# Ensuring (Statistical) Privacy


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Code for this talk: https://github.com/alabid/pre_college_2019

See full course materials here: http://people.seas.harvard.edu/~salil/cs208/


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# The Problem

We have a dataset with sensitive information, such as:

1. Health records (e.g. reveals which disease a patient has)
2. Census data (e.g. reveals income range)
3. Social network activity (e.g. which pages you like)

How can we:

1. Allow the use of the data?
2. Protect the privacy of the data subjects?
3. Achieve both (1) and (2)?

# Some Approaches to Solve the Problem

## Encrypt the Data:

| Name | Sex | Blood | … | HIV? |
| --- | --- | --- | --- | --- |
| James | M | B | … | N |
| Peter | M | O | … | Y |
| … | … | … | … | … |
| Paul | M | A | … | N |
| Eve | F | B | … | Y |

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| Name | Sex | Blood | … | HIV? |
|---|---|---|---|---|
| 10101 | 01010 | 01000 | … | 00001 |
| 11010 | 01101 | 10111 | … | 10111 |
| … | … | … | … | … |
| 10100 | 10000 | 11101 | … | 01111 |
| 11000 | 10001 | 11110 | … | 10001 |

# Some Approaches to Solve the Problem

Encrypt the Data: Are we happy with this solution? Why or why not?

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# Some Approaches to Solve the Problem

"Anonymize the Data": Are we happy with this solution? Why or why not?

| Name | Sex | Blood | … | HIV? |
|---|---|---|---|---|
| James | M | B | … | N |
| Peter | M | O | … | Y |
| … | … | … | … | … |
| Paul | M | A | … | N |
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| Name | Sex | Blood | … | HIV? |
|---|---|---|---|---|
| XXXXX | M | B | … | N |
| XXXXX | M | O | … | Y |
| … | … | … | … | … |
| XXXXX | M | A | … | N |
| XXXXX | F | B | … | Y |

# Some Approaches to Solve the Problem

“Anonymize the Data”: Not sufficient because of linkage attacks!

87% of US population have unique date of birth, gender, and postal code!

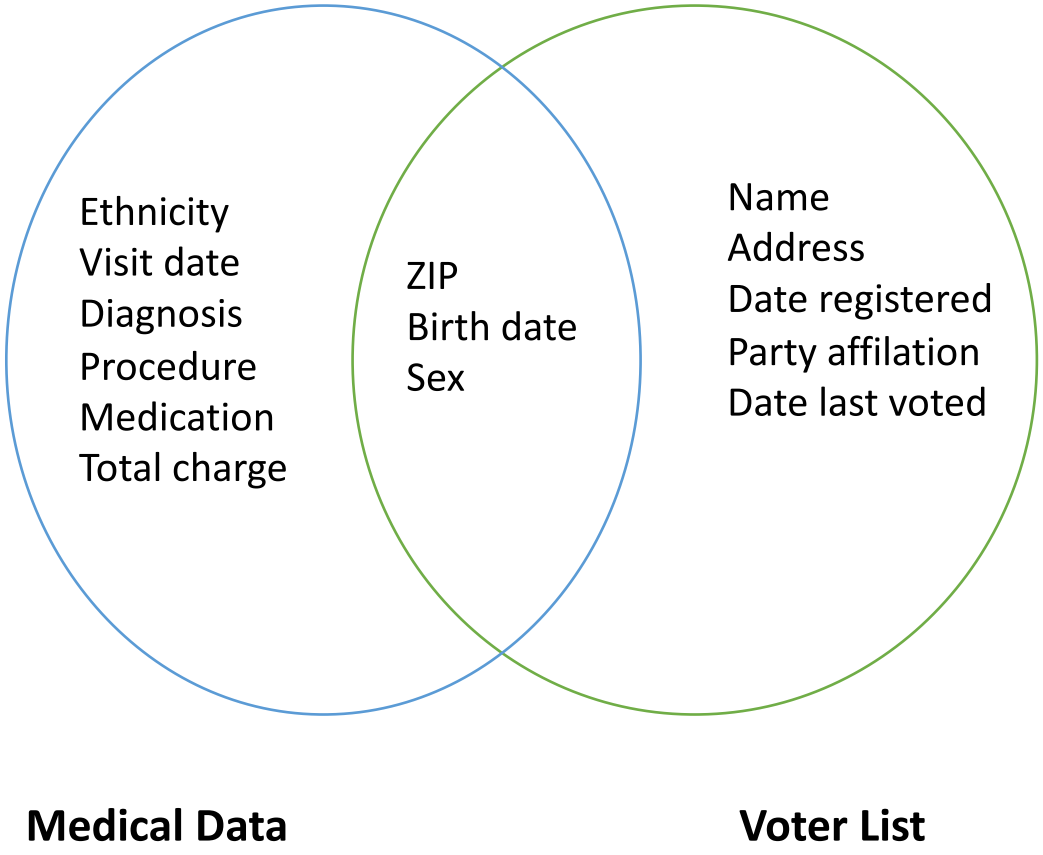
[Golle and Partridge 09]

