Lab1: Pendulum

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1 Objective

The objective of this lab was to gain experience with the simple pendulum and further investigate and understand how the period is affected by the length of the pendulum, the mass, and the amplitude of the oscillation.

2 Description

There are three parts of this experiment: a simple pendulum experiment, a thin tube as a physical pendulum, and a tube with a weight/cylinder as a physical pendulum.

In the simple pendulum experiment, the dependence on mass and length are evaluated. In the thin tube experiment, the center of mass and the moment of inertia are recorded. The results of this experiment are compared to that of the simple pendulum. For the last part of the experiment, the physical pendulum is complicated by the addition of a cylinder, changing the center of mass and the moment of inertia.

3 Theory

The following equations represent simple pendulums, meaning pendulums consisting of a mass on a string with negligible mass. The small angle approximation, where $\sin\theta$ is approximated to θ, is applied to Newton's second law. The length of the string is L, the mass is represented by M, and the angle that string makes with the vertical θ.

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0 \tag{1}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \tag{2}$$

T is the period and ω is the frequency. Both of these do not depend on the mass. The attributes of physical pendulums can be expressed as

$$\frac{d^2\theta}{dt^2} + \frac{MgR}{I}\theta = 0 \tag{3}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MgR}} \tag{4}$$

using I as the moment of inertia and R be the distance between the rotational point and the center of mass. This is calculated using the torque equation $t = I/alpha$. The solution to this second order differential is expressed as $/thetasin(/omegat + /phi)$.

4 Procedure

4.1 Simple Pendulum Experiment

Capstone is used to collect and record data from a photo gate. A pendulum is created using a mass and some string clamped to a bar. The photo gate is positioned in the resting place of the mass in its equilibrium position. The photo gate is configured to measure the Period, Block-to-Block times, speed, and time. These variables are measured for both a 200g mass and 50g mass. Then, the pendulum length was shortened from 78 cm to 38 cm. This tests both the dependence on mass and length.

4.2 Thin Tube As A Physical Pendulum

A 40 cm, 30g, hollow rod was used along with a rotary motion sensor in Capstone as a physical pendulum. The frequency and period were calculated as a function of angular deflection. Three to four cycles were recorded and the line of best fit was calculated.

4.3 Tube Plus Cylinder As A Physical Pendulum

The same thin rod was used as in the previous section, however a 75g mass was attached. The period was measured to be compared to calculated values. The moment of inertia was as described below.

$$I_c = \frac{1}{4}M_c\left(R_1^2 + R_2^2\right) + \frac{1}{12}M_cL^2 + M_cR_c^2 \tag{5}$$

The results were compared to Eq 4.

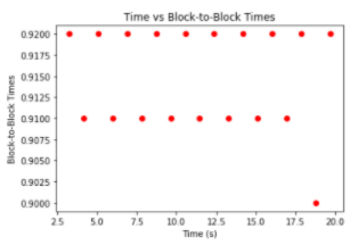
5 Data and Calculations

Python was used to analyze the data, as well as the "calculate line of best fit" function within Capstone.

