Exercises – Parallel Computing and Availability

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1. **Exercise**: Amdahl Law (1)

A program requires 100 sec on a single core architecture, and its sequential fraction is about 1%. How long will it take when executed on a 10 core system? Which is its speed-up? Its efficiency when executed on 100 processors?

Solution:

S_{10}= 10 / (10 x 0.01 + .99) = 10/1.09 = 9.174
T_{10}= T_1/ S_{10}= 100 / 9.174 = 10.9 s
E_{10}= 9.174 / 10 = 91.7%

S_{100}= 100 / (100 x 0.01 + .99) = 100 / 1.99 = 50.25
T_{100}= T_1/ S_{100}= 100/50.25 = 1.99 s
E_{100}= 50.25/ 100 = 50%

2. **Exercise**: Amdahl Law (2)

A program requires 20 sec on a single processor architecture, and 11 sec on a quad-core architecture. How many core would it be required to obtain an execution time of 10 sec? Which will be the speed up and the efficiency in this case?

Solution:

T_s+ T_p= 20;
T_s+ (T_p/4) = 11;

(subtracting the two eq.) 0.75 T_p= 9;

T_p= 12; T_s= 8; T_s+(T_p/ n) = 10; 8 + (12 / n) = 10; n = 12 / (10 – 8) = 6;

S_6= T_1/ T_6= 20 / 10 = 2; E_6= S_6/ 6 = 2 / 6 = 0.333;

3. **Exercise**: Amdahl Law (3)

A computer program, has a serial fraction fs= 20 % , and it takes 24 hours when executed on a single system (single processor). It is necessary to deploy the software on a parallel architecture to obtain a computation time less than 6 hours. Each machine costs 5000$, and the software could be optimized to reduce its serial fraction to fs' = 10 % with an additional expense of 20000$. There are two possibilities: (1) buy several other machines or (2) optimize the software. Which solution is more convenient? Would it be possible to reduce the computation time to 4 hours?

Solution:

1) $f_s = 20\%$; $T_s = 4.8$; $T_p = 19.2$;
2) $f_s' = 10\%$; $T_s' = 2.4$; $T_p' = 21.6$;

Objective: $T_s + (T_p / n) = 6$; $n = T_p / (6 - T_s)$

1) $n = 19.2 / (6 - 4.8) = 16$ $C = 16 \times 5000\$ = 80000\$$ (cost for 16 systems)
2) $n' = 21.6 / (6 - 2.4) = 6$ $C' = 6 \times 5000\$ + 20000\$ =$ **50000$ ->** yes, the optimization is worth!

It is not possible to reduce the computation time to 4 hours, because $T_s = 4.8$ is the lower bound. If the optimization is performed, reducing f_S to 10%, then it will be possible to reduce computation time to 4 hours with n=14.

4. **Exercise:** failure probability of a disk

Compute the probability that a disk with MTTF = 100000 hours fails at least once every 3 years. If instead we have 2 disks, which is the probability that at least one of them fails?

Solution:

For one disk: $P(X \leq t) = 1 - e^{-\frac{26280}{100000}} \approx 23\%$

(also valid the approximate solution: $26280/100000 \approx 26\%$, since $0,26 << 1$)

For two disks: $P(X \leq t) = 1 - e^{-\frac{26280}{100000} \times 2} \approx 41\%$

5. **Exercise:** computation of Reliability

A load balancer(A) is connected in series with a group(B) of three parallel web servers. The load balancer have MTTF of 1000 days and MTTR of 2 days. Each web server have MTTF of 350 days and MTTR of 1 day. Compute the probability of no failures in a t = 7 days period for both (A) and (B) as well as for the whole system (A+B).

Solution:

$R_A(7) = 1-7/1000 = 0.993$
$R_B(7) = 1- (7/350)^3 = 0.999992$ (parallel)
$R_{A+B}(7) = R_A(7) \times R_B(7) = 0.992992$ (serial)

6. **Exercise:** Availability

A load balancer(A) is connected in series with a group(B) of three parallel web servers. The load balancer have MTTF of 1000 days and MTTR of 2 days. Each web server (WS) have MTTF of 800 days and MTTR of 1 day.
Compute the availability of the whole system.

Solution:

A = MTTF / (MTTF+MTTR)
We can compute the availability through availability block formulas:
$A_{Serial} = A_1 A_2 \ldots A_n$ $\quad A_{Parallel} = 1-\prod (1-A_i)^n$

$MTTF_A = 1000$; $MTTF_A = 2$ $\quad A_A = 1000/(1000+2) = 0.998$
$MTTF_{ws} = 800$; $MTTR_{ws} = 1$; $\quad A_{WS} = 800/(800+1) = 0.99875$
$A_B = 1-(1-A_{WS})^3 = 0.99999$
$A_{A+B} = A_A\ A_B = 0.998\ 0.999999 = 0.99799$

7. **Exercise:** Cloud application

To a cloud application are assigned 20 virtual machines, that can be considered as 20 independent systems. Mean execution time for the application (evaluated on multiple running) in such an environment is 8 secs, while the obtained speedup is 10.

For economic problems the user is forced to reduce the virtual machine number to 10.

