**The Elements of Metaphysics**

**Chapter One: Classical Second-Order Quantified Modal Logic**

**1. Sketch of SQML**+

**Lexicon**

*Primitive Terms:*

1. Object constants: *a*1, *a*2 …. (*a*, *b*, *c*)
2. Object variables: *x*1, *x*2 ... (*x, y, z*)
3. *n-*place predicates constants (*n* 0): *Fn*1,*Fn*2 …. (*Fn, Gn, Hn)*
4. *n*-place predicate variables (*n* 0): X*n*1,X*n*2 .… (X*n*, Y*n*, Z*n*)

1-2 are “object terms” and 3-4 are “predicate terms”. In the special case where *n* = 0, a predicate term denotes a proposition. For the sake of convention, however, *p*, with or without subscripts, will be the variable for propositions. Object and predicates are each “types” and are the two most basic types.

*Primitive Connectives, Operators, and Logical Connectives*

1. Connectives: ,

2. Operators: , □

3. Logical Predicates: =

4. Punctuation: (, )

◊

**Defined Connectives, Operators, and Logical Predicates**

1. is short for ( ) (Conjunction)

2. is short for  (Disjunction)

3.  is short for (( )  (Material Biconditional)

4. is short for  (Existential Quantifier)

5. ◊ is short for □ (Possibility)

6. ≠ is short for = (Non-identity)

7. *E!*is short for  = (Existence)

(Where  and  are both object variables or are both predicate variables).

**Grammar**

The Greek letters , , and are used as meta-variables denoting any formula of the language ℒ, *n* is a meta-variable for any *n*-place predicate, α, , and γ are a meta-variables for terms, and is a meta-variable for variable terms in particular. The expression () denotes any formula with at least one free occurrence of .

1. If is an *n*-place predicate, where *n* 0, and α1 … α*n* are object terms, then (α1 … α*n*) is a formula.
2. If α and are terms then α = is a formula, provided that both are object terms or both are predicate terms.
3. If and are formulas then , , and □ are formulas.
4. If is a formula and 1  ... *n* are variables, then 1  ... *n* is a formula.
5. Only strings that can be shown to be WFFs using 1-4 are formulas.

**Logical Axioms and Rules**

**Propositional Logic**

**PL1:**  ( ) (Simplification)

**PL2:** ( ( )) (( ) ( )) (Frege's Axiom)

**PL3:** ( ) ( ) (Transposition)

**MP:** if ├ and ├ , then ├ (Modus Ponens)

**Uniform Substitution (US):** The result of uniformly replacing any formula or formulas 1, … , *n* in a theorem by any formula or formulas 1, … , *n* is itself a theorem.

**Classical Quantification Theory (CQT)**

**CQT1:**  / Universal Instantiation)

Provided that and are both terms of the same type.

**CQT2:** ( ) ( )

Provided that  is not free in .

**UG:** if ├ , then ├  (Universal Generalization)

Provided that is not substituting for any term  that has been introduced by assumption.

**Some Derived Quantification Rules and Theorems**

**UG:** if ├ , then ├  

**UG**if ├  , then ├   

**PC1**: ( ) ( ) (-distribution)

**QI1**   (quantifier interchange 1)

**QI2:**   (quantifier interchange 2)

**The Logic of Identity**

**I1:** α = α (Law of Identity)

**I2:** α =  (() ()) (Leibniz’ Law)

Leibniz’s Law might appear to be substantively stronger than the mere Indiscernibility of Identicals, but, when expressed in terms of meta-variables, this is not really the case, since an instance of might be ⌜ = ⌝, in which case it is a mere tautology that the two are identical. The Identity of Indiscernibles is only an interesting thesis when certain restrictions are made upon the contents of .

**Axioms and Rules of S5 Modal Logic**

**K:** □( ) (□ □) (Kripke’s Axiom)

**T:** □

**S5:** ◊ □◊

**NEC:** if **├** , then**├** □ (Rule of Necessitation)

**Some Derived S5 Rules and Theorems**[[1]](#footnote-1)

**K**◊□) ( ◊)

**T**◊◊

**D:** □ ◊

**S4:** □□□

****□◊

**DR1:** if**├**  , then **├** □ □

**DR2:** if **├**   , then **├ □**  □

**DR3:** if **├**  , then **├** ◊ 

**DR4:** if **├** ◊ , then **├**  □

**Some Modal “Definitions”**

**Df1: Contingency:** c  ( □)

**Df2: Mere Possibility:** m□◊  (□◊ )

**Modal Second Order Logic (MSOL)**

**MSOL1:** X□*x*1 …. *xn*(X*x*1 …. *xn*  (*x*1 …. *xn*)) (Modal Second Order Schema)

Where X does not occur free in .

MMSOL1 stipulates that for any condition that might be satisfied by some ordered *n-*tuple of objects, there exists an *n*-place predicate true of the said *n*-tuple, if and only if it satisfies that condition.

**MSOL2:** X(*x*□(X*x* 𝓨*x*)  𝓨X)

(The difference in font for second-order predication is simply for the purpose of readability).

MSOL2 governs “higher order predication”, i.e., predication of predicates. It stipulates that if it is necessarily the case that everything that is X is also Y, then Y can be predicated of X. Such second-order predication is only well-behaved when the predicates in question are monadic, since 𝓨 must be monadically predicated of X.

**MSOL3:** X*n* = Y*n*  □*x*(X*x*  Y*x*)

The left-to-right conditional follows immediately from I2 but the right side by itself is not sufficient to derive the left side by I2.

**Semantics**

**Definition of a ℒ Model**

A **ℒ** model, ***M*ℒ***,* is triple ***M*ℒ = W, D, I** such that:

**W** is a non-empty set of “possible worlds,” with a distinguished member *w*@, designating the “actual world”.

**D** is a non-empty set of objects: the “domain”

***I*** is a function, called the “interpretation function,” such that:

(a) if  is an object constant then ***I***() **D**

(b) if *n*is an *n*-place predicate then ***I***(*n*) is a set of *n*+1 tuples 1 …. *n*, *w* such that for each *i*, ***I***(*i*) **D** and *w* **W**.

Regarding (a), ***I***assigns each name to a member of the domain. Regarding (b), predicate-valuations are “intensions” of *n*. ***I***(*n*) determines the extension of *n* within each *w* such that *w* **W**.

The *variable assignment* μ assigns each object variable to a member of **D** and each predicate variable to a set of *n*+2 tuples 1 …. *n*, *w, t*.

The *valuation-function* *VM* for ***ML*** and a variable assignment , is a function that assigns 1 or 0 (true or false) to each WFF relative to each *w* **W** according to the following conditions:

**[*V*=]**  *VM*( = ) = 1 iff []*M* = []*M*

**[*V*]** *VM*(α1 … α*n*, *w*) = 1 iff [1]*M*, .... , [*n*]*M*, *w* ***I***()

**[*V*]** *VM*(1211, *w*) = 1 iff ***I***(11) ***I***(12)

**[*V*]** *VM* (, *w*) = 1 iff *VM*(, *w*) = 0

**[*V*]** *VM*( , *w*) = 1 iff *VM*(, *w*) = 0 or *VM*(, *w*) = 1

**[*Vx*]** *VM*(*x*, *w*) = 1 iff for every [1]*M*, **D***,* *VM*(/*x*)((), *w*) = 1

**[*VF*]** *VM*(X, *w*) = 1 iff for every Y replacable for X, *VM*(Y/X)((Y), *w*) = 1

**[*V*□]** *VM*(□, *w*) = 1 iff for every *w*´ **W**, *VM*(, *w*´) = 1

There are derived clauses specifying the truth conditions for the remaining non-primitive connectives as one would expect.

**Definitions of Validity**

1. is valid in **W, D, *I*** iff *V M*(, *w*) = 1 for every *w* **W** and every assignment of .

2. is ***M*L**-valid (written “╞***M*L** ”) iff is valid in all ***ML*** models.

3. A set of formulas **ML**-semantically-implies (written “╞***ML***”) iff for every ***ML*** model **W, D, *I***, every *w* **W**, and for every variable assignment , *V*(, *w*) = 1 for each , entails *VM*(, *w*) = 1.

PL, CQT, CQT plus identity, and S5 are all well known to be both sound and complete. Moreover, the combination of these logics, known as Simplest Quantified Modal Logic (SQML) is also known to be both sound and complete. Second-order logic is known to be incomplete relative to its standard semantics but the above axioms can be shown to be verified by their semantic clauses semantics.

One feature included in the present semantics that is uncommon concerns the **[*V*]** clause. It defines predicate predication in terms of the subset relation rather than the more standard type-theoretic approach whereby the membership relation is employed. The present account is simpler and does not require that we countenance some well-defined hierarchy of predicates. This should not be terribly surprising since higher order logics are reducible to second order logic (Hintikka 1955).

**5. Some General Definitions**

The following are a few useful definitions that can be produced on the basis of SQML+.

**Df3: Singular Formula:**  is singular if and only if it contains at least one object constant.

**Df4. General Formula:**  is general if and only if it is not singular.

**Df5: Impure Predicate:** For any property X, X is impure if and only if:

□*x*(X*x*  (1 …. *n*))

(Whereeach is an object constant)

**Df6: Pure Predicate:** A predicate X is pure if and only if it is not impure.

In what follows some important SQML theorems shall be presented. Many of these theorems are of great significance. The first system of QML was presented by Barcan (1946, 1947). Her system combined classical first-order logic with C.I. Lewis’ S2 and S4 modal axioms along with two additional axioms governing the interaction between quantificational and modal operators, which formulas have come to be known as the Barcan Formula (BF) and Converse Barcan Formula (CBF).[[2]](#footnote-2) Prior (1956) both simplified and strengthened Barcan’s system by combining first order logic with S5 modal logic. An important result of this strengthening is that BF and CBF turn out to be theorems. Hence SQML, when using S5 modality, does not require any additional logical axioms aside from those already employed in classical propositional logic, quantification theory, the logic of identity, and S5 modal logic. It is simply the immediate result of their conjunction.

**1. The Denial of Modal Collapse (A1)**

**Do1: Modal Collapse** **(MC):** ◊  □

MC can be derived from **S5** modal logic plus the following axiom:

□[[3]](#footnote-3)

MC is the doctrine that possibility and necessity are co-extensive.[[4]](#footnote-4) It might also be called *metaphysical determinism* or *metaphysical fatalism*. MC has had some defenders in the past, apparently by Ibn Sina[[5]](#footnote-5) and Spinoza.[[6]](#footnote-6) MC is logically consistent; in particular, MC holds on models in which there is exactly one possible world.

The denial of MC is typically taken for granted in modal metaphysics but, given the axiomatic methodology of the present treatise, it must be explicitly introduced as an axiom. It cannot, of course be stated as ⌜¬(◊  □)⌝, since that would prevent *any* formula from being such that it is possibly true just in case it is necessarily true. The simplest way of expressing DMC is in terms of quantification over proposition (i.e., 0-place predicates).

**A1\*: Denial of Modal-Collapse (DMC):** ¬*p*(◊*p*  □*p*)[[7]](#footnote-7)

**A1** guarantees that there is more than one possible world. In terms of natural science, *any* *indeterminacy* in the world will be sufficient for **A1** to hold.

**2. Necessity and Identity**

**T1: Necessity of Identity (NI):** *xy*(*x* = *y* □*x = y*)

1. *x* = *y*  (□*x* = *x* □*x* = *y*) I2

2. (*x* = *y*  □*x* = *x*) (□*x* = *y*) 1. PL

3. (□*x* = *x* *x* = *y*) (□*x* = *y*) 2. PL

4. □*x* = *x* (*x* = *y* □*x* = *y*) 3. PL

5. *x* = *x* I1

# 6. □*x* = *x* 5. NEC

# 7. *x* = *y* □*x* = *y* 4, 6. MP

# 9. *xy*(*x* = *y* □*x* = *y*) 7. UG

# *Corollary*: XY(X= Y □X= Y)

# Df11: Rigid Designator: A term  designates rigidly if and only if it denotes the same individual in every possible world *w* in which it designates anything at all.

# Semantically speaking, NI is a consequence of rigid designation. Object terms in our semantics are interpreted rigidly in that their interpretation is invariant across possible worlds. Kripke (1972/1980) famously argued that rigid designation entails NI:

If ‘*a*’ and ‘*b*’ are rigid designators, it follows that ‘*a* = *b*’, if true, is a necessary truth. If ‘*a*’ and ‘*b*’ are *not* rigid designators, no such conclusion follows about the statement ‘*a = b*’. (Kripke, 1972/1980: 3)

# An important consequence of this, discussed by Kripke at length, is the inadequacy of the “definite description” theory of reference, according to which the meaning of a proper name is equivalent to a definite description of the object named, where a definite description of an object *a* is any description that is true of *a* and only true of *a*.

# Kripke rightly insists that descriptions play an important role in *fixing* reference but they do not constitute the meaning of a name. For instance, suppose there is a gathering in which only one man is wearing a blue shirt and that person's name happens to be “Bob”. Then the sentence “The guy in the blue shirt” fixes the name “Bob” to Bob. The name “Bob” will not cease to refer to Bob as soon as he changes his shirt or as soon as another blue-shirted man enters the room.

**T2: Necessity of Nonidentity (NNI):** *xy*(*x* ≠ *y* □*x* ≠ *y*)

1. *x = y* □*x* = *y* NI

2. □*x* ≠ *y* *x* ≠ *y* 1 PL

3. ◊*x* ≠ *y*  *x* ≠ *y* 2. DR5

4. *x* ≠ *y* □*x* ≠ *y*  3. DR4

5. *xy*(*x* ≠ *y* □*x* ≠ *y*) 4. UG

While the proof for NI requires only the rule of necessitation, the proof for requires the DR4 rule which, in turn, requires the B axiom. It seems counterintuitive that NI could hold while NNI does not. NNI can fail, however in QMLs in which the *symmetry* between world frames fails, i.e., when the underlying modal logic does not include the B axiom. If *x* and *y* happen to be non-identical in *w*@, there may be some world *wj* in which ⌜*x =* y⌝ is true provided that it is not the case that *wjRw*@, in which case NI would be violated.

**3. Existence and Necessity**

**3.1 The Necessity of Existence**

**T3: Necessity of General Existence (NGE):** □*x* *y=x*

1. *y y* ≠ *x* *x* ≠ *x* CQT1

2. *x = x*  *x y* ≠ *x*  1. PL

3. *x = x* I1

4. *x y* ≠ *x* 2, 3. MP

5. *x* *y = x*  4. QI2

6. □*x* *y = x* 5. NEC

NGEis simply the result of applying the rule of necessitation to the CQT theorem ⌜*x* *y=x*⌝. Since ⌜*E!x*⌝ is short for ⌜*y* *y=x*⌝, NGEtells us that, necessarily, something or other exists. Hence, it could not have been the case that there is nothing at all. It is well known that CQT is not compatible with the existence of empty domains, which has perhaps been the primary motivation for the adoption of Free Logic (to be briefly discussed below).

**T4: The Necessity of Existence (NE):** *x*□*y*(*y* = *x*)

1. □*x* *y=x*  NGE
2. *x*□*y*(*y* = *x*) 6. UG

***First Corollary*:** X□Y(X = Y)

***Second Corollary* (**NNE): **□***x*□*y*(*y* = *x*)

***Third Corollary:*** For any cardinal , if -many objects exist, then, necessarily -many objects exist.

The *second corollary* here is of especial importance, since it tells us that anything that even possibly exists necessarily exists.

In terms of possible world semantics, NE entails that every *x* that exists in the actual world is such that, in every possible world, there exists some *y* with which *x* is identical. Apparently, then, everything necessarily exists. This is, of course, an incredibly strong theorem.

