French Institute of Science and technology for transport, development and networks

Automation strategies for improvement of Traffic Mobility

Control of large scale urban networks

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Module 2: ITS for Smart Mobility

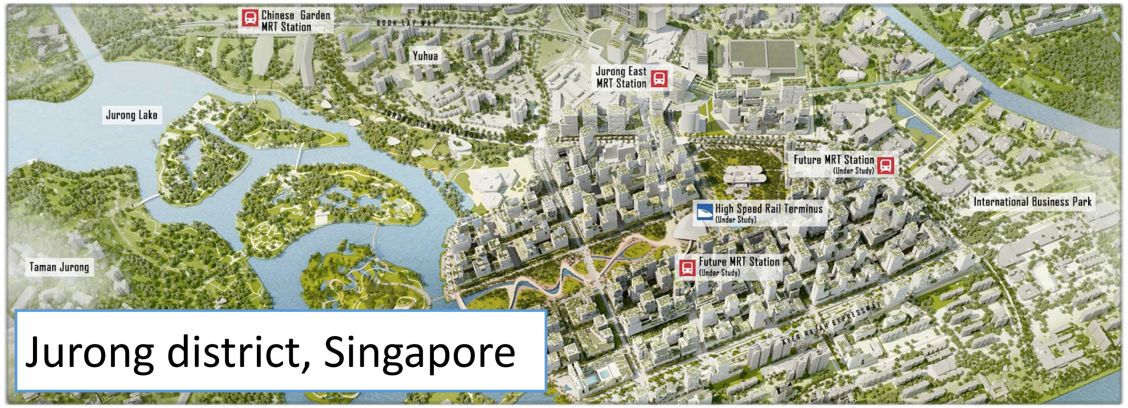
December 5th, 2018

Motivations

- Future of mobility is dynamically changing:


Neom, KSA

-



Jurong district, Singapore

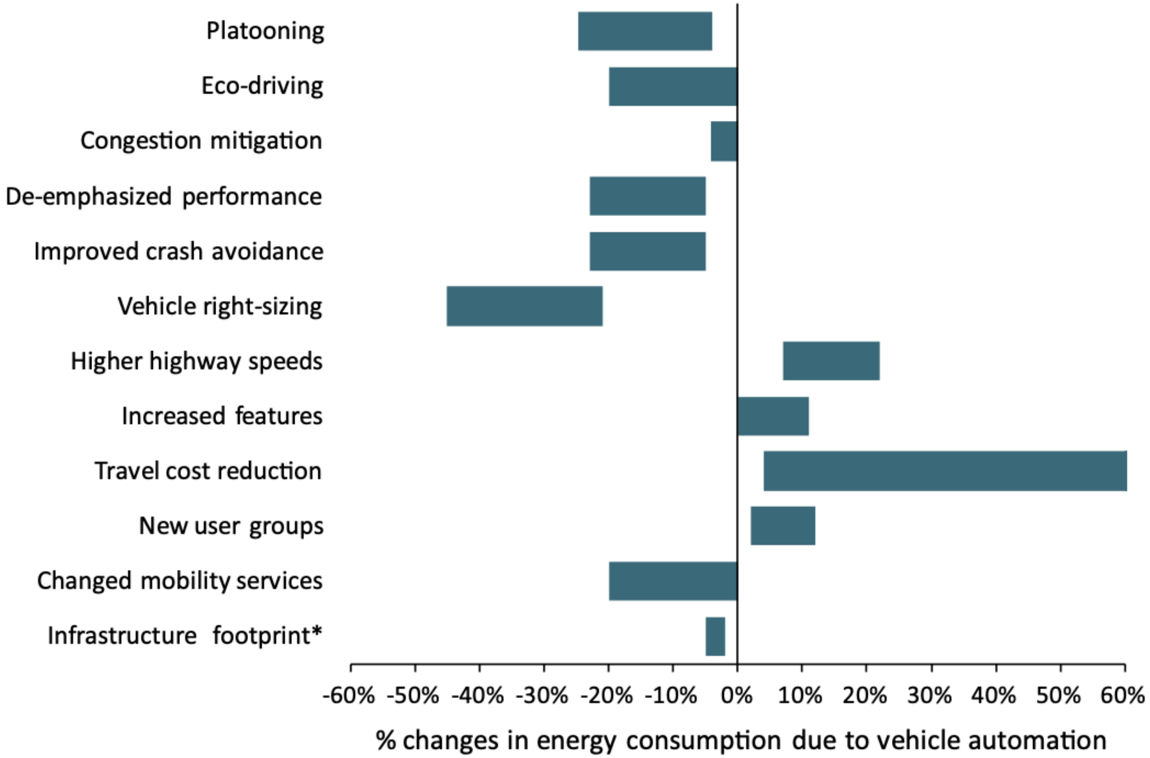
Motivation

- Still there is a price to pay:
  - Improvement of footprint of mobility: **Uncertain.**

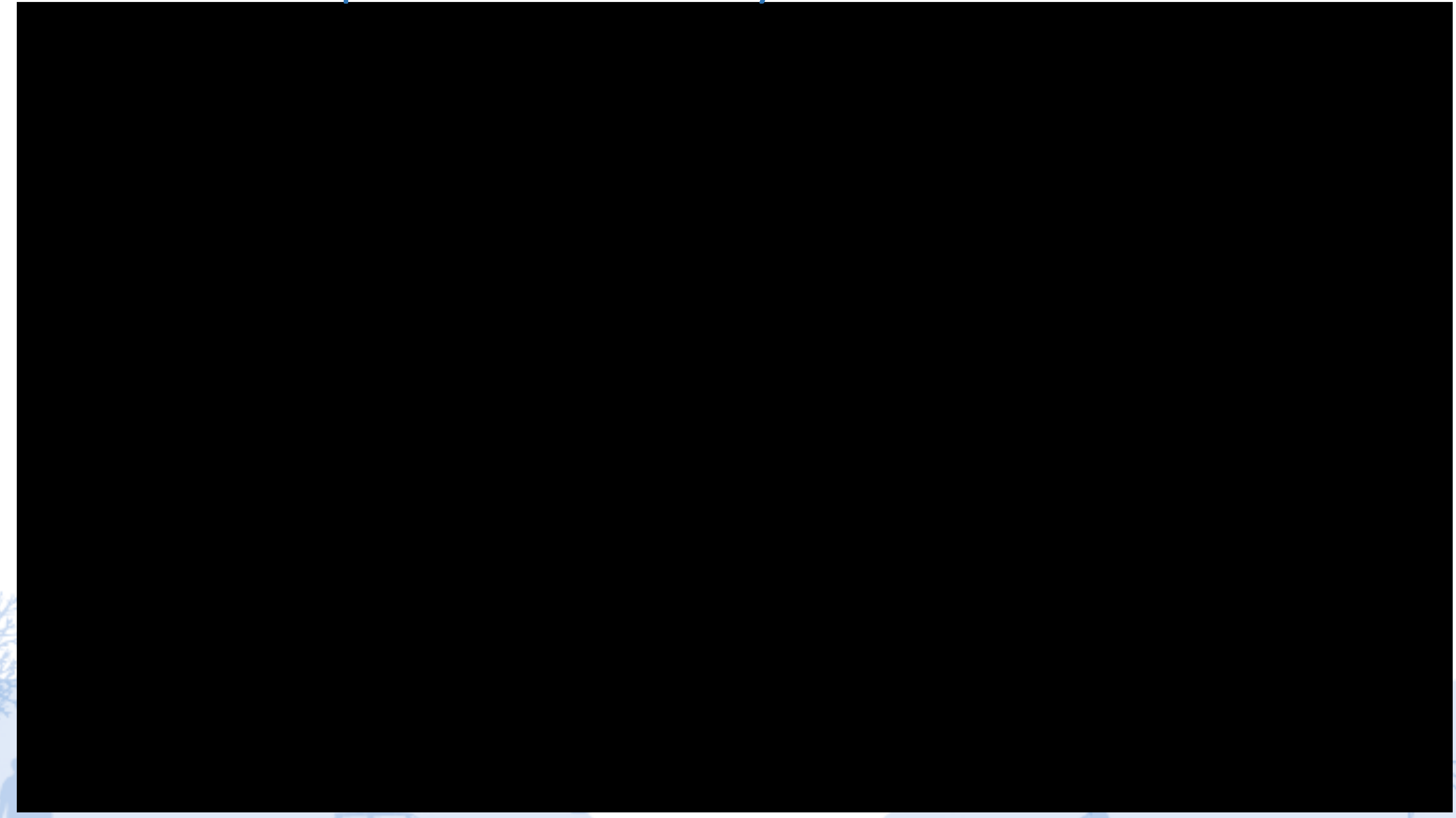
Transportation today:
- **30%** of Energy (US Case)

Improvement in the system may change patterns of mobility

Self driving cars may reduce energy consumption but increase flow



Can we improve mobility? - Tools



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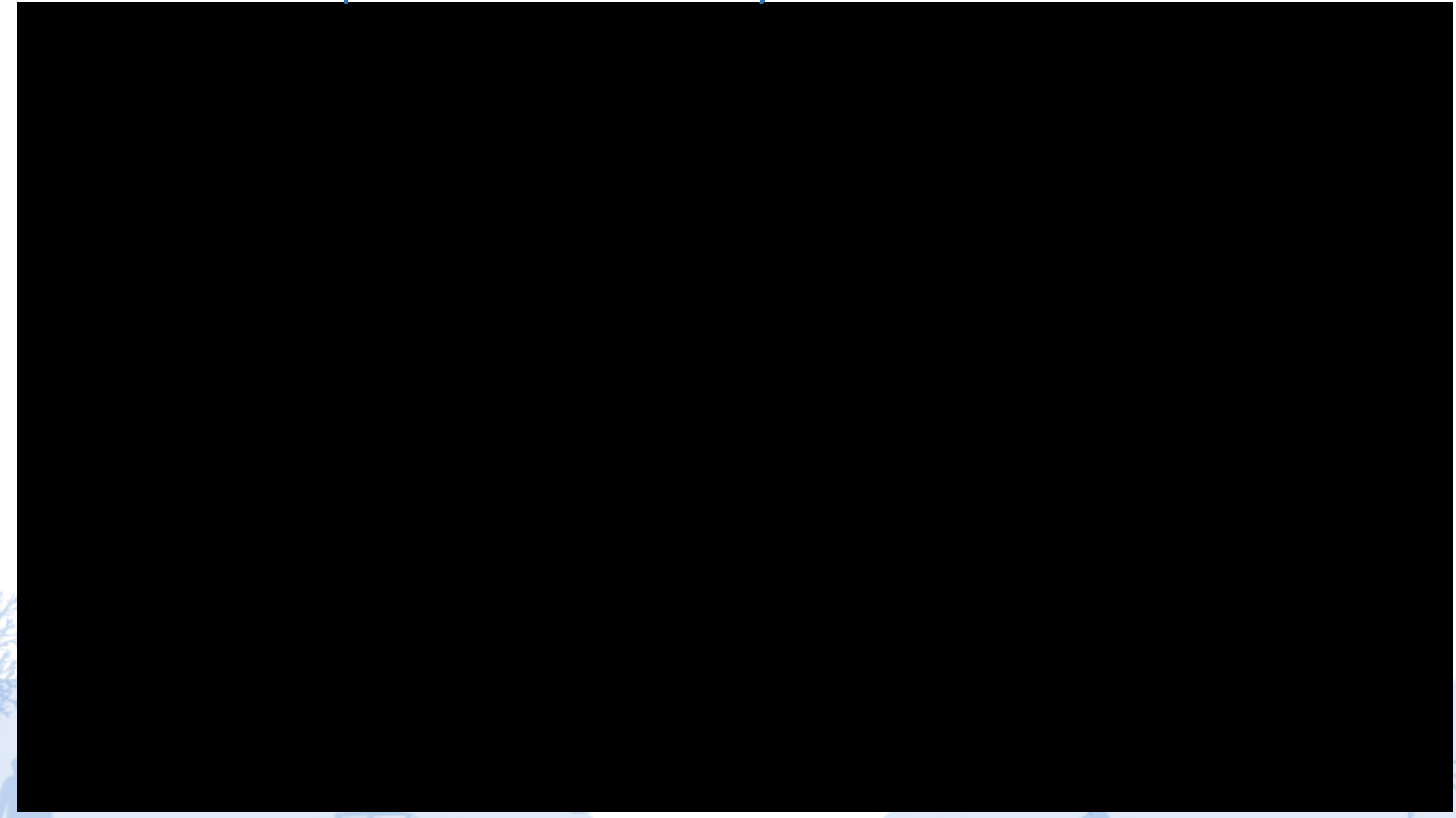

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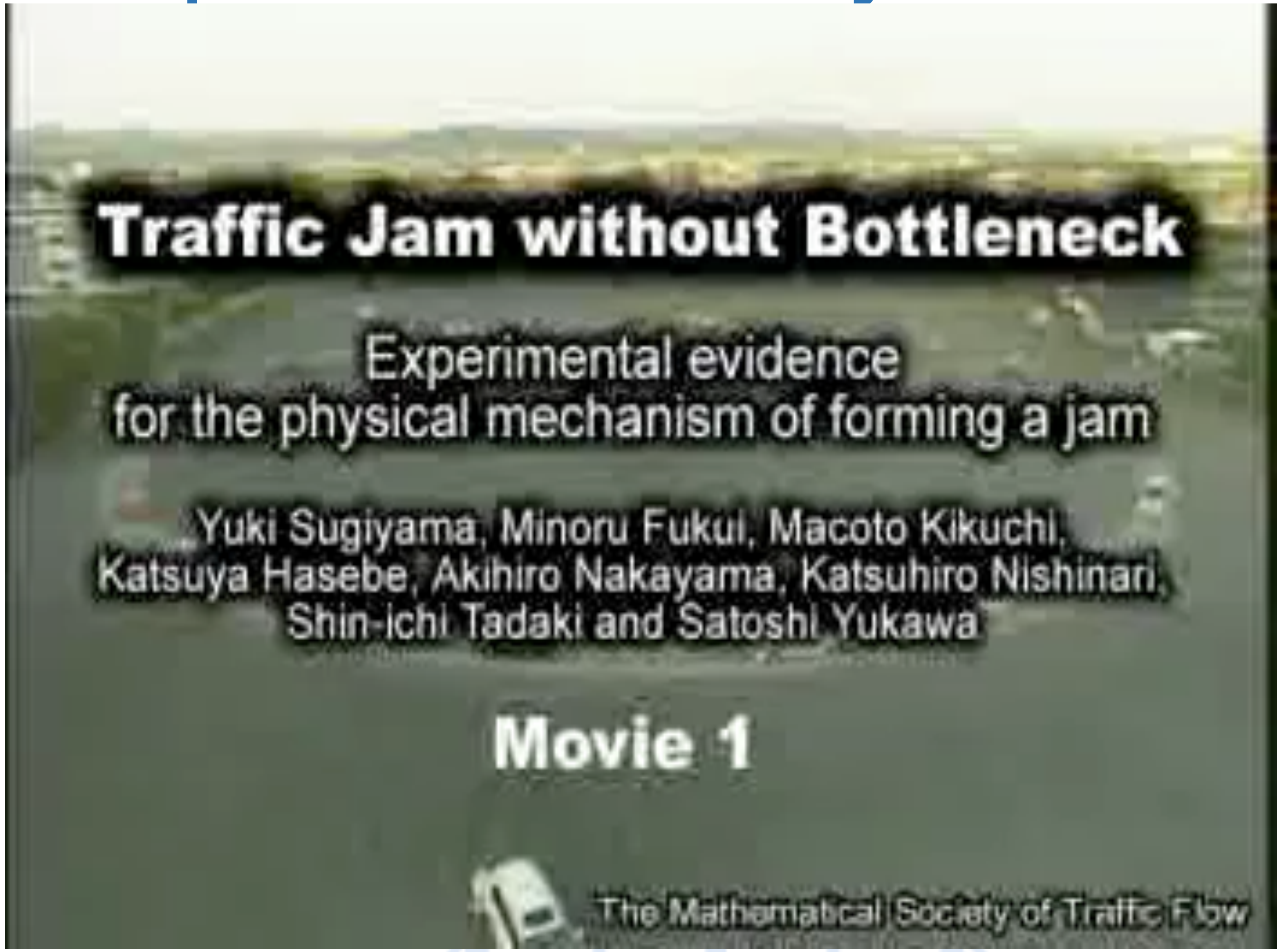

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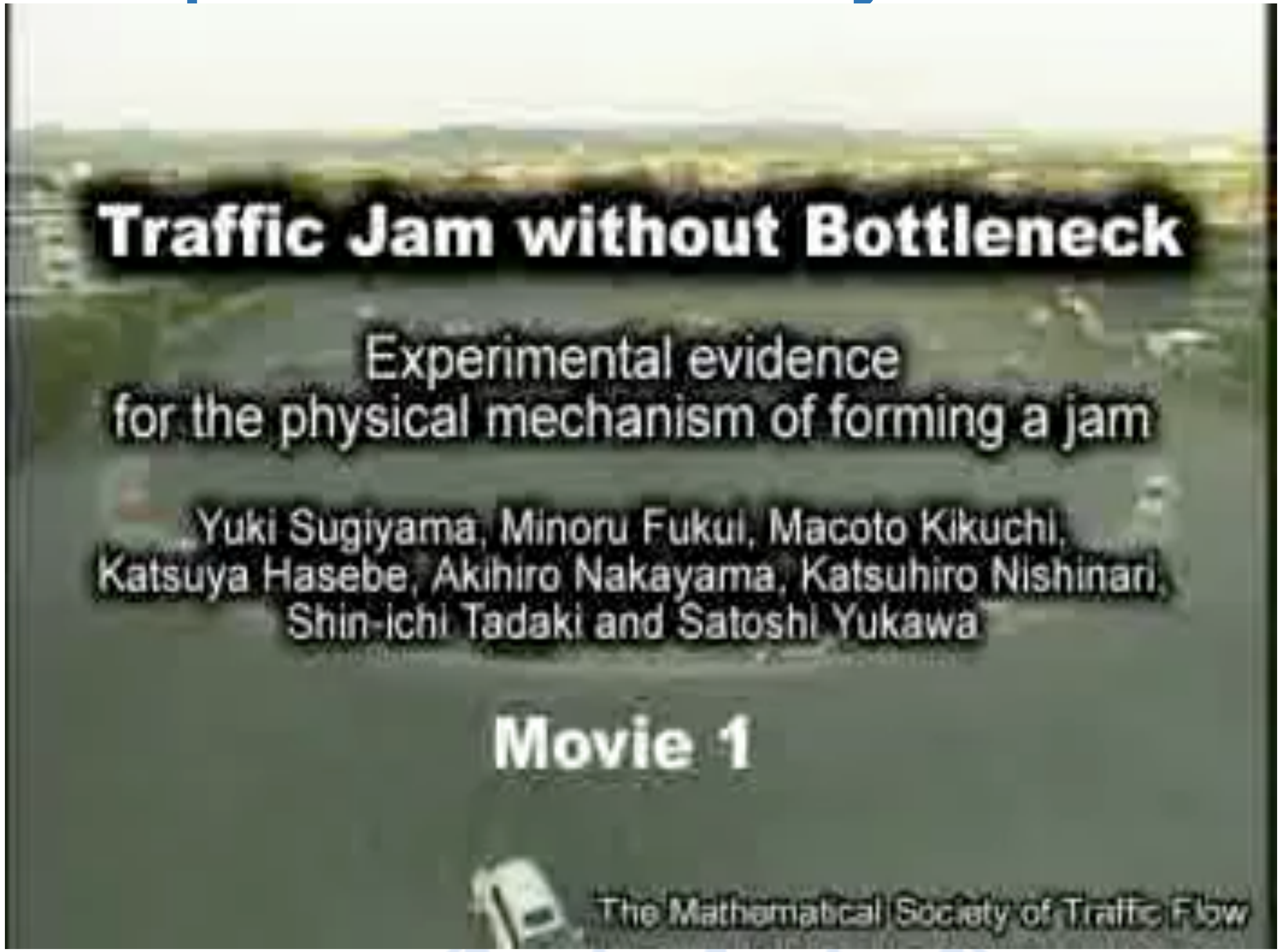