Compute:

a. Execution time for the application when executed on a single system.

b. Serial and parallel fraction of the application execution time.

c. Efficiency with 10 or 20 systems.

d. Speedup gained using 30 systems

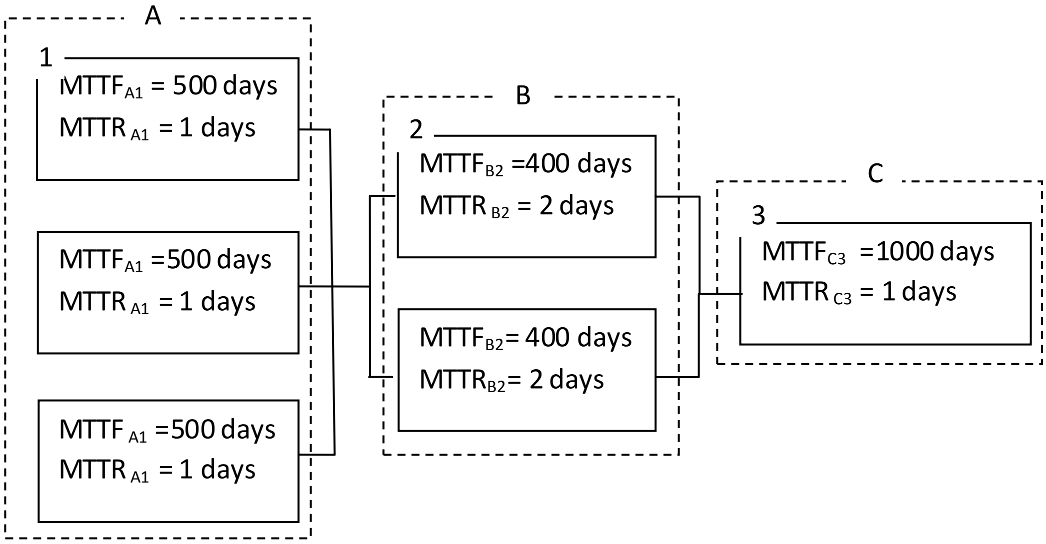
e. Maximum speedup gained without modifying application code but considering an infinite number of systems

Solution:

(a) $T_{20} = 8sec$; $S_{20} = T_1/T_{20}$; $10 = T_1/8$ $T_1 = 80sec$;
(b) $T_{20} = T_s + (T_p/20) = T_s + (1 - T_s)/20$ $T_s = 80/19$ $f_s = T_s/T_1 = 1/19$ $f_p = 18/19$;
(c) $T_{10} = 2240/190 = 11.78s$ $E_{10} = S_{10}/10 = T_1/(T_{10} * 10) = 19/28 = 0.678$ $E_{20} = 10/20 = 0.5$;
(d) $S_{30} = N/(N f_s + (1 - f_s)) = (30 * 19)/48 = 11.875$;
(e) $S_\infty = 1/f_s = 19$;

8. **Exercise:** availability

Consider the following structure where MTTF and MTTR of the components are shown. Compute the availability of each component and of the whole infrastructure.



Solution:

We can compute the availability through availability block formulas:

$A_{Serial} = A_1 A_2 \ldots A_n \quad A_{Parallel} = 1-\prod (1-A_i)^n$

Therefore:

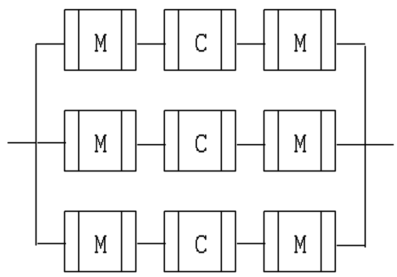
$A_A = 1-(1-A_{A1})^3 = 1-(1-500/(500+1))^3 = 0.999999 \qquad A_B = 1-(1-A_{B2})^2 = 1-(1-400/(400+2))^2 = 0.999975$

$A_C = A_{C3} = 1000/(1000+1) = 0.999$

$A_{A+B+C} = A_A \, A_B \, A_C = 0.998974$

9. **Exercise**: Availability

Consider a communication system, with three trunk in parallel, each one composed by three components in series: two modem and a cable. The system is represented in the following figure:



Let us consider that for the modem: $MTTF_M = 999$ days; $MTTR_M = 1$ days

And for the cable: $MTTF_C = 90$ days; $MTTR_C =$ 10 days

1) Compute the availability of the modem, of the cable, of the trunk and of the entire system.
2) How many trunks should be used to have an availability of the entire system of 99,98% ?
3) If we have a single trunk, with the same modems and a repair time for the cable $MTTR_C = 1$, which should be the $MTTF_C$ to obtain an availability of the entire system of 99,5 ?
4) In the context of exercise 3), would it be possible to have an availability of the trunk of 99,9 ?

Solution

1) $A_M = 999/(999+1) = 0.999$; $A_C = 90/(90+10) = 0.9$; $A_T = 0.999*0.9*0.999 = 0.898201$
   $A_S = 1-(1- A_T)^3 = 0.998945$
2) $A_S = 1-(1- A_T)^n$ $(1- A_T)^n = 1- A$ $n=\ln(1-A)/\ln(1- A_T) = \ln(0.002)/\ln(0.101799)=3.72$ -> n=4
3) $A_T = A_M A_C A_M$ $A_C = 0.995 / A_M^2 = 0.996993$
   $A_C = MTTF_C / (MTTF_{C+} MTTR_C)$ $MTTR_C = 1$ sec therefore $MTTF_C = A_C / (1- A_C) = 331.56$ days
4) It will not be possible, because even if $MTTF_C = \infty$ (thus $A_C = 1$), we will have at most
   $A_T = 0.999*1*0.999 = 0.998001 < 0.999$