Exercises on Dependability
(courtesy of Marco Gribaudo)

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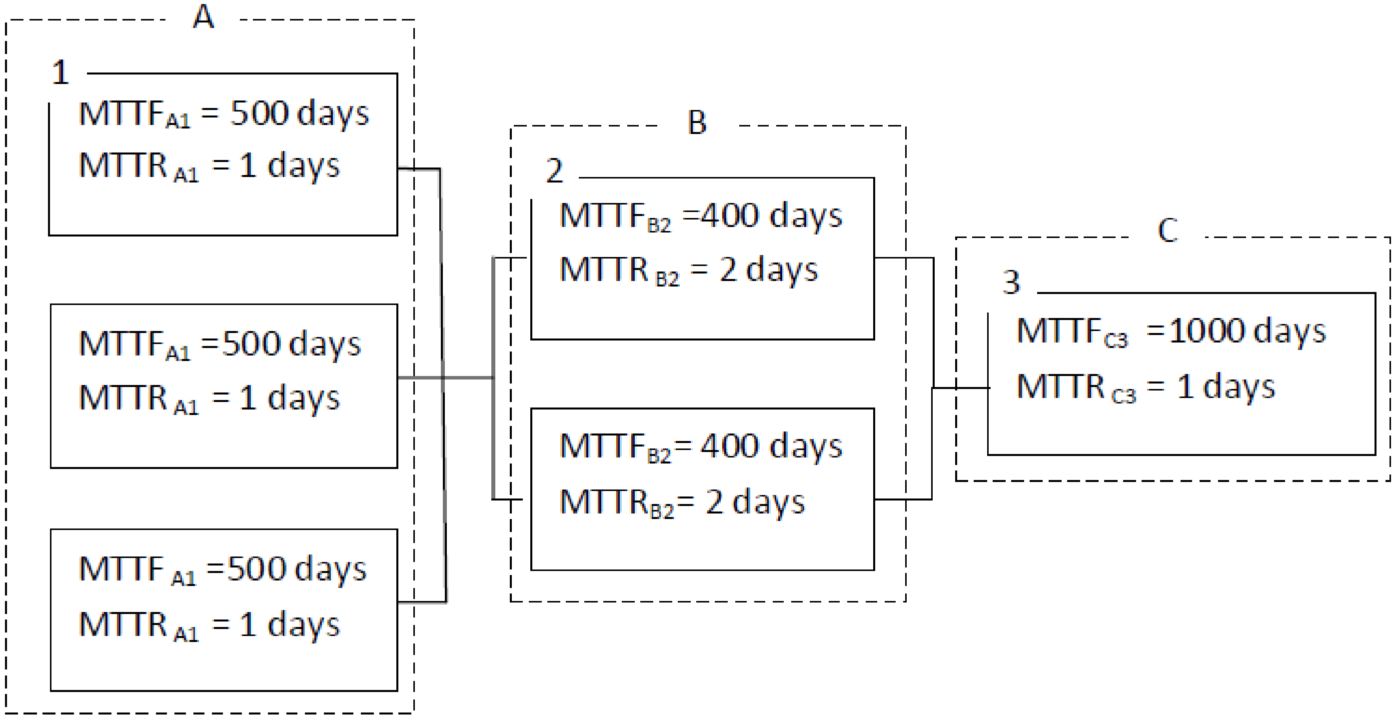
Exercise 1: Availability

- Consider the following structure where MTTF and MTTR of the components are shown. Compute the availability of each component and of the whole infrastructure.



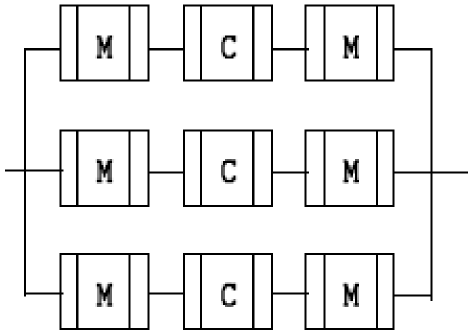
Exercise 1: Availability

- We can compute the availability through availability block formulas:
- $A_{Serial} = A_1 A_2 \ldots A_n$
- $A_{Parallel} = 1 - \Pi\ (1-A_i)^n$

Therefore:

- $A_A = 1-(1-A_{A1})^3 = 1-(1-500/(500+1))^3 = 0.999999$
- $A_B = 1-(1-A_{B2})^2 = 1-(1-400/(400+2))^2 = 0.999975$
- $A_C = A_{C3} = 1000/(1000+1) = 0.999$
- $A_{A+B+C} = A_A\ A_B\ A_C = 0.998974$

Exercise 2: Availability part 1



Consider a communication system, with three trunk in parallel, each one composed of three components in series: two modem and a cable. The system is represented in the above figure.

