



Control of large scale urban traffic networks

Course: Intelligent Transportation Systems (ITS)

Module 2: ITS for Smart Mobility

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Motivation

For today...

Focus: Urban area networks.

Objective: Improve urban traffic conditions via control of traffic light (Macroscopic approach)¹.

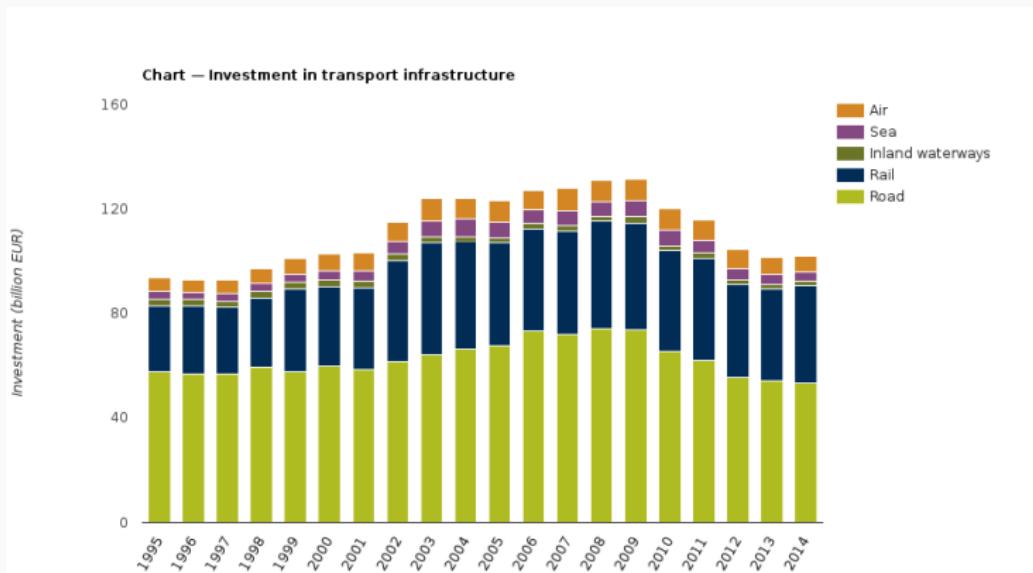
How: Efficient and distributed optimization methods for the aforementioned problems².

¹Pietro Grandinetti, Carlos Canudas de Wit, and Federica Garin. "An efficient one-step-ahead optimal control for urban signalized traffic networks based on an averaged Cell-Transmission-Model". In: *2015 European Control Conference (ECC)*. July 2015, pp. 3478–3483. DOI: 10.1109/ECC.2015.7331072.

²Pietro Grandinetti, Federica Garin, and Carlos Canudas de Wit. "Towards scalable optimal traffic control". In: *2015 54th IEEE Conference on Decision and Control (CDC)*. Oct. 2015, pp. 2175–2180. DOI: 10.1109/CDC.2015.7402529.

Motivation

Top-companies investing in R&D on road traffic management



Some traffic statistics . . .

Are we improving the conditions?



Now →

- ☺ Cities tend to get more urban.
- ☺ Big sources of data.
- ☺ New measurements available for this purpose (e.g. GPS).

Inrix

Lyon: In 2014 drivers waste 36.03 (40H ~2013) hours per year in traffic,
Worst Hour = Tuesday 08:00-09:00 ³

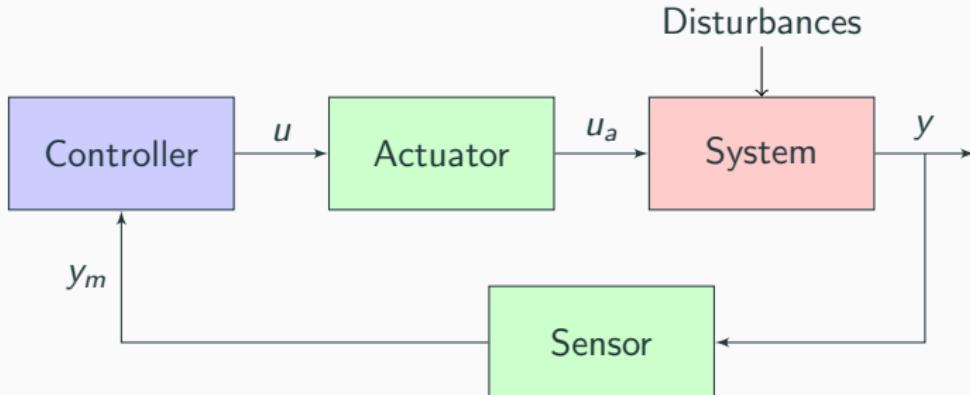
Current condition!

France has moved from 4th to 7th position in the list of most congested countries in Europe with 29 lost hours in congestion during congestions in 2014 - 6th in the last report in 2016⁴.

³INRIX Report 2014

⁴INRIX Report 2015

Basic concepts of control



Objective

u - Control input

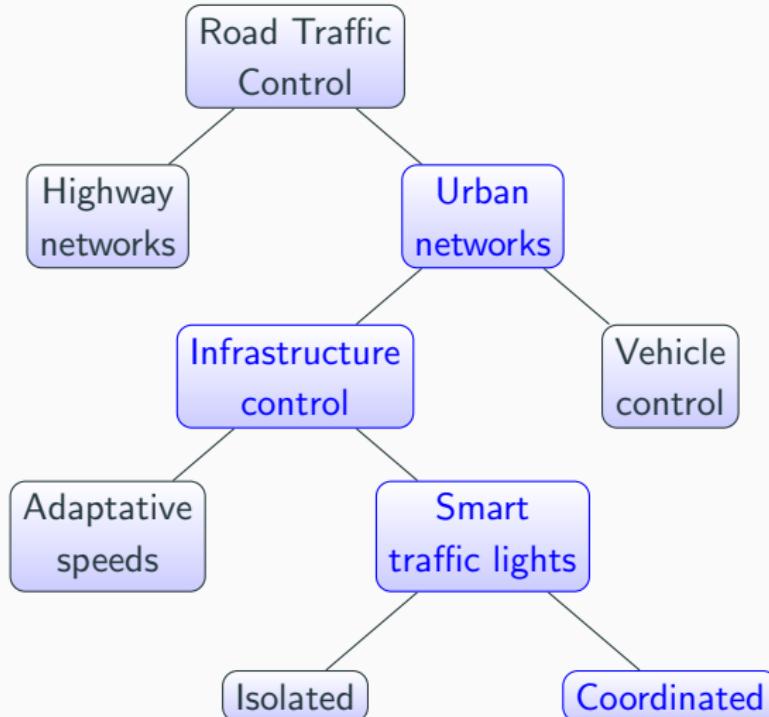
Track a particular value in the output of the system.

