Exercises Enterprise Digital Infrastructures 2014-2015

1. Consider 12 identical HDD, each with a capacity of $1TB$. All disks have an identical $MTTF = 1000days$ and $MTTR = 2days$. State the total capacity, and the $MTTDL$ (mean time to data loss), if the disks are connected in:

   (a) RAID 0

   (b) RAID 1+0 (6 groups of 2 disks)

   (c) RAID 0+1 (2 groups of 6 disks)

   (d) RAID 5

   (e) RAID 6

   SOLUTIONS

   See slides in "L02 - RAIDs.pdf".

   RAID 0: striping - distribute data across several disks, improve performance, it does not affect availability of RAID
   RAID 1: mirroring - replicate data on disks, typically two copies of the data (otherwise the cost is too high), it improve the RAID availability
   RAID X+Y: first apply technique X to each disk of a group, then apply technique Y to all groups considering them as single disks.

   (a) No fault tolerance, a fault in any of the 12 disk (see the minimum time between several concurrent faults "L05 - RAID availability" pp 11-14) causes a failure in the whole RAID.
   $MTTDL = MTTF/12 = 1000/12 = 83.33day$
   All disk are used to store data, thus $C = 12TB$.
   (b) Mirroring to each disk of the group, data stripped over the 6 groups. RAID failure due to: a failure in any disk k followed by a failure in the mirror of disk k before repair of disk k.
   $MTTDL = MTTF^2/(12MTTR) = 1000^2/(12 * 2) = 41666.66day$
   Half of the disks are used to store a copy of the data, thus $C = 6TB$
   (c) Data are stripped over the 6 disks of a group, the whole group is mirrored. RAID failure due to: a failure in any disk k followed by a failure in any of the 6 disks of the mirror group before repair of disk k.
   $MTTDL = 2MTTF^2/(12^2MTTR) = 2 * 1000^2/(12^2 * 2) = 6944.44$
   As in previous case, half of the disks are used to store a copy of the data, thus $C = 6TB$.
   (d) A parity code is uniformly distributed over all disks. RAID failure due to: a failure in any disk k followed by a failure on any of the remaining $12 - 1$ disks before repair of disk k.
   $MTTDL = MTTF^2/((12 * 11)MTTR) = 1000^2/(12 * 11 * 2) = 3787.87days$
   One disk is used to store parity code, thus $C = 11TB$.
   (e) Two parity codes are uniformly distributed over all disks. RAID failure due to: a failure in any disk k followed by a failure on any of the remaining $12-1$ followed by another failure on any of the remaining $12 - 2$ disks before repair of the faulty disks. We assume that when two disks are down a simultaneous repair of the two disks is possible, thus the mean time to repair any of the two faulty disks (i.e the minimum of the two disks) is $MTTR/2$.

$MTTDL = 2MTTF^3/(12 * 11 * 10 * MTTR^2) = 2 * 1000^3/(12 * 11 * 10 * 4) = 378787.87$
Two disks are used to store parity code, thus $C = 10TB$.

2. A HDD spins at $9000RPM$, has a seek time of $6ms$, its transfer rate is $1Gb/s$ and data is divided in blocks of $4KB$. Compute:

   (a) the mean service time of the disk

   (b) the time required to transfer a file of $64MB$ (if no locality is considered).

   (c) the time required to transfer the same file of $64MB$, with a locality of 95% (that is, for which a seek is required only for 5% of the blocks).

   SOLUTIONS:
   (a) We compute the mean time needed to access a single block. Assuming the time of the controller negligible, the mean time is the sum of seek, latency and transfer times.
   $mean\ latency = (60s/min)/(2 * 9000rpm) = 0.000333s = 3.333ms$ (time for 1/2 round)
   $transfer\ time = (4KB)/(1024^2KB/s) = 0.000003815s = 0.003ms$
   $T_{4KB} = 6 + 3.333 + 0.003ms = 9.336ms$
   (b) Given the previously computed mean service time for a single block, the time to access a file of $64MB$ is given by the number of block needed for a $64MB$ multiplied with the time needed for a single block:
   $T_{64MB} = (64 * 1024KB/4KB) * T_{4KB} = 152961ms = 152.961s$
   (c) Considering the effect of locality on the mean time needed to access a single block.
   $T'_{4KB} = (6 + 3.333) * 0.05 + 0.003ms = 0.46965ms$
   The time to access a file of $64MB$ with locality is:
   $T_{64MB} = (64 * 1024KB/4KB) * T'_{4KB} = 7694.7456ms = 7.694s$

3. A company buys a storage server that consists of 4 disks with the same characteristics and capacity of 1 Terabyte each. The MTTF of each disk is 1000 days. The objective is to implement a storage server having a Mean Time To Data Loss (MTTDL) of 10000 days.

   (a) If the disks are interconnected according to a RAID5 architecture, which is the value of Mean Time To Repair (MTTR) that guarantee the achievement of the objective MTTDL ?

   (b) If the disks are interconnected according to a RAID0 architecture, which is the value of Mean Time To Repair (MTTR) that guarantee the achievement of the objective MTTDL ?

   (c) If the disks are interconnected according to a RAID0+1 architecture, (consider two groups of two disks) which is the value of Mean Time To Repair (MTTR) that guarantee the achievement of the objective MTTDL ?

   SOLUTION:
   We have a target $MTTDL$ and we want to compute the value of $MTTR$ that satisfy the objective. Thus, the formulas are just inverted to compute the $MTTR$.

   a) RAID 5
   $MTTR = MTTF^2 * /(n(n-1)MTTDL) = 1000^2/(4 * 3 * 10000) = 8.3333days$
   b) RAID 0: only striping, no fault tolerance. The $MTTDL$ is equal to the $MTTF$ of the 4 disks $MTTDL = MTTF/4 = 1000/4 = 250$. It is impossible to satisfy the desired target

$MTTDL$.
c) 0+1: first stripe then mirror.
$MTTR = MTTF^2 * /(n(n/2)MTTDL) = 1000^2/(4 * 2 * 10000) = 12.5days$

4. The storage server of an enterprise consists of 10 disks with the same MTTF and MTTR.

   (a) The $MTTDL = 100days$ is obtained when the 10 disks are organized in RAID0 architecture. Which is the MTTF of the disks?

   (b) When the same 10 disks are organized according to a RAID 1+0 architecture (5 groups of 2 disks) the MTTDL is 6250 days. Which is the MTTR of the disks in this case?

   (c) Compute the MTTDL of the same 10 disks when connected according to a RAID5 architecture

   (d) If the global capacity of each disk is 1 Terabyte , which is the total storage available for the data (without redundancy) of the previous architectures: RAID0, RAID1+0, RAID5?

   SOLUTIONS:
   (a) $MTTDL = MTTF/N \quad MTTF = MTTDL * N = 1000days$
   (b) $MTTDL = MTTF^2/(MTTR * N) \quad 6250 = 1000000/(10 * MTTR) \quad MTTR = 100000/6250 = 16days$
   (c) $MTTDL = MTTF^2/(N * (N-1) * MTTR) = 1000000/(10 * 9 * 16) = 694,444days$
   (d) $10TB, 5TB, 9TB$