The values for the modem are: $MTTF_M$ = 999 days; $MTTR_M$ = 1 days and for the cable: $MTTF_C$ = 90 days; $MTTR_C$ = 10 days

1) Compute the availability of the modem, of the cable, of the trunk and of the entire system.

Exercise 2: Availability part 1

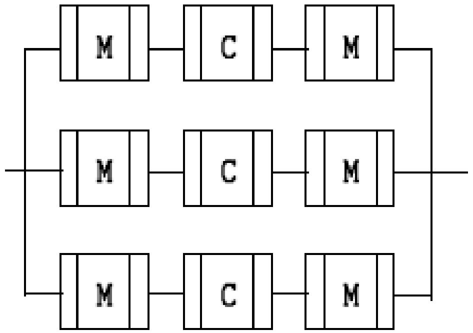
1) $A_M = 999/(999+1) = 0.999$;

$A_C = 90/(90+10) = 0.9$;

$A_T = 0.999*0.9*0.999 = 0.898201$

$A_S = 1-(1- A_T)^3 = 0.998945$

Exercise 2: Availability part 2



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2) How many trunks should be used to have an availability of the entire system of 99,98% ?

Exercise 2: Availability part 2

2) $A = 1-(1- A_T)^n$

$A = 0.9998$

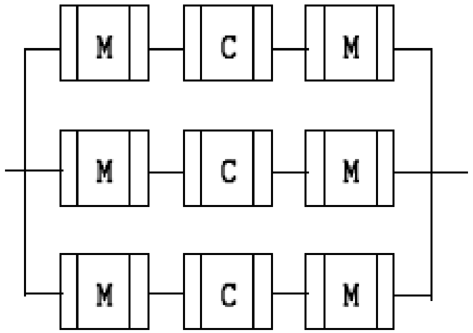
$A_T = 0.898201$

We want to determine *n*

$$(1- A_T)^n = (1- A)$$

$$n = \ln(1-A)/\ln(1- A_T) = \ln(0.0002) / \ln(0.101799) = 3.72 \to n=4$$

Exercise 2: Availability part 3



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The values for the modem are: $MTTF_M$ = 999 days; $MTTR_M$ = 1 days and for the cable: $MTTF_C$ = 90 days; $MTTR_C$ = 10 days

3) If we have a single trunk, with the same modems and a repair time for the cable $MTTR_C$ = 1, which should be the $MTTF_C$ to obtain an availability of the entire system of 99,5 ?

Exercise 2: Availability part 3

3) $A_T = A_M \, A_C \, A_M = A^2{}_M \, A_C$

$MTTR_C = 1$

$A_M = 0.999$

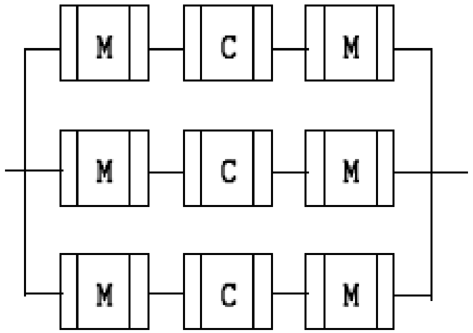
$A_T = 0.995$

$A_C = 0.995 \, / \, A^2{}_M = 0.996993$

$A_C = MTTF_C \, / \, (MTTF_C + MTTR_C)$

therefore $MTTF_C = A_C \, / \, (1- A_C) =$ 331.56 days

Exercise 2: Availability part 4



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The values for the modem are: $MTTF_M$ = 999 days; $MTTR_M$ = 1 days and for the cable: $MTTF_C$ = 90 days; $MTTR_C$ = 10 days

3) If we have a single trunk, with the same modems and a repair time for the cable MTTRC = 1, which should be the $MTTF_C$ to obtain an availability of the entire system of 99,5 ?