u_a - Actuation input

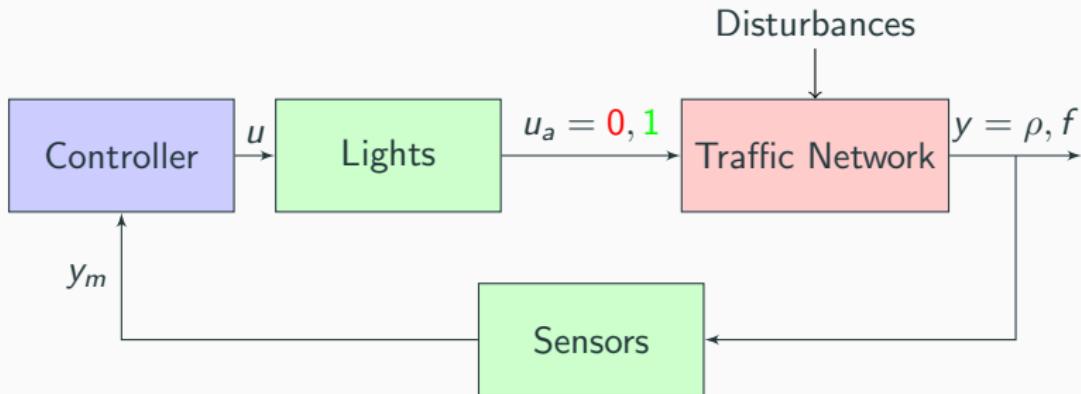
Regularize a value in the output (Stabilize).

y - Output system

Control approach to traffic systems . . .



Control in traffic networks



u - Green times / Red times

u_a - Actuation input
(Green/Red)

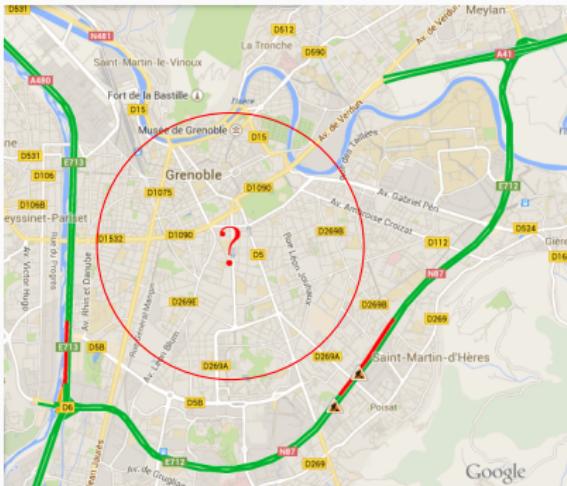
y - Output system (Densities,
Flows)

Objective

Alleviate congestions over the network.

Adapt green times based on measurements.

Motivation – Urban vs Highway model



Topology gap

Control strategies from *highways* can be adapted to urban cases?

Is it possible to design control for the full network?

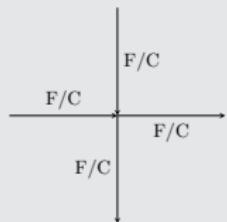
Motivation – Urban vs Highway model

Two level of complexity

Size of the network's model

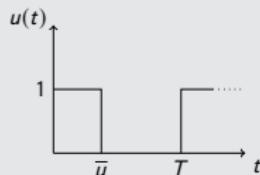
Traffic light almost each intersection

Piece-wise system representation



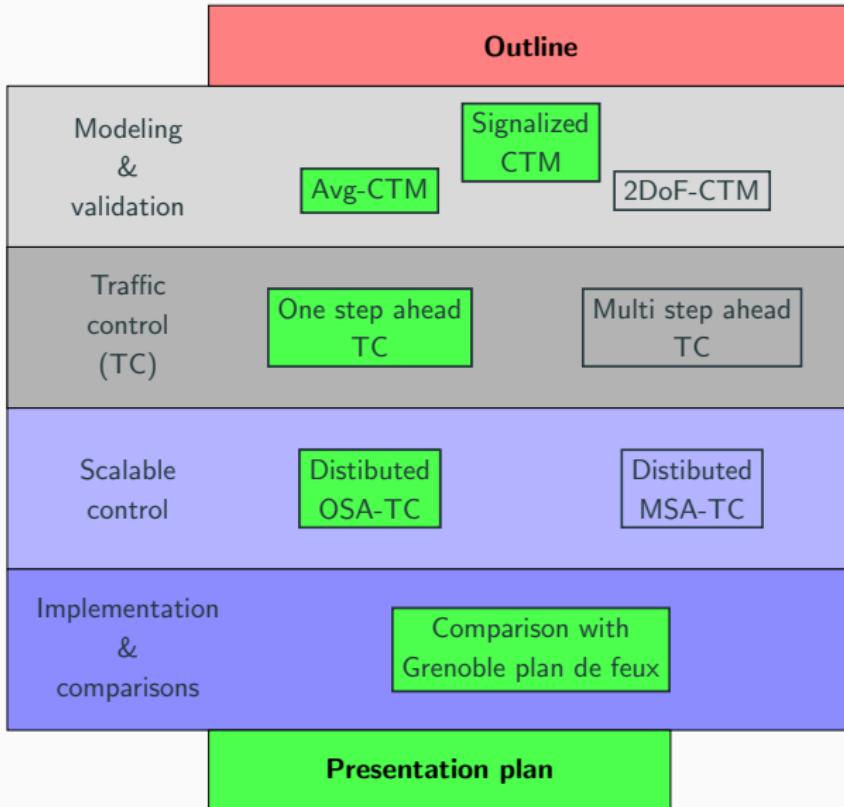
CTM still possible (each road is a cell) but the choice
Free/Congested can give dimensional problem ($\sim 2^{\# \text{roads}}$)

Discontinuities in traffic signal



A traffic light is a T -periodic, discontinuous signal $u(t) \in \{0, 1\}$, with duty cycle $\frac{1}{T} \int_0^T u(t) = \frac{\bar{u}}{T}$

Content



Traffic modeling

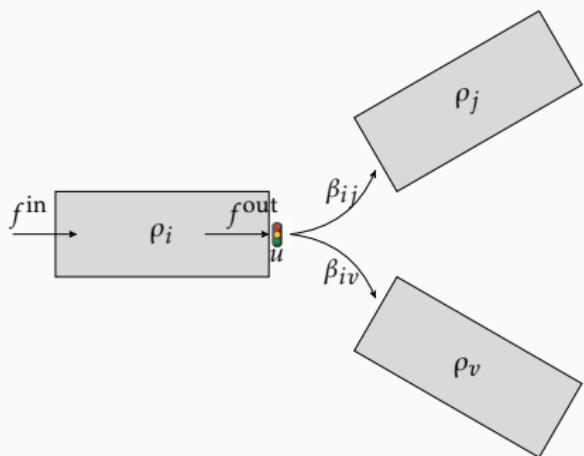
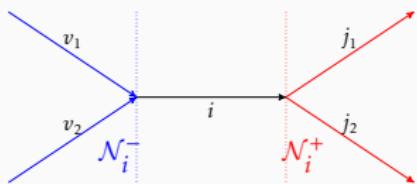
The signalized cell transmission model (S-CTM)

