# 1 Modèle mathématique

Minimiser $$J(\tilde{\mathbf{r}},\lambda) = \sum_{i=1}^{n-1} T^{(i)}(\tilde{p}_i, \tilde{r}_i, \tilde{r}_{i+1}, \lambda) + T^{(n)}(a_n^{des}, \tilde{r}_n, \tilde{p}_n, \lambda) + c_I \sum_{i=1}^{n} \lambda_i \tilde{d}_i$$

subject to

$$(\prod_{j=2}^{i} a_j)\, r_1 + (\sum_{j=2}^{i-1} (\prod_{k=i}^{j+1} a_k)\, b_j) + b_i \leq \tilde{r}_i \leq (\prod_{j=2}^{i} c_k)\, r_1 + (\sum_{j=2}^{i-1} (\prod_{k=i}^{j+1} c_j)\, e_j) + e_i$$

$$\sum_{i=1}^{n-1} \lambda_i = m$$

$$\lambda_i \in \{0,1\} \;\; \forall i, 1 \leq i \leq n-1$$

$$\tilde{r}_1 = r_1 \; \tilde{p}_1 = p_1.$$

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## 1.1 $T^{(i)}$ en fonction de $\tilde{r}_i$, $\tilde{p}_i$, $\tilde{r}_{(i+1)}$

$$T^{(i)}(\tilde{p}_i, \tilde{r}_i, \tilde{r}_{i+1}, \lambda) = A - B - C * ln(D - E) \tag{1}$$

Avec:

$$A = \frac{k_i \tilde{p}_i}{(\tilde{p}_i + \tilde{r}_i)^2 - \frac{k_i \tilde{r}_i r_{i+1} (\tilde{r}_i + \tilde{p}_i)(\tilde{r}_{i+1} - \tilde{r}_i)}{\tilde{d}_i \alpha}} \tag{2}$$

$$B = \frac{k_i p_{i+1} (r_{i+1} - \tilde{r}_{i+1})}{\left( (\tilde{r}_i + \tilde{p}_i)\alpha - \frac{k_i \tilde{r}_i r_{i+1} (\tilde{r}_{i+1} - \tilde{r}_i)}{\tilde{d}_i} \right)} \tag{3}$$

$$C = \frac{\left[ (k_i - \tilde{d}_i) r_{i+1} (\tilde{r}_{i+1} - \tilde{r}_i) - \tilde{d}_i p_{i+1} (r_{i+1} - \tilde{r}_{i+1}) \right] \tilde{d}_i \alpha \left[ \tilde{d}_i (\tilde{p}_i + \tilde{r}_i) - k_i \tilde{r}_i \right]}{\left[ \tilde{d}_i (\tilde{p}_i + \tilde{r}_i)\alpha - k_i \tilde{r}_i r_{i+1} (\tilde{r}_{i+1} - \tilde{r}_i) \right]^2} \tag{4}$$

$$D = \frac{\tilde{p}_i \tilde{d}_i}{\tilde{r}_i \left( k_i \frac{r_{i+1}(\tilde{r}_{i+1} - \tilde{r}_i)}{\alpha} - \tilde{d}_i \right)} \tag{5}$$

$$E = \frac{\alpha \tilde{p}_i (\alpha (\tilde{p}_i + \tilde{r}_i) \tilde{d}_i - k_i \tilde{r}_i r_{i+1} (\tilde{r}_{i+1} - \tilde{r}_i))}{\tilde{r}_i (\tilde{p}_i + \tilde{r}_i) p_{i+1} (r_{i+1} - \tilde{r}_{i+1}) (k_i r_{i+1} (\tilde{r}_{i+1} - \tilde{r}_i) - \tilde{d}_i \alpha)} \tag{6}$$

$\alpha = (r_{i+1}(\tilde{r}_{i+1} - \tilde{r}_i) + p_{i+1}(r_{i+1} - \tilde{r}_{i+1}))$