NE is not, however, as strong as it might seem at first blush. For instance, the fact that there are Kangaroos does not entail that there are necessarily Kangaroos. That is, while ⌜□*xKx*⌝ can be inferred from the conjunction of NE and ⌜*Ka*⌝ we cannot thereby infer ⌜*x*□*Kx*⌝. That is, just because there is necessarily something that happens to be a kangaroo, it does not follow that anything is necessarily a kangaroo. Hence {⌜*x*□*y*(*y* = *x*)⌝, ⌜*Ka*⌝, ⌜◊*xKx*⌝} is a consistent set. The inference to ⌜x□Kx⌝, and hence also ⌜□x□Kx⌝, requires an additional assumption to the effect that ⌜*x*(K*x* □K*x*)⌝. That is to say, one is inclined to make certain *essentialist* assumptions when evaluating the implications of NE and, given such assumptions, NE certainly does seem unacceptable. So the conflict is between classical logic and essentialism. The methodology of this investigation dictates we side with logic.

NE denies a thesis that Nelson (2009) has named Absent:

**Do2:** Absent: *x*◊*y x* = *y*

**T5: Denial of Absent:** ¬*x*◊*y* *x* = *y*

1. *x*□*y* *x* = *y* NE

2. ¬*x*¬□*y* *x* = *y* 1. Def.

3. ¬*x*◊*y* *x* = *y* 2. Def. ◊

Absent is the doctrine that there exists something such that it might not have existed. The denial of Absent is, of course, highly counter intuitive. While it is hardly more controversial to suppose that abstract objects necessarily exist than to suppose that they exist at all, it seems obvious that just about any *concrete* object might not have existed. For instance, one might not have been born if one’s parents had not met and procreated. Broadly speaking, it seems contingent that the Earth exists at all, much less that it turned out to be a life-supporting planet. Indeed, it seems to be contingent that the universe should even support life or, for that matter, physical aggregates. But, again, all of these concerns arise from certain background assumptions that go beyond what is being said in the logic.

# T6: Transworld identity: There exists some object *x* and worlds *w*1 and *w*2 such that *x* exists in *w*1 and *x* exists in *w*2.

*Proof:* Transworld identity is an immediate consequence of **A1**, **NGE**, and **NE**. By **A1** there are at least two possible worlds, *w*1 and *w*2, by NGE some *x* or other must exist, and by NE, we know that *x* exists in every possible world. In particular, we know that *x* exists in both *w*1 and *w*2.

According to transworld identity, when we ask whether Socrates might have been a carpenter and not a philosopher, we are asking if there are any possible worlds that represent Socrates *himself* as a carpenter but not as a philosopher. That is to say, Socrates is represented as *existing* at every possible world in which he may be considered (and by NE he can be considered in every possible world).[[8]](#footnote-8) Transworld Identity is generally taken to be the alternative of Lewis’ Counterpart Theory, according to which all (concrete) objects are world-bound, but have transworld *counterparts*. The counterpart of Socrates at some world *w* is not Socrates himself but some concrete existent resembling Socrates in some pertinent manner. When we ask “could Socrates have been a carpenter?” we are likely asking if someone very much like Socrates, living in very similar circumstances, might have become a carpenter. Hence, counterpart theory takes a distinctly *qualitative* approach to apparently *de res* modal claims.

At first blush counterpart theory seems quite sensible. If someone, in the *actual* world, very similar to Socrates *became* a carpenter, it would be reasonable to infer that Socrates *might have* *become* a carpenter. But here is the problem as I see it. It is *not because* someone very similar to Socrates did become a carpenter that *makes it the case that* Socrates could have. It is because of the way Socrates is (at least at some time during his life) that makes it the case that he might have been a carpenter. He just happens to share this “way” with some other person who did, indeed, become a carpenter. I see no reason why this complaint should vanish when the appropriate similarity relation holds between Socrates and someone in another possible world rather than someone in the actual world. Hence the counterparts of MR seem to be more like epistemic justifiers of modal claims than metaphysical truth-makers but with the peculiarity that these epistemic justifiers are epistemically inaccessible to us and hence do nothing to justify *our* modal claims.

# The precise meaning of transworld identity claims cannot be spelled out until we have discussed the relation between powers and modality, but to put it schematically, I will say that there is a world in which Socrates became a carpenter just in case Socrates possesses the *potency* to become a carpenter. That is, *des res* modality about an object *x* would be grounded in the dispositional properties of *x*

Modal realism comes into conflict with SQML, in that SQML commits us to transworld identity while, as mentioned above, Lewis opts for counterpart theory.[[9]](#footnote-9) Lewis (1986a) argues that combining MR with transworld identity leads into trouble with the Identity of Indiscernibles (**I2**). McDaniel (2004), however, has suggested how MR might be combined with transworld identity provided that one embraces certain non-Lewisian doctrines such as endurentism and mereological pluralism. Accordingly, transworld identity does not quite block the way to MR, but it blocks the conjunction of modal realism and four-dimensionalism.

**Do3: Necessitism** is the doctrine according to which NNE is a logical truth.

**Do4: Contingentism:** Contingentism is the doctrine that NNE is not a logical truth.[[10]](#footnote-10)

Evidently, endorsing SQML commits one to affirming Necessitism and denying Contingentism.

It would seem, then, that possible worlds differ only with respect to their predicate extensions. One might have drawn this conclusion simply be looking at the semantics of , since nothing is said about worlds being assigned different subsets of **D**. In quantified modal logics with varying domains, each possible world *w* is assigned its own domain: **dom**(*w*), and the relevant semantics clauses are modified accordingly.[[11]](#footnote-11) Just as S5 modality makes “world frames” unnecessary, SQML makes it unnecessary to assign a domain of individuals to each possible world. Hence we arrive at the surprising conclusion that the *number* of existing objects is a strictly necessary matter. Since worlds (i.e., state descriptions) do not differ in terms of what exists, they differ only in terms of which objects fall under the extension of which predicates. To cite Williamson:

What objects there are and their identity, distinctness, and number are all necessary matters. Indeed, in LPC=S5 any proposition expressible in terms solely of identity, negation, conjunction, the universal quantifier and necessity is either necessarily true or necessarily false. Contingency has its place within a necessary framework of objects (Williamson 1998: 269).

This permits us to provide an alternative formulation of **A1** (although the best version still awaits):

**A1\*\*:** X*nx*1…*xn*(◊X*nx*1…*xn*  □X*nx*1…*xn*)

It will be noted that, on **NNE**, predicates exist in every possible world. In particular, any predicate X*n* will exist even in worlds in which the extension of X*n* is empty. Given **A1**, there will presumably be some predicates which are not actually instantiated.

Also, inasmuch as propositions may be construed as 0-place predicates, every proposition *p* necessarily exist. This does not mean that every proposition is necessarily true (which would immediately result in MC), since ⌜*p* *p*⌝ is a perfectly grammatical formula. Every proposition necessarily exists as something we can refer to and talk about, as in the case of ordinary predicates.

**3.2 Truth-Makers and the Truth-Maker Principle**

**Df12: Truth-Maker:** For any formula and objects *x*, *x* is the truth-maker of just in case the existence of *x* makes it the case that is true.[[12]](#footnote-12)

**Do5: Truth-Maker Maximalism** is the doctrine that, for every true formula , there exists some *x* such that is true in virtue of the existence of *x*.

**Do6:** **Truth-Maker Necessitism** is the doctrine that, for any formula and any object *x*, if, in the actual world, *x* is sufficient to make true, then in every possible world in which *x* exists, is true.

**Do7: Truth-Maker Principle (TMP):** □( *x*□(*E!x* ))

In words: necessarily, for any true formula , there exists some *x* such that, necessarily, if *x* exists then is true. It is natural to suppose that true formulas must be made true by something and, in the literature, such things have come to be known as “truth-makers.”[[13]](#footnote-13) The doctrine that truths require truth-makers might be regarded as a contemporary form of the correspondence theory of truth, according to which any true formula is true in virtue of corresponding to the world in some appropriate way. Truth-maker theory is a specific kind of correspondence theory of truth; in particular, the truth-maker theorists claims that whenever a proposition *p* is true, there exists some *thing* (or things), in virtue of which *p* is true. That is to say, for the truth-maker theorists, truth does not merely supervene upon *how* the world is (or the conjunction of what exists and how those things are) but upon *which things* exist. Truth-makers are typically thought of as facts or (Armstrong-style) states-of-affairs.[[14]](#footnote-14)

At first glance, Tarskian-style semantics might seem to imply some sort of correspondence theory of truth, but “truth in a model” is typically regarded as a set-theoretical surrogate for “full blooded” truth makers.[[15]](#footnote-15) That is, the model-semantic conception of truth is neutral regarding the question of truth-makers.

It is natural for the truth-maker theorist to be a truth-maker maximalist. If one believes that some true propositions require truth-makers, it is natural to suppose that *all* truths require truth-makers. One difficulty that arises for the truth-maker maximalist, however, is that it is not clear what the truth-makers for *negative* propositions could be. What makes it true that there is no dragon under this table? Is it merely the *absence* of a dragon under this table?[[16]](#footnote-16) But then we are treating such an absence as if it were a thing or a positive feature of the world. In particular, Lewis (1992) has suggested that we not seek truth-makers for negative existentials. Nevertheless, there have been various attempts to preserve Truth-Maker Maximalism in the face of such challenges. Maximalism is also threatened by some necessary truths, such as mathematical truths. What makes it the case that, for every set *S* there exists some set *S*´ such that the cardinality of *S*´ is greater than the cardinality of *S*? One could opt for some sort of trivial truth-makers here, but this seems to betray the spirit of truth-making theory, since truth-making is not supposed to be a trivial matter.

While truth-maker theorists are somewhat divided regarding maximalism, the great majority support Truth-Making Necessitism.[[17]](#footnote-17) The idea is that, if *x* is sufficient to make true, then, in any world in which *x* exists, is true.[[18]](#footnote-18)

TMP, then, is a principle governing the combination of truth-maker maximalism and truth-maker necessitism. As Williamson (1999) has observed, it is not compatible the conjunction of NE and **A1**.

**T7: The Truth-Maker Principle Entails MC**[[19]](#footnote-19)

1. □( *x*□(*E!x* ) TMP

2. *x*□(*E!x* ) 1. T

3. *x*(□*E!x* □ 2. K

4. ( *x*□*E!x*) □ 3. PL

5. *x*□*E!x* NE[[20]](#footnote-20)

6. *x*□*E!x*  5. PL

7. ( *x*□*E!x*) 6. PL

8. □ 4. 7 PL

TMP gets into trouble here precisely because “truth” becomes a matter simply of *what* *objects* exists. In particular, when states-of-affairs (i.e., property instantiations) are denoted by *terms*, it turns out that nothing could have differed from the way things actually are.

The denial of TMP does not mean that we must reject every truth-maker theory as such. Moreover, we may still follow Lewis (1992) in proclaiming that truth supervenes upon being, where “being” is construed broadly so as to include not only *what* there is, but also the *ways* things are. The *way* that some object happens to be is not, itself, some further object.

It is possible to quantify over facts or states of affairs provided that they are not treated as terms but as *formulas*. Again, the problem with Armstrong’s approach to Truth-Makers and states of affairs is that he treats them as “thing like” in a way that encourages quantifying over them as terms. As Williamson (1999) argues, the difficulty might be elevated if we permit ourselves to quantify over propositions, formulas. Again, ⌜*p* ¬*p*⌝ is a perfectly grammatical formula. Accordingly, Williamson (1999) suggests the propositional route by replacing TMP with TMP\*, which also happens to be a theorem of SQML+.

**T8: TMP\*:** □( *p*(*p* □(*p* ))

1. □(  ) PL, NEC

2. ( □(  )) 1, PL

3. ( ( □(  ))) *p*(*p* □(*p*  ) 2. EI

4. *p*(*p* □(*p*  )) 2-3. PL

5. *p*(*p* □(*p* )) 4, CQT, PL

6. □( *p*(*p* □(*p* ))) 5, NEC

The problem with TMP\*, however, is that we are effectively using propositions as truthmakers for other propositions, which is especially evident in the following variation of **TMP\***:

**T9: TMPU:** □*q*(*q* *p*(*p* □(*p* *q*))

Propositions (or sentences) are *truth-bearers*, they are the things that need to be *made true*. In the very least **TMP\*** will not permit a *reductive* analysis of truth whereby truth might be said to supervene upon being or to be grounded in non-truth-bearing facts. A modified version of **TMPU** might provide a principle of *grounding*, whereby some propositions obtain *in virtue of* others.

I shall have occasion to return to the matter of grounding later; as for truth-making, expressing a reductive theory of truth-making in the *object-language* may be possible but it would be cumbersome. It would be necessary to have a theory of non-linguistic *facts* or *events* which make propositions true. But such facts or events are going to have to have conditions under which they obtain anyway, and so there will need to be truth-makers for formulas about their obtaining. The object language does not allow us to say something along the lines of “‘snow is white’ just in case snow is white”. Moreover, NNE cannot be said to be to blame here, since ⌜*E!x*⌝ is itself a truth (or falsity) bearing formula.

# 3.3 Direct Reference

# Df13: Definite Description: is a definite description of *x,* written *x*,just in case

# *y*((*y*/*x*)  *y* = *x*)

# Do8: Definite Description theory of Meaning: The definite description theory of meaning is the doctrine that the meaning of any object *x* is precisely a definite description of *x*.

# Do9: Direct Reference: The doctrine that the meaning of any object term is precisely the element of D to which  is assigned.

# A matter that is closely related to, and easily confused with, rigid designation is that of *direct reference*. Given the combination NI, NNE, and A1, the definite description theory of reference is likely to fail. It is however, epistemically possible for there to be a definite description of each individual that holds rigidly across possible worlds and this description might even be compatible with a tremendous degree of modal flexibility. This would be the case if each object possessed a unique set of essential properties. This is plausibly the case for abstract objects but it seems unlikely to be true of concrete objects. Moreover, even if it is true of concrete objects, it will later be seen that, such descriptions will almost certainly be *elusive*, and hence, it is not clear that such descriptions could serve as the *meanings* of names. This would be a problem for descriptivists, since most are internalists regarding the meaning of names.

# One might try to argue in favor of direct reference by appeal to semantics. The meaning of an object constant is determined by the interpretation function *I* while the meaning of an object variable is determined by the variable assignment 𝜇.Both constants and variables are assigned *directly* to elements of D. Hence the direct theory of reference is practically built into the semantics. The question is whether the content of a theory of reference is exhausted by its model semantics. To be sure, one can produce models in which names are not be rigid and hence do not directly refer. The standard Tarskian treatment of names, which is part of the semantics for SQML, however, seems to favor direct reference.

**3.4 NNE and the denial of “Alien”**

Perhaps more important than NE is NNE, which is a corollary of NE obtained simply by applying the rule of necessitation. In terms of possible world semantics, NNE entails that, in every possible world *w*, everything necessarily exist; that is to say, *everything that possibly exists necessarily exists*. Accordingly, SQML is only compatible with constant domain semantics – every possible world contains the same objects. Not only do we tend to suppose that actual things might have failed to exist, we also tend to presume there could have been individuals that do not in fact exist. Hence NNE appears to deny what Nelson (2009) has named Aliens:

**Do10:** **Aliens:** ◊*x*𝓐*y*(*y* = *x*)

The “𝓐” here is being employed as the actuality operator. “Aliens” is the doctrine that there might have been something that does not actually exist. Semantically speaking, it is the doctrine that in some possible world, there exists something that does not exist in the actual world. As for the actuality operator, Zalta (1999) presents the logic of actuality in terms of the following two axioms:

**LA1:** 𝓐 

**LA2:** 𝓐  □

Formulas involving the actuality operator are logically true but contingent and hence the rule of necessitation cannot be employed on formulas containing such an operator. The valuation for 𝓐 is:

**[*V***𝓐**]** *VM*(𝓐, *w*) = 1 iff *V*(, *w*@) = 1

**T11: Denial of Aliens:** ¬◊*x*¬𝓐*y* *x*=*y*

1. 𝓐*y* *x*=*y* CQT. LA1

2. □𝓐*y* *x*=*y* 1. LA2

3. *x*□𝓐*y* *x*=*y* 2. UG

4. □*x*𝓐*y* *x*=*y* 3. BF

5.¬◊*x*¬𝓐*y* *x*=*y* 4. Def. ◊ and

Despite the fact that NNE is a theorem of SQML, it would be rash to suppose that something has somehow gone wrong with SQML such that it should be construed to suggest MC. First, the S5 modality underlying SQML clearly does not validate MC and the interaction between S5 and CQT validate neither of the following formulas:

*x* □*x*

*x* *x*□

The *metaphysical* question, then, is: how can NNE be true and MC false? The holding of MC is sometimes called “necessitarianism” (Nelson 2009). So another way of putting the question is “how can necessitism be true while necessitarianism is false?” How can it be the case that everything that could possibly exist necessarily exists while it is nevertheless the case that the world might have been different in, presumably, countless many ways? It seems that any metaphysics that affirms NNE while denying MC will be strongly *revisionary* in one way or another.