CTM properties:

Macroscopic model

Network partitioned into
cells

Notation	Value
ρ_i^{\max}	Cell i jam density
v_i	Free-flow speed
w_i	Congestion wave speed
f_i^{\max}	Capacity flow
L_i	Cell i length



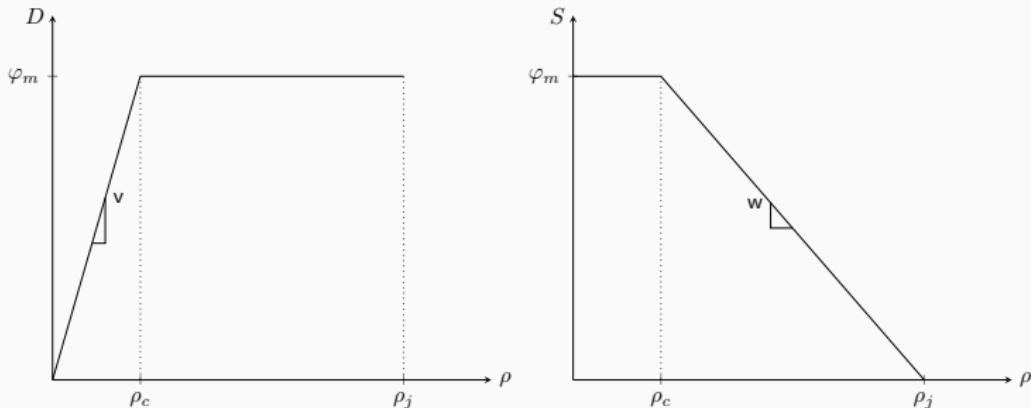
Model for urban traffic network

Demand & Supply paradigm

For a road r we define

Demand of r the flow of vehicles that can go out from r

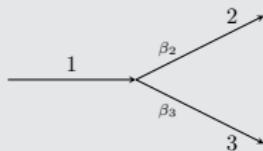
Supply of r the flow of vehicles that r can receive



$$D_r = \min\{v\rho_r, \varphi_m\} \quad S_r = \min\{w(\rho_j - \rho), \varphi_m\}$$

Model for urban traffic network

Diverge network (Daganzo, 1994)



$$f_1^{out} = \max \phi$$

$$\text{s.t. } \phi \leq D_1$$

$$\beta_2 \phi \leq S_2$$

$$\beta_3 \phi \leq S_3$$

$$f_1^{out} = \min \left\{ D_1, \frac{S_2}{\beta_2}, \frac{S_3}{\beta_3} \right\}$$

Split ratio

Several different choices for the β s are possible, e.g. defining β_{ij} which express the percentage of drivers in road i that want to go in road j

The signalized cell transmission model (S-CTM)

$$\rho_i(t + T_s) = \rho_i(t) + \frac{T_s}{L_i} \left(f_i^{\text{in}}(t) - u_i(t) f_i^{\text{out}}(t) \right)$$

$D_i(t), S_i(t)$: demand (supply) of cell i

f^{in} weighted sum of f^{out} $\longrightarrow f_i^{\text{in}}(t) = \sum_{j \in \mathcal{N}^-} \beta_{ji} f_j^{\text{out}}(t) u_j(t)$

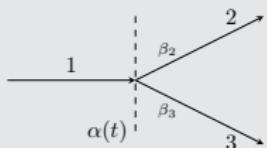
f^{out} with FIFO policy $\longrightarrow f_i^{\text{out}}(t) = \min \left(D_i(t), \left\{ \frac{S_j(t)}{\beta_{ij}} \right\}_{j \in \mathcal{N}_i^+} \right)$



$$\boxed{\begin{aligned} f_i^{\text{out}}(t) &= \max \phi \\ \text{subj. to: } \phi &\leq D_i(t) \\ \beta_{ij} \phi &\leq S_j(t) \forall j \in \mathcal{N}_i^+ \end{aligned}}$$

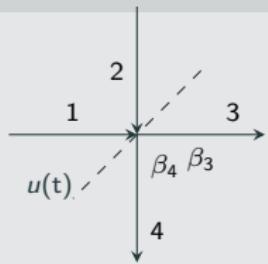
S-CTM - Example

Diverge network



$$\dot{\rho}_1 = \frac{f_1^{in} - f_1^{out} u_1(t)}{L_1}$$

4-roads intersection



$$\begin{aligned}\dot{\rho}_3 &= \frac{1}{L_3} (f_3^{in} - f_3^{out}) = \\ &\frac{1}{L_3} ((u_1(t) f_1^{out} \beta_{13} + (1 - u_1(t)) f_2^{out} \beta_{23})) - f_3^{out}\end{aligned}$$

Model simplification

Why simplification/approximation ?

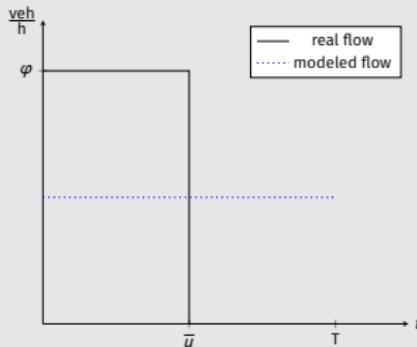
To have a more scalable model (thanks to continuous instead of binary function)

To include duty cycle as a new variable (towards control application)

Store & forward method (Aboudolas et al., 2008)

Provided that spills are avoided (Demand & Supply paradigm) a flow f becomes

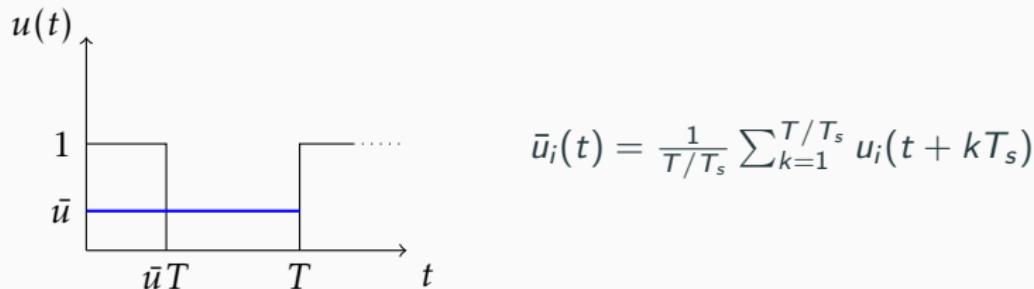
$$f = \begin{cases} 0 & \text{if } u(t) = 0 \\ \varphi & \text{otherwise} \end{cases}$$



The average cell transmission model (Avg-CTM)

Model reduction

Can the binary behavior of the S-CTM be simplified?