4) In the context of exercise 3), would it be possible to have an availability of the trunk of 99,9 ?

Exercise 2: Availability part 4

4) It would not be possible.

Even if the cable never fails ($MTTF_C = \infty$) and we would have full availability ($A_C = 1$), knowing that $A_M = 0.999$, we would have at most:

$$A_T = 0.999 * 1 * 0.999 = 0.998001 < 0.999$$

Exercise 3: A cheap DIMM memory module

A cheap DIMM memory module is characterized by an availability of 99.9% and an MTTR = 1 day.

1. Which is the MTTF of the considered memory module?

Exercise 3: A cheap DIMM memory module

1. $A = MTTF/(MTTF + MTTR)$;

$MTTF = A/(1 - A) \cdot MTTR$;

$MTTF = 0.999/0.001 \cdot 1 = 999$ days.

Exercise 3: A cheap DIMM memory module

A cheap DIMM memory module is characterized by an availability of 99.9% and an MTTR = 1 day.

2. Which are the availability and the MTTF of a system that uses 3 of such memory modules in series?

Exercise 3: A cheap DIMM memory module

2. The components are in series, since it is enough that one breaks for the system to be not operational.

In this case $A = 0.999^3 \approx 0.997 = 99.7\%$.

MTTF = 999/3 = 333 days.

Exercise 3: A cheap DIMM memory module

A cheap DIMM memory module is characterized by an availability of 99.9% and an MTTR = 1 day.

3. Which is the reliability of the system (with three memory banks) after 4 days?

And after 500 days?

Exercise 3: A cheap DIMM memory module

- In the case of $F(t)$ it is exponential, an approximation is available, if t is much smaller than the $MTTF$:

$$F(t) = P(X \le t) = 1 - e^{-\frac{t}{MTTF}}$$

$$\cong \frac{t}{MTTF} \qquad \left(\text{for} \quad \frac{t}{MTTF} << 1\right)$$

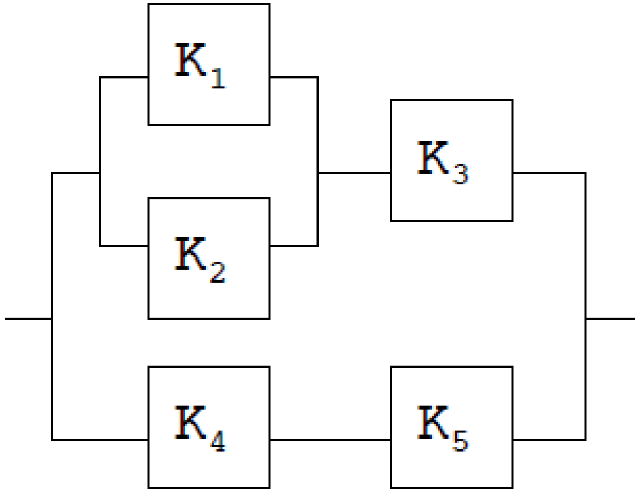
3. Since 4 << 333, we have R(4) = 1 – 4/333 = 1 – 0.012 = 0.988.

We cannot use the approximation for R(500) since 500 > 333.

In this case R(500) = $e^{-500/333}$ = 0.223.

Exercise 4: A system composed of five components

- A system consists of five modules, we assume that the reliability of each module follows an exponential distribution with an identical failure rate of $\lambda = 10^{-4}\ h^{-1}$.
- Calculate the reliability of the system R_S for an up time of t = 1000 hours, if the modules' dependencies are as indicated in the block diagram below.



Exercise 4: A system composed of five components

For a generic module *m,* we have:

$R_m(t) = e^{-\lambda t}$ $R_m = e^{(1/10000)1000} = 0.904837418$

Top Branch Reliability R_t (k_1 and k_2 in parallel, k_3 in series)=

$(1 - (1 - R_m)^2)R_m = 2R^2{}_m - R^3{}_m = 0.896643286$

Bottom Branch Reliability R_b (k_4 and k_5 in series) = $R^2{}_m = 0.818730753$

- The branches are connected in parallel, so we have:
- $R_S = 1 - (1 - R_t)(1 - R_b) = 0.981264606$

Exercise 5: A generic component

- Consider a generic component D with $MTTF_D = 100$. Compute the minimum integer value of **t** such that the failure probability of the component is greater than 0.6.