**4. Contingentism and the Problem of Possibilia**

While the denial of Alien may be counterintuitive, its affirmation also has counterintuitive implications, since it is would seem to commit one to Possibilia:

**Df14: Possibilia:** *x* is a possibilia, written 𝓟*x*,if any only if:

◊*y*(*x* *y*  ¬𝓐*x* = *y*)

**T10: The Class of Possibilia is Empty:** ¬*x*◊*y*(*x**y*  ¬𝓐*x*=*y*)

1. *xy*(*x**y*  𝓐*x*=*y*) LA1, PL, UG

2. □*xy*¬(*x**y*  ¬𝓐*x*=*y*) 1. PL, NEC

3. *x*□*y*¬(*x**y*  ¬𝓐*x*=*y*) 2. CBF

4. ¬*x*¬□¬*y*(*x**y*  ¬𝓐*x*=*y*) 3. Def.

Semantically speaking, possibilia is doctrine that there exists some *x* such that [*x*]μ **D** but it is not the case that [*x*]μ (**d**)*w@*.

While possibilia have not been universally shunned by modal metaphysicians, they are shunned more often than not. Many philosophers, beginning with Parmenides, have denied the intelligibility of referring to that which does not exist. What exactly is wrong with possibilia? First of all, when we speak of possibilia, we seem to be saying that there *are* things that do not exist but might have existed, or, in other words, that there are non-actual things. The required distinction between bare existence and actual existence, however, seems to be dubious. In the very least, if there are things that are non-actual (and if MR is false) then such thing would seem to be rather “shadowy” creatures – we are not quite sure just what it would mean for something to be a possibilia.

While Aliens does not directly entail that there are possibilia, it is difficult to understand how it could express the idea that alien objects exist according to other worlds without forcing one to quantify over merely possible objects. Divers (2002: 211) presents the following question for actualists about possible worlds:

**D**-problem: how can actualists semantically construe formulas such as: ⌜◊*x*𝓐*y*(*y* = *x*)⌝?

Stalnaker has recently produced a monograph dedicated to this problem:

How, on an actualist interpretation of possible worlds as ways a world might be, is one to account for the possibility that there be individuals other than those that actually exist? That is the main focus of this book (Stalnaker 2012: *ix*).

Divers notes three ways in which the actualist may try to circumvent the **D**-problem. One may (i) adopt a QML with constant domains, namely SQML, (ii) construe non-actual possibile individuals as descriptions, or (iii) posit a set of individual essences that exist in every possible world but which are only instantiated in some.

Perhaps the most promising way to be both an actualist and a contingentist is to construe contingently non-existent objects as abstract objects, namely as *definite descriptions*. A name denoting a merely possible object will be construed not so much as referring to an object that does not really exist, but as referring to some description. There are various ways in which one might construe such items. I shall not go into precise details regarding how such objects can be accommodated, since such accommodations involve a number of semantical complications. It is worth noting, however, that there are two general approaches one could take. One could construe non-actual objects as structured abstract objects belonging to **D** or as properly non-referential but meaningful in that each is effectively assigned some set of properties and relations.

It is generally thought that such ersatz individuals fall prey to at least two problems. *First*, as noted in Lewis (1986a) it would deny the possibility of there being indiscernible (actually non-existent) objects. If the meaning of a term denoting an ersatz individual consists in some kind of description and if two names are assigned the same description, then they must be thought of as “referring” to the same possible individual. Second, as noted by McMichael (1983), ersatz individuals raise a problem when interpreting formulas with iterated modalities, such as the following:

◊*x*(*Fx* ◊*Px*)

Suppose *F* is interpreted as “fathered”, while *r* is the name “Ratzinger” and *P* is interpreted as “plumber”. Hence the formula can be read to say “in some world *w*1 there is some *x* that is fathered by Ratzinger and which, in some world *w*2, is a plumber”. Given that the meaning of *x* is some description and supposing that *x* is not a plumber in the first mentioned world, it seems that we fall into a contradiction, since the description will include both ⌜¬*Px*⌝ and ⌜*Px*⌝.

One way to respond to both of these difficulties would be to assign each non-designating name a set of duples {〈*F*, *w*〉,〈*F*´, *w*´〉, …}, where the first term of each member is a one place predicate and the second is a possible world. Hence, a for non-denoting term *a*, ⌜*Fa*⌝ would be true at a world *w* just in case 〈*F*, *w*〉 ***I***(*a*)*.* The problem of indiscernibility at a particular world will be overcome, since two names will “refer” to the same ersatz individual just in case they refer to the same transworld description. While it may be possible for two distinct individuals to be contingently indiscernible, it is implausible to suppose that two distinct individuals could be necessarily indiscernible. Likewise, transworld descriptions permit transworld identity for ersatz individuals.

The limitations of this proposal, however, become apparent when we turn to the evaluation of *n*-place predicates, where *n* >, 1 involving at least one non-actual object and at least one actual object. We cannot evaluate such formulas in the usual Tarskian manner, nor can we evaluate them in the manner of the previous paragraph. A general approach to evaluation is required that holds for both real and ersatz objects while still permitting one to precisely distinguish the two classes.

Antonelli (2000) provides non-Tarskian “proto” semantics for positive Free Logic with a single, actual, domain. The proposed semantics satisfy compactness and completeness relative to its axiomatization, although it is rather complicated and unorthodox. Roughly speaking, non-actual objects do not denote but they are effectively assigned predicates by a proto-interpretation function. The semantics are capable of evaluating formulas containing both real and ersatz objects while still distinguishing them, but the semantics are intended for *non-modal* free logic and hence could not be directly employed in the modal case without further modification. I presume that, with sufficient ingenuity, the required modifications could be made but suspect that the resulting model theory would not only be unorthodox and complicated but also *oblique*. That is, I suspect that any formally adequate solution would do little to provide elucidate of the *meaning* of formulas containing alien objects.

The third general response to the **D**-problem involves the introduction of haecceities. Plantinga (1974), for instance, introduces a set of necessarily existing *individual essences*. Individual essences may be defined as follows:

**Df15: Individual Essences**: A property *E* is an individual essence just in case

*x*(◊*Ex* □((*y* *x* = *y* *Ex*) *y*(*Ey* *y* = *x*)))

A property *E* is an individual essence just in case (i) there is a world *w* and an object *x* such that *E* is exemplified by *x* at *w*, and at any world *w*´, if *x* exists at *w*´ then *x* exemplifies *E* at *w*´ and (ii) at any world *w*´ and any object *y*, if *y* exemplifies *E* at *w*´, then *y* = *x*.

Each individual has exactly one essence and each essence is the essence of exactly one individual. Individual essences are necessary existents and, in the formal semantics provided by Jager (1982), **D** is precisely populated by such essences. In this respect, introducing haecceities (or essences) is very similar to necessitism, only one has populated the domain with the essences of individuals rather than with the individuals themselves. Even if Socrates did not exist, the individual essence, “Socrateity”, or the property of “being Socrates” would still exist. Socrates is a contingent existent precisely because his individual essence is exemplified in some worlds but not in others.

Linsky and Zalta (1994) complain that this revision to semantics is costly, since the objects themselves are no longer directly expressed in the semantics. First-order quantifiers are interpreted as quantifying over subsets of **D** (whichever subset is assigned to the world of evaluation) but such items are not individuals *per se* but properties. Accordingly, a notion of “co-exemplification” must be introduced in order to discuss what is usually understood as exemplification simpliciter. For instance, “Socrates is snub-nosed” must be interpreted as “Being-Socrates is co-exemplified with being-snub-nosed”. As Menzel (2008) notes, many of Plantinga’s definitions cannot be analyzed in terms of the formal semantics that his theory requires.

There are, moreover, some philosophical concerns one might have with the individual essences of non-actual objects. As properties, individual essences will not be qualitative, since in that case they would be no different than descriptions. If they are non-qualitative, it seems they must be construed along the lines of “being identical with Socrates” or simply “being Socrates” (Menzel 2008). In the first case, it seems quite clear that the property of being identical with Socrates is meaningless unless Socrates actually exists. Indeed, both ways involve the singular term Socrates, the meaning of which depends upon reference to Socrates. More generally, Plantinga’s theory of individual essences appears to entail the denial of existentialism:

**Do11: Existentialism: □***p*(*p*(*a*) *x* *a*=*x*)

Existentialism is the doctrine that for any singular proposition *p* including a singular term *a*, if *p* exists then *a* must also exist. On a Russellian theory of propositions, the object named by a is literally a component of p, and hence any proponent of Russellian propositions will be an existentialist. One, however, need not be a Russellian in order to endorse existentialism. Anyone endorsing the “direct reference” theory of names (or something near enough) will also endorse existentialism.[[21]](#footnote-21) At any rate, it does not seem that the property “being identical with Socrates” or “being Socrates” can exist unless Socrates himself exists. Indeed, one might have noticed that the very definition of haecceity leads one into troubles, since in the first clause we still find ourselves quantifying mere possibilia: “a property *E* is an individual essence only if *there is some x* such that, at some world, *x* exemplifies *E*.” It should also be noted that, on Jager’s formalization, BF, CBF, and NE are blocked only at the cost of banishing individual constants and hence a significant loss of the expressive power of QML. Given necessitism, of course, existentialism is necessarily true:

**T11: Affirmation of Existentialism: □***p*(*p*(*a*) *x* *a*=*x*)

1. *x* *a*=*x* CQT

2. *p*(*a*) *x* *a*=*x* 1. PL

3. **□***p*(*p*(*a*) *x* *a*=*x*) 2. UG, NEC

It is especially difficult for a linguistic ersatzer be a contingentist while also being an existentialist. Given existentialism and contingentism, singular propositions will exist contingently. If possible worlds are sets of propositions (or atomic formulas) then possible worlds, even as abstract objects, will be *contingent*. Suppose that world *w*, contains some proposition *p* that does not exist at a world *w*´, we should, I think, conclude that *w*´ is not accessible from *w* and that it is inaccessible precisely because it does not exist at *w*. If this is right, no world containing “alien” objects will be “accessible” and hence *possible relative to the actual world*, which simply means that it is not possible for something to exist if that something is other than anything that actually exists. That is to say, one will be limited to modal semantics in which **D** *may contract but not expand*. The necessity of existence will fail, but it will not be the case that anything actually non-exist object might have existed.

But perhaps I am moving too fast here. There may be ways in which actualists worlds may be contingent without wreaking havoc on our modal semantics. Stalnaker (2012) has proposed an algebraic-based semantics for contingent propositions which is designed to confront these difficulties. His proposal involves a “tactical retreat” from both *extensionality* and *realism* regarding possible worlds. Stalnaker follows McMichael (1983) in his understanding of “extensionality” and “realism” in this context. Extensionality is lost due to the fact that not all instances of modal operators will quantify over the same set of possible worlds **W** (note that we require variable domain of worlds rather than a variable domain of individuals). It will be non-realist in that the semantics will contain elements of **D** which are supposed to represent possibile individuals while denying that any such element is something like a mere possibilia. Note that the failure of full-on realism here is not with respect to concrete possible worlds but with respect to ersatz worlds. Stalnaker concedes that these features of his theory may be unattractive but he regards the theoretical costs to be worth it if we can avoid the ontological costs of possibilism or the metaphysical revisionary costs of necessitism. Indeed, decisions must be made and each choice seems to come with a price.

It will be admitted that Stalnaker’s preference for theoretical costs (i.e., addition of complexity and loss of elegance) over ontological and metaphysical costs is quite sensible. After all, why should we suppose that the right theory will be relatively simple and elegant? Perhaps we have an aesthetic bias in favor of such theories (in fact I am sure that we do) but why should we expect the world to be such that it permits such theories to be the best theories? The history of science lends some credence to our preference for simplicity and elegance but good science can also get very complicated and messy, especially the biological sciences.

At any rate, it is evident that, given necessitism, strict actualism can be easily accommodated without any theoretical sacrifices.

**T12: Strict Actualism:** *x*(◊*y* *x*=*y*  𝓐*y* *x*=*y*)

1. ◊*y* *x*=*y* □*y* *x*=*y* NE, PL

2. □*y* *x*=*y* ◊*y* *x*=*y* T

3. ◊*y* *x*=*y*  □*y* *x*=*y* 1, 2 PL

4. *y* *x*=*y*  𝓐 *y* *x*=*y* LA1

5. ◊*y* *x*=*y*  *y* *x*=*y* 3, T

6. ◊*y* *x*=*y*  𝓐*y* *x*=*y* 4-5 PL

7. *x*(◊*y* *x*=*y*  𝓐*y* *x*=*y*) 6 UG

Semantically speaking, Strict Actualism is the doctrine that for every *x*, if [*x*]μ ***D***, then [*x*]μ (**dom**)*w@*.[[22]](#footnote-22) Necessitists can also easily satisfy the following doctrine:

**Do12: Serious Actualism: □***x*X(X*x* ­ *y* *x=y*)

Serious actualism is the doctrine that objects can only have properties in world in which they exist.

**T13: Serious Actualism:** *x*X(X*x* ­ *y* *x=y*)

1. *y* *x=y* CQT

2. X*x* ­ *y* *x=y* 1. PL

3. *x*X(X*x* ­ *y* *x=y*) 2. UG

4. □*x*X(X*x* ­ *y* *x=y*) 3. NEC

In the absence of NNE, it is awkward to deny serious actualism and it is also awkward to reject it. Even if one has adopted a deviant logic under which the proof for strict actualism fails, it is alarming to say that objects can have properties at worlds in which they do not exist. If the contingentist, however, adopts strict actualism, then his QML will have “truth-gaps” and worlds cannot be maximal under entailment. The necessitist, however, is spared from this dilemma.

**5. The Barcan Formulas**

**T14: Barcan Formula (BF):**[[23]](#footnote-23) *x*□ □*x*

1. *x*□ □ CQT1

2. ◊*x*□ ◊□ 1. DR3

3. ◊□ S5 Theorem

4. ◊*x*□ 2, 3. PL

5. ◊*x*□ *x* 4. CQT2

6. *x*□ □*x* 5. DR4

***First Corollary*:** ◊*x* *x*◊[[24]](#footnote-24)

***Second Corollary*:** X□ □X[[25]](#footnote-25)

***Third Corollary*:**◊X X◊

Apparently BF and its converse, CBF, were initially formulated and endorsed by Ibn Sina, better known in the West as Avicenna. Hence it might be appropriate to call it the Ibn Sina-Barcan Formula.[[26]](#footnote-26) Nevertheless, I shall continue to employ “BF” as its abbreviation. BF can be read as follows: if, in the actual world, every entity necessarily satisfies some condition , then it is necessarily the case that all entities in every possible world satisfy . The problem here is that it seems that there should be possible worlds, perhaps even this one, in which everything is essentially (and hence necessarily) a physical being, but from that it would follow that everything in every possible world is a physical being.

Of course, given NE, this result should not be at all surprising. If Aliens is false and if everything that exists is necessarily a physical being, then it is impossible that there should be any non-physical beings. In Chapter Eight I shall defend a conception of metaphysical modality relative to which this result is not at all surprising: if there is nothing that has the potential to become non-physical or to generate something that is non-physical then it is just not possible for anything to be non-physical.