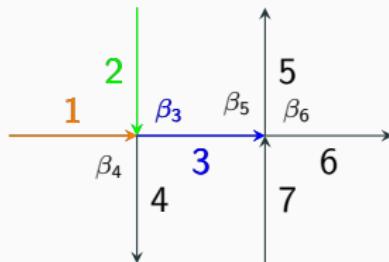


$$\bar{\rho}_i(t + T_s) = \bar{\rho}_i(t) + \frac{T_s}{L_i} \left(f_i^{\text{in}}(t) - \bar{u}_i(t) f_i^{\text{out}}(t) \right)$$

subj. to constraints from the S-CTM

$$\forall i \in \mathcal{R} \setminus \mathcal{R}^{\text{in}}, \forall t \in \mathbb{N}_+ \quad \sum_{j \in \mathcal{N}_i^-} \bar{u}_j(t) \leq 1.$$

Avg-CTM: Example



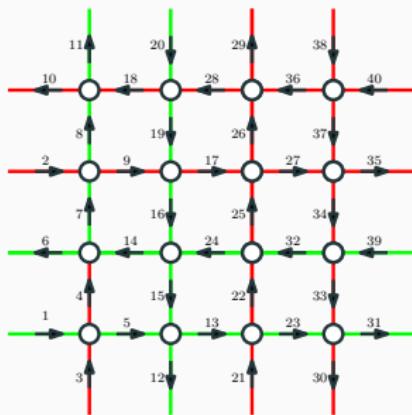
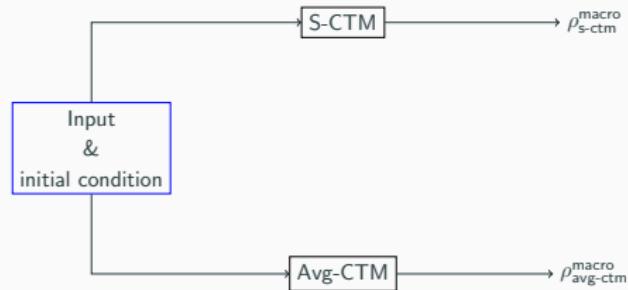
$$\begin{aligned} \frac{L_3}{T_s} (\bar{\rho}_3^+ - \bar{\rho}_3^-) &= f_3^{in} - \bar{u}_3 f_3^{out} = \bar{u}_1 \beta_{13} f_1^{out} + (1 - \bar{u}_1) \beta_{23} f_2^{out} - \bar{u}_3 f_3^{out} = \\ &= \bar{u}_1 \beta_{13} \min \left\{ D_1, \frac{S_3}{\beta_{13}}, \frac{S_4}{\beta_{14}} \right\} + (1 - \bar{u}_1) \beta_3 \min \left\{ D_2, \frac{S_3}{\beta_{23}}, \frac{S_4}{\beta_{24}} \right\} \\ &\quad - \bar{u}_2 \min \left\{ D_3, \frac{S_5}{\beta_5}, \frac{S_6}{\beta_6} \right\} \end{aligned}$$

Consistency of the model

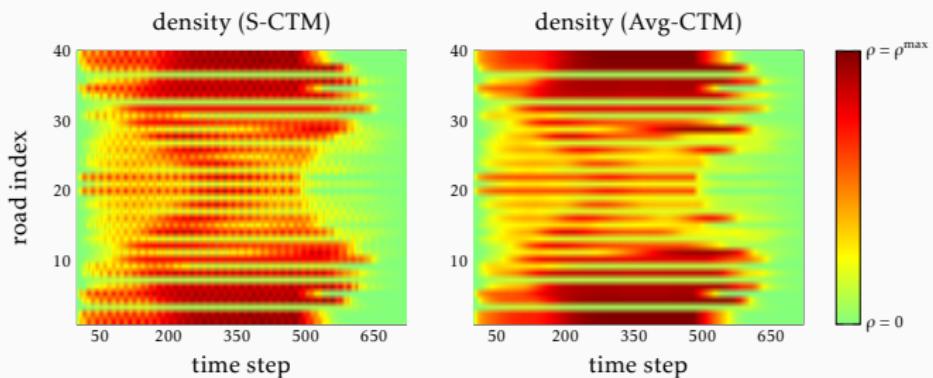
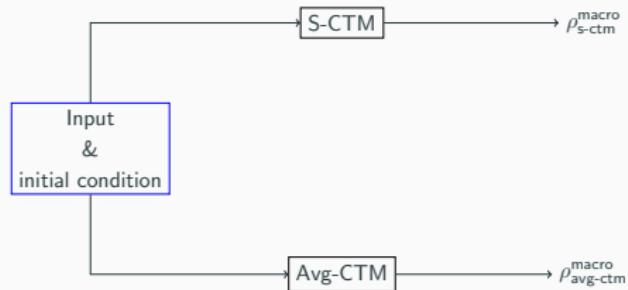
Total outflow never bigger than inflow

Inflow/outflow respect the Demand & Supply paradigm

Avg-CTM — Macroscopic validation



Avg-CTM — Macroscopic validation



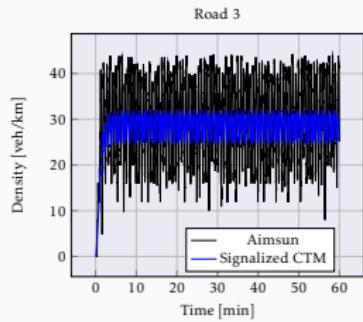
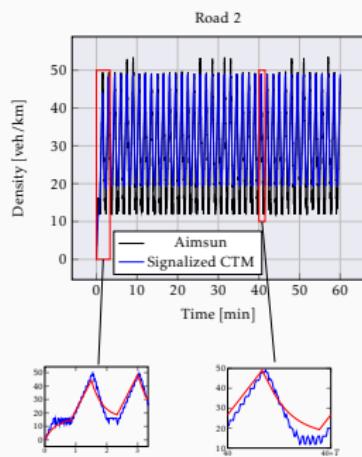
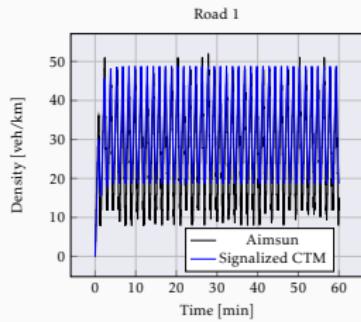
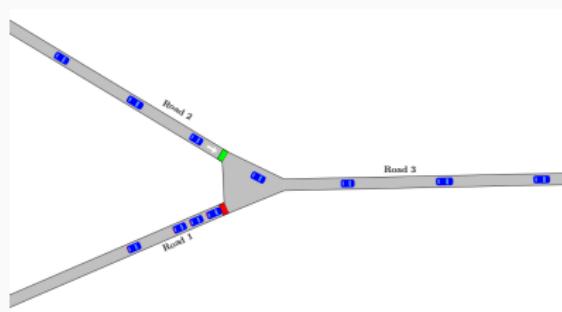
S-CTM — Microscopic validation

Validation objective

How close is the signalized CTM to a realistic behavior (e.g., microscopic models)?