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Can we improve mobility?…after 10 years

Dissipation of stop-and-go traffic waves via control of a single autonomous vehicle

ILLINOIS UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN RUTGERS TEMPLE UNIVERSITY THE UNIVERSITY OF ARIZONA

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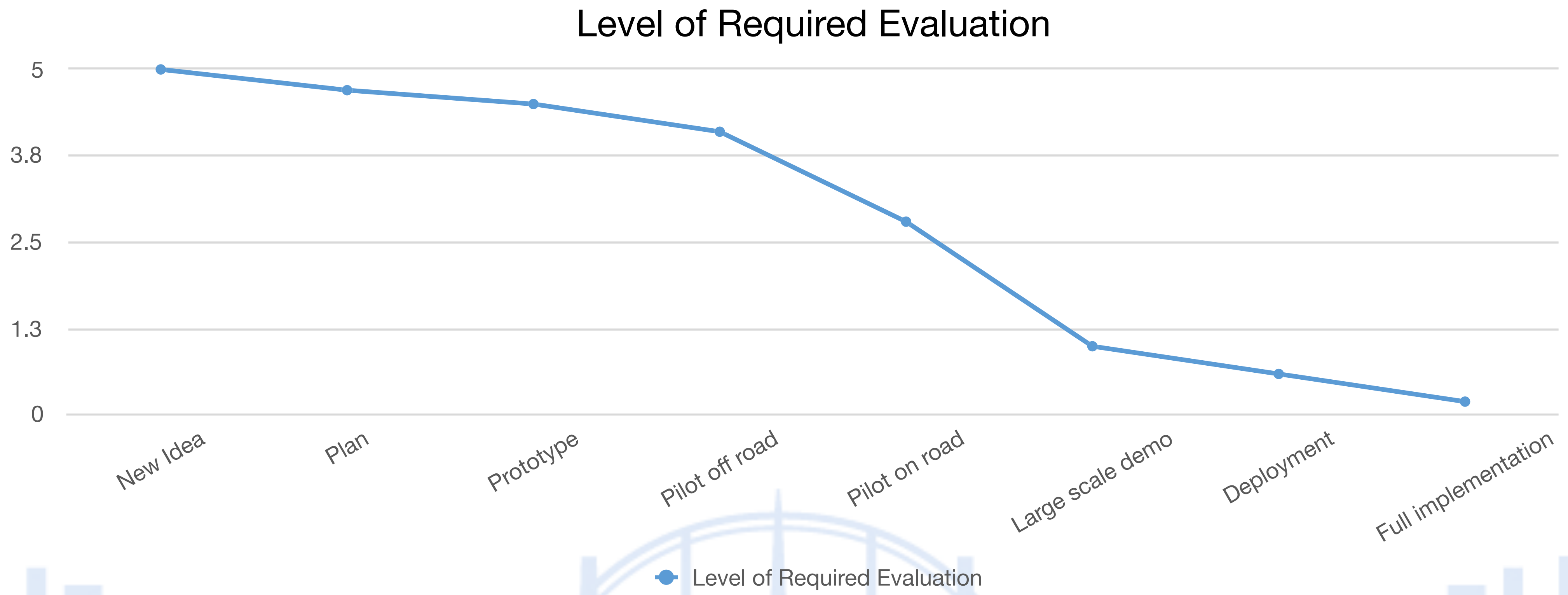
Evaluation needs…

Evaluation is highly required when technology is under development period

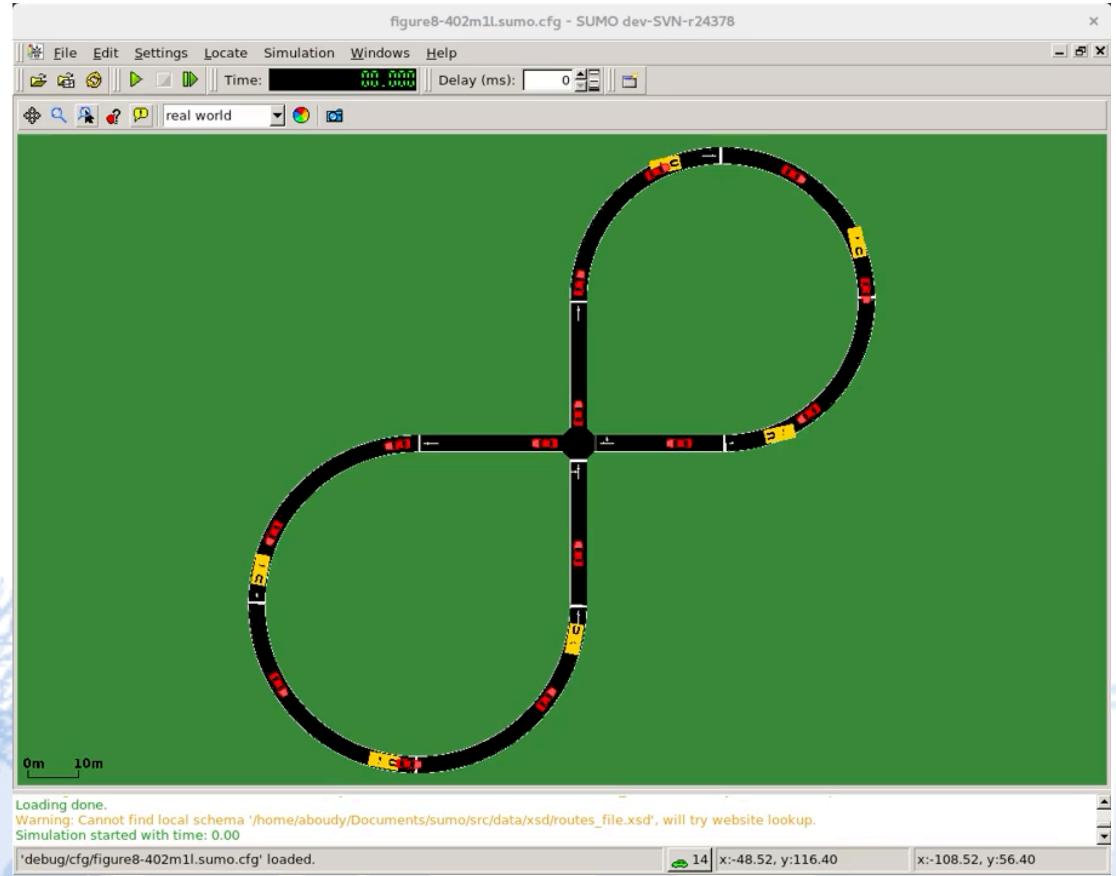


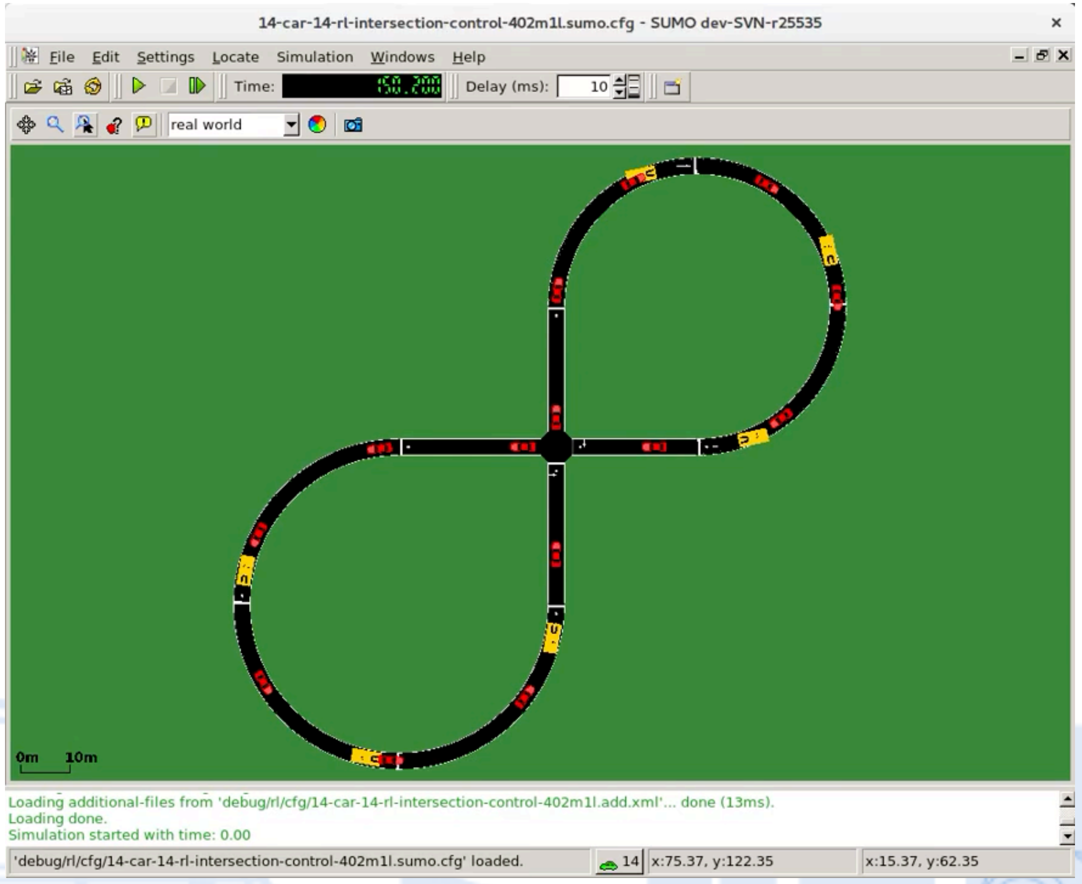
Level of evaluation required at each stage in new services implementation

Lu, M. (2016). *Evaluation of Intelligent Road Transport Systems : Methods and Results.* The Institution of Engineering and Technology.

What is the next?

- Reinforcement learning for automation of complex situations.





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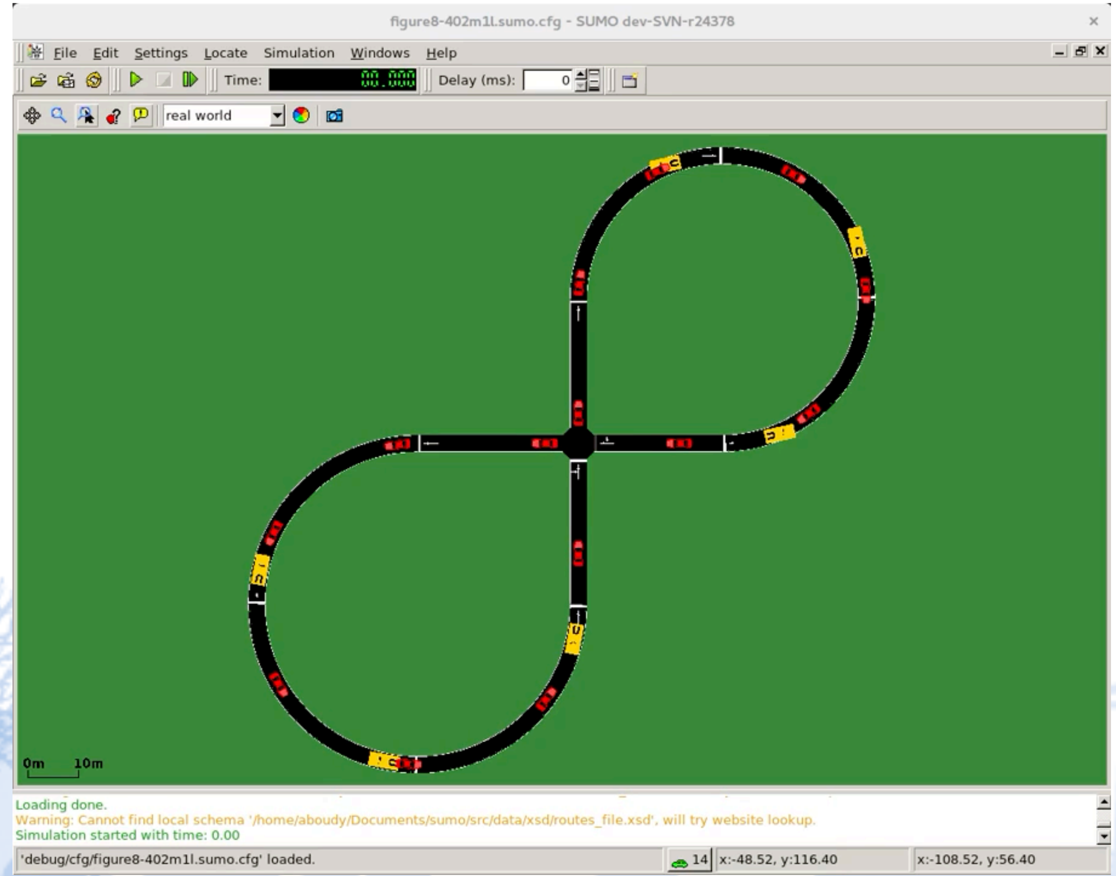

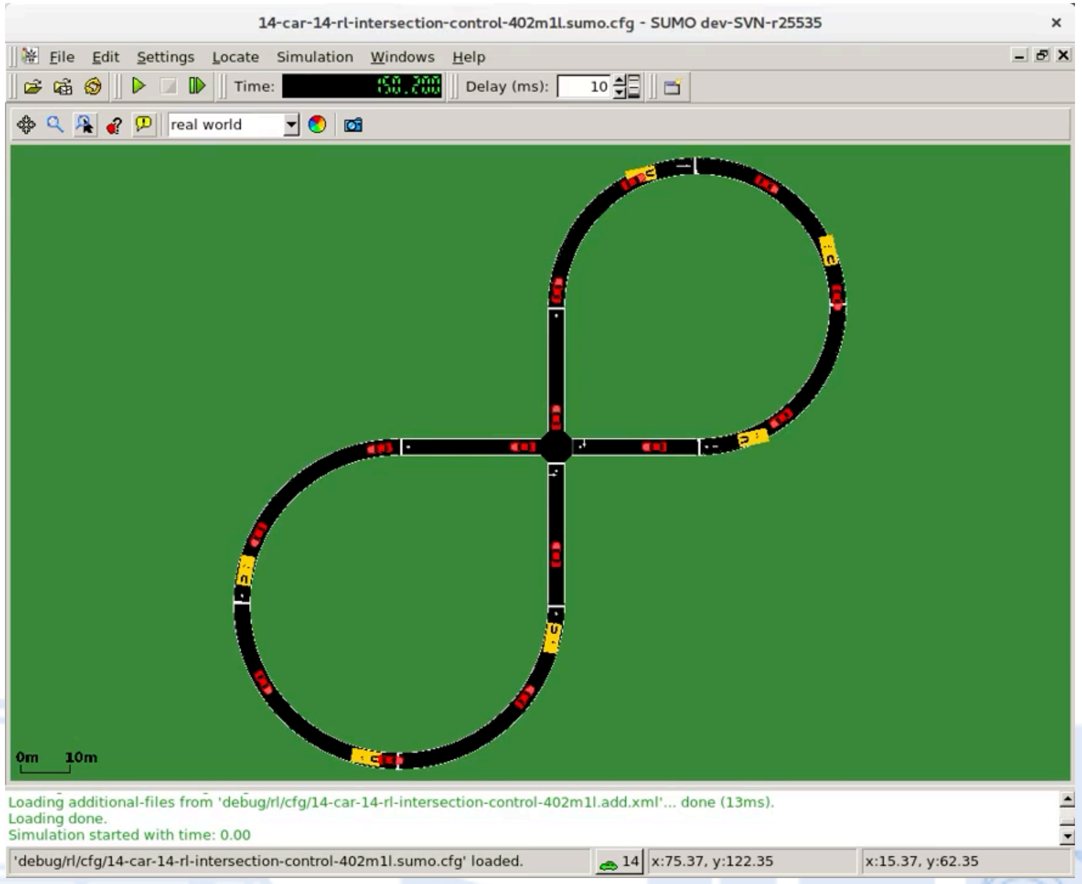

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It is possible to change mobility

- More automation → Better mobility: **Yes!**
- Road map:

Data:
- Floating Car Data (GPS, Cell towrs, CAVs)
- Event data (Closures, Special events, Accidents)

Model:
- Computational time
- Micro/Meso/Macro simulations
- Dimensionality

Estimation:
- Model calibration
- Missing data correction
- Demand forecasting, Routing

Control:
- Control of the traffic models
- Infrastructure based / vehicle based policies
- Integration of AI into control

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Part I

Large Scale Networks - Estimation

How everything started?