The existential form of BF, ⌜◊*x*  *x*◊⌝, states that, if in some possible world, there exists something that is , then there is something in the actual world that could have been . For example, suppose that it is possible for something to be a goblin. On BFit follows that there *is* something in the actual world that in some possible world *is* a goblin. This means that BF conflicts with *sortal essentialism* (SE). Recall that essentialist assumptions also run into trouble when interpreting NE and NNE. By NE, if kangaroos are essentially kangaroos, then every possible world includes all of the kangaroos present in the actual world. Moreover, by NNE every world has precisely those kangaroos present in the actual world. Accordingly, T. Parsons (1995) has written the following regarding BF:

Antiessentialism is required as follows. Certainly there might have been more porcupines than there are. So there must be possible worlds in which there are things that are not porcupines in this world ... but the additional things will not be porcupines in the actual world. We thus need the possibility of a thing that is not a porcupine in this world [but] is one in the other world; that thing cannot essentially be or not be a porcupine (Parsons 1995: 11).[[27]](#footnote-27)

Likewise, Williamson (2010):

‘What there is’ should not be understood as ‘what kinds are instantiated’. The necessitist may agree with the contingentist that it is contingent whether there are rivers. The dispute concerns whether it is contingent or necessary which particular things there are, never mind what kinds they instantiate (Williamson 2010: 8).

Assuming that there really are such things as rivers, when the necessitist says “the St. Laurence necessarily exists,” it is the existence of the particular object, which we call “the St. Laurence”, that is necessary, not its being a river.

Sortal essentialism is difficult to define in a non-circular manner. At present I will make no attempt to provide a rigorous definition of substantial sortal. All that I shall say is that sortals in general are understood to be something like *count nouns* as opposed to *mass nouns* and sortal essentialists typically have in mind those sortals which denote *substantial* kinds.

Regarding count nouns and mass nouns, you can ask someone how many bricks are needed but you cannot ask how many clays are needed; indeed, “clays” is not even an English word. So, in general, we might say that sortals are count-kinds rather than mass-kinds.

Regarding substantial kinds, some might only admit the basic kinds of physical fields or particles. Most sortal essentialists, however, will include the various kinds of living creatures and some might include things like planets, stars, and artifacts as well. Using 𝒮 as a primitive predicate for “substantial sortal”, sortal essentialism may be stated as follows:

**Do: Sortal Essentialism (SE):** X(𝒮X *x*(◊X*x*  □X*x*)[[28]](#footnote-28)

Given NNE, SE would entail that every substantial sortal that is instantiated is necessarily instantiated and, indeed, necessarily instantiated by just those things that actually instantiate it. Moreover, it would be impossible for there to be “alien sortals”, sortals that are not instantiated in the actual world but instantiated in other possible worlds. While the conjunction of NNE and SE does not immediately lead to MC, it certainly places severe restrictions on what might have been the case. The world could not have differed enough to change the general course of biological evolution or which sorts of artefacts were invented![[29]](#footnote-29) Hence, it seems, contrary to appearance, genetic mutation would not be genuinely random, nor the evolution of the cosmos.

Moreover, it will later be demonstrated that classical logic, when extended to time entails permanentism, being the doctrine that everything that exists *always* exists. Given permanentism, SE entails that the universe *never changes* with respect to which ultimate sortals are instantiated. This leaves us with the following disjunction: either none the sortals that we observe with our senses or which are studied by the natural sciences are substantial sortals or SE is false.

BF also appears to conflict with *origins essentialism,* according to which everything necessarily has the origins that it has in fact. For instance, despite the fact that Wittgenstein had no son, presumably Wittgenstein *could* have had a son. According to BF, it follows that there exists something *in this world* that, in some possible world, *is* Wittgenstein’s son.[[30]](#footnote-30) Taking “origin” as primitive, origins essentialism may expressed as follows:

**Do: Origins Essentialism (OE):** *xy*(*Oxy* □*Oxy*)[[31]](#footnote-31)

Origins essentialism is the doctrine that, for every *x* and every *y*, if *x* originates from *y*, then *x* necessarily originates from *y.* Were OE the case, not only would every possible instance of a sortal be a necessary instance, but every possible *circumstance* in which a sortal comes to be instantiated would be a necessary circumstance. OE, however, might be vacuously satisfied by permanentism, since it would not seem that necessary objects would even have origins at all (at least not temporal origins, which is what OE advocates surely have in mind).

**T15: Converse Barcan Formula (CBF)** □*x* *x*□

1. *x* CQT1

2. □(*x* ) 1. NEC

3. □(*x* ) (□*x* □) K

4. □*x* □ 2-3. MP

5. *x*(□*x* □) 4. UG

6. (*x*(□*x* □)) (□*x* *x*□) CQT 2

` 7. □*x* *x*□ 5-6. MP[[32]](#footnote-32)

***First Corollary*:** *x*◊ ◊*x*

***Second Corollary*: □**X X□

***Third Corollary*:** X◊ ◊X

CFB is valid even upon the weakest classical QML, combining first order logic and the modality of system K. *Prima facie,* CBF is uncontroversial. In its universal form it says that if, in every possible world *w*, is true of everything in the domain of *w*, then is necessarily true of everything in the actual world. This seems to be obvious truth. The denial of CBF is equivalent to: ⌜□*x* *x*◊⌝. On possible world semantics, this says that, in every possible world, everything is such that it satisfies and yet, in the actual world, there exists something that is possibly not . Thus, there is something such that, in some possible world is not and hence the second conjunct contradicts the first.

The existential form of CBF, ⌜*x*◊ ◊*x*⌝, states that if is possibly true of something in the actual world then, in some possible world, there is something that is . Again, this seems to be obviously true by the very definition of possibility. Moreover, just as the denial of its universal form entails an absurdity, the denial of the existential form of CBF entails: ⌜*x*◊ ◊*x*⌝: there exists something that might have satisfied and yet it is not the case, in any possible world, that there exists something that satisfies . As Williamson (1998) rightly notes, it is difficult to conceive how CBF could possibly fail.

Semantically, however, CBF entails that there could not have been *fewer* individuals than there in fact are. “Fewer”, if course, is a matter of cardinality, and so CBF does not directly entail NE. Nevertheless, BF and CBF jointly entail NNE.[[33]](#footnote-33)

Bird (2007) reads BF to suggest that possibility is fully grounded upon actual things that exist. Taking its existential form, BF tells us that if it is possible for something to satisfy a given condition, then there is something that could have done so. He suggests that the *x* in the consequent possesses some dispositional property or properties such that, under certain counterfactual circumstances, would result in *x* satisfying the condition in question.

Given, CBF, however, we know that the material conditional goes both ways. If something possibly satisfies then, it is possible for something to satisfy . Simchen (*forthcoming*) argues that, for those of us who share Russell’s “robust sense of reality”, it is more natural to ground general (*de dicto*) modality upon particular (*de res*) modality. Furthermore, he argues that the best reason to accept BF arises not from considerations of modal logic but from how it provides focus on our analysis of metaphysical modality:

If something is generally possible, then it is only so because of a corresponding possibility for something in particular. As an illustration, let us consider the question of what makes it the case that it is possible that there be a chair occupying an actually empty corner. The default intuitive answer is that what makes this generally possible is that something might have been a chair occupying the corner – presumably one of the actual chairs might have been such. And for that to be the case, presumably a counterfactual history ending with a chair in the corner would have had to diverge at some point from the actual history of the empty corner, a counterfactual history involving one of the actual chairs. (Simchen *forthcoming*).

While I am not quite convinced that this use of BF provides the best reason for accepting it, I do, however, agree that it is preferable to ground general modality in particular modality and that BF does suggest as much. The fact that it is possible for something or other to satisfy does not seem to explain why some *particular* might possibly satisfy but the fact that some particular might satisfy does seem to explain why it is that something in general might satisfy .

Later I shall introduce a principle, **A3**, that effectively entails that modality must be grounded in the actual, provided that objects possess powers (and eternalism is true). Simchen’s reference to possible *histories* is also of significance for what is to come, since possible worlds will ultimately be construed as possible, alternative, histories of the actual world.

**6: Some Additional Theorems of SQML**+

**T16:** *x*□ □*x*

**T17:** ◊*x* *x*◊

**T18:** 𝓨X*n*  □𝓨X*n*

1. 𝓨X  □*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) **MSOL2**

2. 𝓨X □*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) 1. PL

3. □*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) □□*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) S4

4. □*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) 𝓨X 1. PL

5. □□*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) □𝓨X 4. K

6. □*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) □□*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) S4

7. □*x*1…*xn*(X*nx*1…*xn* 𝓨*x*1…*xn*) □𝓨X 5-6. PL

8. 𝓨X*n* □𝓨X*n*  2, 7. PL

9. □𝓨X*n* 𝓨X*n* T

10. 𝓨X*n*  □𝓨X*n* 8-9 PL

**7. Alternatives to SQML**

In this work I have made the conservative axiomatic decision to adopt SQML and have begun to explore some of the metaphysical implications of this decision. Nevertheless, a few things might be said regarding alternative forms of QML which do not validate NE, BF, or CBF. In what follows, the two primary alternatives to SQML shall be discussed: Kripke’s Variable Domain Quantified Modal Logic (VDQML) and Free Modal Logic (FML).

Until recently, the most common alternative to SQML has been to adopt “variable domain semantics,” first introduced by Kripke (1963). Variable domains permit one to represent the idea that different possible worlds can include different individuals. Semantically speaking, this involves only a minor complication, as one only needs a function **dom** that assigns a subset of **D**to each world *w*. While the semantics of VDQML remains elegant enough, its axiomatization is both complicated and weakened. CQT1 (Universal Instantiation) is jettisoned and use of the rule of necessitation (NEC) on open formulas is prohibited. CQT1 is not valid because, given a world *w* and a variable assignment 𝜇, 𝜇might not belong to the domain of *w*.[[34]](#footnote-34) The standard proofs for each of the controversial SQML theorems involve the use of CQT1 or applying NEC to open formulas. These same changes, however, also make the proof theory more cumbersome.

In VDQML CQT1 is replaced with the following “generality theorem,” whose standard form is as follows: ⌜*x*(*y* *x/y*)⌝ provided that *x* is substitutable for *y* in . Nevertheless, NE, BF, and CBF can still be derived if one is permitted replace free occurrences of variables with constants. Thus Kripke must banish the use of constants altogether. Regarding this, Menzel has written: “It is surely a sad irony that a system whose motivation is to capture our modal intuitions — most notably, intuitions about contingency — cannot so much as permit us to talk about specific contingent individuals and say of those individuals that they are contingent” (Menzel 2008). In this way, we avoid saying that individual constants are necessary only because we cannot say *anything* about individual constants.[[35]](#footnote-35) In short, VDQML turns out to be more difficult and less expressive than SQML and, while it enjoyed favor for many years, it has lost much of its luster. The decline of VDQML has partly been to the benefit of SQML but it has also been to the benefit of what is currently SQML’s main rival: Free Modal Logic.

Arguably the most promising alternative to SQML involves adopting *Free Logic* (FL), in which CQT1 does not obtain because the domain of objects is understood to be possibly empty. There are three different approaches to construing the semantics of Free Logic: positive, neutral, and negative. On the positive semantics, atomic formulas containing empty terms can be true. For instance, ⌜*a* = *a*⌝ is true even if *a* does not exist (hence the **I1** axiom is retained without restriction, but existentialism and strict actualism fail). In this case **D** is typically said to have an *inner-domain* of objects that are said to exist, and an *outer domain* of non-existing objects. On neutral FL semantics, atomic formulas containing empty terms are not assigned any truth-value. This approach becomes complicated, however, when attempting to evaluate complex formulas containing both empty and nonempty terms and there is disagreement regarding which such formulas should be given truth-values and, if so, how these values should be assigned. On negative FL semantics, atomic formulas containing empty terms are regarded as false.

In Free Modal Logic (FML) positive semantics are adopted, where an outer domain is interpreted to include objects that do not exist in the actual world, but may exist in other possible worlds. This permits one to speak of non-existent things, such as fictional or otherwise non-actual entities without thereby quantifying over them. That is, we can say that Santa Clause is a jolly old man without saying that there exists some *x* such that *x* is a jolly old man. Axiomatically speaking, FML accomplishes this by replacing CQT1 with ⌜*x* (*y* *ya* *a*/*x*)⌝, which reads: if everything is , and *a* exists, then *a* is . Unlike VDQML, FML permits the use of constants and does not restrict the use of necessitation. Various forms of FML have been developed and defended by, among others, Kit Fine (1978) and Garson (2001).

So why not adopt FML rather than SQML? In addition to the fact that the proof theory for FML is more difficult than that of SQML, other difficulties crop up. For instance, substitution instances of co-extensive open formulas typically fail on any semantics for free logics with identity (Nolt 2010). Additionally, FML with an outer domain requires that we quantify over possibilia and hence is not compatible with strict actualism, serious actualism, or existentialism. FML without an outer domain may be possible in principle but would technically difficult. These criticisms are by no means intended to present a decisive case against FML, but rather to highlight some features of FML that might be regarded as unattractive.

**TIMES, EVENTS, AND PROCESSES**

The present chapter introduces temporality as well as events and processes. Regarding temporality as such, it will be demonstrated that existence and identity are both permanent matters. This will mean that the members **D** never come-into-being and never go-out-of-being. One consequence of this is that four-dimensionalism will not be an option. Nevertheless, one can still speak of events and processes, the *occurrences* of which are both contingent and temporary. Indeed, contingency will be a matter of which events and processes occur and talk of contingent objects will ultimately be replaced by talk of contingent processes of a special kind. Moreover, events and processes will be represented as “structured entities” and, to my knowledge, the formalism to be presented will be unique in this respect.[[36]](#footnote-36)

**1. Temporal Logic and the Permanence of Existence**

Presumably anything that exists necessarily exists always exists. After all, it is intuitive to suppose that if something exists necessarily then it must exist *unconditionally*. Williamson (2002) mentions this matter in passing, assuming that necessary existents exist at all times.[[37]](#footnote-37) Given that metaphysical modality, as understood in this text, is considerably different from mere conceptual modality, however, we should not rely too strongly on our intuitions. Fortunately, it is well known that the simplest quantified temporal logic renders objects eternal just as easily as SQML renders them necessary.

Tense logic, beginning with the pioneering work of Prior (1957, 1968), is largely motivated by the intuition that only the present genuinely exists. As Augustine famously wrote, “the past is no longer and the future not yet is.” The doctrine that only the present really exists is known as “presentism” and it is effectively equivalent to endorsing the A-theory of time. Many metaphysicians, however, regard time as literally a fourth dimension. On this view the past and future exist just as much as the present; their difference from the present being no more special than the way “there” differs from “here”. Paris is not spatially present to me, but this gives me no reason, on pains of solipsism, to deny that it exists. Likewise, 429 A.D. is merely “behind us” and 3500 A.D. is merely “ahead of us”. The doctrine that the past and future are just as real as the present is known as “eternalism” and it is effectively equivalent to endorsing the B-theory of time.

According to eternalists, tense is not metaphysically fundamental. While tense logic may be indispensable in the formal analysis of temporal *language* (and our first-person conception of time), the eternalist will not make serious use of it when doing metaphysics. The eternalist, however, does not deny that there is such a thing as time[[38]](#footnote-38) and hence should still have interest in *temporal* logics that do not appeal to tense.

I shall briefly discuss classical quantified tense logic simply in order to demonstrate that it verifies permanentism. Tense operators work much the way as modal operators – both are effectively devices for indexing Tarski models (only in this case the set of said models really must be ordered relative to a frame). The weakest form of temporal logic is known as *Minimal Tense Logic* **Kt,** so named because is the temporal equivalent to the modal system **K**.