# Some Approaches to Solve the Problem

## “Anonymize the Data”: Reidentification via Linkage

Can uniquely identify > 60% of the U.S. population [Sweeny ’00, Golle ‘06, Sweeney ‘97]

| Name | Sex | Blood | … | HIV? |
|---|---|---|---|---|
| XXXXX | M | B | … | N |
| XXXXX | M | O | … | Y |
| … | … | … | … | … |
| XXXXX | M | A | … | N |
| XXXXX | F | B | … | Y |



# The story so far

- Motivations
  - Reidentification via Linkage Attacks
  - Reconstruction and Inference Attacks

Reconstruction attack: If we have dataset $x \in \{0,1\}^n$ and person $i$ has sensitive bit $x_i$ and attacker/adversary gets $q_S(x) = \sum_{i \in S} x_i$ for any $S \subseteq [n]$.

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[Dinur-Nissim '03]: With high probability, adversary can reconstruct 0.99 fraction of the dataset $x \in \{0, 1\}^n$ if noise added to each query is less than $o(\sqrt{n})$ and #queries is $n$.

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Inference attack: Attacker gets $n^2$ answers and needs to know if someone is in dataset or not.

# The story so far

## Message

Releasing too many statistics with too much accuracy can lead to a reconstruction of the entire dataset or inference attacks

# Some Approaches to Solve the Problem

So now what?

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Possible Responses:

- Privacy is an illusion!
- In the long run, it’s better to use data for research! Ignore privacy!
- Never release statistics about any dataset!
- Is there a way to add enough noise to queries and still allow for usefulness?

# Main Message of this Talk

Yes, there is a way to add enough noise to queries and still allow for usefulness!

# An approach: K-anonymity

[Sweeney ‘02]: A mechanism satisfies $k$-anonymity if for every dataset, the output of the mechanism has the property that every distinct row occurs at least $k$ times.

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*Intuition*: Privacy ensured if I can't isolate you!

# An approach: K-anonymity

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3-anonymous dataset:

| Zip code | Age | Nationality |
|---|---|---|
| 021** | < 30 | * |
| 021** | < 30 | * |
| 021** | < 30 | * |
| 021** | > 40 | * |
| 021** | > 40 | * |
| 021** | > 40 | * |
| 021** | 3* | * |
| 021** | 3* | * |
| 021** | 3* | * |

# An approach: K-anonymity

It’s a nice approach but doesn’t:

- Compose well (e.g. if you have two $k$-anonymous datasets)
- Utility not as quantifiable as other approaches

| Zip code | Age | Nationality |
|---|---|---|
| 021** | < 30 | * |
| 021** | < 30 | * |
| 021** | < 30 | * |
| 021** | > 40 | * |
| 021** | > 40 | * |
| 021** | > 40 | * |
| 021** | 3* | * |
| 021** | 3* | * |
| 021** | 3* | * |

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# Differential Privacy

- Utility
- Privacy
- Definition

# Differential Privacy

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  - Now and in the future!
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- Definition: pure and approximate

# Differential Privacy

<u>Definition</u>: pure and approximate

[Dwork-McSherry-Nissim-Smith ‘06]

Other references:

Motivated from and based off of work in

[Dinur-Nissim ‘03, Dwork-Nissim ‘04, Blum-Dwork-McSherry-Nissim ‘05]

# Differential Privacy

<u>Definition</u>: pure and approximate

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*Intuition*: for a statistic, the effect of each individual (whether in the dataset or not) should be close to nothing.

