundancy,
- Having some spare component to rapidly recover from failures.

The behaviour of a system can be represented using **Reliability block diagram** (RBD), components can be connected in serial or in parallel.

1. Serial components
   a. $F_{TOT}(t) = 1 - \prod_i [1 - F_i(t)]$
   b. $R_{TOT}(t) = \prod_i R_i(t)$
   c. $MTTF = \frac{1}{\sum_i 1/MTTF_i}$, if they are identical it becomes $MTTF = \frac{MTTF_i}{n}$
2. Parallel components
   a. $F_{TOT}(t) = \prod_i F_i(t)$,
   b. $R_{TOT}(t) = 1 - \prod_i [1 - R_i(t)]$
   c. the mean time to failure is
      i. $MTTF_{TOT} = MTTF_1 + MTTF_2 + \frac{1}{\frac{1}{MTTF_1} + \frac{1}{MTTF_2}}$ for two different components
      ii. $MTTF_{TOT} = MTTF \left(\frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{2} + 1\right)$

The **mean time to repair** (MMTR) is the time required to repair a failed component, it includes the time to discover, detect and repair the failure and to reset the system to make it work again, it can be very expensive to keep it small.
The concept leads to that of **mean time between failures** (MTBF), which is the sum of the MMTR and the MTTF and it is calculated in different ways depending on the component:

- Software: restarting usually hides the problem, but it is not the real solution;
- RAID: replace the disk and, if possible, restore the data;
- Memory: identify, replace it and restart the system;
- …

The **availability** is the probability that a system is working at a certain time t, it differs from the reliability as the latter does not include the possibility of repairing it, which the former does. It is calculated as $A = \frac{MTTF}{MTTF+MTTR}$. The formula is correct also system-wise, but the computation can be done also starting from the values for the single components:

- $A_{SERIAL} = \prod_k \frac{MTTF_K}{MTTF_K + MTTR_K}$
- $A_{PARALLEL} = 1 - \prod_k \left(1 - \frac{MTTF_K}{MTTF_K + MTTR_K}\right)$