S-CTM — Microscopic validation



Comments

The intrinsic complexity of the most descriptive traffic models (microscopic) makes them unsuitable for control synthesis

We rely on macroscopic representations, starting off with the CTM and extending it to account for signalized intersections (S-CTM)

The binary dynamics of traffic lights can be approximated with an average trajectory (Avg-CTM) that proves in practice to be a good approximation of the former

The Avg-CTM is a smoother model, suitable for control design with duty cycle as control variables

Traffic control

Control history

1772 - Manual of traffic flow control

1866 - Heritage from railway systems

1960 - Driver Aided Information and Routing (DAIR)

1990 - Loop detectors + Message signs

1991 - ALINEA

2000 - Hierarchical control

Recent years - Autonomy



TODAY'S COMPLEX ROADWAYS, increased vehicle speeds, and heavy traffic intensity the driver's need for frequent directions and information. DAIR meets this need for increased safety and driving enjoyment with a simple, low-cost communications system. Features include two-way radio communication, a display panel with warnings to supplement upcoming traffic signs, messages about the road ahead, and an in-car route direction indicator.

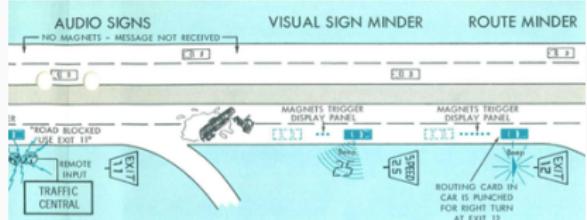
Picture yourself on a long, lonely segment of highway. It's a rainy night, and you're trying to stretch your gasline to the next service station.

Soon enough, the engine begins to sputter. You crawl to the shoulder and stop. Your wife, who suggested a stop at the last town, gives you the special look she saves for such occasions.

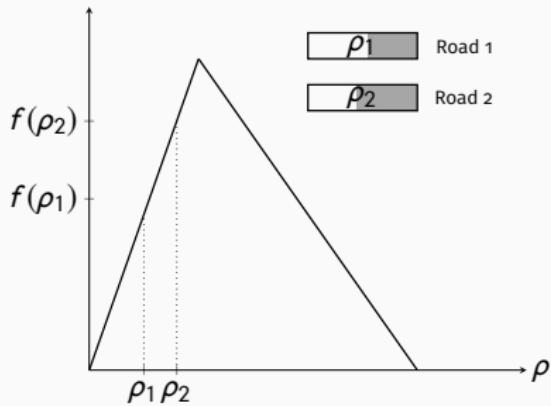
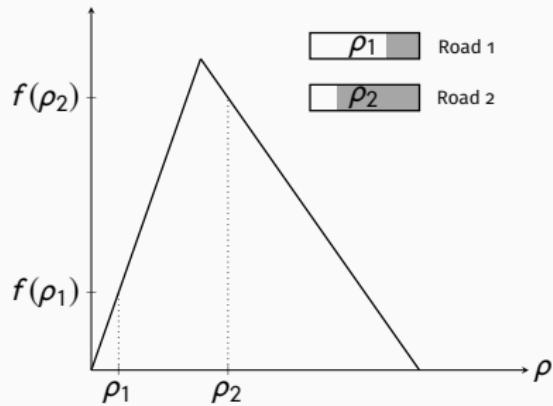
It's a bad situation at best. But if the car is equipped with GMR DAIR, you simply dial a series of numbers on a small instrument panel. The message is received



The Visual Sign Minder alerts Clark Quinn with a "beep" as it repeats an upcoming Stop sign on the display panel inside the car. The inset shows the panel with all features lit for a better view. The DAIR console fits between the seat and the dashboard next to the driver.



Control objective



Objectives of u

Maximize the throughput of the network.

Homogenize the use of the network (when possible)

Traffic metrics

Total travel distance

$$\text{TTD}(t) = \sum_{k=0}^{\lfloor t/T_s \rfloor} \sum_{i \in \mathcal{R}} f_i(kT_s)$$

Density balancing

$$\text{Bal}(t) = \sum_{k=0}^{\lfloor t/T_s \rfloor} \sum_{i \in \mathcal{R}} \sum_{j \in \mathcal{N}_i^+} (\rho_i(kT_s) - \rho_j(kT_s))^2 = \sum_{k=0}^{\lfloor t/T_s \rfloor} \rho'(kT_s) \mathcal{L} \rho(kT_s)$$

where

$$\mathcal{L}_{ii} = |\mathcal{N}_i|,$$

$$\mathcal{L}_{ij} = \begin{cases} -1 & \text{if } j \in \mathcal{N}_i \\ 0 & \text{elsewhere.} \end{cases}$$

Traffic metrics

Traffic lights regularization

$$\|\bar{u}(t) - \bar{u}(t - T)\|_2^2$$

Other indexes:

Service of demand

Total travel time

Queues length

Stop time

Control problem formulation

$$\underset{u(kT_s)}{\text{optimize}} \sum_{k=1}^K J(t + kT_s)$$

subj. to signalized traffic dynamics over K .

Duty cycles as controlled variables

Weighted sum of TTD and Bal as objective function, with regularization

Model-based predictions computed by using the Avg-CTM

Control problem formulation

$$\min_{\bar{u}} \quad \sum_{k=1}^K \left(k_{\text{bal}} \bar{\rho}'(t + kT_s) \mathcal{L} \bar{\rho}(t + kT_s) - k_{\text{ttd}} \sum_{i \in \mathcal{R}} f_i(t + kT_s) \right) + \|\bar{u} - \bar{u}(t - T)\|$$

subj. to: $\forall i, \forall t \quad 0 \leq \bar{u}_i(t) \leq 1$

$$\sum_{j \in \mathcal{N}_i^-} \bar{u}_j(t) \leq 1.$$

Minimize density balancing

Maximize total travel distance

A convex formulation is achievable when $K = 1$

Convexity of the formulation — ideas

One step ahead density:

$$\bar{\rho}_i(t + T_s) = \bar{\rho}_i(t) + \frac{T_s}{L_i} \left(\sum_{j \in \mathcal{N}_i^-} \beta_{ji} f_j^{\text{out}}(t) \bar{u}_j - \bar{u}_i f_i^{\text{out}}(t) \right) \quad (1)$$

