CS-UY-1134 Fall 2022 Homework #2 Q1 and Q2

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TOTAL POINTS

9 / 10

QUESTION 1

1 Question 1 4 / 5

✓ **- 2 pts** *Part C: Not a fully correct proof using definition of big O (finding values for c and n0). One example case does not constitute a proof./ incorrect*

+ 1 *Point adjustment*

Part C adjustment

1 n value should be the max of the two n values derived from the individual coeffecients

QUESTION 2

2 Question 2 5 / 5

✓ **- 0 pts** *Did not give explanations*

Question 1:

(a) Show that $5n^3+2n^2+3n = O(n^3)$

$\Leftrightarrow \exists C>0$ and $\exists n_0 \in \mathbb{Z}^+$ s.t. $f(n) \le c \cdot g(n)$ for all $n \ge n_0$

$\Rightarrow 5n^3+2n^2+3n \le Cn^3$

$\Rightarrow 5n^3+2n^2+3n \le 5n^3+2n^3+3n^3 \le 10n^3$

$C=10$

for all n

QED.

(b) Show that $\sqrt{7n^2+2n-8} = \Theta(n)$

$\Leftrightarrow \exists c_1, c_2 > 0$ and $\exists n_0 \in \mathbb{Z}^+$ s.t. $c_1 g(n) \le f(n) \le c_2 g(n)$

Let's first show that $\exists c_1 > 0$ s.t. $\sqrt{7n^2+2n-8} \ge c_1 n$

$\sqrt{7n^2+2n-8} \ge \sqrt{7n^2}$ for $2n-8 \ge 0$

$2n \ge 8$

$n \ge 4$

$\sqrt{7n^2+2n-8} \ge \sqrt{7} \cdot n$

$c_1 = \sqrt{7}$ $n \ge 4$

Now let's show that $\exists c_2 > 0$ s.t. $\sqrt{7n^2+2n-8} \le c_2 n$

$\sqrt{7n^2+2n-8} \le \sqrt{7n^2+2n} \le \sqrt{7n^2+2n^2} \le \sqrt{9n^2} \le 3n$

$c_2 = 3$

QED

(c) Show that if $d(n) = O(f(n))$ and $e(n) = O(g(n))$, then the product $d(n)e(n)$ is $O(f(n)\,g(n))$

since $d(n) = O(f(n)) \Leftrightarrow \exists c_1$ s.t. $d(n) \le c_1 f(n)$

and $e(n) = O(g(n)) \Leftrightarrow \exists c_2$ s.t. $e(n) \le c_2 g(n)$

Hence, we can derive that $d(n) \cdot e(n) \le c_1 c_2 f(n) \cdot g(n)$

where $c_1 \cdot c_2$ is some positive constant. Therefore, by definition, $d(n)\,e(n) = O(f(n)\,g(n))$.

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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

1 Question 1 4 / 5

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Part C adjustment

1 n value should be the max of the two n values derived from the individual coeffecients

Question 2:

Ex 1: $\theta(n^2)$

Ex 2: $\theta(n)$

Ex 3: $\theta(\log n)$

Ex 4: $\theta(n)$

2 Question 2 **5 / 5**

✓ **- 0 pts** *Did not give explanations*