Exercise 5: A generic component

$$1 - e^{-t/100} \geq 0.6$$

$$0.4 \geq e^{-t/100}$$

$$\ln 0.4 \geq -t/100$$

$$t \geq -100 \ln 0.4$$

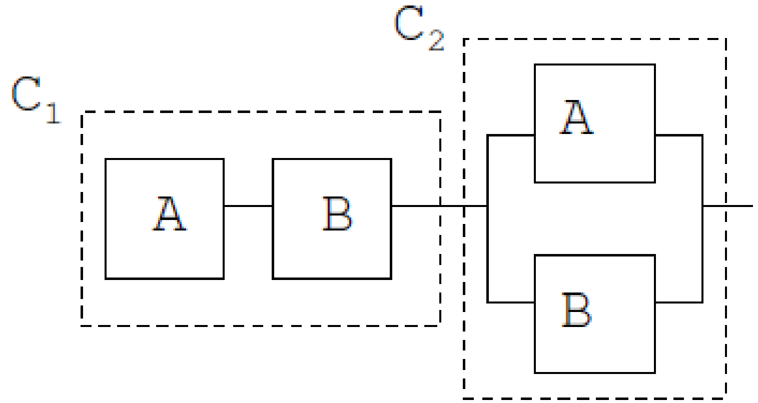
$$t \geq 92$$

Exercise 6: A generic component part 1

In the following questions we will assume that both failure and repair events follow exponential distributions.

1) Consider two components A and B in a serial configuration as in block C_1 of the Figure. They have the following characteristics: $MTTF_A$ = 500 days, $MTTR_A$ = 5 days;

   $MTTF_B$ = 200 days, $MTTR_B$ = 2 days.



Compute the reliability of sub-system C1 at

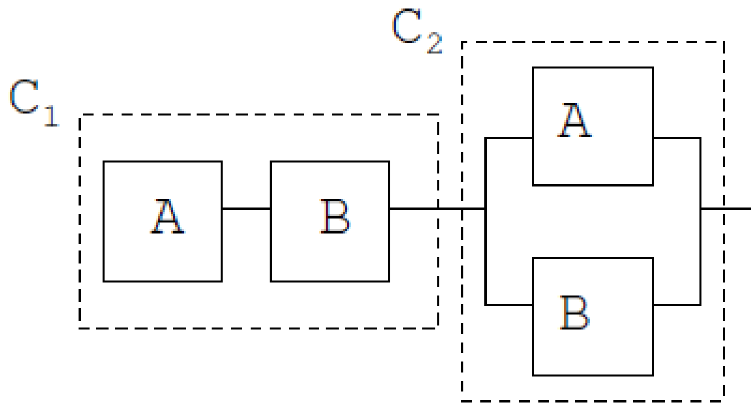
t = 50 days

Exercise 6: A generic component part 1

$R_A(50) = e^{-50/500} = 0.9048$

$R_B(50) = e^{-50/200} = 0.7788$

$R_C = R_A\, R_B = 0.7046$

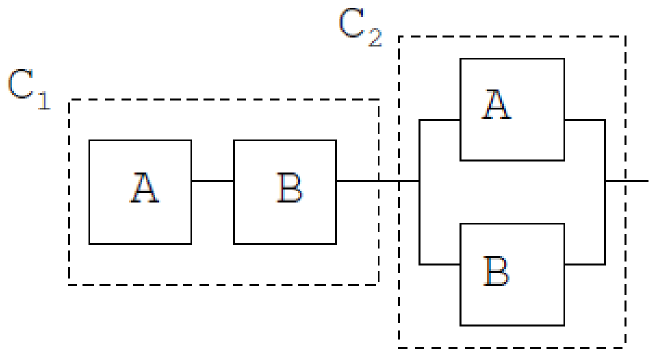


Exercise 6: A generic component part 2

1) Consider two components A and B in a serial configuration as in block C_1 of the Figure. They have the following characteristics: $MTTF_A$ = 500 days, $MTTR_A$ = 5 days;