$$\bar{\rho}(t + T_s) = H(t) \bar{u} + c(t) \quad (2)$$

Bal + regularization:

$$\bar{u}' Q \bar{u} + p' \bar{u} \quad (3)$$

where

$$Q = k_{\text{bal}} H'(t) \mathcal{L} H(t) + I, \quad (4)$$

$$p = (2k_{\text{bal}} c'(t) \mathcal{L} H(t) - \bar{u}'(t - T))'. \quad (5)$$

Convexity of the formulation — ideas

Problem formulation (equivalent):

$$\min_{\bar{u}} \quad \bar{u}' Q \bar{u} + p' \bar{u} - k_{\text{ttd}} \mathbf{1}' f(\bar{u})$$

$$\text{subj. to, } \forall i : l_i \leq \bar{u}_i \leq 1$$

$$\sum_{j \in \mathcal{N}_i^-} \bar{u}_j \leq 1$$

TTD:

$$f_i(\bar{u}) = \min \left(v_i \left(\bar{\rho}_i(t) + \frac{T_s}{L_i} \left(\sum_{j \in \mathcal{N}_i^-} \bar{u}_j \beta_{ji} f_j^{\text{out}}(t) - \bar{u}_i f_i^{\text{out}}(t) \right) \right) \right),$$

$$w_i \left(\rho_i^{\max} - \bar{\rho}_i(t) - \frac{T_s}{L_i} \left(\sum_{j \in \mathcal{N}_i^-} \bar{u}_j \beta_{ji} f_j^{\text{out}}(t) + \bar{u}_i f_i^{\text{out}}(t) \right) \right).$$

Convexity of the formulation — ideas

Problem formulation (relaxed):

$$\min_{\bar{u}, y \geq 0} \quad \bar{u}' Q \bar{u} + p' \bar{u} - k_{\text{ttd}} \mathbf{1}' y$$

subj. to, $\forall i : l_i \leq \bar{u}_i \leq 1$

$$\sum_{j \in \mathcal{N}_i^-} \bar{u}_j \leq 1$$

$$y_i \leq v_i \left(\bar{\rho}_i(t) + \frac{T_s}{L_i} \left(\sum_{j \in \mathcal{N}_i^-} \bar{u}_j f_j^{\text{out}}(t) - \bar{u}_i f_i^{\text{out}}(t) \right) \right)$$

$$y_i \leq w_i \left(\rho_i^{\max} - \bar{\rho}_i(t) - \frac{T_s}{L_i} \left(\sum_{j \in \mathcal{N}_i^-} \bar{u}_j f_j^{\text{out}}(t) + \bar{u}_i f_i^{\text{out}}(t) \right) \right).$$

Comments

Pros:

The relaxed formulation is provably *equivalent* to the original problem

Convex problems are generally considered "easy" to solve

Cons:

Although efficient, the solution does not scale well with the size of the network

Scalable traffic control

Distributed optimal traffic control

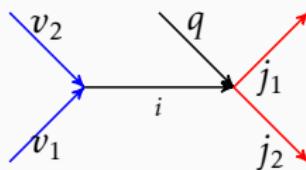
Decomposition in subproblems

Communication graph

Iterative (distributed) algorithm

Optimality of the distributed algorithm

Communication graph



Set of roads that can "talk" to i :

$$\mathcal{S}_i = \mathcal{N}_i^- \cup \mathcal{N}_i^+ \cup \mathcal{I}_i,$$

where

$$\mathcal{I}_i = \{q : \mathcal{N}_q^+ \equiv \mathcal{N}_i^+\}.$$

Why?

\mathcal{N}_i^- and \mathcal{N}_i^+ are needed for density prediction. \mathcal{I}_i is needed for constraints over traffic lights.

Problem decomposition

Problem set-up (e.g., problem i):

$$\begin{aligned} \min_{\bar{u}} \quad & \sum_{i \in \mathcal{R}} g_i(\bar{u}_{[p \in \mathcal{S}_i]}^{(i)}) \\ \text{s.t.} \quad & \bar{u}_{[p \in \mathcal{S}_i]}^{(i)} \in \mathcal{X}_i, \forall i \in \mathcal{R}, \\ & \bar{u}_i^{(i)} = \bar{u}_i^{(p)}, \quad \forall i \in \mathcal{R}, \forall p \in \mathcal{S}_i \setminus i \\ & \bar{u}_p^{(i)} = \bar{u}_p^{(p)}, \quad \forall i \in \mathcal{R}, \forall p \in \mathcal{S}_i \setminus i. \end{aligned}$$

where $\bar{u}_p^{(i)}$ is the copy of the global variable \bar{u}_p kept in memory locally by subproblem i .

Why?

Each subproblem needs to be self-contained: variables will be requested from others according to the communication graph

Distributed algorithm

Partial (separable) Lagrangian:

$$L = \sum_{i \in \mathcal{R}} L_i = \sum_{i \in \mathcal{R}} g_i(\bar{u}_{[p \in \mathcal{S}_i]}) + \bar{u}_i^{(i)} \sum_{p \in \mathcal{S}_i \setminus i} (\lambda_i^{(i,p)} - \lambda_i^{(p,i)}) + \sum_{p \in \mathcal{S}_i \setminus i} \bar{u}_p^{(i)} (\lambda_p^{(i,p)} - \lambda_p^{(p,i)}).$$

Initialization. Set $k = 0$. Create local variables for each subproblem

Primal update. Update primal variables $\bar{u}_p(k+1)$ by minimizing the i -th term of the partial Lagrangian

Transmission. Send primal local variables to requiring neighbors $p \in \mathcal{S}_i$, and collect the most recent values of $\bar{u}_i^{(p)}$, $\bar{u}_p^{(p)}$ from them

Dual update. Update the Lagrangian multipliers with a gradient update of step α

Stop condition. If the solution has numerically stabilized then stop, otherwise increment k and go to 1

Proposition

Let \bar{u}^* be the unique optimal solution of the centralized optimization problem. There exists $\alpha^* > 0$ such that, if $0 < \alpha < \alpha^*$, then

$$\lim_{k \rightarrow \infty} \bar{u}_i^{(i)} = \bar{u}_i^*,$$

for all $i \in \mathcal{R}$.