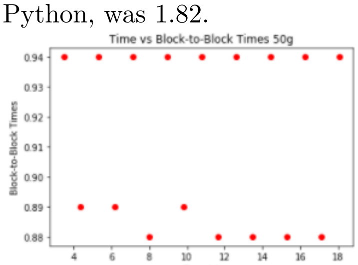
	mass (g) Set	Date and Time Run #1	Time (s) Run #1	State Run #1	Block Event Times (s) Run #1	Period (s) Run #1	Block-to-Block Times (s) Run #1	Speed (m/s) Run #1	Time in Gate (s) Run #1	Date and Time Run #2	...	Speed (m/s) Run #3	Time in Gate (s) Run #3	Date and Time Run #4	Time (s) Run #4	State Run #4	Block Event Times (s) Run #4	Period (s) Run #4	Block-to-Block Times (s) Run #4	Speed (m/s Run #
0	200.0	09/23/2020 01:58:32 pm	1.450	0.0	NaN	NaN	NaN	NaN	NaN	09/23/2020 02:03:35 pm	...	NaN	NaN	09/23/2020 02:09:22 pm	0.038	1.0	0.04	NaN	NaN	Na
1	200.0	09/23/2020 01:58:33 pm	2.342	1.0	2.34	NaN	NaN	NaN	NaN	09/23/2020 02:03:36 pm	...	NaN	NaN	09/23/2020 02:09:22 pm	0.612	0.0	NaN	NaN	NaN	0.0
2	200.0	09/23/2020 01:58:33 pm	2.448	0.0	NaN	NaN	NaN	0.15	0.11	09/23/2020 02:03:36 pm	...	0.03	0.64	09/23/2020 02:09:23 pm	1.372	1.0	1.37	NaN	1.33	Na
3	200.0	09/23/2020 01:58:34 pm	3.264	1.0	3.26	NaN	0.92	NaN	NaN	09/23/2020 02:03:37 pm	...	NaN	NaN	09/23/2020 02:09:23 pm	1.915	0.0	NaN	NaN	NaN	0.0
4	200.0	09/23/2020 01:58:34 pm	3.370	0.0	NaN	NaN	NaN	0.15	0.11	09/23/2020 02:03:37 pm	...	0.74	0.02	09/23/2020 02:09:24 pm	2.039	1.0	2.04	2.0	0.67	Na

Attached is some sample data

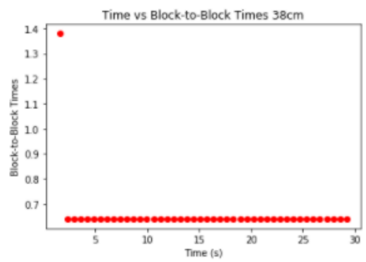
5.1 Simple Pendulum Experiment



The line of best fit graphing angle vs time within Capstone for the simple, 78cm long, pendulum with the 200g mass was $Asin(/omegat + /phi)$. The values for each were as follows $0.191sin(6.35t + -5.28)$ The average period, as calculated by Python, was 1.82.



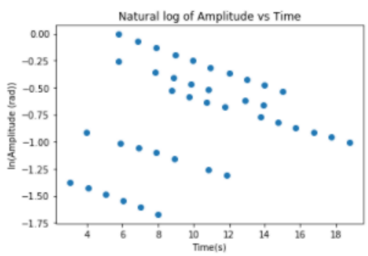
The line of best fit graphing angle vs time within Capstone for the simple, 78cm long, pendulum with the 50g mass was $Asin(/omegat + /phi)$. The values for each were as follows $-0.24sin(6.33t + -20.1)$ The average period, as calculated by Python, was 1.82.



The average period when the length of the string was decreased by 40cm was 1.32.

5.2 Thin Tube As A Physical Pendulum

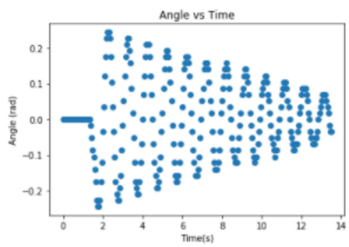
For this part of the experiment, a thin tube of 40 cm and 30g was used as a physical pendulum. For each trial, the initial angle of oscillation was increased by 0.2 rad starting at 0.2 rads. The graph below shows multiple trials.



The natural logs of the amplitude was compared to the time to show straight lines for each trials. This shows that the damping is proportional to the angular velocity since the amplitude is exponential.