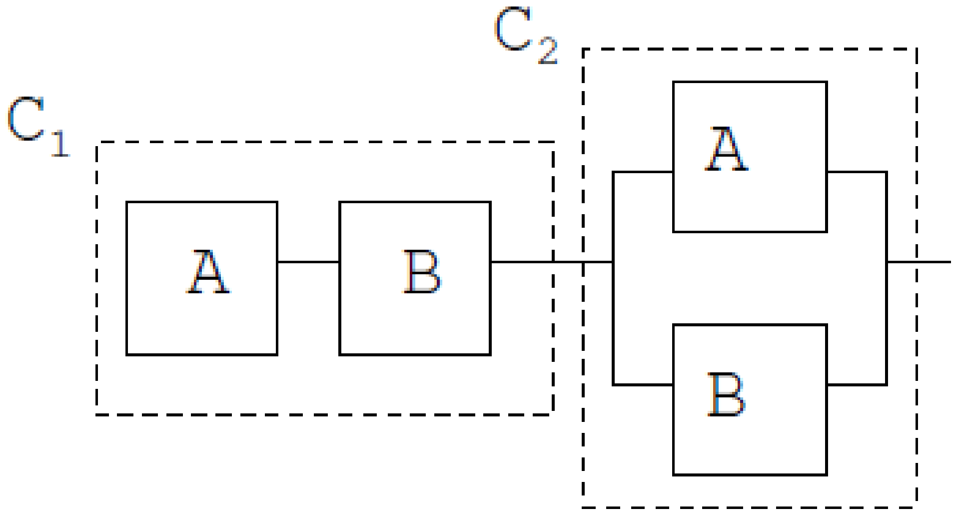
   $MTTF_B$ = 200 days, $MTTR_B$ = 2 days.

2) In the previous configuration is required to change component B in order to achieve an MTTF of block C_1 = 187.5, computed without repair. Calculate the required update.



Exercise 6: A generic component part 2

- $MTTF_{C1} = 187.5 = 1/(1/500+1/MTTF_B)$
- $500MTTF_B/(MTTF_B + 500) = 187.5$
- $MTTF_B = 300$



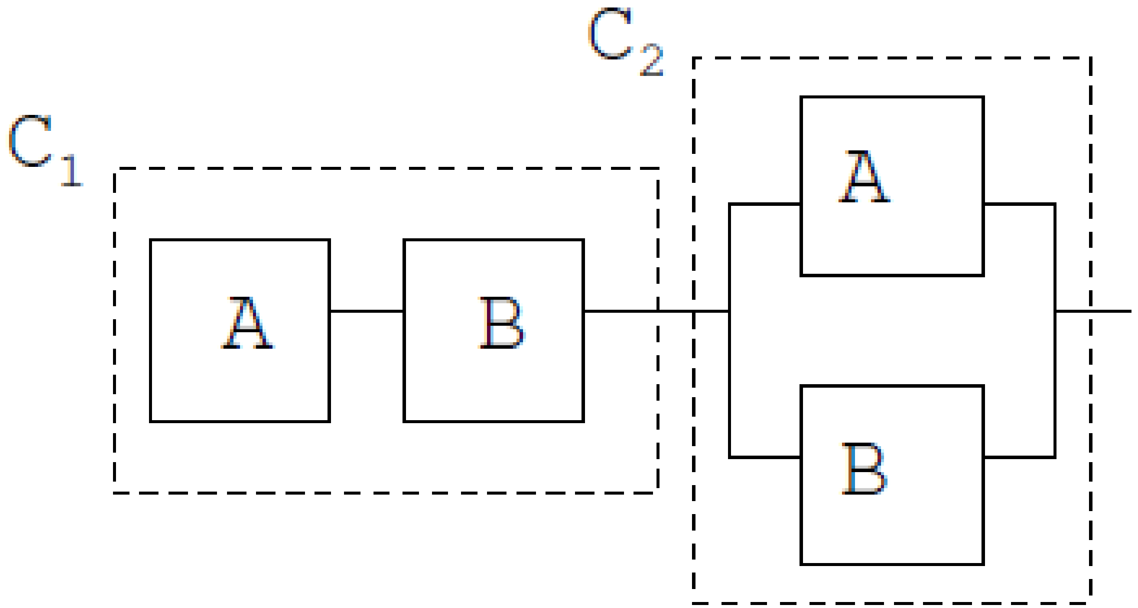
Exercise 6: A generic component part 3

3) Consider the same components of question 1 in a parallel configuration as in block C_2:

$MTTF_A$ = 500 days, $MTTR_A$ = 5 days;

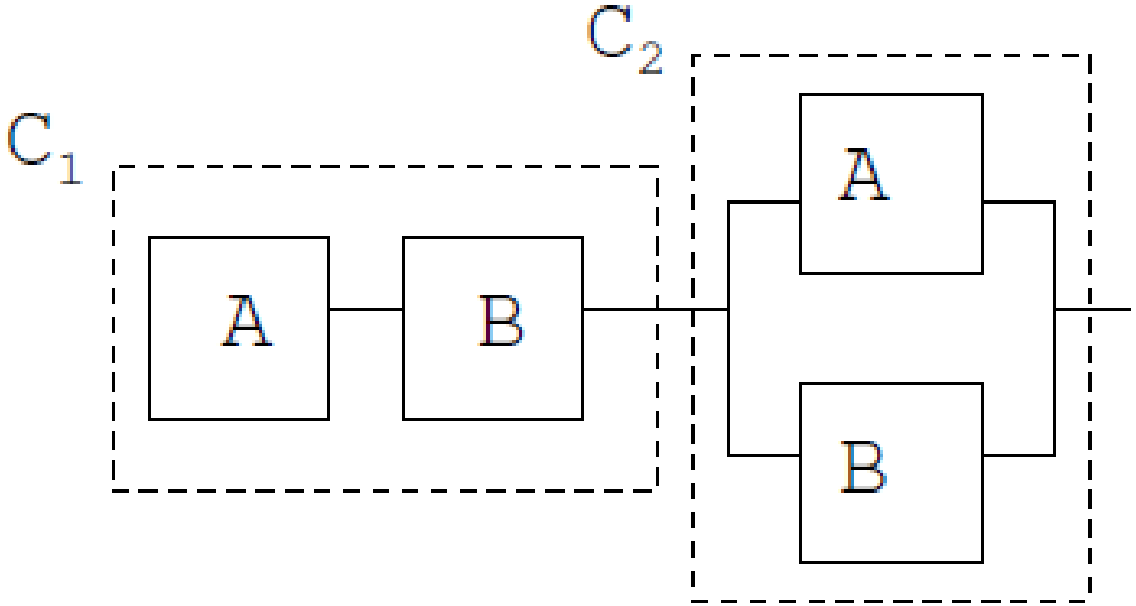
$MTTF_B$ = 200 days, $MTTR_B$ = 2 days

Calculate the MTTF of C_2 computed without repair.