Proof elements

g_i strictly convex

\mathcal{X}_i compact and convex

Numerical simulations

Two-fold evaluation

Time to convergence of the distributed algorithm

Number of iteration before the iterative procedure stabilizes,
measured in different numerical scenarios

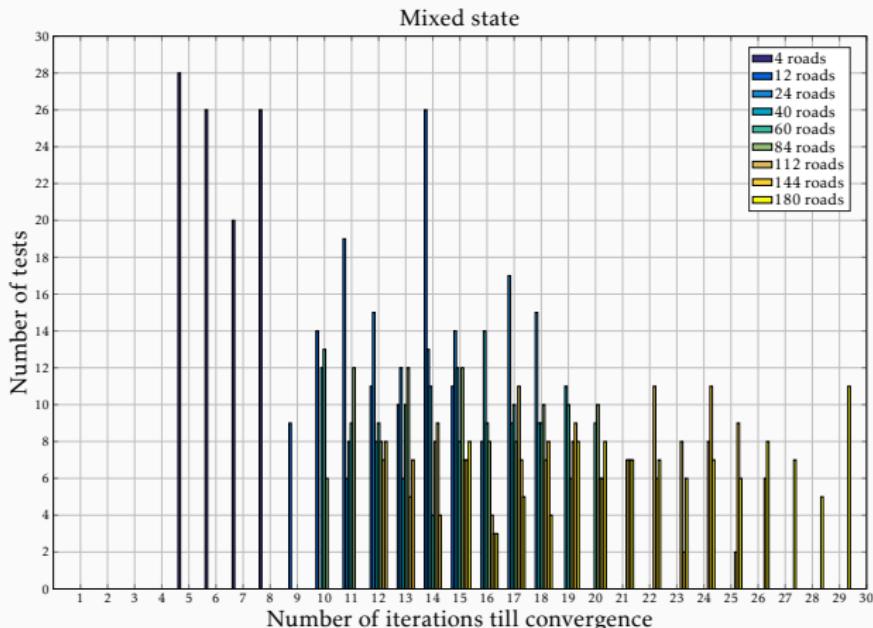
Traffic performance

Measure of traffic metrics, both macroscopic and microscopic

Convergence of the distributed algorithm

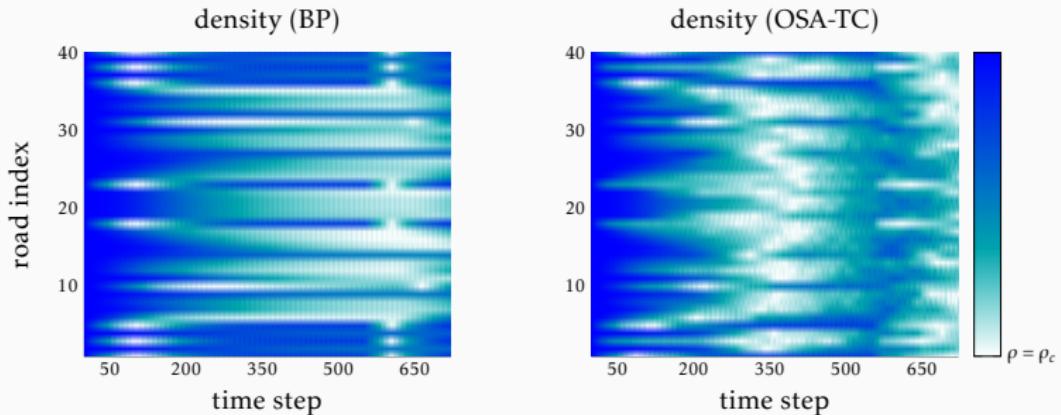
Several network dimensions (from 4 to 180 roads)

Initial state randomly selected (in 300 simulations)

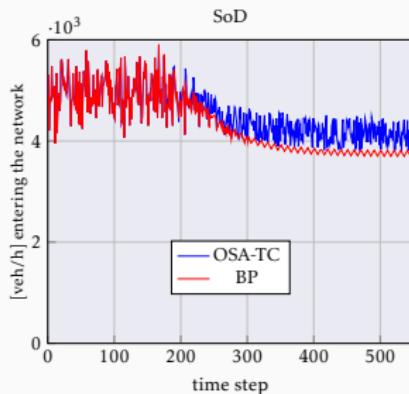
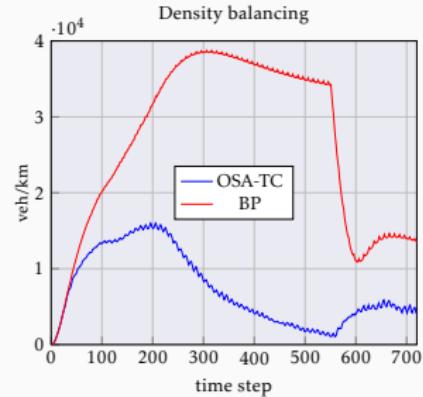
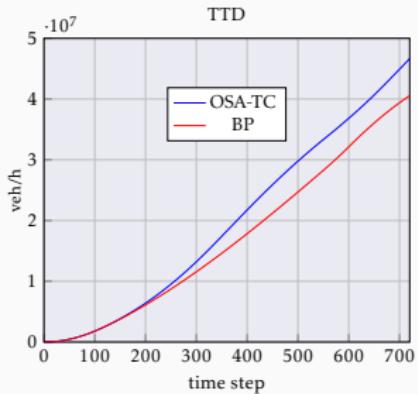


Traffic performance

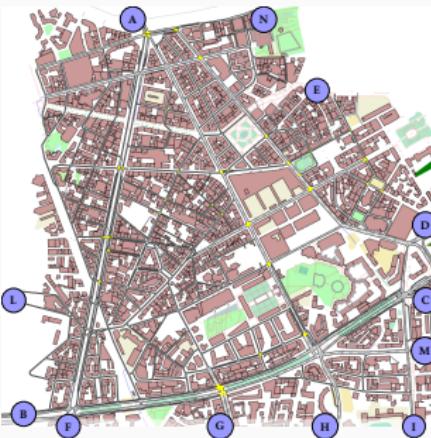
Comparisons with a best practice (BP) schedule, chosen to optimize the statistical traffic behavior



Traffic performance



Miscroscopic simulation



Index	Scenario 1		Scenario 2	
	BP	OSA-TC	BP	OSA-TC
Travelled distance [km]	23396	26471	19772	17003
Travel time [h]	1775	1462	1955	1583
Mean queue [veh]	496	441	627	564
Stop time [sec/km]	123	97	172	139

