

$$\text{Res} = \lim_{z \rightarrow a} f(z) (z-a)$$

$$\oint_{\gamma} f(z) dz = 2\pi i \sum_{k=1}^n \text{Res}_{s_k}(f)$$

$$\|G\|_2^2 = \frac{1}{2\pi i} \oint G(-s)G(s) = \sum k$$

$$\begin{aligned} G(-s)G(s) &= \frac{s-3}{s^2+4s+13} \cdot \frac{-s-3}{s^2-4s+13} \\ &= \frac{9-3s+3s-s^2}{(s^2+4s)(s^2-4s) + 13(s^2+4s) + 13(s^2-4s) + 13 \cdot 13} \\ &= \frac{-s^2+9}{s^4-16s^2+13s^2+52s+13s^2-52s+169} = \frac{-s^2+9}{s^4+10s^2+169} \\ &= \frac{-s^2+9}{(s+2+3i)(s-2+3i)(s+2-3i)(s-2-3i)} \end{aligned}$$

$$\begin{aligned} b &= s^2 \quad b^2+10b+169=0 \quad b_{1,2} = \frac{10 \pm \sqrt{100-4 \cdot 169}}{2} \\ b &= \sqrt{b} = \sqrt{5 \pm 12i} = 5 \pm 12i \\ b &= \sqrt{s^2+12^2} e^{i \tan(14s)} = 13 e^{i 1.176 + 2\pi k} \\ s_{1,2} &= \sqrt{13} \cdot e^{i \frac{1.176}{2} + \pi k} \quad s_1 = \sqrt{13} e^{i 0.588} \quad s_2 = \sqrt{13} e^{i 0.588 + \pi} \\ & \quad s_1 = 2+3i \quad s_2 = -2-3i \\ \text{same for } s_{3,4} \quad & s_3 = -2+3i \quad s_4 = 2-3i \end{aligned}$$

$$\begin{aligned} \text{Res}_{s=-2-3i} &= \lim_{s \rightarrow -2-3i} (s+2+3i) \frac{-s^2+9}{(s+2+3i)(s+2-3i)(s-2-3i)(s-2+3i)} \\ &= \frac{-(-2-3i)(-2-3i)+9}{(-2-3i+2-3i)(-2-3i-2-3i)(-2-3i-2+3i)} \\ &= \frac{14-12i}{-6i \cdot -4(-4-6i)} = \frac{14-12i}{144-96i} \end{aligned}$$

$$= \frac{(14-12i)(144+96i)}{144^2+96^2} = \underline{\underline{\frac{11}{104} - \frac{1}{78}i}}$$

Same procedure for residue at $s = -2+3i$

$$\begin{aligned} \text{Res}_{s=-2+3i} &= \lim_{s \rightarrow -2+3i} (s+2-3i) \frac{-s^2+9}{(s+2+3i)(s+2-3i)(s-2-3i)(s-2+3i)} \\ &= \underline{\underline{\frac{11}{104} + \frac{1}{78}i}} \end{aligned}$$

$$\text{The two norm is } \|G\|_{\infty} = \sqrt{\sum \text{Res}} = \sqrt{\frac{22}{104}} = \underline{\underline{0.4599331}}$$