# Differential Privacy

<u>Definition</u>: pure and approximate

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*Intuition*: for a statistic, the effect of each individual (whether in the dataset or not) should be close to nothing.

*Worst-case notion*: protects against all possible adversaries and any kind of individual.

# Differential Privacy

<u>Definition</u>: pure and approximate
[Dwork-McSherry-Nissim-Smith ‘06]

For any algorithm $\mathcal{A}$, it satisfies $\epsilon$ differential privacy if
For all datasets $D, D'$ differing in exactly one row all queries $q$
Distribution of $\mathcal{A}(D, q)$ is at most $\epsilon$ away from $\mathcal{A}(D', q)$

The smaller $\epsilon$ is, the more privacy is ensured!

# Differential Privacy

Definition: pure and approximate

[Dwork-McSherry-Nissim-Smith '06]

For any algorithm $\mathcal{A}$, it satisfies $\epsilon$ differential privacy if

For all datasets $D, D'$ differing in exactly one row all queries $q$

Distribution of $\mathcal{A}(D, q)$ is at most $\epsilon$ away from $\mathcal{A}(D', q)$

For all sets $T$,

$\Pr[\mathcal{A}(D, q) \in T] \leq (1 + \epsilon) \Pr[\mathcal{A}(D', q) \in T]$

# Differential Privacy

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[Dwork-McSherry-Nissim-Smith ‘06]

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For all datasets $D, D'$ differing in exactly one row all queries $q$
Distribution of $\mathcal{A}(D, q)$ is at most $\epsilon$ away from $\mathcal{A}(D', q)$

The probability is only over the randomness of the algorithm $\mathcal{A}$

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# Mechanisms for Differential Privacy

Examples:

1. Laplace Mechanism [we’ll discuss and implement this one!]
2. Gaussian Mechanism
3. Exponential Mechanism
4. Geometric Mechanism

….

# Laplace Mechanism

$\mathrm{Lap}(s)$ is the Laplace Distribution with scale $s$.

Some properties:

- Has mean 0
- Has standard deviation $\sqrt{2} \cdot s$
- It's a “double-exponential” distribution

# Laplace Mechanism for Sum and Average

1. $\mathcal{A}(x) = \sum_{i=1} x_i + \mathrm{Lap}(\frac{1}{\epsilon})$

where $x_i \in [0, 1]$ for all $i \in [n]$.