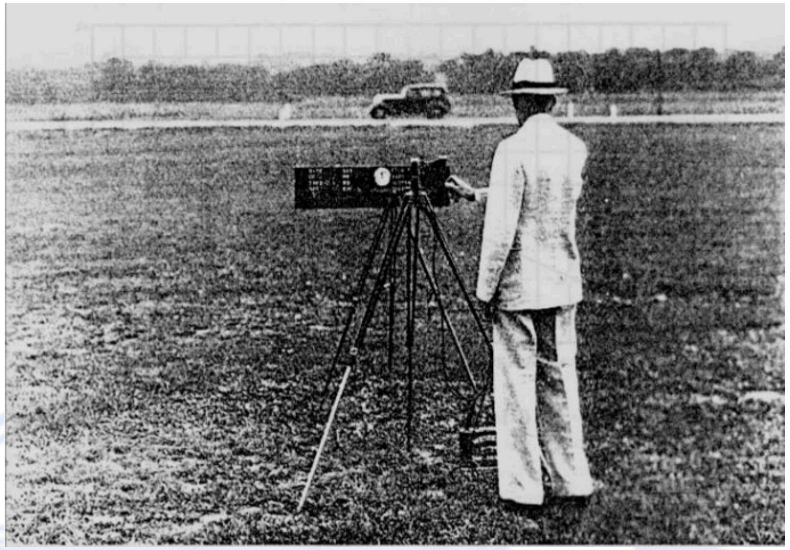
1935

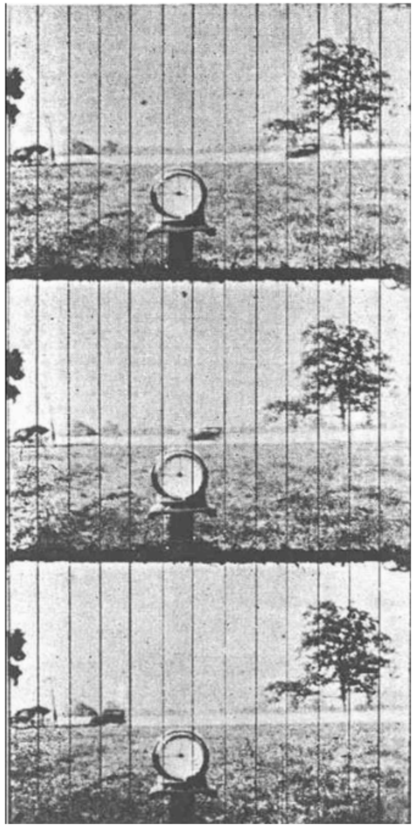
First aggregate model for congestion

1955

First PDE model for traffic congestion

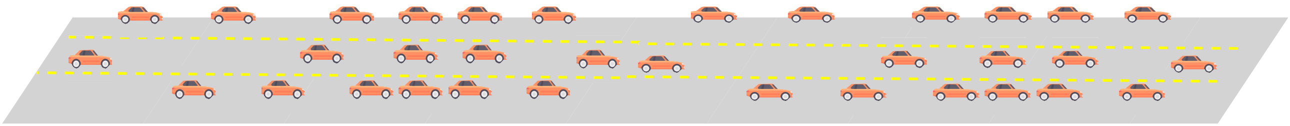
$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial \phi(\rho)}{\partial x} = 0}$$





Mechanism
vehicle counting

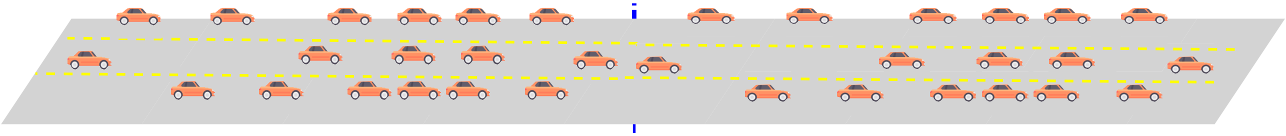
Real world



Modeling problem

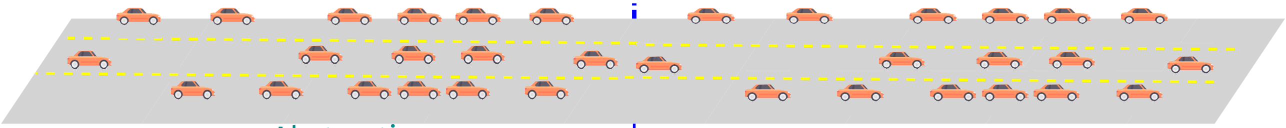
Estimation problem

Real world

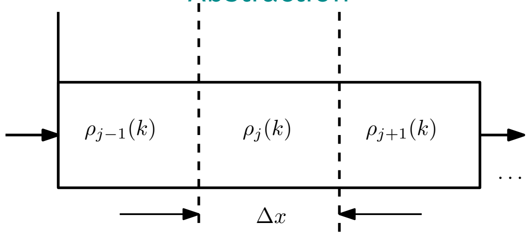


Modeling problem | *Estimation problem*

Real world



Abstraction



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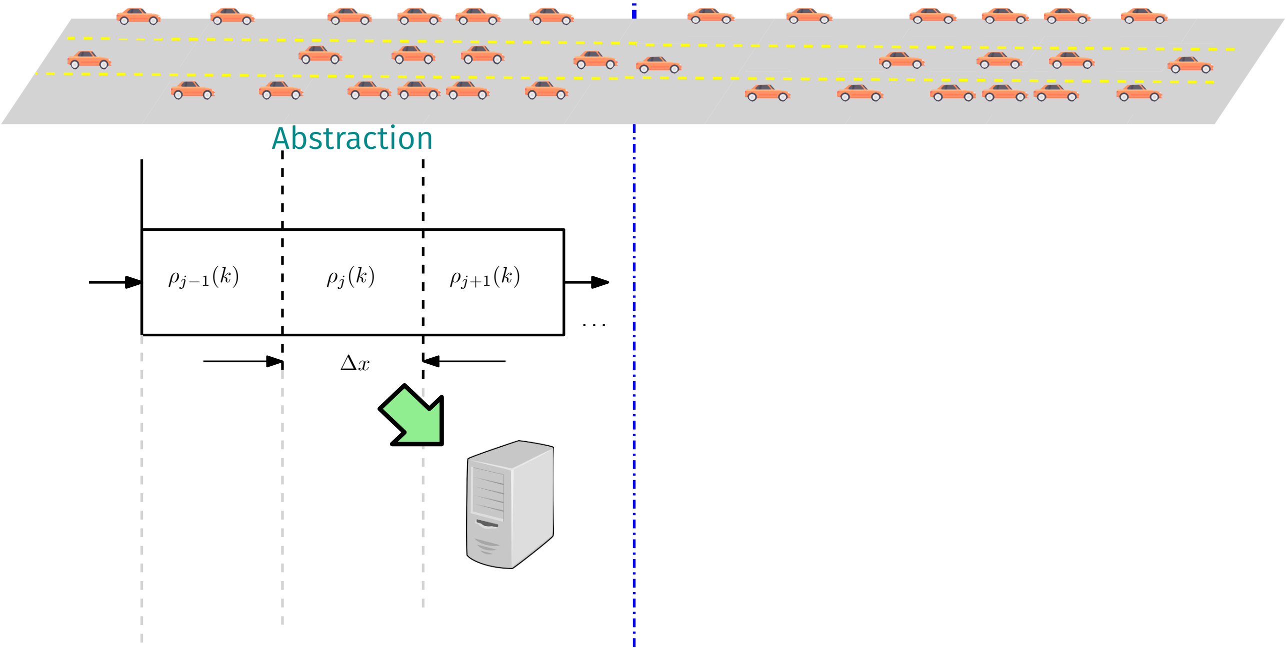

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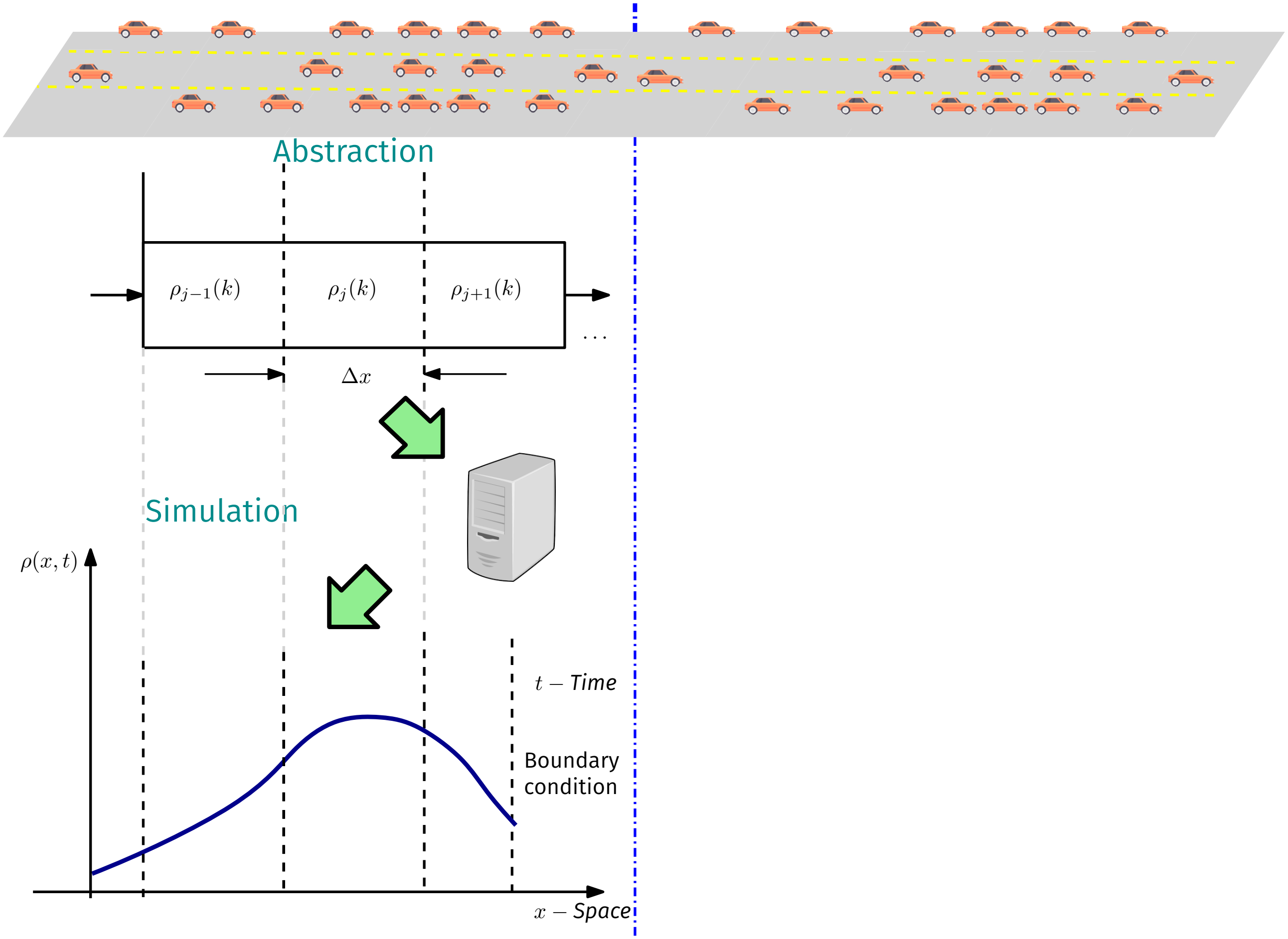

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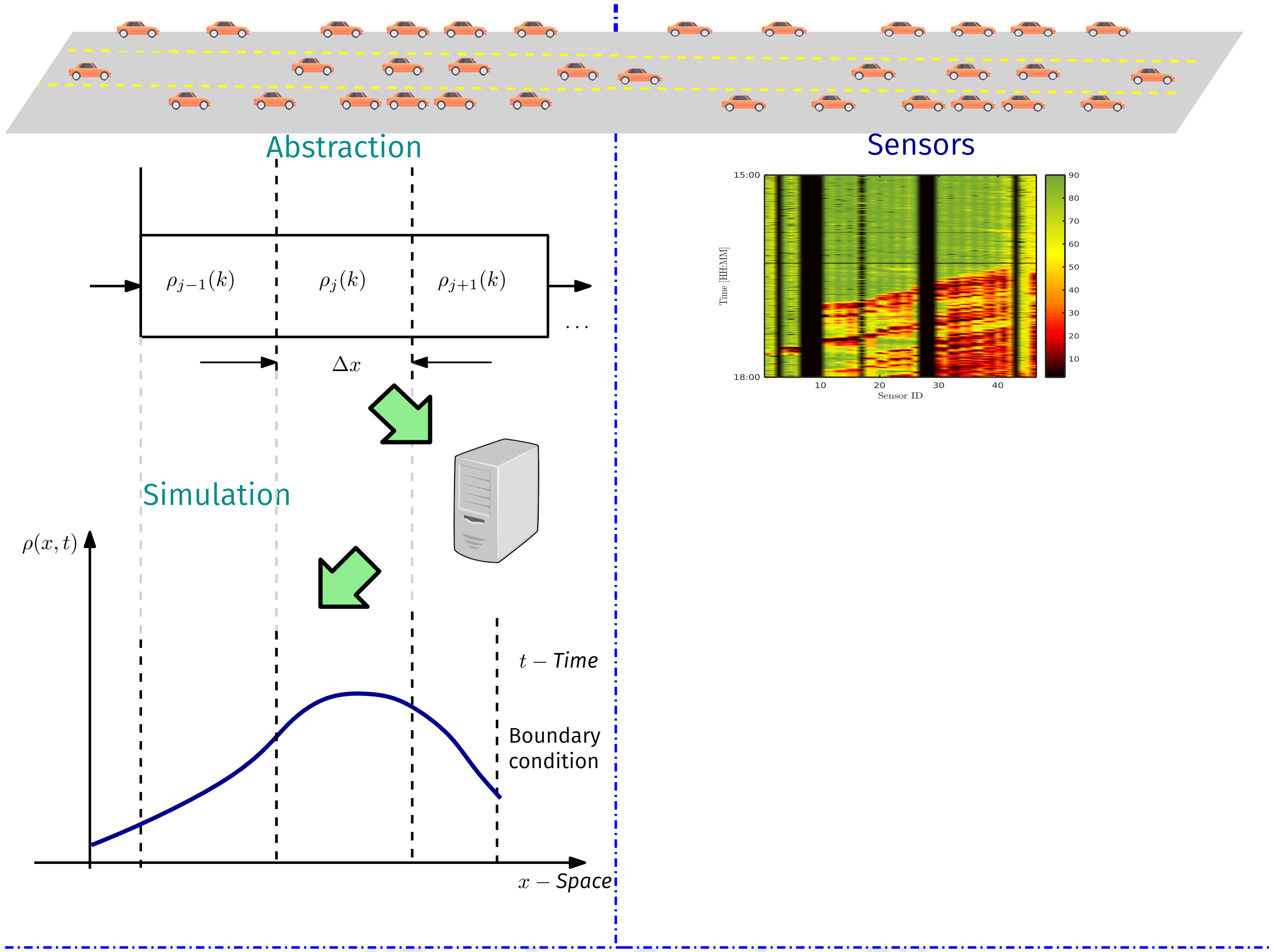
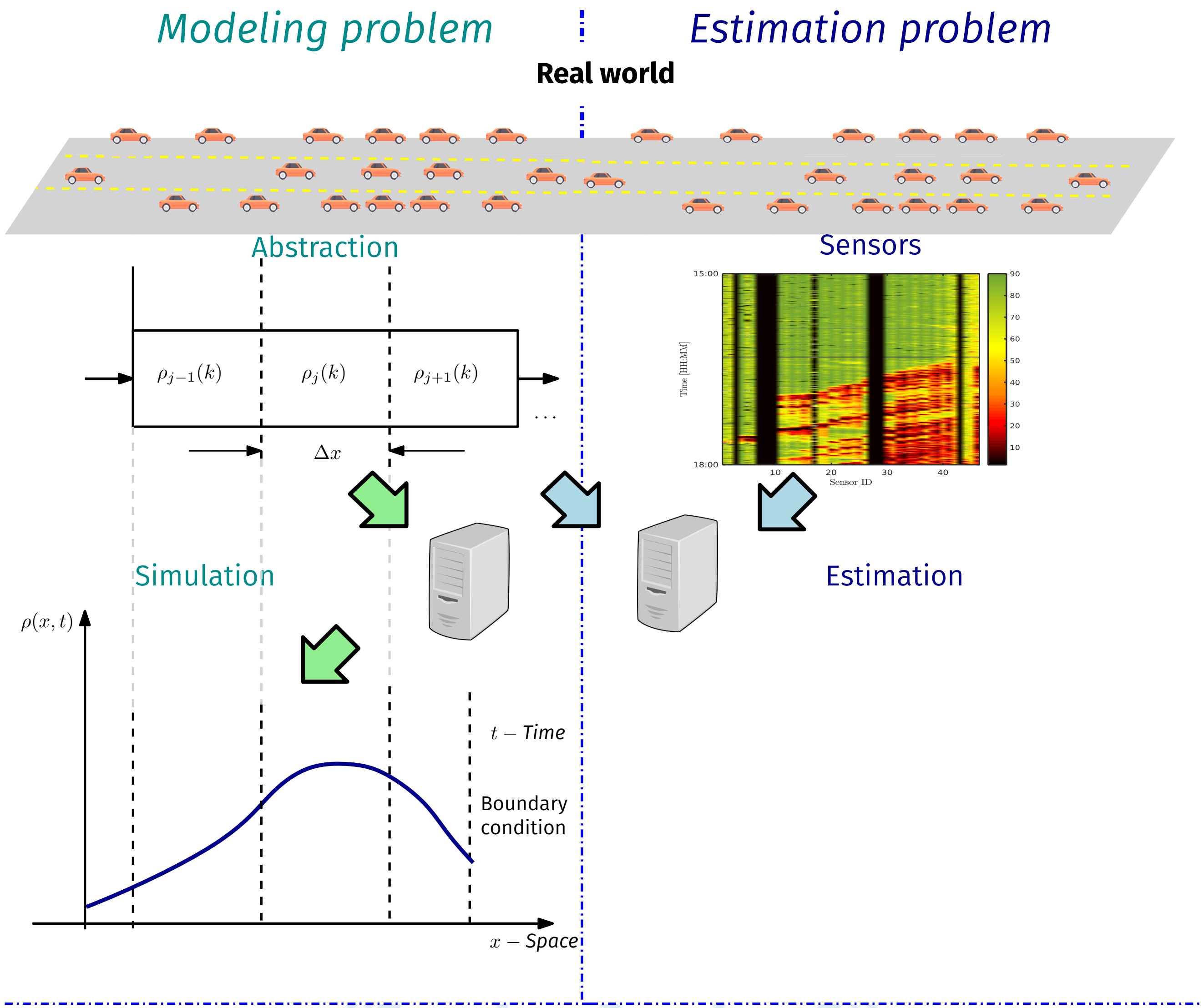
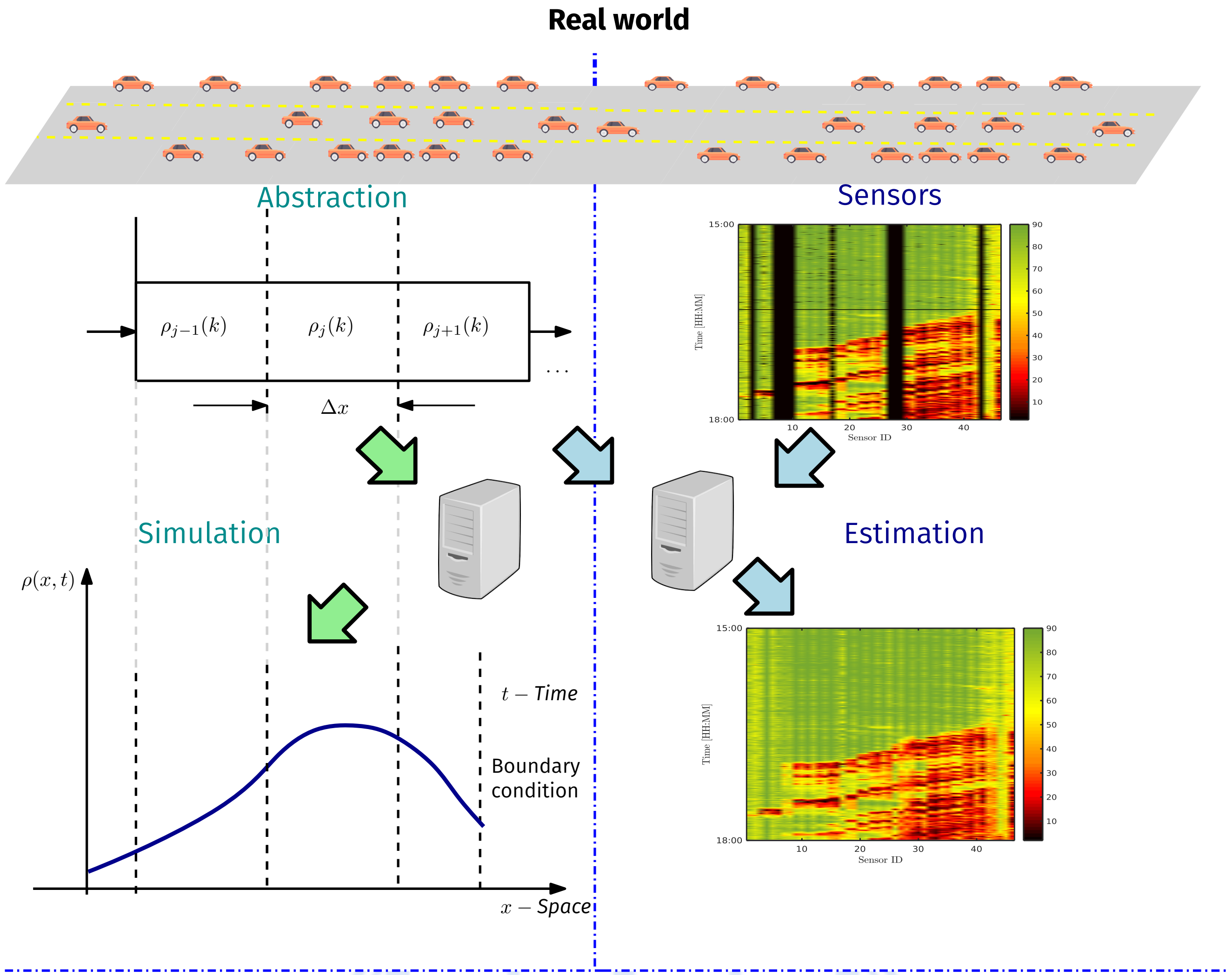




