1. A coffee company requires its clients to first get a ticket with a progressive number, then numbers are called by one of three free counters, and finally costumers are offered a free coffee. We imagine that the three counters operators are all with identical service speed, and that only one customers out of two accept the free coffee, which is served by another single operator. The ticket machine can provide 12 tickets per minute, the mean service time for a costumer is 2 minutes, and the time required to prepare the free coffee is 30 seconds.

   (a) Draw a visual representation of the system.

   (b) Compute the visits and the demands of all the service centers.

   (c) Identify the bottleneck.

   (d) Which is the maximum number of costumers per minutes that the system is able to serve?

   (e) How many operators at counters and at the free coffee point should be present to make the ticket machine the bottleneck?

   (f) If costumers arrive at a rate of 1 per minute, the mean service time is 3 minutes at the counters. Which is the mean number of costumers in the shop in this case?

   SOLUTIONS:

   (a)

   (b) $D_{ticket} = \frac{1}{X_{ticket}}; D_{counter} = \frac{1}{3}120 = 40sec; D_{free} = \frac{1}{2}30 = 15sec$

   (c) Bottleneck is counter

   (d) $X_{max} = \frac{1}{D_{max}} = \frac{1}{40} = 0.025rec/sec = 1.5req/min$

   (e) Bisogna far scendere la D di ogni stazione sotto la D_{ticket}, aumentando il numero di stazioni. 6 counter e 4 free.

   (f) $N = XR$ con R somma dei tempi delle singole stazioni. $X_{max} = \frac{1}{60} = 0.01666$; $R = 60 + 60 + 15 = 135$; $N = 2.25$

3. A storage server, has two power supply units (PSU), two network adapters, and four disks in RAID 1+0 (two groups of two disks each). The parameters of the components are the following: $MTTF_{PSU} = 998$ days, $MTTR_{PSU} = 2$ days, $MTTF_{net} = 4995$ days, $MTTR_{net} = 5$ days, $MTTF_{disk} = 990$ days, $MTTR_{disk} = 10$ days.

   - Draw the reliability block diagram of the system
   - Compute the MTTDL of the RAID
   - Compute the availability of the system

   SOLUTIONS
   1)
   2) $MTTDL_{RAID} = MTTF_{disk}^2/(4 * MTTR_{disk}) = 990^2/(4 * 10) = 24502.5\ days$
   3) Assuming $A_{RAID} = MTTDL_{RAID}/(MTTDL_{RAID} + MTTR_{disk}) =$
   $A_{PSU} = MTTF_{PSU}/(MTTF_{PSU} + MTTR_{PSU}) =$
   $A_{net} = MTTF_{net}/(MTTF_{net} + MTTR_{net}) =$
   $A_{Paral\ PSU} = 1 - (1 - A_{PSU})^2 =$
   $A_{Paral\ net} = 1 - (1 - A_{net})^2 =$
   $A_{Sys} = A_{RAID} * A_{Paral\ PSU} * A_{Paral\ net} =$

1. A medium size company wants to evaluate the performances provided to their customers. The intranet of the company includes a web server, an application server and a database, called A, B and C, respectively. The intranet is represented as a closed model. In order to evaluate the network performance, a resources monitoring procedure was set-up and the results are the following:

2. For the system with all servers the following data were detected for an interval time of $T = 2h.$ and $N = 10jobs$:
   C number of interactions completed by the system: $3600jobs$
   C_A number of operations completed by server A: $5400jobs$
   B_A busy time of server A: $7200s$
   B_C busy time of server C: $1800s$
   U_B utilization of server B: 0.625
   S_B service time of server $B : 0.25s$
   S_C service time of server $C : 0.1s$
   Z think time: $4s$

   For the complete system (i.e. with all servers) compute:

   (a) the service demand of each station.

   (b) the utilization of each server

   (c) the visits of each server

   (d) the average network response time.

   (e) Moreover, write the equations of the asymptotes of system throughput and compute for which value of N the system switches from light to heavy load.

   SOLUTIONS:

   (a) $D_k = V_k S_k = \frac{C_k}{C}\frac{B_k}{C_k} = \frac{B_k}{C}$. $D_A = 7200/3600 = 2$s, $D_C = 1800/3600 = 0.5$s. $B_B = U_B \cdot T = 0.625 \cdot 7200$. $D_B = 0.625 \cdot 7200/3600 = 1.25$s.
   (b) $U_k = B_k/T$. $U_A = 7200/7200 = 1$, $U_C = 1800/7200 = 0.25$, $U_B = 0.625$.
   (c) $v_A = 5400/3600 = 1.5, v_B = D_B/S_B = 1.25/0.25 = 5, v_C = 0.5/0.1 = 5$
   (d) $R = N/X - Z$. $X = C/T = 3600/7200 = 0.5$j/s. $Z = 10/0.5 - 4 = 16$s.
   (e) $\frac{N}{3.75 \cdot N+4} \leq X \leq \min\left(0.5, \frac{N}{3.75}\right)$. $N* = (D+Z)/D_{Max} = (2+1.25+0.5+4)/2 = 3.875$

3. According to a consolidation policy, the 10 local disks of the 10 servers of a intranet will be concentrated in a centralized storage server NAS. The capacity of the 10 disks is utilized at 60% (mean value).