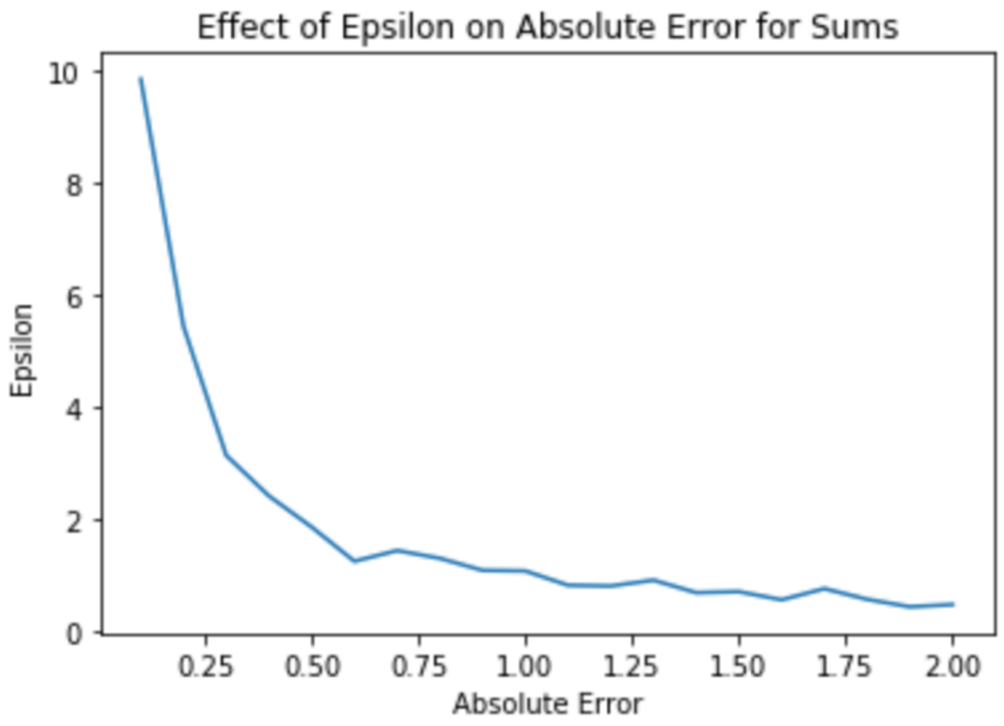
# Laplace Mechanism for Sum and Average

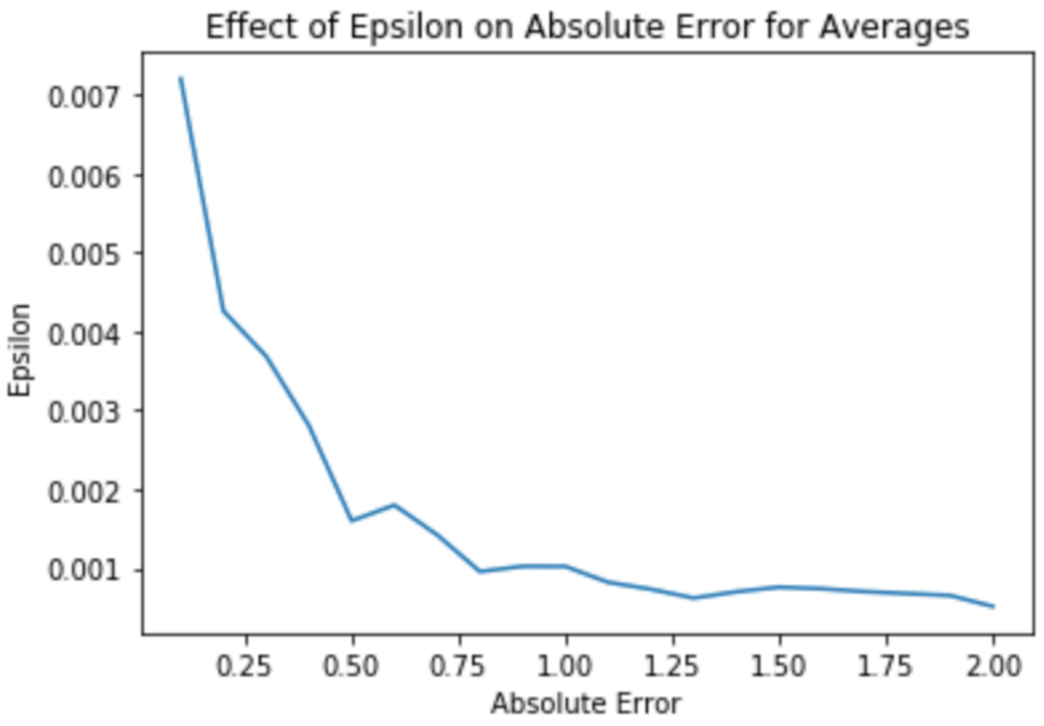
*1.* $\mathcal{A}(x) = \sum_{i=1} x_i + \mathrm{Lap}(\frac{1}{\epsilon})$
where $x_i \in [0,1]$ for all $i \in [n]$.

*2.* $\mathcal{A}(x) = \frac{1}{n} \cdot \sum_{i=1} x_i + \mathrm{Lap}(\frac{1}{n \cdot \epsilon})$
where $x_i \in [0,1]$ for all $i \in [n]$.

# Code Demonstration

- https://github.com/alabid/pre_college_2019



# Conclusion

- Differential Privacy is a mathematically rigorous definition of individual data privacy.
- YOU can code it up. See GitHub page – clone it and play with the code there! https://github.com/alabid/pre_college_2019
- It's being used by the U.S. Census Bureau (for the 2020 Decennial Census), Google, Apple.