Modeling problem
Estimation problem
Real world
Abstraction
Sensors
$\rho_{j-1}(k)$
$\rho_j(k)$
$\rho_{j+1}(k)$
Δx
Simulation
Estimation
$\rho(x,t)$
t – Time
Boundary condition
x – Space
Time [HH:MM]
Sensor ID

Modeling problem | *Estimation problem*



Traffic estimation approaches

Approach to perform traffic estimation and prediction

Traffic estimation approaches

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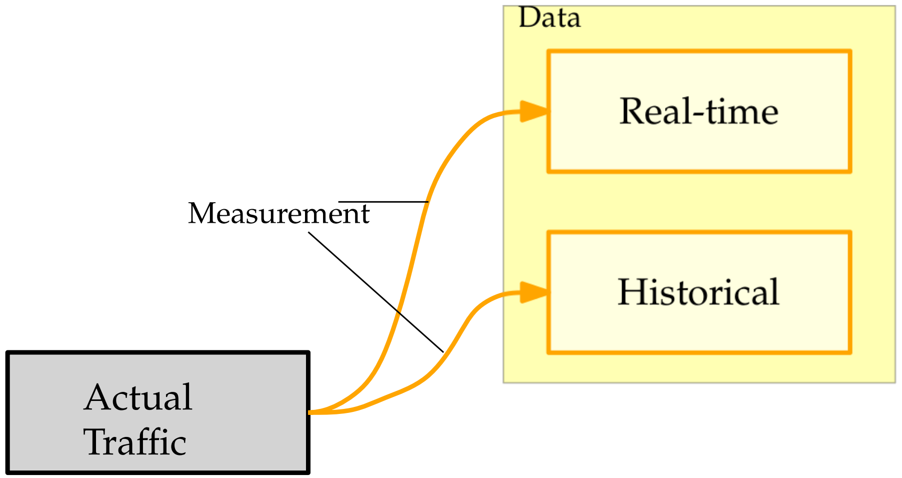
Actual Traffic

Traffic estimation approaches

Approach to perform traffic estimation and prediction



Traffic estimation approaches

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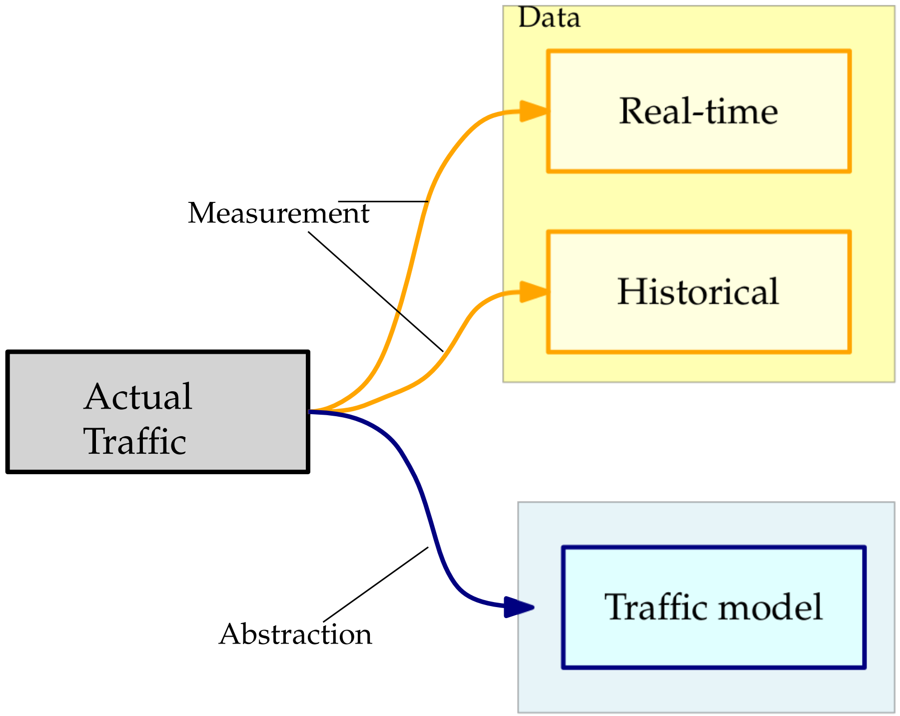

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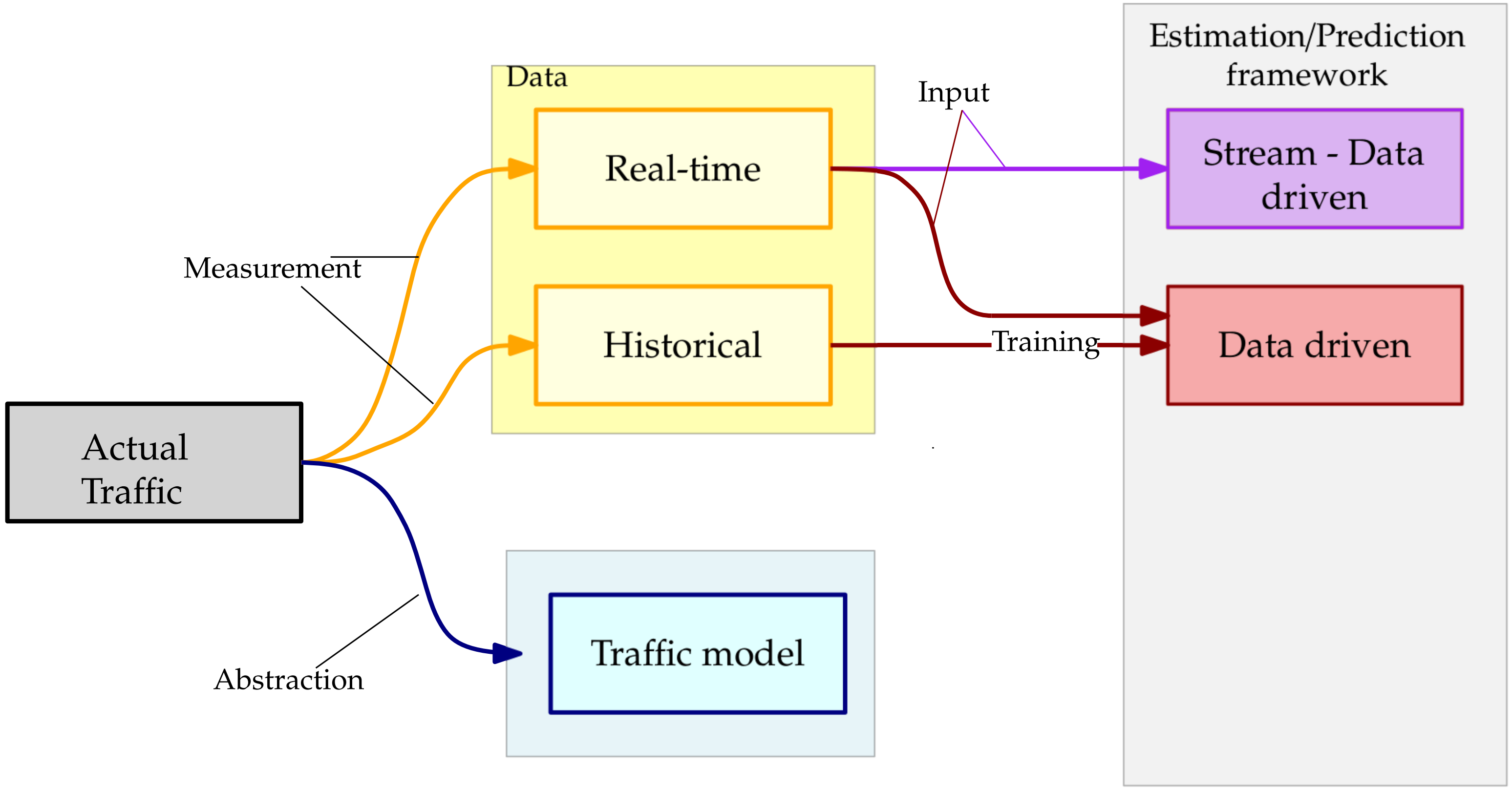

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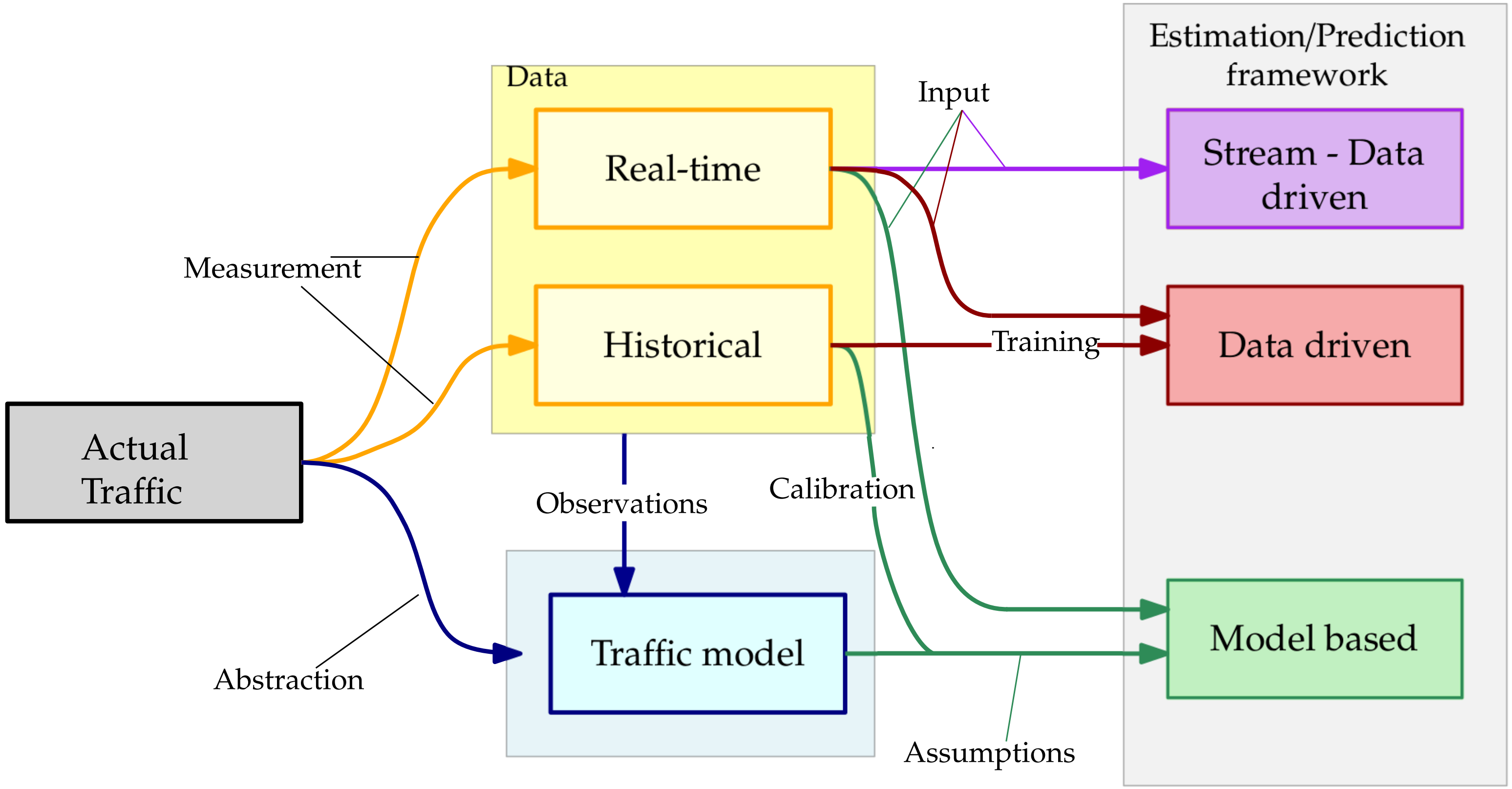

Challenges and objectives Traffic Estimation

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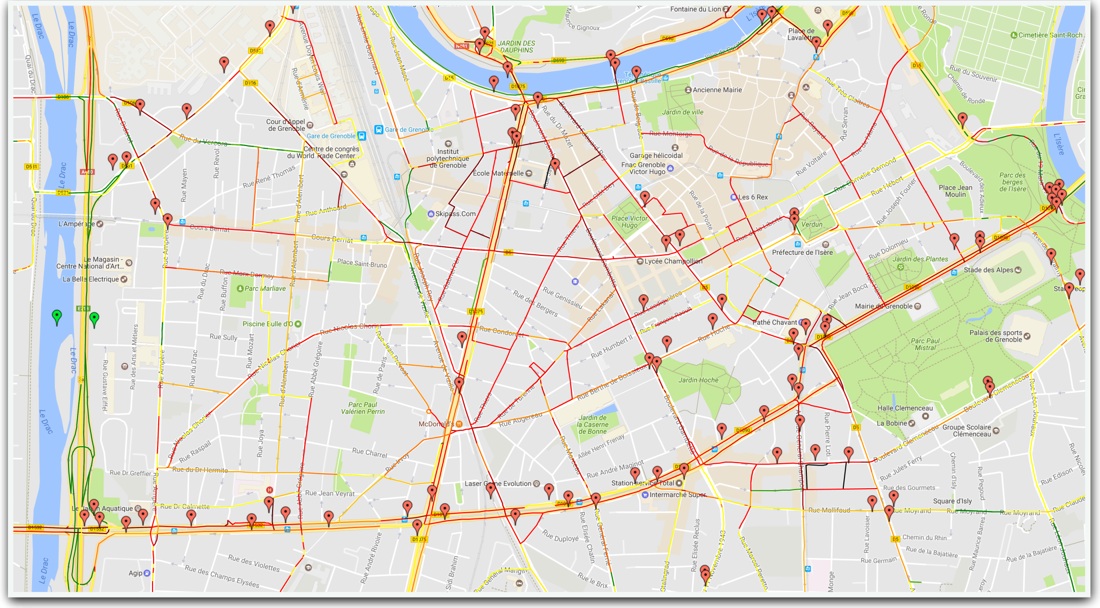
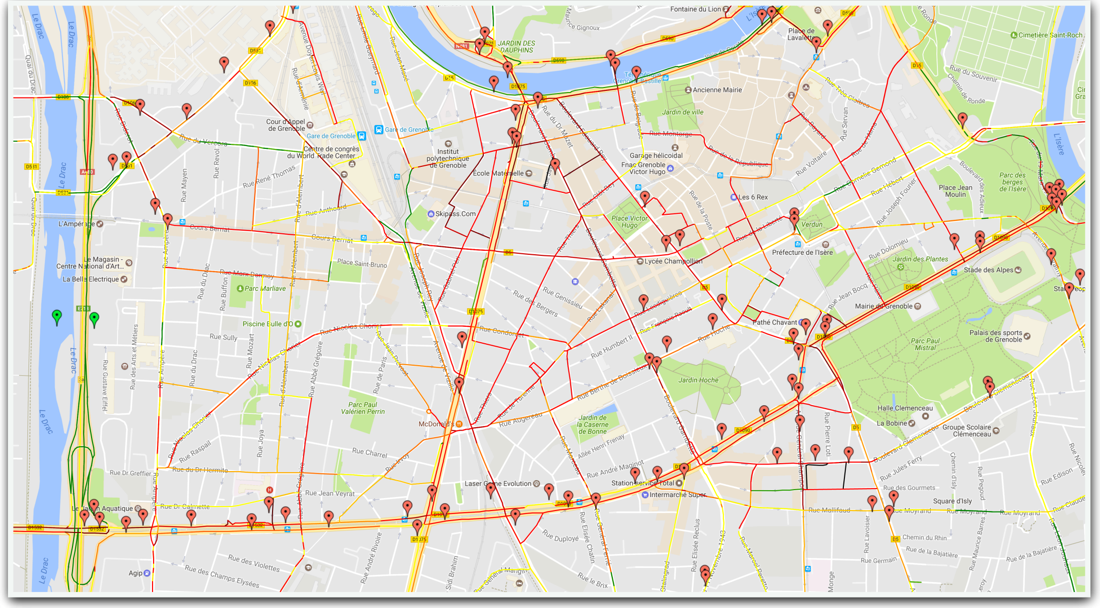


Grenoble:
(FCD) Floating Car Data. Source Google +
(MLD) Magnetic Loop Data

Challenges and objectives Traffic Estimation



Grenoble:
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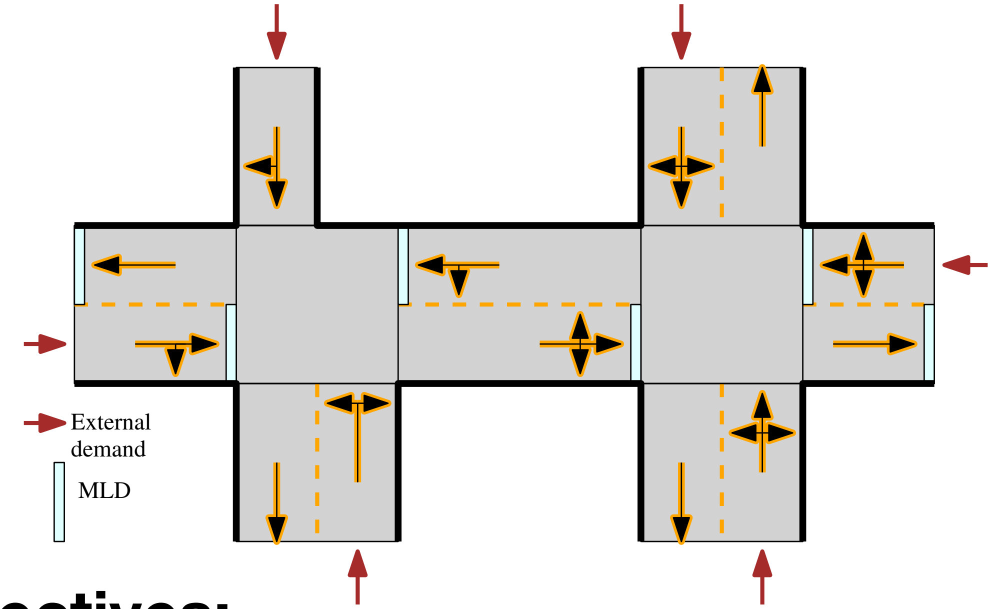
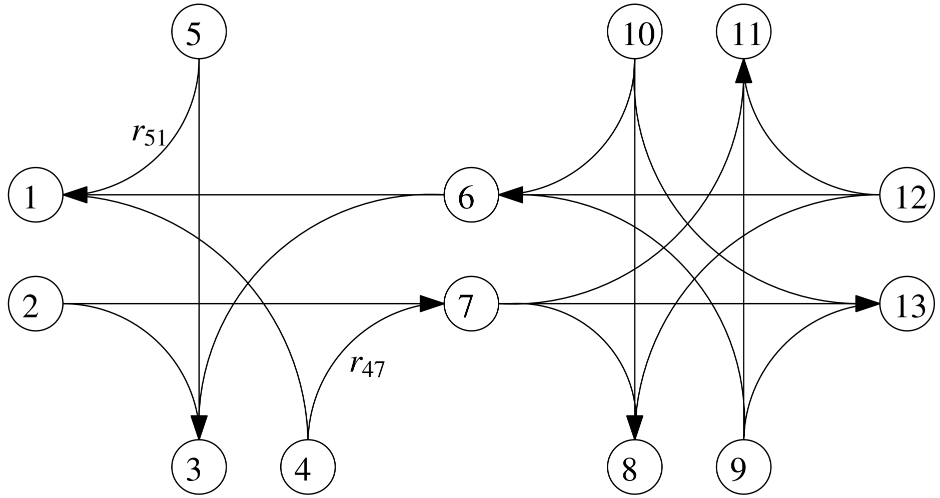
- Estimation:
  - *Heterogeneous sources* are collected data in traffic systems.
    - Nature of the measurement.
    - Level of aggregation (time, space)
  - Focus: Urban networks
    - *Challenge*: Non-linearities of the traffic model