   (a) in the conditions above described, which are, among the followings, the RAID configurations of the NAS that can be implemented with the 10 disks: RAID 0, RAID 0+1, RAID 5? Motivate the answer.

   (b) for a single disk the MTTF is 300 days and the MTTR is 1 day. Find which one among the possible configurations (identified in the previous question) has the maximum MTTDL and compute its value.

   (c) identify which configuration (among those identified in question (a)) has the best performance (minimum response time), motivate the answer and compute the MTTDL with the parameters above described

   SOLUTIONS:
   a) RAID 0+1 cannot be used since it requires at least 50% of the space for redundancy, while only 40% is available.
   b) RAID 5 is the most reliable with $MTTDL = 300^2/(10 \cdot 9 \cdot 1) = 1000$ days
   c) RAID 0 is the fastest with $MTTDL = 300/10 = 30$ days

1. Consider the reliability block diagram in Figure 1. Assume that the components have the following characteristics:
A: $MTTF_A = 150\ days$
B: $MTTF_B = 100\ days\ MTTR_B = 5\ days.$
and that both the time to failure and to repair are exponentially distributed.

Assuming the system WITHOUT repair compute:

(a) the probability of no failures in a $t = 30$ days period for each component and for the whole system;

(b) the $MTTF$ of the system without component A.

Assuming the system WITH repair compute:

(c) the availability of the system without component A.

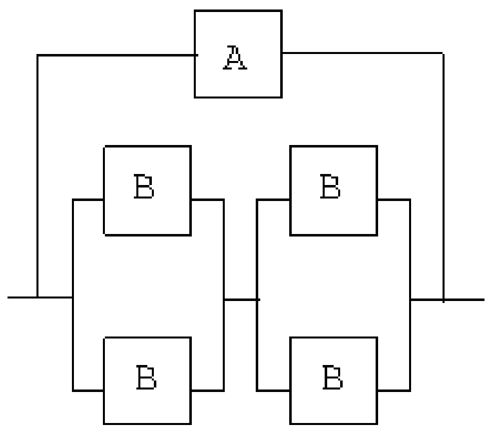

Figure 1: RBD

SOLUTIONS:
(a) $R_A(30) = e^{-t/MTTF} = e^{-30/150} = 0.818730753$
$R_B(30) = e^{-t/MTTF} = e^{-30/100} = 0.740818221$
$R_C(30) = R_{B||B} = 1 - (1 - R_B(30))^2 = 1 - (1 - 0.7408182)^2 = 0.932824805$
$R_D(30) = R_{C-C} = R_C(30) \cdot R_C(30) = 0.870162118$
$R_{sys}(30) = 1-(1-R_A(30))\cdot(1-R_D(30)) = 1-(1-0.818730753)\cdot(1-0.870162118) =$
$= 0,976464385$

(b) $MTTF_C = MTTF_{B||B} = MTTF_B(1 + 1/2) = 150$
$MTTF_{C-C} = MTTF_C/2 = 75$

(c) $A_B = MTTF_B/(MTTF_B + MTTR_B) = 100/(100 + 5) = 0.9523$
$A_C = A_{B||B} = 1 - (1 - A_B)^2 = 1 - (1 - 0.9523)^2 = 0.99772471$
$A_{sys} = A_C^2 = 0.99772471^2 = 0.995454597$

2. A secure system is composed of a dispatcher (D), a token server T, and a computation server (P) elaborating critical process, and it is used by $N = 10$ users. The system throughput is $X = 0.366\ job/s$. The utilization of the dispatcher is $U_D = 0.7686$, and its service time is $S_D = 0.6\ s$. Each jobs is served two times by the token server to acquire and release the token ($V_T = 2$), and each operation takes $S_T = 1\ s$. On the average, the critical process needs to be run only for one job every two ($V_P = 0.5$), but the computation server has the same utilization of the token server.

   (a) Compute the visits V_D to the dispatcher and its demand D_D.

   (b) Compute the utilization U_T and the demand D_T of the token server. Determine also its expected busy time on an interval of $T = 30\ min$.

   (c) Compute the demand D_P and the mean service time S_P of the computation server.

   (d) Knowing that the system response time is $R = 17.3224\ s$, which is the think time of the clients? Which is the average number of clients in the system (that are not thinking)?

   (e) Which are minimum and maximum throughput that the system is expected to show for $N = 20$?