## 1.2 $T^{(n)}$ en fonction de $\tilde{r}_n$, $\tilde{p}_n$, $a_n^{des}$

$$\begin{aligned} T^{(n)}(a_n^{des}, \tilde{r}_n, \tilde{p}_n, \lambda) &= c_p \left( K_n \frac{a_n^{des}(\tilde{p}_n + \tilde{r}_n) - \tilde{r}_n}{(\tilde{p}_n + \tilde{r}_n)(1-\rho_n)\tilde{p}_n} \right) \\ &+ \left( \frac{(\tilde{p}_n + \tilde{r}_n)(1 - a_n^{des})}{\mu_n (1-\rho_n)^2 \tilde{p}_n} - \frac{1}{\mu_n(1-\rho_n)} \right) \ln \left[ \frac{1}{\rho_n} \left( 1 - \frac{(1-\rho_n)}{(1-a_n^{des})(\frac{(\tilde{p}_n + \tilde{r}_n)}{\tilde{p}_n})} \right) \right] \end{aligned}$$

with: $\rho_n = \frac{\tilde{r}_n (k_n - \frac{\tilde{d}_n}{a_n^{des}})}{\tilde{p}_n \frac{\tilde{d}_n}{a_n^{des}}}$, $\mu_n = \frac{\tilde{p}_n}{(k_n - \frac{\tilde{d}_n}{a_n^{des}})}$

## 1.3 Les bornes de $\tilde{r}_i$

$$a_i \tilde{r}_{i-1} + b_i \leq \tilde{r}_i \leq c_i \tilde{r}_{i-1} + e_i, \forall i \tag{7}$$

avec

$a_i = \frac{\frac{1-M}{M} r_i}{(r_i \frac{1-N}{N}) + p_i}$ , $b_i = \frac{p_i r_i}{(r_i \frac{1-N}{N}) + p_i}$ , $c_i = \frac{\frac{1-N}{N} r_i}{(r_i \frac{1-M}{M}) + p_i}$ et $e_i = \frac{p_i r_i}{(r_i \frac{1-M}{M}) + p_i}$

et

$$N = max \left( \prod_{j=1}^{i-1} \left[ \frac{r_j}{r_j + p_j} \right], \frac{r_i + p_i}{r_i k_i} d \right) \tag{8}$$

$$M = min \left( \prod_{j=i}^{n} \left[ \frac{r_j + p_j}{r_j} \right] a_n^{des}, 1 \right) \tag{9}$$

Et donc

$$(\prod_{j=2}^{i} a_j) r_1 + (\sum_{j=2}^{i-1} (\prod_{k=i}^{j+1} a_k) b_j) + b_i \leq \tilde{r}_i \leq (\prod_{j=2}^{i} c_k) r_1 + (\sum_{j=2}^{i-1} (\prod_{k=i}^{j+1} c_j) e_j) + e_i \tag{10}$$

De plus

$$\tilde{d}_i = d \prod_{i=j}^{n} (1 + \lambda_j q_j) \tag{11}$$

$$q_i = (1 - \lambda_{i-1}) q_{i-1} (1 + \beta_i) + \beta_i, \ i \geq 2. \tag{12}$$

$$\tilde{p}_i = (\frac{(p_i + r_i)}{a_{i-1} r_i} - 1) \tilde{r}_i \tag{13}$$

$q_1 = \beta_1$;

# 2 Les données

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\beta_i$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| $k_i$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| $p_i$ | 0.1 | 0.2 | 0.3 | 0.15 | 0.1 | 0.25 | 0.05 | 0.27 | 0.12 | 0.18 |
| $r_i$ | 0.8 | 0.6 | 0.7 | 0.9 | 0.85 | 0.65 | 0.75 | 0.95 | 0.85 | 0.92 |

| n | d | $c_p$ | $c_I$ | $a_n^{des}$ |
|---|---|---|---|---|
| 10 | 1 | 1 | 2 | 0.95 |

# 3 Le Travail demandé

1. Programmer les fonctions nécessaires pour ce problème en C

2. Tester la validation de votre programme ( je vous fournis les variables de décisions: $\tilde{r}$ et $\lambda$)

3. Optimiser le modèle à l'aide d'un logiciel ( je vous aide dans cette partie)