Density and flow reconstruction

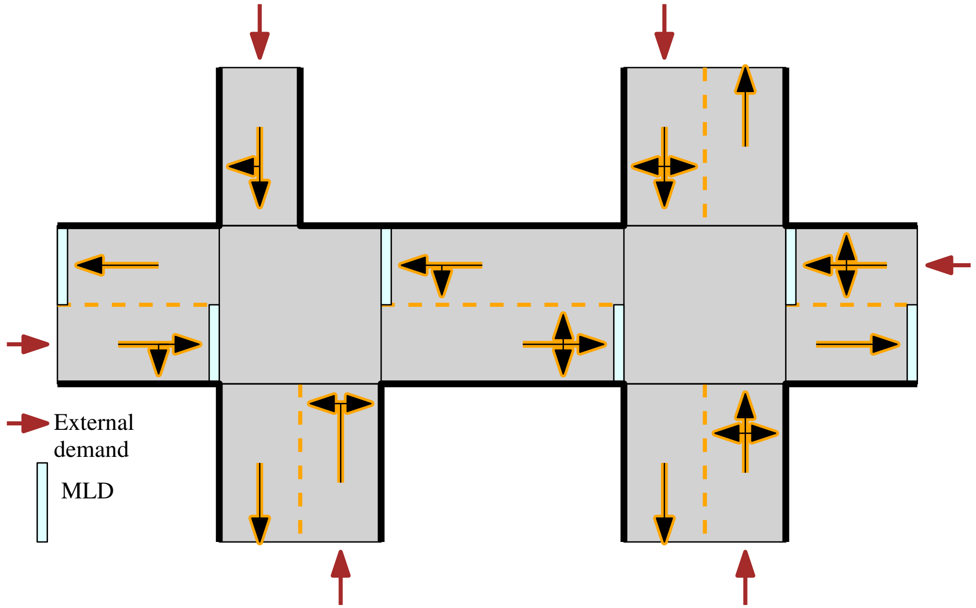


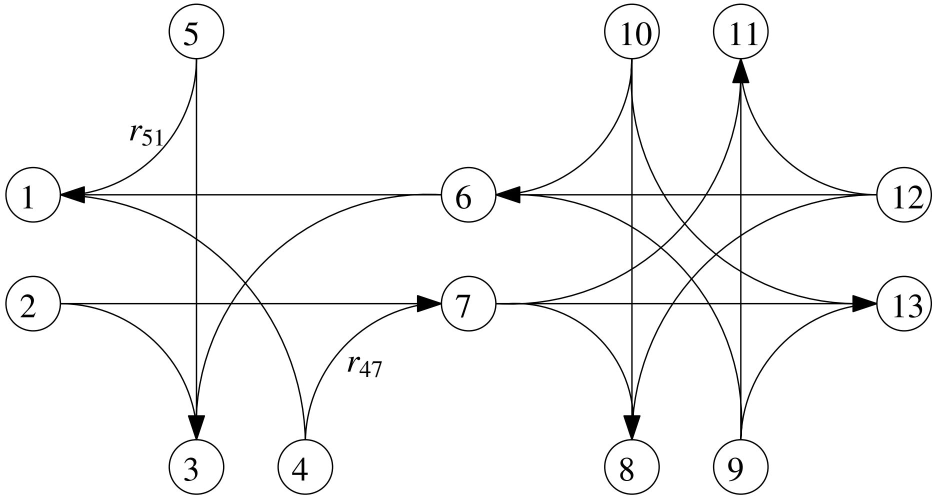


Objectives:

- Use and integrate two sources of information:
  - Floating Car Data (FCD): Speed measurements
  - Magnetic Loop Data (MLD): Flow measurements
- Use FCD in order to determine state of the network

Density and flow reconstruction





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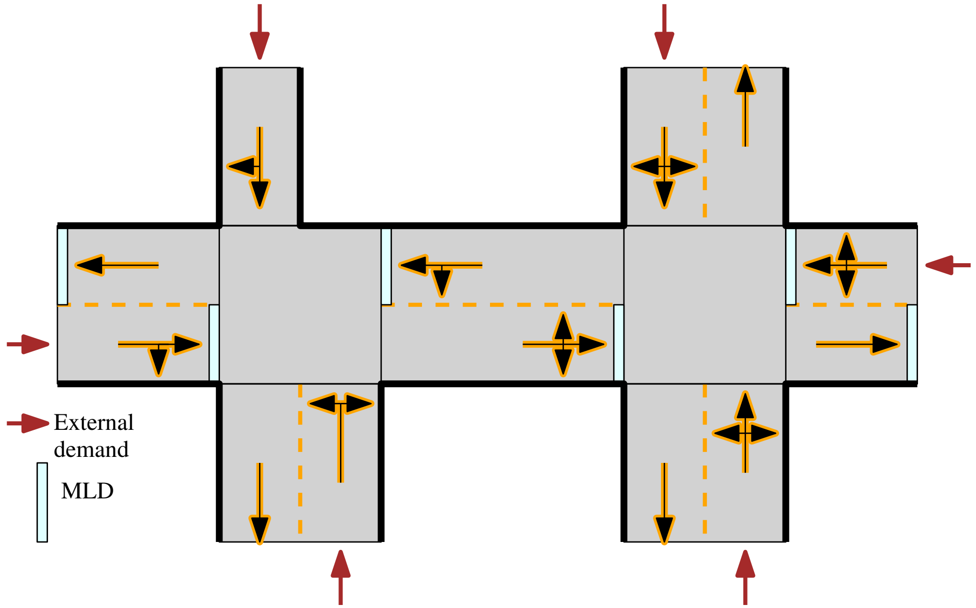

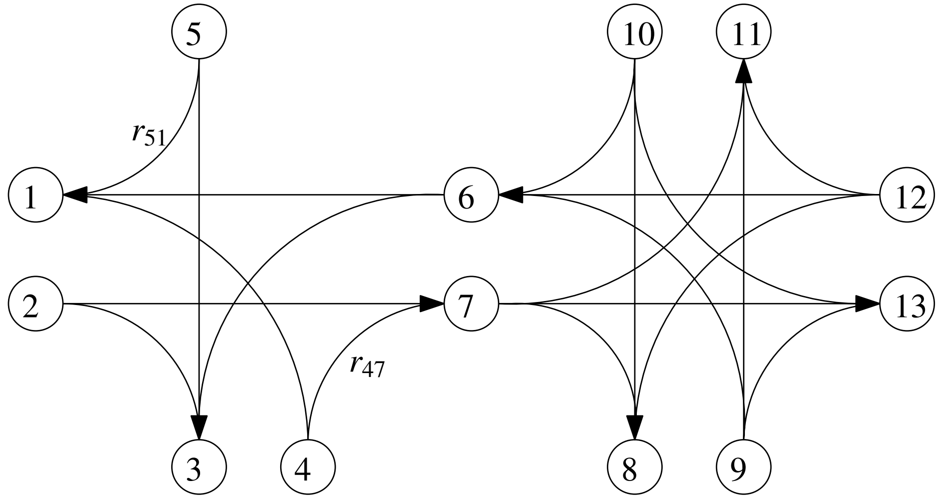

- *Challenge:* Map density-speed is unobservable
    - *Key*: Introduce flow information and steady state conditions.
- Urban or semi urban networks.

Traffic model: One dimensional road

Lighthill - William- Richards (LWR) Partial differential equation

Traffic model: One dimensional road

Lighthill - William- Richards (LWR) Partial differential equation

$$\partial_t \rho + \partial_x \Phi(\rho) = 0$$

ρ : Density

$\Phi(\rho)$: Flux function

Traffic model: One dimensional road

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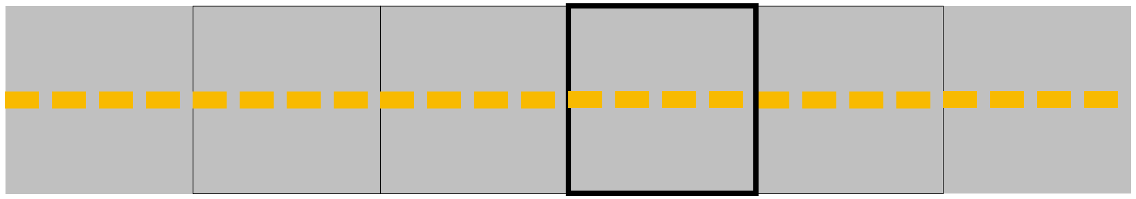
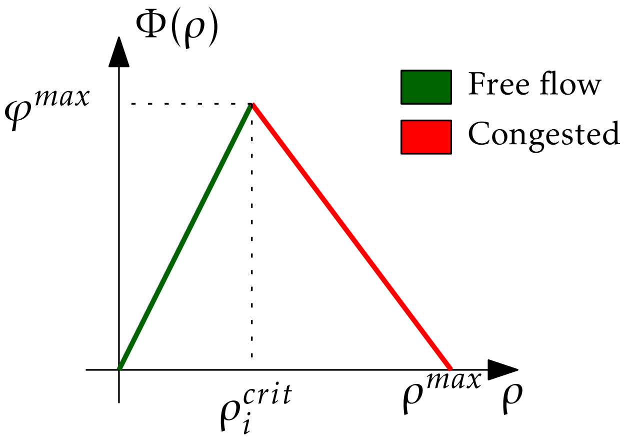
Cell transmission model (CTM)





[Daganzo, Transportation Research Part B 1994]

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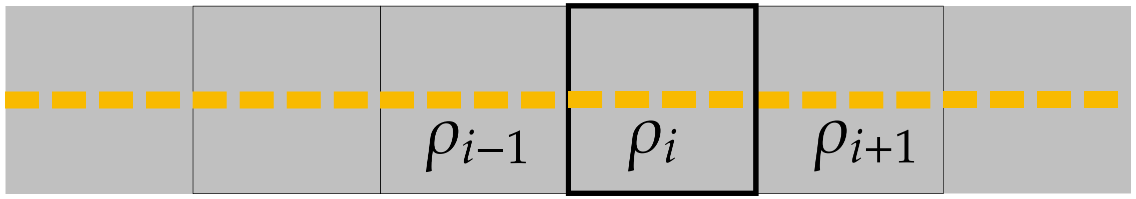

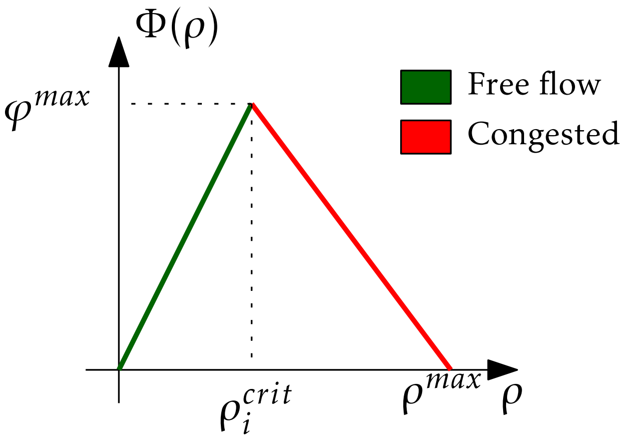

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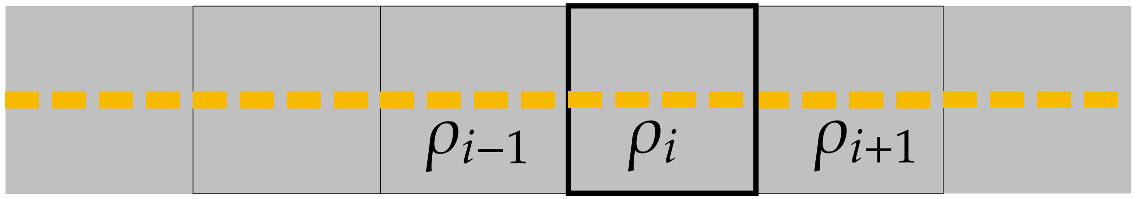

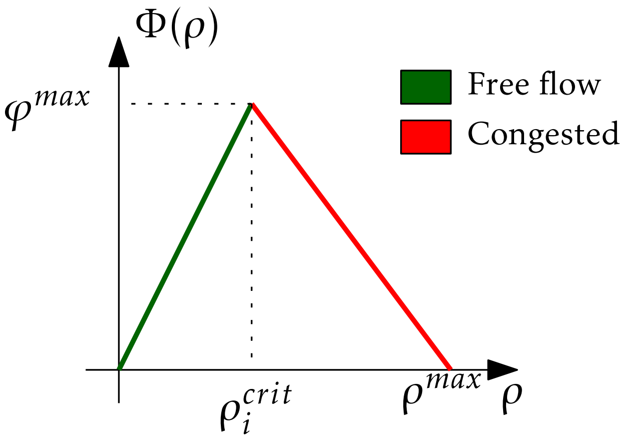

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{l_i}(\varphi_{i-1}(k) - \varphi_i(k))$$

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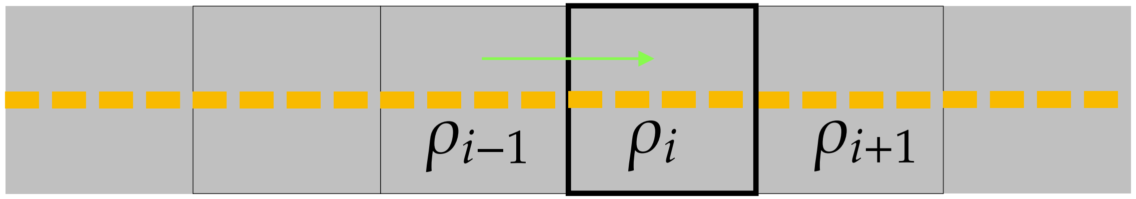

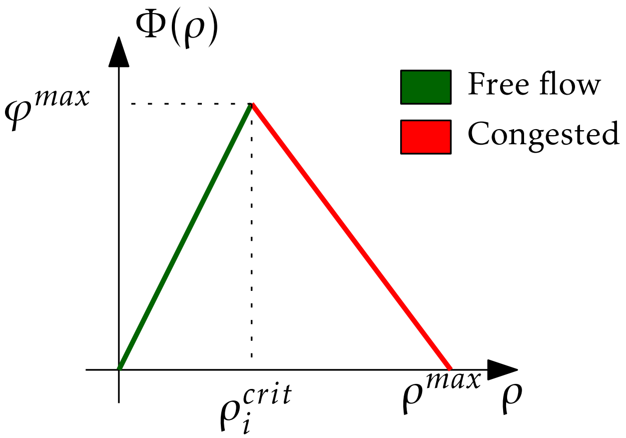

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$$\varphi_{i-1}(k) = \min\left(v^{\max}\rho_{i-1}(k), \varphi^{\max}, w(\rho^{\max} - \rho_i(k))\right)$$

[Daganzo, Transportation Research Part B 1994]

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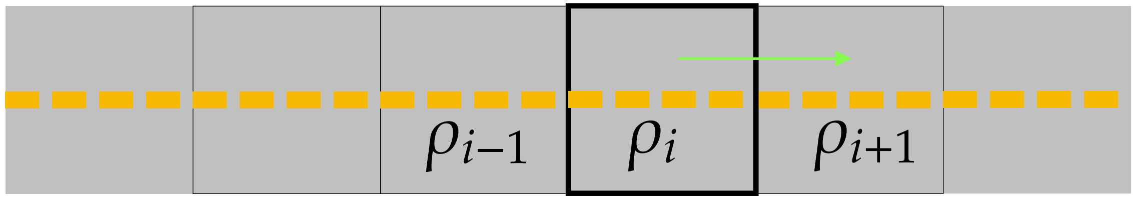

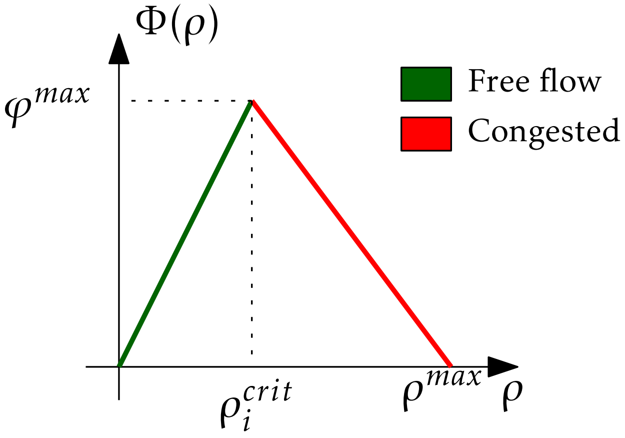

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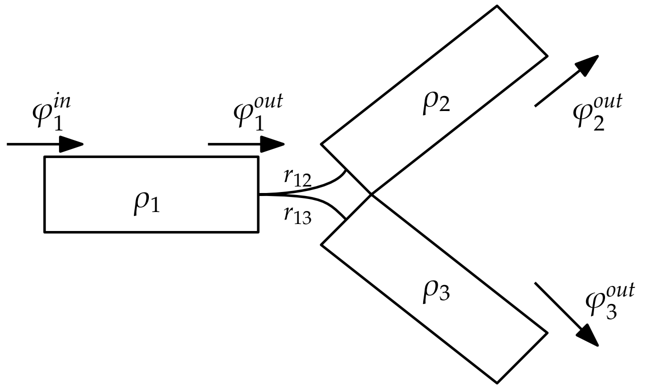
Junction model

Single junction problem:

$$\sum_{i=1}^{n} \varphi_i^{\text{out}}(\rho_i(k)) = \sum_{j=n+1}^{n+m} \varphi_j^{\text{in}}(\rho_j(k)).$$



Determine values for φ_i^{out} *and* φ_i^{in} *consistent with the LWR model, in particular its discretized counterpart*

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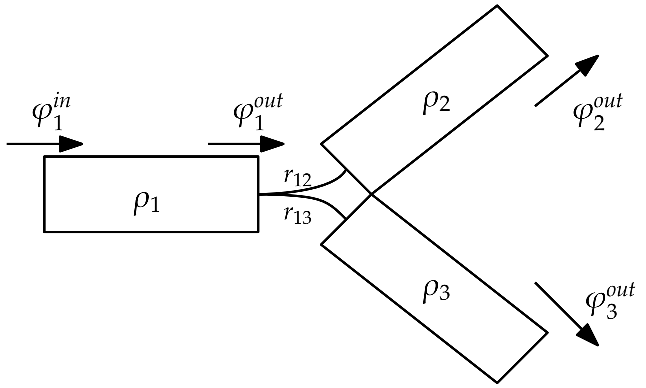