SOLUTIONS:

(a) $X_D = U_D/S_D = 0.768/0.6 = 1.28\ job/s$; $V_D = X_D/X = 3.5$; $D_D = V_D S_D = 2.1s$

(b) $D_T = V_T S_T = 2s$; $U_T = XD_T = 0.732$; $B_T = TU_T = 30 * 0.732 = 21.96min = 1317.6s$

(c) $U_P = U_T$; $D_P = U_P/X = 0.732/0.366 = 2s$; $S_P = D_P/V_P = 2/0.5 = 4s$

(d) $R = N/X - Z$; $Z = N/X - R = 10/0.366 - 17.3224 = 10s$

$N_{elab} = XR = 0.366 * 17.3224 = 6.34\ job$

(e) $N^* = (D + Z)/D_{max} = (10 + 2.1 + 2 + 2)/2.1 = 16.1/2.1 = 7.666\ job$; we are in heavy-load regime.

$X_{min} = 20/(20 * 6.1 + 10) = 0.1515\ job/s$;

$X_{max} = 1/D_{max} = 1/2.1 = 0.47619\ job/s$

2. Consider the following RAID 6 disk array:

Block A1	Block A2	Block A3	Block A4	Block A5	Block A6
D0 1	D1 0	D2 3	D3 2	P	Q

Block B1	Block B2	Block B3	Block B4	Block B5	Block B6
D0 5	D1 6	D2 2	P 14	Q	D3

Block C1	Block C2	Block C3	Block C4	Block C5	Block C6
D0 0	D1 0	P 6	Q 32	D2	D3

Figure 1: RAID 6 disk

Assuming that the generator used to perform calculations is $g = 2$ (using standard arithmetic and NOT Golois fields),

(a) Compute the data on the missing blocks (A5, A6, B5, B6, C5, C6).

(b) Data in block A3 is replaced with 7. Compute the new parity P in block A5, and the new parity Q in block A6 without using the data in blocks A1, A2 and A4.

SOLUTIONS:
a)
P on A5: $D0 + D1 + D2 + D3 = 1 + 0 + 3 + 2 = 6$
Q on A6: $D0 + 2 \cdot D1 + 4 \cdot D2 + 8 \cdot D3 = 1 + 2 \cdot 0 + 4 \cdot 3 + 8 \cdot 2 = 1 + 12 + 16 = 29$
D3 on B6: $P - D0 - D1 - D2 = 14 - 5 - 2 - 6 = 1$
Q on B5: $D0 + 2 \cdot D1 + 4 \cdot D2 + 8 \cdot D3 = 5 + 2 \cdot 6 + 4 \cdot 2 + 8 \cdot 1 = 33$
D2 on C5 and D3 on C6:

$$\begin{cases} D2 + D3 = 6 \\ 4 \cdot D2 + 8 \cdot D3 = 32 \end{cases} \qquad \begin{cases} D2 = 6 - D3 \\ 24 - 6 \cdot D3 + 8 \cdot D3 = 32 \end{cases}$$

$$\begin{cases} D3 = (32 - 24)/2 = 4 \\ D2 = 6 - 4 = 2 \end{cases}$$

b)
$P' = P + A3' - A3 = 6 + 7 - 3 = 10$
$Q' = Q + 4 \cdot (7 - 3) = 29 + 16 = 45$

3. In an embedded system, it is known that the bottleneck is the disk, with a demand of $D_{disk} = 45ms$, while the total demand of the system is $D_{tot} = 140ms$. The system is run with an increasing number of jobs $N = 1$, $N = 5$, $N = 20$, and the throughput is measured as: $X(1) = 7.143 job/s$, $X(5) = 17.18 job/s$ and $X(20) = 21.798 job/s$.

   (a) Compute the utilization of the disk for the three considered workloads

   (b) Compute the minimum response time that could be achieved in the three case, considering that it is a batch system (i.e. there are no terminal, and the think time can be considered $Z = 0$).

   (c) Compute the exact response time for the three cases.

   (d) Now consider the same system serving an open workload: which is the maximum arrival rate it can handle?

SOLUTIONS

a) $U(1) = 0.045 \cdot 7.143 = 0.32$, $U(5) = 0.045 \cdot 17.18 = 0.77$, $U(20) = 0.045 \cdot 21.798 = 0.98$.

b) $N^* = \frac{D_{tot}}{D_{max}} = \frac{140}{45} = 3.11$

$R_{min}(1) = D_{tot} = 0.14s$,

$R_{min}(5) = 5 \cdot D_{max} = 5 \cdot 45ms = 0.225s$,

$R_{min}(20) = 20 \cdot 45ms = 0.9s$

c) $R(1) = 1/7.143 = 0.14s$, $R(5) = 5/17.18 = 0.291s$, $R(20) = 20/21.798 = 0.918s$.

d) $\lambda_{max} = 1/0.045 = 22.2 job/s$.