Exercises –Availability

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1. **Exercise:** failure probability of a disk

Compute the probability that a disk with MTTF = 100000 hours fails at least once every 3 years. If instead we have 2 disks, which is the probability that at least one of them fails?

Solution:

For one disk: $P(X \leq t) = 1 - e^{-\frac{26280}{100000}}$ ≈ 23%

(also valid the approximate solution: 26280/100000 ≈ 26%, since 0,26 << 1)

For two disks: $P(X \leq t) = 1 - e^{-\frac{26280}{100000} \times 2}$ ≈ 41%

2. **Exercise:** computation of Reliability

A load balancer(A) is connected in series with a group(B) of three parallel web servers. The load balancer have MTTF of 1000 days and MTTR of 2 days. Each web server have MTTF of 350 days and MTTR of 1 day. Compute the probability of no failures in a t = 7 days period for both (A) and (B) as well as for the whole system (A+B).

Solution:

$R_A(7) = 1-7/1000 = 0.993$
$R_B(7) = 1- (7/350)^3 = 0.999992$ (parallel)
$R_{A+B}(7) = R_A(7) \times R_B(7) = 0.992992$ (serial)

3. **Exercise:** Availability

A load balancer(A) is connected in series with a group(B) of three parallel web servers. The load balancer have MTTF of 1000 days and MTTR of 2 days. Each web server (WS) have MTTF of 800 days and MTTR of 1 day.
Compute the availability of the whole system.

Solution:

$A = MTTF / (MTTF+MTTR)$

We can compute the availability through availability block formulas:

$A_{Serial} = A_1A_2…A_n$ $A_{Parallel} = 1-\prod (1-A_i)^n$

$MTTF_A = 1000$; $MTTF_A= 2$ $A_A=1000/(1000+2)=0.998$

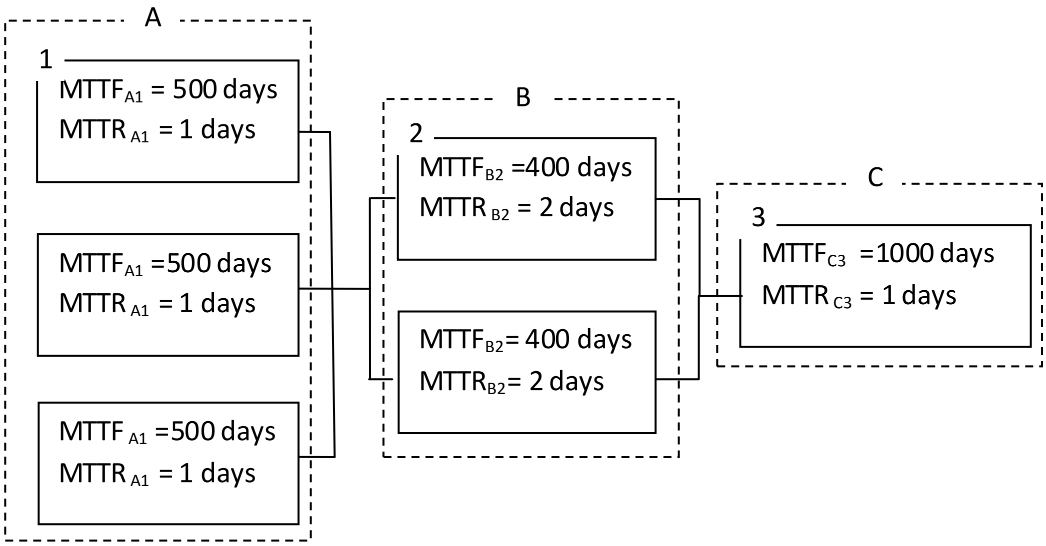
$MTTF_{ws} = 800$; $MTTR_{ws}= 1$; $A_{WS}= 800/(800+1)=0.99875$

$A_B = 1-(1-A_{WS})^3 = 0.99999$

$A_{A+B} = A_A\ A_B = 0.998\ 0.999999 = 0.99799$

4. **Exercise:** availability

Consider the following structure where MTTF and MTTR of the components are shown. Compute the availability of each component and of the whole infrastructure.



Solution:

We can compute the availability through availability block formulas:

$A_{Serial} = A_1A_2…A_n$ $A_{Parallel} = 1-\prod (1-A_i)^n$

Therefore:

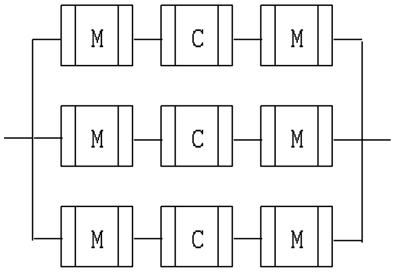
$A_A = 1-(1-A_{A1})^3 = 1-(1-500/(500+1))^3 = 0.999999$ $A_B = 1-(1-A_{B2})^2 = 1-(1-400/(400+2))^2 = 0.999975$

$A_C = A_{C3} = 1000/(1000+1) = 0.999$

$A_{A+B+C} = A_A\,A_B\,A_C = 0.998974$

5. **Exercise**: Availability

Consider a communication system, with three trunk in parallel, each one composed by three components in series: two modem and a cable. The system is represented in the following figure:



Let us consider that for the modem: $MTTF_M = 999$ days; $MTTR_M = 1$ days
And for the cable: $MTTF_C = 90$ days; $MTTR_C =$ 10 days

1) Compute the availability of the modem, of the cable, of the trunk and of the entire system.
2) How many trunks should be used to have an availability of the entire system of 99,98% ?
3) If we have a single trunk, with the same modems and a repair time for the cable $MTTR_C = 1$, which should be the $MTTF_C$ to obtain an availability of the entire system of 99,5 ?
4) In the context of exercise 3), would it be possible to have an availability of the trunk of 99,9 ?

Solution

1) $A_M = 999/(999+1) = 0.999$; $A_C = 90/(90+10) = 0.9$; $A_T = 0.999*0.9*0.999 = 0.898201$
   $A_S = 1-(1- A_T)^3 = 0.998945$
2) $A_S = 1-(1- A_T)^n$ $(1- A_T)^n= 1- A$ $n=\ln(1-A)/\ln(1- A_T) = \ln(0.002)/\ln(0.101799)=3.72$ -> n=4
3) $A_T = A_M A_C A_M$ $A_C = 0.995 / A_M^2 = 0.996993$
   $A_C = MTTF_C / (MTTF_{C+} MTTR_C)$ $MTTR_C = 1$ sec therefore $MTTF_C = A_C / (1- A_C) = 331.56$ days
4) It will not be possible, because even if $MTTF_C = \infty$ (thus $A_C = 1$), we will have at most
   $A_T = 0.999*1*0.999 = 0.998001 < 0.999$