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A. <u>Drivers follow fixed routes:</u> There exists traffic coefficients $r_{ij} \in (0,1]$ representing the splitting ratio to go from road I_i to road I_j

$$\varphi^{\text{in}} = \mathcal{R}_g^T \varphi^{\text{out}} + \varphi^{\text{ext}}$$

[Coclite et. al, SIAM J. Math. Anal. 2005]

Junction model

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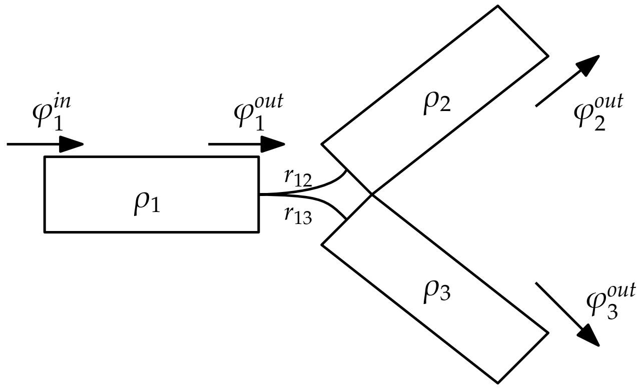

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$$\varphi^{\text{in}} = \mathcal{R}_g^T \varphi^{\text{out}} + \varphi^{\text{ext}}$$

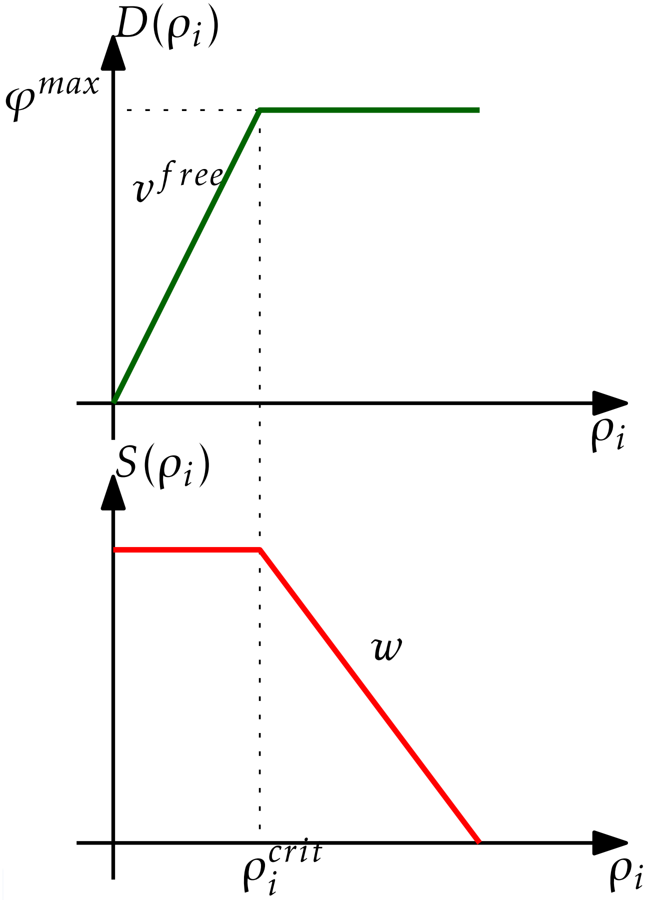
B. Drivers tend to maximize the network throughput:

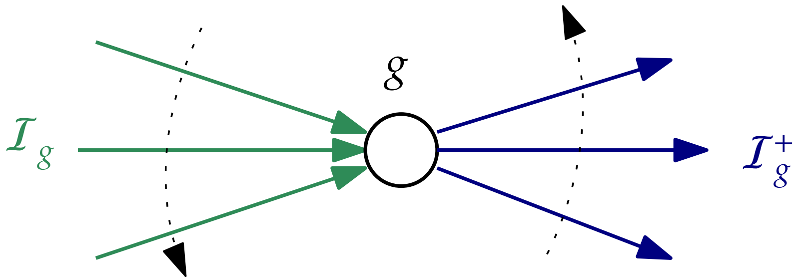
$$\varphi^{\text{out}} = \arg\max \sum_{i=1}^{n} \varphi_i^{\text{out}}$$

[Coclite et. al, SIAM J. Math. Anal. 2005]

Solving the junction model

Solutions for the φ^{out} are given boundary value problems





$$D(\rho_i) = \min\left(v^{free}\rho_i, \varphi^{max}\right)$$

$$S(\rho_i) = \min\left(\varphi^{\max}, w(\rho^{\max} - \rho_i)\right)$$

Solving the junction model

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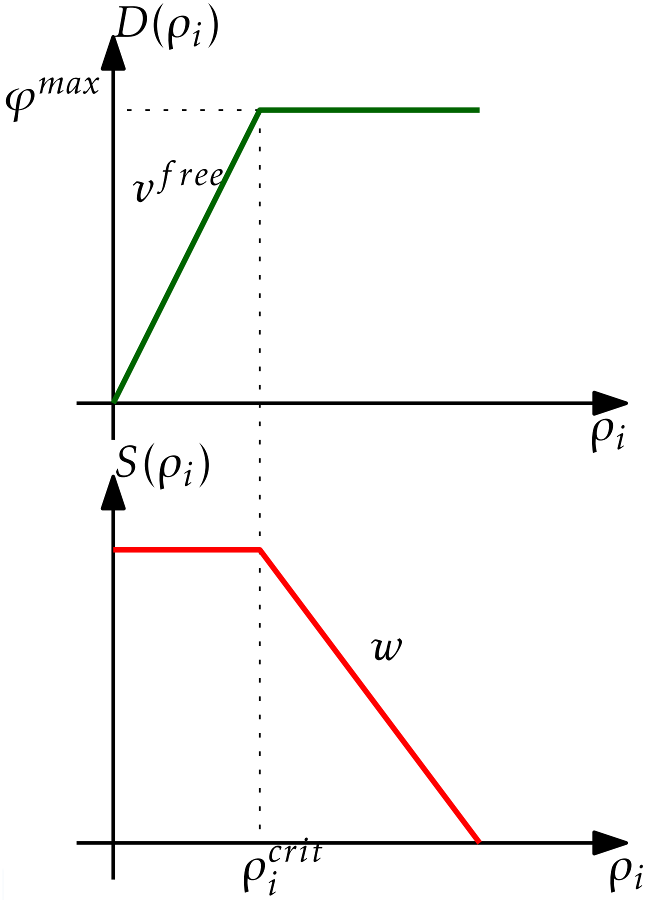

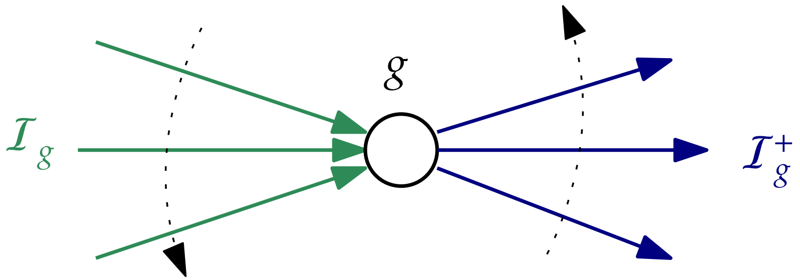

Constraints can be integrated into rule B

$$
\begin{aligned}
\max_{\varphi_i^{\text{out}}} \quad & \sum_{i\in\mathcal{I}_g^-} \varphi_i^{\text{out}} \\
\text{s.t} \quad & \varphi_i^{\text{out}} \leq D(\rho_i) \quad i \in \mathcal{I}_g^- \\
& \varphi_i^{\text{out}} \leq \varphi^{max} \\
& \sum_{i\in\mathcal{I}_g^-} r_{ij}\varphi_j^{\text{out}} \leq S(\rho_j) \quad j \in \mathcal{I}_g^+ \\
& \sum_{i\in\mathcal{I}_g^-} r_{ij}\varphi_j^{\text{out}} \leq \varphi^{max}
\end{aligned}
$$

Solving the junction model

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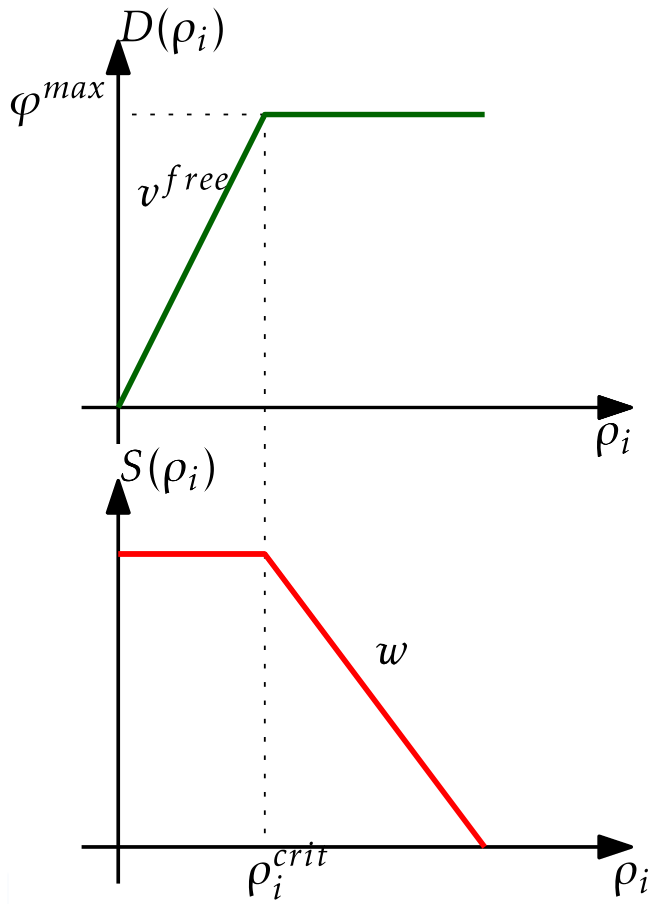

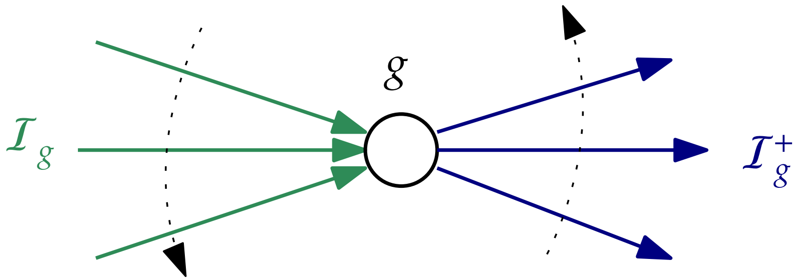

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$$
\begin{array}{lll}
\max\limits_{\varphi_i^{\text{out}}} & \displaystyle\sum_{i\in\mathcal{I}_g^-} \varphi_i^{\text{out}} & \\
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 & \displaystyle\sum_{i\in\mathcal{I}_g^-} r_{ij}\varphi_j^{\text{out}} \le S(\rho_j) \quad j \in \mathcal{I}_g^+ & \\
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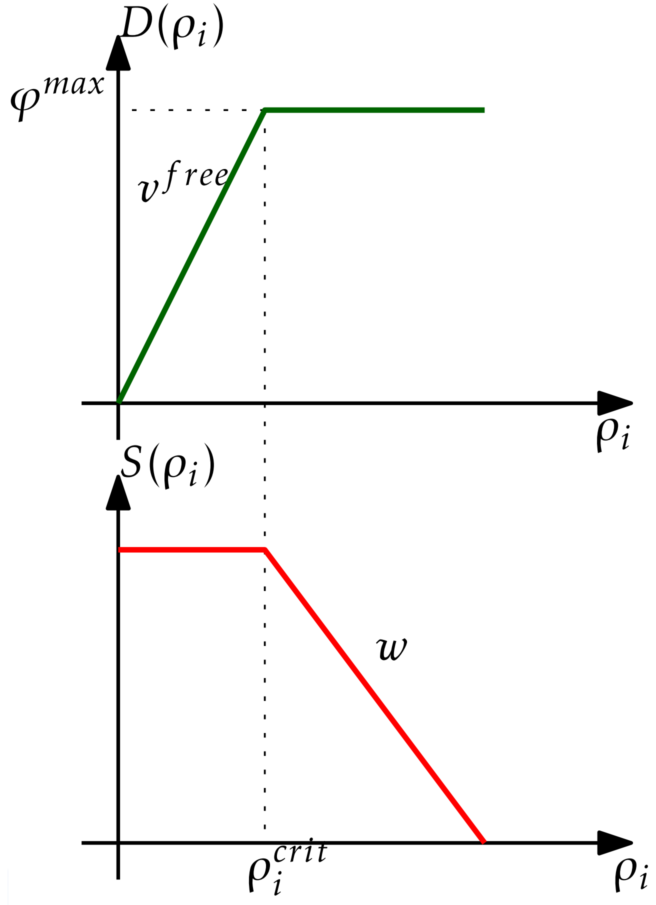
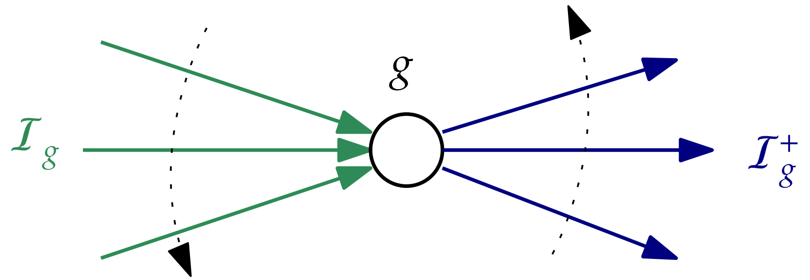
Solving the junction model

Solutions for the φ^{out} are given boundary value problems





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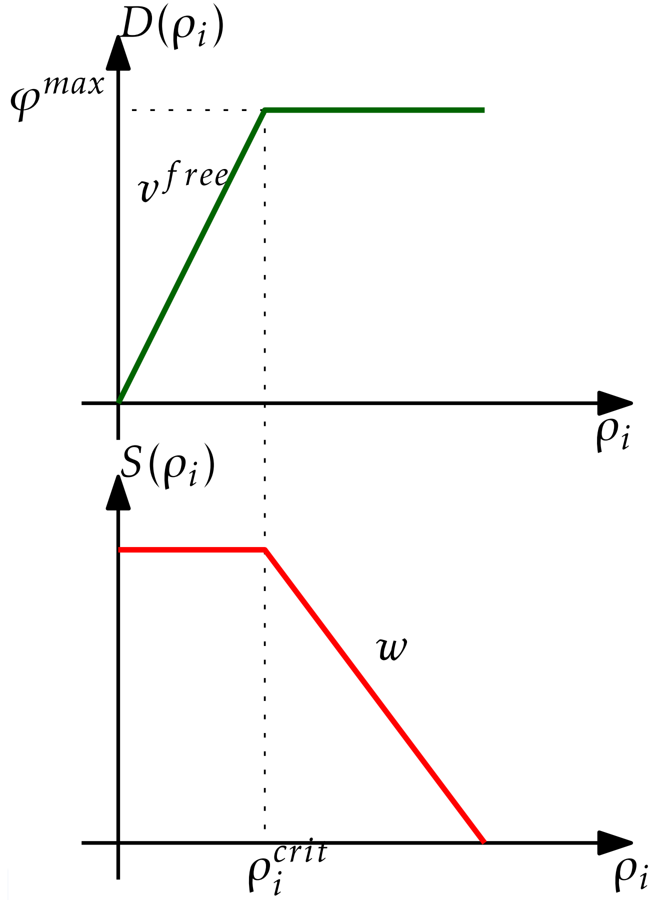

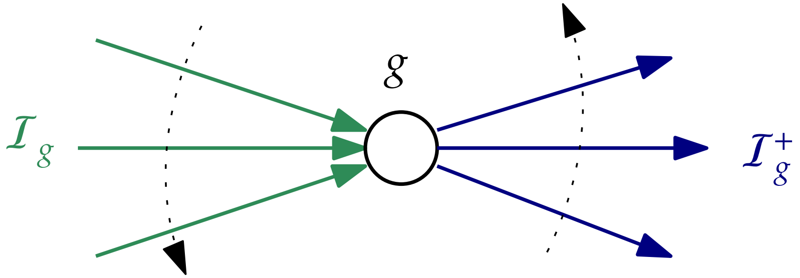

Constraints can be integrated into rule B

$$\max_{\varphi_i^{\text{out}}} \sum_{i \in \mathcal{I}_g^-} \varphi_i^{\text{out}} \qquad \mathcal{P}_g^{\varphi}$$

s.t	$\varphi_i^{\text{out}} \leq D(\rho_i) \quad i \in \mathcal{I}_g^-$	$\check{\mathcal{D}}_g$
	$\varphi_i^{\text{out}} \leq \varphi^{max}$	$\mathcal{D}_g^{\max}$
	$\sum_{i \in \mathcal{I}_g^-} r_{ij}\varphi_j^{\text{out}} \leq S(\rho_j) \quad j \in \mathcal{I}_g^+$	$\check{\mathcal{S}}_g$
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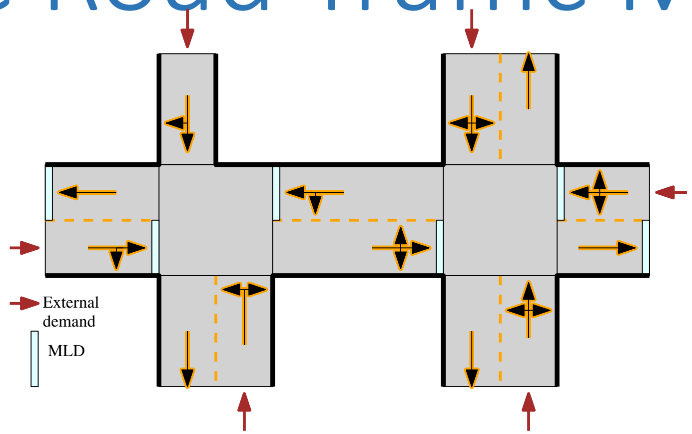
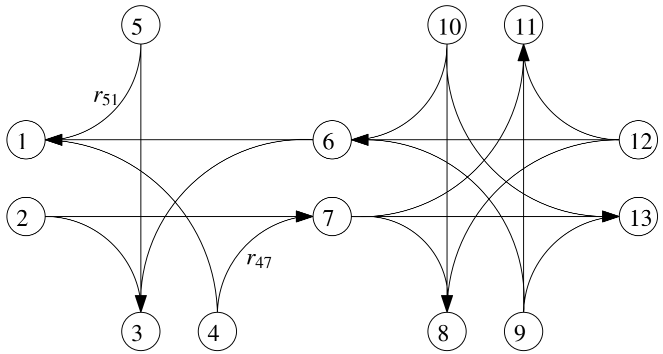
Dynamic Road Traffic Model