5.3 Tube Plus Cylinder As A Physical Pendulum

	Time (s) Run #1	Angle (rad) Run #1	Angular Velocity (rad/s) Run #1	Angular Acceleration (rad/s²) Run #1	Position (m) Run #1	Velocity (m/s) Run #1	Acceleration (m/s²) Run #1	Time (s) Run #2	Angle (rad) Run #2	Angular Velocity (rad/s) Run #2	...	Position (m) Run #6	Velocity (m/s) Run #6	Acceleration (m/s²) Run #6	Time (s) Run #7	Angle (rad) Run #7	Angular Velocity (rad/s) Run #7
0	0.00	0.0	NaN	NaN	0.0	NaN	NaN	0.00	0.0	NaN	...	0.000000	NaN	NaN	0.00	0.0	NaN
1	0.05	0.0	0.0	NaN	0.0	0.0	NaN	0.05	0.0	0.0	...	0.000000	-0.004	NaN	0.05	0.0	0.0
2	0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.10	0.0	0.0	...	-0.000417	-0.003	0.028	0.10	0.0	0.0
3	0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.15	0.0	0.0	...	-0.000417	-0.001	0.023	0.15	0.0	0.0
4	0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.20	0.0	0.0	...	-0.000417	0.000	0.014	0.20	0.0	0.0



$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MgR}} = 2\pi\sqrt{\frac{\frac{1}{4}M_c\left(R_1^2 + R_2^2\right) + \frac{1}{12}M_cL^2 + M_cR_c^2}{MgR}} = \tag{6}$$

Above a sample of the data can be seen. The period was calculated within Capstone and found to be 1.33. The calculated value was 1.27.

6 Error Analysis

$$T_c = \frac{T}{1 - \frac{1}{4}\sin^2\left(\frac{\theta_m}{2}\right)} \tag{7}$$

Above is the equation representing the correction of the period, T, as a result of the use of the small angle approximation for sine in Eq 1. Most oscillations were started at 0.2 radians.

Error propagation can be used in the future to determine the error as a result of the variance in each variable.

6.1 Simple Pendulum Experiment

6.1.1 200 Grams

The error for $/omega$, as calculated by Capstone, was $/pm$ 0.014.

6.1.2 50 Grams

The error for $/omega$, as calculated by Capstone, was $/pm$ 0.002.

6.1.3 Shortened String

The error for */omega*, as calculated by Capstone, was */pm* 0.0015.

7 Conclusions

7.1 Simple Pendulum Experiment

The period did not depend on the mass, but instead the length between the mass and the point of rotation.

7.1.1 Period of a Point Mass

The equation below reflects the period of a point mass

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MgR}} \tag{8}$$

This is derived from the following equation.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MgR}} \tag{9}$$

The moment of inertia, I, for a point mass is mr^2. Substitute into the previous equation, this becomes:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MgR}} = 2\pi\sqrt{\frac{MR^2}{MgR}} \tag{10}$$

The M and and the radius cancel out to form the initial equation.

7.2 Thin Tube As A Physical Pendulum

7.2.1 Center of Mass

The distance, R, of the center of mass from the pivot is given by

$$R_t = \frac{b-a}{2} \tag{11}$$

where b is the length of the longer segment and a is the length of the shorter segment. The center of mass, C, for a thin stick is usually given by

$$C = \frac{L}{2} \tag{12}$$

where L is the length of the stick. Relative to the pivot, $a/2$ is subtracted from the pivot point as this portion of the tube is above the pivot point, therefore is negligible.

7.2.2 Moment of Inertia of the tube

The formula for moment of Inertia is the following:

$$I = \int_0^M r^2 dm \tag{13}$$

Since it was determined that the center of mass given the pivot point is

$$R_t = \frac{b-a}{2} \tag{14}$$

This can be substitute into the equation for moment of inertia to find the moment of inertia for the hollow tube below.

$$I_t = \frac{M_t}{3}\left(\frac{a^3+b^3}{a+b}\right) \tag{15}$$

7.3 Tube Plus Cylinder As A Physical Pendulum

The moment of inertia of the cylinder is significantly greater than that of the hollow tube due to the distribution of weight and the amount of it. The cylinder is a large mass distant from the point of rotation, maximizing the moment of inertia, while the hollow tube is a lesser weight distributed across the entire radius. The period of this pendulum was greater as compared to the thin tube alone, which is expected due to the moment of inertia. This makes sense as the torque produced by gravity on the pendulum is greater due to the greater moment of inertia.