Comparisons with the Grenoble traffic lights plan

Scenario description



Scenario description

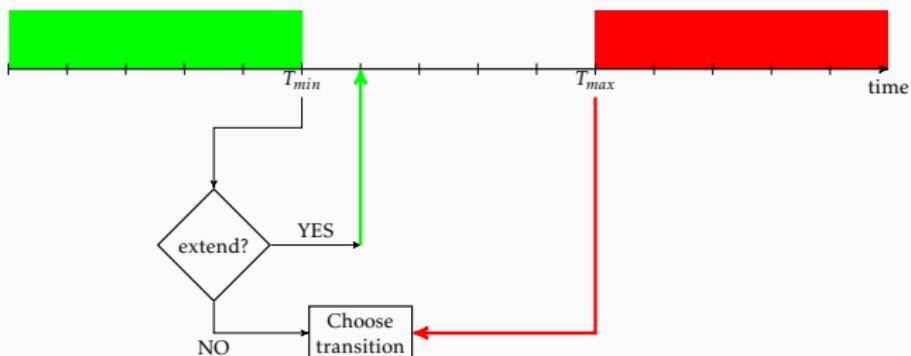
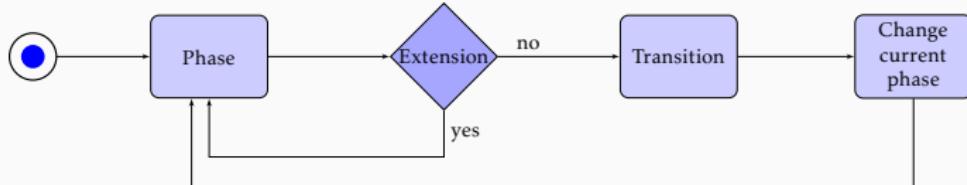
Intersection are equipped with several on–off detectors

Every intersection is controlled by a single–intersection controller

A fusion of data from GPS and magnetic detectors is used in order to say to the controller how far the trams are

Several constant parameters (min/max green time, amber time) are decided as function of the speed limit, the number and size of lanes, the size of the intersections

Grenoble traffic lights plan (plan de feux)



Numerical comparisons

Set up:

Traffic lights plan virtually replicated into Aimsun

Input flow and supply estimated by real data extracted by loop detectors...

... as well as split ratios at intersections...

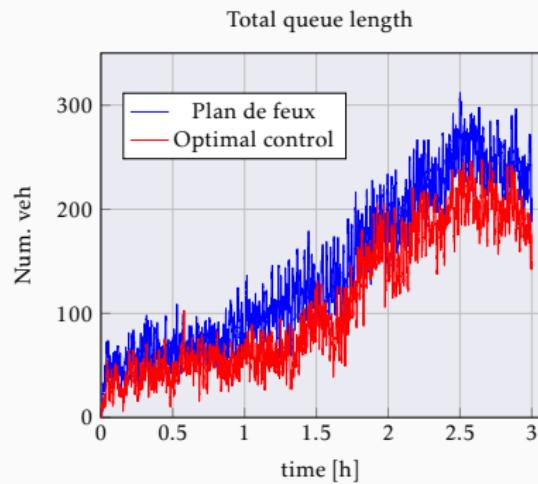
... from 7am to 10am

Adaptation of the OSA-TC to fit this scenario's technological requirements

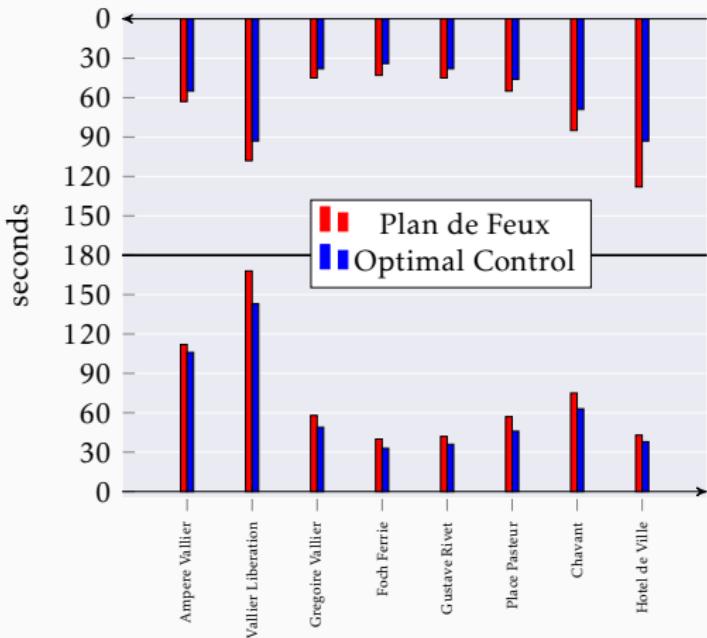
A posteriori analysis of on-line measured data (within Aimsun)

Numerical results

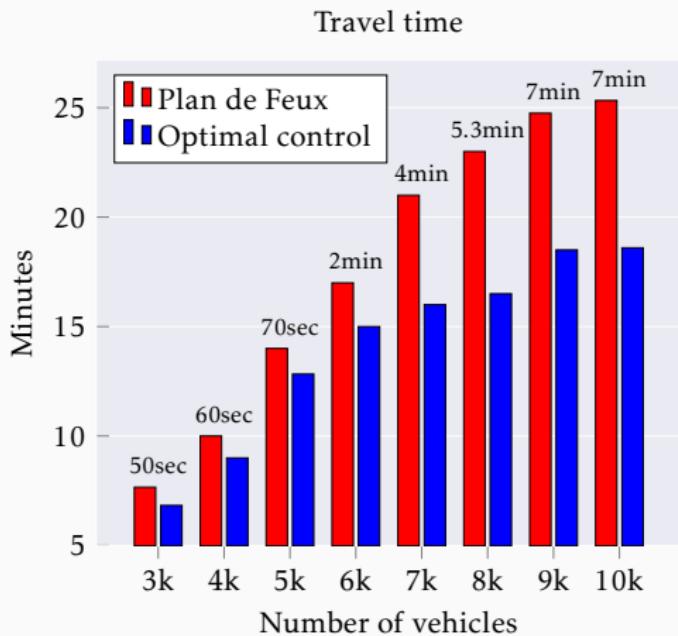
Index	Plan de feux	Optimal control
Input flow [veh/h]	6097	6151
Mean queue [veh]	152	113
Stop time [sec/km]	110	83
TTD [km]	16307	16411
TTT [h]	794	680
Veh. waiting [veh]	333	240



Numerical results



Numerical results



Final comments

We have designed an optimization-based algorithm to control green split of traffic lights

The control algorithm is applicable to large cities thanks to its scalability property

Numerical simulations show the improvements w.r.t. standard fixed-time policies

The algorithm can be adapted to real scenarios and in simulations it performs better than "real world" traffic lights scheduling algorithms

Discussion

The binary traffic lights

- can help (two incoming flows will never collide)
- make future optimizations more difficult

The averaged-based approximation

- preserves the consistency
- avoid combinatorial issues while adding the duty cycle inside the model

Several representations are possible for the traffic lights:

- Uni-coordinated: $\alpha_i(t) = \alpha_j(t), \forall i, j, \forall t$
- With delay: $\alpha_i(t) = \alpha_j(t - k\Delta T)$

Control advices:

Control of duty cycle

- Control of *green times* $\bar{\alpha}_i$
- Control of both $\bar{\alpha}_i$ and T_i

Control of split ratio

Mixed strategy

Questions?

References

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