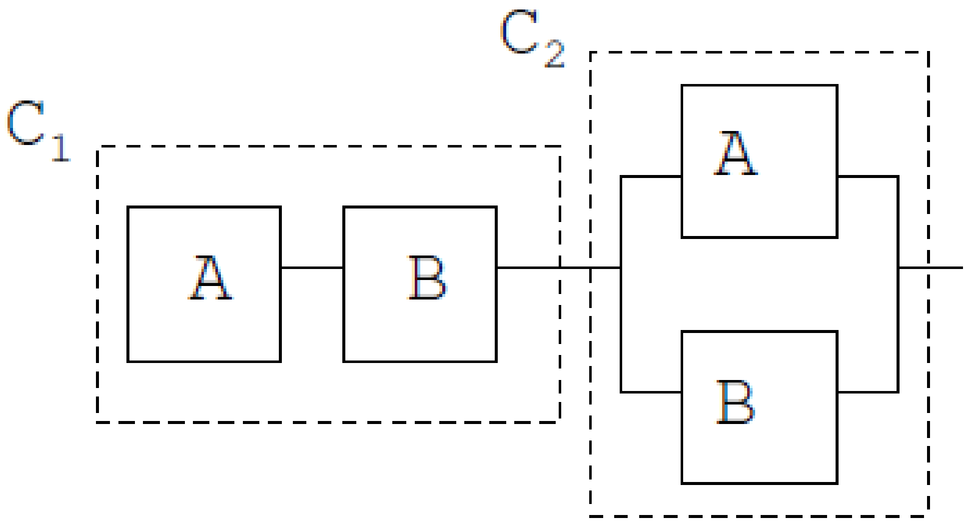
Exercise 6: A generic component part 3

$MTTF_{C2} = MTTF_A + MTTF_B - MTTF_A * MTTF_B/(MTTF_A + MTTF_B) =$ 557.1428



Exercise 6: A generic component part 4

4) Compute the availability of block C_2

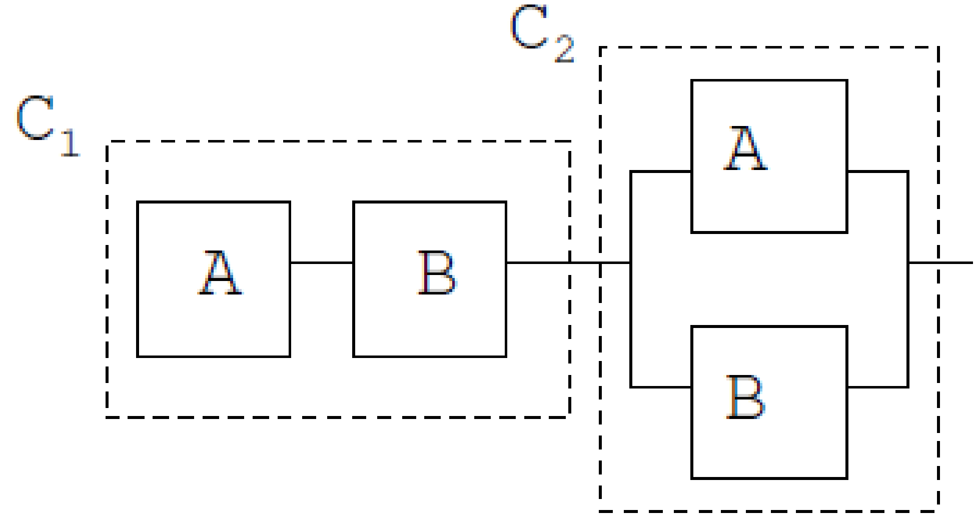


Exercise 6: A generic component part 4

$A_A = MTTF_A/(MTTF_A + MTTR_A) = 500/(500 + 5) = 0.9900$

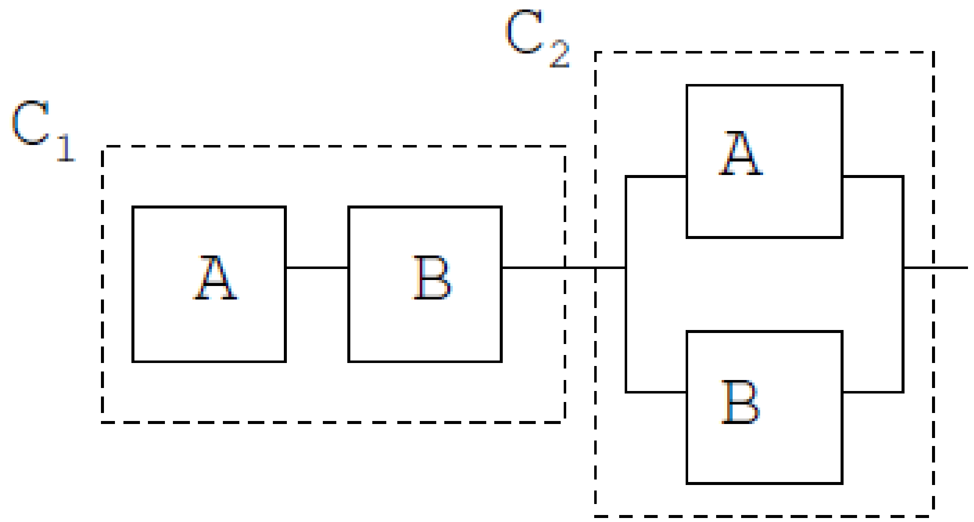
$A_B = MTTF_B/(MTTF_B + MTTR_B) = 200/(200 + 2) = 0.9900$