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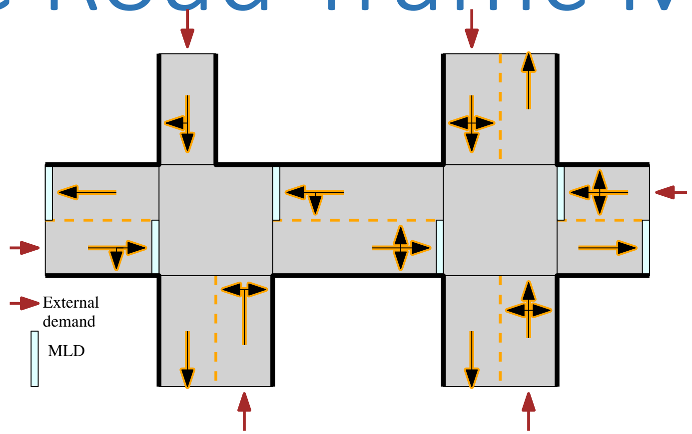
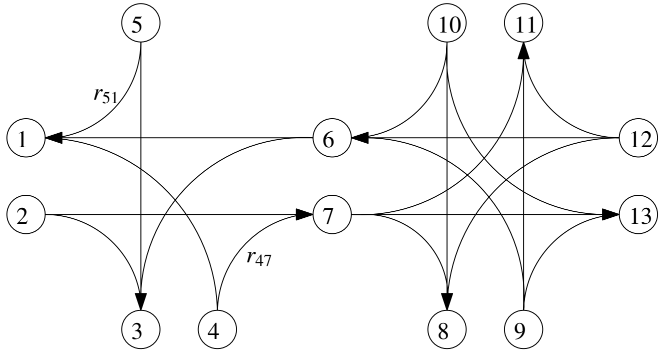
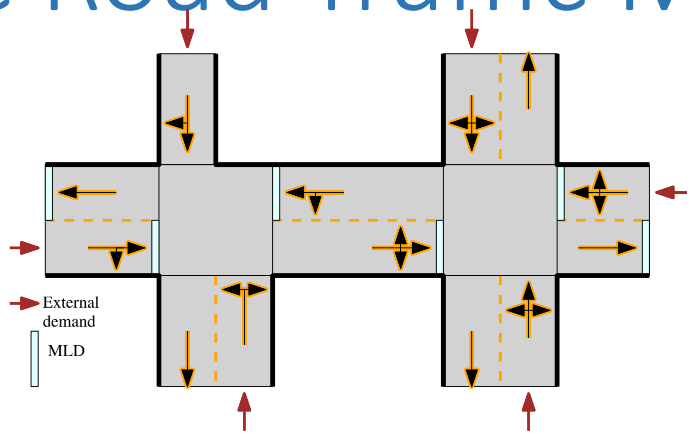


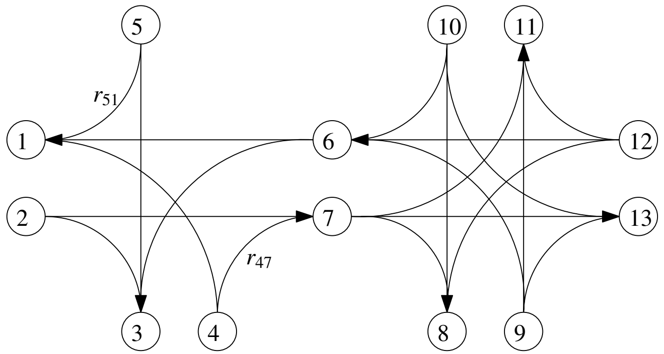


Conservation law at the network level is given by:

$$\rho(k+1) = \rho(k) + TL^{-1}(\varphi^{\text{in}}(k) - \varphi^{\text{out}}(k))$$

Dynamic Road Traffic Model





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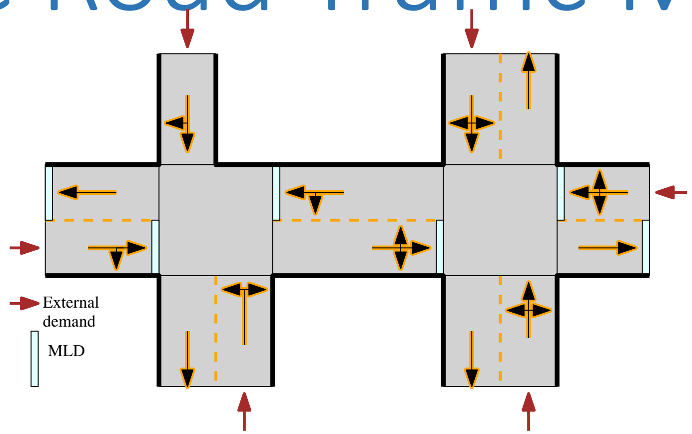

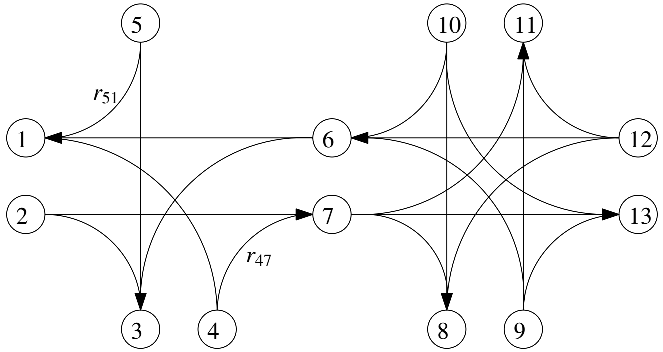

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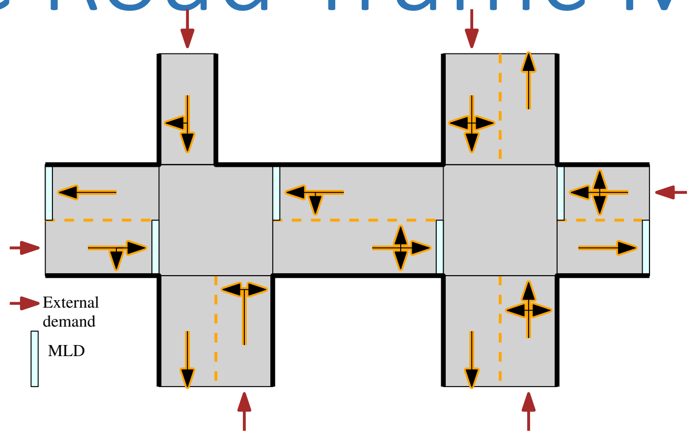
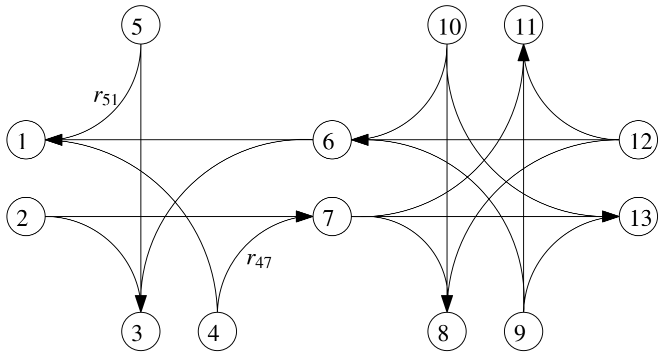
Network junction problem

Dynamic Road Traffic Model





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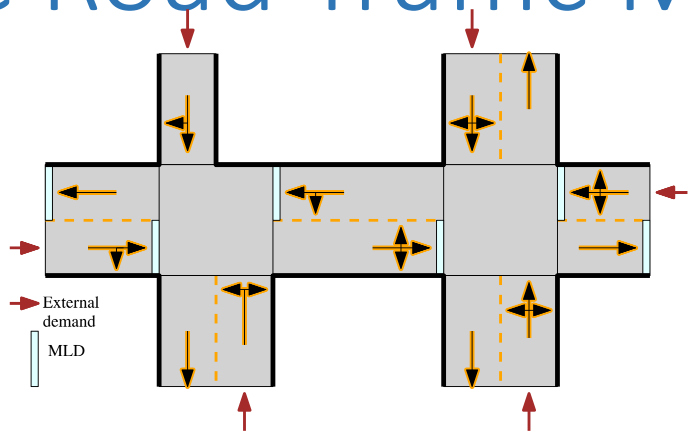
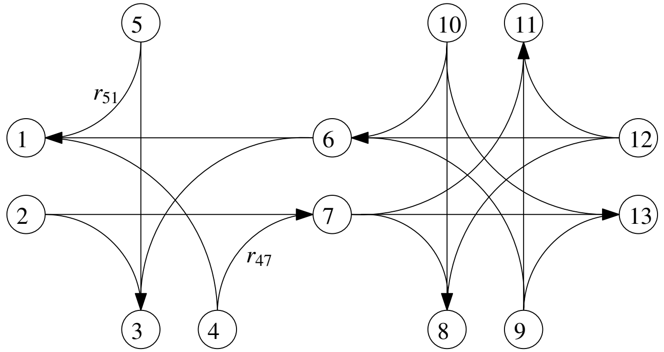
Network junction problem

The solution can be found by solving the optimal problem

$$\max_{\varphi^{\text{out}}} \quad \mathbb{1}^T \varphi^{\text{out}} \quad \text{s.t.} \quad \varphi^{\text{out}} \in \mathcal{P}^{\varphi}$$

Dynamic Road Traffic Model





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Network junction problem

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$$\max_{\varphi^{\text{out}}} \quad \mathbb{1}^T \varphi^{\text{out}} \quad \text{s.t.} \quad \boxed{\varphi^{\text{out}} \in \mathcal{P}^{\varphi}} \longrightarrow \text{Bounded} \quad \mathcal{P}^{\varphi} := \bigcup_{g \in \mathcal{G}} \mathcal{P}_g^{\varphi}$$

Measures

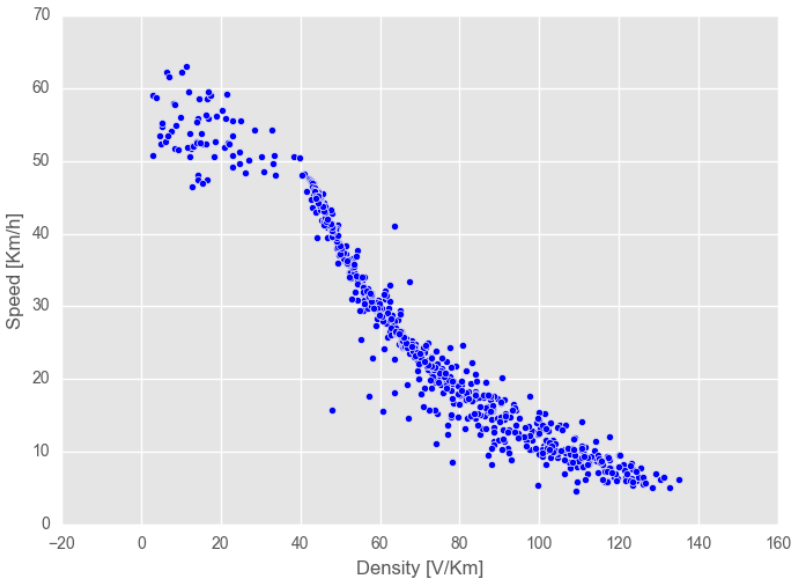
$$\bar{v}_i(k) = \Psi(\rho_i, k) + \eta_v(\rho_i, k), \quad i \in \mathcal{I}_{FCD}$$

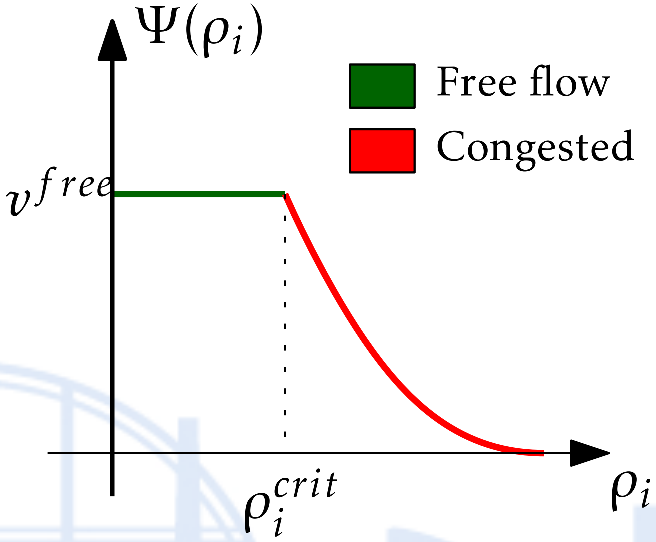
$$\bar{\varphi}_l^{\text{out}}(k) = \Phi(\rho_l, k) + \eta_{\varphi^{\text{out}}}(\rho_l, k), \quad l \in \mathcal{I}_{MLD}$$

$$\Psi(\rho_i) = \begin{cases} v^{\text{free}} & 0 \leq \rho_i \leq \rho^{\text{crit}} \\ w\left(\dfrac{\rho^{\max}}{\rho_i} - 1\right) & \rho^{\text{crit}} < \rho_i \leq \rho^{\max} \end{cases}$$

$0 \leq \rho_i \leq \rho^{\text{crit}} \longrightarrow$ *Unobservable*

$\rho^{\text{crit}} < \rho_i \leq \rho^{\max} \longrightarrow \bar{\rho}_i = \Psi^{-1}(\bar{v}_i) = \dfrac{\rho^{\max}}{1 + \bar{v}_i / w}$





Considerations on reconstruction

Assumption 1 (Boundary flows):
All inflows and outflows in the boundaries of the network are measurable.

Assumption 2 (Speeds measurements):
Speeds measured by floating car data (FCD) are captured everywhere in the network.

Assumption 3 (Density pseudo observations):
Based on the speed-density map. For congested cells there exists a *density pseudo observation* which can be recovered from the map

$$\bar{\rho}_i = \frac{\rho^{\max}}{1 + \bar{v}_i / w}, \ \forall \ \bar{v}_i < v^{\text{free}}$$

Assumption 4 (Network equilibrium):
The reconstruction is performed in a time scale such that the network can be considered to be at equilibrium.

$$(R^T - I)\varphi^{\text{out}} + B\lambda_e = 0.$$

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$$(R^T - I)\varphi^{\text{out}} + B\lambda_e = 0.$$

Reconstruction problem

Estimation problem

$$\min_{\hat{\varphi}^{\text{out}},\hat{\rho}} \quad J_{\varphi^{\text{out}}} + J_{\rho}$$
$$\text{s.t} \quad \mathcal{M}_{\varphi,\rho}$$

Non-linearities appear within $\mathcal{M}_{\varphi,\rho}$ due to the *min* term in the fundamental diagram. No explicit solution is known to the problem.

Problem relaxation

Reconstruction problem

Estimation problem

$$\min_{\hat{\varphi}^{\text{out}},\hat{\rho}} \quad \sum_{i=1}^{N_M} (\hat{\varphi}_l^{\text{out}} - \bar{\varphi}_l^{\text{out}})^2 + \sum_{j=1}^{N_H} \left(S_{\bar{v}_j} (\hat{\rho}_j - \bar{\rho}_j) \right)^2$$

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$$\min_{\hat{\varphi}^{\text{out}},\hat{\rho}} \left\| C_M \hat{\varphi}^{\text{out}} - \bar{\varphi}^{\text{out}} \right\|_{\gamma_\varphi}^2 + \left\| S_{\bar{v}} \left(\hat{\rho} - \Psi^{-1}(\bar{v}) \right) \right\|_{\gamma_\rho}^2 + \left\| (R^T - I) \hat{\varphi}^{\text{out}} + B \lambda_e \right\|_{\gamma}^2$$

Reconstruction problem

Estimation problem

$$\min_{\hat{\varphi}^{\text{out}},\hat{\rho}} \sum_{i=1}^{N_M} (\hat{\varphi}_l^{\text{out}} - \bar{\varphi}_l^{\text{out}})^2 + \sum_{j=1}^{N_H} \left(S_{\bar{v}_j}(\hat{\rho}_j - \bar{\rho}_j) \right)^2 \qquad S_{\bar{v}_i} = \begin{cases} 1, & \bar{v}_i < v^{\text{free}} \\ 0, & \text{elsewhere} \end{cases}$$

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$$\min_{\hat{\varphi}^{\text{out}},\hat{\rho}} \underbrace{\left\| C_M \hat{\varphi}^{\text{out}} - \bar{\varphi}^{\text{out}} \right\|_{\gamma_\varphi}^2}_{\text{Internal flow measurements}} + \left\| S_{\bar{v}} \left(\hat{\rho} - \Psi^{-1}(\bar{v}) \right) \right\|_{\gamma_\rho}^2 + \left\| (R^T - I)\hat{\varphi}^{\text{out}} + B\lambda_e \right\|_{\gamma}^2$$

Reconstruction problem

Estimation problem

$$\min_{\hat{\varphi}^{\text{out}},\hat{\rho}} \sum_{i=1}^{N_M} (\hat{\varphi}_l^{\text{out}} - \bar{\varphi}_l^{\text{out}})^2 + \sum_{j=1}^{N_H} \left(S_{\bar{v}_j}(\hat{\rho}_j - \bar{\rho}_j)\right)^2 \qquad S_{\bar{v}_i} = \begin{cases} 1, & \bar{v}_i < v^{\text{free}} \\ 0, & \text{elsewhere} \end{cases}$$

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Reconstruction problem

Estimation problem

$$\min_{\hat{\varphi}^{out},\hat{\rho}} \sum_{i=1}^{N_M} (\hat{\varphi}_l^{out} - \bar{\varphi}_l^{out})^2 + \sum_{j=1}^{N_H} \left(S_{\bar{v}_j}(\hat{\rho}_j - \bar{\rho}_j) \right)^2 \qquad S_{\bar{v}_i} = \begin{cases} 1, & \bar{v}_i < v^{free} \\ 0, & \text{elsewhere} \end{cases}$$

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Internal flow measurements | Density pseudo observations | Network equilibrium

Reconstruction problem

Estimation problem

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$$\text{s.t} \quad A(\bar{v})\varphi^{\text{out}} = B(\bar{v})\rho + C(\bar{v})$$

Reconstruction problem

Estimation problem

$$\min_{\hat{\varphi}^{\text{out}},\hat{\rho}} \quad \sum_{i=1}^{N_M} (\hat{\varphi}_l^{\text{out}} - \bar{\varphi}_l^{\text{out}})^2 + \sum_{j=1}^{N_H} \left(S_{\bar{v}_j}(\hat{\rho}_j - \bar{\rho}_j) \right)^2 \qquad S_{\bar{v}_i} = \begin{cases} 1, & \bar{v}_i < v^{\text{free}} \\ 0, & \text{elsewhere} \end{cases}$$

$$\text{s.t} \quad \mathcal{M}_{\hat{\varphi}^{\text{out}},\hat{\rho}}$$

Non-linearities appear within $\mathcal{M}_{\varphi,\rho}$ due to the *min* term in the fundamental diagram. No explicit solution is known to the problem.