***Sketch of Kt:***

*Primitive Terms*:

*H*: “It has always been the case that”

*G*: “It will always be the case that”

*Defined Terms*:

*P*ϕ ¬*H*¬ “It was the case that”

*F*ϕ ¬*G*¬ “It will be the case that”

∎ϕ (ϕ∧(*H*ϕ∧*G*)) “It is always the case that”

ϕ (ϕ∨(*P*ϕ∨*F*)) “It is at some time that case that”

*Axioms and Rule*:

**TL1:** *H*(ϕ¬ ) ¬ (*H*ϕ¬ *H*)

**TL2:** *G*(ϕ¬ ) ¬ (*G*ϕ¬ *G*)

**TL3:** ϕ¬ *HF*

**TL4:** ϕ¬ *GP*

**ETR:** If⊦ , then ⊦ *H*ϕand ⊦ *G*

*H* and *G* are both analogous to the necessity operator, where *H* ranges over the entire past and *G* ranges over the entire future. They are often referred to as the “strong” temporal quantifiers. Likewise, *P* and *F* are analogous to the possibility operator, where *P* rangers over some past time and *F* rangers over some future time. They are often referred to as the “weak” temporal quantifiers and are defined in terms of *H* and *G* respectively (although ∎and are more tightly analogous to □ and ).

TL1 and TL2 are direct analogs of the modal K axioms while TL3 and TL4 ensure that the strong and weak operators are interdefinable. Finally ETR, is short for “eternalization” is the temporal analogue of the rule of necessitation. Temporal logic requires the addition of two new model elements: a non-empty set **T** and a binary relation <. Intuitively speaking, **T** is the set of all moments of time while < is the “before” relation.[[39]](#footnote-39)

Expressions that are indexed according to both worlds and times are effectively “two-dimensional” or “multi-modal”. When both modalities are in effect, the intensions of *n*-place predicate are treated as *n*+2 tuples, indexed to both a world *w* and a time *t*. Accordingly, the truth valuation for atomic formulas would be modified simply by adding the temporal index:

**[*V*t]** *VM*(α1 … α*n*, *w*, *t*) = 1 iff [1]*M*, .... , [*n*]*M*, *w*, *t* ***I***()

The other valuation clauses would be modified as expected. The truth valuation clauses for the temporal operators are as follow:

**[*VH*]** *VM*(*H, w, t*) = 1 iff for every *t*´such that *t*´ < *t*, *VM*(, *w*, *t*´) = 1

**[*VG*]** *VM*(*G*, *w,* *t*) = 1 iff for every *t*´ such that *t*  < *t*´, *VM*(, *w*, *t*´) = 1

Within this minimal tense logic it is easy to show that all objects permanently exist.

**T\*: Permanence of Existence (PE):** ∀*x*∎*y*(*x* = *y*)

1. *y* *x*=*y* CQT

2. *Gy* *x*=*y* 1. ETR

3. *Fy* *x*=*y* 1. ETR

4. ∎*y x*=*y* 1-3 Def. ∎

5. ∀*x*∎*y* *x=y* 4. UG

One will have noticed that the proof is effectively the same as the proof for NE. Combining temporality with ordinary modality, one can easily derive ⌜∀*x*□ ∎*y*(*x*=*y*)⌝ by applying the rule of necessitation between lines 4 and 5.

The *official* proof for PE, however, will not make use of tense logic as such, but will involve *direct* quantification over times. This decision is made for two reasons. First, the logic of events will be more expressive if quantification over times is permitted. Second, I shall ultimately need to adopt eternalism, and so there is not much need for tense operators (although the eternalist operators by themselves would have been fine).

Let *t*, with or without subscripts, serve as the variable for times or moments. Non-italicized t, with or without subscripts will be used on the rare occasion in which time constants might be required.

How much metaphysical weight should be placed upon these times or moments? Not too much and not too little! Times are semantically *indexical* *devises*, much like worlds. Now, of course, they are to be intrepreted as *representing* something or other and in the very least they represent successive states of a given world.

The set **T** of times will be left *neutral* in several respects and hence it will not say that much about the nature of time. As for those features of time which will be decided upon, I do not want to *insist* that all of them characterize the nature of time as such. For instance, if time is interdefinable with space in such a way that there is no such thing as absolute simultaneity, then quantification over times will be misleading.

Should we be very doubtful regarding absolute simultaneity? While there is no absolutely simultaneity between events in the spacetime of Special Relativity (SR), there are a number of scientifically motivated reasons for supposing some absolute time-frame:

1. QM assumes an absolute time-frame for the evolution of the wavefunction.

2. In cosmological applications of General Relativity (GR), which effectively supersedes SR, absolute simultaneity re-emerges. For instance, statements about the age of the universe are made relative to a privileged cosmological time-frame (Swinburne 2007).

3. Relative simultaneity might well be experimentally challenged by observations of Bell’s inequalities, such as Aspect et. al. (1982). This is because wave collapse of a physical system must be *simultaneous* for each particle of the system, regardless of the distance between them.

Accordingly, the wide-spread belief that SR did away with absolute simultaneity once and for all is misguided.[[40]](#footnote-40) From considerations of total physics the case for absolute simultaneity looks stronger than the case against it. If, however, SR is right about absolute simultaneity, then any interpretation will have to be understood to be relative to an arbitrarily chosen timeframe.

Time will also be construed as *discrete*. This condition will later be important when analyzing modality in terms of “diverging” worlds, but I also believe that time is probably discrete. When I stake my bets that time is discrete I affirm the following disjunction: if there is such a thing as spacetime at all, it is made up of discrete Plank-scale units *or*, if there is no such thing as spacetime, there is a minimal amount of time during which a physical event can be said to take place. In each case the unit itself might be contingent, but there would necessarily be some unit or other. Again, the supposition of continuous spacetime leads to infinities in QFT and the infinities that arise from gravitational fields cannot even be renormalized without quantizing gravity itself, and hence spacetime. If string theory is true, and all intrinsic properties are string-vibrations or oscillations, then it would be meaningless to speak of properties being possessed at some literal point of time, since no oscillation will occur during such an “instant”. Indeed, this is a lesson one might have already learned from Zeno’s arrow argument. If I happen to be mistaken, and time is genuinely continuous, then, once again, the “unit” here must be interpreted as an arbitrary but appropriate choice, perhaps Plank time would do.

Nevertheless, I do not want to make too much of the times or moments of the temporal logic. Their primary purpose is to relativize predication to a linear sequence of “moments”. This is partly because I am uncertain that time is really fundamental at all. It might well be the case that spatiotemporal relations can be reduced to ordered *causal* *relations*, as proposed by the Causal Sets theory of quantum gravity.

In terms of model theory, then, we shall introduce a non-empty set **T** of times (or moments) and a relation < on **T** requiring that **T** should be linear and discrete. Axiomatically speaking, quantification over times will be just like quantification over objects. It will be necessary to update the logic of identity so as to accommodate times:

**I1:** α =*t* α (Law of Identity)

**I2:** α =*t* ¬ ∀*t*´((, *t*´) (, *t*´)) (Indiscernibility of Identicals)

The *t* subscript is added to denote that  is identical with itself, or , at *t*. The < relation will also be introduced into the object language. The following axioms will govern <:

**T1:** ∀*t* ¬*t*  < *t* (Time Irreflexivity)

**T2:** ∀*t*1∀*t*2∀*t*3((*t*1 < *t*2 ∧*t*2 < *t*3) ¬ *t*1 < *t*3) (Time Transitivity)

**T3:** ∀*t*1∀*t*2((*t*1 < *t*2 ∨*t*2 < *t*1)∨*t*1 = *t*2) (Linear Ordering)

**T4a:** ∀*t*1∀*t*2(*t*1 < *t*2 ¬ *t*3(*t*1 < *t*3 ∧¬*t*4 *t*1 < *t*4 < *t*3)) (Discreteness-a)

**T4b:** ∀*t*1∀*t*2(*t*1 < *t*2 ¬ *t*3(*t*3 < *t*2 ∧¬*t*4 *t*3 < *t*4 < *t*2)) (Discreteness-b)

**T5:** *t*1 < *t*2 ¬ □  *t*1 < *t*2 (Necessity of Priority)

This sort of language is, in many respects, more expressive than tense logic. For instance, one can express “there is a beginning of time” and “there is no beginning of time” as follows:

*t*1¬*t*2 *t*2 < *t*1

∀*t*1*t*2 *t*2 < *t*1

One can similarly express the propositions “there is an end of time” and “there is no end of time”.

*t*1¬*t*2 *t*1 < *t*2

∀*t*1*t*2 *t*1 < *t*2

Given discreteness, we can also define a *successor* function between times.

**Df16: Successor:** *s*(*t*1) = *t*2  (*t*1 < *t*2 ∧¬*t*3 *t*1 < *t*3 < *t*2)

The *inverse* of the successor function will get us the *predecessor* of a time:

**Df17: Predecessor:** *p*(*t*1) = *t*2  *s*(*t*2) = *t*1

Having introduced times, ordinary first-order formulas will officially be replaced by their temporal counterparts. That is to say, ⌜*Fxy*⌝ will now be understood as an elliptical formula requiring that a time be added to the relata: ⌜*Fxyt*⌝, which is to be read as “*x* is *F*-related to *y* at *t*”. More precisely, where *n* is an *n*-place predicate and 1…*n* and *n*-tuple of objects, ⌜*n*1…*nt*⌝ is a temporal atomic formula, or a *simple event*. If ⌜*Fxt*1⌝ and ⌜*Gxyt*2⌝ are both true and ⌜*t*1 < *t*2⌝ we may also say that ⌜*Fxt*1 < *Gxyt*2⌝.[[41]](#footnote-41) That is, events inherit temporal order from their time-relata. The primary difference, however is that the ordering of events is not linear, since more than one event may occur at a given time. In the treatment of events below a simultaneity operator will be introduced to indicate events which occur at the same time.

Some might complain that this way of temporally relativizing relation instances effectively renders each *n*-place relation as an *n*+1 place relation. “Being taller than” is no longer a two place relation, but a three place relation, where the first two relata are objects and the third is a moment of time. Nevertheless, it might still be regarded as a two-place relation precisely because it takes two *objects*. After all, in the standard semantics for SQML relations are already relative to worlds, the only difference is that we do not include world variables within the object language (since modal operators are used rather than direct quantification over worlds). Moreover, I might have decided that times are directly predicated by abstracts formed from first-order atomic formulas. Where [ ] is an abstraction operator that turns a formula into a complex *term*, [*Fxy*] would be a complex predicate, which might be regarded as a state of affairs. Such complex terms can be attributed to times such that ⌜[*Fxy*]*t*⌝ would read something like: “*x* bearing *F* to *y* is instantiated at *t*”. While in some respects I prefer this approach, it would require additional syntactical terms, namely a variable for states of affairs, and the abstraction operator.

**2. Permanence of Existence and the Denial of Four-Dimensionalism**

**T19: Permanence of Existence (PE):** ∀*x*∀*ty* *x* =*t y*

1. ∀*y x* ≠*t* *y* ¬ *x* ≠*t* *x* CQT1

2. *x* =*t x* ¬ ¬∀*y x* ≠*t y*  1. PL

3. *x* =*t x* I1

4. ¬∀*y x* ≠*t y* 2,3. MP

5. *y* *x* =*t y* 4. QI2

6. ∀*x*∀*ty* *x* =*t y*  5. UG

***Corollary*:** ∀*x*□ ∀*ty* *x* =*t y*

No object ever comes into existence or goes out of existence – there is no “becoming” with respect to existence. The corollary effectively combines PE with NE: everything exist in every possible world at every time. The temporal UG on line 6 is clearly justified, since the initial *t* is perfectly general (i.e., *t* was not introduced by assumption). When something or other is both necessary and permanent I shall say that it is “fixed”. Hence we may now speak of the fixity of existence across all 〈*w*, *t*〉 pairs.

PE is suggestive of Leibniz’s doctrine that substances have no beginning or end in time but, rather, undergo *transformations.*

We must then know that the machines of nature have a truly infinite number of organs, and are so well supplied and so resistant to all accidents that it is not possible to destroy them. A natural machine still remains a machine in its least parts, and moreover, it always remains the same machine that it has been, being merely transformed through the different enfolding it undergoes, sometimes extended, sometimes compressed and concentrated, as it were, when it is thought to have perished (Leibniz 1989 (1695): 142).

Indeed, in **Chapter Three** something along these lines was alluded to when speaking of the “modes” of necessary bare particulars. At present it would be premature to begin speculating on how a new theory of transformations might be construed, since there are other ontological questions, yet to be addressed, which may well preclude the need for such a theory. Perhaps there are no such things as living beings at all and hence nothing for a theory of transformations to explain. Likewise, if existence monism should turn out to be true, then we have at most one animal that would undergo such transformations.

**Df19: Endurent:** For every *x*, *x* is an endurent if and only if:

*t*1*t*2(*t*1 ≠ *t*2 ∧*y*(*x* =*t*1 *y* ∧*x* =*t*2 *y*)

An endurent is an object that strictly exists over time, relative to which one may speak of *identity over time*.

**Do14: Endurentism** is the doctrine that some objects are endurents.

**Do15: Perdurantism** is the doctrine that there are no endurents.

**Do16: Universal Endurentism** is the doctrine that all objects are endurents.

**T20: Permanence of Identity (PI):** ∀*x*∀*yt*1(*x* =*t*1 *y* ¬ ∀*t*2 *x =t*2 *y*)

1. ∀*t*2 *x =t*2 *y* I1. UG

2. *x* =*t*1 *y* ¬ ∀*t*2 *x =t*2 *y* 1. PL

3. *t*1(*x* =*t*1 *y* ¬ ∀*t*2 *x =t*2 *y*) 2. EI

4. ∀*x*∀*yt*1(*x* =*t*1 *y* ¬ ∀*t*2 *x =t*2 *y*) 3. UG

***Corollary*:** *Universal Endurentism is affirmed unless* |**T**| = 1.

***Corollary***: *Perdurantism is denied unless* |**T**| = 1.

At first one might have supposed PI would directly entail both universal endurentism and the denial of perdurantism, but perdurantism would be vacuously true if **T** contains but a single member. Nevertheless, perdurantism is just as well as defeated by **T22**, since it can only be true if the universe is practically timeless. That is, to say that there is but a single moment of time is just as much as to say that time is an illusion.

Endurentism as such may seem to be obviously true, but it happens to be controversial in the contemporary literature on the metaphysics of material objects. Recall that Quine (1950), Lewis (1986a), Armstrong (1997), and Sider (2001) endorse the doctrine of supersubstantialism, according to which the fundamental objects are all spacetime points. By PI, however, we have seen that spacetime points cannot be included in **D**.

The controversy over endurentism arises due to several paradoxes regarding change, where two alternatives to endurentism, namely perdurantism and exdurantism respond to said paradoxes by denying that there is any such thing as strict identity over time. Given PI, retreating from endurentism will not be an option when responding to such paradoxes. The problem of temporary intrinsic properties will be discussed in **Chapter Nine**.

**Basic Mereology**[[42]](#footnote-42)

**P: Part:** ⌜*Pxy*⌝ is read “*x* is a part of *y*”.

“Part” is a primitive term.

**M1: Transitivity of Parthood:** (*Pxy* ∧*Pyz*) ¬ *Pxz*

**M2: Reflexivity:** *Pxx*

**M3: Antisymmetry:** (*Pxy* ∧*Pyx*) ¬ *x* = *y*

**Df19: Proper Part:** *x* is a proper part of *y,* written *PPxy*, if and only if:

*Pxy* ∧¬*x* = *y*

**Df20: Overlap:** *x* overlaps *y*, written *Oxy* if and only if:

*z*(*Pzx* ∧*Pzy*)

**M4: Strong Supplementation:** ¬Pyx ¬∃z(Pzy ∧ ¬Ozx)

**Df21: Disjoint:** *x* and *y* are disjoint, written *Dxy* if and only if:

¬*Oxy*

**Df22: Underlap:** *x* and *y* underlap, written *Uxy*if and only if:

*z*(*Pxz* ∧*Pyz*)

**Do17: Presentism:** ∀*t*1 *t*1 = *t*@

(Where *t*@ is the “present moment, by analogy with the “actual world”)

**Do18: Eternalism:** ∀*t*1*t*2*t*1 = *t*2

**T21\*: Eternalism:** ∀*t*1*t*2*t*1 = *t*2

⌜∀*t*1*t*2*t*1 = *t*2⌝ is a simple theorem of CQT when quantifying over times. But using it as a refutation of presentism is perhaps a little too fast. The presentist might insist that classical quantification over times is permissible provided that we are talking about *ersatz* times. After all, *had* we decided to quantify over worlds themselves, we would not immediately commit ourselves to possibilism (i.e. modal realism). Hence I have placed an asterisk to note that the “eternalism” here need not be taken to exclude presentism. Presentism, however, will surely be in trouble if past or future events can serve as *truth-makers* for either tensed or non-tensed formulas. It will also be in trouble if there is a fact of the matter as to which events actually occur not only in the past, but also in the future.