$A_{A||B} = 1 – (1 – A_A)^2 = 0.9999$



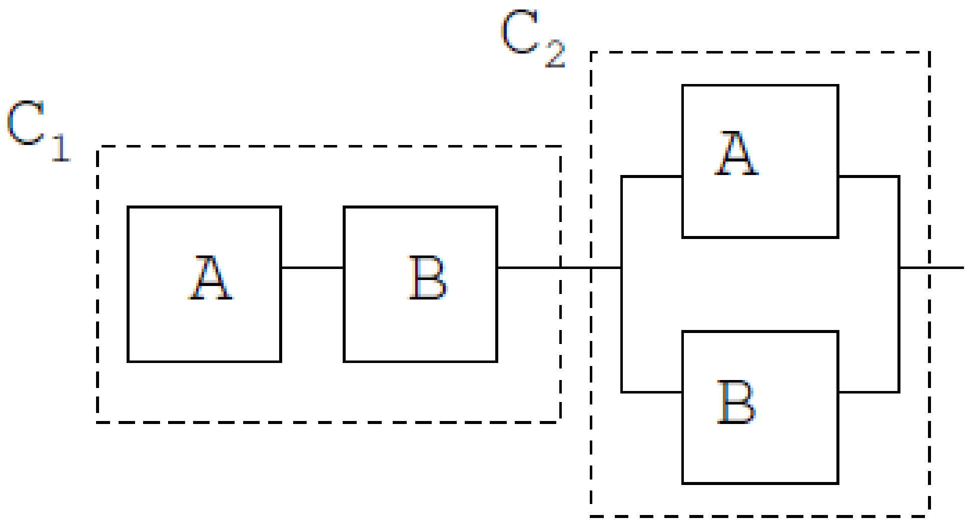
Exercise 6: A generic component part 5

Compute the availability of the whole system.



Exercise 6: A generic component part 5

$A_{Sys} = 0.9999 * 0.992 = 0.98$



Exercise 7: Intranet Part 1

An Intranet consists of a web server WS, an application server AS and a storage server SS, all connected in serial. Let us consider the following values:

For the WS: MTTF = 600 days; MTTR = 2 days.

For the AS: MTTF = 300 days; MTTR = 1 days

For the SS: MTTF = 200 days; MTTR = 4 days.

(a) Compute the probability of no failures in a t = 5 days period for each component and for the whole system.

Exercise 7: Intranet Part 1

$R_{WS} = e^{-5/600} = 0.991701$; $R_{AS} = e^{-5/300} = 0.983471$;

$R_{SS} = e^{-5/200} = 0.97531$ $R_{sys} = R_{WS}R_{AS}R_{DS} = 0.\ 951229$

Exercise 7: Intranet Part 2

An Intranet consists of a web server WS, an application server AS and a storage server SS, all connected in serial. Let us consider the following values:

For the WS: MTTF = 600 days; MTTR = 2 days.

For the AS: MTTF = 300 days; MTTR = 1 days

For the SS: MTTF = 200 days; MTTR = 4 days.

(b) A storage server with the same characteristics was added in **parallel to the existing one**. Compute the exact steady state availability of the resulting entire system;

Exercise 7: Intranet Part 2

A_{WS} = 600/(600+2) = 0.996678; A_{AS} = 300/(300+1) = 0.996678;

A_{SS} = 200/(200+4) = 0.980392;

$A_{Paral} = 1 - (1 - A_{SS})^2 = 0.999616$;

$A_{sys} = A_{WS}A_{AS}A_{Paral} = 0.992987$

Exercise 7: Intranet Part 3

An Intranet consists of a web server WS, an application server AS and a storage server SS, all connected in serial. Let us consider the following values:

For the WS: MTTF = 600 days; MTTR = 2 days.

For the AS: MTTF = 300 days; MTTR = 1 days

For the SS: MTTF = 200 days; MTTR = 4 days.

(c) Adding further storage servers in parallel, it is possible to reach an availability of the entire system of 0.999 ? Motivate your answer.

Exercise 7: Intranet Part 3

Even if we assume $A_{Paral} = 1$, we have $A_{sys} = A_{WS}A_{AS} = 0.993366$.
Therefore it is not possible to reach an availability of 0.999.