Problem relaxation

$$\min_{\hat{\varphi}^{\text{out}},\hat{\rho}} \quad \left\| C_M \hat{\varphi}^{\text{out}} - \bar{\varphi}^{\text{out}} \right\|_{\gamma_\varphi}^2 + \left\| S_{\bar{v}} \left(\hat{\rho} - \Psi^{-1}(\bar{v}) \right) \right\|_{\gamma_\rho}^2 + \left\| (R^T - I)\hat{\varphi}^{\text{out}} + B\lambda_e \right\|_{\gamma}^2$$

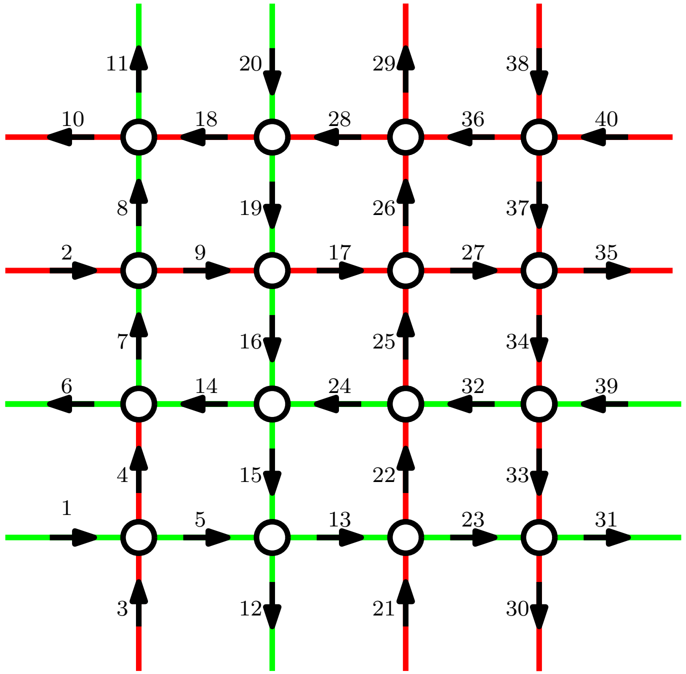
$$\text{s.t} \quad A(\bar{v})\varphi^{\text{out}} = B(\bar{v})\rho + C(\bar{v})$$

$$\hat{\rho} \in \mathcal{P}^\rho$$

$$\mathcal{P}^\rho := \bigcup_{g \in \mathcal{G}} \mathcal{P}_g^\rho$$

Subspace of densities

Scenario & Simulations



$$AE_x(k) = \frac{1}{N}\sum_{i\in\mathcal{I}} |\hat{x}(k) - x(k)|$$

Test characteristics

Manhattan grid:

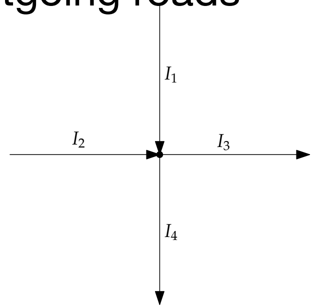
- 2 incoming roads - 2 outgoing roads

$$\rho_1(k+1) = \rho_1(k) + \frac{T}{l_1}(\varphi_1^{\text{in}} - \varphi_1^{\text{out}})$$

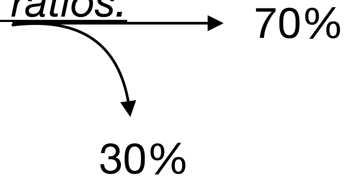
$$\rho_2(k+1) = \rho_2(k) + \frac{T}{l_2}(\varphi_2^{\text{in}} - \varphi_2^{\text{out}})$$

$$\rho_3(k+1) = \rho_3(k) + \frac{T}{l_3}(r_{13}\varphi_1^{\text{in}} + r_{23}\varphi_2^{\text{in}}) - \varphi_3^{\text{out}}$$

$$\rho_3(k+1) = \rho_4(k) + \frac{T}{l_4}(r_{14}\varphi_1^{\text{in}} + r_{24}\varphi_2^{\text{in}}) - \varphi_4^{\text{out}}$$



Splitting ratios:

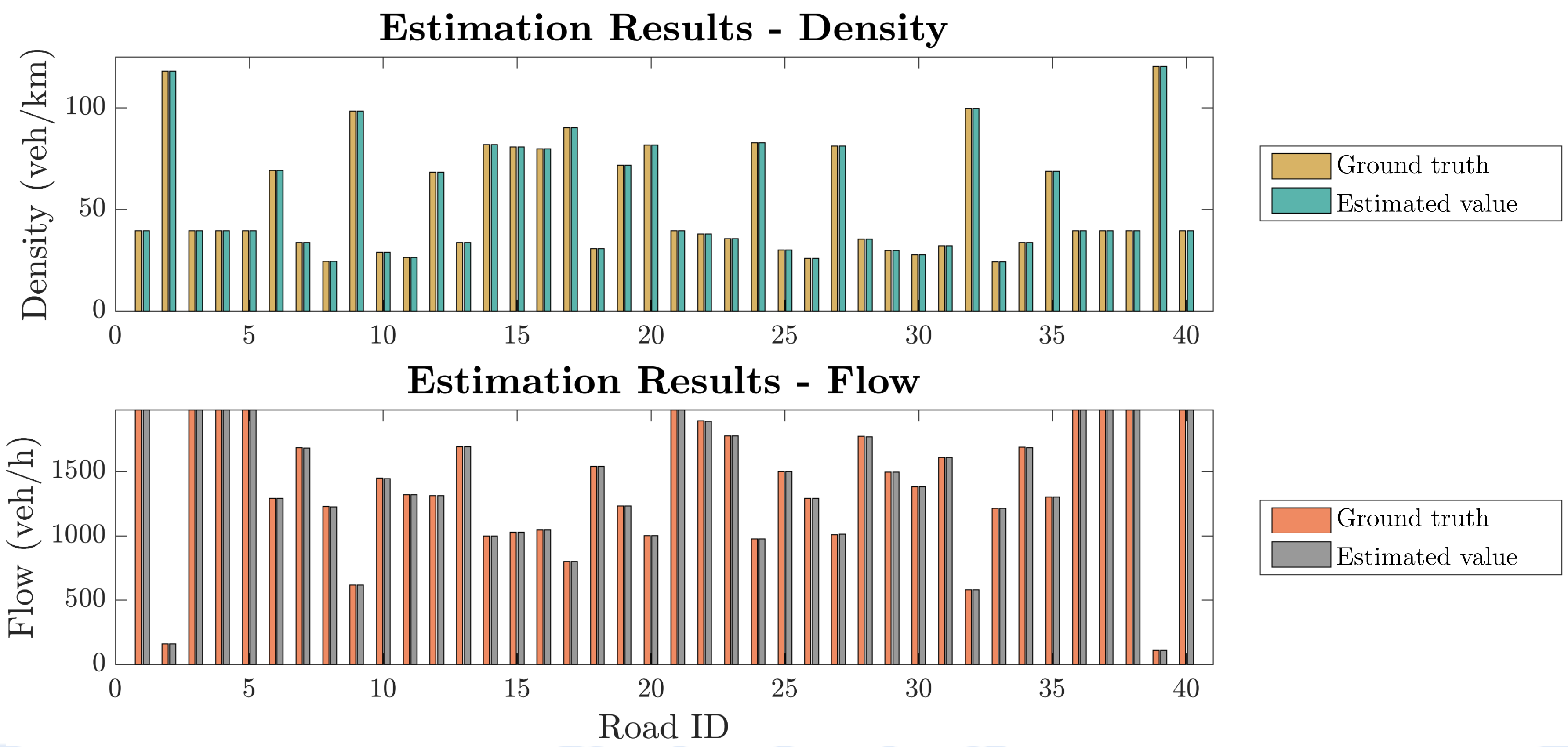


Parameters:

- Uniform cells: 500m
- Sampling time: 15s
- Flow measured only at the external boundary.
- Constant random demands.

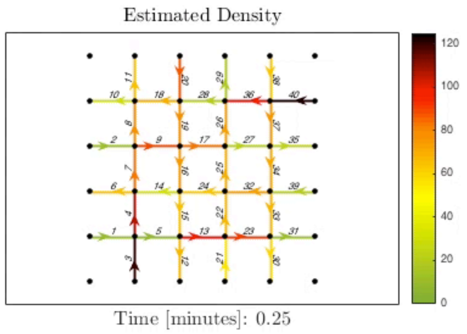
Scenario & Simulations

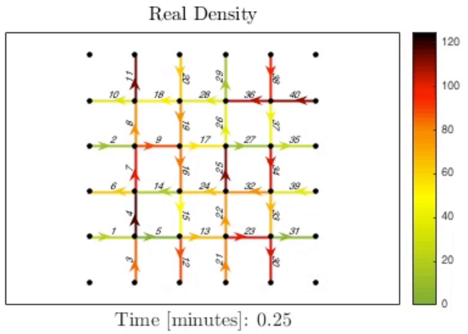
Network in at steady state:

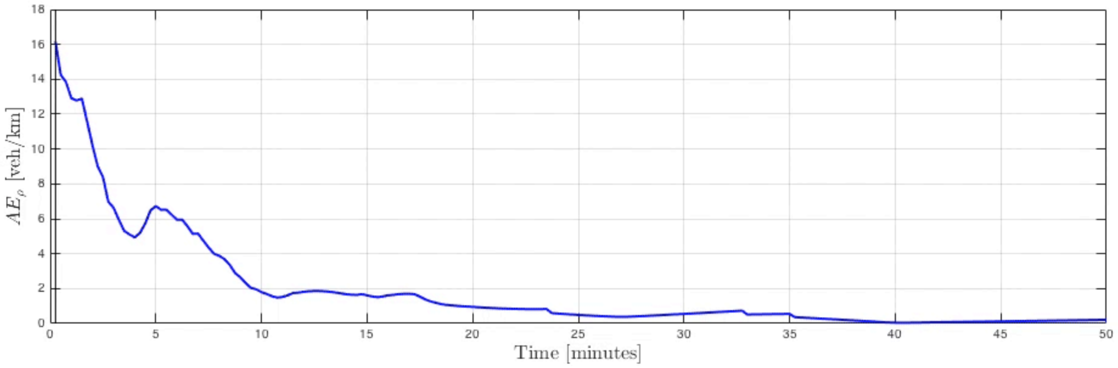


Scenario & Simulations

Dynamic density reconstruction:

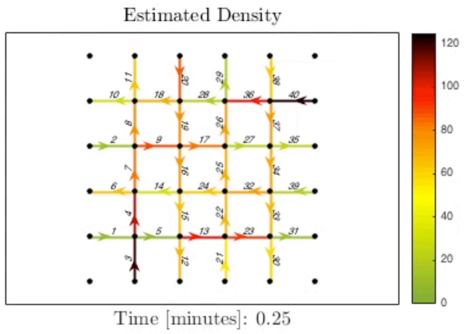


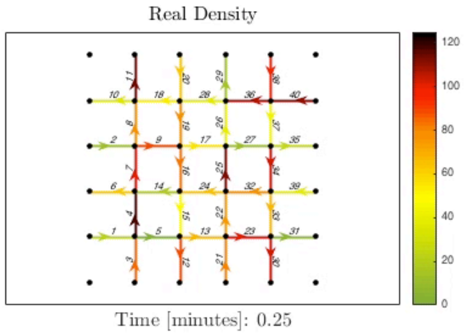


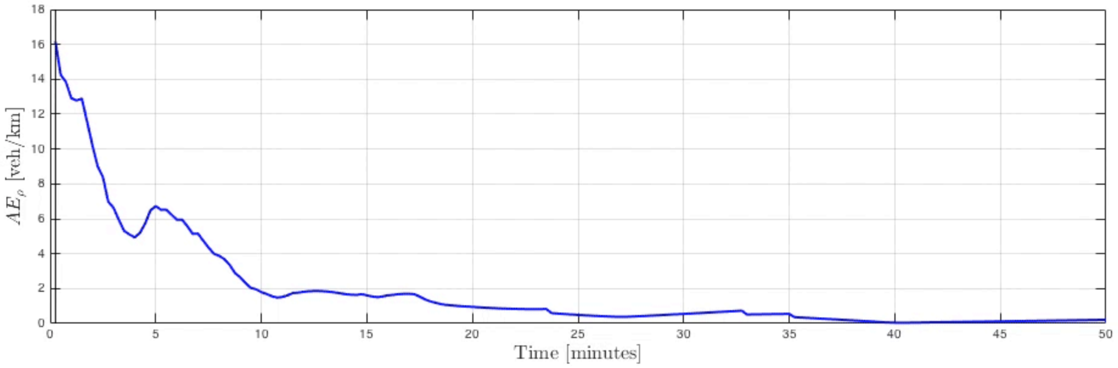


Scenario & Simulations

Dynamic density reconstruction:

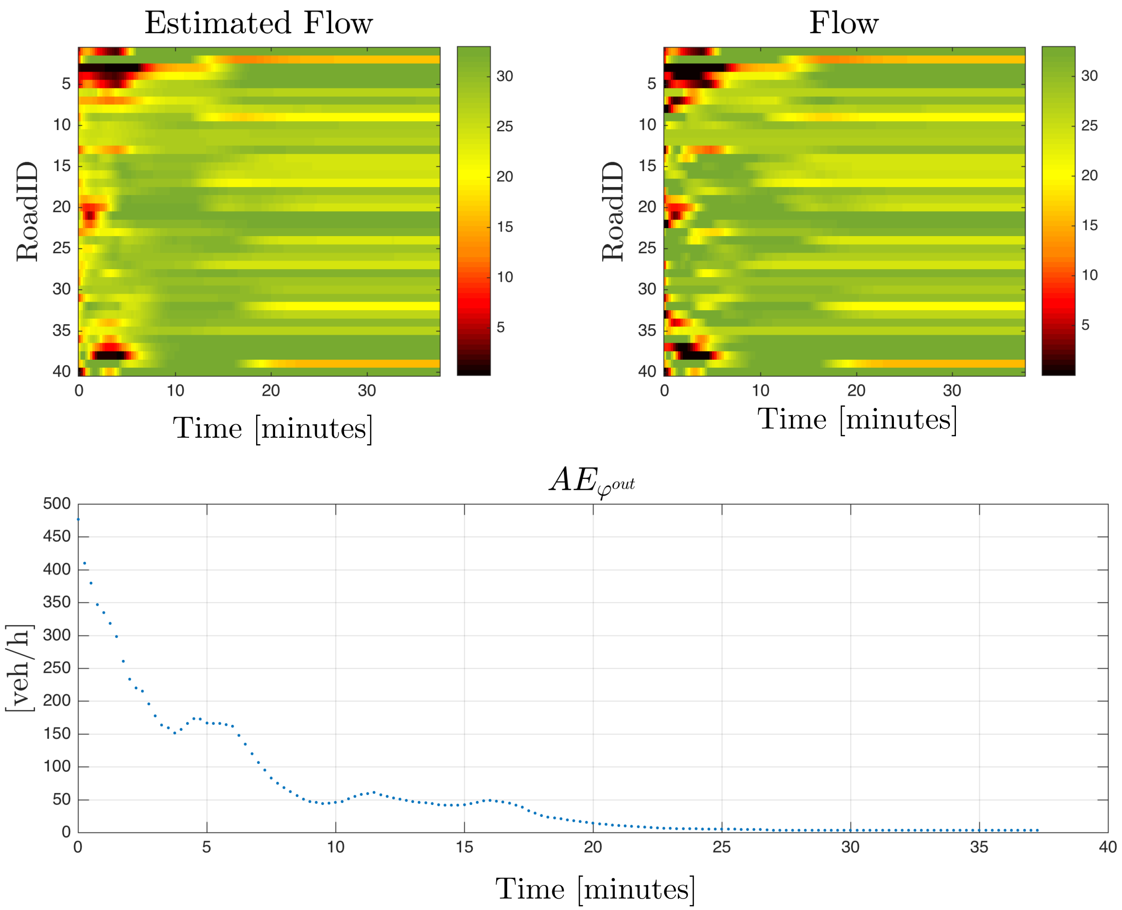




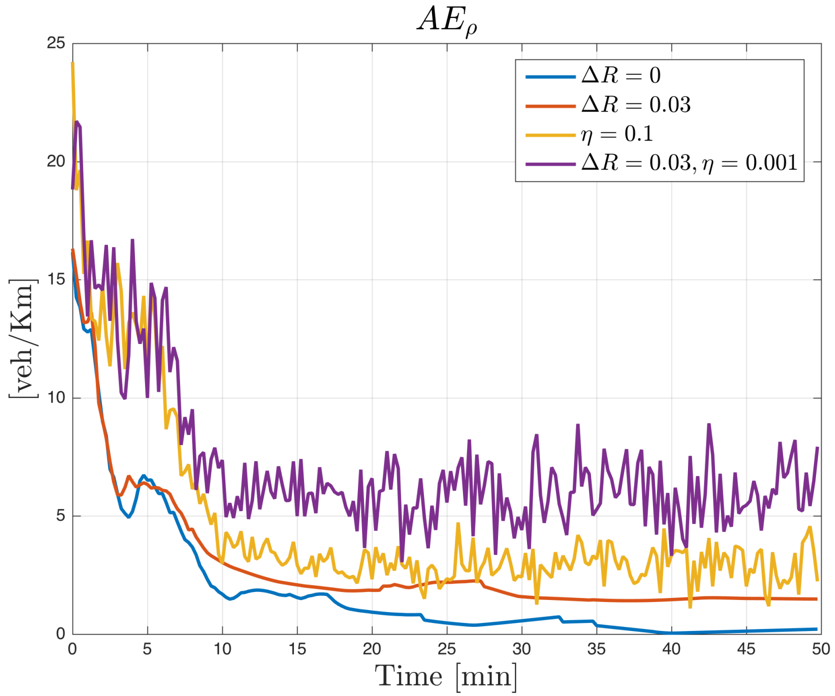


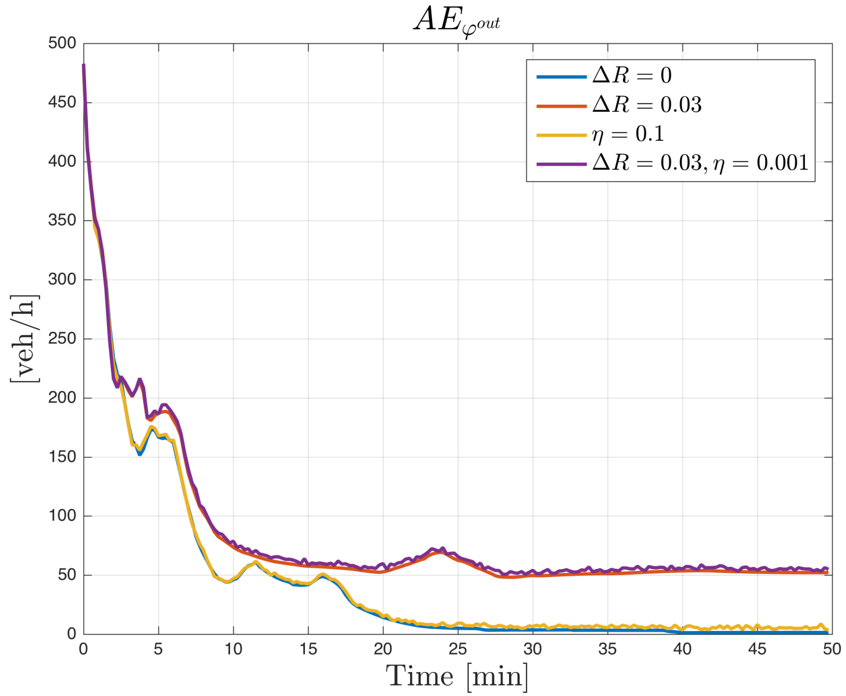
Scenario & Simulations

Dynamic flow reconstruction:



Noise & Uncertainty





Case 1: Nominal case
Case 2: Perturbation on the drivers preference matrix
Case 3: Noise within the measurements (speed + flow)
Case 4: Perturbation drivers preference + noise

Summary of Reconstruction

1. A method for simultaneous flow/density estimation was presented based on model driven approaches.

2. The resulting formulation is cast as a quadratic optimization problem with equality constraints.

3. Results at simulation level show the that density/flow can be effectively reconstructed at equilibrium point.

4. The method was tested also on dynamic conditions with satisfying results.

5. Robustness of the method was tested to variation of the drivers' preference parameter as well as noisy conditions.

Part II

Control of large scale traffic networks