The following definition of instantaneous temporal part is taken from Sider (2001):

**Df23: Instantaneous Temporal Part:** *x* is an instantaneous temporal part of *y* at time *t*1 if and only if:

*Pxyt*1 ∧(∀*t*2(*E*!*xt*2 *t*2 = *t*1) ∧∀*z*(*Pzyt*1 ¬ *Oxzt*1))

In words: *x* is an instantaneous temporal part of *y* at time *t*1 just in case, *x* is a part of *y* at *t*1and for every time *t*2, if *x* exists at *t*2, then *t*2 is identical with *t*1and for every *z* if *z* is a part of *y* at *t*1, then *x* overlaps *z* at time *t*1. This definition is intended to capture the idea that each temporal part of an object is a temporal *slice* of some purdeurnt object.

**T22:** There are no instantaneous temporal parts unless for every *t*1, *t*2 **T**, *t*1 = *t*2 (i.e., unless |**T**| = 1)

This is an immediate consequence of PE and the definition of instantaneous temporal part. Every object exists at every time, hence there can be instantaneous objects only if there is but a single instant of time.

It is quite questionable whether it is even possible for there to be but a single moment of time. That is, a model in which there is only a single member of **T** does not seem to differ from a model a world that is *eternal* in the sense in which there is no time at all: there is but a single unqualified state of the world. Moreover, it is quite questionable whether time could come to an end in a naturalistic manner. At any rate, unless time is *wholly* illusionary, there cannot be any instantaneous temporal parts.

**Do19: Four-Dimensionalism (4D)** is the doctrine that (1) Eternalism is true and (2) all objects are either temporal parts or composed of temporal parts.

**T23:** Four-dimensionalism fails on any model in which there is more than one member of **T**.

This is an immediate result of the definition of 4D and **T24**.

No possible world is an interesting four-dimensional world, since 4D can only obtain in worlds that are practically timeless.

**Do20: Spacetime Substantialism** is the doctrine that spacetime points are first-order objects.

Spacetime points, no matter how they might be defined, are going to be temporary existents and so:

**T24:** *Spacetime Substantialism is false unless* |**T**| = 1.

Perhaps I have been too quick to conditionally refute 4D. Perhaps 4D theorists need not construe temporal parts as first-order-objects. After all, on 4D, objects are construed rather more like *occurents*. That is, perhaps quantification over temporal parts is really quantification over *events* while fusions of temporal parts are like processes. Events and processes will be discussed below and have first-order objects as constituents and hence cannot exist without such objects. This will not help matters for the 4D theorists, since the higher-order items *are going to have to be built upon the basis of permanent first-order objects*, while 4D denies that there are any such things.

**3. Events and Processes**

Now that temporality has been introduced, events may be introduced by rendering each *n*-place relation as an *n*+1 relation, where the extra relata is a member of **T**. The result is similar to the manner in which Kim (1973, 1976) treats events. It is arguably the case that atomic first-order formulas implicitly involve a temporal relata or indexes in the first place. What do we mean when we simply say *x* is *F*? Sometimes we implicitly refer to the time of the utterance, so that “*x* is *F*” is construed as “*x is F now*”*.* This is not, however, the case in such propositions as “Socrates is snub-nosed”. Hence, in general, ⌜*Fx*⌝ might be construed as ⌜*t Fxt*⌝.

Many may find the Kim-style an analysis of “event” to be objectionable but I do not intend to be giving a general analysis of how the word “event” behaves in ordinary language. Nor am I trying to provide an analysis of “our concept of ‘event’”. I basically want to be able to talk about predicates being instantiated by objects at times and I am naming such instantiations “events”. An event need not even indicate that any change has occurred. For instance, suppose *t*1 = *p*(*t*2) and both ⌜*Fxyt*1⌝ and ⌜*Fxyt*2⌝ are true. ⌜*Fxyt*2⌝ names an event no less than ⌜*Fxyt*1⌝.

I will, however, note that, some the most glaring difficulties that arise on Kim’s analysis[[43]](#footnote-43) will be removable once the notion of natural property is officially introduced. Moreover, Kim’s approach to analyzing events is by far the easiest to logically model.[[44]](#footnote-44) It will, however, be useful, to model not only simple events, involving a single predicate instantiation, but also events involving several such instantiations occurring at a given time. In addition to these, we will eventually want to model *processes*, or events which unfold over a span of time.

A *simple* event is effectively an atomic formula with the addition of a time, i.e., ⌜*n*1…*nt*⌝. I shall use *e*, with or without subscripts, as the variable for simple events. While event variables count as formulas, it will be useful to assign them a limited kind of quasi-predication, which I shall call *involvement.* If *e* is in event variable substituting for ⌜X*nx*1….*xnt*⌝, then *e* can be said to involve X*n*, *x*1….*xn*, and *t*: ⌜(*e*〈X*n*〉 ∧*e*〈*x*1…*xn*〉 ∧*e*〈*t*〉⌝. Involvement is syntactically distinct from ordinary predication in that ⌜*e*〈*t*〉⌝ does not entail *e*. Moreover, there will be some additional *logically defined* predicates that may be assigned to event variables. Extra-logical predicates (i.e., those which are not logically defined), however, can never be predicted of an event.

Quantification over events is *not* quantification into name position but quantification into *formula* position and in this respect it is similar to quantification over propositions. Just as ⌜X*nx*1….*xnt*⌝ can stand alone as a formula, so can *e*, since one can substitute the one for the other. Again, while events will “exist” necessarily, they only *occur* (or take place) in some worlds. This is because ⌜*e* ¬*e*⌝ is a perfectly coherent formula.

It must be admitted that treating events in this way comes at some cost. Non-occurring events would appear to be abstract objects, while occurent events do not seem to be abstract, and hence concrete. It should be noted, however, that the object language does not attribute such predicates to events. Nevertheless, it must be admitted that, meta-linguistically speaking, non-occurent events would be construed as abstract objects while occurring events are, in some lose sense, concrete. If this is so, am I not treating occurent events as contingently concrete objects and non-occurent events as contingently abstract objects? Again, strictly speaking no, since in the strict sense of the term events are not the sort of things that can really be abstract or concrete; there is something of a category mistake at work here. More importantly, the fact that simple events, being equivalent to temporalized atomic formulas, are naturally denoted by formulas rather than terms means that the distinction between existent and occurent events is a *natural* result of logic itself.[[45]](#footnote-45) I do, however, admit something *like* contingent concreteness and abstractness is being attributed to events and that it is of some concern. The primary appeals here is that our intuitions about the metaphysics of *events* are not terribly well-formed in the first place and that treating events in this way is a direct result of logical considerations and not of some additional theoretical decision, as in the case of positing contingently non-concrete objects.

Also, some defenders of four-dimensionalism may wonder why they could not help themselves to this distinction when quantifying over temporal parts. My response is that they *could*, but not in the context of a first-orderizable theory, where first-order quantification is certainly over terms admitting of non-logically defined predication. So four-dimensionalism might be salvaged within a fundamentally *propositional* language, but this comes at a considerable cost to expressiveness.

One decision that must be made in the formation of any theory of events (or facts) is whether events that necessarily obtain in the same possible worlds are identical. If so, then we can replace identity with the necessitated bi-conditional ⌜□ (X*nx*1…*xnt*  *e*)⌝. This is sometimes referred to as “logical identity”. While this would be convenient, I am not convinced that necessary co-occurrence must yield identity For instance, if non-identical *a* and *b* are essentially *F* and *G* respectively, then, for any *t*, ⌜□ (*Fa*  *Gb*)⌝ will be true. Perhaps more importantly, logical identity raises the threat of the dreaded Slingshot argument. Events are fact-like objects and so they are potentially threated by the Slingshot. One of the standard premises of the Slingshot argument is that facts which are logically equivalent are identical. Denying this, however, requires that we *discern* another standard of identity for events. This is one place where the introduction of structure (involvement) comes in handy: *two events are identical just in case they involve all the same items.*

Rather than introduce separate variables for compound events and processes, I shall introduce a *plural event* variable: *ee* (with or without subscripts). Plural events in general are utterly gerrymandered but various sorts of occurent objects can be obtained by placing various *restrictions* on which sort of atomic events may be contained in a plural event.

Two events will be said to be *simultaneous*, written ⌜*e* ⪅ *e*´⌝ just in case for any *t* **T**, *e*〈*t*〉 if and only if *e*´〈*t*〉. A *compound event,* written *Eee*, is just a plural event *ee* such that any two simple events *contained* in *ee* are simultaneous. Containment will be written ⌜*e* ⋿ *ee*⌝ or ⌜X*nx*1….*xnt* ⋿ *ee*⌝ and is formally like the membership relation of set-theory. Furthermore, a plural event involves anything that is involved by any simple event that it contains.

Let a *span*, written 〈*tm* … *tn*〉,be a set of linearly ordered discrete times (or moments) without gaps (i.e., any *ti* that is after *tm*and before *tn* it is a member of 〈*tm* … *tn*〉). A *thin* process is a sequence of simple events along a span of time: ⌜*em*〈*tm*〉…*en*〈*tn*〉⌝. As shorthand, one may leave out the involved times and just write ⌜[*em*…*en*]⌝. Likewise, a *thick* process in general is a sequence of compound events: ⌜*eem*…*een*]⌝. *P* and *p* will be used as “predicates” for thick process and thin process respectively.

The difference between a thin process and a compound event is that no two simple events contained in a thin process involve the same time while every simple event contained in a compound event involves the same time. All of the definitions mentioned in the last few paragraphs (and others besides) will be clearly laid out below.

**4: The Logic of Times, Events, and Processes**

**New Primitive Terms:**

1. Time Variables: *t*1, *t*2 ….

2. Singular Event variables: *e*1, *e*2 ….

3. Plural Event variables: *ee*1, *ee*2 ….

**New Primitive Connectives:**

⋿, <, =

(Using identity for formulas is technically an operator as opposed to the already introduced identity relation)

**Expanded Grammar:**

1. If *n* is an *n*-place predicate, where *n* 0, α1 … α*n* are terms, and *t* is a time, then α1 … α*nt* is a formula (namely an atomic formula).
2. If σ and σ´ both denote events then σ = σ´ if a formula
3. If σ denotes a singular event and σ´ denotes a plural event, then σ ⋿ σ´ is a formula.

**Defined Terms and Relations:**

**Df24: e-Involvement:**  *e*〈〉  ((*e* = *n*1…*nt*) ∧(( = *n* ∨ = 1…*n*) ∨ = *t*))

**Df25: e-Monadic:** *Me* *e*⟨1⟩

**Df26: Inclusion:** (*ee*1 ⊏ *ee*2)  ∀*e**e* ⋿ *ee*1 ¬ *e* ⋿ *ee*2

Inclusion is to containment what the subset relation is to the membership relation.

**Df27: Span:** *S*〈*t*1…*tn*〉  ∀*ti*(*t*i *<* *t*i < *tn*¬ (*ti* ⋿ 〈*t*1…*tn*〉))

**Df28: Thin Process “Constant”:** [*e*1…*en*]  *t*1…*tn*(*S*〈*t*1…*tn*〉 ∧(*e*1〈*t*1〉 ∧*e*2〈*t*2〉 …… *en*〈*tn*〉))

[*e*1…*en*] may be used to denote a sequence of simple events ordered along a time span.

**Df29: ee-(Weak) Involvement:** *ee*〈〉  *e*((*e* ⋿ *ee*) ∧*e*〈〉)

**Df30: ee-Strong Involvement:** *ee*[]  ∀*e*((*e* ⋿ *ee*) ∧*e*〈〉)

**Df31: Compound Event:** *Eee*  *t*(*ee*[*t*])

A compound event is simply a plurality of events that strongly involves a particular time.

**Df32: Thick Process “Constant”:** [*ee*1…*een*]  *t*1…*tn*(*S*〈*t*1…*tn*〉 ∧(*ee*1[*t*1] ∧*ee*2[*t*2] …… *een*[*tn*]))

**Df33: Event Priority:** (*e*1 < *e*2)  ((*e*1〈*t*1〉 ∧*e*2〈*t*2〉) ∧*t*1 < *t*2)

**Df34: Event Simultaneity:** (*e*1 ⪅ *e*2)  (¬(*e*1 < *e*2) ∧¬(*e*2 < *e*1)

Definitions 34-38 are to be understood relative to compound events.

**Df35: Strong-ee Temporal Priority:** (*ee*1 <*S* *ee*2) ∀*e*1∀*e*2(((*e*1 ⋿ *ee*1) ∧((*e*2 ⋿ *ee*2) ¬ *e*1< *e*2)))

One plurality of events is strongly prior to a second just in case every event contained in the first is temporally prior to every event contained in the second.

**Df36: ee-Weak Temporal Priority:** (*ee*1 <*W* *ee*2) *e*1((*e*1 ⋿*ee*1) ∧∀*e*2((*e*2 ⋿ *ee*2) ¬ (*e*1 < *e*2)))

One plurality of events is weakly prior to a second just in case at least one event contained in the first is prior to every event contained in the second.

**Df37: ee-Temporal Overlap:** (*ee*1 <*O* *ee*2)  (¬(*ee*1 <*S* *ee*2) ∧¬(*ee*2 <*S* *ee*1))

Plural events *ee*1 and *ee*2temporally overlap just in case there is some time *t* such that ⌜(*e*1 ⋿ *ee*1) ∧(*e*2 ⋿ *ee*2)⌝ and ⌜*e*1⟨*t*⟩ ∧*e*2⟨*t*⟩⌝.

**Df38: ee-Temporal Underlap:** (*ee*1 <*U* *ee*2)  ((*ee*1 <*W* *ee*2) ∧*e*1((*e*1 ⋿*ee*1) ∧∀*e*2((*e*2 ⋿ *ee*2) ¬ (*e*2 < *e*1)))

One plural event temporally underlaps a second just in case the earliest member of the first is earlier than the earliest member of the second and the latest member of the first is later than the latest member of the second.

**Df39: ee-Temporal Simultaneity:** (*ee*1 ⪅ *ee*2)  (¬(*ee*1 <*W* *ee*2) ∧¬(*ee*2 <*W* *ee*1))

Two plural events are effectively simultaneous just in case the earliest members of each occur at the same time and it the latest members of each occurs at the same time.

**Df40: Process:** *Pee*1  *t*1…*tn*(*S*〈*t*1…*tn*〉 ∧∀*ee*2((*ee*2 ⋿ *ee*1)  (*ee*2[*t*1]…… *ee*2[*tn*])))

**Df41: Thin Process:** *pee*  (*Pee* ∧∀*t*∀*e*1∀*e*2((((*e*1 ⋿ *ee*) ∧*e*1〈*t*〉) ∧(((*e*2 ⋿ *ee*) ∧*e*2〈*t*〉)))

¬ *e*1 = *e*2))

**Df42: p-Stage:** (*e* ⋿*t* *ee*)  ((*e* ⋿ *ee*) ∧*pee*)

**Df43: P-Stage:** (*ee*1 ⊏*t**ee*2)  ((*ee*1 ⊏ *ee*2) ∧(*Pee*2∧*Eee*1))

Stages are like “instantaneous temporal parts” of processes.

**Df44: Sub-Process:** (*ee*1 ⊏*P* *ee*2)  ((*ee*1 ⊏ *ee*2) ∧(*Pee*1 ∧*Pee*2))

**Df45: Segment:** (*ee*1 ⊏*S ee*2)  ((*ee*1 ⊏*P* *ee*2) ∧((*Pee*2∧*Pee*1) ∧(*ee*2 <*U* *ee*1)))

**Df46: Embedded-Process:** (*ee*1 ⊏*E* *ee*2) ((*ee*1 ⊏*P* *ee*2) ∧(*ee*1 ⪅ *ee*2))

**Axioms**

The standard axioms for classical quantification and identity are extended to the treatment of times, events, and plural events, although the extension of CQT1 to events requires some spelling out due to the complex nature of event formulas.

**Temporal Logic:**

**t1:** ¬*t* < *t* (**t**-Irreflexivity)

**t2:** (*t*1 < *t*2 ∧*t*2 < *t*3) ¬ *t*1 < *t*3 (**t**-Transitivity)

**t3:** (*t*1 < *t*2 ∨*t*2 < *t*1)∨*t*1 = *t*2 (**t**-Linearity)

**t4a:** *t*1 < *t*2 ¬ *t*3(*t*1 < *t*3 ∧¬*t*4 *t*1 < *t*4 < *t*3) (**t**-Discreteness (a))

**t4b:** *t*1 < *t*2 ¬ *t*3(*t*3 < *t*2 ∧¬*t*4 *t*3 < *t*4 < *t*2) (**t**-Discreteness (b)

**t5:** *t*1 < *t*2 ¬ □  *t*1 < *t*2 (Necessity of Priority)

**Event Logic:**

**e1:** ∀*e*(*e*) ¬ (1…*nt*/*e*)(**e-**CQT1)

**Derived: *e*-Existential Introduction** (1…*nt*) ¬ *e*(*e*/1…*nt*)

**ee1:** ∀*ee*(*ee*) ¬ ((*e*1……*en*)/*ee*) (**ee**-CQT1)

**ee2:**  *e*(*e*) ¬ *ee*∀*e*´((*e*´ ⋿ *ee*) (*e*´)) (**ee**-Comprehension)

**ee3:** ∀*eee*(*e* ⋿ *ee*) (**ee-**Non-Vacuity)

**ee4:** ∀*e*((*e* ⋿ *ee*)  (*e* ⋿ *ee*´)) ¬ (*ee* = *ee*´) (**ee**-Identity)

**ee4:** (*e* ⋿ *ee*) ¬ □ (*e* ⋿ *ee*)(Nec, of Containment)

**ee5:** *ee*  ∀*e*((*e* ⋿ *ee*) ¬ *e*) (**ee**-Occurrence)

**Pee1:** *ee*((*ee* ∧(*pee* ∧*ee*[]) ¬ *ee*´((*Pee*´ ∧(*ee* ⪅ *ee*´)) ∧∀*e*(*e* ⋿ *ee*´  (*e* ∧*e*〈〉))))

(**Pee**-Comprehension)

The logic of plural events is, of course, very similar to plural logic for terms except for ee5, which arises on the basis of the fact that events variables are formulas rather than terms (ee5 is something like a principle of “analytic entailment” – see van Fraassen (1969))*.* **Pee1** is required to specify processes centered around the inclusion of a specific *n-*tuple of objects or property. If there is a thin process *ee* that strongly involves  then there is a (typically properly-thick) process *ee*´ that contains every -involving event that occurs during the span of *ee*.

**Semantics**

**Definition of a ℒ Model**

A model, ***ML****,* is five-tuple ***ML* = D, W, T,** <**, I*,*** such that:

**D** is a non-empty set of objects: the “domain”

**W** is a non-empty set of worlds

**T** is a non-empty set of “times”

**<** is a relation on **T** satisfying the following conditions:

(1) **T** is a strict linear ordering:

(a) ∀*t* ¬*t* < *t* (b) ∀*t*1∀*t*2∀*t*3((*t*1 < *t*2 ∧*t*2 < *t*3) ¬ *t*1 < *t*3)

(c) ∀*t*1∀*t*2(*t*1 < *t*2 ∨*t*2 < *t1* ∨*t*1 = *t*2)

(2) For any *t*1, *t*2 **T**,if *t*1 < *t*2, then there exists some *t*3  **T** such that *t*1 < *t*3 and there is no *t*4 **T** such that *t*1 < *t*4 and *t*4 < *t*3 (i.e., **T** is discrete).

***I*** is the interpretation function satisfying the following conditions:

(a) if  is an object constant then ***I***() **D**

(b) if t is a time constant then ***I***(t) **T**

(b) if *n*is an *n*-place predicate constant then ***I***(*n*) assigns *n*  to a set of *n*+2 tuples 〈*x*1, …,*xn*, *t, w*〉[[46]](#footnote-46)

Let 𝝨 be a set such that, for each positive integer *n*, if *n* is an *n*-place predicate, 〈 ***I***(1), …, ***I***(*n*)〉 **D***n*, and *t* **T**, then 〈***I***( 1), …, ***I***(*n*), ***I***(*n*), ***I***(t)〉 𝝨. 𝝨 is the set of all events, possible or otherwise.

Let μ be the *variable assignment* subject to the following conditions:

1. For each time variable *t,* [*t*]*M* **T**

2. For each simple event variable *e*, [*e*]*M* 𝝨

3. For each plural event variable *ee*, [*ee*]*M* 𝝨

The *valuation-function* *VM* for ***ML*** and a variable assignment , is a function that assigns 1 or 0 (true or false) to each formula relative to each world according to the following conditions:

**[*V*∀*t*]** *VM*(∀*t*, *w*) = 1 iff for every *t*´ **T**, *VM*(*t´*/*t*)(, *w*) = 1

**[*Vt*]** *VM*(α1 … α*nt*, *w*) = 1 iff 〈[α1]*M*, … [α*n*]*M*, [*t*]*M*〉 ***I***()(*w*)[[47]](#footnote-47)

**[*V*□ ]** *VMμ*(□ , *w*) = 1 iff for every 〈*w*´, *t´*〉, *w*´ **W** and *t*´ **T**, *V*((*t*´), *w*´) = 1

**[*V*∀*e*]** *VM*(∀*e*, *w*) = 1 iff for every *e*´ 𝝨, *VM*(*e´*/*e*)(, *w*) = 1

**[*Ve*=]** *VM*(*e* = α1 … α*nt*, *w*) = 1 iff [*e*]*M* = 〈*n*, 〈[α1]*M*, … [α*n*]*M*〉,[*t*]*M*𝜇〉

**[*Ve*]** *VM*(*e*, *w*) = 1 iff [*e*]*M* = 〈*n*, 〈[α1]*M*, … [α*n*]*M*〉,[*t*]*M*𝜇〉 and 〈[α1]*M*, … [α*n*]*M*, [*t*]*M*〉 ***I***(*n*)(*w*)

**[*V*∀*ee*]** *VM*(∀*ee*, *w*) = 1 iff for every *ee*´ 𝝨, *VM*(*ee*´/*ee*)(, *w*) = 1

**[*V***⋿**]** *VM*(*e* ⋿ *ee*) iff [*e*]*M* [*ee*]*M*

**[*Vee*]** *VM*(*ee*, *w*) = 1 iff for each *e* such that *e* ⋿ *ee*, *VM*(*e*, *w*) = 1

Compound events, thin processes, and thick processes, might have been provided with their own variables and quantifiers by defining the respective derived domains. These definitions will be useful for keeping track of the semantics of some of the more sophisticated formulas of the object language.

1. Let 𝝨*E* a subset of ℘(𝝨) such that *S* 𝝨*E* if and only if:

For any *s*1, *s*2  𝝨, if *s*1, *s*2  *S* then, for any time *t*, *t* *s*1 if and only if *t* *s*2. That is, 𝝨*E* is the set of all possible compound events.

2. Where **T** is the set of all times, let ***C*** be a subset of ℘(**T**) subject to the following condition:

Where *B* ℘(***T***), *B* ***C***if and only if, for all *t*1,  *t*2 such that *t*1 < *t*2, *t*1  *B* and *t*2 *B* implies that for any *t*3, if *t*1 < *t*3 <*t*2, *t*3 *B*. Roughly speaking, ***C*** is the set of convex subsets of **T**,[[48]](#footnote-48) that is, each member of ***C***is a time-span.

3. Let 𝝨*p* be a set of subsets of {𝝨1, 𝝨2, …× 𝝨*n*} such that, for any positive integer *n*, *sn* 𝝨*p* if and only if:

There exists some *B* ***C*** such that │*B*│= *n* and for any *t*, if *t* *B* there is exactly one *s* *sn* such that *t* *s*.

4. Let 𝝨*P* be a set of subsets of {℘(𝝨1), ℘(𝝨2), …× ℘(𝝨*n*)} such that, for any positive integer *n*, *Sn* 𝝨*P* if and only if:

There exists some *B* ***C*** such that │*B*│= *n* and for each *t*  **T**, *t* *B* there is some *S* *Sn* such that each *s* *S* is such that *t* *s*.

**5. Some Event Logic Theorems**

**T25:** ∀*x*∀*t*X X*xt*

1. ∀*x*∀*ty* *x* =*t* *y*  PE

2. *y* *x* =*t* *y* 1. CQT

3. *E!xt* 2. Def. *E!*

4. ∀*x*∀*t*XX*xt* 3.CQT

Every object always has some property or other (i.e., namely “existence”).

**T26: *e*-Comprehension:** ∀X*n*∀*x*1…*xn*∀*te*□ (X*nx*1…*xnt* = *e*)

1.∀*ee*´□ (*e* = *e*´) ¬ *e*´□ (X *nx*1…*xnt* = *e*´) **e1**

2.∀*ee*´□ (*e* = *e*´) NE

3. *e*´□ (X *nx*1…*xnt* = *e*´) 1-2. PL

4. ∀X*n*∀*x*1…*xn*∀*te*´□ (X*nx*1…*xnt* = *e*´) 3. UG

For every *n*-place predicate X*n*, *n*-tuple of objects, *x*1…*xn* and time *t*, there is some event that is identical to *x*1…*xn* instantiating X*n* at *t*. This is something like a comprehension principle for simple events.

**T27:** ∀*e*((X*x*1…*xnt* = *e*) ¬ □ (X*x*1…*xnt*  *e*))

1. (X*x*1…*xnt* = *e*) ¬ ((X*x*1…*xnt*)  (*e*)) I2

2. (X*x*1…*xnt* = *e*) ¬ (X*x*1…*xnt*  *e*)1. {}/

3. □ (X*x*1…*xnt* = *e*) ¬ □ (X*x*1…*xnt*  *e*) 2. NEC, K, MP

4. (X*x*1…*xnt* = *e*) ¬ □ (X*x*1…*xnt* = *e*) NI

5. (X*x*1…*xnt* = *e*) ¬ □ (X*x*1…*xnt*  *e*) 3, 4. MP

6. ∀*e*((X*x*1…*xnt* = *e*) ¬ □ (X*x*1…*xnt*  *e*)) 5. UI

***Corollary****:* ∀*e*(*e* = *e*) ¬ □ (*e*  *e*)

While necessary logical equivalence is not to be regarded as identity, identity does entail necessary logical equivalence.

**T28: □** ∀*e*X*x*1…*xnt*□ (*e*  X*x*1…*xnt*)

1. ∀*x*□ (*e*  *e*) ¬ □ (*e*  1…*n*𝜏) **e1** □ (*e*  *e*)/

2. □ (*e*  1…*n*𝜏) ¬ X*x*1…*xnt*□ (*e*  X*x*1…*xnt*) EI

3. ∀*x*□ (*e*  *e*) PL, NEC, UG

4. X*x*1…*xnt*□ (*e*  X*x*1…*xnt*) 1-3. PL

5. □ ∀*e*X*x*1…*xnt*□ (*e*  X*x*1…*xnt*) 4. UG, NEC

**T29:** ∀X*nx*1…*xn*∀*te*□ (*e*  X*x*1…*xnt*)

1. (X*x*1…*xnt*) ¬ *e*(*e*/X*x*1…*xnt*) EI

2. □ (X*x*1…*xnt*  X*x*1…*xnt*) ¬ *e*□ (*e* X*x*1…*xnt*) (1, □ (1…*nt*  X*x*1…*xnt*)/)

3. □ (X*x*1…*xnt*  X*x*1…*xnt*) PL, NEC

4. ∀X∀*x*1…*xnte*□ (*e*  X*x*1…*xnt*) 2, 3. MP, UG

**T30:** □ (X*x*1…*xnt* ¬ *e*(*e* ∧□ (*e* ¬ X*x*1…*xnt*)))

1. □ (X*x*1…*xnt*  X*x*1…*xnt*) PL, NEC

2. X*x*1…*xnt* ¬ (X*x*1…*xnt* ∧□ ( X*x*1…*xnt*  X*x*1…*xnt*)) 1. PL

3. (X*x*1…*xnt* ∧□ ( X*x*1…*xnt*  X*x*1…*xnt*)) ¬ *e*(*e* ∧□ (*e* X*x*1…*xnt*)) EI

4. X*x*1…*xnt* ¬ *e*(*e* ∧□ (*e* X*x*1…*xnt*)) 2, 3. PL

5. X*x*1…*xnt* ¬ *e*(*e* ∧□ (*e*¬ X*x*1…*xnt*)) 4 PL

6. □ ∀X∀*x*1…*x*n(X*x*1…*xnt* ¬ *e*(*e* ∧□ (*e*¬ X*x*1…*xnt*))) 5. UG, NEC

**T31: Necessity of e-Involvement:** *e*〈〉  □ *e*〈〉

1. *e*〈〉  ((*e* = X*x*1…*xnt*) ∧(( = X ∨ = *x*1…*xn*) ∨ = *t*) Def 〈〉

2. *e*〈〉  ((*e* = X*x*1…*xnt*) ∧(( = X ∨ = *x*1…*xn*) ∨ = *t*) 1. PL

3. ((*e* = X*x*1…*xnt*) ∧(( = X ∨ = *x*1…*xn*) ∨ = *t*) ¬ (□ (*e* = X*x*1…*xnt*) ∧□ (( = X ∨ = *x*1…*xn*) ∨ = *t*) NI

4. ((*e* = X*x*1…*xnt*) ∧(( = X ∨ = *x*1…*xn*) ∨ = *t*)¬ *e*〈〉 1. PL

5. □ ((*e* = X*x*1…*xnt*) ∧□ (( = X ∨ = *x*1…*xn*) ∨ = *t*)¬ *e*〈〉 4. K

6. *e*〈〉 □*e*〈〉 2-3, 5. PL

**T32: Analytic Entailment:** ∀*ee*∀*e*((*e* ⋿ *ee*) ¬ (*ee* ¬ *e*))

(From **ee5** by PL)

**T33: Priority Anti-Reflexivity:** ¬(*e* < *e*)

By Def. Event Priority and **t1**.

**T34: Priority Transitivity:** ((*e*1 < *e*2) ∧(*e*2 < *e*3)) ¬ (*e*1 < *e*3)

By Def. Event Priority and **t2**.

**T35:** ((*e*1 < *e*2) ∨(*e*2 < *e*1))∨(*e*1 ⪅ *e*2)

By Def. Event Simultaneity and **t3**.

**T36: Eee-Comprehension:** *e*1((*e*1) ∧*e*1〈*t*〉) ¬ *ee*∀*e*2((*e*2 ⋿ *ee*)  ((*e*2) ∧*e*2〈*t*〉))

This is an instance of the **ee2** comprehension schema.

**T37: Non-Vacuity of Time: □** ∀*te*(*e* ∧*e*〈*t*〉)

1. ∀*x*∀*t*X X*xt* **T27**

2. X(X*xt* ∧(X*xt* = X*xt*)) 1. CQT, I1.

3. *e*X(*e* ∧(X*xt* = *e*)) 2. **T26**

4. ((*e* = X*xt*) ∧(( = X ∨ = *x*) ∨ = *t*)) ¬ *e*〈〉 Def 〈〉, PL

5. *e*(*e* ∧*e*〈*t*〉) 3-4. PL

6. □ ∀*te*(*e* ∧*e*〈*t*〉) 5. UG, NEC

It is impossible for there to be a time at which no event occurs.

**T38: Maximal *t*-Event: □** ∀*tee*∀*e*((*e* ⋿ *ee*)  (*e* ∧*e*〈*t*〉))

1. *e*1(*e*1∧*e*1〈*t*〉) ¬ *ee*∀*e*2((*e*2 ⋿ *ee*)  (*e*2 ∧*e2*〈*t*〉)) **ee2**

2. ∀*te*1(*e*1∧*e*1〈*t*〉) ¬ ∀*tee*∀*e*2((*e*2 ⋿ *ee*)  (*e*2 ∧*e2*〈*t*〉)) 1. **UG¬**

3. ∀*te*(*e* ∧*e*〈*t*〉) **T38**

4. ∀*tee*∀*e*2((*e*2 ⋿ *ee*)  (*e*2 ∧*e2*〈*t*〉)) 2-3. PL, NEC

At any world *w* and time *t* there is some compound event that includes every event involving *t* at *w.* Such an event is effectively a *stage* of the world in question.

The next few theorems are basically direct implications of the **ee** comprehension schema.

**T39: *x*-Stage: □** ∀*xee*∀*e*((*e* ⋿ *ee*)  (*e* ∧(*e*〈*t*〉 ∧*e*〈*x*〉)))

At any world *w* and time *t*, and object *x*, there is some compound event that includes every event involving both *t* and *x* at *w.* Such an event is a complete description of *x* a given moment in time (in a given world).

**T40: Maximal *x*-Process: □** ∀*xee*∀*e*((*e* ⋿ *ee*)  (*e* ∧*e*〈*x*〉))

A maximal *x*-process is effectively a process to which every simple event involving *x* belongs. It provides a complete description of *x* relative to the world in question.

**T41: World Process: □***ee*∀*e*((*e* ⋿ *ee*)  *e*)

A world process contains every occurent event within a world. Each world *w* can effectively be identified with the world process that occurs in *w*.

The next two theorems are instances of the **Pee** schema.

**T42: *x-*Segment:** *ee*((*ee* ∧(*pee* ∧*ee*[*x*]) ¬ *ee*´((*Pee*´ ∧(*ee* ⪅ *ee*´)) ∧∀*e*(*e* ⋿ *ee*´  (*e e*〈*x*〉))))

An *x-*segment is a process that contains every event involving an object *x* during a certain span of time.

**T43: World Segment:** *ee*((*ee* ∧*pee*) ¬ *ee*´((*Pee*´ ∧(*ee* ⪅ *ee*´)) ∧∀*e*(*e* ⋿ *ee*´  *e*)))

The next three theorems are analogs of the reflexivity, antisymmetry, and transitivity of the subset relation of set they.

**T44: Reflexivity of Inclusion:** ∀*ee*(*ee* ⊏ *ee*)

1. *ee* = *ee* I1

2. (*ee* = *ee*) ¬ ((*e* ⋿ *ee*)  (*e* ⋿ *ee*)) I2

3. (*e* ⋿ *ee*)  (*e* ⋿ *ee*) 1-2. PL

4. (*e* ⋿ *ee*)  (*e* ⋿ *ee*) 3. PL

5. ∀*e*((*e* ⋿ *ee*)  (*e* ⋿ *ee*)) 4. UG

6. ∀*ee*(*ee* ⊏ *ee*) 5. Def ⊏, UG

**T45: Antisymmetry of Inclusion:** ((*ee*1 ⊏ *ee*2) ∧(*ee*2 ⊏ *ee*1)) ¬ (*ee*1= *ee*2)

1. (*ee*1 ⊏ *ee*2) ¬ ((*e* ⋿ *ee*1) ¬ (*e* ⋿ *ee*2)) Def. ⊏

2. (*ee*2 ⊏ *ee*1) ¬ ((*e* ⋿ *ee*2) ¬ (*e* ⋿ *ee*1)) Def. ⊏

3. ((*ee*1 ⊏ *ee*2) ∧(*ee*2 ⊏ *ee*1)) ¬ (((*e* ⋿ *ee*1) ¬ (*e* ⋿ *ee*2)) ∧((*e* ⋿ *ee*2) ¬ (*e* ⋿ *ee*1)))

1, 2. PL

4. (((*e* ⋿ *ee*1) ¬ (*e* ⋿ *ee*2)) ∧((*e* ⋿ *ee*2) ¬ (*e* ⋿ *ee*1))) ¬ ∀*e*((*e* ⋿ *ee*1)  (*e* ⋿ *ee*2)) PL,CQT

5. ∀*e*((*e* ⋿ *ee*1)  (*e* ⋿ *ee*2)) ¬ (*ee*1= *ee*2) **ee4**

6. ((*ee*1 ⊏ *ee*2) ∧(*ee*2 ⊏ *ee*1)) ¬ (*ee*1= *ee*2) 3-5. PL

**T46: Transitivity of Inclusion:** ((*ee*1 ⊏ *ee*2) ∧(*ee*2 ⊏ *ee*2)) ¬ (*ee*1 ⊏ *ee*3)

1. (*ee*1 ⊏ *ee*2) ¬ ((*e* ⋿ *ee*1) ¬ (*e* ⋿ *ee*2)) Def. ⊏

2. (*ee*2 ⊏ *ee*3) ¬ ((*e* ⋿ *ee*2) ¬ (*e* ⋿ *ee*3)) Def. ⊏

3. ((*ee*1 ⊏ *ee*2)∧(*ee*2 ⊏ *ee*3)) ¬ (((*e* ⋿ *ee*1) ¬ (*e* ⋿ *ee*2)) ∧((*e* ⋿ *ee*2) ¬ (*e* ⋿ *ee*3)) ) 1, 2. PL

4. (((*e* ⋿ *ee*1) ¬ (*e* ⋿ *ee*2)) ∧((*e* ⋿ *ee*2) ¬ (*e* ⋿ *ee*3)) ¬ ((*e* ⋿ *ee*1) ¬ (*e* ⋿ *ee*3)) PL

5. ((*e* ⋿ *ee*1) ¬ (*e* ⋿ *ee*3)) ¬ (*ee*1 ⊏ *ee*3) Def. ⊏

6. ((*ee*1 ⊏ *ee*2)∧(*ee*2 ⊏ *ee*3)) ¬ (*ee*1 ⊏ *ee*3) 3-5. PL

Finally, with the introduction of events, the most precise formulation of DMC can be introduced:

**A1: DMC:** ¬∀*e*(*e*  □ *e*)

The logic of events herein presented is quite rich in both its syntax and semantics. This is, of course, due to the fact that events possess a relatively fine-grained structure, namely the structure of atomic temporal first-order formulas. Further explorations of its theorems must be saved for another time (save for those which are of direct relevance the general theory of necessary substances). The logic is also semantically very rich and it is both beyond the task of this treatise and my present abilities to rigorously discuss its semantic properties. It is, of course, quite obvious that any axiomization will be incomplete since it involves both second-order quantification over predicates and second-order quantification over events themselves.

1. The derived rules presented here are taken from Hughes and Cresswell (1996). [↑](#footnote-ref-1)
2. Williamson (2006) has noted that Movahed has observed that Ibn Sina (better known in the West as Avicenna) anticipated both the Barcan Formula (BF) and the Converse Barcan Formula (CBF). See Movahed (2006). [↑](#footnote-ref-2)
3. *Proof*:

   1. □ Assumption

   2. □ 1. K

   3. □□ S5 theorem

   □ 2, 3. PL

   □ D

   6.  □ 4, 5. PL

   Henceforth, □ ⊢ □ will be known as DR 5 [↑](#footnote-ref-3)
4. Hughes and Cresswell (1996) have named any modal system that validates MC “**Triv.**”, since the modal operators are rendered “trivial” in any system in which MC is valid. [↑](#footnote-ref-4)
5. Ibn Sina distinguishes between that which is necessary in itself and that which is necessitated by something else, this is basically to distinguish that which pertains to the divine nature from everything else. [↑](#footnote-ref-5)
6. MC was probably implicitly held by Greek Neoplatonists such as Plotinus and Proclus. [↑](#footnote-ref-6)
7. I have written **A1\*** rather than **A1** because it will ultimately be replaced with a more precise formulation. [↑](#footnote-ref-7)
8. It should be noted, however, that we have not really established that even possibly Socrates exists. But some sort of examples must be given when discussing these theorems. To anticipate a doctrine according to which it is not the case that Socrates exists, it might be that Mereological Nihilism is true and that nothing like a soul or immaterial mind exits. [↑](#footnote-ref-8)
9. David Lewis (1968). [↑](#footnote-ref-9)
10. These two terms are coined by Williamson (2010). [↑](#footnote-ref-10)
11. In particular: **[*Vx*]** becomes: “*VM*(*x*, *w*) = 1 iff for every **dom**(*w),* *V*(/*x*)(, *w*) =1”. [↑](#footnote-ref-11)
12. In articulating Truth Maker one might prefer to use plural rather than singular variables. [↑](#footnote-ref-12)
13. The best known truth-maker theorist has been Armstrong (1997, 2004). Other prominent truth-maker theorists include Martin (1996) and Cameron (2005, 2008b). The contemporary discussion of truthmakers was initiated by Mulligan, Simons, and Smith (1984). [↑](#footnote-ref-13)
14. This is Armstrong’s (1997, 2004) position, but one could seek other sorts of truthmakers, such as tropes. [↑](#footnote-ref-14)
15. See: Tarski (1944) and Mulligan, Simons, and Smith (1984). [↑](#footnote-ref-15)
16. Armstrong’s favored solution is to appeal to a maximal state of affairs, which would effectively contain every other state of affair as a part. So the proposition “there is no dragon under the table” would be made true by the fact that the maximal fact does not include the state of affairs “there is a dragon under the table.” [↑](#footnote-ref-16)
17. Although Heil (2000) is a truth-maker theorists who denies necessitism. [↑](#footnote-ref-17)
18. Armstrong (1997) presents an argument in favor of Truth-Maker Necessitism but Cameron (2008b), who happens to be a proponent of the doctrine, argues persuasively that Armstrong’s argument, and any evident modification thereof, ultimately begs the question. [↑](#footnote-ref-18)
19. The proof is run using singular variables, but could also be run using plural variables. [↑](#footnote-ref-19)
20. Recall that *E!* is a logic predicate defined as ⌜*y* *x=y*⌝. [↑](#footnote-ref-20)
21. Marcus (1986) argues along these lines. [↑](#footnote-ref-21)
22. This definition of “actualism” is somewhat stronger than is the norm but I think that Williamson is correct in his judgment that the actualism vs. possibilism, as typically formulated is rather obscure (Williamson 1998, 2010). Nevertheless, the above definition of actualism follows Adams (1981) where he writes “Actualism is the doctrine that there *are* no things that do not exist in the actual world” (7). Barcan (1986) and Bennett (2006) also endorse this position. [↑](#footnote-ref-22)
23. The following proof was first put forward by Prior (1956). [↑](#footnote-ref-23)
24. *Proof:*

    1. □*x* *x*□ BF

    2. *x*□ □*x* 1. US [/]

    3. □*x* *x*□ 2. PL

    4. *x* *x* 3. Def.

    5. *x* *x* 4. QI2 [↑](#footnote-ref-24)
25. Williamson (2009) shows that this second-order version of BF holds even on quantified modal logics with variable domains. [↑](#footnote-ref-25)
26. Williamson (2006) cites Movahed (2004), in which this discovery is presented. [↑](#footnote-ref-26)
27. It might be noted that Parsons himself reads NE to merely say “necessarily, everything is something” rather than to say that everything necessarily exists. This, however, requires the rejection of Existentialism. [↑](#footnote-ref-27)
28. Any doctrine or definition that relies upon non-official primitives shall not be numbered, since they are not officially part of the formal theory. Their use here is for purely dialectical purposes. [↑](#footnote-ref-28)
29. Some essentialists might deny that sortal essentialism applies to artifacts by denying that artifacts are genuine substances. [↑](#footnote-ref-29)
30. It is rather surprising that that Quine (1953.1961: 156) took SQML in to entail Aristotelian essentialism (all that he in fact shows is that it involves a theorem equivalent to the necessity of identity. Parsons (1969), however, rightly shows that SQML does not entail any kind of interesting essentialism. [↑](#footnote-ref-30)
31. One might prefer to use a plural variable in the place of *y*. [↑](#footnote-ref-31)
32. Employing CBF enables the following alternative proof for NE (Deutsch 1990):

    1. *xy* *x=y* CQT

    2. □*xy* *x=y* 1. NEC

    3. □*xy* *x*=*y* *x*□*y* *x=y* CBF

    4. *x*□*y* *x=y* 2, 3. MP

    5. □*x*□*y* *x=y* 4. NEC) [↑](#footnote-ref-32)
33. Williamson (2010) provides the following axiomatic proof that the conjunction of BF and CBF entail NNE:

    1. *xy* *x=y* *xy* *x=y* (CBF)

    2. *xy* *x=y* (CQT)

    3. *xy* *x=y* (2. NEC, Def. )

    4. *xy* *x=y* (1,3. PL)

    5. *xy* *x=y* (4, S5)

    6. *x y* *x=y* *x y* *x=y* (BF)

    7. *xy* *x=y* (5, 6. PL)

    8. □*x*□*y* *x=y* (7. Def. ) [↑](#footnote-ref-33)
34. Williamson (1998) 262. [↑](#footnote-ref-34)
35. It is also ironic that the great advocate of the necessity of identity should banish the use of names from QML. We can express the idea that if *x* any *y* are identical, then they are necessarily identical, then they are necessarily identical, but we immediately go beyond the confines of logic when we say “Hesperus is identical with Phosphorus”. [↑](#footnote-ref-35)
36. Kim’s analysis, upon which mine is partially based, is also structured, but he does not provide a logic of events, much less a quantified logic of both singular and plural events. [↑](#footnote-ref-36)
37. Williamson (2013), however, recognizes that necessitism could hold while permanentism fails to hold, but suggests that most necessitists will also be permantentists. [↑](#footnote-ref-37)
38. Whether eternalism amounts to the elimination of our *ordinary* conception of time is quite another question and one on which I have no strong opinion (it should be noted that Augustine’s analysis of time was phenomenological in character). What is quite certain, however, is that it does not amount to the elimination of our *scientific* conception of time, which is relatively flexible. [↑](#footnote-ref-38)
39. **Kt**is neutral regarding a number of questions regarding **T**. It is indifferent as to whether or not **T** is dense, or discrete, has a minimal element or a maximal element etc. Indeed, **Kt** alone does not even guarantee that **<** is a partial ordering over **T,** much less a total ordering. When analyzing events it will be presumed that time is not dense. [↑](#footnote-ref-39)
40. See Craig and Smith (eds.) (2007) for a collection of papers defending absolute simultaneity in the context of contemporary physics. [↑](#footnote-ref-40)
41. Although in the latter case < is effectively an operator rather than a relation. [↑](#footnote-ref-41)
42. Contemporary mereology has its origins in Leśniewski (1916) and Lenard and Goodman (1940). [↑](#footnote-ref-42)
43. In particular, I have in mind that, on Kim’s analysis, “Brutus stabbing Caesar” and “Brutus stabling Caesar violently” come out as two separate events when it seem that they are really just two distinct ways of describing one and the same event. [↑](#footnote-ref-43)
44. The more sophisticated analysis of events found in Davidson (1967, 1969) is focused upon action verbs, instances of which are analyzed as three-place relations involving an agent, a patient, an event. Hence events themselves are provided with a primitive domain. [↑](#footnote-ref-44)
45. This would still have been the case if events were construed as state of affairs instantiated by times. [↑](#footnote-ref-45)
46. Alternatively, it could be said that ***I*** assigns each p to a function **g**: **T**×**W** ℘(**D***n*). [↑](#footnote-ref-46)
47. Where “***I***()(*w*)” is understood to be the interpretation of  restricted to a world *w* (i.e., ***I***()(*w*) includes all and only those members of ***I***() of which *w* is a member. [↑](#footnote-ref-47)
48. Although “convex” is typically reserved for real-valued sets. [↑](#footnote-ref-48)